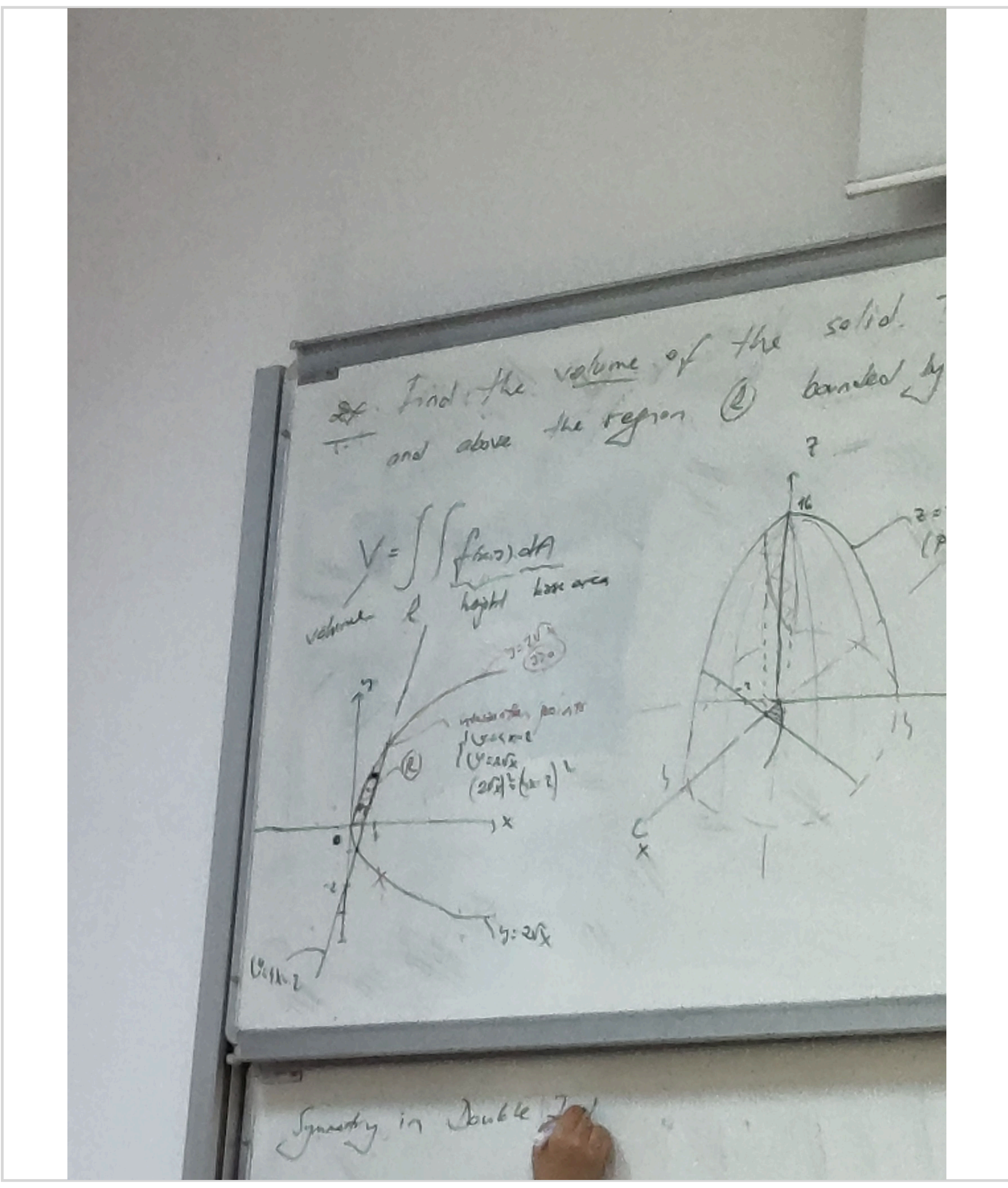
Example the volume of the solid, that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region

(R) bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$ and x-axis

$$V = \iint \underbrace{f(x,y)}_{\text{height}} \, \underbrace{dA}_{\text{basis area}}$$



Yandaki soru alan hesaplama üzerine tekrar et

* Symmetry in Double Integration:

① Suppose that Ⓡ is symmetric about y-axis:

ⓐ If Ⓕ is add in ⓧ [if $f(-x,y) = -f(x,y)$], then

$$\iint_R f(x,y)\,dx\,dy = 0$$

ⓑ If Ⓕ is even in ⓧ [if $f(-x,y) = f(x,y)$],

then $\iint f(x,y)\,dx\,dy = 2 \cdot \iint f(x,y)\,dx\,dy$ R
right half

of (R)

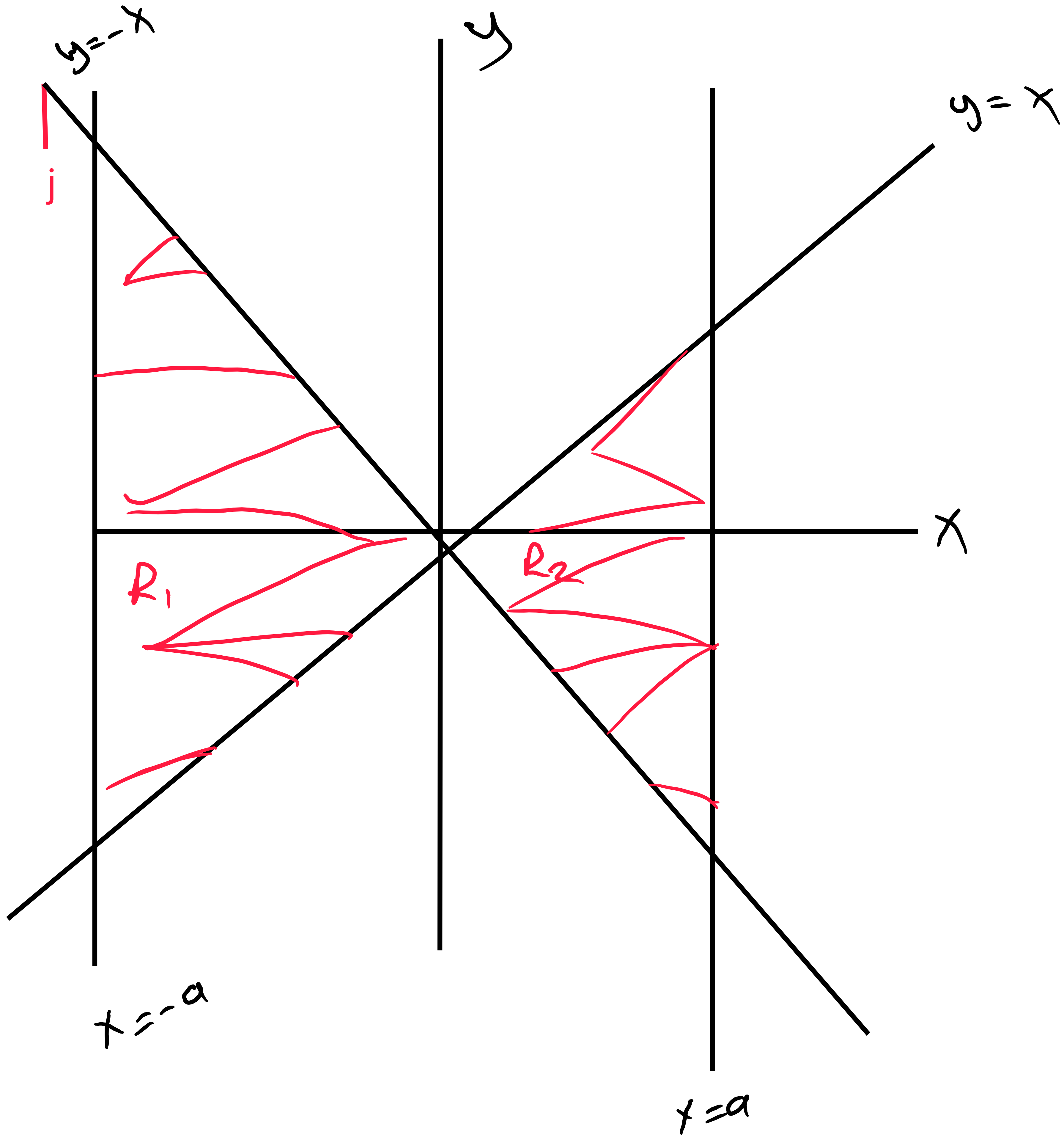
(2) Suppose that (R) is symmetric about
x-axis:

(a) If (f) is odd in (y) [if $f(x,-y) = -f(x,y)$],
then $\iint_R f(x,y)\,dy\,dy = 0$.

(b) If (f) is even in (y) [if $f(x,-y) = f(x,y)$],
then $\iint_R f(x,y) = dx\,dy = 2 \iint_{\substack{\text{upper half} \\ \text{of } (R)}} f(x,y)\,dx\,dy$

Example Take the region branded by x
+y=0, x-y=0, x = a, x = -a Suppose that we
want to evolvate

$$\iint_R (ax - \sin(x^2 y))\,dx\,dy.$$



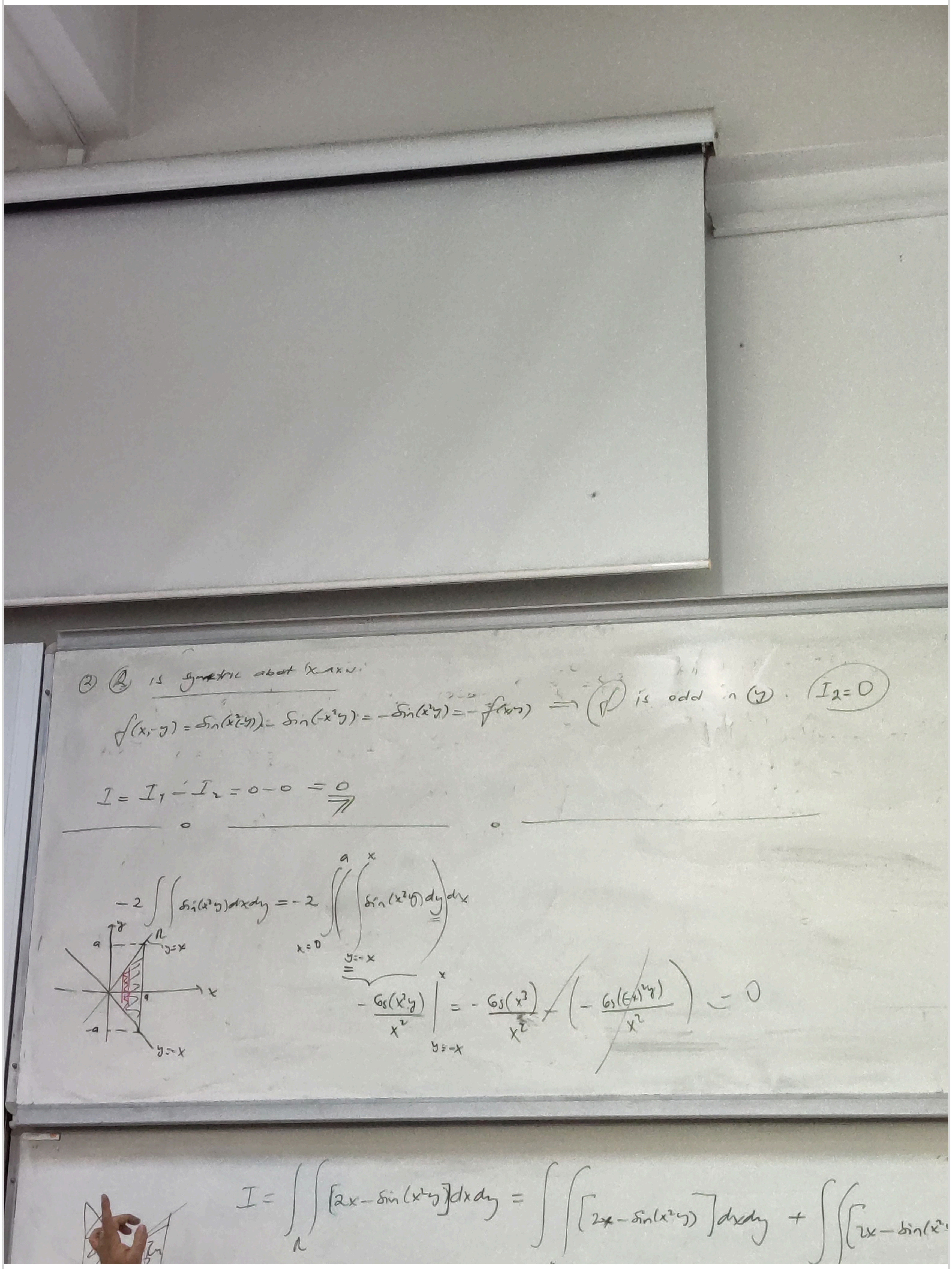
$$I = \underbrace{\iint_R \underbrace{ax}_{f(x,y)}\,dx\,dy}_{I_1} - \underbrace{\iint_R \underbrace{\sin(x^2 y)}_{f(x,y)}\,dx\,dy}_{I_2}$$

1 R is symetric about y-axis ;

F_1 : $f(-x,y) = 2(-x) = -2x = -f(x,y) \Rightarrow f$ is odd in x

F_2 :

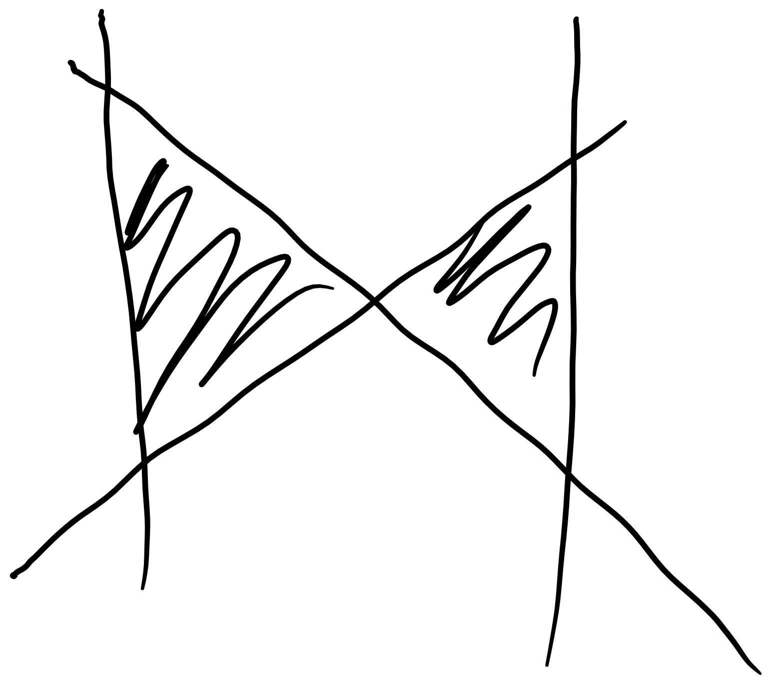
δ



Aşağıdaki soru çözümü

I_2 nin

2. integraline ait.



$$I = \iint_R \left[2x - \sin(x^2 y)\right] dx\,dy$$

$$= \iint_{R_1} \left[2x - \sin(x^2 y)\right] dx\,dy \; +$$

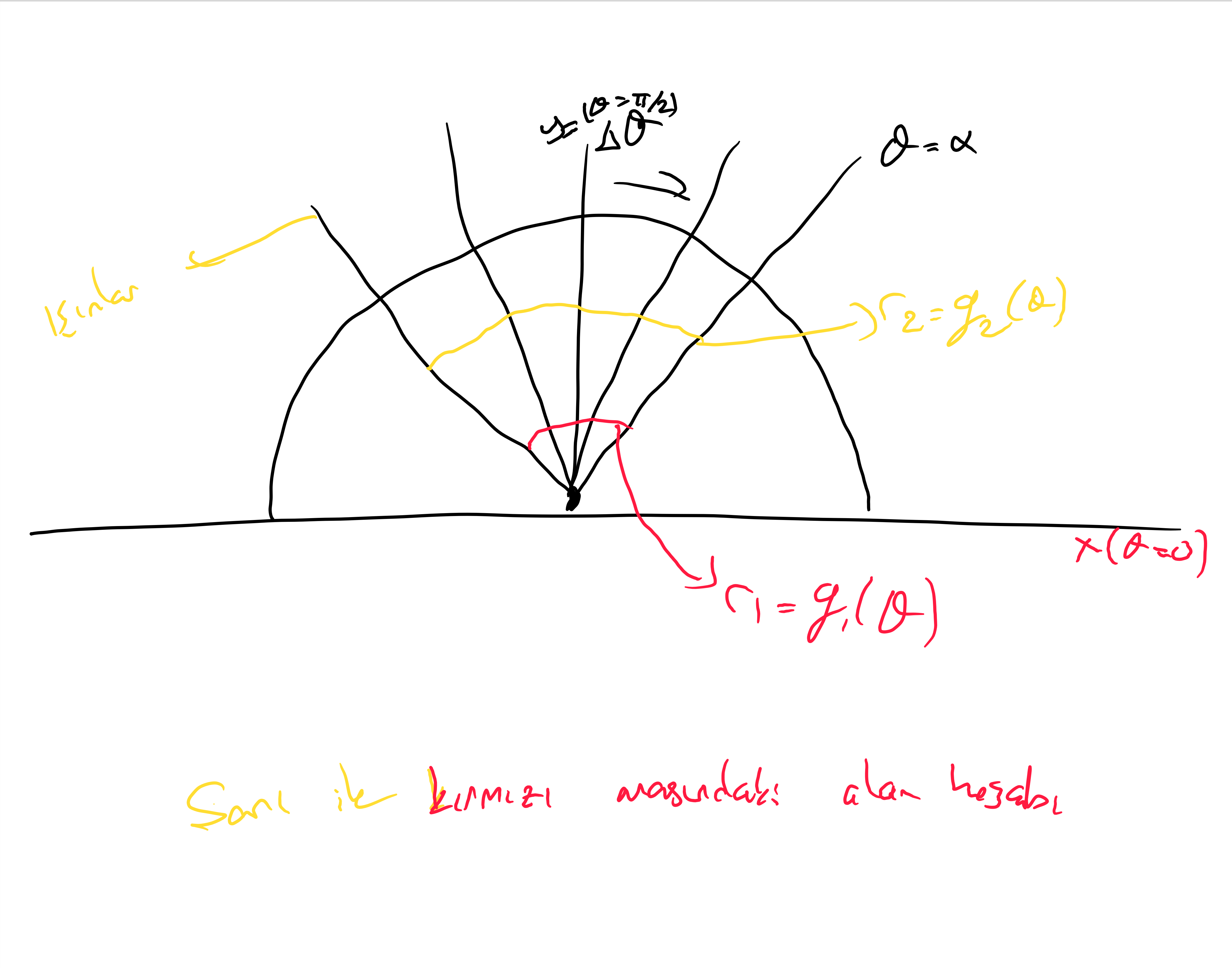
$$\iint_{R_2} \left[2x - \sin(x^2 y)\right] dx\,dy$$

$$= \int_{x=-a}^{0} \left(\int_{\underline{\underline{y=x}}}^{-x} \left[2y - \sin(x^2 y)\right] dy \right) dx + \int_{y=0}^{0} \left(\int_{y=-x}^{x} \left[2x - \sin(x^2 y)\right] dy \right) dx$$

Double Integrals in Polar forms:

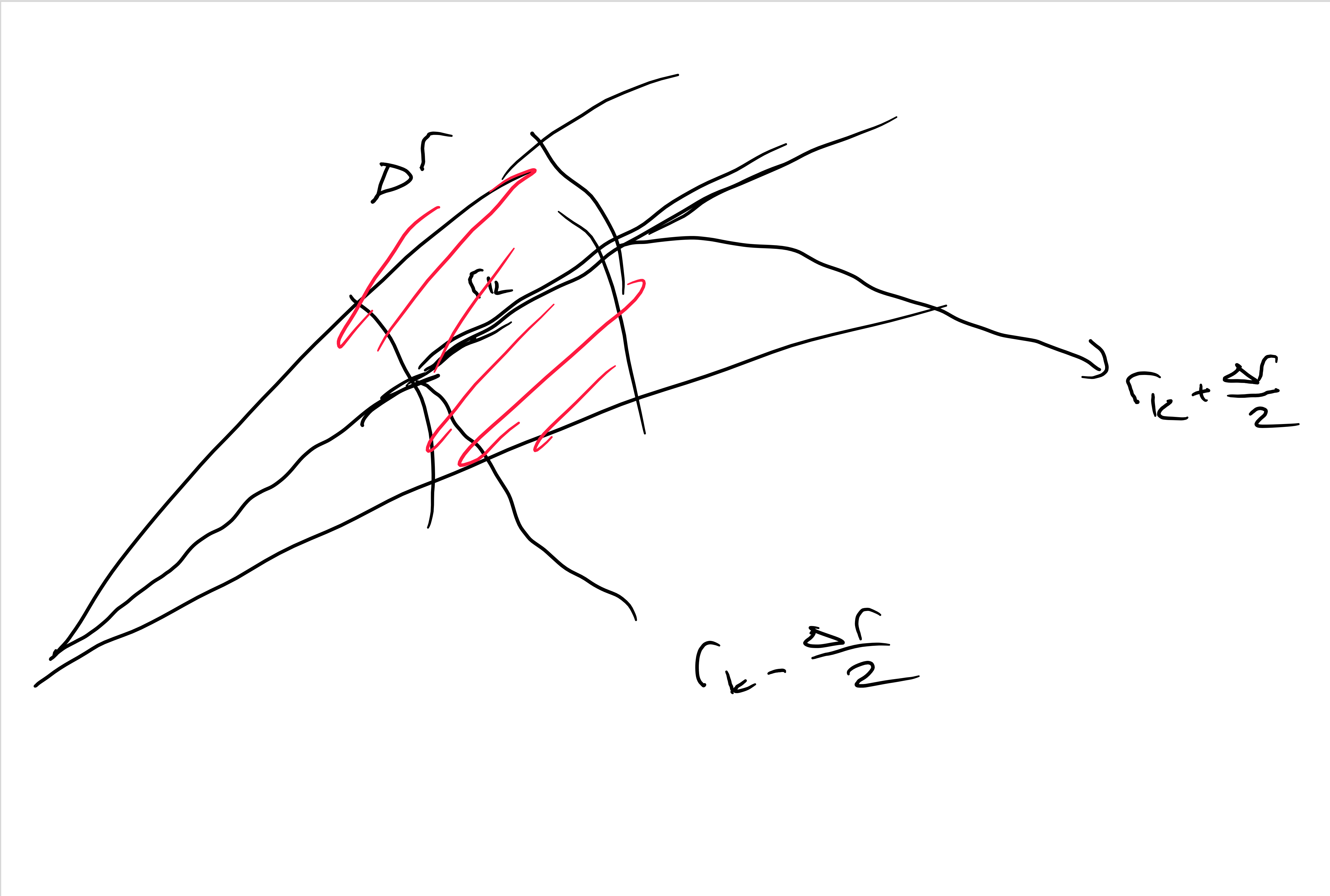
Integrals in Polar Coordinates:

Suppose that a function $f(r,a)$ is defined over a region R that is bounded by the rays $\theta=\alpha$ and $\theta=\beta$ and by the continuous curves $r=g_1(\theta)$ and $g_2(\theta)$ $(0\leq g_1(\theta)\leq g_2(\theta)\leq a)$



To evaluate this limit we first

have to write the sum S_n in a way that express ΔA_k in terms of Δr and $\Delta \theta$



The area of a wedge shaped sector of a circle having radius r and angle

θ is $Area = \frac{1}{2}\theta r^2$ $\left(A = \frac{\pi r^2 \cdot \theta}{2\pi} \right)$

So the areas of the circular sectors subtended by these areas at the origin are.

$\Rightarrow$ inner radius : $\frac{1}{2}\Delta\theta.\left(r_k - \frac{\Delta r}{2}\right)^2$

$\Rightarrow$ outer radius : $\frac{1}{2}\Delta\theta\left(r_k + \frac{\Delta r}{2}\right)^2$

Therefore, ΔA_k = area of large sector - area of small sector

$$= \frac{1}{2}\Delta\theta.\left(r_k + \frac{\Delta r}{2}\right)^2 - \frac{1}{2}\Delta\theta\left(r_k - \frac{\Delta r}{2}\right)$$

$$= r_k.\Delta r.\Delta\theta$$

As $n \to \infty$ and values of Δr and $\Delta\theta$ approaches zero, these sum is the double integral