SQUARE ROOT OF A COMLLEX NUMBER

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November 20, 2024

To compute the square root of a complex number $a + bi$:

1. Express the square root: Assume $\sqrt{a+bi} = x + yi$, where x and y are real numbers.
2. Square both sides: $(x + yi)^2 = a + bi$.
3. Expand the square: $x^2 + 2xyi - y^2 = a + bi$.
4. Separate real and imaginary parts:
– Real part: $x^2 - y^2 = a$ (Equation 1)
– Imaginary part: $2xy = b$ (Equation 2)

From Equation 2, solve for y:

$$y = \frac{b}{2x}$$

(Equation 3)
Substitute Equation 3 into Equation 1:

$$x^2 - \left(\frac{b}{2x}\right)^2 = a$$

$$x^2 - \frac{b^2}{4x^2} = a$$

$$4x^4 - 4ax^2 - b^2 = 0$$

This is a quadratic equation in x^2:

$$x^2 = \frac{a \pm \sqrt{a^2 + b^2}}{2}$$

(Equation 4)
Since a and b are real numbers, $a^2 + b^2 \geq 0$ (i.e., positive), and $\sqrt{a^2 + b^2} > a$ (since $b \neq 0$):

$$x^2 = \frac{a + \sqrt{a^2 + b^2}}{2}$$

Let $k = \frac{a+\sqrt{a^2+b^2}}{2}$. Since $k \geq 0$, $\sqrt{k} \in \mathbb{R}$:

$$x = \pm\sqrt{k}$$

Substitute $x = \pm\sqrt{k}$ into Equation 3:

$$y = \pm\frac{b}{2\sqrt{k}}$$

Therefore, the two square roots of the complex number $a + bi$ are $\pm(x + yi)$.

Since a complex number only has two square roots, the roots found by using $x^2 = \frac{a-\sqrt{a^2+b^2}}{2}$ will also be identical.