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Report
on the practical task No. 4

“Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms”

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Goal

The use of stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution, Particle Swarm Optimization) in the tasks of unconstrained nonlinear optimization and the experimental comparison of them with Nelder-Mead and Levenberg-Marquardt algorithms

Problems

I. Generate the noisy data (x_k , y_k), where k = 0, ... ,1000, according to the rule:

$$y_k = \begin{cases} -100 + \delta_k, \ f(x_k) < -100, \\ f(x_k) + \delta_k, \ -100 \leq f(x_k) \leq 100, \\ 100 + \delta_k, \ f(x_k) > 100, \end{cases} \quad x_k = \frac{3k}{1000} \ (1)$$

where $\delta_k \sim N(0, 1)$ are values of a random variable with standard normal distribution. Approximate the data by the rational function

$$F(x, a, b, c, d) = \frac{ax+b}{x^2+cx+d} \ (2)$$

by means of least squares through the numerical minimization of the following function:

$$D(a, b, c, d) = \sum_{k=0}^{1000} (F(x_k, a, b, c, d) - y_k)^2 \ (3)$$

To solve the minimization problem, use Nelder-Mead algorithm, Levenberg- Marquardt algorithm and at least two of the methods among Simulated Annealing, Differential Evolution and Particle Swarm Optimization. If necessary, set the initial approximations and other parameters of the methods. Use ε = 0.001 as the precision; at most 1000 iterations are allowed. Visualize the data and the approximants obtained in a single plot. Analyze and compare the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).

II. Choose at least 15 cities in the world having land transport connections between them. Calculate the distance matrix for them and then apply the Simulated Annealing method to solve the corresponding Travelling Salesman Problem. Visualize the results at the first and the last iteration. If necessary, use the city dataset from https://people.sc.fsu.edu/~jburkardt/datasets/cities/cities.html

Brief theoretical part

Stochastic (Monte Carlo) algorithms are a broad class of algorithms that are based on repeated random sampling to solve an optimization problem. These methods are most useful when it is impossible or difficult to apply others (for example, there is no information about the differentiability of the function being optimized, or the problem is discrete).

Metaheuristic algorithms are algorithms inspired by natural phenomena that solve an optimization problem by trial and error. Metaheuristic methods, generally speaking, do not guarantee that a solution to an optimization problem will be found.

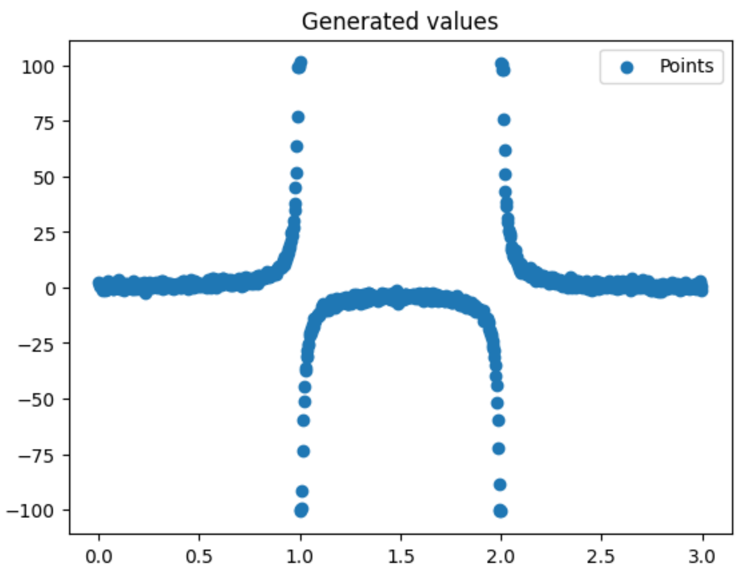
Simulated annealing is a metaheuristic algorithm that solves an optimization problem similar to the annealing process in metallurgy (heating and controlled cooling of a material to increase its crystal size and reduce defects).

Differential evolution is a metaheuristic algorithm that solves the optimization problem through the evolution of a population of agents, that is, possible solutions, creating new generations of agents by combining existing ones and further selecting the best ones.

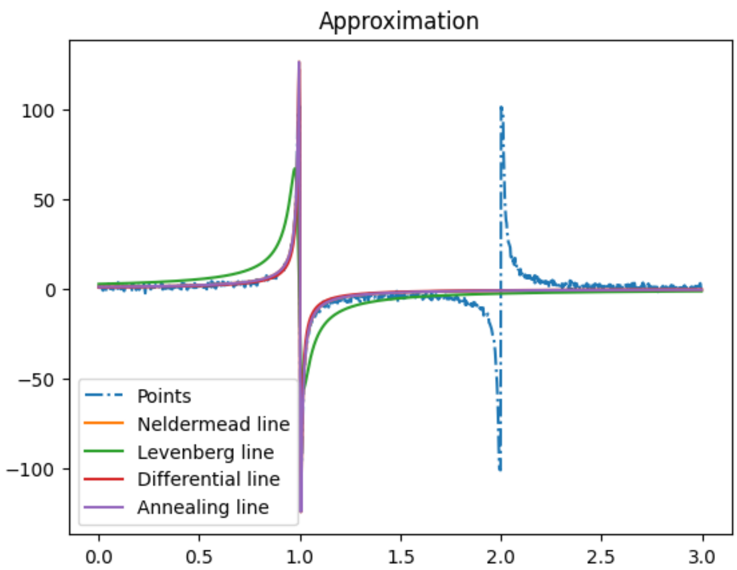
The particle swarm method is a metaheuristic algorithm that solves an optimization problem by iteratively changing the position of possible solutions (particles) at a certain rate. The change in the position of each particle is affected by its best known position and the best known positions of other particles.

Solution of the minimization problem.

Generated the noisy data (x_k , y_k), where k = 0, ... ,1000, according to the rule (1).



Comparison approximation plot

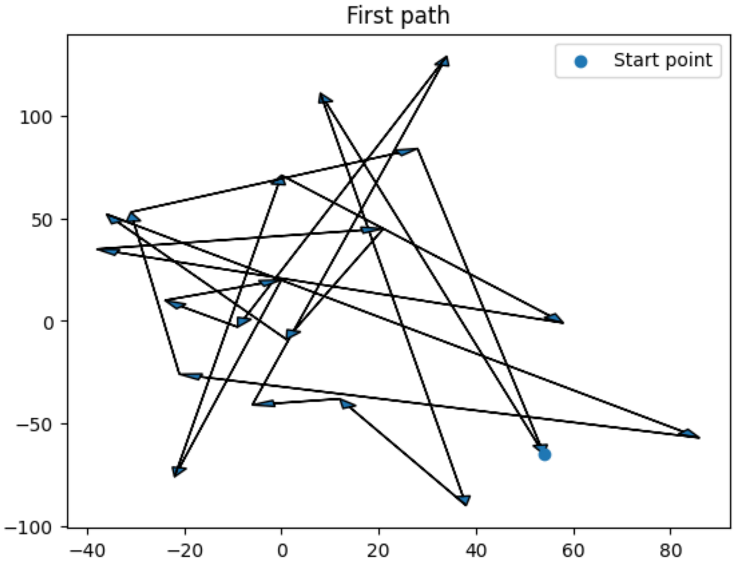


Comparison approximation table

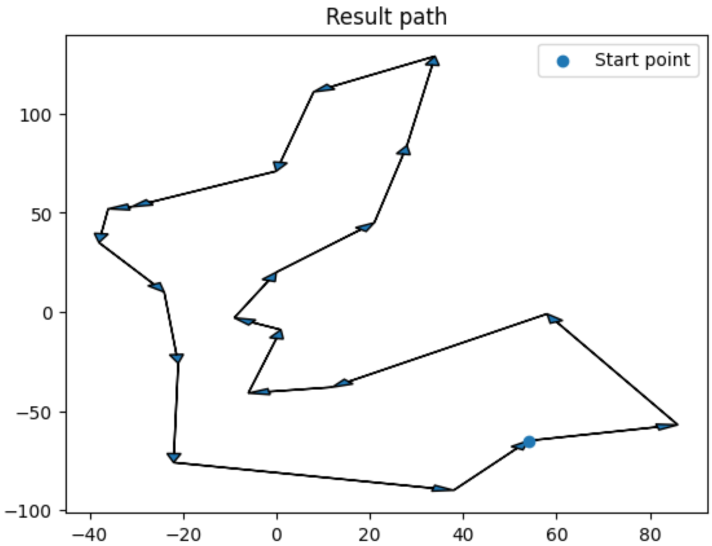
	iterations	calculations	loss	a	b	c	d
Neldermead	416	712	135736.724987	-1.002793	1.003289	-2.00092	1.000936
Levenberg	1000	1000	[222220.01402055746]	-2.645533	2.636815	-1.989283	0.989785
Differential	4	1135	137285.603494	-0.848279	0.848693	-2.000914	1.000926
Annealing	1000	9246	135736.728494	-1.002623	1.003118	-2.000921	1.000937

Solution of Travelling Salesman Problem with the Simulated Annealing method.

First iteration results



Last iteration results



The result of solving the traveling salesman problem by simulating annealing.

```
{'fist_path': array([ 0,  3, 14, 10, 12,  8, 19, 15,  7,  6,  1, 18, 16, 13,  4,  5, 17,
        11,  2,  9,  0]),
 'first_dist': 2024.2538003659486,
 'result_path': array([ 0, 17, 18, 10, 12,  4, 19,  7, 13,  9,  8,  3,  1,  2,  5, 16, 15,
        11,  6, 14,  0]),
 'result_dist': 817.9505785039942}
```

Conclusions

The worst result was shown by the Levenberg-Marquardt algorithm, its loss is 1.6 more than other algorithms.

The Simulated annealing optimization is intended to provide more control over the search process, as opposed to a fixed-rule gradient descent algorithm.
During the laboratory work, two main rules of the Simulating Annealing method were determined:
First, the direction of movement must be determined probabilistically at every step with the hope of not falling into the trap of a local optimum and moving towards a global optimum.
And the second. The search step should decrease as the search process progresses and approaches the final result. This helps to move vigorously in the first steps and carefully in the

next. The Simulating Annealing method is a heuristic search algorithm and, therefore, it is sensitive to its initial point in the search space.

Appendix

https://github.com/OnlyOneUseAcc/algorithms-RD-practise