Quantum machine learning 1: qubits

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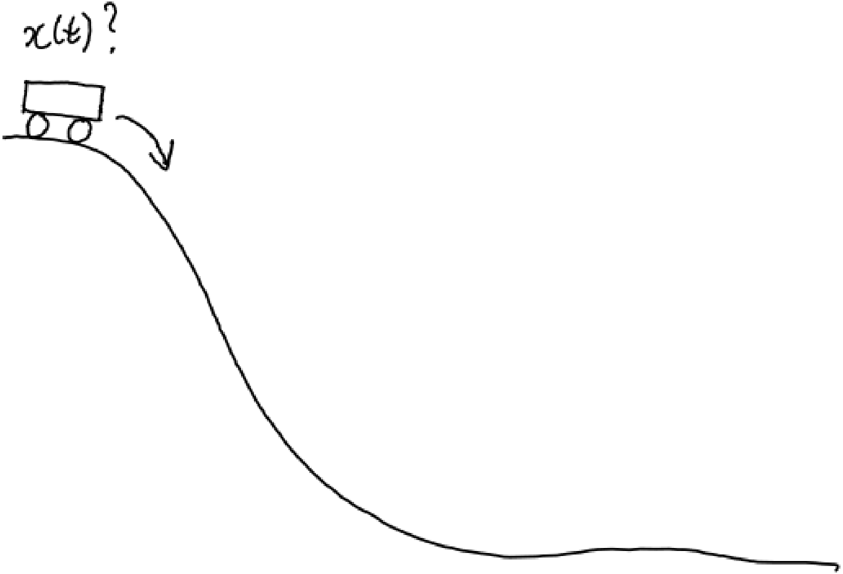
Introduction

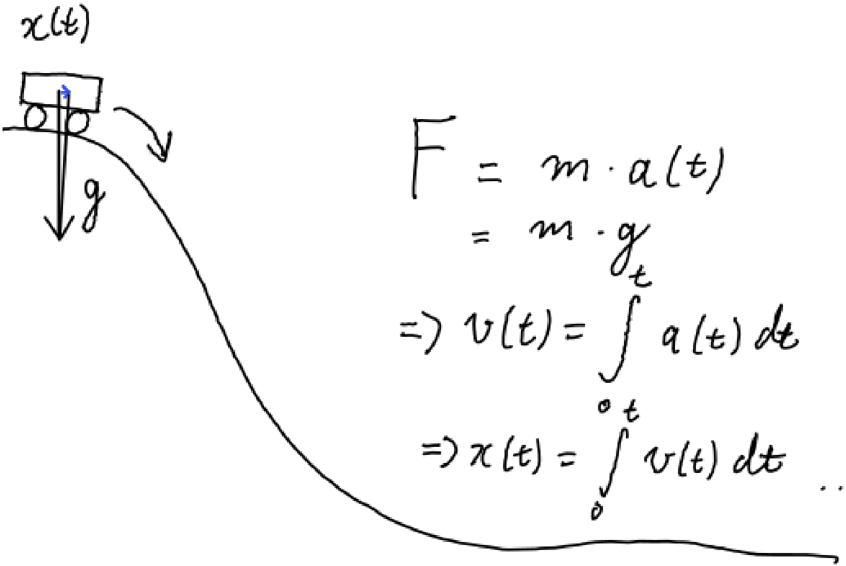
- Onno Huijgen
- Studied theoretical physics
- Work as lecturer/researcher at ADSAI
- (slowly) finishing PhD in QML at Radboud

Classical physics

Classical systems



Classical systems



$$F = m \cdot a(t) = m \cdot g$$

$$\Rightarrow v(t) = \int_0^t a(t)\, dt$$

$$\Rightarrow x(t) = \int_0^t v(t)\, dt \quad \ldots$$

Quantum systems

For small particles, different equation that determines behaviour:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \tag{1}$$

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- ψ: Wave function

Quantum systems

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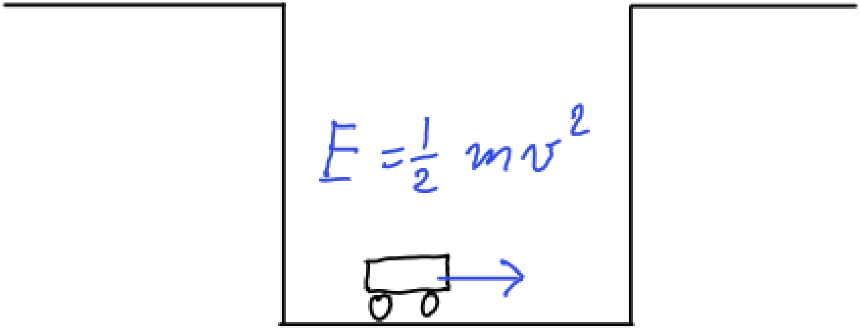
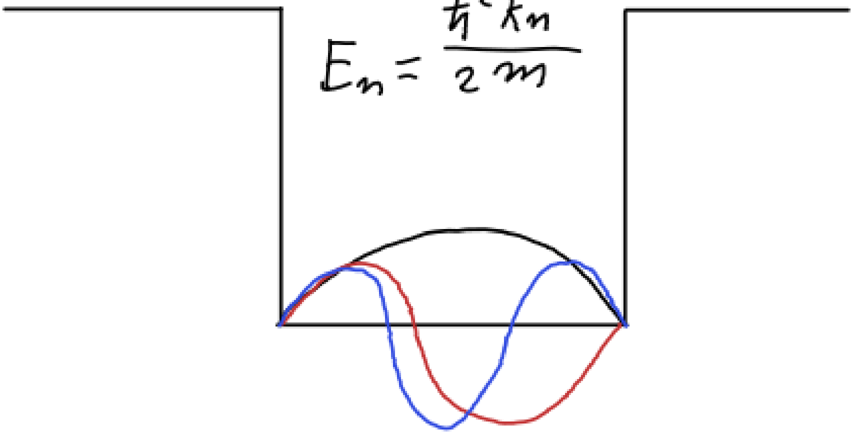
- $i = \sqrt{-1}$
- $\hbar$: constant
- ψ: Wave function
- $\hat{H}$: Hamiltonian, or **energy** operator

Classical: continuous energy

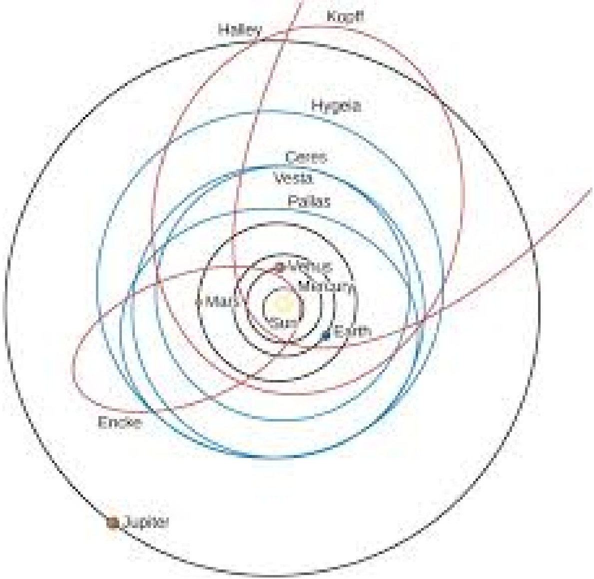


Quantum: Discrete energies

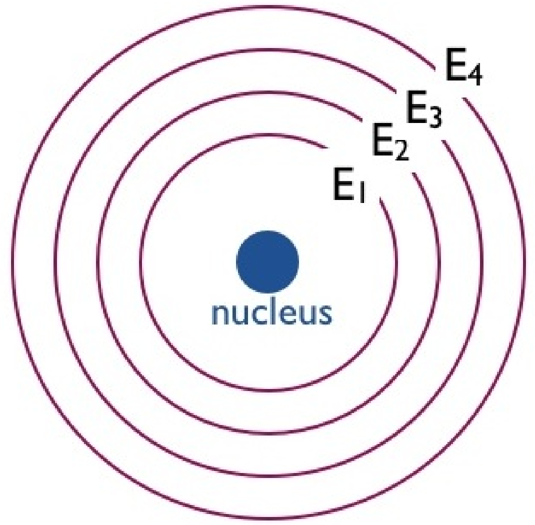


$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

Classical: Orbits



Quantum: Orbitals



Meten = interactie

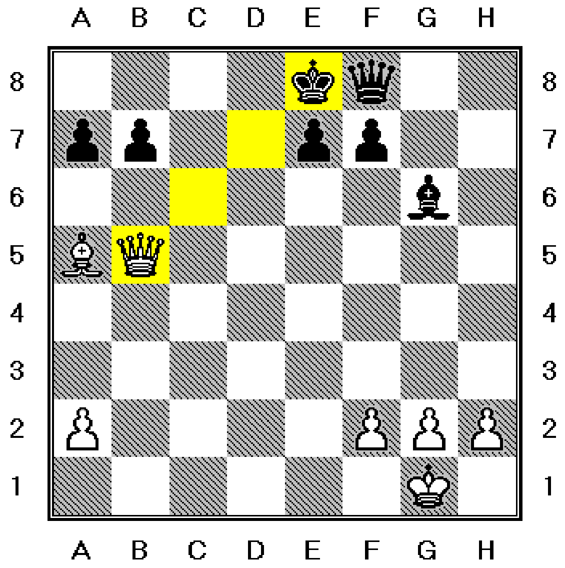
Klassiek meten

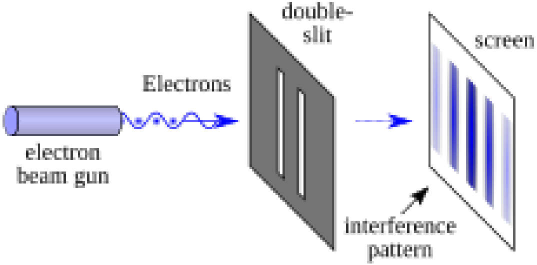
Auto heeft:

- snelheid
- positie
- energie
- magnetizatie

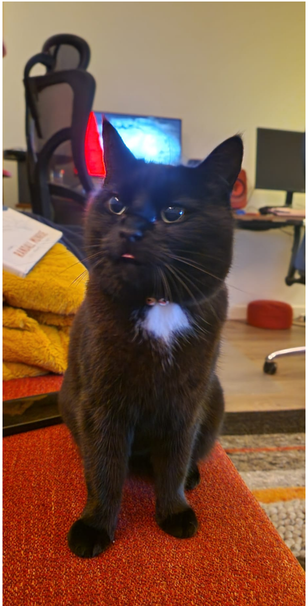
Superposition

Superposition

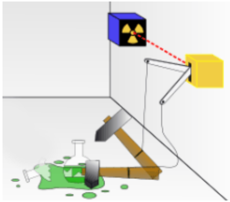



double-
slit
screen
Electrons
electron
beam gun
interference
pattern

Schödinger's cat



$\Leftarrow$



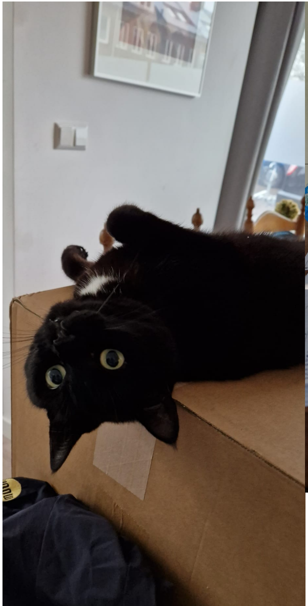
$\Rightarrow$

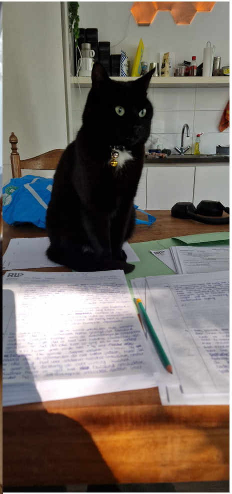
Quantum measurements - Heisenberg's uncertainty principle

- Superposition: properties are not always well-defined
- Sometimes QM particles can be 'in between'
- More precise: some properties are not well-defined at the same time

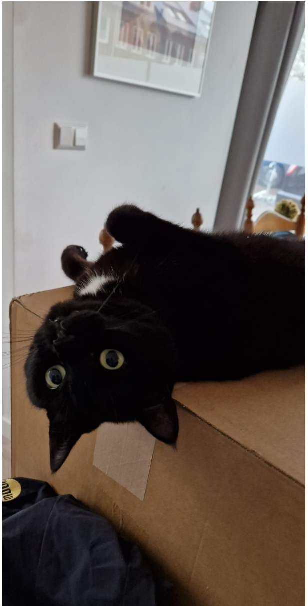
$$\Delta\hat{x}\Delta\hat{p} \geq \frac{\hbar}{2}$$

Measuring is influencing





$\Rightarrow$



Vectors

scalar:

$$10, 3 + 5i, \sqrt{2}, \pi$$

vector:

$$\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{2} \end{pmatrix}$$

matrix:

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Complex numbers

$$
\begin{aligned}
z &= 10 + 3i \\
\bar{z} &= 10 - 3i \\
|z| &= z * \bar{z} = 10^2 + \cancel{10 * 3i} - \cancel{10 * 3i} - 3 * 3 * i^2 \\
&= 10^2 + 3^2
\end{aligned}
$$

Dirac notation

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\langle\uparrow| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

Measurements

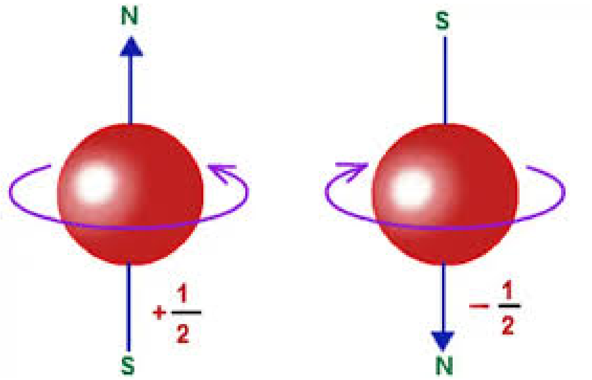
Observables in QM: operators/matrices

$$H\,|\uparrow\rangle = E_1\,|\uparrow\rangle$$

Classical bits

```
11110100 11011001 01000101 01001110 11010100 10111100 00010110 01011110
10000110 00110100 11110001 10111111 11000001 00100101 01101110 00010111
00000101 01111110 11011100 00100001 00101000 10111000 00011101 10000011
11000101 10111111 11110001 00010000 01101010 10000101 11011110 10001000
11011001 11010011 01110011 10011111 10110101 00111100 01101111 00001000
01010010 10100011 00000000 01100111 01110100 00100110 11110011 11011000
10101010 01111111 10100100 01110100 01101110 01110001 01010010 11011111
01111010 01101111 01100101 01111110 01011110 01010101 10010010 10101111
00010011 10000101 10101001 00100111 10000101 11110010 10101111 11001101
10011101 11011111 10011011 00010001 11001100 11100011 10100100 10111001
00010111 01111100 01010010 11011010 11011011 11001001 10010001 11111000
10100001 00101011 00001111 11111110 01001010 11011001 01100011 11101000
01100111 10100011 00110001 00000101 00101111 11100011 00011000 11110010
01101111 00100010 01010110 10111011 00110010 11101010 10110001 10001010
10011110 00000010 11111010 00011000 00111101 00100101 01001111 10100001
```

Spin

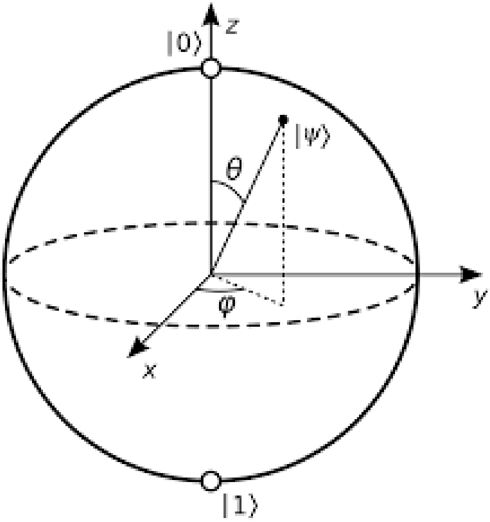


Spin

- Spin is quantized: discrete values
- Whole or half integer
- Spin-n particle: $\{-n, -n+1, -n+2, ...n-1, n\}$
- For $n = \frac{1}{2}$: $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$

example: electron

Bloch sphere



Qubit states

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$|\downarrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Spin operators

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Measurements

$$\langle \sigma_z \rangle = \langle \psi | \sigma_z | \psi \rangle$$

Possible outcomes are given by eigenvectors of operators:

$$\sigma_z \left| \downarrow \right\rangle = -1 * \left| down \right\rangle \tag{5}$$

Measurements 2

For those that did linear algebra:

$$\sigma_z = 1 * |\uparrow\rangle \langle\uparrow| - 1 * |\downarrow\rangle \langle\downarrow|$$

show all possible outcomes when measuring σ_z

Born's rule

$$\sigma_z \left(\frac{1}{\sqrt{2}} \left|\uparrow\right\rangle + \frac{1}{\sqrt{2}} \left|\downarrow\right\rangle \right) = \begin{cases} 1 * \left|\uparrow\right\rangle & \text{with } p = \frac{1}{2} \\ -1 * \left|\uparrow\right\rangle & \text{with } p = \frac{1}{2} \end{cases}$$

Measuring the state destroys the state! Also,

$$\left(\frac{1}{\sqrt{2}} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2 = 1$$

Two particles

So far, we have dealt with one particle or qubit. Multiple qubits?

$$|\uparrow\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Four possibilities:

$$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$$

entanglement

Suppose we have a superposition state:

$$\frac{1}{\sqrt{2}}\left|\uparrow\uparrow\right\rangle + \frac{1}{\sqrt{2}}\left|\downarrow\downarrow\right\rangle \tag{6}$$

And we measure the first particle to be $\left|\uparrow\right\rangle$. What do we know about the second particle?

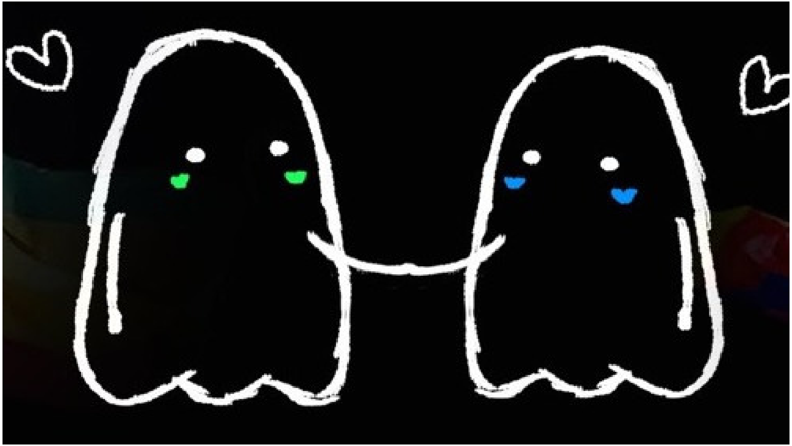
entanglement

Suppose we have a superposition state:

$$\frac{1}{\sqrt{2}} |\uparrow\uparrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\downarrow\rangle \tag{6}$$

And we measure the first particle to be $|\uparrow\rangle$. What do we know about the second particle? Measuring the first particle **changed** the second particle

Spooky action at a distance



Python environment

Need:

- Python ¿= 3.8
- numpy
- scipy

Assignment 1

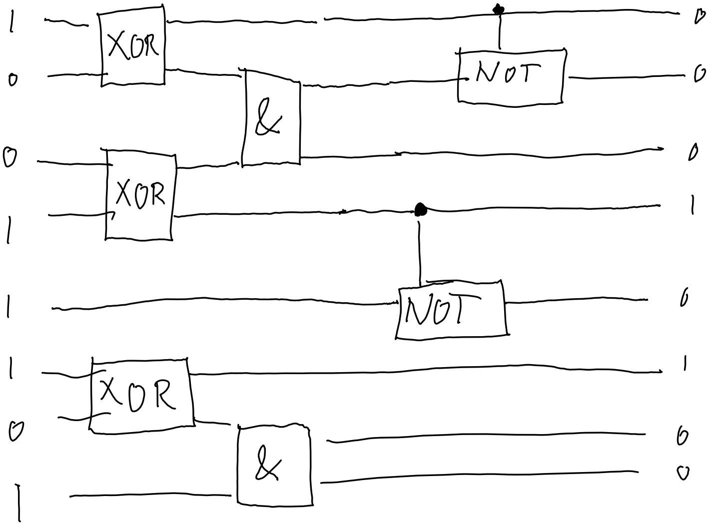
- states: `up = np.array([[1], [0]])`
- Transpose: `up.T`
- Operators: `sigma_x = np.array([[0, 1], [1, 0]])`
- Matrix product: `sigma_x @ up`
- Expectation value: `up.T @ \sigma_x @ up`

Logical operations on bits

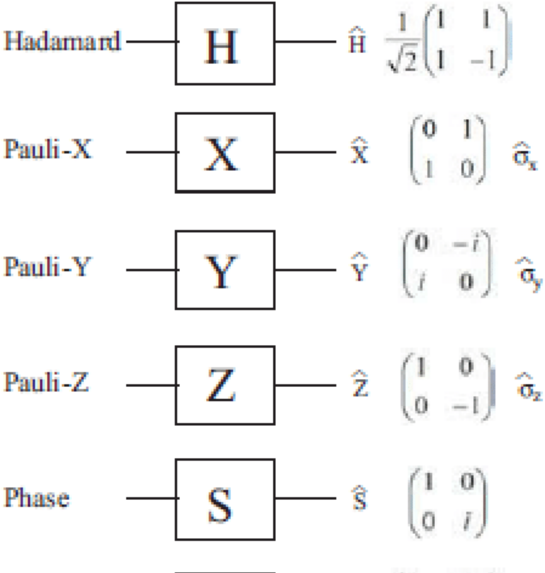
<table>
<tr><th>Bit 1</th><th>Bit 2</th><th>&</th><th>|</th><th>^</th></tr>
<tr><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

bit	~
1	0
0	1

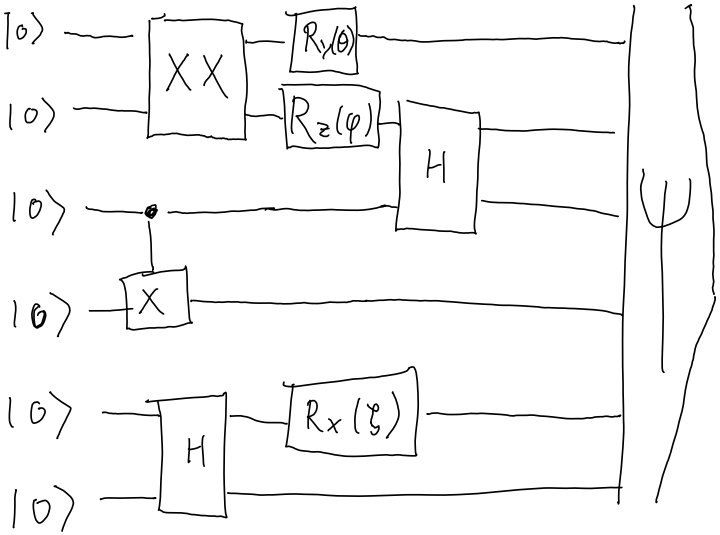
Classical computer programs



Operations on qubits

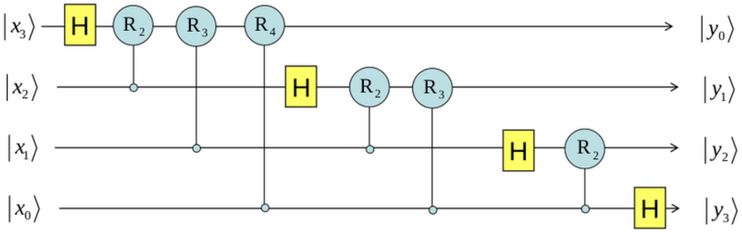


Gated quantum computers



Quantum fourier transform

Quantum Fourier Transform

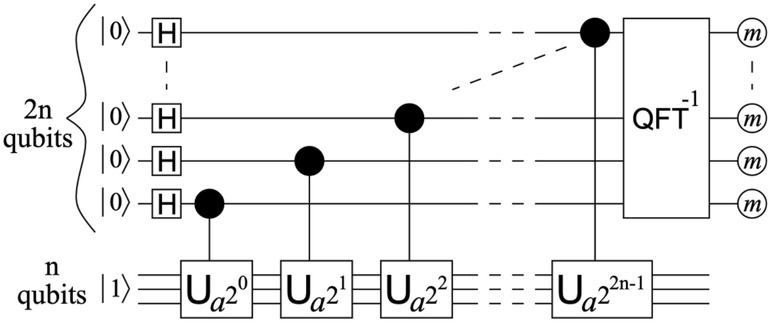


$$|x\rangle \rightarrow \frac{1}{\sqrt{2^n}} \sum_y e^{\frac{2\pi i}{2^n}xy} |y\rangle$$

Uniform family of networks

n Hadamard gates and n(n-1)/2 phase shifts, the size of the network = n(n+1)/2

Shor's algorithm



Variational algorithms

VQE

Remember: Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

H: energy operator. so:

$$\langle\psi|H|\psi\rangle$$

measures energy of a state.

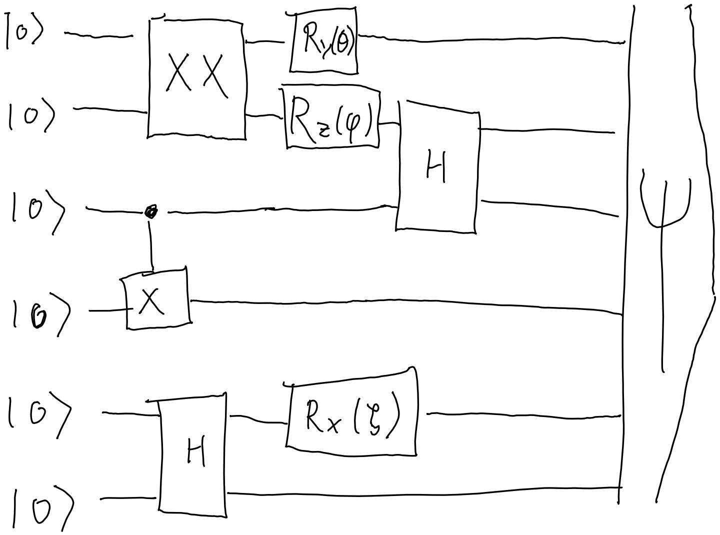
VQE - optimization

Look for a state with a low energy:

$$\min_{\theta} \langle \phi(\theta) | H | \phi(\theta) \rangle$$

Done with gradient descent.

Difficulties in gated QC



Running code on a QPU

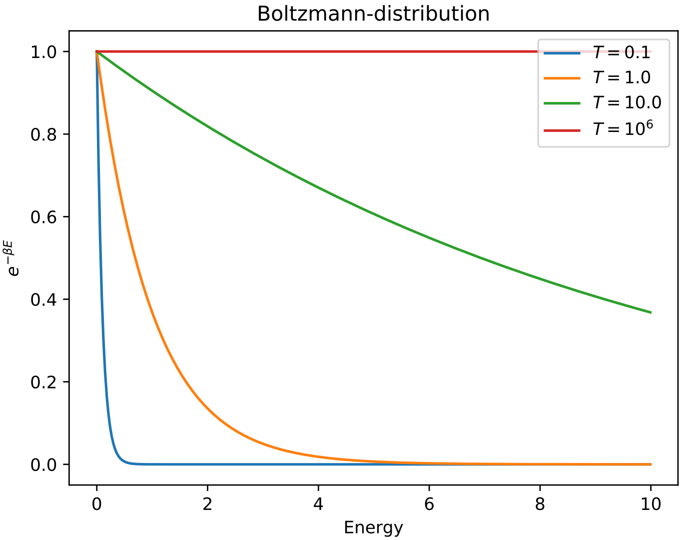
Nobel prize in Physics, 2024

Boltzmann distribution

$$p(s) = e^{-\frac{1}{T}E(s)} / \sum_{s'} e^{-\beta \frac{1}{T} E(s')}$$

where $E(s)$ is the energy function.

Boltzmann disribution



Connected BM