Image segmentation

Onofre Martorell, Lidia Talavera

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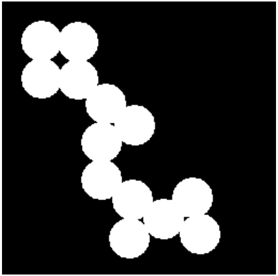
Introduction

Goal

We want to segment an image f through the Chan-Vese Segmentation.

An example of image to segment:



Chan-Vese segmentation

The Chan-Vese segmentation finds the curve C that is the boundary of the segmentation. The way to find that curve is minimizing the following functional:

$$\text{arg min}_{c_1,\ c_2,\ C}\ \mu \text{Lenght}(C) + \nu \text{Area}(inside(C)) +$$

$$\lambda_1 \int_{inside(C)} |f(x) - c_1|^2 dx + \lambda_2 \int_{outside(C)} |f(x) - c_2|^2 dx,$$

where C is the boundary of a closed set and $c1$, $c2$ are the values of u respectively inside and outside of C.

Level set functions

As it is difficult to manipulate C we will use a function ϕ and the C will be the zero crossing of ϕ that is

$$C = \{x \in \Omega : \phi(x) = 0\}.$$

With this the functional is rewritten as:

$$\arg\min_{c_1,c_2,\phi} \mu \int_\Omega \delta(\phi(x))|\nabla\phi(x)|dx + \nu \int_\Omega H(\phi(x))dx +$$

$$\lambda_1 \int_\Omega |f(x) - c_1|^2 H(\phi(x))dx + \lambda_2 \int_\Omega |f(x) - c_2|^2 (1 - H(\phi(x)))dx,$$

Level set functions

In the previous formula H denotes the Heaviside function and δ the Dirac mass, its distributional derivative:

$$H = \begin{cases} 1 & t \geq 0, \\ 0 & t < 0 \end{cases}, \quad \delta(t) = \frac{d}{dt}H(t).$$

Note that we cannot derive $H(t)$. Because of that, in the implementation we take the Heaviside function as

$$H_\epsilon(t) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{t}{\epsilon}\right)\right)$$

and the Dirac mass as

$$\delta_\epsilon = \frac{\epsilon}{\pi(\epsilon^2) + t^2)}$$

Implementation

Now we have to minimize the functional respect to c_1, c_2 and ϕ. The way to do it is the following:

1. Update c_1 and c_2 as

$$c_1 =$$

and

$$c_2$$

.

2. Evolve ϕ using the semi-implicit gradient descent

$$\phi_{i,j} =$$

.

Results