Efficient algorithms for Flock Detection in Moving Object Data-Supplement

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1 Theorems

1.1 Triangle cells Vs Square cells for Range query

Let a_{sq} be area of square cell, a_{tr} be area of triangular cell given that, side of cell is ϵ (flock daimeter). Assume that points are distributed equally over the entire area.Hence area under computation is directly proportional to computation complexity. Calculations:

- For range query in square cell, Figure 2 , we need to consider each point for calculating ϵ-neighbourhood in 9 cells (in yellow).Therefore total search space for neighbourhood calculation is,
  $= 9 * \epsilon * \epsilon$
  $= 9 * a_{sq}$
- For triangular cells, there are 2 cases based on location of point p ,
  1. p lies on any vertex of triangular cell Figure 3a,
     All points marked in yellow triangles are at a distance $<= \epsilon$. Therefore all those points can be directly added to range. For the points in the triangles marked blue, we need to check for ϵ-neighbourhood . Thus search space under calculations is reduced to 2 triangles. $= 2 * a_{tr}$
     $= 2 * 0.5 * \epsilon * \sqrt{3}/2 * \epsilon$
     $= \epsilon * \sqrt{3}/2 * \epsilon$
     $=0.866 * a_{sq}$

2. If p lies on face of triangle or any edge Figure 3b,
   All points marked yellow are directly taken and ones in blue required ϵ-neighbourhood calculation.During this calculation, we always check the x coordinate, if it exceeds p.x+ϵ then search for that triangle is halted. .For any point p on face of triangle, maximum triangles under blue will be 7.5, Therefore search space is,
   =7.5 *a_{tr}
   = 7 * 0.5 * ϵ * $\sqrt{3}/2\ * \epsilon$
   = 3.5 * ϵ * $\sqrt{3}/2\ * \epsilon$
   = 3.03 * a_{sq}

Both cases of triangular cells have less search space than square cells for range query.

1.2 Any two points on line segment, and either of the point is to be reached then ,there is higher probability of reaching from midpoint than from start of segment.(Given that two points cannot interchange the order)

Say line segment is pp' having midpoint at m. Let k be mid point of pm. Let a ,b be the point that is to be reached.
There are 4 cases ,

- a, b lie on either sides of m (figure 4a)
  Probablity that a lies left to k given (b-m) > (a-p) = 0.25(a is reachable faster from p)
  Probablity that a lies left to k given (b-m) < (a-p) = 0.25(b is reachable faster from m)
  Probablity that a lies right to k given it is in left of m = 0.5 (a is reachable faster from m)

- a, b lie on right side of m (figure 4b)
  Probablity of m reaching a faster =1
  Probablity of p reaching a faster =0

- a, b lie on left side of m (figure 4c,4d)
  b is never faster reachable from p, hence, there are 2 cases, Probablity that a is faster reachable from p=0.25 Probablity that a is faster reachable from m=0.75

Hence probablity of m reaching either a or b faster
=0.33(0.75+1+0.75)
=0.825
and probablity of p reaching a faster is,
=0.33(0.25+0+0.25)
=0.165
Hence for two points on line segment, and either of the point is to be reached then ,there is higher probability of reaching from midpoint than from start of segment.(Given that two points cannot interchange the order)