Research 3

Explore how SVD (i.e., PCA) can help figure out the "intrinsic dimension" for discrete data on the following Klein Bottle (Ans = 2):

>> [u,v]=meshgrid(linspace(0,2*pi));

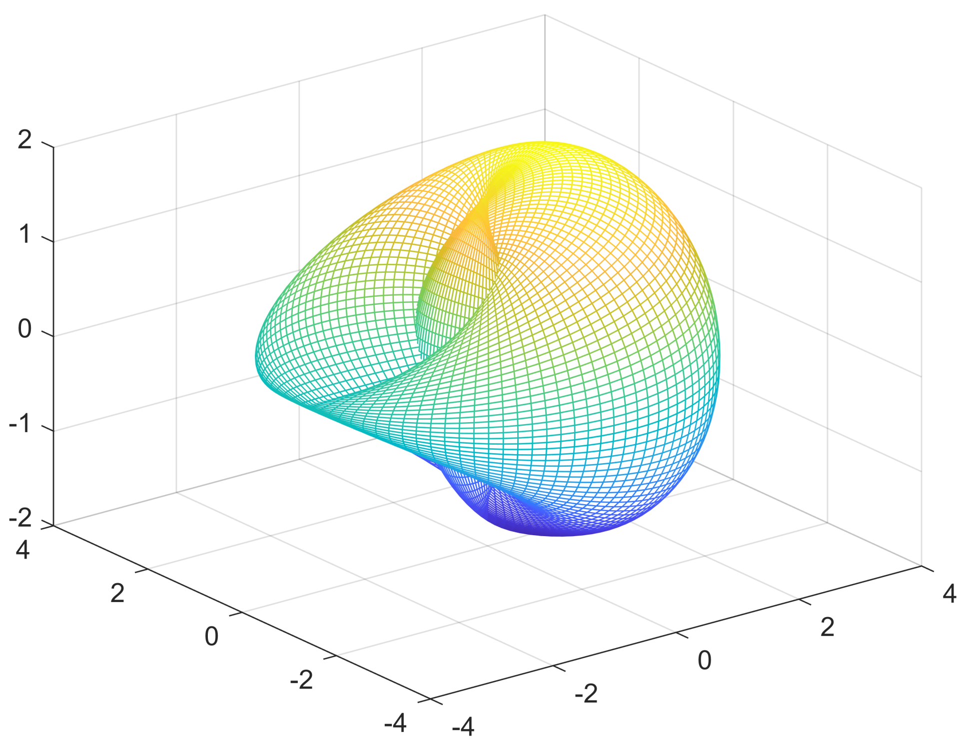
>> x = (2*cos(v)+1).*cos(u); y = (2*cos(v)+1).*sin(u); z = 2*sin(v).*cos(u/2); w = 2*sin(v).*sin(u/2);

>> mesh(x,y,z)

Propose possible ways (i.e., pseudocode) to deal with d-dimensional data embedded in R^D with d<<D. Also discuss computational cost.

Genearting raw data

```
[u,v]=meshgrid(linspace(0,2*pi));
x = (2*cos(v)+1).*cos(u);
y = (2*cos(v)+1).*sin(u);
z = 2*sin(v).*cos(u/2);
w = 2*sin(v).*sin(u/2);
mesh(x,y,z)
```

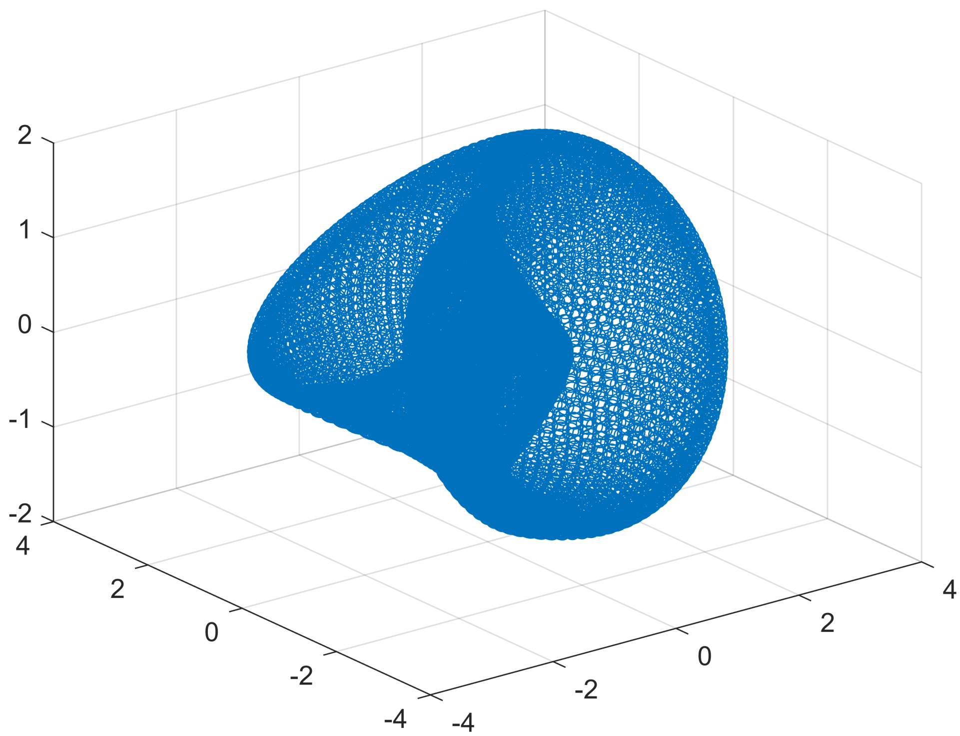


Converting x, y, z to a 10000 by 3 matrix

```
A0 = zeros(10000, 3);
```

```
for inx = 1:100
    i_start = 1 + 100*(inx - 1);
    i_end = 100*inx;
    A0(i_start:i_end, 1) = x(:, inx);
    A0(i_start:i_end, 2) = y(:, inx);
    A0(i_start:i_end, 3) = z(:, inx);
end
```

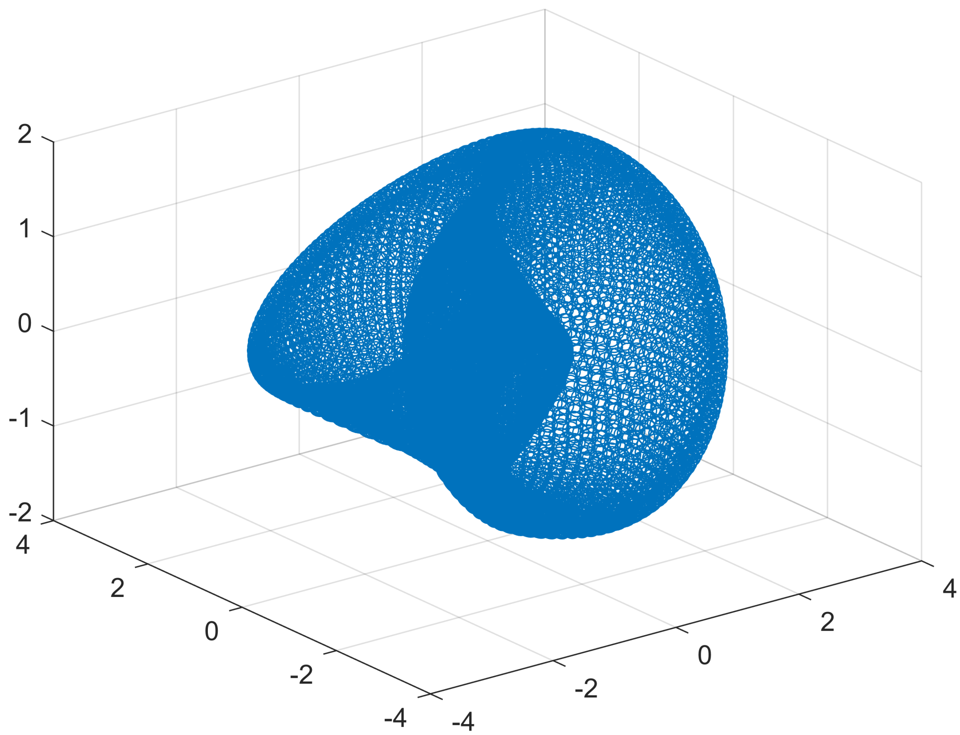
```
scatter3(A0(:, 1), A0(:, 2), A0(:, 3))
```



SVD

Singular value decomposition and verifying the result by U*S*V'

```
[U, S, V] = svd(A0);
A1 = U * S * V';
scatter3(A1(:, 1), A1(:, 2), A1(:, 3))
```



Check **Singular values** of each dimension after SVD

```
S
```

```
S = 10000×3
  124.3101         0         0
         0  123.0731         0
         0         0   99.9950
         0         0         0
         0         0         0
         0         0         0
         0         0         0
         0         0         0
         0         0         0
         0         0         0
        ⋮
```

Since all 3 singular values are big, I cannot figure out why the intrinsic dimension is 2.

Principal component analysis (PCA)

```
[coeff,score,latent,tsquared,explained] = pca(A0);
explained
```

```
explained = 3×1
   38.0609
   37.3098
```

```
24.6293
```

As we can see, the percent variability explained by principal components of all 3 dimensions is big and similar, so I also cannot get the reault: the intrinsic dimension of A0 is 2.

Propose possible ways (i.e., pseudocode) to deal with d-dimensional data embedded in R^D with d<<D

Basic idea: Just like KNN (k-nearest neighbors) model, we can reduce the dimension of points in R^D to d by dealing with a small local part each step.

Pseudocode:

Assume there are n points and the whole points form a set N,

D(n by n matrix) <- calculate the distance between each point pair in R^D

for x = 1:len(N)

x <- Select 1 point from N randomly

C1 <- Find all nearest neighbors of point x which the distance less than a specific value

Reduce the dimension of cluster C1 to R^d by PCA

N <- N - C1 (remove cluster C1 from N)

end

Computational cost:

The computional cost of D is O(n^2), PCA of n points from R^D is O(n*D^2). Therefore the total comuptational cost is O(n^2 + n*D^2).