xplanations for your answers. For at least half of the examples,

a) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{2^{2n}}{2^n} = \lim_{n\to\infty} 2^n = \infty$

$f(n) = o(g(n))$

b) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n^3}{n^2} = \lim_{n\to\infty} n = \infty$

$f(n) = o(g(n))$

c) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{2n-5}{3n+1} = \frac{2}{3}$ } constant

$f(n) = \Theta(g(n))$

d) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{n^2}{4n^2} = \frac{1}{4}$ } constant

$f(n) = \Theta(g(n))$

e) $\lim_{n\to\infty} \frac{\log_{10} n}{\log_2 n} = \frac{\frac{\log n}{\log 10}}{\frac{\log n}{\log 2}} = \frac{\cancel{\log n}}{\log 10} \cdot \frac{\log 2}{\cancel{\log n}} = \frac{\log 2}{\log 10}$ } constant

$f(n) = \Theta(g(n))$

f) $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n = \infty$

$f(n) = o(g(n))$

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$0(n)$

g) $\lim_{x\to\infty} \frac{g(x)}{f(x)} = \frac{1000n^2}{n^3}$

$= \lim_{n\to\infty} \frac{1000}{n} = 0$

$f(x) = \omega(g(x))$

h) $\lim_{n\to\infty} \frac{f(x)}{g(x)} = \frac{5n+4}{2n+2} = 2$

$f(x) = \Theta(g(x))$

n^2

- 2

(n)

i) $\lim_{n\to\infty} \frac{f(x)}{g(x)} = \lim_{n\to\infty} \frac{\sqrt{n}}{\log_2 n}$

$\Rightarrow \lim_{n\to\infty} \frac{n^{1/2}}{\log n} \cdot \log 2 = \frac{\frac{1}{2} n^{-1/2}}{n^{-1}}$

$= \lim_{n\to\infty} \frac{1}{2} n^{1/2} = \infty$

$\Rightarrow f(x) = \omega(g(x))$

j) $\lim_{n\to\infty} \frac{f(x)}{g(x)} = \frac{2^n}{2^{n+1}} = \frac{1}{2}$

$f(x) = \Theta(g(x))$

2) $\lim_{n\to\infty} \frac{\frac{1}{2n}}{\log n} = \lim_{n\to\infty} \frac{1}{2n \cdot \log n} = 0$

$\frac{1}{2n} < \log n < \sqrt{n+5} < n+1 < n^2 \log n < 2^n < 10^n < n!$

$\frac{1}{2n} < \log n$

$\lim_{n\to\infty} \frac{\log n}{\sqrt{n+5}} \overset{\text{L-hospital}}{=} \frac{1/n}{\frac{1}{2}(n+5)^{-\frac{1}{2}}} = \frac{2}{n}(n+5)^{\frac{1}{2}}$

$= \frac{2 \cdot \frac{1}{2}(n+5)^{-\frac{1}{2}}}{1}$ } L-hospital

$= \lim_{n\to\infty} \frac{1}{\sqrt{n+5}} = 0$

$\log n < \sqrt{n+5}$

$\lim_{n\to\infty} \frac{\sqrt{n+5}}{n+1} \overset{\text{L-hospital}}{=} \frac{1}{2 \cdot \sqrt{n+5}}$

$= \lim_{n\to\infty} \frac{1}{2\sqrt{n+5}} = 0$

$\sqrt{n+5} < n+1$

$\lim_{n\to\infty} \frac{n+1}{10^n} \overset{\text{L hospital}}{=} \frac{1}{n \cdot 10^{n-1}} = \lim_{n\to\infty} \frac{1}{n 10^{n-1}} = 0$

$n+1 < 10^n$

$\lim_{n\to\infty} \frac{10^n}{n^2 \log n} = \frac{n \cdot 10^{n-1}}{2n \log n + n^2 \cdot \frac{1}{n}} = \lim_{n\to\infty} \frac{10^{n-1}}{2\log n + 1}$ } L-hospital

$n^2 \log n < 10^n$

$= \lim_{n\to\infty} \frac{(n-1) \cdot 10^{n-2}}{\frac{2}{n}}$

$= \lim_{n\to\infty} \frac{(n^2-n) \cdot 10^{n-2}}{2}$

$= \infty$

$\lim_{n\to\infty} \frac{n+1}{n^2 \log n} = \frac{\infty}{\infty}$

$= \lim_{n\to\infty} \frac{1}{2n \cdot \log n + n^2 \frac{1}{n}}$

$= \frac{1}{2n \log n + n} = \frac{1}{\infty} = 0$

$n+1 < n^2 \log n$

$\lim_{n\to\infty} = \frac{10^n}{2^n} = 5^n = \infty$

$2^n < 10^n$

$\lim_{n\to\infty} \frac{n^2 \log n}{2^n} = \frac{\infty}{\infty}$ } L-hospital

$= \frac{2n \cdot \log n + n^2 \frac{1}{n}}{n \cdot 2^{n-1}} = \lim_{n\to\infty} \frac{2\log n + 1}{n-1}$ } L-hospital

$n^2 \log n < 2^n$

$= \lim_{n\to\infty} \frac{2 \cdot \frac{1}{n}}{1} = 0$

$\lim_{n\to\infty} \frac{10^n}{n!} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{10^n}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} \Rightarrow \frac{(10 \cdot e)^n}{\sqrt{2\pi}\ n^{n+\frac{1}{2}}}$

$10^n < n!$

$\Rightarrow \lim_{n\to\infty} \frac{\left(\frac{10 e}{n}\right)^n}{\sqrt{2\pi} \cdot \sqrt{n}} = \frac{1}{\infty} = 0$

$\lim_{n\to\infty} \frac{n!}{n^{2^n}} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{n^{2^n}}$

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Worst case

* First condition can not terminate loop.
* Both conditional statement will executed sequential.
* So if we suppose there is only second conditional statement, we can solve and then multiply with two.
* i is increasing with it's square. So it will continue until it reach n.

$2^1, 2^2, 2^{2^2}, 2^{2^3} \ldots$

$(i)^{2^k} \le n$

$2^k \le \log n$

$k \le \log(\log(n)) \Rightarrow T_w(n) = \Theta(\log(\log(n)))$

Best Case

If i is larger than n

$T_B(n) = \Theta(1)$ Constant time

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If probability of being even is %20

* it is %20/n for each index.

%20/n for $1 \le i \le n-1$

%20/n + %80/n for $i = n$

$\Rightarrow A(n) = \sum_{i=1}^{n} i \cdot p_i$

$\Rightarrow A(n) = \sum_{i=1}^{n-1} i \cdot \frac{2}{10 n} + n \cdot \left(\frac{2}{10n} + \frac{8}{10n}\right)$

$= \frac{n \cdot (n-1)}{2} \cdot \frac{2}{10n} + n \cdot \frac{1}{n}$

$= n-1+1$

$A(n) = n$ $\quad A(n) = \Theta(n)$

3

if the height of tree is n it means it has $2^n - 1$ elements.

* My algorithm is finding the smallest element which is leftmost element. It takes n operations, since it's height is n. And then it will traverse k more elements. and k is $1 \le k \le 2^n - 1$.

Best Case

For the best case k is 1 so

$T(n) = \Theta(n+1) = \Theta(n)$

Worst Case

For the worst case k is $2^n - 1$

$T(n) = \Theta(2^n - 1 + n) = \Theta(2^n)$

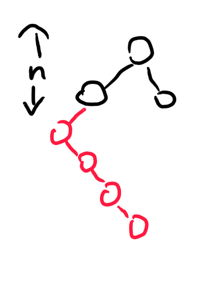
* In merging algorithm it will traverse one of trees inorder traversal. And then it will add to the other tree sequentially. So my algorithm will add $2^n - 1$ elements to the other BST.

Best Case

It will add new elements and it will traverse $\log n$ times. New tree will be perfect. So it is $T(n) = \Theta((2^n - 1) \cdot \log n)$

Worst Case

For the worst case all the elements of second BST are smaller or larger than other. And new BST will not be perfect. It will be getting longer.



So for first adding operation it will perform n operations.

$\sum_{i=0}^{i=2^n-2} n+i$

$\underbrace{\overbrace{1 + \ldots\ldots + n}\ + (n+1) + (n+2) + \ldots + (n+2^n-2)}$

$\frac{(n+2^n-\cancel{2})(n+2^n-\cancel{1})}{2} - \frac{n(n-1)}{2}$

$\frac{2^{2n}}{2} - \frac{n^2}{2} = \quad T(n) = O(2^n - n^2)$

$= O(2^n)$