

# Adaptive Full State Observer for Nonsalient PMSM with Noised Measurements of the Current and Voltage <sup>★</sup>

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**Abstract:** An algorithm of adaptive estimation of the magnetic flux for the non-salient permanent magnet synchronous motor (PMSM) for the case when measurable electrical signals are corrupted by a constant offset is presented. A new nonlinear parameterization of the electric drive model based on dynamical regressor extension and mixing (DREM) procedure is proposed. Due to this parameterization the problem of flux estimation is translated to the auxiliary task of identification of unknown constant parameters related to measurement errors. It is proved that when both current and voltage measurements are biased the proposed algorithm ensures convergence of the flux observation error to a bounded set. At the same time the position error converges to zero. The observer provides global exponential convergence if the corresponding regression function satisfies the persistent excitation condition. If the regression function is not square integrable the global asymptotic convergence is ensured. In comparison with known analogues this paper gives a constructive way of the flux reconstruction for a nonsalient PMSM with guaranteed performance (low oscillation, convergence rate regulation) and, from other hand, a straightforwardly easy implementation of the proposed observer to embedded systems.

Keywords: Nonlinear control systems, robust observers, synchro motors, flux estimator, speed estimator, sensorless approach, voltage offset.

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## 1. INTRODUCTION

The problem of so-called “sensorless” control is very resonating and challenging today. The main difficulty is a nonlinear model of the PMSM in which the magnetic flux is unmeasurable variable. Very popular strategy in literature is to reconstruct mechanical variables of the drive using knowledge of the total flux. And usually the magnetic flux observation is a key problem, which attracts a lot of scientists from adaptive control and electric drive societies, including L. Praly, R. Ortega, R. Marino, P. Tomei, K. Nam, A. Stankovic, and many other famous researchers.

Although a lot of different approaches are existed and even already implemented as preset feature in distributed actuators, however, the performance of such control strategy is still an open “hot” problem. Existed, known for authors, methods do not guarantee the convergence of regulation error to zero in scenarios with measurement errors. Some approaches give robust estimates that acceptable in practical applications. Estimators which ensure the asymptotic convergence of estimates to observable states of electrical

drives usually are not robust with respect to measurement noise because were designed with strong assumptions regarding this issue.

The approaches Bobtsov et al. [2015] and Bazylev et al. [2018] require only measurements of the voltage  $v$  and the current  $i$ , however it was assumed that there is no bias or noise in electrical signals. Experimental study on the setup with noisy measurements demonstrate that both observers ensure bounded errors in the low speed region. Nevertheless, the robustness against noises is still need to be guaranteed. In simulation results we present advantages of the new method compared to the mentioned observers.

The paper Pyrkin et al. [2018] was devoted to the case when the measured signals  $v$  and  $i$  contain uncertain biases that are assumed to be constant. Two observers were presented, robust version with reduced dimension and adaptive one designed with a classical gradient approach. In this paper we present a reduced order observer and show its efficiency for a higher value of biases in comparison with Pyrkin et al. [2018]. Also we apply the DREM procedure Aranovskiy et al. [2016] to get performance enhancement.

In this brief paper we focus on the problem of observer design of the flux in PMSM which is, from one hand, is robust with respect to biases in measurements and does not contain any open-loop integration schemes, and, from other hand, provides convergence of all estimation

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errors to zero with such performance properties as low oscillations of estimates and possibility of convergence rate regulation. Also for the case when both current and voltage measurements are biased the presented approach ensures global convergence of the flux observation error to a low bounded set which depends on stator resistance and inductance and voltage offset.

## 2. PROBLEM FORMULATION

Consider the classical, two phase  $\alpha\beta$  model of the unsaturated, non-salient, PMSM described by Krause [1986], Nam [2010]

$$\begin{aligned}\dot{\lambda} &= v - Ri, \\ j\dot{\omega} &= -f\omega + \tau_e - \tau_L, \\ \dot{\theta} &= \omega,\end{aligned}\quad (1)$$

where  $\lambda \in \mathbb{R}^2$  is the total flux,  $i \in \mathbb{R}^2$  are the currents,  $v \in \mathbb{R}^2$  are the voltages,  $R > 0$  is the stator windings resistance,  $j > 0$  is the rotor inertia.  $\theta \in \mathbb{S} := [0, 2\pi]$  is the rotor phase,  $\omega$  is the mechanical angular velocity,  $f > 0$  is the viscous friction coefficient,  $\tau_L \in \mathbb{R}$  is the – possibly time-varying – load torque,  $\tau_e$  is the torque of electrical origin, given by

$$\tau_e = n_p i^\top J \lambda$$

with  $n_p \in \mathbb{N}$  the number of pole pairs and  $J \in \mathbb{R}^{2 \times 2}$  is the rotation matrix

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

For surface-mounted PMSM's the total flux verifies

$$\lambda = Li + \lambda_m \mathcal{C}(\theta), \quad (2)$$

where  $L > 0$  is the stator inductance and

$$\mathcal{C}(\theta) := \begin{pmatrix} \cos(n_p \theta) \\ \sin(n_p \theta) \end{pmatrix}.$$

Assume the only signals available for measurements are the current  $i$  and the voltage  $v$ , which are corrupted by constant unknown bias terms  $\delta_i \in \mathbb{R}^2$  and  $\delta_v \in \mathbb{R}^2$ , respectively, that is

$$i_m = i + \delta_i, \quad v_m = v + \delta_v, \quad (3)$$

where  $i_m$  and  $v_m$  are actually measured signals. The resistance  $R$  and the inductance  $L$  are assumed to be known.

The goal is to reconstruct asymptotically the total flux  $\lambda$  with asymptotic convergence of estimation errors to 0.

## 3. MAIN RESULT

The adaptive flux observer for PMSM was proposed in Bobtsov et al. [2015] and was based on the equation

$$|\lambda - Li|^2 - \lambda_m^2 = 0, \quad (4)$$

which follows from (2).

Expression (4) may be rewritten as

$$\lambda^\top \lambda - 2L\lambda^\top i_m + L^2 i_m^\top i_m + \lambda^\top \eta_1 + i_m^\top \eta_2 + \eta_3 = 0, \quad (5)$$

where  $\eta_1 = 2L\delta_i$ ,  $\eta_2 = -2L^2\delta_i$ ,  $\eta_3 = L^2\delta_i^\top \delta_i - \lambda_m^2$  are constants that are unknown.

The approach presented in Pyrkin et al. [2018] extends the method firstly appeared in Bernard and Praly [2017].

The key feature of extension is the possibility to find a linear regressor equation depending on uncertain flux  $\lambda$ , set of unknown parameters and measurable signals. The following proposition establishes this fact.

*Proposition 1.* Consider the model of PMSM (1) with measurable signals (3) corrupted by uncertain offsets. The following regression model holds

$$\begin{aligned}\dot{\lambda} &= -Ri_m + v_m + \eta_m, \\ y &= \Phi^\top (\lambda + \frac{1}{2}\eta_1) + \Psi^\top \eta + \varepsilon_t,\end{aligned}\quad (6)$$

where the known functions  $y$ ,  $\Phi$ ,  $\Psi$  are computed from available signals,  $\eta_m := R\delta_i - \delta_v$  and  $\eta := (\eta_{m1} \ \eta_{m2} \ \eta_m^\top \eta_m)^\top$  are unknown constant vectors and  $\varepsilon_t$  is an exponentially decaying term.

*Proof.* The proof is based on [Pyrkin et al., 2018, Lemma 3.3] and may be easily repeated with the following steps.

**Step 1.** Differentiate (5).

**Step 2.** Apply the filter  $\frac{\nu}{p+\nu}$ , where  $p := \frac{d}{dt}$  is a differential operator.

**Step 3.** Using the Swapping Lemma [Sastry and Bodson, 2011, Lemma 3.6.5]

$\frac{\nu}{p+\nu} (x^\top z) = z^\top \left( \frac{\nu}{p+\nu} x \right) - \frac{1}{p+\nu} \left( \dot{z}^\top \left( \frac{\nu}{p+\nu} x \right) \right)$   
get the regressor equation where the flux  $\lambda$  and vector  $\eta$  enter linearly only.

**Step 4.** Apply the filter  $\frac{p}{p+\nu}$ .

**Step 5.** Rewrite obtained operator expression as ordinary differential equations.

After 5 steps we introduce the following filters

$$\begin{aligned}\dot{\xi}_1 &= -\nu\xi_1 + 2\nu y_m + 2\nu^2 Li_m, \\ \dot{\xi}_2 &= -\nu\xi_2 + \xi_1 + 2y_m, \\ \dot{\xi}_3 &= -\nu\xi_3 + y_m^\top \xi_1 + \nu^2 L^2 i_m^\top i_m, \\ \dot{\xi}_4 &= -\nu\xi_4 + \nu\xi_2 - \xi_1, \\ \dot{\xi}_5 &= -\nu\xi_5 + \nu\xi_3 - \nu^2 L^2 i_m^\top i_m + y_m^\top (\nu\xi_2 - \xi_1),\end{aligned}\quad (7)$$

that operate the known signal

$$y_m = -Ri_m + v_m.$$

The proof is completed by picking

$$y = \xi_3 - \nu L^2 i_m^\top i_m - \xi_5,$$

$$\Phi = 2\xi_1 - 2\nu Li_m - \nu\xi_2,$$

$$\Psi = \begin{pmatrix} 2\xi_4 \\ 2\nu^{-1} \end{pmatrix}.$$

Then we will use the basic result of the Proposition 1 in design the adaptive observer of the magnetic flux and apply the DREM procedure Aranovskiy et al. [2016].

Directly, the DREM procedure is not applicable to the model (6) since  $\lambda$  is a function of time. Neglecting the

exponential term  $\varepsilon_t$  we apply one more filter with the coefficient  $\alpha > 0$  to the second equation of (6):

$$\begin{aligned} \frac{\alpha}{p+\alpha}y &= \frac{\alpha}{p+\alpha}\Phi^\top(\lambda + \frac{1}{2}\eta_1) + \frac{\alpha}{p+\alpha}\Psi^\top\eta \\ &= (\lambda + \frac{1}{2}\eta_1)^\top \frac{\alpha}{p+\alpha}\Phi - \frac{1}{p+\alpha} \left( \dot{\lambda}^\top \frac{\alpha}{p+\alpha}\Phi \right) + \eta^\top \frac{\alpha}{p+\alpha}\Psi \quad \text{where} \\ &= (\lambda + \frac{1}{2}\eta_1)^\top \frac{\alpha}{p+\alpha}\Phi - \frac{1}{p+\alpha} \left( (v_m - Ri_m + \eta_m)^\top \frac{\alpha}{p+\alpha}\Phi \right) \\ &+ \eta^\top \frac{\alpha}{p+\alpha}\Psi = (\lambda + \frac{1}{2}\eta_1)^\top \frac{\alpha}{p+\alpha}\Phi - \frac{1}{p+\alpha} \left( y_m^\top \frac{\alpha}{p+\alpha}\Phi \right) \\ &- \eta_m^\top \frac{\alpha}{(p+\alpha)^2}\Phi + \eta_m^\top \frac{\alpha}{p+\alpha}(2\xi_4) + \eta_m^\top \eta_m \frac{\alpha}{p+\alpha}(2\nu^{-1}) \end{aligned} \quad (8)$$

and rewrite as

$$z_\alpha = \bar{\Phi}_\alpha^\top(\lambda + \frac{1}{2}\eta_1) + \bar{\Psi}_\alpha^\top\eta \quad (9)$$

where

$$\begin{aligned} z_\alpha &= \frac{\alpha}{p+\alpha}y + \frac{1}{p+\alpha} \left( y_m^\top \frac{\alpha}{p+\alpha}\Phi \right), \\ \bar{\Phi} &= \frac{\alpha}{p+\alpha}\Phi, \\ \bar{\Psi} &= \begin{pmatrix} \frac{\alpha}{p+\alpha}(2\xi_4) - \frac{\alpha}{(p+\alpha)^2}\Phi \\ \frac{\alpha}{p+\alpha}(2\nu^{-1}) \end{pmatrix}. \end{aligned}$$

Following the concept of DREM we consider the set of linear filters  $\frac{\alpha_k}{p+\alpha_k}$  with different gains  $\alpha_k > 0$ ,  $k = \overline{1, N-1}$ , where  $N := \dim(\lambda + \frac{1}{2}\eta_1) + \dim(\eta) = 5$  and get a system of  $N$  linear equations

$$z_k(t) = \bar{\Phi}_k^\top(t)(\lambda + \frac{1}{2}\eta_1) + \bar{\Psi}_k^\top(t)\eta, \quad (10)$$

where index  $k$  denotes the coefficient  $\alpha_k$  of the corresponding filter. Combining 5 new regression models (10) with the original one (6) we form extended regression

$$Z(t) = M(t) \begin{pmatrix} \lambda(t) + \frac{1}{2}\eta_1 \\ \eta \end{pmatrix},$$

where

$$Z(t) := \begin{pmatrix} y(t) \\ z_1(t) \\ \vdots \\ z_N(t) \end{pmatrix}, \quad M(t) := \begin{pmatrix} \Phi^\top & \Psi^\top \\ \bar{\Phi}_1^\top & \bar{\Psi}_1^\top \\ \vdots & \vdots \\ \bar{\Phi}_N^\top & \bar{\Psi}_N^\top \end{pmatrix}.$$

The next step is to obtain a set of scalar  $N$  equations as follows.

$$\begin{aligned} \text{adj}M(t)Z(t) &= \text{adj}M(t)M(t) \begin{pmatrix} \lambda(t) + \frac{1}{2}\eta_1 \\ \eta \end{pmatrix} \\ &= \det M(t) \begin{pmatrix} \lambda(t) + \frac{1}{2}\eta_1 \\ \eta \end{pmatrix} \end{aligned}$$

or

$$Y_\lambda(t) = \Delta(t)(\lambda(t) + \frac{1}{2}\eta_1), \quad (11)$$

$$Y_\eta(t) = \Delta(t)\eta, \quad (12)$$

$$Y(t) := \begin{pmatrix} Y_\lambda(t) \\ Y_\eta(t) \end{pmatrix} = \text{adj}M(t)Z(t),$$

$$\Delta(t) := \det M(t).$$

*Proposition 2.* Consider the parameterized model of PMSM (6). The update law

$$\dot{\hat{\eta}} = \gamma_\eta \Delta(Y_\eta - \Delta\hat{\eta}), \quad (13)$$

provides exponential convergence of  $\tilde{\eta} = \eta - \hat{\eta}$  to 0 for  $\gamma_\eta > 0$  if  $\Delta(t)$  is persistently excited,  $\Delta(t) \in \text{PE}$ . If  $\Delta(t)$  is not square integrable,  $\Delta(t) \notin \mathcal{L}_2$ , then  $\tilde{\eta}$  tends to 0 asymptotically.

*Proof.* Compute the derivative of error  $\tilde{\eta}$

$$\dot{\tilde{\eta}}(t) = -\gamma_\eta \Delta^2(t)\tilde{\eta}.$$

Solving the latter equation we immediately see that

$$\tilde{\eta}(t) = e^{-\gamma_\eta \int_0^t \Delta^2(s)ds} \tilde{\eta}(0),$$

which completes the proof.

As seen from (11) the estimate of flux is affected by the unknown parameter  $\eta_1$  which is proportional to the unknown constant bias of currents  $\delta_i$ . Therefore, we consider the following three scenarios to construct the flux observer:

- 1)  $\delta_i$  is known and  $\delta_v$  is unknown;
- 2)  $\delta_i$  is unknown and  $\delta_v$  is known;
- 3) both  $\delta_i$  and  $\delta_v$  are unknown.

The assumption of known bias terms  $\delta_i$  and  $\delta_v$  is equal to the case when the signals  $i$  and  $v$  are measured (calculated) precisely.

The following Proposition establishes the main result of the paper for the first scenario.

*Proposition 3.* Consider the parameterized model of PMSM (6) and parameter estimator (13). Let  $\delta_i$  to be known. The update law

$$\begin{aligned} \hat{\lambda} &= \chi - L\delta_i, \\ \dot{\chi} &= -Ri_m + v_m + \hat{\eta}_m + \gamma_\lambda \Delta(Y_\lambda - \Delta\chi) \end{aligned} \quad (14)$$

ensures

$$\lim_{t \rightarrow \infty} \tilde{\lambda}(t) = 0, \quad \text{if } \Delta(t) \notin \mathcal{L}_2,$$

$$|\tilde{\lambda}_{1,2}(t)| \leq \rho e^{-\sigma t} |\tilde{\lambda}_{1,2}(0)|, \quad \text{if } \Delta(t) \in \text{PE},$$

for some  $\gamma_\lambda > 0$  and  $\rho \geq 1$ ,  $\sigma > 0$ .

*Proof.* The flux error model takes the form

$$\begin{aligned}
\dot{\tilde{\lambda}}(t) &= \dot{\hat{\lambda}} - \dot{\lambda} = \dot{\chi} - v_m + Ri_m - \eta_m \\
&= \gamma_\lambda \Delta(Y_\lambda - \Delta\chi) + \tilde{\eta}_m \\
&= \gamma_\lambda \Delta\left(\Delta(\lambda + \frac{1}{2}\eta_1) - \Delta(\hat{\lambda} + L\delta_i)\right) + \tilde{\eta}_m \\
&= -\gamma_\lambda \Delta^2(t)\tilde{\lambda} + \tilde{\eta}_m.
\end{aligned}$$

Convergence of  $\tilde{\eta}_m$  depends on properties of  $\Delta(t)$  and the gain  $\gamma_\eta$ . If  $\Delta(t)$  is persistently excited (PE), then  $\tilde{\eta}$  converges to 0 exponentially, which guarantees convergence of  $\tilde{\lambda}$  to 0 as proved in [Aranovskiy et al., 2015, Lemma 1]. If there is a lack of excitation but  $\Delta(t) \notin \mathcal{L}_2$  then  $\tilde{\lambda}$  tends to 0 asymptotically. Otherwise  $\tilde{\lambda}$  converges to a bounded set about 0.

Propositions 4 and 5 correspond to the second and the third scenarios, respectively.

*Proposition 4.* Consider the parameterized model of PMSM (6) and parameter estimator (13). Let  $\delta_v$  to be known. The update law

$$\begin{aligned}
\dot{\hat{\lambda}} &= \chi - \frac{L}{R}(\hat{\eta}_m + \delta_v), \\
\dot{\chi} &= -Ri_m + v_m + \hat{\eta}_m + \gamma_\lambda \Delta(Y_\lambda - \Delta\chi)
\end{aligned} \quad (15)$$

ensures

$$\lim_{t \rightarrow \infty} \tilde{\lambda}(t) = 0, \quad \text{if } \Delta(t) \notin \mathcal{L}_2,$$

$$|\tilde{\lambda}_{1,2}(t)| \leq \rho e^{-\sigma t} |\tilde{\lambda}_{1,2}(0)|, \quad \text{if } \Delta(t) \in \text{PE},$$

for some  $\gamma_\lambda > 0$  and  $\rho \geq 1$ ,  $\sigma > 0$ .

*Proof.* First, we notice that the error dynamics and convergence properties of  $\tilde{\eta}$  are the same as in the proof of Proposition 2. Secondly, since  $\delta_v$  is known we get

$$\tilde{\eta}_m = \hat{\eta}_m - \eta_m = R\hat{\delta}_i - R\delta_i = R\tilde{\delta}_i.$$

Then calculation of the error model of  $\tilde{\lambda}$  yields

$$\begin{aligned}
\dot{\tilde{\lambda}}(t) &= \dot{\hat{\lambda}} - \dot{\lambda} = \gamma_\lambda \Delta(Y_\lambda - \Delta\chi) + \tilde{\eta}_m - \frac{L}{R}\dot{\hat{\eta}}_m \\
&= \gamma_\lambda \Delta^2\left(\lambda + \frac{1}{2}\eta_1 - \hat{\lambda} - \frac{L}{R}(\hat{\eta}_m + \delta_v)\right) + \tilde{\eta}_m - \frac{L}{R}\dot{\hat{\eta}}_m \\
&= -\gamma_\lambda \Delta^2(t)\tilde{\lambda} - \gamma_\lambda \Delta^2(t)\tilde{\eta}_m + \tilde{\eta}_m - \gamma_\lambda \frac{L}{R}\Delta^2(t)\tilde{\eta}_m,
\end{aligned}$$

The proof is completed using [Aranovskiy et al., 2015, Lemma 1] invoking that  $\frac{L}{R} > 0$ .

*Proposition 5.* Consider the parameterized model of PMSM (6) and parameter estimator (13). Let  $\delta_i$  and  $\delta_v$  are unknown. The update law

$$\begin{aligned}
\dot{\hat{\lambda}} &= \chi - \frac{L}{R}\hat{\eta}_m, \\
\dot{\chi} &= -Ri_m + v_m + \hat{\eta}_m + \gamma_\lambda \Delta(Y_\lambda - \Delta\chi)
\end{aligned} \quad (16)$$

ensures exponential convergence of  $\tilde{\lambda}$  to  $\frac{L}{R}\delta_v$  if  $\Delta(t) \in \text{PE}$ . If  $\Delta(t) \notin \mathcal{L}_2$  then  $\tilde{\lambda}$  converges to provides asymptotic convergence of  $\tilde{\eta} = \eta - \hat{\eta}$  to 0 for  $\gamma_\eta > 0$  and  $\tilde{\lambda} = \lambda - \hat{\lambda}$  to a bounded set  $\frac{L}{R}\delta_v$  for  $\gamma_\lambda > 0$ .

*Proof.* Compute the derivative of the flux error

$$\begin{aligned}
\dot{\tilde{\lambda}}(t) &= \dot{\hat{\lambda}} - \dot{\lambda} = \gamma_\lambda \Delta(Y_\lambda - \Delta\chi) + \tilde{\eta}_m - \frac{L}{R}\dot{\hat{\eta}}_m \\
&= \gamma_\lambda \Delta^2\left(\lambda + \frac{1}{2}\eta_1 - \hat{\lambda} - \frac{L}{R}\hat{\eta}_m\right) + \tilde{\eta}_m - \frac{L}{R}\dot{\hat{\eta}}_m \\
&= \gamma_\lambda \Delta^2\left(-\tilde{\lambda} + \frac{L}{R}(\eta_m + \delta_v) - \frac{L}{R}\hat{\eta}_m\right) + \tilde{\eta}_m - \frac{L}{R}\dot{\hat{\eta}}_m \\
&= \gamma_\lambda \Delta^2(t)(-\tilde{\lambda} + \frac{L}{R}\delta_v) - \gamma_\lambda \Delta^2(t)\tilde{\eta}_m \\
&\quad - \gamma_\lambda \frac{L}{R}\Delta^2(t)\tilde{\eta}_m + \tilde{\eta}_m,
\end{aligned}$$

where  $\frac{L}{R}\delta_v$  is a vector of unknown constant parameters.

Assuming zero  $\tilde{\eta}_m$  we get

$$\dot{\tilde{\lambda}}(t) = \gamma_\lambda \Delta^2(t)(-\tilde{\lambda} + \frac{L}{R}\delta_v).$$

Introducing the following new denotations

$$x := \tilde{\lambda}, \quad g(t) := \gamma_\lambda \Delta^2(t), \quad a := \frac{L}{R}\delta_v, \quad (17)$$

we rewrite differential equation (17) as

$$\frac{dx}{dt} = g(t)(-x + a),$$

which is a linear differential equation with separable variables.

Making a replacement  $w = x - a$ ,  $dw = dx$  we obtain

$$\frac{1}{w}dw = -g(t)dt,$$

with a solution  $w(t) = w(0)e^{\int_0^t -g(s)ds}$ .

Performing the reverse replacement yields

$$\tilde{\lambda}(t) = (\tilde{\lambda}(0) - \frac{L}{R}\delta_v)e^{-\gamma_\lambda \int_0^t \Delta^2(s)ds} + \frac{L}{R}\delta_v,$$

which completes the proof.

The unknown position is reconstructed by the following. Replacing (3) in (2) we obtain

$$\lambda = Li_m - L\delta_i + \lambda_m C(\theta). \quad (18)$$

Since  $\eta_1 = 2L\delta_i$  the latter can be rewritten in the form

$$\lambda_m C(\theta) = \lambda + \frac{1}{2}\eta_1 - Li_m. \quad (19)$$

As seen from (14), (15) and (16) the estimate of the signal  $(\lambda + \frac{1}{2}\eta_1)$  is generated by the signal  $\chi$  the convergence of which depends on properties of  $\Delta(t)$  and the gains  $\gamma_\eta$  and  $\gamma_\lambda$ . Consequently, the unknown position is reconstructed by

$$\hat{\theta}(t) = \frac{1}{n_p} \arctan\left(\frac{\chi_2 - Li_{m2}}{\chi_1 - Li_{m1}}\right). \quad (20)$$

Notice, that in contrast to the flux observer, the position error is not affected by the unknown current and voltage biases and converges to zero asymptotically if  $\Delta$  is PE.

The rotor speed can be estimated from the observed position  $\hat{\theta}$  using a PLL estimator [Nam, 2010]

$$\begin{aligned}
\dot{\varsigma}_1 &= K_p(\hat{\theta} - \varsigma_1) + K_i \varsigma_2, \quad \varsigma_2 = \hat{\theta} - \varsigma_1 \\
\dot{\omega} &= K_p(\hat{\theta} - \varsigma_1) + K_i \varsigma_2,
\end{aligned}$$

where  $K_p > 0$  and  $K_i > 0$  are proportional and integral gains, respectively.

#### 4. NUMERICAL EXAMPLE

In this section we consider the motor as in Bobtsov et al. [2017], which parameters are listed in Table 1. The motor is controlled by classical field-oriented controller described by Nam [2010]. The desired motor speed increases linearly from zero to 523 rad/sec during the time interval  $t \in (0; 0.2)$  sec and then remains constant. The external load torque applied to the motor is zero at the start and  $\tau_L = 1$  N·m after 0.3 sec.

Parameter (units)	Motor
Inductance $L$ (mH)	40.03
Resistance $R$ ( $\Omega$ )	8.875
Drive inertia $j$ (kgm <sup>2</sup> )	$60 \times 10^{-6}$
Pairs of poles $n_p$ (-)	5
Magnetic flux $\lambda_m$ (Wb)	0.2086

Table 1. Parameters of the first motor BMP0701F

The initial motor parameters  $\lambda^\top(0) = [0.2086 \ 0]$ ,  $\theta(0) = 0$  with zero initial currents and speed.

The initial parameters for the estimation algorithms (13), (14), (15), (16) are the following:  $\hat{\eta}^\top(0) = [0 \ 0 \ 0]$  and  $\chi^\top(0) = [0 \ 0]$ . The design parameters  $\nu = 1400$ ,  $\alpha_1 = 80$ ,  $\alpha_2 = 200$ ,  $\alpha_3 = 360$ ,  $\alpha_4 = 520$ ,  $\gamma_\eta = \gamma_\lambda = 1$ ,  $K_p = 2 \cdot 10^3$ ,  $K_i = 10^4$ . The constant biases in currents and voltages  $\delta_i^\top = [0.4 \ -0.3]$  and  $\delta_v^\top = [0.2 \ -0.1]$ .

In Figs. 1a and 1b the transient processes for the estimates  $\hat{\eta}^\top = [\hat{\eta}_1 \ \hat{\eta}_2 \ \hat{\eta}_3] = [\hat{\eta}_{m1} \ \hat{\eta}_{m2} \ \hat{\eta}_m^\top \hat{\eta}_m]$  are shown. As seen, the parameter estimates converge to their true values fast. The estimates  $\hat{\eta}_1$  and  $\hat{\eta}_2$  perform very low oscillations. The estimate  $\hat{\eta}_3$  exhibits an overshoot at the beginning, then shows small fluctuations and goes to the true value.

Figs. 1c illustrate the transients of the absolute value of the flux observation error for the algorithms (14), (15) and (16), respectively. As seen from simulation results, observers (14) and (15) perform fast convergence of the flux error to zero. The transient behavior of the observer (16) is similar to the observers (14) and (15). However, there is a low stationary error in the steady state after  $t = 0.035$  sec  $\tilde{\lambda}_s \approx [9.02 \ 4.51] \cdot 10^{-4}$  which coincides with analytically calculated bounded set  $\frac{L}{R}\delta_v$ .

In Figs. 2 and 3 the proposed observer (13), (16) and (20) is compared with the observers of Bobtsov et al. [2015] (I) and Bazylev et al. [2018] (II). The initial motor parameters are the same and the initial parameters of the observers are set to zero.

The design parameters for the observer I:  $\alpha = 100$  and  $\gamma = 1$ . The design parameters for the observer II:  $\alpha = 100$ ,  $\beta = 800$  and  $\gamma_1 = \gamma_2 = 1$ .

Fig. 2 demonstrates the transients of the position estimation error for the three mentioned observers. In Fig. 2a the observer I shows oscillations of the error near zero. The observer II performs faster convergence with higher steady oscillations in Fig.2b. It should be mentioned that using lower value of  $\alpha$  results in higher oscillations for both I and II observers. At the same time, the proposed solution performs fast convergence of the position error to zero after  $t = 0.04$  sec.

The transients of the speed estimation error is illustrated in Fig. 3. Fig. 3a shows relatively high steady oscillations of the speed error for the observer I. The behavior of the observer II is similar in Fig. 3b.

As seen from Fig. 3c the designed observer ensures steady state speed error that goes to zero value. The first overshoot is reasoned by the transients of position estimation. The next two are caused by the change of the speed reference and the load torque applied to the motor.

#### 5. CONCLUSION

In this paper we propose the new adaptive observer design algorithm which allows to parametrize the model of disturbed PMSM as a linear regressor equation with respect to observable flux shifted by the term which depends on current bias and some constants depending on measurement errors (biases or offset). Using DREM procedure the vector regressor equation may be splitted to a set of scalar regressor equations with a common measurable regressor and unknown variables or parameters. Such decomposition allows to guarantee convergence of estimation errors to zero for the case when currents are measured properly and voltage measures are biased and vice versa. Also proposed solution allows to regulate the convergence rate via adaptation gains. As proven, for the challenging case when measured currents and voltages are both noised the flux observer perform low stationary error in the steady state. Proposed method ensures global asymptotic convergence of position error to zero. Our future work will be devoted to a full state observer design in conditions of parametric uncertainty of the motor.

#### REFERENCES

- Stanislav Aranovskiy, Alexey Bobtsov, Anton Pyrkin, Romeo Ortega, and Antoine Chaillet. Flux and position observer of permanent magnet synchronous motors with relaxed persistency of excitation conditions. *IFAC-PapersOnLine*, 48(11):301–306, 2015.
- Stanislav Aranovskiy, Alexey Bobtsov, Romeo Ortega, and Anton Pyrkin. Parameters estimation via dynamic regressor extension and mixing. In *American Control Conference (ACC), 2016*, pages 6971–6976. IEEE, 2016.
- Dmitry Bazylev, Slobodan Vukosavic, Alexey Bobtsov, Anton Pyrkin, Aleksandar Stankovic, and Romeo Ortega. Sensorless control of pm synchronous motors with a robust nonlinear observer. *Proceedings of 2018 IEEE Industrial Cyber-Physical Systems (ICPS)*, pages 304–309, 2018.
- Pauline Bernard and Laurent Praly. Robustness of rotor position observer for permanent magnet synchronous motors with unknown magnet flux. *IFAC-PapersOnLine*, 50(1):15403–15408, 2017.
- Alexey Bobtsov, Dmitry Bazylev, Anton Pyrkin, Stanislav Aranovskiy, and Romeo Ortega. A robust nonlinear position observer for synchronous motors with relaxed excitation conditions. *International Journal of Control*, 90(4):813–824, 2017.
- Alexey A Bobtsov, Anton A Pyrkin, Romeo Ortega, Slobodan N Vukosavic, Aleksandar M Stankovic, and Elena V Panteley. A robust globally convergent position observer for the permanent magnet synchronous motor. *Automatica*, 61:47–54, 2015.

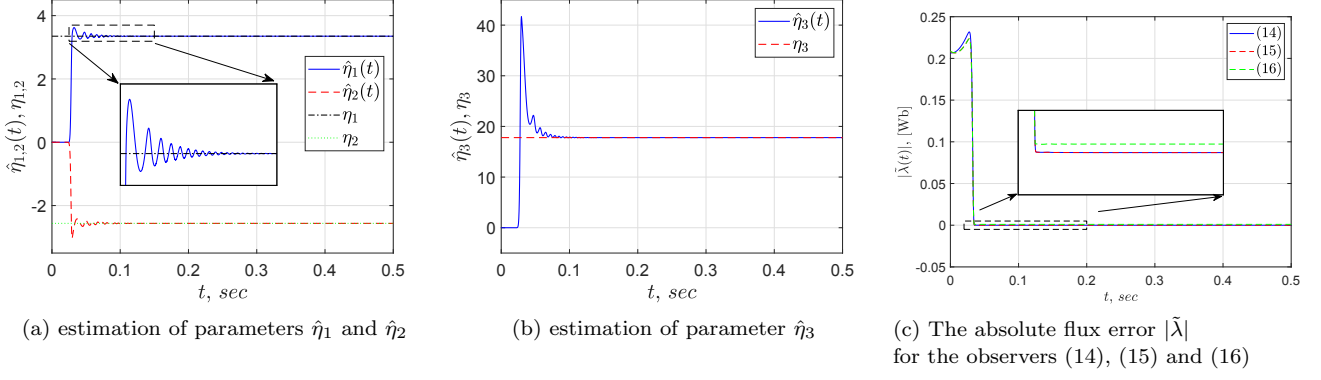


Fig. 1. Transients of parameter estimation and the absolute value of the flux error  $|\tilde{\lambda}|$  for  $\delta_i^\top = [0.4 \ -0.3]$  and  $\delta_v^\top = [0.2 \ -0.1]$

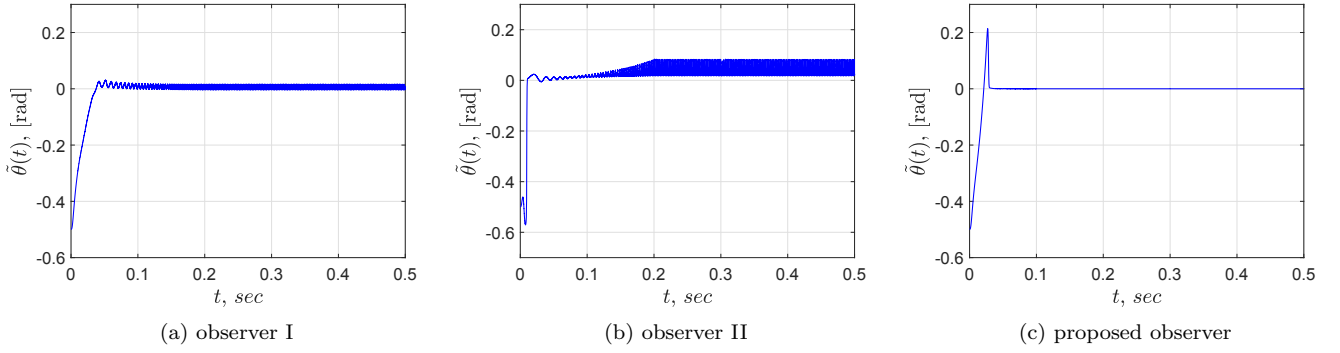


Fig. 2. Transients of the position estimation error for observers with  $\delta_i^\top = [0.4 \ -0.3]$  and  $\delta_v^\top = [0.2 \ -0.1]$

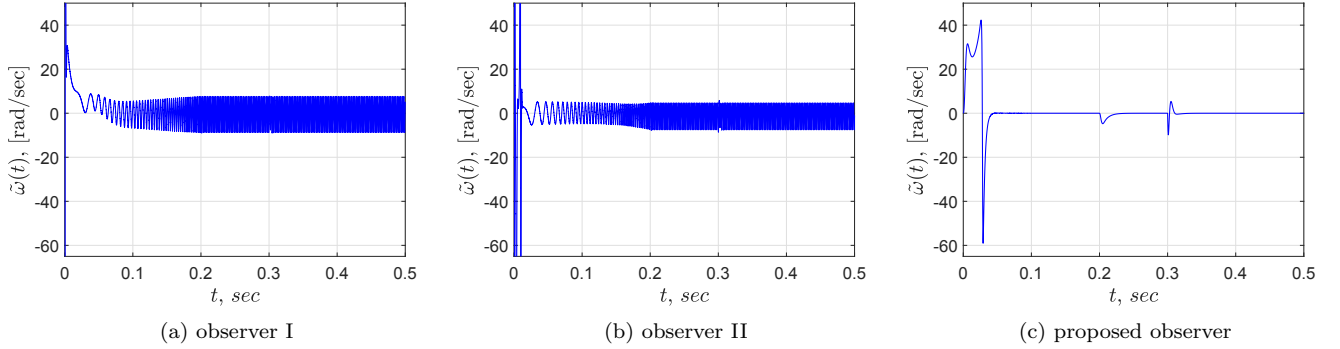


Fig. 3. Transients of the speed estimation error for observers with  $\delta_i^\top = [0.4 \ -0.3]$  and  $\delta_v^\top = [0.2 \ -0.1]$

Paul C Krause. Analysis of electric machinery. McGraw-Hill, New York, 1986.

Kwang Hee Nam. AC motor control and electrical vehicle applications. CRC press, 2010.

Anton A Pyrkin, Alexey A Vedyakov, Romeo Ortega, and Alexey A Bobtsov. A robust adaptive flux observer for a class of electromechanical systems. *International Journal of Control*, pages 1–11, 2018.

Shankar Sastry and Marc Bodson. Adaptive control: stability, convergence and robustness. Courier Corporation, 2011.