A Variational Framework for Active and Adaptative Segmentation of Vector Valued Images

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Abstract

During the last few years, many efforts have been done in integrating different informations in a variational framework to segment images. Recent works on curve propagation were able to incorporate stochastic informations [9, 6] and prior knowledge on shapes [3, 7]. The information inserted in these studies is most of the time extracted offline. Meanwhile, other approaches have proposed to extract region information during the segmentation process itself [2, 4, 8].

Following these new approaches and extending the work in [6] to vector-valued images, we propose in this paper an entirely variational framework to approach the segmentation problem. Both, the image partition and the statistical parameters for each region are unkown.

After a brief reminder on recent segmenting methods, we will present a variational formulation obtained from a bayesian model. After that, we will show two different differentiations driving to the same evolution equations. Detailed studies on gray and color images of the 2-phase case will follow. We will finish on an application to tracking which shows benefits of our dynamical framework.

1 Introduction

Considering the last studies in the domain, a variational formulation seems adequate to tackle the problem of segmenting images. In particular, this approach has shown its ability to integrate various cues. Hence, different kind of informations can be used as direct constraint on the boundaries (regularity, tension, high gradient, shape prior) or indirect constraint induced by modules accounting for region integrity (homogeneity, texture classification, prior on spatial intensity distribution,...).

In most of the cases, a preprocessing step is necessary to extract the relevant information from the image before the partitioning process itself. It can be creation of an edge image to extract boundary informations or data clustering for a simple region module.

In this paper, no prior extraction of information is made. An entirely variational formulation is proposed, where region cues extraction is made jointly with the partitioning process. Related works can be found. In [2], T. Chan and L. Vese propose to use the mean to distinguish the regions, the mean of the regions being dynamically estimated during the evolution of the curve which delimits the regions. In [4], the criteria used to separate two smooth regions is their entropy, this one depending again on the moving border position. Yezzi *et al.* have also proposed similar approaches. In [8], bimodal images are segmented using Gaussian distributions with fixed variances and adaptative means.

The method proposed in the next section can be seen as a generalization of the *CV model* [2]. Actually, a more general variational formulation is obtained from the maximization of the a posteriori segmentation probability given an observed data. First, limiting our study to the 2-phase case, we will present two different ways of minimizing the functional driving both to the same evolution equations. One use a modified energy by an early introduction of level set functions, whereas the other method consists in differentiating directly the functional using recent results on shape derivation. Next, we will show some experimental results on gray-valued and color images, and also on sequences of images. Then, we will finish by making some comments on the limitations of the method and the possible extensions to an arbitrary number of regions.

2 The segmentation problem

Segmenting images consists of finding a partition of an observed data I into an unknown number of regions. These regions are characterized by statistical properties which represent a visual consistency. The interface between the regions is supposed to be regular.

Let $\Omega \in \Re^2$ be open and bounded, and let $I : \Omega \rightarrow \Re^p$ be the observed data. Let $\mathcal{P}(\Omega)$ be a partition of the domain

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and $\partial\Omega$ be the boundaries between the regions. We make the following assumptions: 1) I is composed by a maximum of N regions Ω_i verifying an hypothesis h_i, 2) the interface between the regions $\partial\Omega$ is regular.

Let $p_{\Theta_i}(I(x))$ be the conditional probability density function of a given value $I(x)$ with respect to the hypothesis h_i. The good segmentation of a given observed data respecting the hypotheses is obtained by solving the optimization problem with respect to the *a posteriori* segmentation probability, given the observation set: $p(P(\Omega)|I)$.

Following the Geodesic Active Regions [6], we can reformulate the problem in terms of energy. Only two assumptions are made in [6]: all the partitions are equally possible and the pixels within each region are independent. Since these assumptions are reasonable in our case, the optimal frame partition is obtained by minimizing the energy:

$$F(\Sigma, \mu, \partial\Omega) = \sum_{i=1}^{N} \int_{\Omega_i} -\log p_{\Theta_i}(I) dx + \text{length}(\partial\Omega) \quad (1)$$

To describe the visual consistency of a region, we need to define a *family of density function*. The choice of this family must be done such that it can represent/approximate the information of each region and it should be able to discriminate two different regions. For smooth non-textured images, a common choice is to use Gaussian distributions. It means that the conditional probability with respect to h_i for a value I is:

$$p_{\theta_i}(I) = p_{\Sigma_i,\mu_i}(I) = \frac{1}{(2\pi)^{\frac{p}{2}} |\Sigma_i|^{1/2}} e^{-\frac{1}{2}(I-\mu_i)^T \Sigma_i^{-1}(I-\mu_i)}$$

Remark : As we will see on some experiments, the use of Gaussian densities may be also efficient on particular textured images. Actually, these images can be segmented by fitting Gaussian models even if these models do not give a good estimation of the region properties.

The energy (1) with gaussian density probability functions has already been studied in [5] by looking for the minimum length description and in [9] with a region completion scheme.

Here, we propose a global variational framework using the level set theory. The energy minimum is obtained via a gradient descent with respect to the statistical parameters and the boundary position.

3 On finding the minima

We study two different approaches driving to a local minima of this functional when considering the 2-phase problem for smooth, non-textured images. The energy corresponding to this particular case is:

$$F(\partial\Omega, \Theta) = \int_{\Omega_1} e_1(x)\, \mathrm{d}x + \int_{\Omega_2} e_2(x)\, \mathrm{d}x + \text{length}(\partial\Omega) \quad (2)$$

where $\Theta = (\Sigma_1, \mu_1, \Sigma_2, \mu_2)$
and $e_i(x) = \log|\Sigma_i| + (I(x) - \mu_i)^T \Sigma_i^{-1}(I(x) - \mu_i)$.
Remark : Active contours without edges for vector-valued images presented in [2] can be seen as a particular case of this formulation where Σ_1 and Σ_2 are set to the identity matrix.

3.1 First approach: extension of the integral terms to all the domain

A difficulty encountered when we want to derive (2) is the dependence with respect to the border position of the integration domains Ω_1 and Ω_2. As it has been done in the past [2], we extend these integrals to all the domain by using the level set function $\phi : \Omega \rightarrow \Re$ defined as:

$$\begin{cases} \phi(x) = \mathcal{D}(x, \partial\Omega), & \text{if } x \in \Omega_1 \\ \phi(x) = -\mathcal{D}(x, \partial\Omega), & \text{if } x \in \Omega_2 \end{cases} \quad (3)$$

Using the same regularized form $H_\epsilon(z)$ of the heaviside function as in [2], the functional (2) can be written as:

$$E(\phi, \Theta) = \int_{\Omega} \left(e_1(x) H_\epsilon(\phi) + e_2(x)(1 - H_\epsilon(\phi))\right) \mathrm{d}x + \text{length}(\partial\Omega) \quad (4)$$

The length term can also be expressed with respect to ϕ:

$$\text{length}(\partial\Omega) = \int_{\Omega} |\nabla H_\epsilon(\phi(x))|\, \mathrm{d}x$$

The Euler-Lagrange equations obtained for the Gaussian paramaters μ_i and Σ_i can be directly solved. The solution gives expressions of μ_i and Σ_i with repect to the level set function ϕ:

$$\begin{cases} \mu_i(\phi) = \frac{\int_\Omega I(x)\chi_i(\phi(x))dx}{\int_\Omega \chi_i(\phi(x))dx} \\ \Sigma_i(\phi) = \frac{\int_\Omega (\mu_i - I(x))(\mu_i - I(x))^T \chi_i(\phi(x))dx}{\int_\Omega \chi_i(\phi(x))dx} \end{cases} \quad (5)$$

where $\chi_1(\phi) = H_\epsilon(\phi)$ and $\chi_2(\phi) = 1 - H_\epsilon(\phi)$ (see annex for details on the derivation). These expressions are the estimation of the gaussian parameters into the respective region which is coherent with the formulation of the problem.
Hence the energy depends only on ϕ :

$$E(\phi, \Theta) = E(\phi, \Theta(\phi)) = G(\phi)$$

Then, it is possible to compute the virst variations of G with respect to ϕ. A detailed description of the derivation can be found in annex. Using the result of the annex, we can write the following evolution equation for ϕ:

$$\phi_t(x) = \delta_\epsilon(\phi(x)) \left[\nu.\text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + e_2(x) - e_1(x)\right] \quad (6)$$

with $\delta_\epsilon(\phi) = H'_\epsilon(\phi)$.
The implementation is straightforward: **evolve ϕ using the equation (6) while the Gaussian parameters are updated at each iteration with resect to (5)**.

3.2 Second approach: direct derivation using shape derivation principle

We can wonder how the use of a regularized function in the energy modify our objective function. Using the shape derivative tool introduced in [1], it is possible to differentiate directly the functional (2).
We suppose that the statistical parameters are obtained using an estimation on the respective region as in (5).
Then the functional depends only on the border position $\partial\Omega$:

$$E(\partial\Omega) = \int_{\Omega_1} e_1(x, \Omega_1)dx + \int_{\Omega_2} e_2(x, \Omega_2)dx + \nu \int_{\partial\Omega} \partial\dot{\Omega}(x)dx \quad (7)$$

The integrals $D(\Omega_i) = \int_{\Omega_i} e_i(x, \Omega_i)dx$ may be differenciated with respect to the position of Ω_i using shape derivative. Concerning our case, the derivative gives a simple term (see annex):

$$< D'(\Omega_i), V >= +/- \int_{\partial\Omega} e_i(x, \Omega_i)(V(x).N(x))da(x)$$

where $N(x)$ is the unit normal vector to $\partial\Omega$ at point x. The sign depends on the direction of N, i.e. if Ω_i is the inside region, the sign is minus otherwise it is positive. Now we can write the new evolving equation:

$$\partial\Omega(x)_t = (e_2(x) - e_1(x) + \nu\kappa)\mathbf{N}$$

where κ is the curvature of the curve ($\partial\Omega$).
It is common to use the level set theory to implement this equation. The level set function is defined as the distance function like in (3) and we get the following level set evolution equation:

$$\phi_t(x) = \left[e_2(x) - e_1(x) + \nu \text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\right]|\nabla\phi| \quad (8)$$

and the equation (6) is obtained: 1) by approximating $|\nabla\phi|$ by 1 since ϕ should be the distance function; 2) by introducing $\delta_\alpha(x)$ to impose the Narrow Band hypothesis: only local pixels have influence on the curve propagation.

4 Experiments

The complexity of the variational method presented in the last section is high. The unkown parameters for region information can be up to 9 per region for color images (6 for the covariance matrix and 3 more for the mean).

4.1 Gray-valued images

For scalar images, the complexity is smaller. Only two parameters (mean and variance) are necessary to represent a region. The simplified evolving equation for this case is:

$$\phi_t(x) = \delta_\epsilon(\phi(x))\Big(\nu.\text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + \log\frac{\sigma_2^2}{\sigma_1^2} - \frac{(I(x)-\mu_1)^2}{\sigma_1^2} + \frac{(I(x)-\mu_2)^2}{\sigma_2^2}\Big)$$

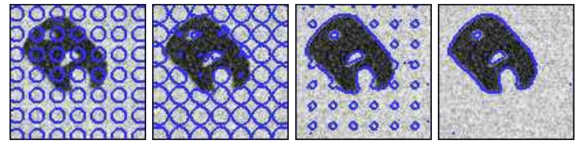


Figure 1 : Two regions with different means - Contour evolution

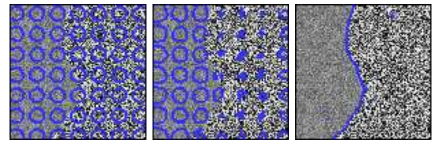
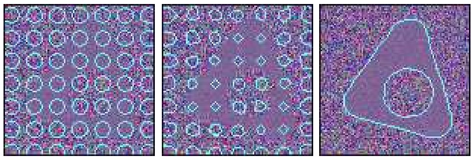


Figure 2 : Two regions with different variances - Contour evolution

We show results on two synthetical images. The first one is composed by two regions with different means but same variances [Fig.1] while the second has two regions different only by their variances [Fig.2].

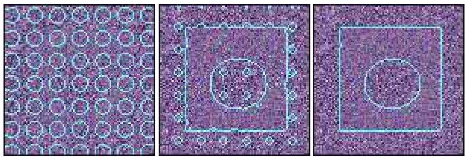
4.2 Color images

Concerning color images, the complexity is higher but images composed by complex regions can be considered. First, we show results on synthetical data where 3-dimensional gaussian models were used to generate data for each region. In both examples, the regions have the same mean but their covariance matrix is different. For the first example, auto-correlation coefficients (between color components) are different while only cross-correlation coefficients are different for the second example. Contrary to more classical approaches as the one presented in [2], our method is able to capture these differences [Fig.3].
Experiments on real images gave interesting results. Let us precise that we used the CIE-Lab color space such that distance between 3D points corresponds to perceptual color difference.
The first test consists in segmenting a hand photography placed on different backgrounds. Three backgrounds are considered: uniform (hue close to the hand's) [Fig.4(a)], noisy [Fig.4(b)] and textured [Fig.4(c)]. The algorithm succeeded in giving the expected partitionning for each test. Two more tests on *natural* images are shown. First, the "squirrel" image: the regions are textured but they have a really different color. The partionning result looks very accurate [Fig.8]. Second example, the "Rocks on mars" image:



$$\mu_1 = \mu_2$$

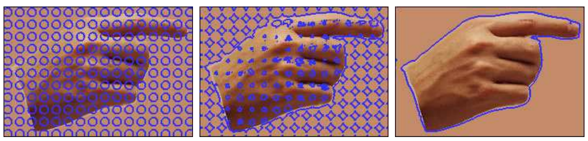
$$\Sigma_1 = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \beta_1 & 0 \\ 0 & 0 & \gamma_1 \end{pmatrix} \text{ and } \Sigma_1 = \begin{pmatrix} \alpha & a_1 & b_1 \\ b_1 & \beta & c_1 \\ c_1 & b_1 & \gamma \end{pmatrix}$$



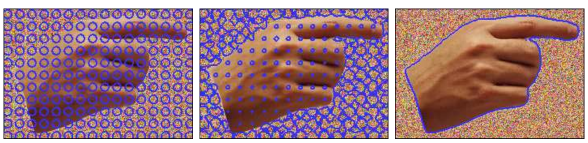
$$\mu_1 = \mu_2$$

$$\Sigma_2 = \begin{pmatrix} \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \gamma_2 \end{pmatrix} \text{ and } \Sigma_2 = \begin{pmatrix} \alpha & a_2 & b_2 \\ b_2 & \beta & c_2 \\ c_2 & b_2 & \gamma \end{pmatrix}$$

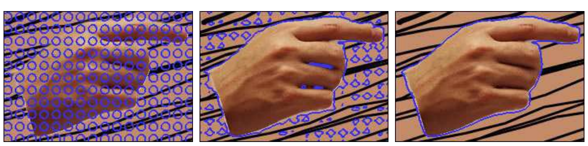
Figure 3 : Synthetic test for color images - Contour evolution



(a) Uniform background - Contour evolution



(b) Noisy background - Contour evolution



(c) Textured background - Contour evolution

Figure 4 : "Hand"image (left: initial contour, center: evolving contour, right: final segmentation)

the rock is slightly darker than the sand and it has a different texture. The obtained result is still satisfactory [Fig.6].

4.3 Color image sequences

The dynamical property of our method can be usefull to segment a sequence of images. The hand examples presented here is composed by a moving object (a hand) placed in front of a textured background. Since the hand motion is composed by various rotations and translations, the luminosity of the object varies very much during the sequence. The result of the previous frame is used as initialization for the next frame. The method does not integrate additional terms based on motion properties as optical flow. Hence, the method is not limited to small displacements. As you can see on the example, the algorithm can naturaly integrate luminosity modifications thanks to the dynamical Gaussian parameters. We can also remark that a Gaussian approximation for the backround is not good at all but it is sufficient to differanciate it from the hand [Fig.7].

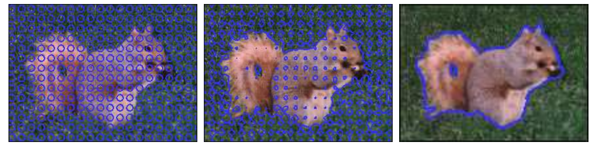


Figure 5 : "Squirell"image - Contour evolution

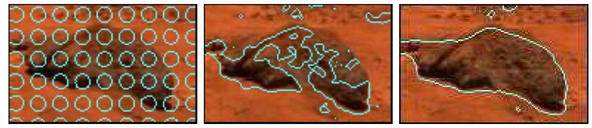


Figure 6 : "Rocks on mars" image - Contour evolution

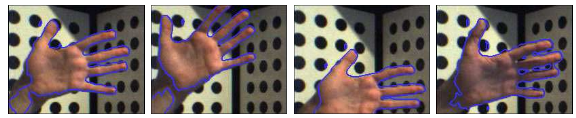


Figure 7 : Hand sequence

4.4 Implementation remarks

For all the tests, except the sequence, the same initialization with tiny circles was used. Such an initialization has been already used in the past. The reasons why we choosed this initialization was its ability to detect easily holes and the convergence speed. However, other initializations have been tested such as a single circle. It still gives good results but the convergence speed decreases dramatically. Moreover, we have to set only two parameters: the time step and the regularization weight ν. For all the color examples, we used the same values: $dt = 1$ and $\nu = 1$.
Concerning the speed, we use an explicit discretization in time, then the evolution must be done slowly to be stable. Nevertheless less than 30 iterations are necessary in most of our examples. Then, on 100x100 images, less than five seconds are required for scalar images while color images need around twenty seconds on a 1 Ghz CPU. To deal with

bigger images, we are in the process of developping a multi-scale approach.

5 Generalization to N regions

We start from the functional (1) and we would like to extend each integral over Ω_i to all the image domain Ω. We must find characteristic functions $\{\chi_i, i = 1, .., N\}$ such that:

$$\begin{cases} \phi_i(x) > 0 & \text{if x} \in \Omega_i \\ \phi_i(x) = 0 & \text{if x} \in \partial\Omega_i \\ \phi_i(x) < 0 & \text{otherwise} \end{cases}$$

Actually, two different kind of characteristic functions have been proposed for this problem. One can simply associate one level set per region but pixels can be associated to multiple regions or to no region. An additional coupling term must be added to avoid that. A second way of defining χ_i has been propose by Chan and Vese by associating a characteristic function to each combinatation of level set signs. Hence only $\log(N)$ level sets are needed to segment an image into N regions and each pixel is associated to one and only one region. For example, an image can be segmented in 4 regions using 2 level sets ϕ_1 and ϕ_2 by minimizing the following functional:

$$E(\phi_1, \phi_2, \Theta) = \sum_{i=1}^{4} \int_\Omega -\log p_{\theta_i}(I(x))\chi_i(\phi_1, \phi_2)dx + \text{length}(\partial\Omega_1 \cup \partial\Omega_2) \tag{9}$$

$$\text{with} \begin{cases} \chi_1(\phi_1, \phi_2) = H_\epsilon(\phi_1)H_\epsilon(\phi_2), \\ \chi_2(\phi_1, \phi_2) = (1 - H_\epsilon(\phi_1))H_\epsilon(\phi_2), \\ \chi_3(\phi_1, \phi_2) = H_\epsilon(\phi_1)(1 - H_\epsilon(\phi_2)), \\ \chi_4(\phi_1, \phi_2) = (1 - H_\epsilon(\phi_1))(1 - H_\epsilon(\phi_2)) \end{cases}$$

The derivation of (9) is similar to the one for the 2-phase case and two evolution equations are obtained for ϕ_1 and ϕ_2. A nice example is shown [Fig.8] but the method becomes more sensible to the initial condition for more complicated images. Moreover, the complexity becomes very high when the number of regions increase. The exact number of regions has not to be known (the segmentation can give several regions with very close statistical parameters which can be merged). But a close upper bound of this number is good for speed efficiency.

6 Conclusion and future works

We have presented a totally dynamical framework to segment scalar or vector-valued images. Two different approaches have been considered to tackle the problem of energy minimization, both drives to the same evolution equations. Convincing results were shown on synthetical and real images. The method looks particularly powerful in segmenting various type of images. A simple multi-dimensional Gaussian to represent each region is often sufficient to discriminate complex regions. Regarding the tracking example, improvements can be done by using for instance priors on the shape or by integrating a more complex model for the backgroung and a skin model for the hand. Our future studies will concentrate on these last issues to improve the method.

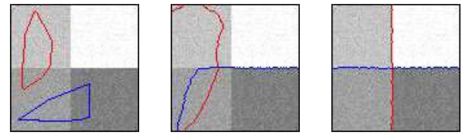


Figure 8 : Four different means - Contour evolution

A Derivation details

A.1 Energy extended over all the domain Ω

To show how the Euler-Lagrange equations are obtained, let us differentiate the following functional:

$$\begin{aligned} F(\phi) &= -\int_\Omega \log p(I(x)|\mu, \Sigma)\chi(\phi)dx \\ &= \frac{1}{2}\int_\Omega (\log|\Sigma| + (I(x) - \mu)^T \Sigma^{-1}(I(x) - \mu))\chi(\phi)dx \end{aligned} \tag{10}$$

where $p(x|\mu, \Sigma)$ is the Gaussian density function with mean μ and covariance matrix Σ (μ and Σ depend on ϕ (5))
We compute the first variation of F :

$$\begin{aligned} \frac{\partial F(\phi + \epsilon\psi)}{\partial \epsilon}\Big|_{\epsilon=0} =& \frac{1}{2}\int_\Omega \log p(I(x)|\mu, \Sigma)\chi'(\phi)dx \\ &+ \frac{1}{2}\int_\Omega \frac{\partial G(x, \phi + \epsilon\psi)}{\partial \epsilon}\Big|_{\epsilon=0}\chi(\phi)dx \end{aligned}$$

where $G(x, \phi + \epsilon\psi) = \log|\Sigma(\phi + \epsilon\psi)| + (I(x) - \mu(\phi + \epsilon\psi))^T\Sigma(\phi + \epsilon\psi)^{-1}(I(x) - \mu(\phi + \epsilon\psi)))$
First we compute the gradient with restpect to the mean vector μ:

$$\frac{\partial G}{\partial \mu} = \Sigma^{-1}(I(x) - \mu)$$

We can compute the gradient with repect to the covariance matrix Σ in the same manner:

$$\frac{\partial}{\partial \Sigma}G = \Sigma^{-1}(\Sigma - (I - \mu)(I - \mu)^T)\Sigma^{-1}$$

The second term of the first variation of F can be simplified:

$$\begin{aligned} &\int_\Omega \frac{\partial G(x, \phi + \epsilon\psi)}{\partial \epsilon}\Big|_{\epsilon=0}\chi(\phi)dx \\ &= \frac{\partial \mu}{\partial \epsilon}\Big|_{\epsilon=0}\int_\Omega \frac{\partial G}{\partial \mu}\chi(\phi)dx \quad + \frac{\partial \Sigma}{\partial \epsilon}\Big|_{\epsilon=0}\int_\Omega \frac{\partial G}{\partial \Sigma}\chi(\phi)dx \end{aligned}$$

and we can compute the two integrals over Ω:

$$\int_\Omega \frac{\partial G}{\partial \mu}\chi(\phi)dx = \int_\Omega \Sigma^{-1}(I(x)-\mu)\chi(\phi)dx$$
$$= \Sigma^{-1}\left(\int_\Omega I(x)\chi(\phi)dx - \mu\int_\Omega \chi(\phi)dx\right) = 0$$

and

$$\int_\Omega \frac{\partial G}{\partial \Sigma}\chi(\phi)\,\mathrm{d}x$$
$$= \int_\Omega \Sigma^{-1}(\Sigma - (I(x)-\mu)(I(x)-\mu)^T)\Sigma^{-1}\chi(\phi)\,\mathrm{d}x$$
$$= \Sigma^{-1}\Big(\Sigma\int_\Omega \chi(\phi)\,\mathrm{d}x - \int_\Omega (I(x)-\mu)(I(x)-\mu)^T\chi(\phi)\,\mathrm{d}x\Big)\Sigma^{-1}$$
$$= 0$$

It follows that the second term is null. The first variation of F can simply be expressed as:

$$\frac{\partial F(\phi+\epsilon\psi)}{\partial\epsilon}\Big|_{\epsilon=0} = \frac{1}{2}\int_\Omega \log p(I(x)|\mu,\Sigma)\chi'(\phi)dx$$

A.2 Shape derivative method

We suppose that we have the following functional depending on the domain Ω:

$$D(\Omega) = \int_\Omega k(x,\Omega)dx$$
$$= \int_\Omega \Big(\log|\Sigma(\Omega)| + \big(I(x)-\mu(\Omega)\big)^T\Sigma(\Omega)^{-1}\big(I(x)-\mu(\Omega)\big)\Big)dx$$

$$\text{with}\begin{cases}\mu(\Omega) = \frac{1}{V(\Omega)}\int_\Omega I(x)dx\\ S(\Omega) = \frac{1}{V(\Omega)}\int_\Omega (I(x)-\mu(\Omega))(I(x)-\mu(\Omega))^T dx\\ V(\Omega) = \int_\Omega dx\end{cases}$$

The Gâteaux derivative of $D(\Omega)$ in the direction of V is:

$$< D'(\Omega), V >= \int_\Omega k_s(x,\Omega,V)dx - \int_\Gamma k(x,\Omega)(V(x).N(x))da(x)$$

where $k_s(x,\Omega,V)$ is the shape derivative of $k(x,\Omega)$.

$$k_s(x,\Omega,V) = f_s(x,\mu(\Omega),\Sigma(\Omega),V)$$
$$= f_\mu < \mu'(\Omega), V > + f_\Sigma < \Sigma'(\Omega), V >$$

Since $\mu(\Omega)$ and $S(\Omega)$ do not depend on x, we have:

$$\int_\Omega f_s(x,\ldots,V)dx = < \mu'(\Omega), V > \int_\Omega f_\mu(x,\ldots)dx + < \Sigma'(\Omega), V > \int_\Omega f_\Sigma(x,\ldots)dx$$

Simarily to the first method, both integrals over Ω of f_μ and f_Σ are equal to zero. Then, only the second term of the derivative has to be considered:

$$< D'(\Omega), V >= -\int_\Gamma k(x,\Omega)(V(x).N(x))da(x)$$

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