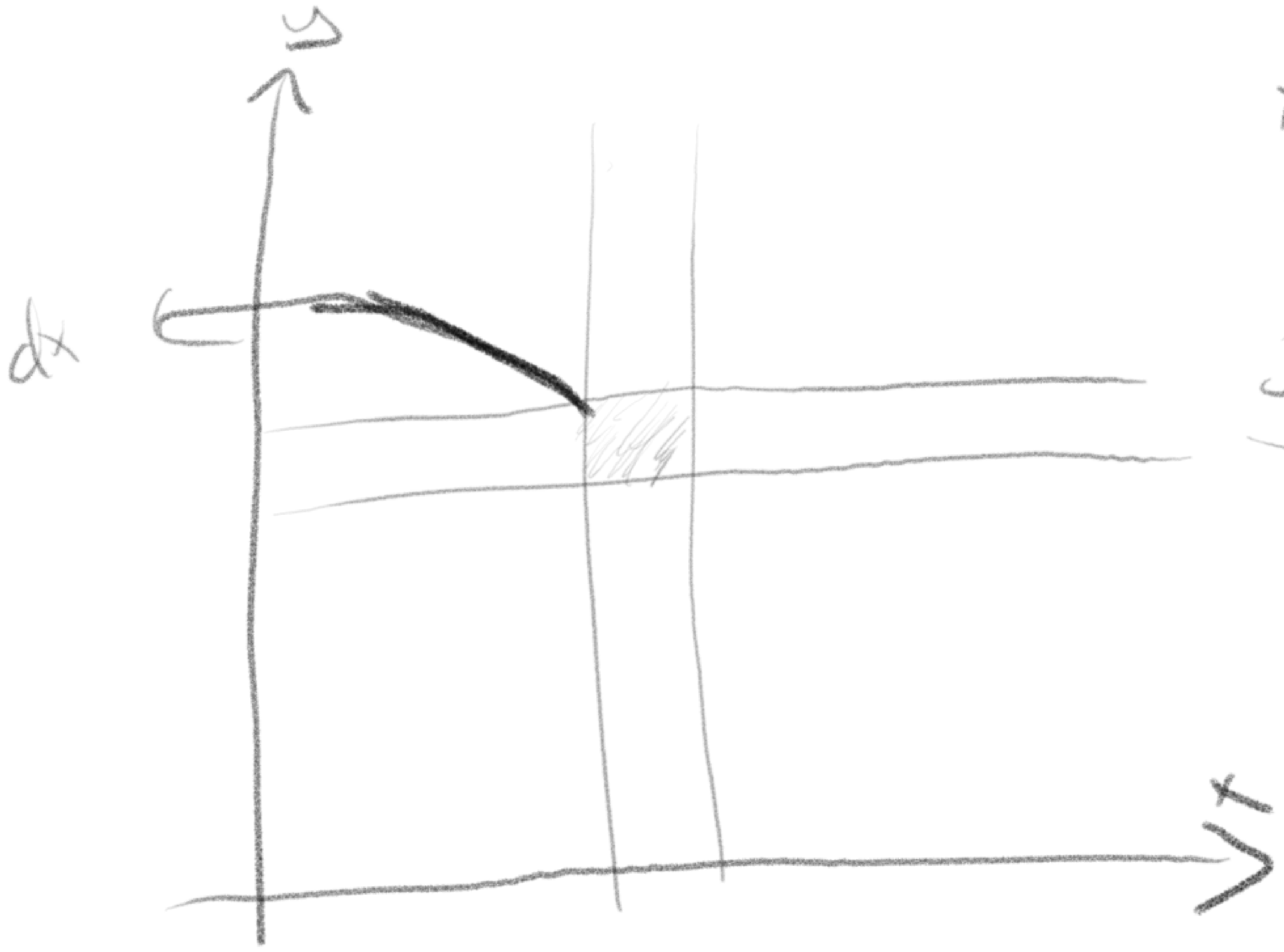
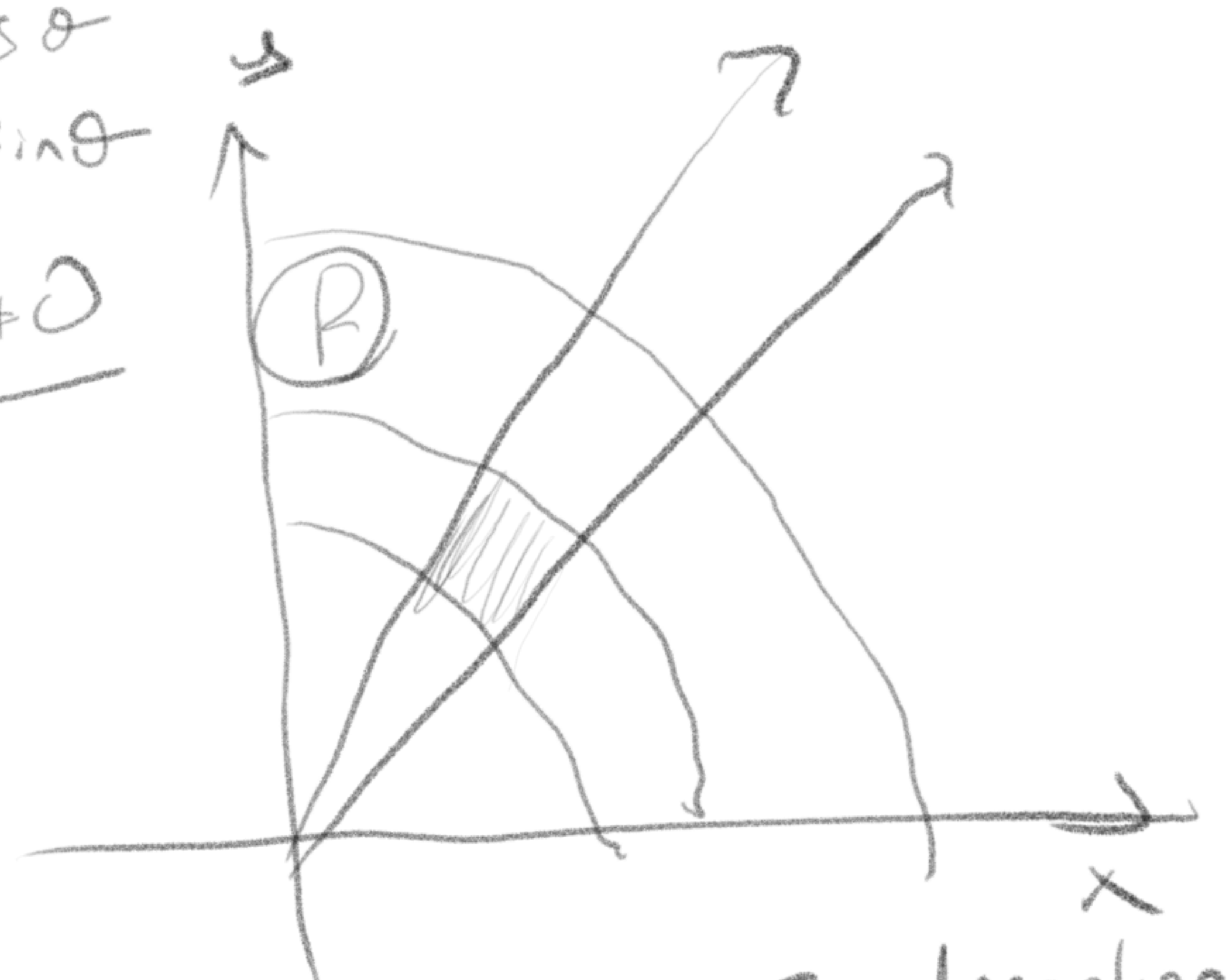


$x = r\cos\theta$

$y = r\sin\theta$

$J(r,\theta) \neq 0$



Polar Cartesian Transformation

$x = r\cos\theta$

$y = \sin\theta$

$x^2 + y^2 = r^2$

$dA = dx\,dy = r\,dr\,d\theta$

$r\theta$ plane

$(x,y) \longrightarrow (r,\theta)$

Ex: Evaluate $\displaystyle\int_{y=0}^{4}\left(\int_{x=y/2}^{x=y/2+1}\left(\frac{2x-y}{2}\right)dx\right)dy$

$u = \dfrac{2x-y}{2}$

$v = 2x-2$

$\boxed{\text{use transformation}}$

$x = u$

$\dfrac{y}{2} = v$

$x = \frac{y}{2}$	$y = 0$	$\dfrac{2x-y}{2}$
$x = \frac{y}{2} + 1$	$y = 4$	

$y = 2x$

$y = 2x - 2$

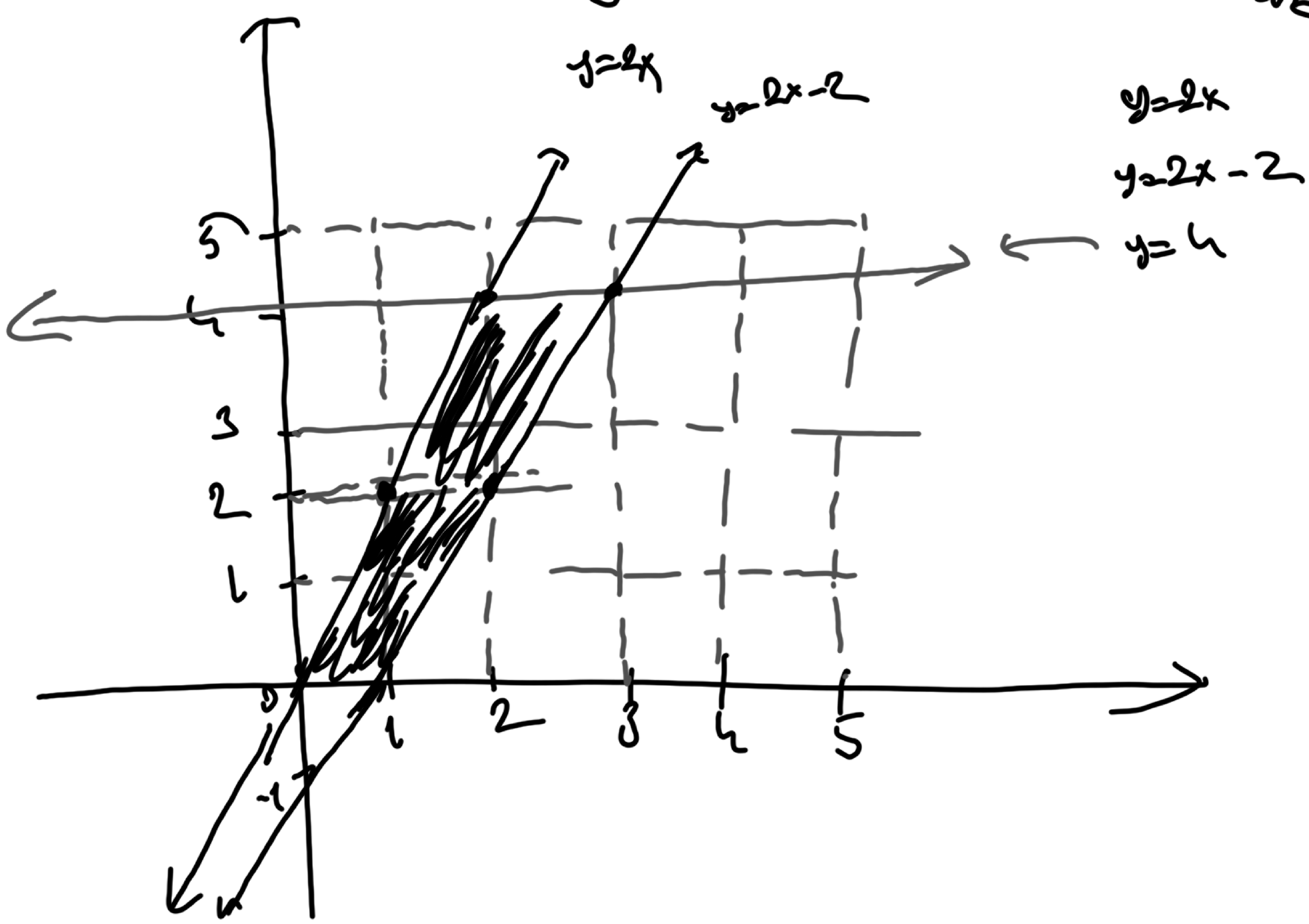
$$2x - y = 2u \qquad y = 2v \Rightarrow 2x - 2v = 2u \Rightarrow x = \frac{2u + 2v}{2} = u + v$$

$$\Rightarrow \begin{array}{l} x = u + v \\ y = 2v \end{array} \Rightarrow \begin{array}{l} (x,y) \longrightarrow (u,v) \\ J(u,v) \neq 0 \end{array}$$

$$J(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \dots \longrightarrow = 2 \neq 0$$

$$\int_{y=0}^{4} \int_{x=y/2}^{\frac{y}{2}+1} \frac{(2x-y)}{2}\, dx\, dy = \iint (u)\, |J(u,v)|\, du\, dv$$

$$= \iint (u) \cdot |2|\, du\, dv \Rightarrow$$ devamı arkada

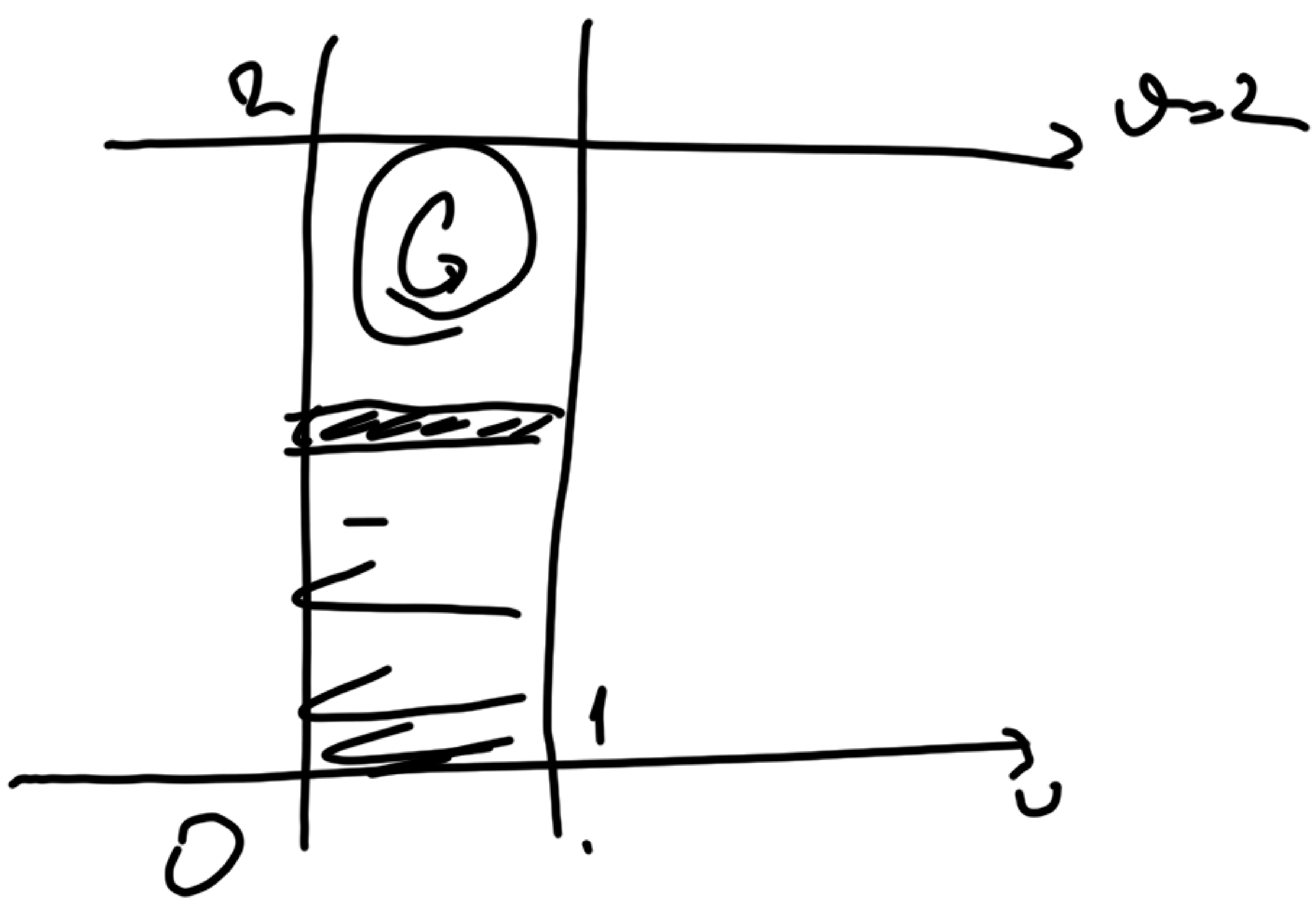


$$x = \frac{y}{2} \Rightarrow x + v \Rightarrow \frac{2v}{2} \Rightarrow u = 0$$

$$x = \frac{y}{2} + 1 \Rightarrow u + v = \frac{2v}{2} + 1 \Rightarrow u = 1$$

$$y = 0 \Rightarrow v = 2v \Rightarrow v = 0$$

$$y = 4 \Rightarrow 4 = 2v \Rightarrow v = 2$$



Arkadaki işlemi devam $\Rightarrow$

$$2\int_{v=0}^{2}\left(\int_{u=0}^{1} u\,du\right)dv = 2\int_{0}^{2} \frac{u^2}{2}\Big|_0^1 dv = \int_0^2 dv$$

$$= v\Big|_0^2 = 2 \;\cancel{4}$$

Ex

find the volume of the solid lying inside the sphere

$x^2+y^2+z^2=4a^2$ and the cylinder

$x^2+y^2+z^2=\rho^2$ ρ : radius of sphere

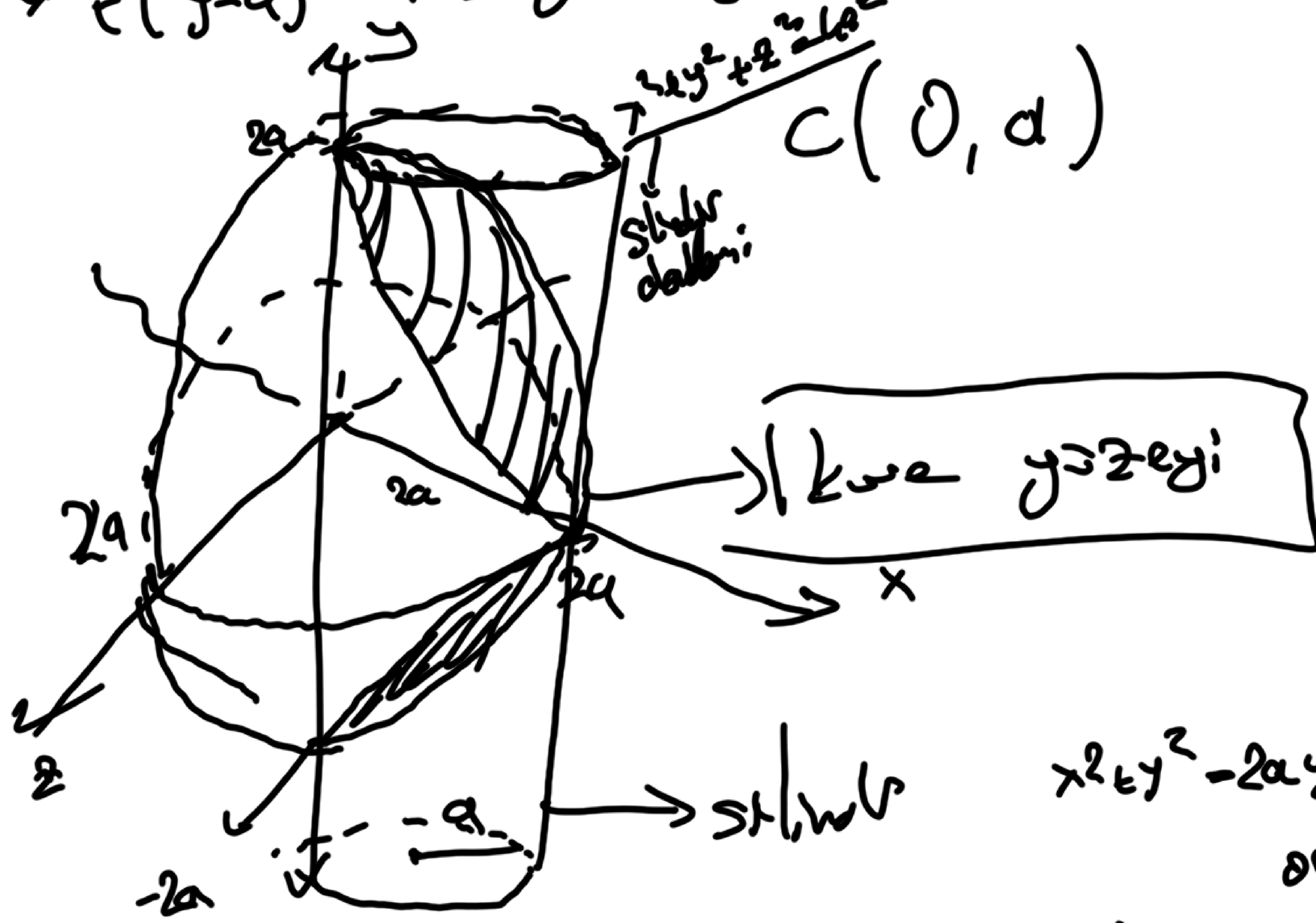
$\rho^2=4a^2$ $\rho=2a$ $\rho\geq 0$!

$x^2+y^2-2ay=0$

$x^2+(y-a)^2-a^2=0 \Rightarrow x^2+(y-a)^2=a^2$ $r=a$

küre yarıçapı

$C(0,a)$

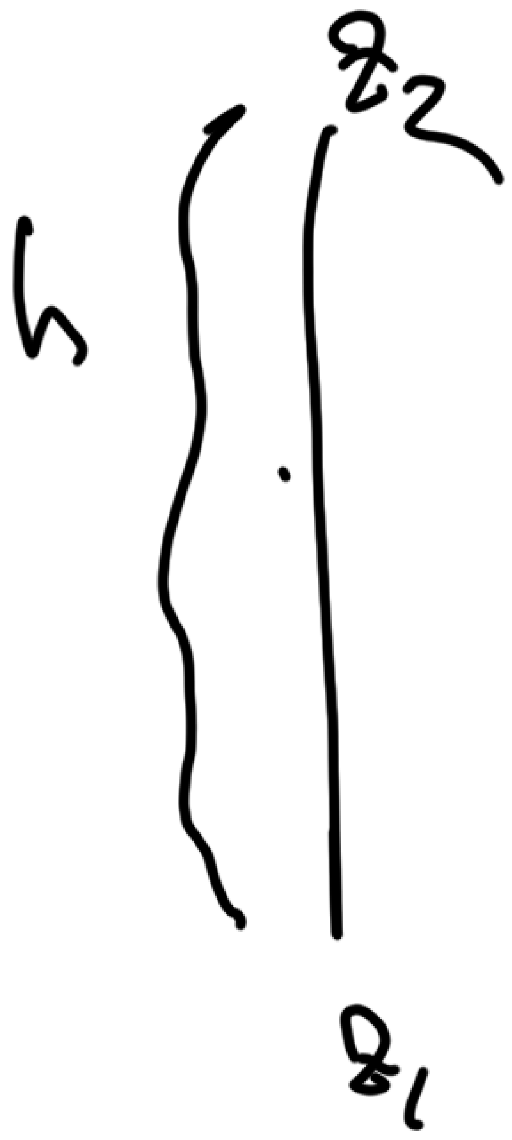


$x^2+y^2-2ay=0$

or

$x^2+(y-a)^2=a^2$

küre yüzeyi tarafı.



silindir denklemi

$$h = z_2 - z_1 = 2\sqrt{4a^2-x^2-y^2}$$

Kutupsal koordinatları kullanabiliriz !

$$z = \pm\sqrt{4a^2-x^2-y^2}$$

$$V = \iint_R \underbrace{f(x,y)}_{\text{height}} \overbrace{\underbrace{dx\,dy}_{\text{base area}}}^{dA} = 2\iint_R \sqrt{4a^2-x^2-y^2}\,dx\,dy$$

use polar coordinate transformation

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

$dA = r\,dr\,d\theta$

$(dx\,dy)$

$0 \le r \le 2a\sin\theta$

$0 \le \theta \le \pi$

$$\int_{\theta=0}^{\pi} = 2\int_{0}^{\pi/2}$$

$$V = 2\iint_R \sqrt{4a^2 - x^2 - y^2}\,dx\,dy = (2).(2)\int_{\theta=0}^{\pi/4}\left(\int_{r=0}^{2a\sin\theta}\sqrt{4a^2 - r^2}\; r\,dr\right)d\theta$$

Exmp

Evaluate $I = \int_1^{\sqrt{2}}\left(\int_0^x xy\,dy\right)dx + \int_{\sqrt{2}}^{2}\left(\int_0^{\sqrt{4-x^2}} xy\,dy\right)dx$ by using Polar Coordinates

$x = 1$ | $y = 0$

$x = \sqrt{2}$ | $y = x$

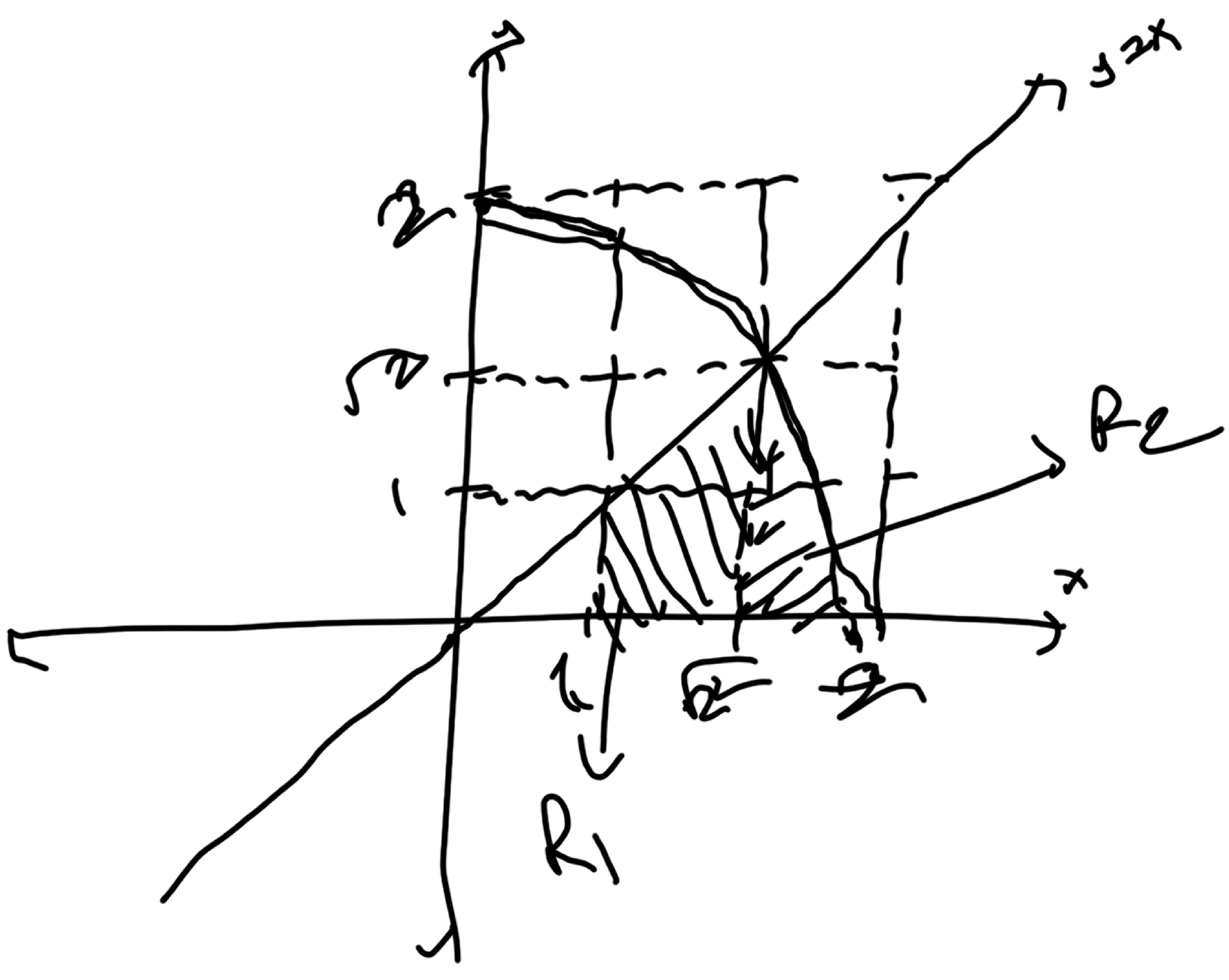
R_1

$x = \sqrt{2}$ | $y = 0$

$x = 2$ | $y = \sqrt{4 - x^2}$

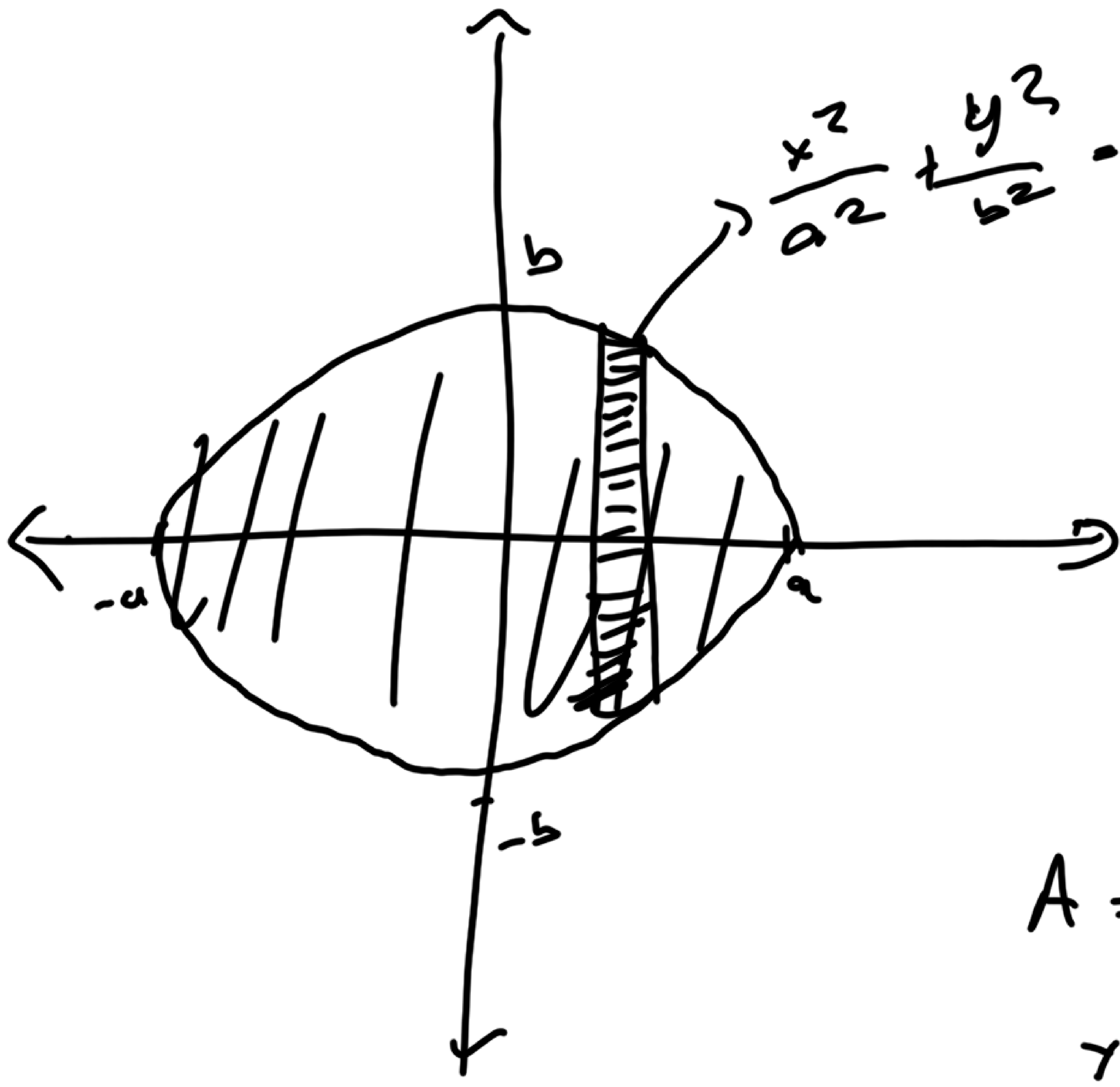
R_2

$0 \le \theta \le \pi/4$



Ex Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by using substitution in double Integral

a > b



$$A = \iint_R \underbrace{dx\,dy}_{dA}$$

Area of R

$$= \iint_R r\,dr\,d\theta$$ in polar coordinates

$$A = \int_{x=-a}^{a}\left(\int_{y=-b\sqrt{1-\frac{x^2}{a^2}}}^{b\left(\sqrt{1-\frac{x^2}{a^2}}\right)} dy\right)dx$$

$y = \pm b\sqrt{1-\frac{x^2}{a^2}}$

$$= \int_{-a}^{a} x\Big|_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\left(1-\frac{x^2}{a^2}\right)} dx$$

use transform

$$\begin{cases}\frac{x}{a}=v\\ \frac{y}{b}=\theta\end{cases}$$

$(x,y) \longrightarrow (v,\theta)$

$J(v,\theta)\neq 0$

$v^2+\theta^2=1$ cember

$2.y\theta)$

foti cektim