Master in Computer Vision
Barcelona

Module: Optimization and Inference Techniques in CV
Project: Segmentation

Vadim Fedorov, J.F. Garamendi

Image Segmentation

To decompose an image using active contours without edges The student will learn:

- Level sets.
- Mumford-Shah Energy Functional.
- Active Contours without Edges segmentation.
- The relationship between the Mumford-Shah Model and the ROF Denoising Model (Optional).
- Anisotropic Mumford Shah Energy Functional (Optional).
- The relationship between the Anisotropic Mumford-Shah Model and the ROF Denoising Model (Optional).
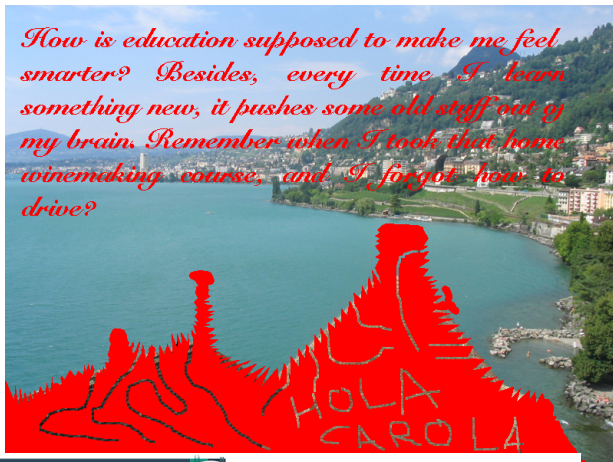
Goal

GOAL

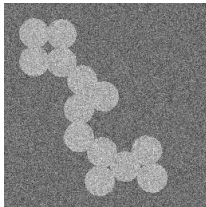To smooth the boundary of the region to inapaint on the roof.

How

Using the Chan-Vese segmentation model.

Segmentation

Mumford and Shah Model

Let $f \in L^{\infty}(\Omega)$ be the given image.

D. Mumford and J. Shah

Optimal approximations by piecewise smooth functions and associated variational problems.

Commun. Pure Appl. Math. 42 (1989) 577-685

Segmentation
Mumford and Shah Model

Let $f \in L^\infty(\Omega)$ be the given image. We search for a pair (u, Γ) where $\Gamma \subset \Omega$ is the set of discontinuities.

$$J_{ms}(u, \Gamma) = \mu \int_{\Omega \backslash \Gamma} |\nabla u|^2 dx + \nu \text{length}(\Gamma) + \int_\Omega |u - f|^2 dx$$

where μ and ν are nonnegative constants and length(Γ) is defined as $\int_\Gamma d\sigma$
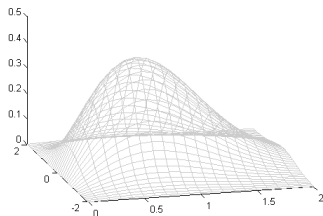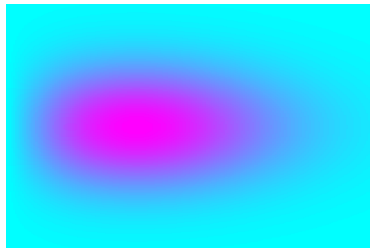
D. Mumford and J. Shah
Optimal approximations by piecewise smooth functions and associated variational problems.
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Coarea formula

Suppose $u \in BV(\Omega)$, then

$$\int_\Omega |Du| = \int_{-\infty}^{\infty} \text{Per}(E_s)\,ds \qquad \text{where} \quad E_s = \{(x,y) \in \Omega \mid u > s\}$$
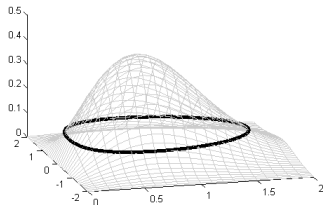

For smooth images, the co-area formula shows that $TV[u]$ is to sum up the lenghts of all level curves (level sets) weighted by the Lebesgue element ds.

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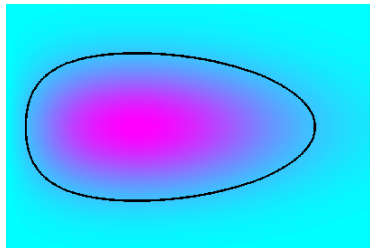

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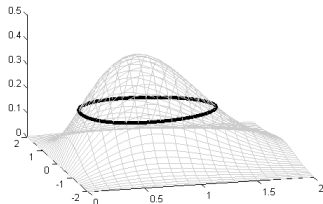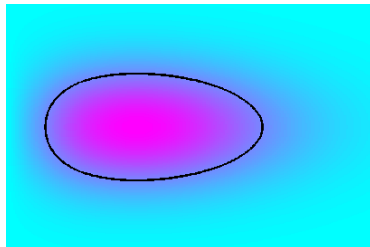

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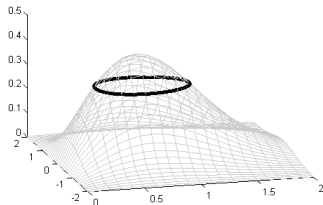

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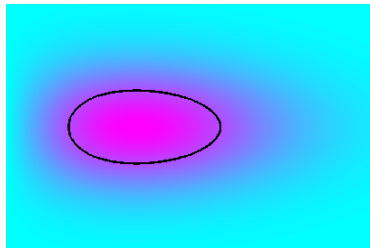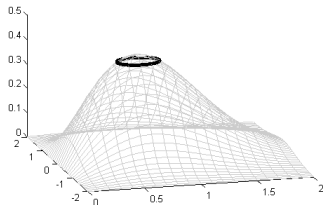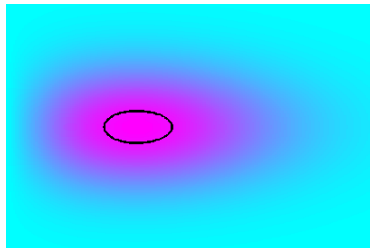

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Level Sets and Chan-Vese Model

📑 T.F.Chan L.Vese

Active Contours without edges

IEEE Trans. Image Process. 10 (2) (2001)
266-277

Level Set Dictionary

$$\Gamma \equiv \{\phi = 0\} \qquad \text{length}(\Gamma) = \int_{\Omega} |DH(\phi)|$$

$$H(\phi) = \left\{ \begin{array}{ll} 1 & \text{if} \quad \phi \geq 0 \\ 0 & \text{if} \quad \phi < 0 \end{array} \right.$$

Level Sets and Chan-Vese Model

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$$J_{cv}(\bar{c}, E) = \text{Per}(E) + \frac{1}{2\lambda} \int_{E} |c_1 - f|^2 dx + \frac{1}{2\lambda} \int_{CE} |c_2 - f|^2 dx$$

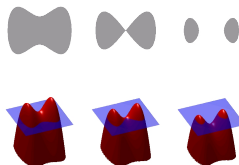# Level Sets and Chan-Vese Model

📄 T.F.Chan L.Vese
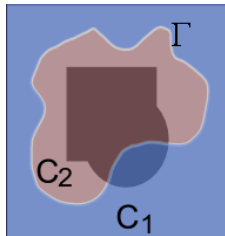*Active Contours without edges*
IEEE Trans. Image Process. 10 (2) (2001)
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Level Set Dictionary

$$\Gamma \equiv \{\phi = 0\} \qquad \text{length}(\Gamma) = \int_{\Omega} |DH(\phi)|$$

$$H(\phi) = \begin{cases} 1 & \text{if} & \phi \geq 0 \\ 0 & \text{if} & \phi < 0 \end{cases}$$
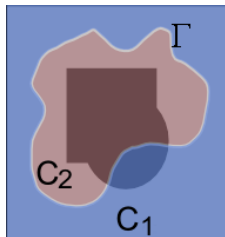
$$J_{cv}(\bar{c}, \phi) = \int_{\Omega} |DH(\phi)| + \frac{1}{2\lambda_{cv}} \int_{\Omega} |c_2 - f|^2 H(\phi) dx + \frac{1}{2\lambda_{cv}} \int_{\Omega} |c_1 - f|^2 (1 - H(\phi)) dx$$

$$c_1 = \frac{\int_{\Omega} f(1 - H(\phi)) \, dx}{\int_{\Omega} 1 - H(\phi) dx} \qquad c_2 = \frac{\int_{\Omega} f H(\phi) dx}{\int_{\Omega} H(\phi) dx}$$

Active Contours without Edges: Chan-Vese Model

Active Contours Without Edges

Tony F. Chan, *Member, IEEE,* and Luminita A. Vese

$$F(c_1, c_2, C) = \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(inside(C))$$
$$+ \lambda_1 \int_{inside(C)} |u_0(x, y) - c_1|^2 \, dx \, dy$$
$$+ \lambda_2 \int_{outside(C)} |u_0(x, y) - c_2|^2 \, dx \, dy,$$

where $\mu \geq 0, \nu \geq 0, \lambda_1, \lambda_2 > 0$ are fixed parameters. In almost all our numerical calculations (see further), we fix $\lambda_1 = \lambda_2 = 1$ and $\nu = 0$.

Therefore, we consider the minimization problem:

$$\inf_{c_1, c_2, C} F(c_1, c_2, C).$$

Active Contours without Edges: Chan-Vese Model

Level Set Dictionary

$$\text{Length}\{\phi = 0\} = \int_{\Omega} |\nabla H(\phi(x, y))| \, dx \, dy$$

$$= \int_{\Omega} \delta_0(\phi(x, y)) |\nabla \phi(x, y)| \, dx \, dy,$$

$$\text{Area}\{\phi \geq 0\} = \int_{\Omega} H(\phi(x, y)) \, dx \, dy,$$

Active Contours without Edges: Chan-Vese Model

Level Set Dictionary

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$$\text{Area}\{\phi \geq 0\} = \int_{\Omega} H(\phi(x, y)) \, dx \, dy,$$

$$\int_{\phi > 0} |u_0(x, y) - c_1|^2 \, dx \, dy$$
$$= \int_{\Omega} |u_0(x, y) - c_1|^2 \, H(\phi(x, y)) \, dx \, dy,$$
$$\int_{\phi < 0} |u_0(x, y) - c_2|^2 \, dx \, dy$$
$$= \int_{\Omega} |u_0(x, y) - c_2|^2 \, (1 - H(\phi(x, y))) \, dx \, dy.$$

Then, the energy $F(c_1, c_2, \phi)$ can be written as

$$F(c_1, c_2, \phi)$$
$$= \mu \int_{\Omega} \delta(\phi(x, y))|\nabla \phi(x, y)| \, dx \, dy$$
$$+ \nu \int_{\Omega} H(\phi(x, y)) \, dx \, dy$$
$$+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 \, H(\phi(x, y)) \, dx \, dy$$
$$+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 \, (1 - H(\phi(x, y))) \, dx \, dy.$$

Active Contours without Edges: Chan-Vese Model

Gradient Descent

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi) \left[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 (u_0 - c_1)^2 \right.$$

$$\left. + \lambda_2 (u_0 - c_2)^2 \right] = 0 \text{ in } (0, \infty) \times \Omega,$$

$$\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega,$$

$$\frac{\delta_\varepsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial \vec{n}} = 0 \text{ on } \partial\Omega$$

Active Contours without Edges: Chan-Vese Model

Discretization

$$\frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\Delta t}$$

$$= \delta_h(\phi_{i,j}^n)\left[\frac{\mu}{h^2}\Delta_-^x\right.$$

Gradient Descent

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu \operatorname{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \nu - \lambda_1(u_0 - c_1)^2\right.$$

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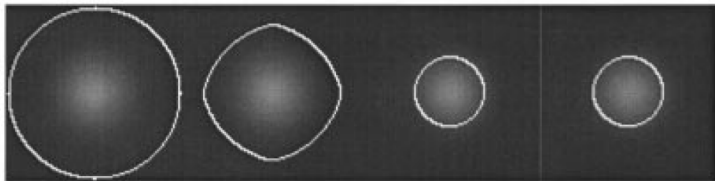
$$\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega,$$

$$\frac{\delta_\varepsilon(\phi)}{|\nabla\phi|}\frac{\partial\phi}{\partial\vec{n}} = 0 \text{ on } \partial\Omega$$

$$\cdot\left(\frac{\Delta_+^x\phi_{i,j}^{n+1}}{\sqrt{(\Delta_+^x\phi_{i,j}^n)^2/(h^2) + (\phi_{i,j+1}^n - \phi_{i,j-1}^n)^2/(2h)^2}}\right)$$

$$+ \frac{\mu}{h^2}\Delta_-^y$$

$$\cdot\left(\frac{\Delta_+^y\phi_{i,j}^{n+1}}{\sqrt{(\phi_{i+1,j}^n - \phi_{i-1,j}^n)^2/(2h)^2 + (\Delta_+^y\phi_{i,j}^n)^2/(h^2)}}\right)$$

$$- \nu - \lambda_1(u_{0,i,j} - c_1(\phi^n))^2 + \lambda_2(u_{0,i,j} - c_2(\phi^n))^2\right],$$

where
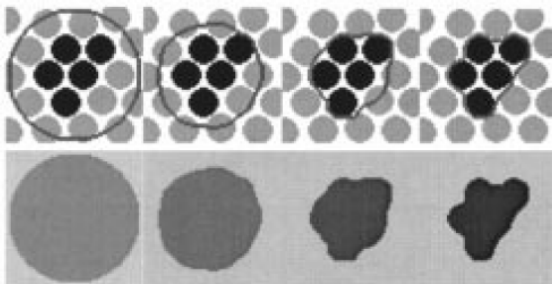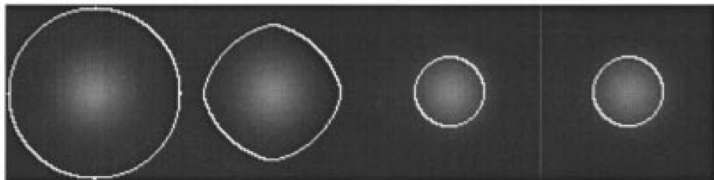$$\Delta_-^x\phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}, \quad \Delta_+^x\phi_{i,j} = \phi_{i+1,j} - \phi_{i,j},$$

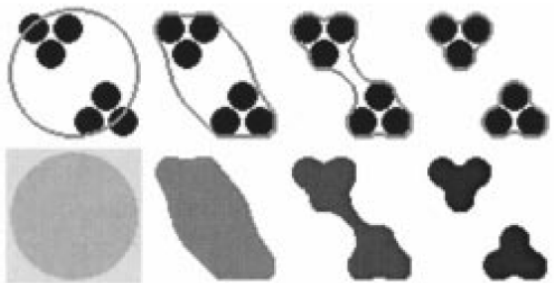$$\Delta_-^y\phi_{i,j} = \phi_{i,j} - \phi_{i,j-1}, \quad \Delta_+^y\phi_{i,j} = \phi_{i,j+1} - \phi_{i,j}.$$

Active Contours without Edges: Results

Active Contours without Edges: Results

Active Contours without Edges: Results

Deliverable: Mandatory

Mandatory means if there any point that it is not done, then the weekly task will FAIL.

- ▶ Implement the Chan-Vese algorithm and test it with the given images.
- ▶ Video showing the curve evolution.
- ▶ To discover under which assumptions Chan-Vese works and when it does not.

Maximum: 9 points. The evaluation will depend on the document and the code. **Deliver Wed. Oct. 26th. at 09:00 hours.**

Hints: Creating the initial function ϕ

```
[ni, nj]=size(I);
[X, Y]=meshgrid(1:ni, 1:nj);
modelParams.initialPhi=(-sqrt( ( X-round(ni/2)).^2 +
(Y-round(nj/2)).^2)+50)*h;
```

Deliverable: Optional

- ► O1: Test with your own real images. Up to $+1$ point.
- ► 02: Implement the Chambolle's convexification for the Chan-Vese energy functional. Up to $+5$ points.
- ► O3: Implement the Convexification for the Anisotropic Mumford-Shah proposed in "A Fast Anisotropic Mumford-Shah Functional Based. Segmentation". Up to $+5$ points.
- ► O4: Implement the minimization of a functional that works for ALL given images. Up to $+5$ points.
- ► O5: Solve the inpainting problem for the Tampa and GR images. Up to $+1$ points.
- ► O6: Implement the Coupled Pairwise scheme proposed in "Box Relaxation Schemes in Staggered Discretizations for the Dual Formulation of Total Variation Minimization". Up to $+5$ points.

Deliverable of mandatory and O1. Wed. OCt. 26th. 9:00h