FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION OF HIGHER EDUCATION ITMO UNIVERSITY

Report
on the practical task No. 2

"Algorithms for unconstrained nonlinear optimization. Direct methods"

Performed by
Kozlov Alexey
Dmitriy Koryakov
J4133c
Accepted by
Dr Petr Chunaev

St. Petersburg
2022

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Goal

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization.

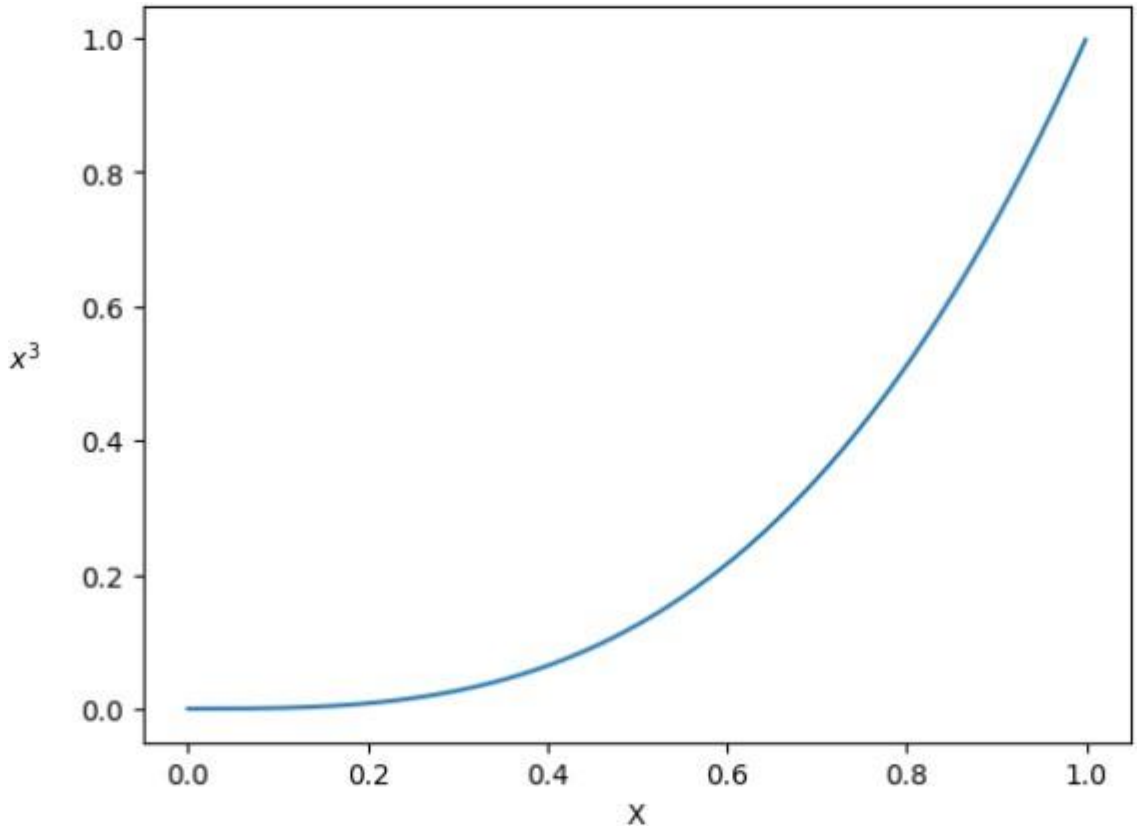
One-dimensional

Problem

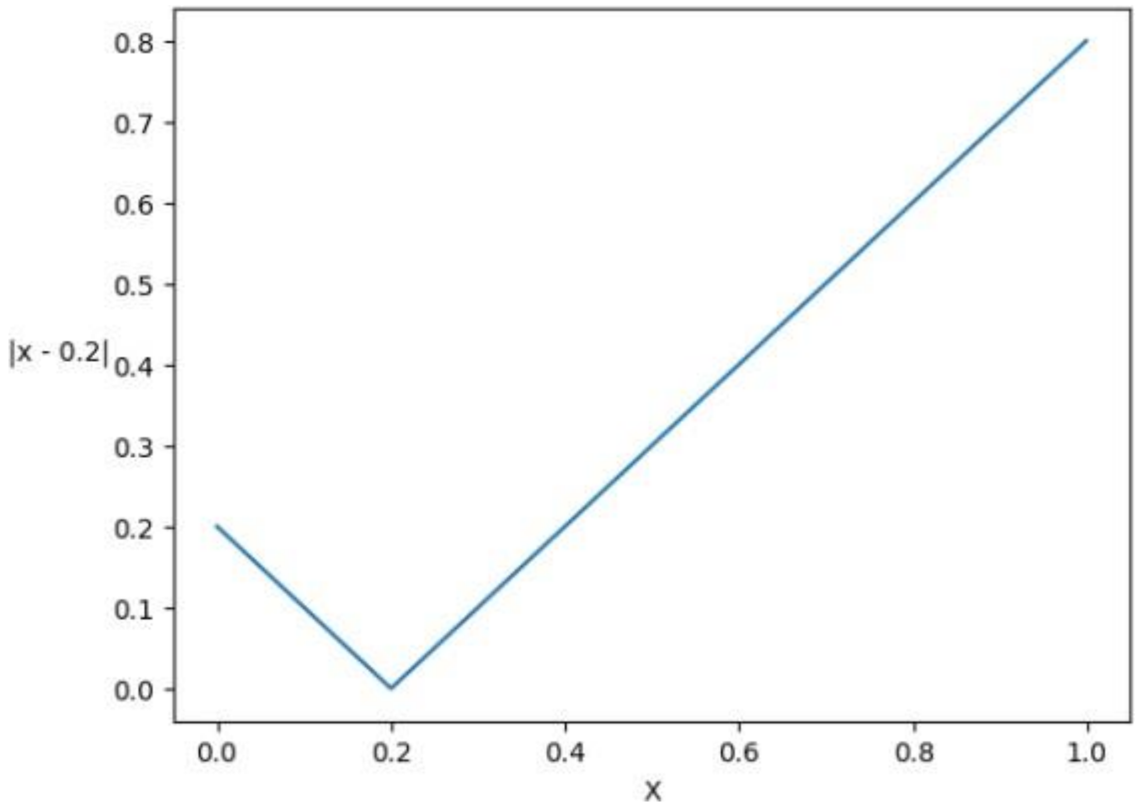
Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision eps= 0.001) solution x: $f(x) \rightarrow min$ for the following functions and domains.

Functions

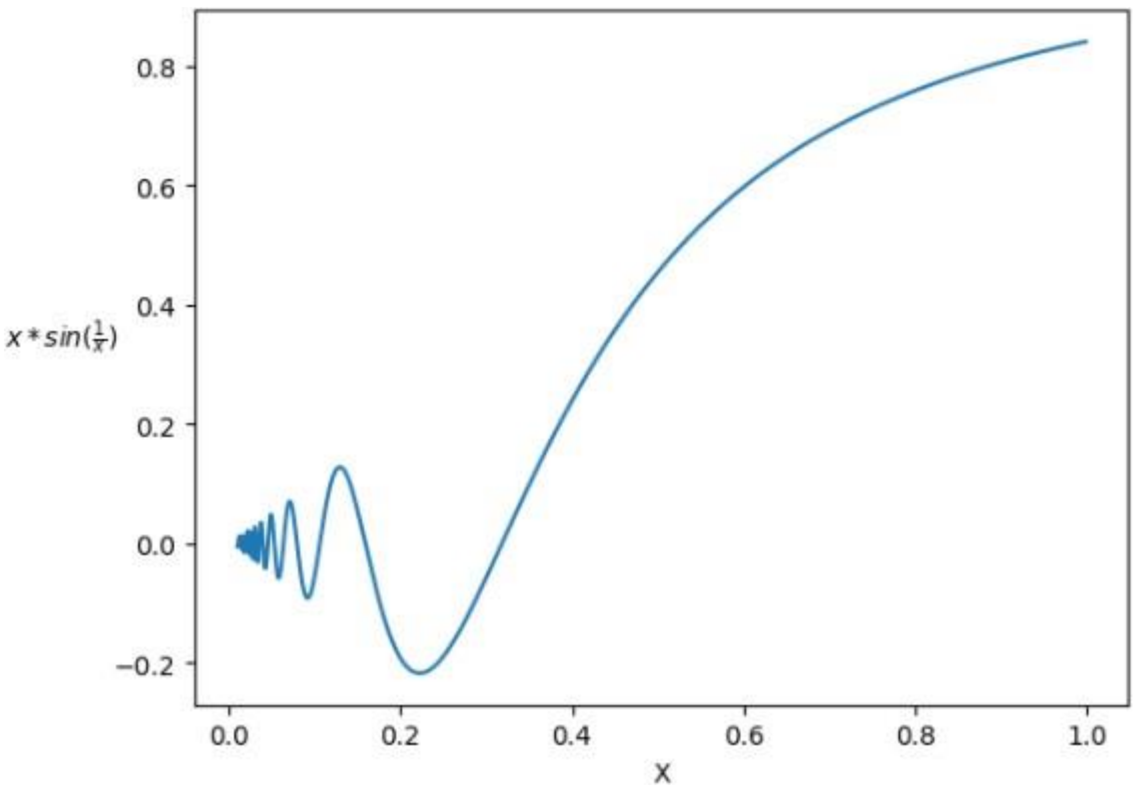
Cubic function



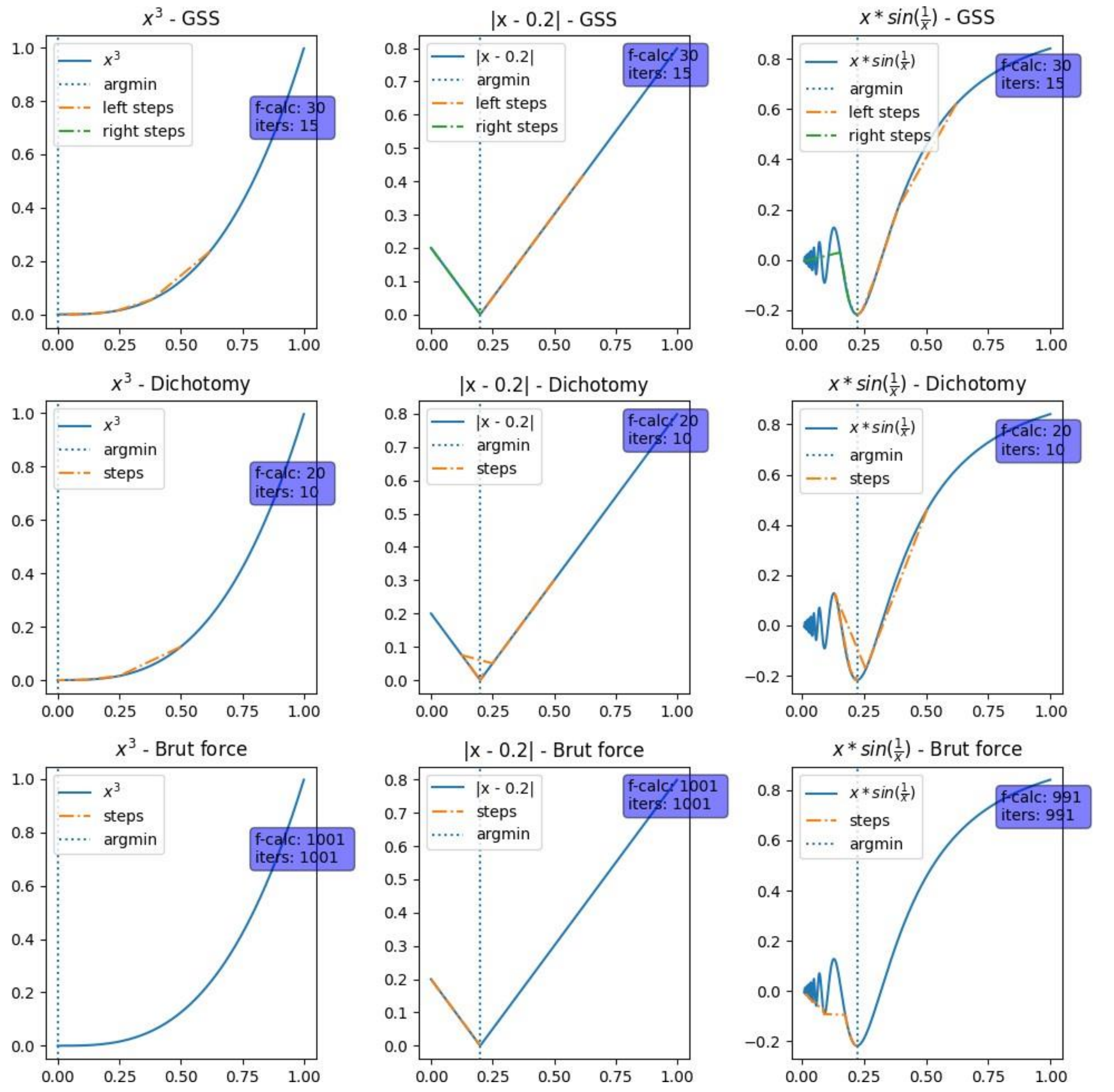
Abs function



Sine function



Comparison



Conclusion

All algorithms found minimum of all functions with specified error. Brute-force algorithm has fixed number of iterations and function calculations, these indicators depend only on error and axis bounds. Brute-force algorithm has the most iterations and function calculations. As for dichotomy and GSS, number of iterations and function calculations are the same on different functions, but dichotomy algorithm has the smallest number of iterations and function calculations

Two-dimensional

Problem

Generate random numbers $\alpha \in (0,1)$ and $\beta \in (0,1)$. Furthermore, generate the noisy data $\{x_k, y_k\}$, where k = 0, … ,100, according to the following rule $y_k = \alpha x_k + \beta + \delta_k$, $x_k = \frac{k}{100}$, where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution.

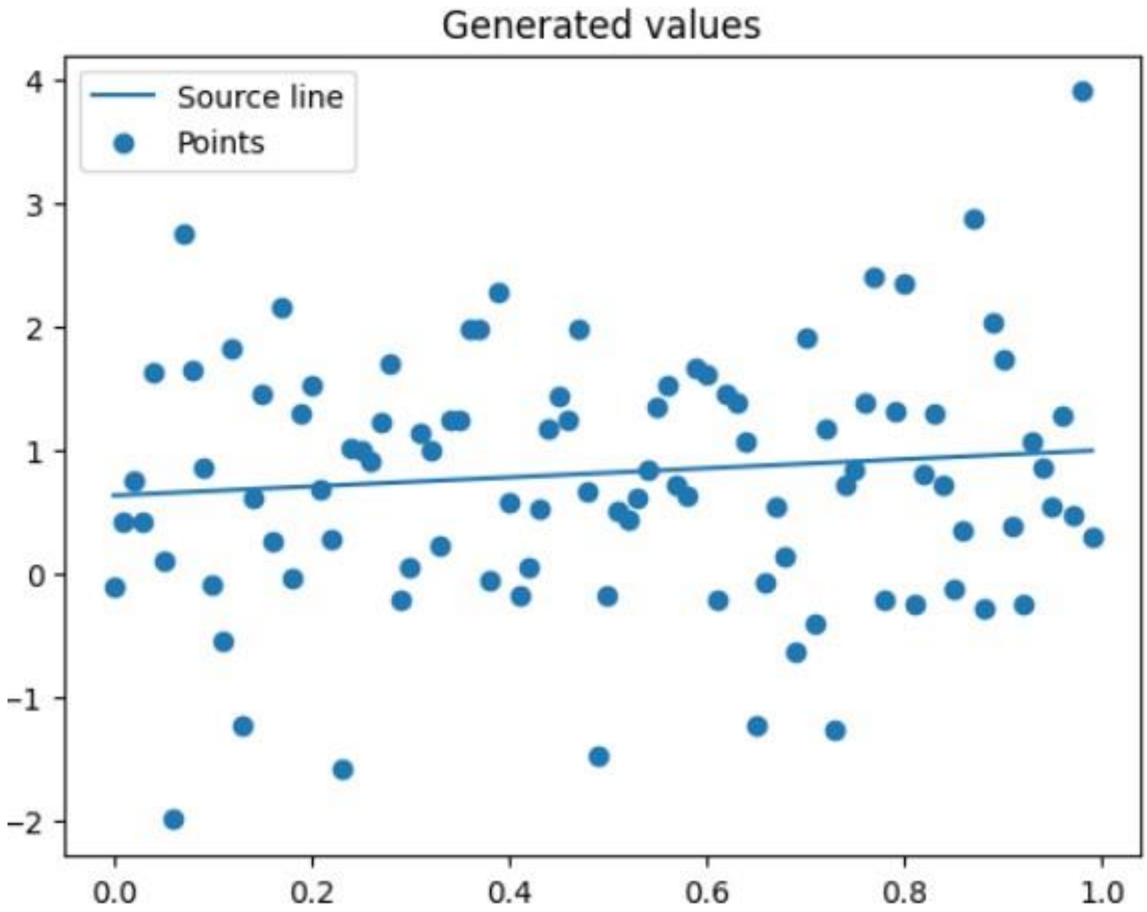
Approximate the data by the following linear and rational functions:

1. $F(x, a, b) = ax + b$ (linear approximant),

2. $F(x, a, b) = \frac{a}{1+bx}$(rational approximant),

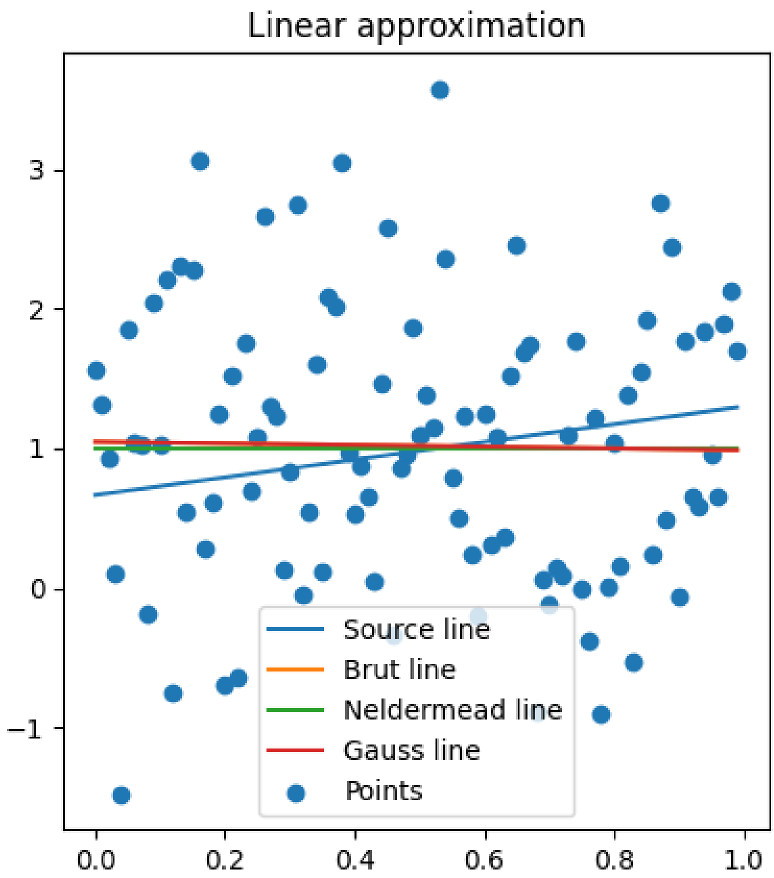
by means of least squares through the numerical minimization (with precision eps = 0.001) of the following function: $D(a, b) = \sum_{k=0}^{100}(F(x_k, a, b) - y_k)^2$

Generated data



Linear approximation

Results



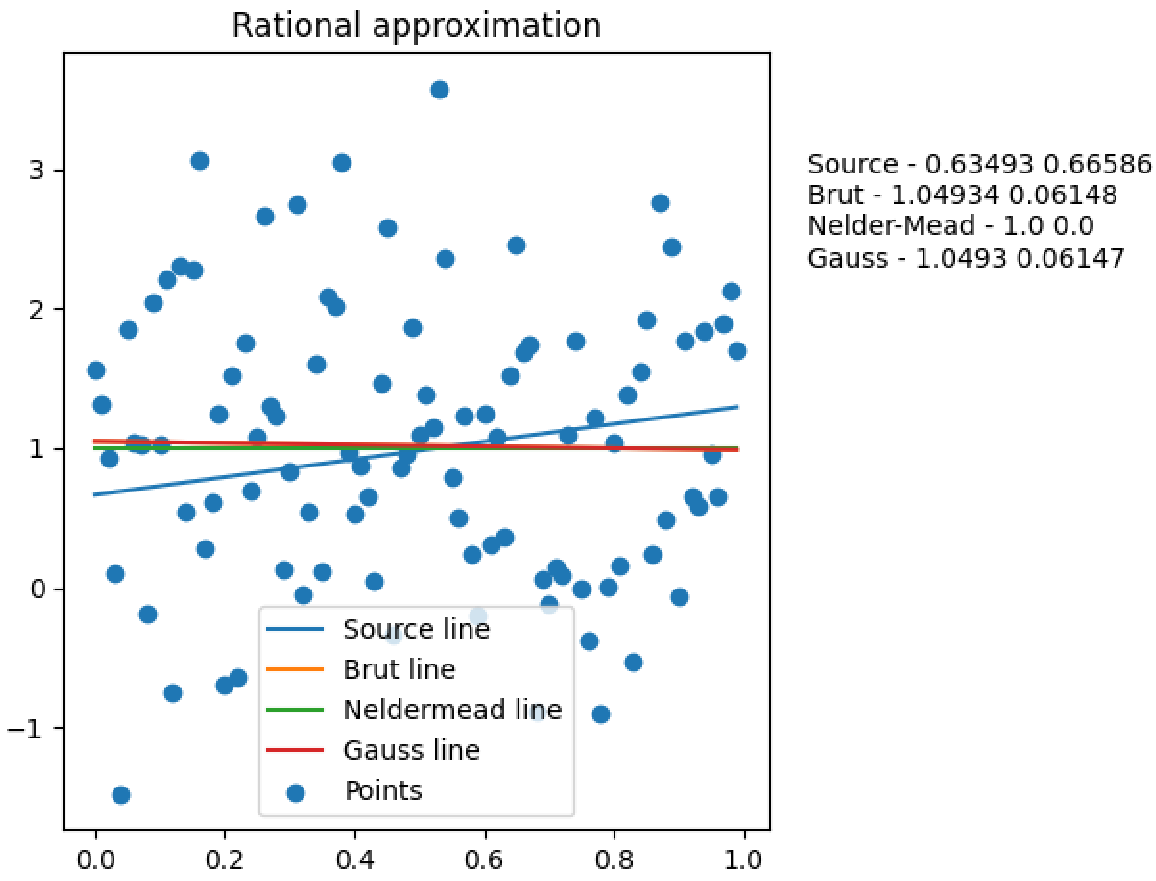
Source - 0.63493 0.66586
Brut - -0.0601 1.04838
Nelder-Mead - 0.0 1.0
Gauss - -0.06012 1.04838

Comparison

	Brut force	**Nelder-Mead**	**Gauss**
iterations	1000000	28	2
functions calc	1000000	55	15
loss	98.96941	99.034198	98.96941

Rational approximation

Results



Comparison

	Brut force	**Nelder-Mead**	**Gauss**
iterations	1000000	25	7
functions calc	1000000	40	42
loss	98.968992	99.034198	98.968992

Conclusion

As a results, linear approximations are closer to source line, however amount of loss is approximately the same with rational approximation. Count of iterations is smaller for Gauss algorithm during linear and rational approximation. Count of function calculations during Nelder-Mead and Gauss search depends on approximation type. Brute-force algorithm has fixed number ofiterations and function calculations, these indicators depend only on error and axis bounds. Brute-force algorithm has the most iterations and function calculations.

Appendix

Link to github - https://github.com/OnlyOneUseAcc/algorithms-RD-practise