

Siena College's 31st Annual High School Programming Contest

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Gold Problem #6: Perfect Riffle Shuffles

Background Information: The Hindu, Pile, Corgi, Mongean, Weave and Faro, and Mexican Spiral are some of the many ways to shuffle a deck of 52 (or any number) playing cards. A **perfect riffle shuffle** is another way to shuffle a deck of cards. For this shuffle, the deck is divided into two equal-sized packets of cards and then the cards are interleaved, always starting with the first packet. For a perfect riffle shuffle, the interleaving requires a strict alternation between the two packets.



The perfect riffle shuffle can be described mathematically as a transformation of a sequence into a new sequence. The starting sequence of N distinct objects is divided into two parts: the first $N/2$ elements and the second $N/2$ elements. If N is odd, then the first part gets the extra element. To complete the transformation, the new sequence is an interleaving of the elements of the first part and the second part. For example, $[1\ 2\ 3\ 4\ 5\ 6]$ would be transformed into $[1\ 4\ 2\ 5\ 3\ 6]$, and $[1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9]$ would be transformed into $[1\ 6\ 2\ 7\ 3\ 8\ 4\ 9\ 5]$. If the sequence is perfectly reshuffled repeatedly, you will eventually recreate the original sequence.

Magicians and card sharks are interested in the number of perfect riffle shuffles to return a deck of cards back to its original configuration.

Programming Problem:

Input: A positive integer N ($1 \leq N \leq 20,000$), which represents a sequence $[1\ 2 \dots N]$.

Output: The minimum positive number of riffle shuffles needed to return to the original sequence.

Example 1: Input:
 2
 Output:
 1

Example 3: Input:
 1235
 Output:
 36

Example 2: Input:
 52
 Output:
 8

Example 4: Input:
 11738
 Output:
 2640