NLPP : Gradient Search Method

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GRADIENT-BASED NONLINEAR OPTIMIZATION METHODS

When the objective function and/or some or all of the constraints of a problem are nonlinear, other problem solution methods must be used. A class of such solution techniques used information about the problem geometry obtained from the gradient of the objective function. These solution methods are collectively categorized here as “gradient-based” methods.

The purpose of this section is to provide a simple introduction to gradient solution methods. Those interested in greater detail regarding the many gradient-based methods and the mathemati?cal theory upon which they are based should refer to Wagner (1975), MacMillan (1975), among many others.

INTRODUCTION TO NONLINEAR gradient-based PROBLEMS

A number of gradient-based methods are available for solving constrained and unconstrained
nonlinear optimization problems. A common characteristic of all of these methods is that they
employ a numerical technique to calculate a direction in n-space in which to search for a better
estimate of the optimum solution to a nonlinear problem. This search direction relies on the
estimation of the value of the gradient of the objective function at a given point.

Gradient-based methods are used to solve nonlinear constrained or unconstrained problems
where other techniques: are not feasible (e.g., LP) do not yield desired information about the problem geometry (e.g., DP)

Gradient-based methods have the advantages that they are applicable to a broader class of prob?lems than LP and they provide much more information

Common Gradient-Based Methods

The most commonly used gradient techniques are:

1. steepest ascent
2. conjugate gradient
3. reduced gradient

Each of these methods can be found in commercially available mathematical programming soft?ware. The method of steepest ascent is mathematically simple and easy to program, but con?verges to an optimal solution only slowly. On the other end of the spectrum is the reduced gradient method, which has a high rate of convergence, but is much more mathematically com?plex and difficult to program.

In GSM We now learning...

Lagrange's Multipliers with Problem

Kuhn-Tucker Conditions with Problem

NON-LINEAR PROGRAMMING PROBLEM WITH ONE INEQUALITY CONSTRAINT

Consider a general non-linear programming problem having one inequality constraint of the type,

$$\text{Maximize } Z = f(X)$$
$$\text{subject to } g(X) \le b \qquad X \ge 0, X = x_1, x_2, \dots, x_n$$

Introducing a slack variable S in the form of S^2 so as to ensure that it is always non-negative, the constraint equation can be modified

$$h(X) + S^2 = 0$$
$$\text{Where } h(X) = g(X) - b$$

The problem can now be expressed as,

$$\text{maximize } Z = f(X)$$
$$h(X) + S^2 = 0$$
$$\text{subject to}$$
$$X \ge 0$$

Lagrange's Multipliers

THE LAGRANGEAN FUNCTION FOR THE NON-LINEAR PROGRAMMING PROBLEM WITH ONE INEQUALITY CONSTRAINT IS GIVEN BY:

$$L(X, S, \lambda) = f(X) - \lambda[h(X) + S^2]$$

THE NECESSARY CONDITIONS FOR A STATIONARY POINT INCLUDE:

$$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0, j = 1,2, \ldots, n,$$

$$\frac{\partial L}{\partial \lambda} = -[h(X) + S^2] = 0,$$

$$\frac{\partial L}{\partial S} = -2\, S\lambda = 0.$$

SINCE S^2 IS A NON-NEGATIVE SLACK VARIABLE,

This implies that when $h(X) < 0, \lambda = 0$;
and when $\lambda > 0, h(X) = 0$.

Kuhn-Tucker Conditions

THE NECESSARY CONDITIONS FOR A MAXIMIZATION PROBLEM CAN BE SUMMARIZED AS:

$$\begin{aligned} \frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} &= 0, \\ \lambda h(\mathrm{X}) &= 0, \\ h(\mathrm{X}) &\leq 0, \\ \lambda &\geq 0. \end{aligned}$$

THESE NECESSARY CONDITIONS ARE ALSO CALLED KUHN-TUCKER CONDITIONS.

A SIMILAR ARGUMENT HOLDS FOR THE MINIMIZATION NON-LINEAR PROGRAMMING PROBLEM:

$$\begin{aligned} &\text{Minimize} && \mathrm{Z} = f(\mathrm{X}), \\ &\text{subject to} && g(\mathrm{X}) \geq b, \\ & && X \geq 0 \end{aligned}$$

Lagrange's Multipliers Problem

Maximize the function $f(x,y) = \sqrt{xy}$ subject to the constraint $20x + 10y = 200$.

Let's set $g(x,y) = 20x + 10y - 200$. *We have*

$$\frac{\partial f}{\partial x} = \frac{1}{2}\sqrt{\frac{y}{x}} \qquad \frac{\partial g}{\partial x} = 20$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\sqrt{\frac{x}{y}} \qquad \frac{\partial g}{\partial y} = 10$$

So the equations we need to solve are

$$\frac{1}{2}\sqrt{\frac{y}{x}} = 20\lambda \qquad \frac{1}{2}\sqrt{\frac{x}{y}} = 10\lambda$$

$$20x + 10y = 200.$$

Dividing the first by the second gives us

$$\frac{y}{x} = 2,$$

which means $y = 2x$. We plug this into the equation of constraint to get

$$20x + 10(2x) = 200 \implies x = 5 \implies y = 10.$$

When dividing equations, one must take care that the equation we divide by is not equal to zero. So we should verify that there is no solution where

$$\frac{1}{2}\sqrt{\frac{x}{y}} = 10\lambda = 0$$

If this were true, then $\lambda = 0$. Since $y = 800\lambda^2 x$, we get $y = 0$. Since $x = 200\lambda^2 y$, we get $x = 0$. But then the equation of constraint is not satisfied. So we're safe.

Make sure you account for these because you can lose solutions!

Kuhn-Tucker Conditions Problem

Example : Solve the following NLPP using the Kuhn-Tucker conditions:

$$\begin{aligned} \text{Maximize } Z = &\ 2x_1^2 - 7x_2^2 + 12x_1x_2, \\ \text{subject to } &\ 2x_1 + 5x_2 \leq 98, \\ &\ x_1, x_2 \geq 0. \end{aligned}$$

- The given problem can be put as
- $$\begin{aligned} f(\mathrm{X}) &= 2x_1^2 - 7x_2^2 + 12x_1x_2, \\ h(\mathrm{X}) &= 2x_1 + 5x_2 - 98 \end{aligned}$$
- The Kuhn-Tucker conditions are
- $$\begin{aligned} \frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} &= 0 \\ \frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} &= 0 \\ \lambda h(\mathrm{X}) &= 0 \end{aligned}$$
- $h(\mathrm{X}) \leq 0 \ \lambda \geq 0.$

- Applying these conditions, we get

$$\begin{aligned} 4x_1 + 12x_2 - 2\lambda &= 0 \\ 12x_1 - 14x_2 - 5\lambda &= 0 \end{aligned}$$

- $\lambda(2x_1 + 5x_2 - 98) = 0$

$$\begin{aligned} 2x_1 + 5x_2 - 98 &\leq 0 \\ \lambda &\geq 0 \end{aligned}$$

- From equation (iii) either $\lambda = 0$ or $2x_1 + 5x_2 - 98 = 0$.
- When $\lambda = 0$, equations (i) and (ii) give $x_1 = x_2 = 0$, which does not satisfy condition (iv). Thus a feasible solution cannot be obtained for $\lambda = 0$.
- When $2x_1 + 5x_2 - 98 = 0$, this equation along with (i) and (ii) gives the solution, $x_1 = 44$ and $x_2 = 2$ with $\lambda = 100$ and $Z_{\max} = 4{,}900$.

Feel free to get in touch

Contact Teacher Morgan for any questions or clarifications

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