Curve shapes

- POLYNOMIALS
  1. Straight Line
  2. 2nd order polynomial
- CONVEX/CONCAVE CURVES
  1. Exponential model
  2. Power curve
  3. Logarithmic equation
  4. Rectangular hyperbola
  5. Asymptotic regression
- SIGMOID CURVES
  1. Logistic symmetric
  2. Gompertz
  3. Extreme value
  4. Log-logistic (Hill function)
  5. Weibull-1 (log-Gompertz)
  6. Weibull-2 (log-Extreme)
- PEACKED CURVES
  1. Brain-Cousens model

Linear model

$$Y = a + b \cdot X$$

- PARAMETERS: a is the value of Y when X=0; b is the growth rate (constant; unit increase in y when x increases of one unit)
- SHAPE: Straight line
- USAGE: local approximation of biological processes
- Very simple to parameterise!!!

2nd order polynomial

$$Y = a + b \cdot X + c \cdot X^2$$

- PARAMETERS: a is the value of Y when X=0; b is the initial growth rate (when $X = 0$). The slope is $b + 2cX$. The maximum (minimum) is reached when $b + 2cX = 0$, i.e. at $X = -b/(2c)$.
- SHAPE: local approximation of curvilinear biological processes.
- USAGE: degradation kinetics, exponential growth
- Very simple to parameterise!!!

Exponential model

$$Y = a \cdot b^X = a \cdot exp(cX) = exp(d + cX)$$

- PARAMETERS: A is the value of Y when X=0; b is the relative growth rate (constant)
- SHAPE: Exponential growth (concave increasing; $c > 0$) and exponential decay (concave decreasing $c < 0$)
- USAGE: degradation kinetics, exponential growth
- SELF STARTER FUNCTIONS: NLSexponential.1(), NLSexponential.2(), DRCexponential.1(), DRCexponential.2()

Rectangular hyperbola

Definition

$$y = \frac{a \cdot x}{b + x}$$

- ALSO KNOWN AS: Michaelis-Menten model
- PARAMETERS: a: maximum value of the response; b x value at which half the maximum response is attained
- SHAPE: convex increasing
- USAGE: species-area curves, enzyme kinetics
- SELF STARTER FUNCTIONS: SSmicmen(); MM.2()

Rectangular hyperbola - 2

Definition

$$y = \frac{i \cdot x}{1 + \frac{i \cdot x}{A}}$$

- PARAMETERS: i: initial slope; A maximum value
- SHAPE: convex increasing
- USAGE: competition studies (Cousens, 1985)
- SELF STARTER FUNCTIONS: DRCcousens85, NLScousens85

Power curve

Definition

$$y = a \cdot X^b = a \cdot exp[(b \cdot log(X)]$$

- ALSO KNOWN AS: Allometric function, Freundlich equation
- PARAMETERS: X must be > 0, b dictates the shape
- SHAPE: concave increasing ($b > 1$), concave decreasing ($b < 0$) or convex increasing ($0 < b < 1$).
- USAGE: sorption of pesticides, species-area curves
- SELF STARTER FUNCTIONS: NLSpowerCurve(); DRCpowerCurve()

Logarithmic equation

IT IS A LINEAR MODEL (on log-transformed x)!

Definition

$$y = a + b \cdot log(x)$$

- PARAMETERS: X must be > 0, b dictates the shape
- SHAPE: concave increasing ($b > 0$), concave decreasing ($b < 0$)
- USAGE: species-area curves and many others
- SELF STARTER FUNCTIONS: NLSlogCurve(); DRClogCurve()

Asymptotic regression

Definition

$$plateau - (plateau - init) \cdot e^{-m \cdot x}$$

$$plateau - range \cdot e^{-m \cdot x}$$

$$plateau(1 - b \cdot e^{-m \cdot x}$$

- ALSO KNOWN AS: monomolecular growth, Mitscherlich law, negative exponential
- PARAMETERS: *plateau*: maximum attainable y; *init* y at x=0; *range* equals to *plateu* − *init*; *b* is the ratio *range*/*plateau* *m* relates to the RGR
- SHAPE: convex increasing
- USAGE: growth studies
- SELF STARTER FUNCTIONS: DRCmonoGrowth.1, DRCmonoGrowth.2, DRCmonoGrowth.3

Asymptotic regression - other parameterisations

Definition

$$plateau - range \cdot e^{-m \cdot x}$$

$$plateau(1 - b) \cdot e^{-m \cdot x}$$

- PARAMETERS: *plateau*: maximum attainable y; *range*: $plateau - init$; b: $range/plateau$; m relates to the RGR
- SELF STARTER FUNCTIONS: DRCmonoGrowth.1, NLSmonoGrowth.1, DRCmonoGrowth.3, NLSmonoGrowth.3

Logistic Curve

COUNTLESS PARAMETERISATIONS!

Definition

$$c + \frac{d - c}{1 + exp(b(x - a))}$$

- PARAMETERS: d: maximum attainable y, c: minimum attainable y a X50, b slope at x50;
- SHAPE: symmetric sigmoid increasing ($b > 0$) or decreasing ($b < 0$)
- USAGE: growth studies and bioassay work (log-logistic o Hill's function)
- SELF STARTER FUNCTIONS SSfpl(), SSlogis ($d = 0$) (they give $1/b$, instead of b)

Gompertz Curve

COUNTLESS PARAMETERISATIONS!

Definition

$$c + (d - c) * exp(-exp(b(x - e))$$

- PARAMETERS: d: maximum attainable y, c: minimum attainable y a X50, b slope around inflection;
- SHAPE: asymmetric sigmoid increasing ($b > 0$) or decreasing ($b < 0$)
- USAGE: growth studies and bioassay work (log-Gompertz or Weibull-1 function)
- SELF STARTER FUNCTION: SSGompertz ($d = 0$)

Model parameterisation

A model is parameterised when its parameters are known. This is possible by using:

- local experience
- literature information
- measurements in appropriately planned experiments