

gcd calculation with division algorithm

$$1 \quad x = q_1 \cdot y + r_1 \quad 0 \leq r_1 < |y|$$

$$2 \quad y = q_2 \cdot r_1 + r_2 \quad 0 \leq r_2 < r_1$$

$$3 \quad r_1 = q_3 \cdot r_2 + r_3 \quad 0 \leq r_3 < r_2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$n. \quad r_{n-2} = q_n \cdot r_{n-1} + \boxed{r_n} \rightarrow \text{gcd} \quad 0 \leq r_n < r_{n-1}$$

$$n+1 \quad r_{n-1} = q_{n+1} \cdot r_n + 0 \rightarrow \text{if value equal zero, stop the algorithm.}$$

Example: 45 and 126 numbers Let find to greatest common divider with division algorithm. And
 $\text{gcd}(45, 126) = s \cdot 45 + t \cdot 126$ to be condition "s" and "t" number's

find.

	q	r
$126 = 2 \cdot 45 + 36$	q_1	
$45 = 1 \cdot 36 + 9$	q_2	
$36 = 4 \cdot 9 + 0$	q_3	

$\boxed{\text{gcd}}$

$$\text{gcd}(45, 126) = 45 \cdot s + 126 \cdot t$$

$$g = 45 \cdot s + 126 \cdot t$$

$$g = 45 \cdot 1 - 1 \cdot 36$$

$$= 45 \cdot 1 - 1 \cdot (126 - 2 \cdot 45)$$

$$= 1 \cdot 45 + 2 \cdot 45 - 126 \cdot 1$$

$$g = 3 \cdot 45 -$$

$$\boxed{s = 3}$$

$$\boxed{t = -1}$$

Example Determine the integers "x" and "y" provided

that $512x + 320y = 64$

$$\underline{\gcd(x, y) = 64} \quad \div 64 / \quad 512x + 320y = 64$$

$$\underline{8x + 5y = 1}$$

$$x = x_0 + 5k$$

$$y = y_0 - 8k$$

$$k=0 \rightarrow x=$$

$$k=1 \rightarrow x=4$$

$$k=-1 \rightarrow$$

Exmp Determine $\text{lcm}(120, 500)$ and $\text{gcd}(120, 500)$

$$\left. \begin{array}{l} 120 = 2^3 \cdot 3 \cdot 5 \\ 500 = 2^2 \cdot 5^3 \end{array} \right\} \text{gcd}(120, 500) = 2^2 \cdot 3^0 \cdot 5^1 = \underline{20}$$

$$\rightarrow \text{lcm}(120, 500) = 2^3 \cdot 3 \cdot 5^3 = \underline{3000}$$