

Report Jan 2020 – Attempts at parameterising an empirical AUF

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ABSTRACT

Given the depths that LSST will reach, we possibly require a different approach than previously undertaken to compute the perturbation component of the Astrometric Uncertainty Function (AUF) for probabilistic photometric catalogue cross-matching. We document here an attempt to derive this AUF – along with any other non-statistical AUF component, such as proper motions or binary reflex motion – from the data themselves, creating in-situ AUFs which do not rely on simulation.

1 INTRODUCTION

The Astrometric Uncertainty Function (AUF) is the probability density function (PDF) describing the probability of a source being detected at some position given its true location. The probability of two sources being the same intrinsic object detected twice given their respective locations – or the distance between their given positions – is the convolution of their respective AUFs. Each AUF is, itself, a convolution of the individual components describing its offset from its “true” location. The first of these, and typically the only component used, is its statistical component, in which Poisson noise in a photometric image causes the centroiding of a source imperfectly.

A second critical component in crowded regions, or in photometric surveys which probe significant depths relative to the length scale of the survey PSF (of which LSST and *WISE* are both subject), is the perturbation from blended sources. These objects, while hidden beneath the flux from a brighter source, contaminant the central object by both adding additional flux, and by tugging on the center-of-light of the composite object. Thus, in this simple two-component model, the AUF is

$$h(\mathbf{x}) = (f_{\text{stat}} * f_{\text{blend}})(\mathbf{x}) \quad (1)$$

where the notation $(f * g)(x)$ denotes a convolution.

Previously, [Wilson & Naylor \(2018\)](#) produced f_{blend} by a Monte Carlo simulation, via Galactic stellar densities from a TRILEGAL simulation ([Girardi et al. 2005](#)). However, the limiting magnitude of these simulations – via the public API – is 32nd magnitude; this was fine for *WISE* where we still had access to stars 10 magnitudes fainter than the limit of the survey (necessary for probing the faintest flux ratios and sufficiently small flux weighted astrometric perturbations), being brighter than $W1 = 32$. However, the limiting magnitude of LSST is about 27th magnitude, and thus the magnitude limit is out of the range of the TRILEGAL simulations.

2 IN SITU AUF

Thus the need for deriving empirical AUFs was born. Instead of modelling the perturbations from each component individually –

simulating blended source position shifts, unknown proper motions, etc. – we could extract from the data themselves the AUF describing *all* components, minus the statistical noise component, and use the data-driven AUF subsequently.

The idea here was that we can reverse the equation above, and compute f_{blend} directly from knowing the prescription for the statistical AUF and the total PDF describing the probability of two detections being the same source detected twice given their separation. To this end we matched the *WISE* data, thought to be well understood and accurately described by its quoted uncertainties, to *Gaia* data, again using the high precision on the *Gaia* dataset to effectively reduce the convolution of the two datasets to that of just the *WISE* data, allowing for a one-sided probe of the test.

We are therefore looking for the deconvolution of f_{stat} and f_{blend} , for which we can turn to Fourier transforms. Where $\mathcal{F}[f(x)](\rho)$ denotes the Fourier transform of f , we have

$$h = f * g \quad (2)$$

and, if we take the Fourier transform,

$$H = F \times G \quad (3)$$

where in Fourier space the convolution is denoted by a simple multiplication. Thus we can solve for G – or f_{blend} – as

$$G = H/F \quad (4)$$

and inverse Fourier transform, giving

$$f_{\text{blend}}(\mathbf{x}) = \mathcal{F}^{-1} [\mathcal{F}[h(\mathbf{x})](\rho) / \mathcal{F}[f_{\text{stat}}(\mathbf{x})](\rho)](\mathbf{x}). \quad (5)$$

2.1 Computing the Fourier Transforms

To reduce the computational expense of a two-dimensional convolution by simplifying it to a 1D integral, we use the Hankel transform,

$$\mathcal{F}[f(r, \theta)](\rho) = \mathcal{F}[f(r)](\rho) = 2\pi \int_0^\infty r f(r) J_0(2\pi r \rho) dr \quad (6)$$

where we assume circular symmetry (i.e., the Hankel transform, or Fourier-Bessel transform, is a two-dimensional Fourier transform under the assumption of a radially symmetric integral kernel) and J_0 is the zeroth order Bessel function of first kind. This is computed via a series of small Δr bins with each function evaluated at small r (or ρ for an inverse transform) offsets, for a given ρ (or r , for inverse transforms).

There is, however, a small instability in this function computed numerically — no matter how finely you evaluate the functions, J_0 , effectively a combination of a $1/\sqrt{x}$ function and a $\cos(x)$ function, cannot be reasonably computed with sufficient precision at high ρ in the Fourier transform. At large ρ , the $\cos(x)$ component of J_0 becomes very rapid, cycling completely every $1/r$ steps (remember it is $J_0(2\pi r\rho)$ and thus goes as $\cos(2\pi r\rho)$). While this numerical instability is acceptable for a *convolution* (being of order 10^{-5}), as a convolution *multiplies* Fourier transforms, and thus our large scale Fourier transform values, which should be effectively zero, are truncated at 10^{-10} , and thus do not contribute much when inverse Fourier transformed. Unfortunately, for a *deconvolution*, the functions are *divided*, resulting in an inverse Fourier transform which, for large ρ , has a constant *but significant* value – tending to of order unity. We therefore cannot compute the deconvolution numerically.

2.2 Mixture of Gaussian Model

REFERENCES

- Girardi L., Groenewegen M. A. T., Hatziminaoglou E., da Costa L., 2005, *A&A*, 436, 895
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