Experiment 1, The Stefan-Boltzman Radiation Law

Onuray SANCAR

November 18, 2019

with **Yaren AKSEL**, **Experiment Date**:Novermber 8, 2019

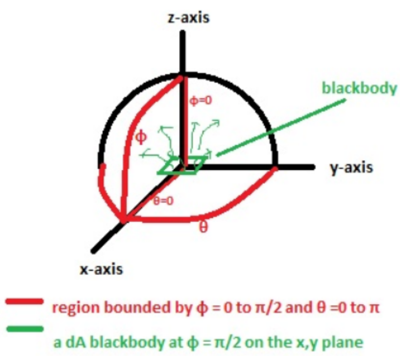
Abstract

The purpose of our experiment is to find the famous Stefan-Boltzman relation. To do this we have first calculated the resistance of our filament, verified the inverse square law and looked for the famous relation both for high and low temperatures. We have found the proportionality constant between the intensity of radiation and the temperature as 2.73 ± 0.45 which is 2.82σ away from the real value.

Theory

The relation between the radiation and temperature had been a hot topic for centuries. There was a definet divergence from a linear behavior as the temperature grew. Josef Stefan introduced a model that stated the relation as $I \propto T^4$, later Boltzmann showed that by treating electromagnetic radiation as the working fluid in a Carnot cycle the T^4 law was correct. The law applies only to blackbodies, theoretical surfaces that absord all incident heat radiation.[1]

The law can be derived from Planck's law by considering a small flat blackbody surface radiating out into a half-sphere.[2]



The intensity of the light emitted from the blackbody surface is given by Planck's law:

$$I\left(\nu,T\right)=\frac{2h\nu^{3}}{c^{2}}\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{1}$$

where I(ν ,T) is the amount of power per unit surface area per unit solid angle per unit frequency emitted at a frequency ν emitted by a black body at temperature T, h is Planck's constant, c is speed of light, k is Boltzmann's constant.[2]
The quantity $I(\nu,T)Ad\nu d\Omega$ is the power radiated by a surface of area A through a solid angle $d\Omega$ in the frequency range between v and v+dv.
The Stefan-Boltzmann law gives the power emitted per unit area:

$$\frac{P}{A}=\int\limits_{0}^{\infty}I\left(\nu,T\right)d\nu\int\cos\theta d\Omega \tag{2}$$

The cosine appears because black bodies are Lambertian meaning that the intensity observed along the sphere will be the actual intensity times the cosine of the zenith angle.[2]

$$\frac{P}{A}=\int\limits_{0}^{\infty}I\left(\nu,T\right)d\nu\int\limits_{0}^{2\pi}d\varphi\int\limits_{0}^{\pi/2}\cos\theta\sin\theta d\theta=\pi\int\limits_{0}^{\infty}I\left(\nu,T\right)d\nu \tag{3}$$

Making a substitution, $u=\frac{h\nu}{kT}$ and $du=\frac{h}{kT}d\nu$

$$\frac{P}{A}=\frac{2\pi h}{c^{2}}\left(\frac{kT}{h}\right)^{4}\int\limits_{0}^{\infty}\frac{u^{3}}{e^{u}-1}du \tag{4}$$

Which gives

$$\frac{P}{A}=\sigma T^{4},\sigma=\frac{2\pi^{5}k^{4}}{15c^{2}h^{3}}. \tag{5}$$

Apparatus

- **Stefan-Bolztman Lamp**: Used as radiation source in first parts of the experiment.
- **Ammeter**: Used to measure the current in the system.
- **Voltmeter**: Used to measure the voltage in the system.
- **Electrical Oven**: Used to increase temperature steadily in the last part of the experiment.
- **Temperature Probe with Display**: Used to measure temperature.
- **Radiation Sensor**: Read radiation is turned to voltage through means.
- **Ruler**: Used to measure distance between between the lamp and the radiation sensor.
- **Multimeters**: Used to measure/apply smaller values of voltages and currents than otherwise possible.
- **Voltage Amplifier**: Used to amplify low voltage.

Procedure

1. We connected radiation sensor and the Stefan-Boltzman lamp to voltmeter and a power source respectively.
2. We have adjusted our read values to get rid of the effects of background but at some range the voltmeter did not work the way we wanted so we did not direclty see the effects of the background radiation.
3. We have connected multimeters to the set-up and then applied voltage at 0-20mV range. We have measured the resistance from these voltages and measured voltage from the sensor.
4. Inverse Square Law Part: We have created a substantial radiation from the lamp then have taken datas at different distances from the sensor to see the inverse square law in the sensor voltage region of 3×10^{-2}V and 10^{-3}V.

5. High temperature part: We have measured the output voltage at the 3 different distances between the lamp and the sensor. We have applied various different voltages and currents.

6. Low temperature part: By using an electrical oven we have increased temperature of our system and measured the voltage output in our sensor. We have placed the sensor to an appropriate distance from the oven and also we placed a shielding between them to reduce reflections. We collected data up to $400^o C$.

Data

Table 12.1 Temperature and Resistivity as a function of the Relative resistance of the Tungsten filament

R/R_{300K}	Temp. (°K)	Resistivity $\mu\Omega$·cm	R/R_{300K}	Temp. (°K)	Resistivity $\mu\Omega$·cm
1.0	300	5.65	10.63	2100	60.06
1.43	400	8.06	11.24	2200	63.48
1.87	500	10.56	11.84	2300	66.91
2.34	600	13.23	12.46	2400	70.39
2.85	700	16.09	13.08	2500	73.91
3.36	800	19.00	13.72	2600	77.49
3.88	900	21.94	14.34	2700	81.04
4.41	1000	24.93	14.99	2800	84.70
4.95	1100	27.94	15.63	2900	88.33
5.48	1200	30.98	16.29	3000	92.04
6.03	1300	34.08	16.95	3100	95.76
6.58	1400	37.19	17.62	3200	99.54
7.14	1500	40.36	18.28	3300	103.3
7.71	1600	43.55	18.97	3400	107.2
8.28	1700	46.78	19.66	3500	111.1
8.86	1800	50.05	26.35	3600	115.0
9.44	1900	53.35			
10.03	2000	56.67			

Potential Difference (mV)	Current (mA)	Potential Difference (mV)	Current (mA)
0.8	2.75	9.2	31.31
1.0	3.53	9.9	33.73
1.7	5.76	12.6	42.90
2.3	7.40	14.5	49.40
4.1	14.09	16.4	55.70
5.5	18.66	17.8	6.40
7.4	25.23	19.5	66.60

Figure 1: Data for initial part of the experiment. Errors for potential difference is 0.1mV and the error for the current is 0.01mA.

Potential Difference (V)	Distance (cm)	Potential Difference (V)	Distance (cm)
$0.45\times10^{-2} \pm 0.05 \times 10^{-2}$	3.0±0.1	$0.9\times10^{-3} \pm 0.05 \times 10^{-3}$	11.0±0.1
$0.35\times10^{-2} \pm 0.05 \times 10^{-2}$	4.0±0.1	$0.85\times10^{-3} \pm 0.05 \times 10^{-3}$	11.5±0.1
$0.25\times10^{-2} \pm 0.05 \times 10^{-2}$	5.0±0.1	$0.75\times10^{-3} \pm 0.05 \times 10^{-3}$	12.0±0.1
$0.20\times10^{-2} \pm 0.05 \times 10^{-2}$	6.0±0.1	$0.7\times10^{-3} \pm 0.05 \times 10^{-3}$	13.0±0.1
$1.8\times10^{-3} \pm 0.2 \times 10^{-3}$	7.0±0.1		
$1.6\times10^{-3} \pm 0.2 \times 10^{-3}$	7.5±0.1		
$1.4\times10^{-3} \pm 0.2 \times 10^{-3}$	8.0±0.1		
$1.4\times10^{-3} \pm 0.2 \times 10^{-3}$	8.5±0.1		
$1.2\times10^{-3} \pm 0.2 \times 10^{-3}$	9.0±0.1		
$1.0\times10^{-3} \pm 0.2 \times 10^{-3}$	10.0±0.1		

Figure 2: Data for the inverse square law part of the experiment with 0.9V of applied voltage and 2.3A of applied current.

Applied Potential Difference (V)	Applied Current(A)	Measured Potential Difference (V)
6.77	2.232	1.8×10−3
7.15	2.287	2.2×10−3
5.60	2.015	1.4×10−3
3.93	1.688	0.8×10−3
3.088	1.509	0.6×10−3

Figure 3: Data for the high temperature part of the experiment for distance between the lamp and the sensor 5.4±0.1cm.The uncertainty in applied voltage is 0.01 Volts, The uncertainty in applied current is 0.001 Ampere and the uncertainty in measured voltage is 0.0001 Volts.

Applied Potential Difference (V)	Applied Current(A)	Measured Potential Difference (V)
3.511	1.601	$0.4 \times 10{-}3$
5.45	1.995	$0.8 \times 10{-}3$
6.40	2.162	$1.0 \times 10{-}3$
7.14	2.285	$1.2 \times 10{-}3$
7.75	2.386	$1.4 \times 10{-}3$

Figure 4: Data for the high temperature part of the experiment for distance between the lamp and the sensor 8.4 ± 0.1cm.The uncertainty in applied voltage is 0.01 Volts, The uncertainty in applied current is 0.001 Ampere and the uncertainty in measured voltage is 0.0001 Volts.

Applied Potential Difference (V)	Applied Current(A)	Measured Potential Difference (V)
7.76	2.386	$0.8 \times 10{-}3$
7.09	2.274	$0.8 \times 10{-}3$
8.22	2.458	$1.0 \times 10{-}3$
5.94	2.075	$0.6 \times 10{-}3$
4.64	1.832	$0.4 \times 10{-}3$

Figure 5: Data for the high temperature part of the experiment for distance between the lamp and the sensor 11.4 ± 0.1cm.The uncertainty in applied voltage is 0.01 Volts, The uncertainty in applied current is 0.001 Ampere and the uncertainty in measured voltage is 0.0001 Volts.

Temperature(^oC)	Potential Difference(V)	Temperature(^oC)	Potential Difference(V)
29.5 ±0.1	0.05×10−3	79.5±0.1	1.2×10−3
31.1 ±0.1	0.1×10−3	85.5±0.1	1.4×10−3
33.5 ±0.1	0.15×10−3	93.6±0.1	1.6×10−3
36.0 ±0.1	0.20×10−3	100.0±0.1	1.8×10−3
38.3±0.1	0.25×10−3	112.8±0.1	2.2×10−3
41.3±0.1	0.30×10−3	124.3±0.1	2.6×10−3
43.9±0.1	0.35×10−3	139.4±0.1	3.0×10−3
45.6±0.1	0.40×10−3	162.7±0.1	4.0×10−3
48.4±0.1	0.45×10−3	203.7±0.1	6.0×10−3
50.8±0.1	0.50×10−3	220.4±0.1	7.0×10−3
53.3±0.1	0.55×10−3	236.5±0.1	8.0×10−3
55.8±0.1	0.60×10−3	252.3±0.1	9.0×10−3
58.0±0.1	0.65 ×10−3	303.6±0.1	14.0×10−3
62.8±0.1	0.75×10−3	342.4±0.1	18.0×10−3
64.4±0.1	0.80×10−3	374.8±0.1	22.0×10−3
69.1±0.1	0.90×10−3		

Figure 6: Data for the low temperature part of the experiment.Errors for potential difference are 0.01×10−3 for the left part and 0.1×10−3 for the right part.

s

Analysis

Some useful formulas:
Error propagation

$$\sigma_f = \sqrt{\sum_i^n \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2 + \dots} \tag{6}$$

Weighted average

$$f_{weighted} = \frac{\sum_i \frac{f_i}{\sigma_i{}^2}}{\sum_i \frac{1}{\sigma_i{}^2}} \tag{7}$$

Where σ_i is the uncertainty in f_i

By taking the weighted sum of the $R_{300} = voltage/current$ we get R_{300} (the error in R_{300} is not written because it is not needed for rest of the analysis)

$$R_{300} = \frac{\sum_i \frac{R_i}{\sigma_i{}^2}}{\sum_i \frac{1}{\sigma_i{}^2}} = 0.2938\Omega \tag{8}$$

We have used the table's datas, given in the book, to fit a line to the graph of temperature vs. R/R_{300}. This way we can convert the resistance we have found in the later parts of the experiment to the corresponding temperature. There is no errorbar in this graph because the values are taken as given.

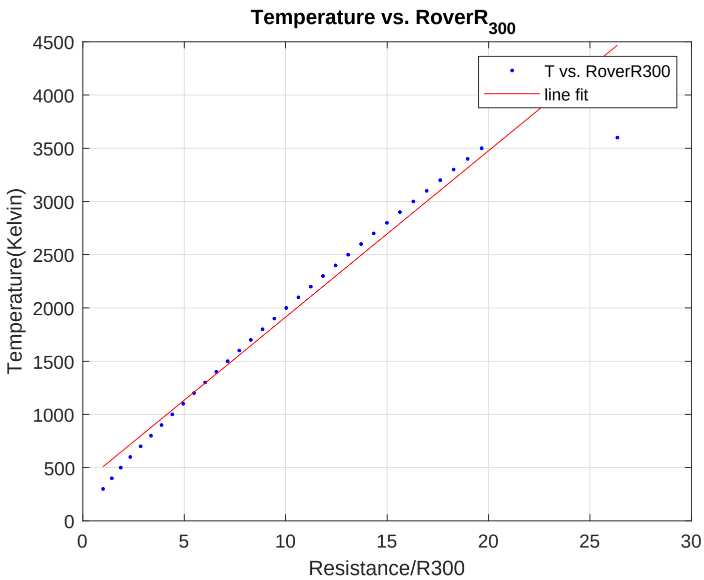


Figure 7: $f(x) = p_1 x^{-2} + p_2$ with $p_1 = 156.2 \pm 10.3$, $p_2 = 352.8 \pm 122.3$. R-square=0.9679

For the inverse square law part we first fit a polynomial to see look fo what to do next.

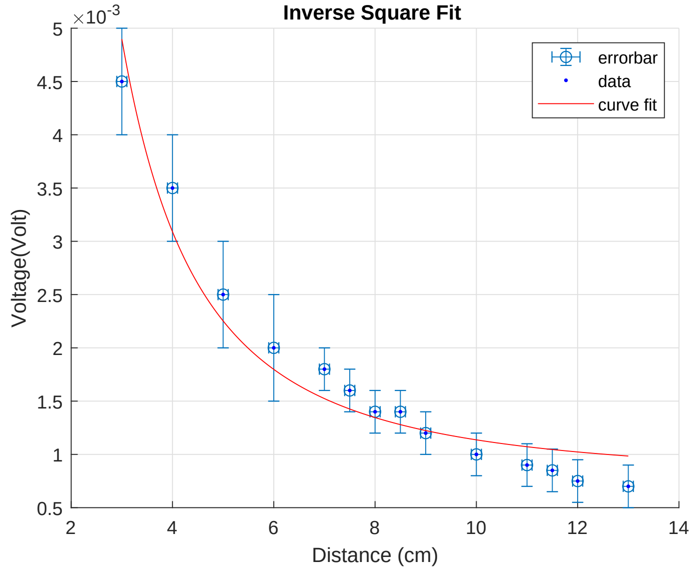


Figure 8: $f(x) = p_1 x^{-2} + p_2$ with $p_1 = 0.0372 \pm 0.0054$, $p_2 = 0.0007646 \pm 0.0002035$. R-square=0.9503

We create log-log graph for the inverse square fit so we can see the relation between the read voltage(intesity) and the distance.

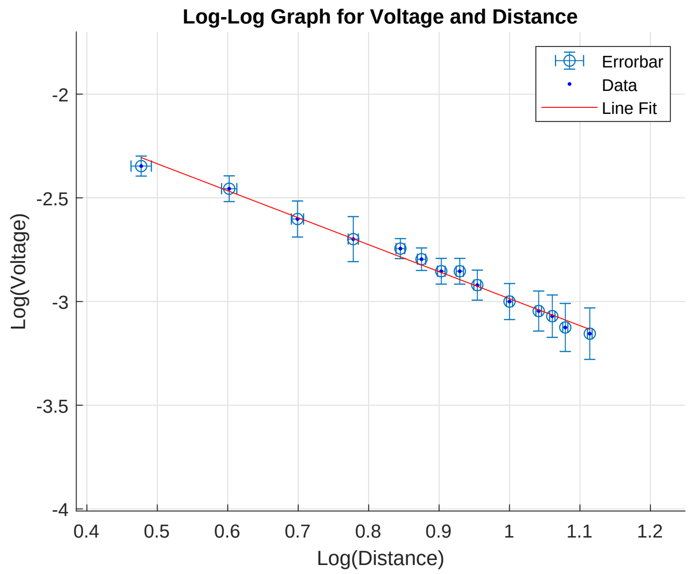


Figure 9: $f(x) = p_1 x + p_2$ with $p_1 = -1.301 \pm 0.082$, $p_2 = -1.685 \pm 0.075$. R-square=0.9899

We have found the proportionality constant n in $I \propto D^n$ as

-1.301 ± 0.082 which is 8.5σ away from the real value.

By using the temperature vs. R/R_{300} fit, we have obtained the Log(Voltage) vs. Log(Temperature) using the data for the d=5.4cm part:

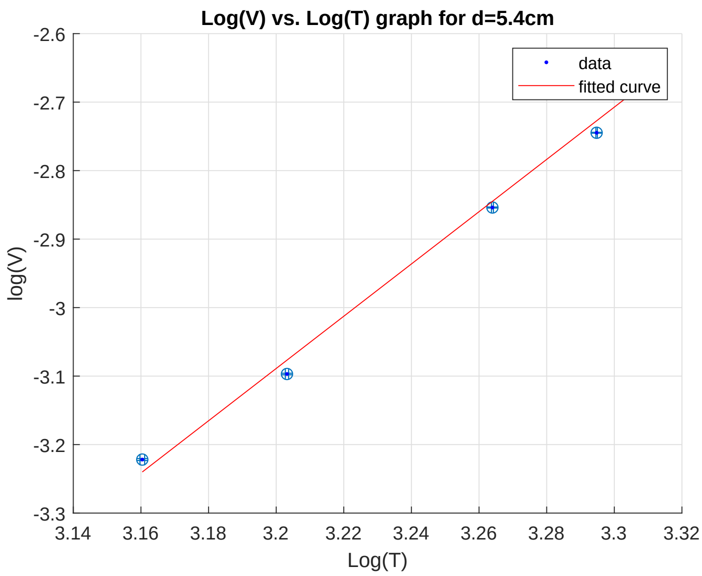


Figure 10: $f(x) = p_1 x + p_2$ with $p_1 = 5.742 \pm 0.871$, $p_2 = -19.5 \pm 2.5$. R-square=0.9932

From this graph we see the proportionality constant n in $I \propto T^n$ as $n = 5.742 \pm 0.871$.

By using the temperature vs. R/R_{300} fit, we have obtained the Log(Voltage) vs. Log(Temperature) using the data for the d=8.4cm part:

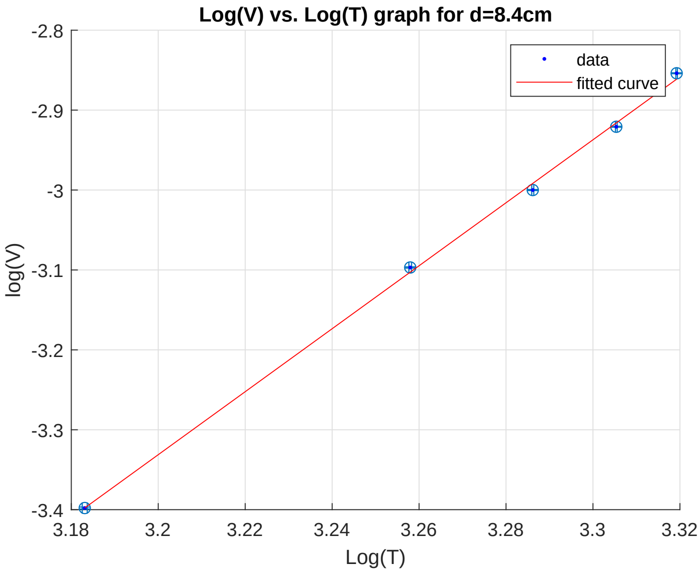


Figure 11: $f(x) = p_1 x + p_2$ with $p_1 = 5.859 \pm 0.349$, $p_2 = -20.07 \pm 1.01$. R-square= 0.999

From this graph we see the proportionality constant n in $I \propto T^n$ as $n = 5.859 \pm 0.349$.

By using the temperature vs. R/R_{300} fit, we have obtained the Log(Voltage) vs. Log(Temperature) using the data for the d=11.4cm part:

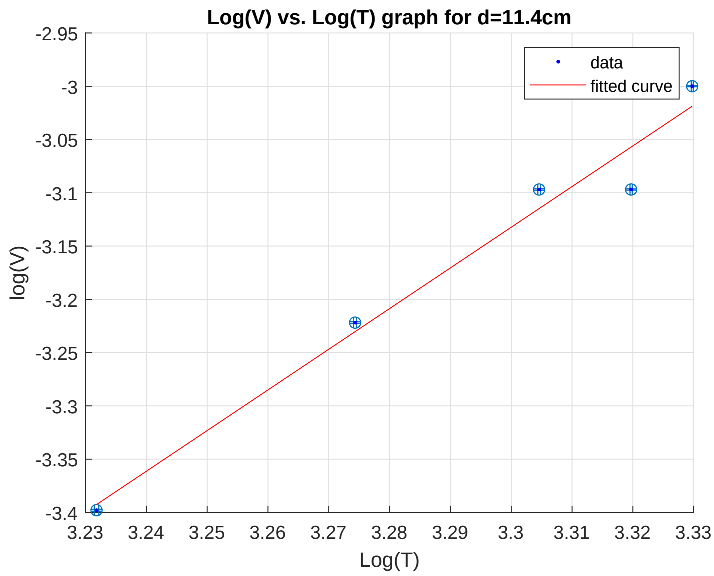


Figure 12: $f(x) = p_1 x + p_2$ with $p_1 = 5.543 \pm 1.655$, $p_2 = -19.34 \pm 4.83$. R-square= 0.9743

From this graph we see the proportionality constant n in $I \propto T^n$ as $n = 5.543 \pm 1.655$.

By taking their weighted average we get:

$$n_{weighted} = 5.832 \pm 0.338 \tag{9}$$

First we transform our Celsius datas to Kelvins by just adding 273.15 to Celsius datas. Then we fit a line to log(Voltage) vs. log(Temperature) graph to see the behavior at lower temperatures.

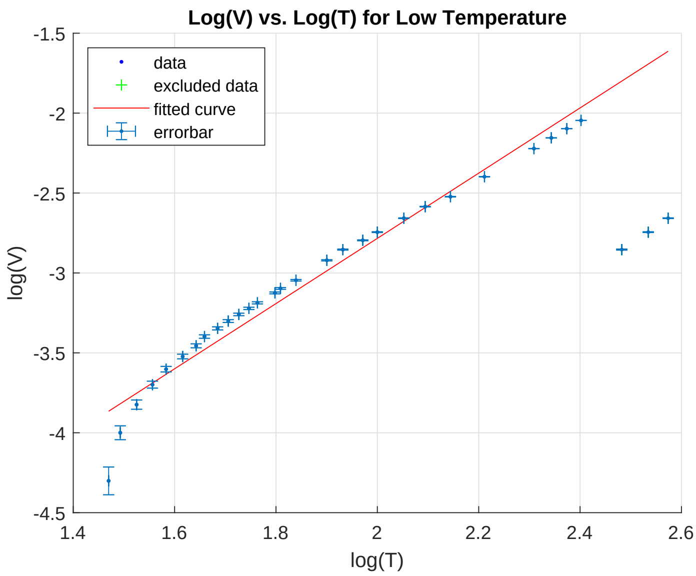


Figure 13: $f(x) = p_1x + p_2$ with $p_1 = 2.04 \pm 0.16$, $p_2 = -6.864 \pm 0.298$. R-square=0.9633

From this graph we see the proportionality constant n in $I \propto T^n$ as $n = 2.04 \pm 0.16$ for lower temperatures.

Taking weighted averages of part 2 and part 3 we get:

$$n_{weighted} = 2.73 \pm 0.45 \tag{10}$$

We have found the proportionality constant 2.82σ away from the expected value, 4.

Conclusion

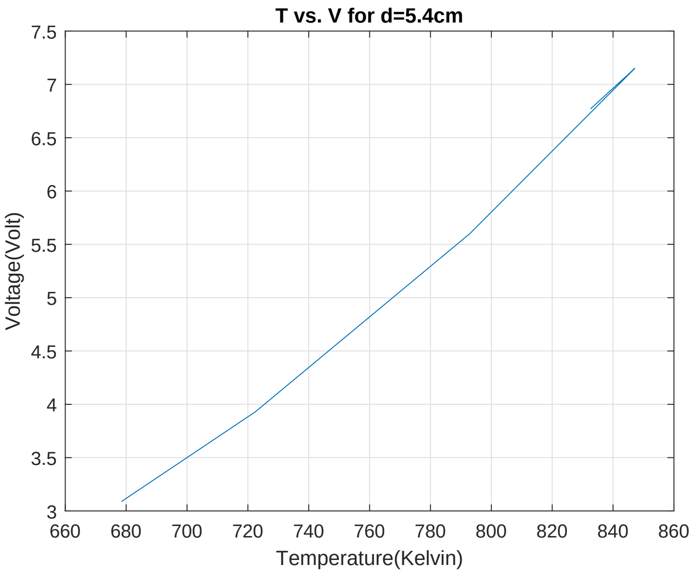
The proportionality constant we have found is not that close to the real value. But this was to be expected. We were not really careful about some parts of the experiment. We sometimes lean over to the lamp's side differing the value read by the sensor. Also the reflective surface all around the setup might have caused some problems. For low temperature part the electric oven was not examined beforehand so this might have caused a problem in that part. For the same reasons stated earlier the inverse square law part came as terrible. To improve these, we can isolate our setup from the background and human interventions and cover it with nonreflective surfaces; and the electric oven can be replaced with a new one.

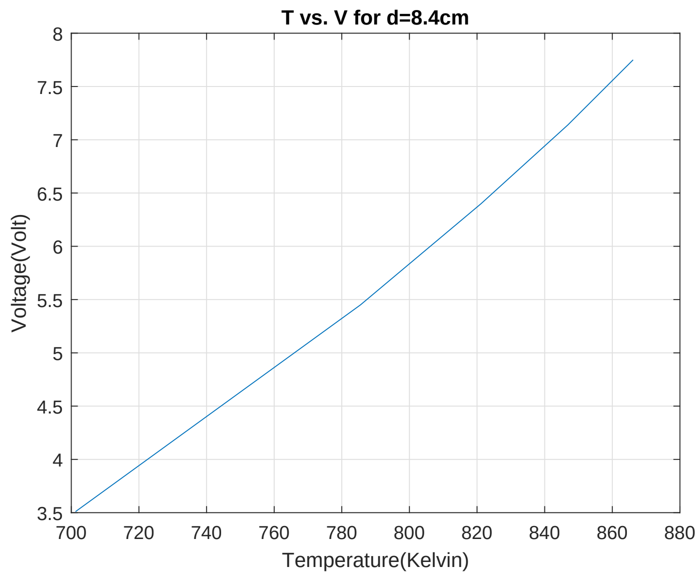
References

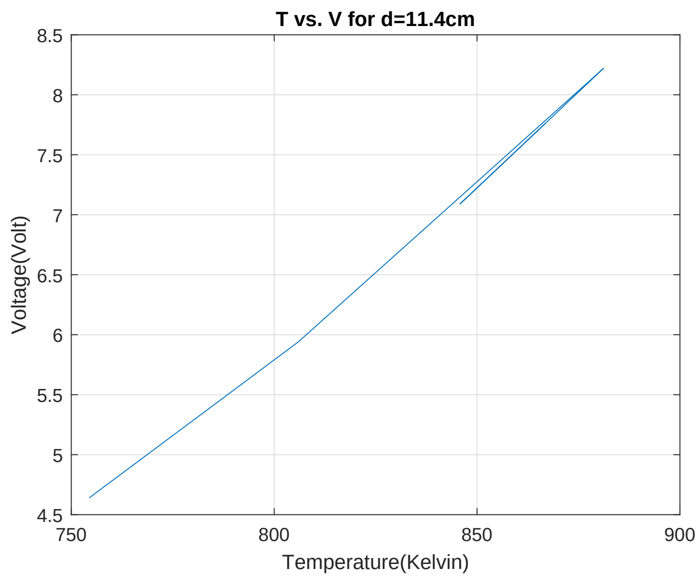
[1]A Brief History of the T^4 Radiation Law - Crepeau, John.
https://pdfs.semanticscholar.org/748d/6a04e78f8dd3bdc15bdb74365e1a9db0667d.pdf
[2]Wikipedia, Stefan-Boltzmann Law.
[3]Advanced Physics Experiments - Gülmez, Prof. Dr. Erhan.

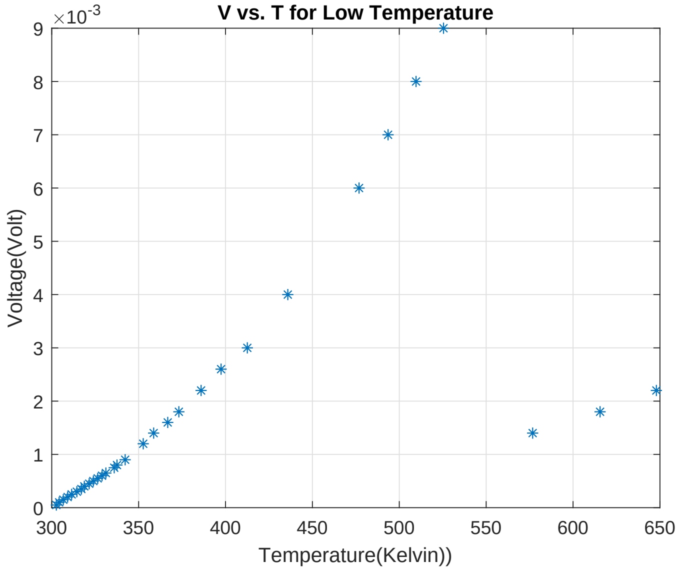
Appendix

The non logartihmic graphs of some parts that was not needed in the analysis:




T vs. V for d=8.4cm
Voltage(Volt)
Temperature(Kelvin)
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
700
720
740
760
780
800
820
840
860
880


T vs. V for d=11.4cm
Voltage(Volt)
Temperature(Kelvin)
8.5
8
7.5
7
6.5
6
5.5
5
4.5
750
800
850
900


V vs. T for Low Temperature
×10-3
9
8
7
6
5
4
3
2
1
0
Voltage(Volt)
300
350
400
450
500
550
600
650
Temperature(Kelvin))