

FEDERAL STATE AUTONOMOUS EDUCATIONAL
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Report
on the practical task No. 2
“Algorithms for unconstrained nonlinear optimization.
Direct methods”

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St. Petersburg
2022

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Goal

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization.

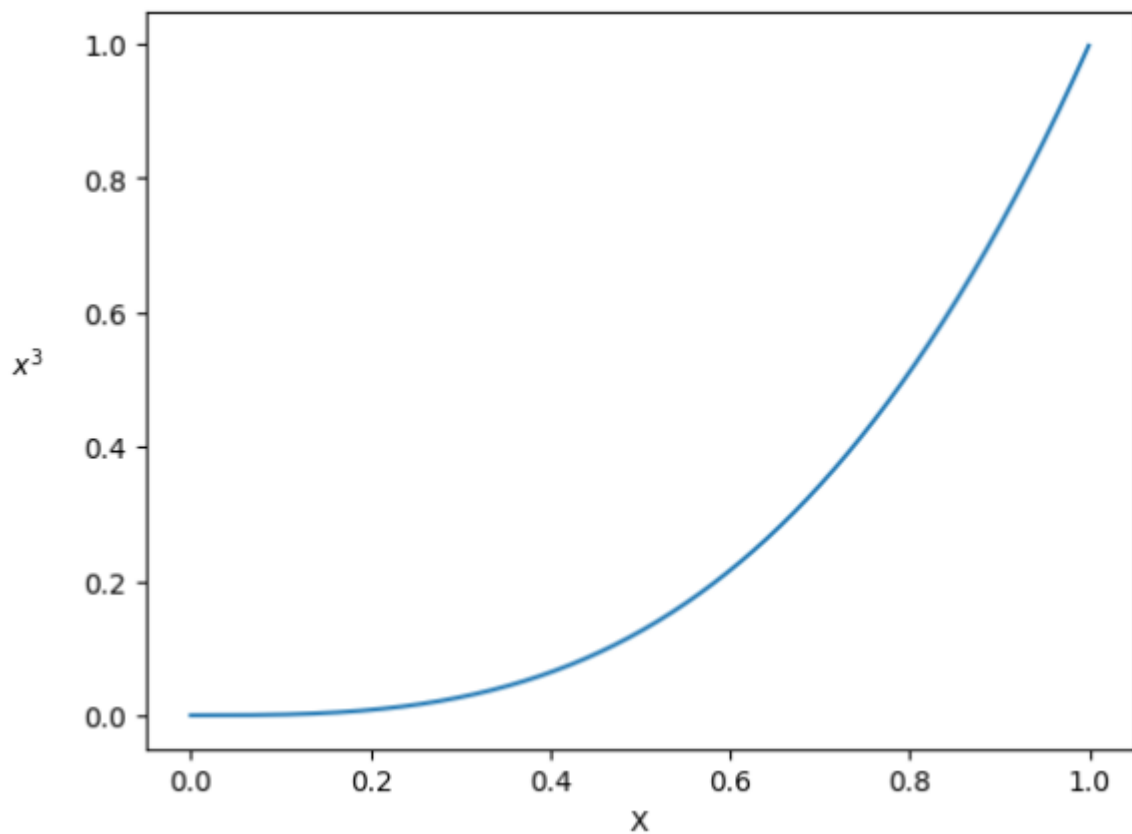
One-dimensional

Problem

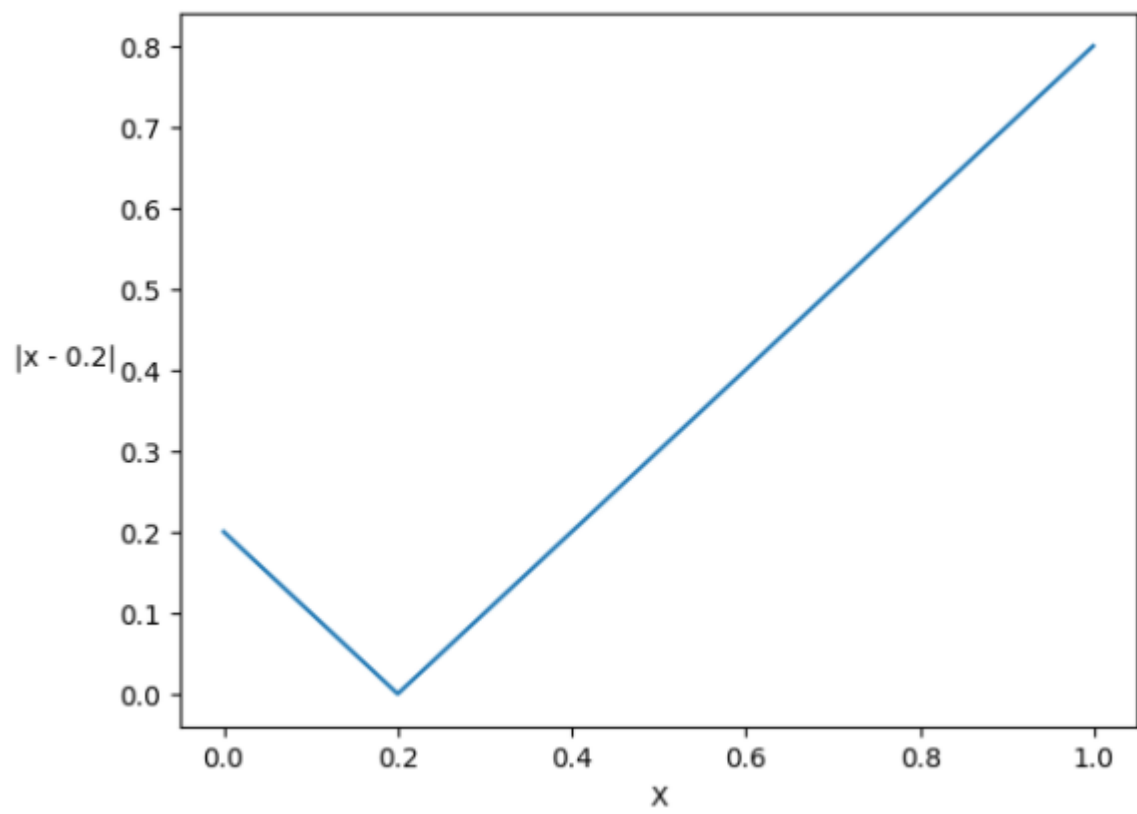
Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision $\text{eps} = 0.001$) solution $x: f(x) \rightarrow \min$ for the following functions and domains.

Functions

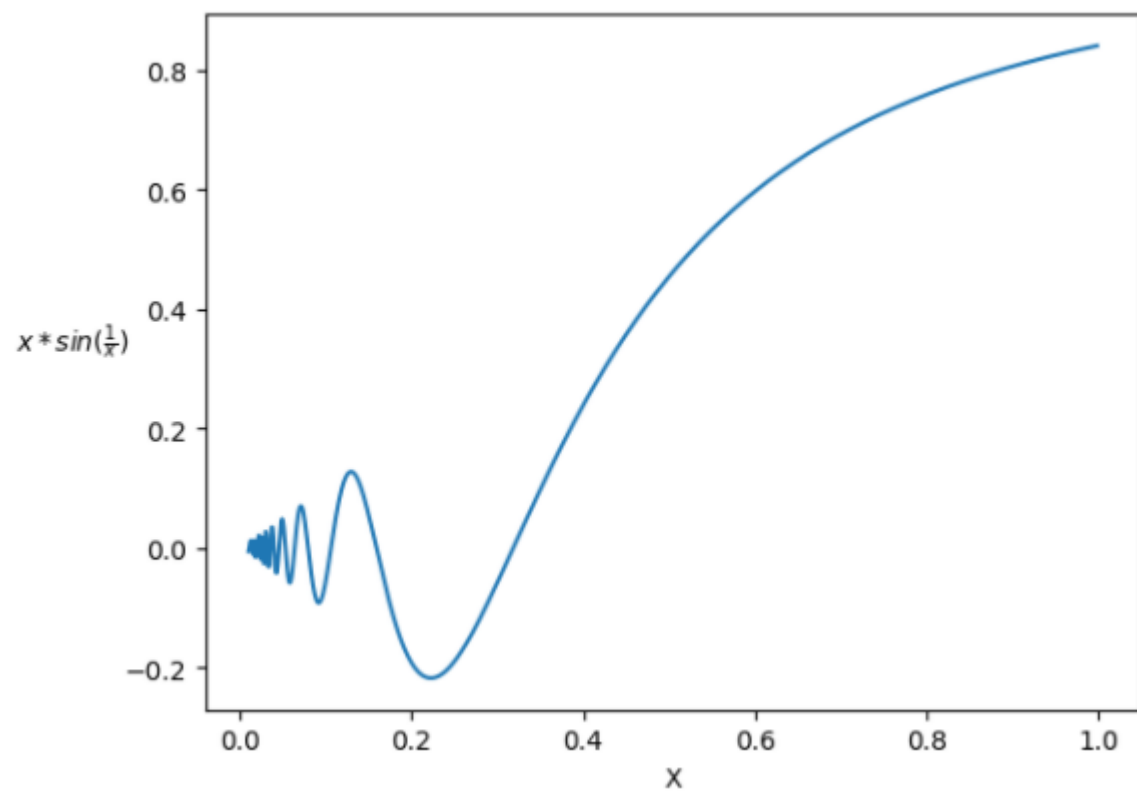
Cubic function



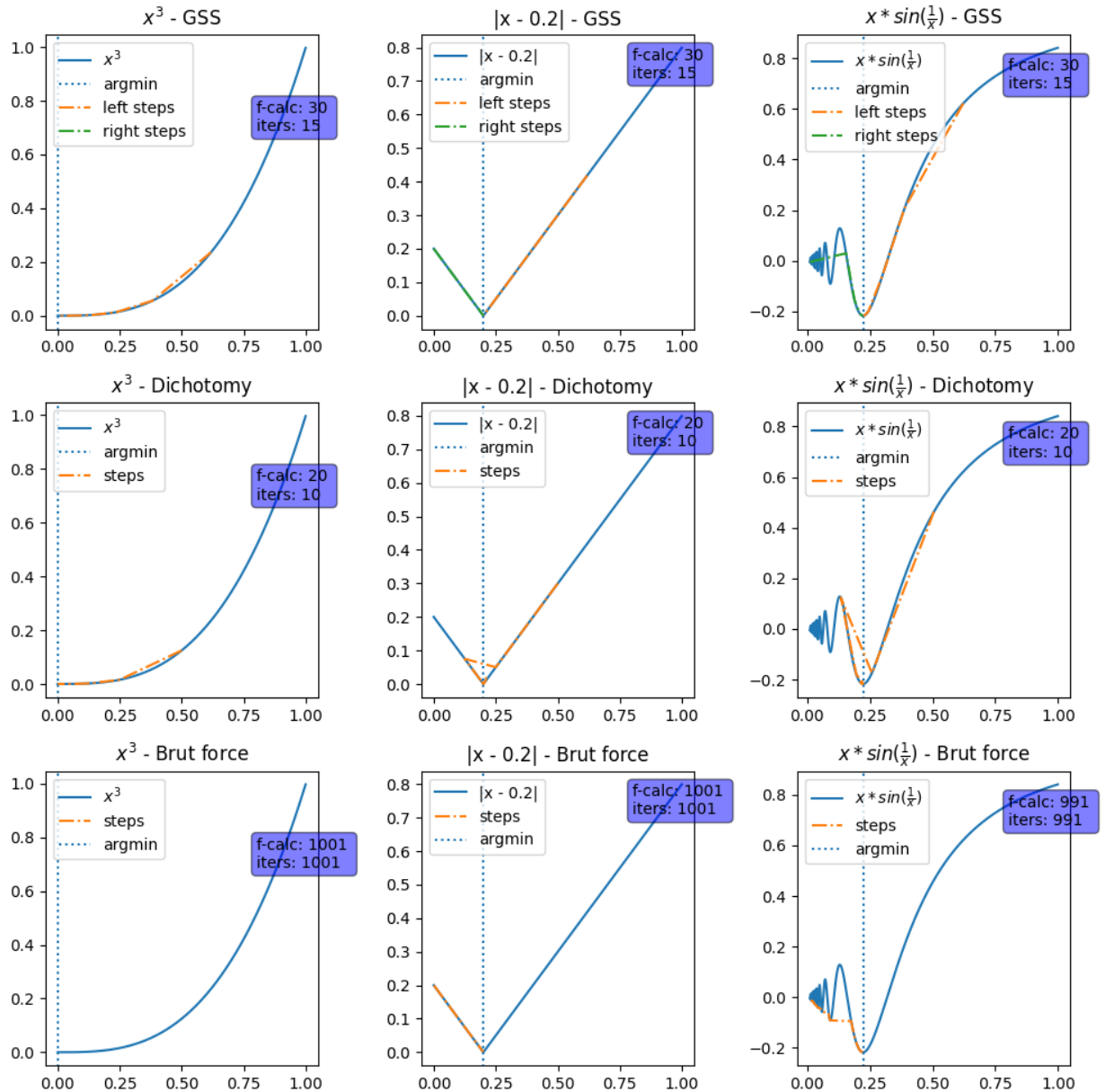
Abs function



Sine function



Comparison



Conclusion

All algorithms found minimum of all functions with specified error. Brute-force algorithm has fixed number of iterations and function calculations, these indicators depend only on error and axis bounds. Brute-force algorithm has the most iterations and function calculations. As for dichotomy and GSS, number of iterations and function calculations are the same on different functions, but dichotomy algorithm has the smallest number of iterations and function calculations

Two-dimensional

Problem

Generate random numbers $\alpha \in (0,1)$ and $\beta \in (0,1)$. Furthermore, generate the noisy data $\{x_k, y_k\}$, where $k = 0, \dots, 100$, according to the following rule $y_k = \alpha x_k + \beta + \delta_k, x_k = \frac{k}{100}$, where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution.

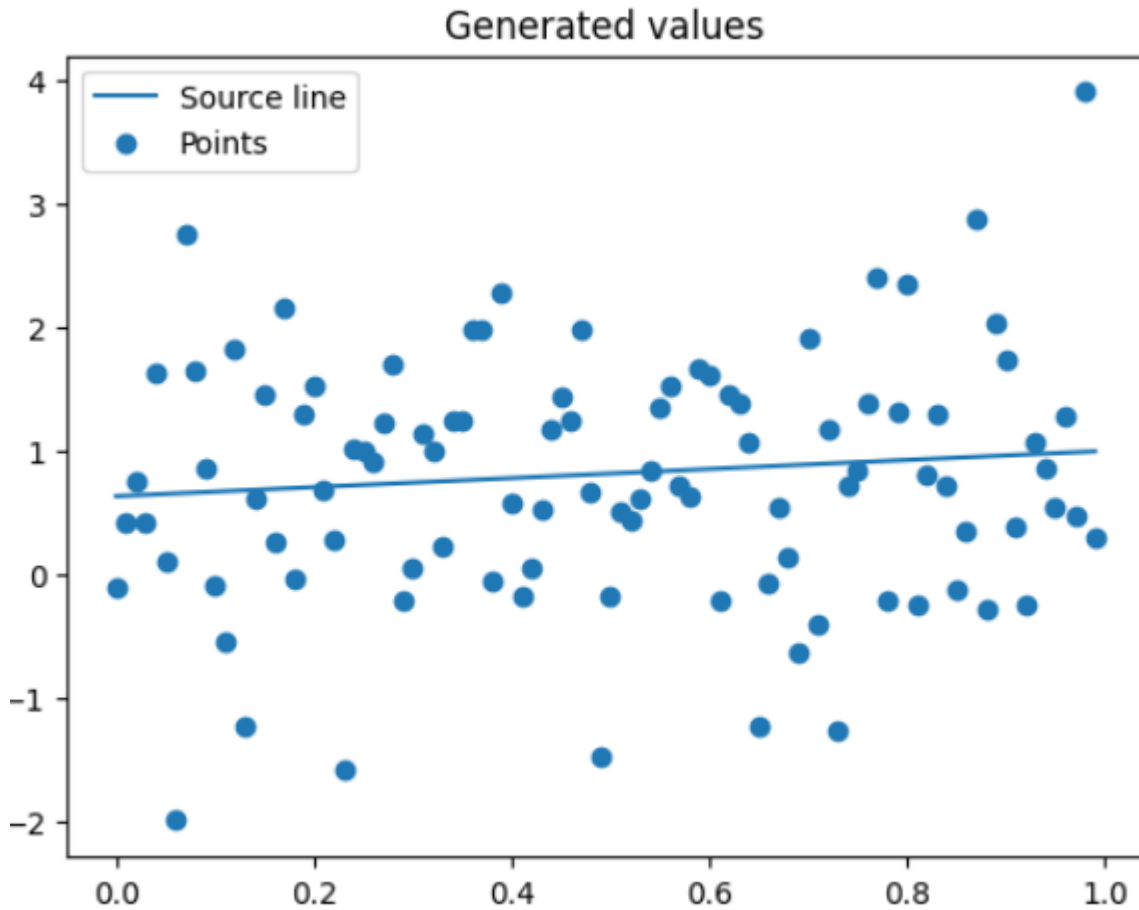
Approximate the data by the following linear and rational functions:

1. $F(x, a, b) = ax + b$ (linear approximant),

2. $F(x, a, b) = \frac{a}{1+bx}$ (rational approximant),

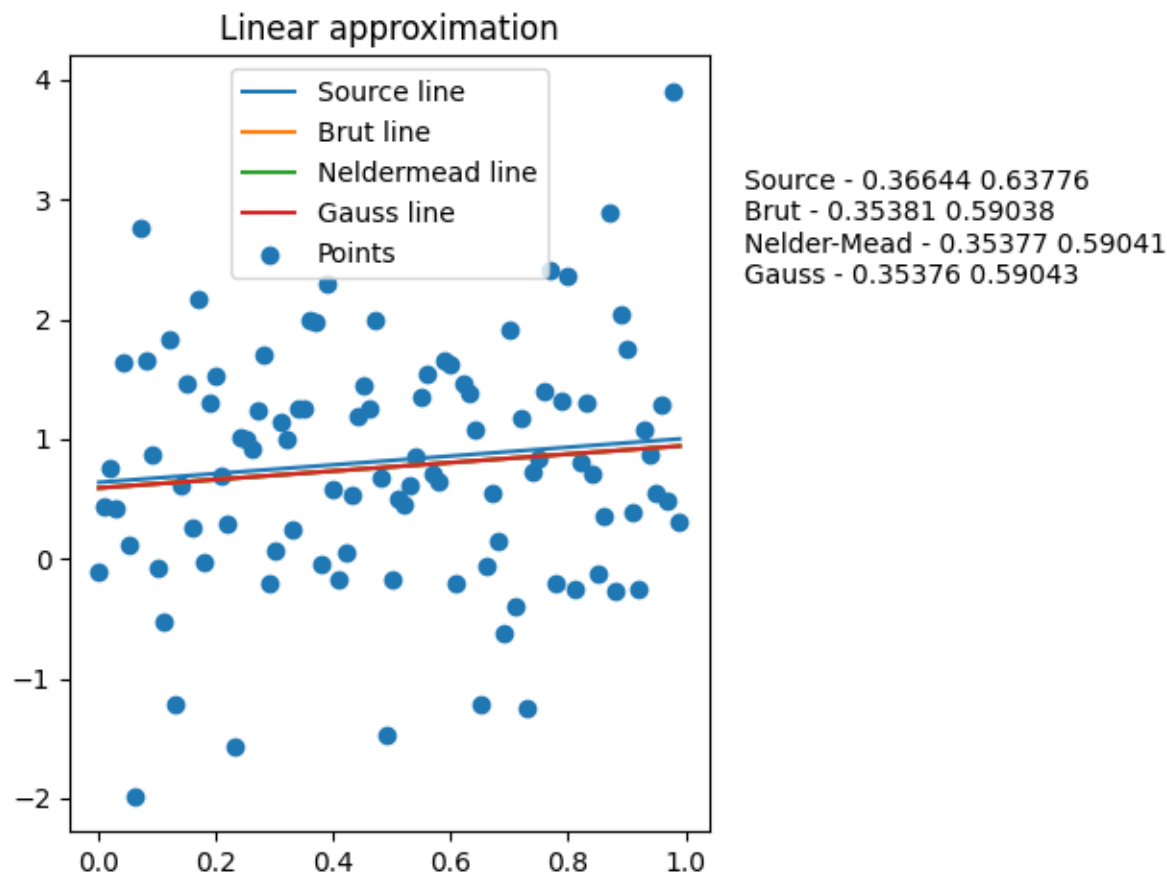
by means of least squares through the numerical minimization (with precision $\text{eps} = 0.001$) of the following function: $D(a, b) = \sum_{k=0}^{100} (F(x_k, a, b) - y_k)^2$

Generated data



Linear approximation

Results

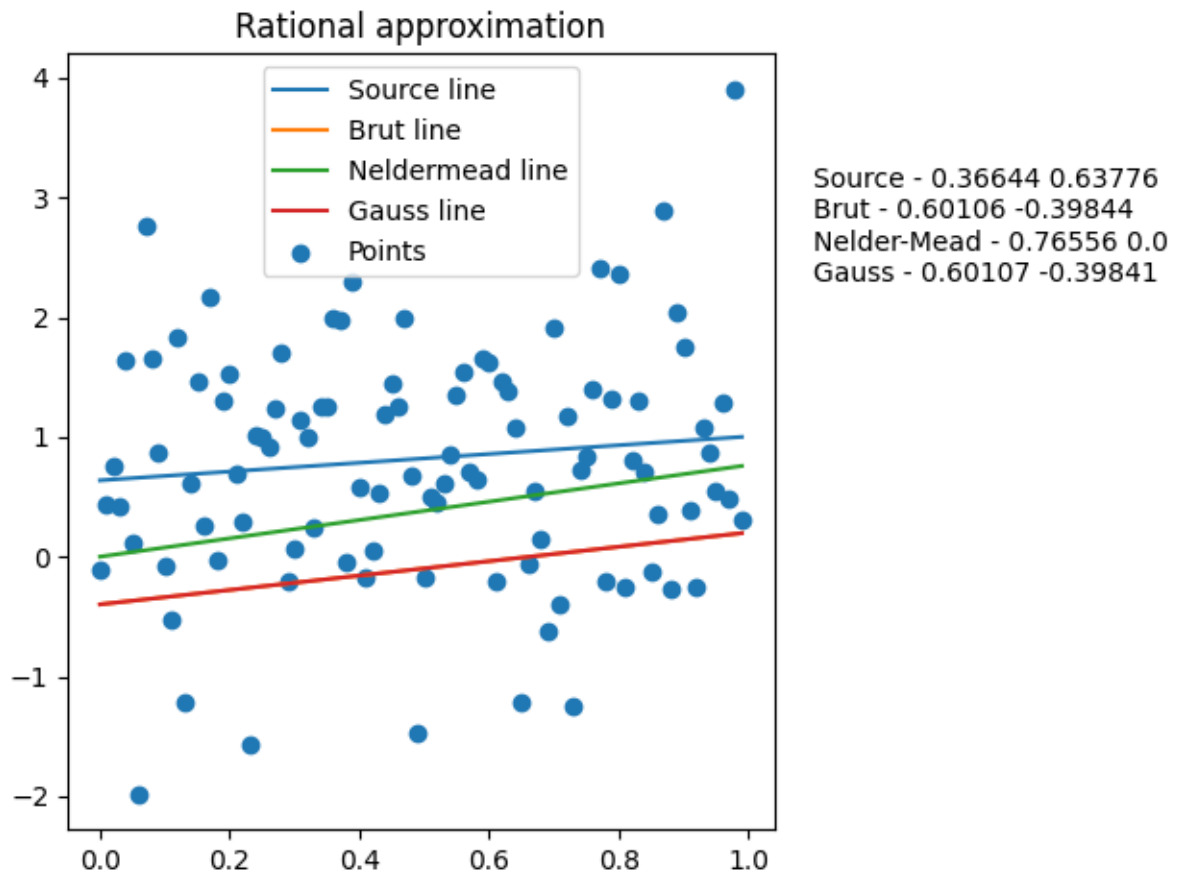


Comparison

	Brut force	Nelder- Mead	Gauss
iterations	1000000	60	2
functions calc	1000000	116	15
loss	97,77662	97,77662	97,77662

Rational approximation

Results



Comparison

	Brut force	Nelder- Mead	Gauss
iterations	1000000	41	11
functions calc	1000000	67	75
loss	97,68556	98,81939	97,68556

Conclusion

As a results, linear approximations are closer to source line, however amount of loss is approximately the same with rational approximation. Count of iterations is smaller for Gauss algorithm during linear and rational approximation. Brute-force algorithm has fixed number of iterations and function calculations, these indicators depend only on error and axis bounds. Brute-force algorithm has the most iterations and function calculations.