Integer division

Definition: $x, y \in \mathbb{Z}$ and $x \neq 0$

x divides to y OR x was divider to y

$$x \mid y \Rightarrow y = q \cdot x \quad \exists q \in \mathbb{Z}$$

$$3 \mid 12 \Rightarrow 12 = 3 \cdot x \quad \exists x \in \mathbb{Z}$$

$$3 \nmid 10 \Rightarrow 10 = 3 \cdot y \quad \nexists y \in \mathbb{Z}$$

Theorem:

a, b and $c \in \mathbb{Z}$

i. $\mp 1 \mid a$

ii. $a \mid b \Rightarrow a \mid -b,\ -a \mid b,\ -a \mid -b$

iii. $a \mid b \Rightarrow a \mid bx \quad \forall x \in \mathbb{Z}.$

IV. $a \mid b$ and $b \mid c \Rightarrow a \mid c$

Proof

$$\left.\begin{array}{l} b = a \cdot x \quad \exists x \in \mathbb{Z} \\ c = b \cdot y \quad \exists y \in \mathbb{Z} \end{array}\right\} \quad \begin{array}{l} c = b \cdot y \\ c = a \cdot x \cdot y \\ a \mid c \end{array}$$

V. $a|b$, $b|a$ $\Rightarrow$ $a = \mp b$

VI. $a|b$ and $a|c$ $\Rightarrow$ $a|(b+c)$

$\hookrightarrow$ proof: $a|b \rightarrow b = ax \; \exists x \in \mathbb{Z}$

$a|c \rightarrow c = a \cdot y \; \exists y \in \mathbb{Z}$

$b+c = a(y+x)$

$a|(b+c)$

VII. $a|b$ and $a|c$ $\Rightarrow$ $a|(bx+cy)$ $\forall x, y \in \mathbb{Z}$

Theorem (Division algorithm)

x and $y \in \mathbb{Z}$ to be condition $y \neq 0$.