Experiment 6, Scattering in 2D

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Abstract

The purpose of the experiment is to find the radius(or diameter) of the cylinder at the middle of our setup. To do this, we shoot balls that we consider as point particles to our cylinder and look at the scattering angles on the scattering tray. We find the cross section (i.e. half the radius of the cylinder) with two methods and compare them with the direct result we obtained from the vernier caliper.

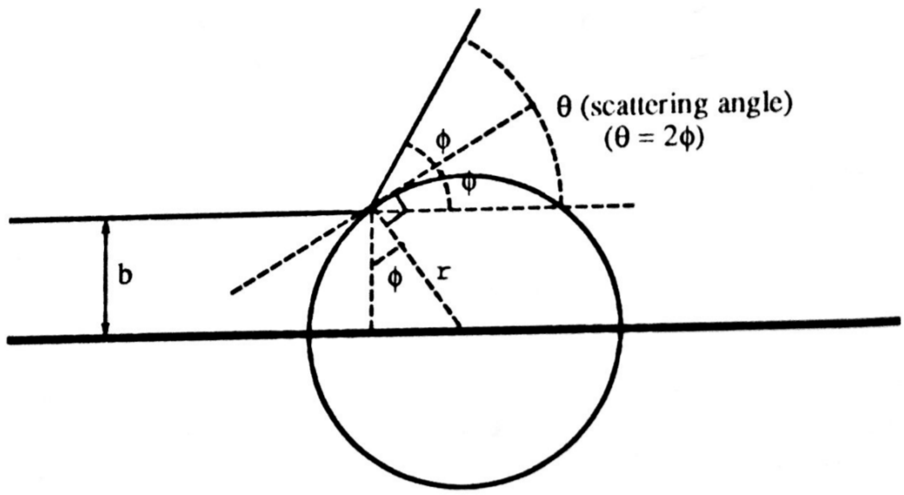
Theoretical Motivation

Scattering is basically getting the needed information from a source by throwing some things at it and reading the angles that those things scatter. Scattering is an integral part of the physics. For an example, when we want to find the radius of an atom, we shoot electrons to the atom (of course since atoms usually don't touch to each other, what we really determine is how far can electrons get close to the atom). Or in rare cases like fusion, we need to know the atom's/particle's scattering parameter so we can calculate how close we need to shoot at the chosen particle to maximize our chances. The examples go on.

The differential cross section is defined simply as the ratio of the number of scattered particles observed in a region normalized by corresponding solid angle to the total number of particles shot towards the target.

$$\frac{d\sigma}{d\Omega} = \frac{Y}{Id\Omega}$$

where Y is the yield in $d\Omega$, $d\Omega$ is the solid angle and I is the incident flux. This is the most general way we can express ourselves in this experiment.



The scattering angle is given by the relation $\frac{b}{r} = \cos(\phi)$ where $\theta = 2\phi$

$$\text{Then } \frac{b}{r} = \cos\left(\frac{\theta}{2}\right)$$

$$db = -\frac{1}{2} r \sin\left(\frac{\theta}{2}\right) d\theta$$

Then the number of particles scattered at a given angle is

$$dN = -Idb = \frac{1}{2} r I d\theta \sin\left(\frac{\theta}{2}\right)$$

There is a minus sign because N can not be a negative number and db is negative!

Another way of calculating the radius is the following:

$$\frac{d\sigma}{d\theta} = \frac{dN}{Id\theta}$$

Since we did not consider 3rd dimesion, solid angle equals to θ.

$$d\sigma = \frac{dN}{I}$$

$$\int_{boundary} d\sigma = \int_0^{2\pi} \frac{r}{2} \sin\left(\frac{\theta}{2}\right) d\theta$$

Then $\sigma = 2r = \sum_\theta \frac{dN}{I}$

Finally the error propagation

$$\sigma_f = \sqrt{\sum_i^n \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2 + \dots}$$

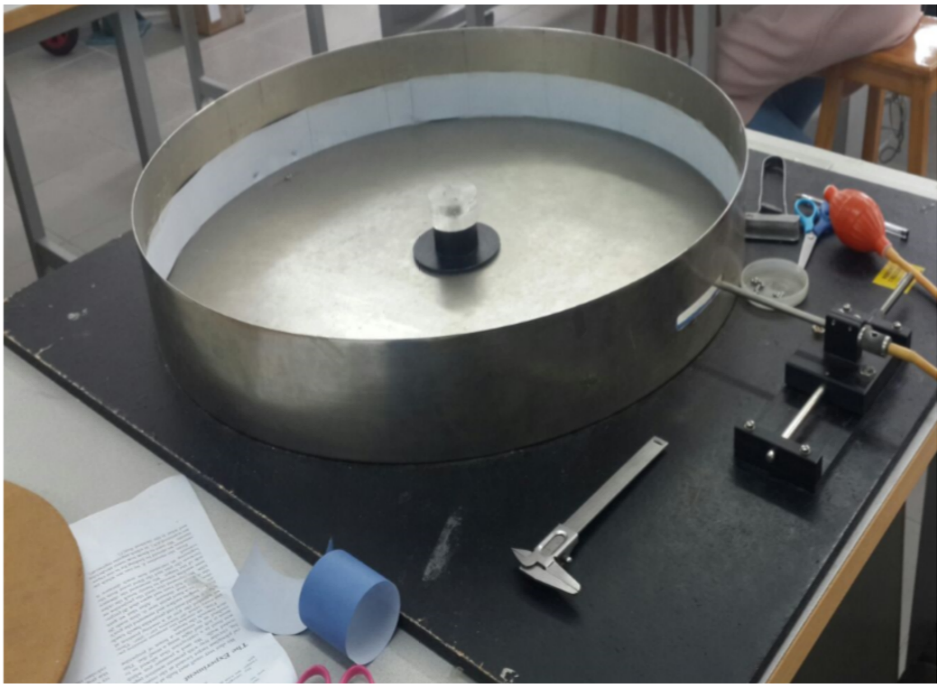
Then we find error on sine

$$\sigma_{sin\frac{\theta_i}{2}} = \sqrt{\left(\frac{1}{2}\cos\left(\frac{\theta_i}{2}\right)\right)^2 . \sigma_{\theta_i}^2}$$

And for $g = a \cdot \frac{x}{y}$ or $g = a.x.y$ there is a special case:

$$\frac{\sigma_g^2}{g^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2}$$

Apparatus



Apparatus	Description
20 Steel Balls	Used to shoot from our gun to the target and to measure scattering angle. Taken as point particles.
Scattering Tray	Consist of our target, the gun we use to shoot balls at the target and the screw that changes the position of the gun.
Pressure Sensitive Paper Tape	Steel balls leave a mark after hitting the tape stretch along the rim. These marks are used to take data accordingly
Vernier Caliper	Used to measure the diameter of the cylinder.
Ruler	Used to measure the initial and final position of the gun.

Experimental Procedure

- We stretch the tape along the rim carefully to not leave any marks and measure the gun's initial position with respect to some origin
- We shoot 20 balls from the gun with the same pressure for every location of the gun that hits the cylinder.
- After shooting 20 balls, change the location of the gun by turning the screw at the side of it. After ,again, we shoot 20 balls, we repeat the procerure until we no longer observe scattering.
- We record the number of screws and measure the final position of the gun.
- We mark the certaing angles, namely 10 30 50 etc, on the tape and count the number of marks caused by balls in each interval.
- We measure the diameter of the cylinder with vernier caliper.

Data

dθ	dN	Error of dN
20	45	6.7
40	21	4.6
60	27	5.2
80	36	6.0
100	45	6.7
120	59	7.7
140	45	6.7
160	63	7.9
180	62	7.9
200	41	6.4
220	20	4.5
240	29	5.4
260	44	6.6
280	48	6.9
300	33	5.7
320	9	3
340	14	3.7

Figure 1: The Data of $d\theta$, dN and error of dN for the full circle

dθ	dN	Error of dN
20	59	7.7
40	30	5.5
60	60	7.7
80	84	9.2
100	89	9.4
120	88	9.4
140	65	8.1
160	104	10.2
180	124	11.1

Figure 2: The Data of $d\theta$, dN and error of dN by taking the symmetry about 180 degree

Distance between the initial and the final position of the gun: $\Delta = 0.068 \mp 0.001 meters$

Diameter of the cylinder as measured by the vernier caliper: $2r = 0.0577 \mp 0.0001 meters$

Number of screws:48

Analysis

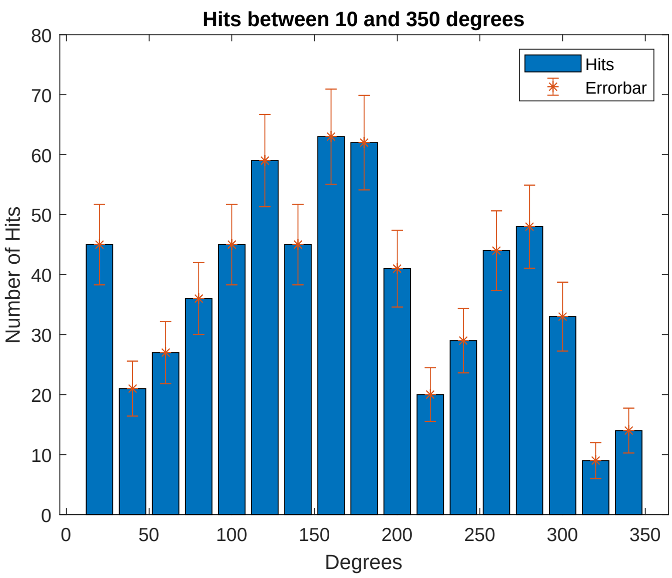


Figure 3: Unsymmetrized scattering histogram with errorbars. Values of errors are given in the data part.

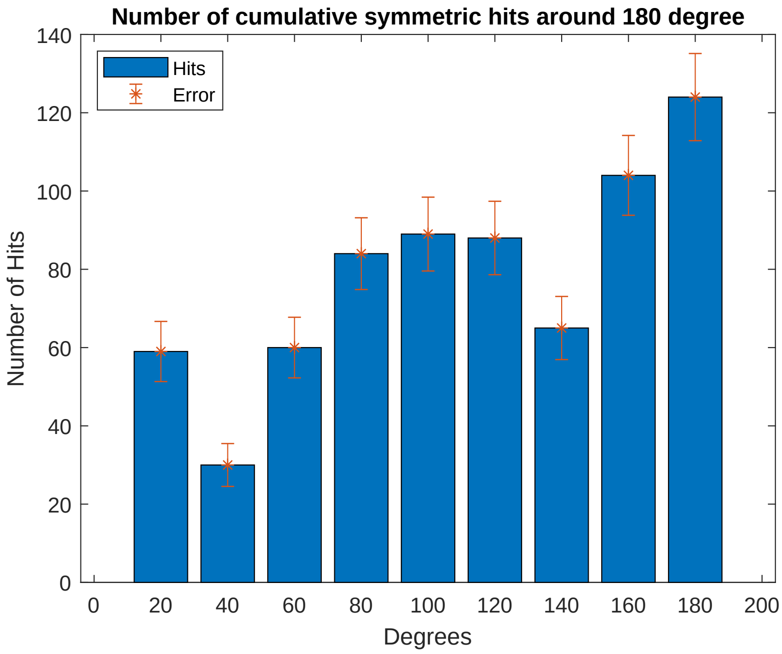


Figure 4: The symmetrized histogram of scattering around 180 degrees with errorbars. Values of errors are ,again, given in the data part.

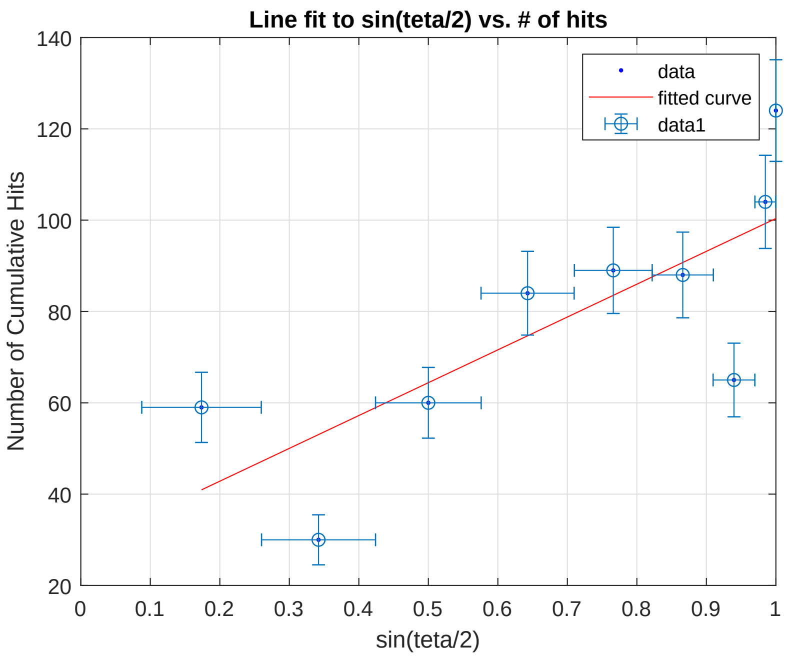


Figure 5: Sine vs dN, R-square: 0.59, Adjusted R-square: 0.5315, RMSE: 19.07, SSE: 2545.

$$I = \frac{\frac{number\ of\ shots\ in\ each\ turn}{\Delta}}{total\ number\ of\ screws}$$

Then (with error propagation) $I = 1.4\times10^4 \mp 2.0\times10^2\ shots\ per\ meter$

By looking at the slope of the line fit to our sine data, we calculate the radius:

$$dN = \frac{1}{2} r I d\theta \sin\left(\frac{\theta}{2}\right)$$

Here the slope is $\frac{1}{2} r I d\theta$

The slope of the sine fit is $p1 = 71.91 \mp 26.78$. Then

$$p1 = \frac{1}{2} r I d\theta$$

where $d\theta = \frac{\pi}{9} \mp \frac{\pi}{18} radians$

Then (with error propagation): $2r = 0.059 \mp 0.037 meters$

Another way to find 2r: $2r = \sum_\theta \frac{dN}{I}$

Since the flux is constant $2r = \frac{total\ number\ of\ marks}{I} = \frac{703}{1.4\times10^2} meters$

Then (with error propagation): $2r = 0.050 \mp 0.0020 meters$

Error of the first method: $\frac{|0.0577-0.059|}{0.037} = 0.035$

Error of the second method $\frac{|0.0577-0.050|}{0.0020} = 3.8$

Conclusion

We found the diameter of the cylinder with two methods. Compared to the value we take from the vernier caliper, first method yield better result and we were only 0.035σ away from the vernier caliper value. Second method ,however, gave a much worse result and we were 3.8 σ away from the vernier caliper value. This result may be caused by the low number of marks we got on the tape.

We should have gotten 960 marks (because we shot 20 in each turn and there are 48 turns) but we have only read 703 marks on the tape. To fix this we may insert an additional uncertainty on our histograms by distributing the missing number of hits equally on each bar. Other type of approaches can be tried as well. Same error may have caused some of the data points on the line fit graph to be too away from the line because R-square of the line is not so close to 1. Especially first two data points seem to be affected greatly from this error. Another error, this time affecting both of the methods, is us(humans). We may not have squeezed equally in each shot, we may have forgotten to shoot 20 balls in each turn, we may have left additional marks on the tape by touching them etc. Another type of error can be caused by our system. The rim, the cylinder and the balls may not be perfectly circular. To reduce these systematic and human-caused errors, we can make our sytem a more precise and technologial one that does not involve human interaction as much.

Github link: https://github.com/OnuraySancar/phys442-2d-scattering.git

References

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https://en.wikipedia.org/wiki/Scattering

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https://www.mathworks.com/help/matlab/ref/errorbar.html