Experiment 5, The Poisson Statistics

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with **Yaren AKSEL**, **Experiment Date**:March 8, 2019

Abstract

The purpose of our experiment is to measure and observe the behavior of the number of emissions from two different sources in given time intervals (namely 1 and 10 seconds) by using Geiger counter. We plot histograms of these datas and observe that they are distributed with Poisson or/and Gaussian distribution. They are distributed in these ways because decaying of gamma rays is an independent process and we can histogram them in a meanigful way.

Theoritical Motivation

Gamma ray decaying is a discrete process and shows a nondeterministic statistical behavior. This is due to the radioactivity being a quantum mechanical construct. We use ^{137}Cs source for our experiment because its decaying process can be expressed in a predictable average rate. Because of the reasons stated in abstract, we model it with a Poisson distribution. From the distribution we can calculate the half time (or decaying rate) of our material.Now let's define Poisson distribution!

Poisson distribution can be obtain with modifications on Binomial distributions. The Binomial distribution models the success of an event x with a given probability p over n measurements, and is given by the equation:

$$Pr(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \tag{1}$$

If we take $\lambda = p.n$, as n goes to infinity :

$$\lim_{x\to\infty} \frac{n!}{x!(n-x)!} \frac{\lambda}{n}^x (1-\frac{\lambda}{n})^{n-x} \tag{2}$$

$$\lim_{x\to\infty} \underbrace{\frac{n!}{n^x(n-x)!}}_{\approx 1} \frac{\lambda^x}{x!} \underbrace{(1-\frac{\lambda}{n})^n}_{\approx e^{-\lambda}} \underbrace{(1-\frac{\lambda}{n})^{-x}}_{\approx 1} \tag{3}$$

$$\approx \frac{\lambda^x e^{-\lambda}}{x!} \tag{4}$$

Which gives the Poisson distribution. i.e.

$$P(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{5}$$

where λ is the mean of the probability distribution.

Standard deviation of Poisson distribution is extremely useful!

$$< x^2 >= \sum_{x=0}^{\infty} \frac{x^2 \lambda^x e^{-\lambda}}{x!}$$

$$< x^2 >= e^{-\lambda} \sum_{x=1}^{\infty} \left(x\left(x-1\right)+x\right) \frac{\lambda^x}{x!}$$

$$< x^2 >= \lambda^2 e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{(x)!} + e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$< x^2 >= \lambda^2 + \lambda$$

Then $\sigma^2 =< x^2 > - < x >^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$
This is a really interesting result!

Poisson distibution approaches the Gaussian distribution as λ becomes large. A normalized distribution with the mean μ and the standard deviation σ is expressed as (From the Sterling's formula for x!):

$$x! \to \sqrt{2\pi x}\, e^{-x} x^x \quad \text{as} \quad x \to \infty$$

$$\begin{aligned} p(x) &= \frac{\lambda^{\lambda(1+\delta)} e^{-\lambda}}{\sqrt{2\pi} e^{-\lambda(1+\delta)} [\lambda(1+\delta)]^{\lambda(1+\delta)+1/2}} \\ &= \frac{e^{\lambda\delta}(1+\delta)^{-\lambda(1+\delta)-1/2}}{\sqrt{2\pi\lambda}} \\ &= \frac{e^{-\lambda\delta^2/2}}{\sqrt{2\pi\lambda}} \end{aligned}$$

Substituting back for x, with $\delta = \frac{x-\lambda}{\lambda}$, yields

$$p(x) = \frac{e^{-(x-\lambda)^2/(2\lambda)}}{\sqrt{2\pi\lambda}}$$

Hence, it is not easy to distinguish between the Poisson and Gaussian distribution as λ becomes large. However, the Poisson processes show a different behavior when it comes to distribution of successive events.

The probability of observing n counts during a time interval t is (where $\alpha = \frac{\lambda}{t}$)

$$P(\lambda, n) = P(\alpha, t, n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$$

The probability of having one event in time interval dt is

$$P(\alpha, dt, 1) = \frac{(\alpha dt)^n e^{-\alpha dt}}{1!}$$

The probability of having n events in t interval plus another event within a time dt is

$$\begin{aligned} P_q(n+1, t)dt &= P(\alpha, t, n) P(\alpha, dt, 1) \\ &= \frac{(\alpha t)^n e^{-\alpha t}}{n!} \frac{(\alpha dt)^n e^{-\alpha dt}}{1!} \\ &\simeq \frac{(\alpha t)^n e^{-\alpha t} \alpha dt}{n!} \end{aligned}$$

$$P_q(n+1, t) = \frac{(\alpha t)^n e^{-\alpha t} \alpha}{n!}$$

For our purposes n will be limited to 0 and 1.

χ^2 is a useful tool for evaluating whether our fits are good or not. We are mainly ineterested in χ^2_ν where ν is degrees of freedom and $\chi^2_\nu = \frac{\chi^2}{\nu}$ is chi squared per degrees of freedom $(N - k = \nu)$.

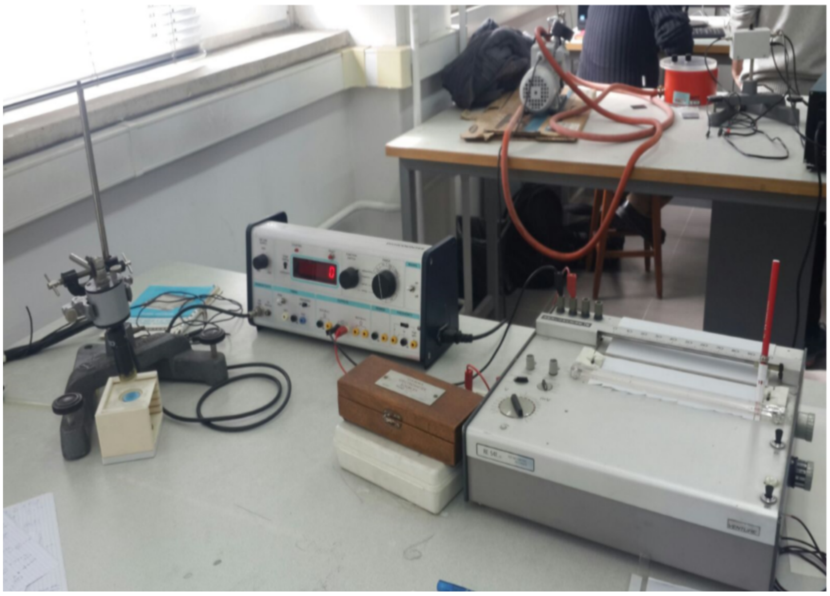
Chi squared of a normalized distribution is $\chi^2_{normalized} = \sum_a \frac{(y_a - y(x_a))^2}{\sigma_a^2}$

$\chi^2_{poisson} = \sum_{i=n} \frac{(y(n) - B.P(\lambda,n))^2}{\sigma_{y_i}^2}$ and $\chi^2_{gaussian} = \sum_{i=n} \frac{(y(n) - B.G(\lambda,n))^2}{\sigma_{y_i}^2}$

Where B is the bin width, N is number of bins, P and G are values of normalized fits and k is number of parameters in each fit.

Apparatus

Apparatus	Description
Gamma Ray Sources	Sources of our gamma rays. We picked CS137(10 UC) and CS137(1.0 UC)
Sample Holder	Consists of Trays we put our sources onto.
Chart Recorder	Records the peaks caused by gamma rays
Geiger Counter with Scaler	Gamma rays are hitting in the Geiger tube. There is a potential difference in the system and gamma rays are causing some type of photoelectric effect and the scaler changes potential and reads the number of hits in a given interval
Lead Absorbers	Absorbs the radiation.



Experimental Procedure

First we calibrate the voltage in the system. To do this we set our time intervals as 100 seconds and put a source emitting about 1000 rays in that interval. Then we change the values of potential difference by 20V in each 100 seconds starting from the 300V. We take the record of the number of rays counted after every interval is finished. We observe that we reach plateau at some point and after than that increasing voltage does not really affect the counts read. We call this region a plateau region. We pick a voltage value in this region as an operating voltage We picked 460V as operating voltage.

We place CS137(10 UC) in the top 3rd tray and record the number of counts for 100 10 seconds intervals. Repeat for 100 1 second intervals.

We place CS137(1.0 UC) in the top tray and record the number of counts for 100 10 seconds intervals. Repeat for 100 1 second intervals.

Finally we put a source that is emitting approximately 1 ray in each secon on the trays and let the chart recorder record 100 peaks. These 100 peaks will help us find n=0 and n=1 Poisson disribution.

Data

Raw 400 data and raw peak distances will be shown in the histograms of the analysis part!

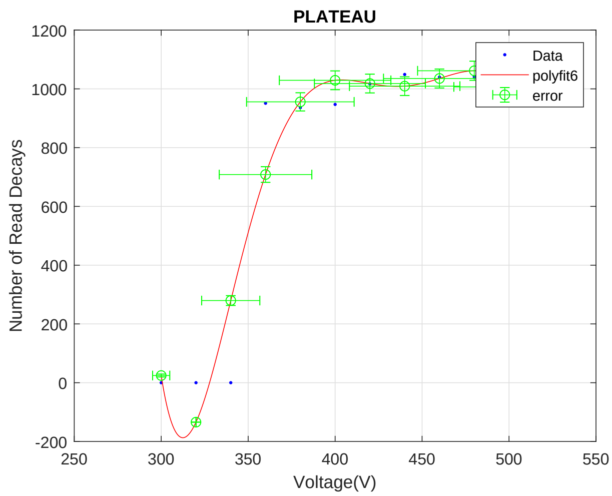
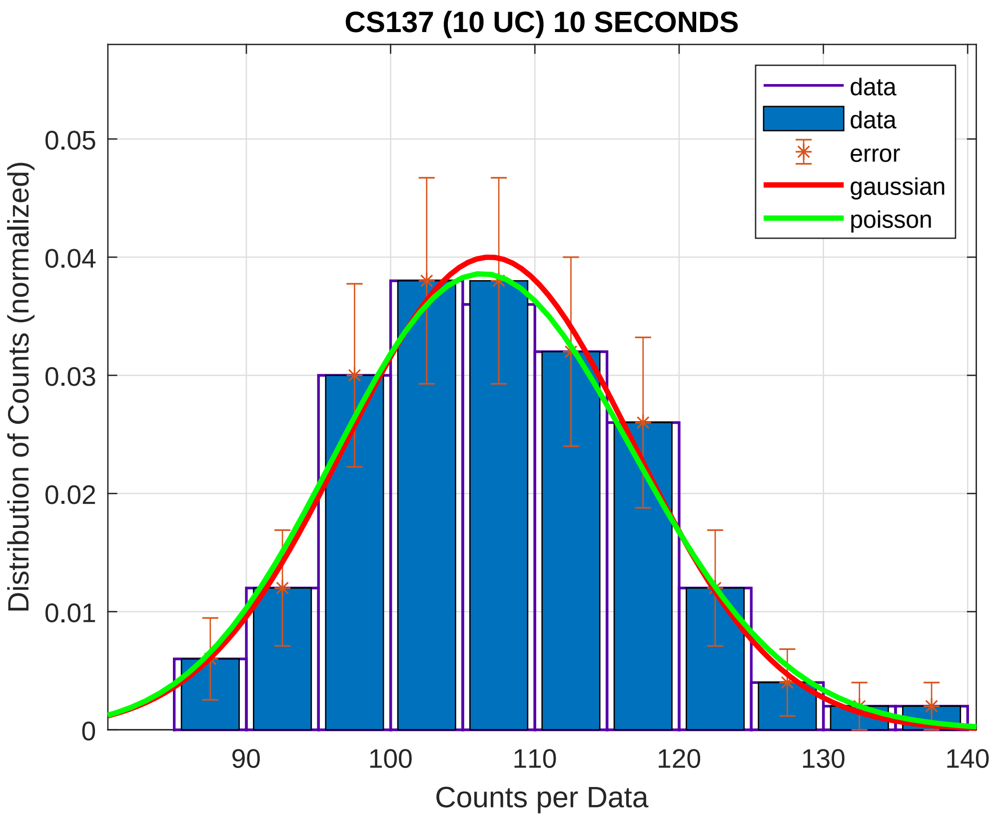


Figure 1: Observation of plateau region. We picked 460V from plateau! Error bars look bad because of the scaling of x and y axis.

Analysis

For CS137(10UC) 10 second;



$\chi^2_{poisson} = 11.2223$, we have 11 bins and Poisson distribution has 1 parameters. So

$\chi^2_{\nu,poisson} = 1.12223$
$\lambda = 106.86 \mp 1.01$

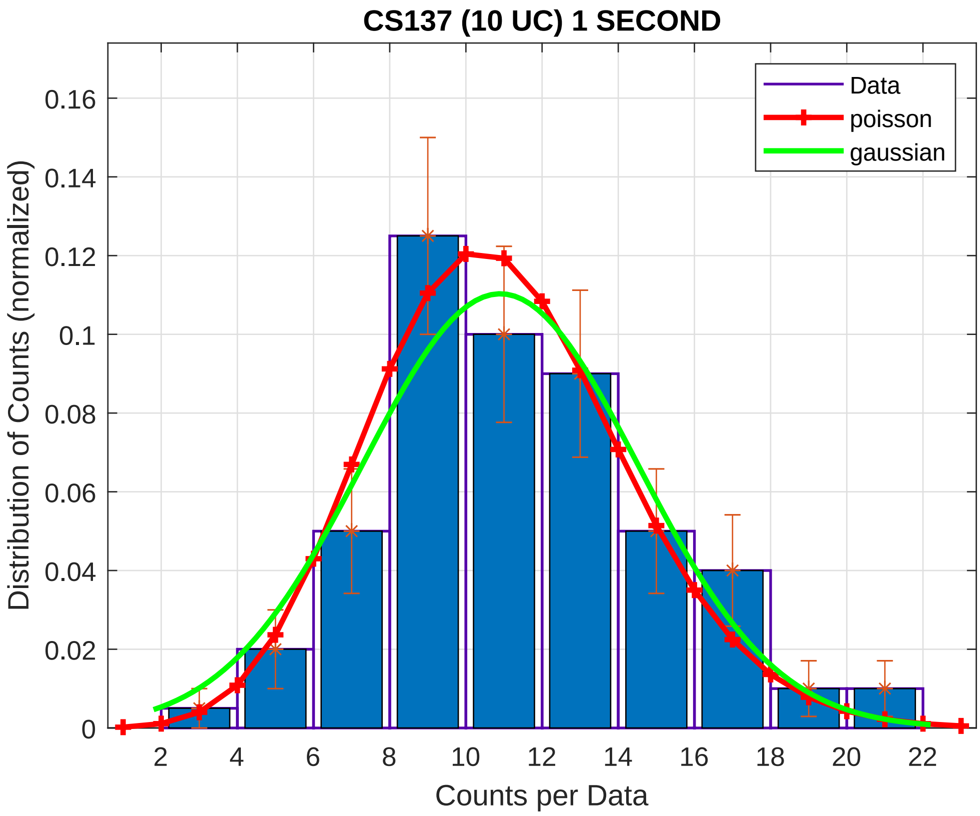
$\chi^2_{gaussian} = 11.6763$, we have 11 bins and Gaussian distribution has 2 parameters. So

$\chi^2_{\nu,gaussian} = 1.29737$

$\mu = 106.86 \mp 0.99$

$\sigma = 9.97271 \mp 0.60830$

For CS137(10UC) 1 second;



$\chi^2_{poisson} = 2.3335$, we have 10 bins and Poisson distribution has 1 parameters. So

$\chi^2_{\nu,poisson} = 0.25928$

$\lambda = 10.9 \mp 0.3$

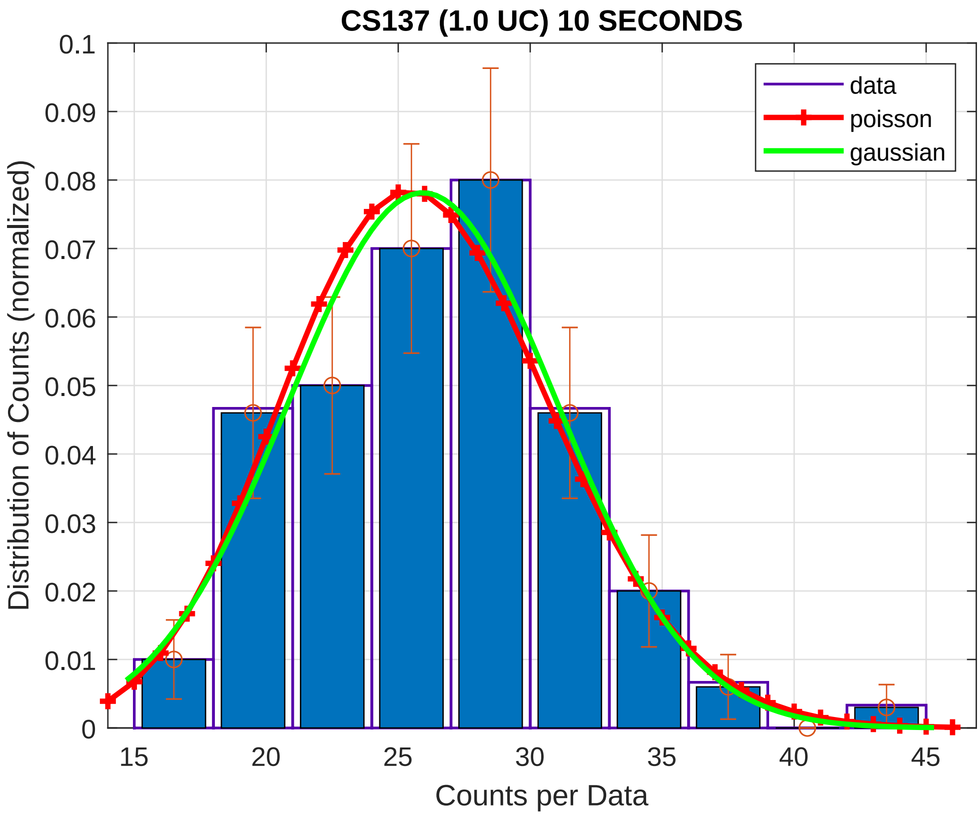
$\chi^2_{gaussian} = 2.1036$, we have 10 bins and Gaussian distribution has 2 parameters. So

$\chi^2_{\nu,gaussian} = 0.26295$

$\mu = 10.9 \mp 0.4$

$\sigma = 3.61674 \mp 0.22061$

For CS137(1.0 UC) 10 second;



$\chi^2_{poisson} = 5.4963$, we have 10 bins and Poisson distribution has 1 parameters. So

$\chi^2_{\nu,poisson} = 0.61070$

$\lambda = 25.93 \mp 0.50$

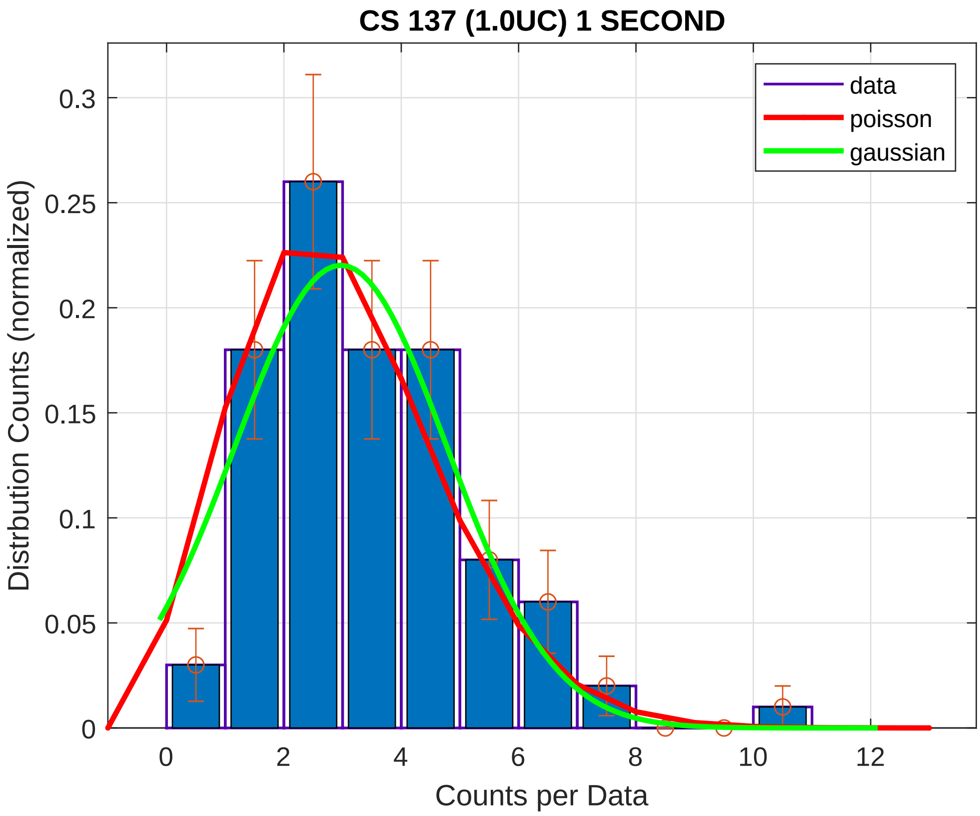
$\chi^2_{gaussian} = 5.4334$, we have 10 bins and Gaussian distribution has 2 parameters. So

$\chi^2_{\nu,gaussian} = 0.67918$

$\mu = 25.93 \mp 0.51$

$\sigma = 5.10546 \mp 0.31142$

For CS137(1.0 UC) 1 second;



$\chi^2_{poisson} = 0.1734$, we have 11 bins and Poisson distribution has 1 parameters. So

$\chi^2_{\nu,poisson} = 0.01734$

$\lambda = 2.97 \mp 0.17$

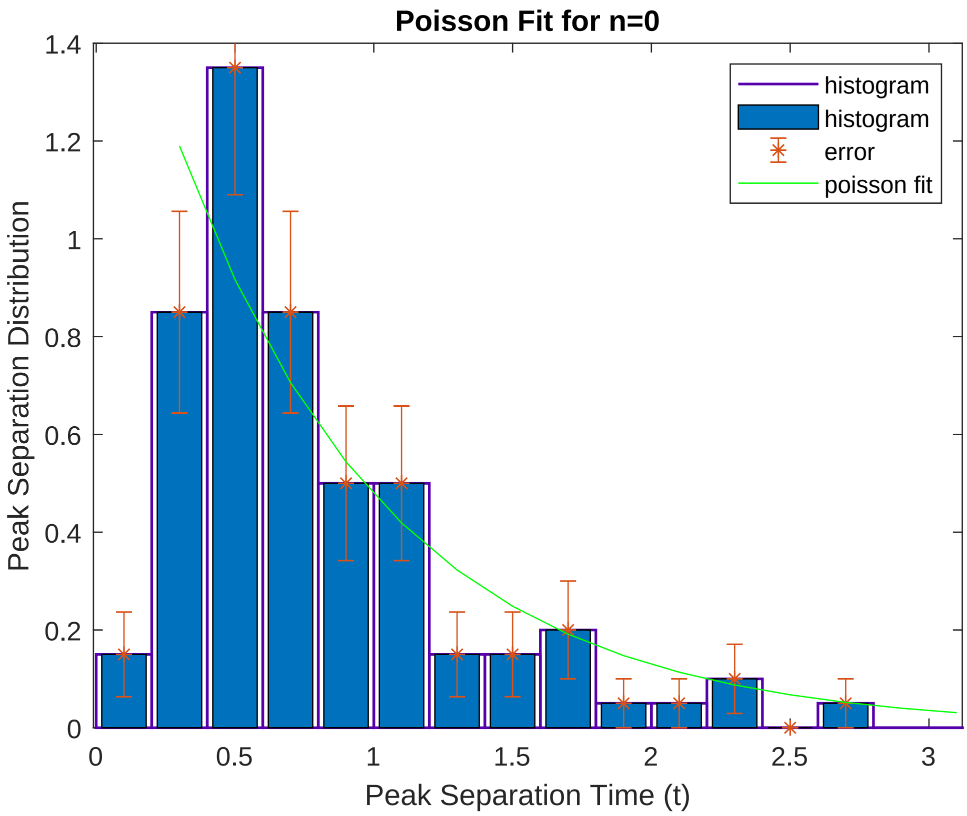
$\chi^2_{gaussian} = 0.32744$, we have 11 bins and Gaussian distribution has 2 parameters. So

$\chi^2_{\nu,gaussian} = 0.03641$

$\mu = 2.97 \mp 0.18$

$\sigma = 1.8116 \mp 0.1105$

Now let's look at the data from the peak datas.
For n=0



To this histogram we fit $P(1,t) = \frac{(\alpha t)^0 e^{-\alpha t}\alpha}{0!}$ with some normalization constant because n=0 corresponds to the situation where there is nothing between the end points. Which here, happens to be the distance between two consecutive peaks!

$\alpha = 1.305 \mp 0.269$

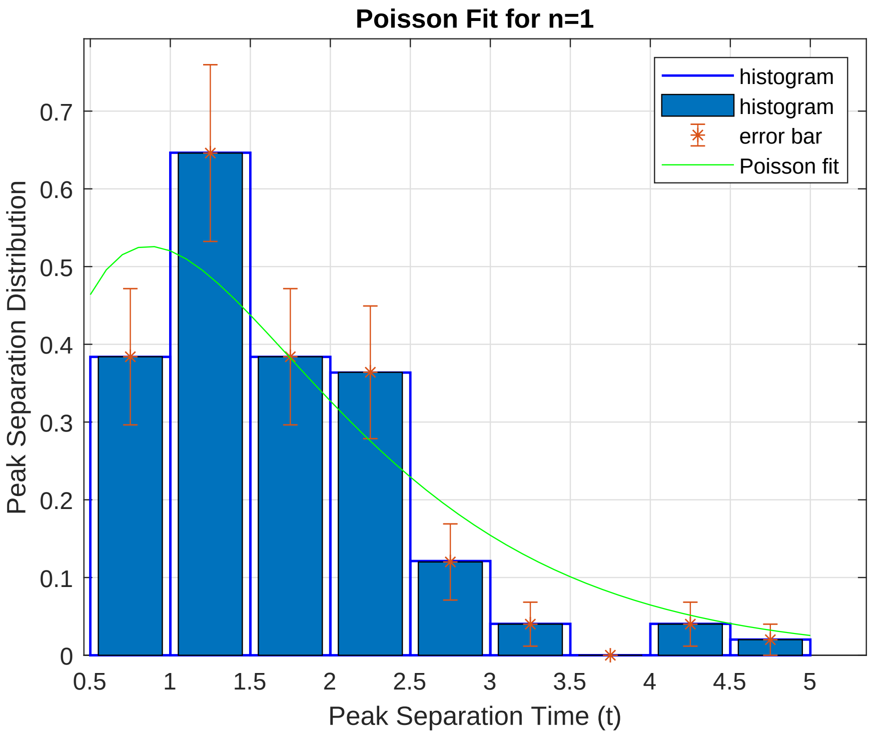
SSE: 0.393

R-square: 0.8316

Adjusted R-square: 0.8187

RMSE: 0.1739

For n=1



To this histogram we fit $P(2,t) = \frac{(\alpha t)^1 e^{-\alpha t}\alpha}{1!}$ with some normalization constant because n=1 corresponds to the situation where there is 1 event between the endpoints (endpoint is another event, adding to the total of 2). Here, this happens to be the distance between 3 neighboring peaks (i.e. one peak is contained in the interval)!

$\alpha = 1.157 \mp 0.399$

SSE: 0.07073

R-square: 0.8312

Adjusted R-square: 0.8071

RMSE: 0.1005

In both of these cases (n=0 and n=1) R squared value is close to 1 which means our fits did a good job at representing the behavior of the histogram.

Conclusion

The purpose the experiment was to show that gamma ray decaying process can be expressed as a Poisson distribution and was to look at general qualities of the Poisson distribution. To do these we histogrammed our datas and did a Poisson and Gaussian fit to the histograms. These fits ,as seen from their chi-squared and R-squared values, do fairly represent the data, except maybe second source's 1 second histogram. Since the number of decaying was small there the fact that it gave a worse fit is to be expected. We saw that as mean becomes large ,indeed, Poisson distribution approaches Gaussian distribution. But for small means Poisson distribution represents the data better! Last part of the experiment shows the independence and Poisson behavior of the gamma ray decay process. We were able to add the nehigboring distances and get a 2 step poisson distribution.

CODE LINK:

https://github.com/OnuraySancar/phys442-poisson.git

References

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