The Method of Integrating Factors for FO Linear PDEs

→ Recall the first order linear PDE

$$a(x,y)u_x + b(x,y)u_y + c(x,y)u = d(x,y) \quad (2.1)$$

→ Let us define the differential operator

$$L = a(x,y)\frac{\partial}{\partial x} + b(x,y)\frac{\partial}{\partial x} + c(x,y) \quad (2.2)$$

→ which us to rewrite (2.1), in short, as

$$Lu = d(x,y) \quad (2.3)$$

→ Any operator is said to be linear if it satisfies the following, for all $c_1, c_2 \in \mathbb{R}$ and all partially differentiable functions f_1 and f_2

$$L(c_1 f_1 + c_2 f_2) = c_1 L(f_1) + c_2 L(f_2) \quad (2.4)$$

** Definition

↳ If $d(x,y) \equiv 0$ in (2.1), then the corresponding equation

$$Lu = 0 \quad (2.5)$$

is called the homogeneous equation corresponding to (2.4).

More over, if $u = \phi(x,y)$ is particular solution of (2.1), then it satisfies the following identity:

$$a(x,y)\phi_x + b(x,y)\phi_y + c(x,y)\phi \equiv d(x,y) \quad (2.6)$$

** The partially differentiable function $u = \phi(x,y)$ is said to be the integral surface of (2.1), if it satisfies (2.1).

** Any family of surfaces is called the general solution of the homogeneous equation if it contains an arbitrary function which satisfies (2.5)

** Let u_h be the general soltion of the homogeneous equation (2.5) and u_p be the particular solution of (2.4). Then, $u = u_h + u_p$ is called the general solution of the nonhomogeneous equation.