Middle East Technical University Department of Computer Engineering

CENG 460

Introduction to Robotics for Computer Engineering

Spring 2021-2022

Assignment 2 - Arm Kinematics & Motions of Coordinate Frames

Due date: May 9th Monday, 23:55

1 Theory (70pts)

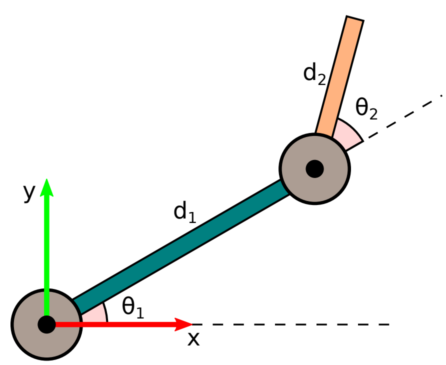

Figure 1: A 2-link arm situated in the origin with respect to the global frame.

Figure 1 visualizes a 2-link planar arm situated at the origin with respect to the global frame, with 2 revolute joints ($\theta_1, \theta_2 \in (-\pi,\ \pi]$). The turquoise link acts as a prismatic joint with length $d_1 \in [d_{min},\ d_{max}]$. The orange link has a fixed length of $d_2 < d_{min}$ and also $d_2 < \frac{d_{max}-d_{min}}{2}$.

1. Draw the configuration space of the end effector position at the tip of the orange arm:
   (a) (5pts) with the given specifications.
   (b) (5pts) with the given specifications, assuming length of turquoise link is fixed.
   (c) (5pts) with the given specifications, assuming $\theta_1 \in [0,\ \pi],\ \theta_2 \in [\frac{-\pi}{2},\ \frac{\pi}{2}]$.
2. (10pts) Establish the local poses of each joint and their variables with the DH convention. Derive the forward kinematics for the end effector with these poses.
3. Derive the inverse kinematics (that is, specify the set of joint variables that satisfy the condition on the end effector - it might be empty for some conditions!) for:
   (a) (5pts) the case of global position of the end effector being $[x_0,\ y_0]$, assuming d_1 is fixed.
   (b) (7.5pts) the case of global 2D homogeneous transform of the end effector being T, with its global position being $[x_0,\ y_0]$, s.t. $d_{min} + d_2 < \sqrt{x_0^2 + y_0^2} < d_{max} - d_2$.

(c) (7.5pts) the case of global position of the end effector being $[x_0, y_0]$, s.t. $d_{min} + d_2 < \sqrt{x_0^2 + y_0^2} < d_{max} - d_2$.

(d) (7.5pts) the case of global position of the end effector being $[x_0,\ y_0]$ for the default specification *(Hint & Simplification: Just show it for* $[x_0,\ y_0]$ *s.t.* $\sqrt{x_0^2 + y_0^2} > d_{max} - d_2$*. Seeing how it behaves for any* $[x_0,\ y_0]$ *is trivial after this. Think of intersecting circles situated on the joints and the end effector with specific radii).*

4. Calculate the angular velocity vector (w.r.t global frame) of the following rotation matrices:

(a) (5pts) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(at) & \sin(at) \\ 0 & -\sin(at) & \cos(at) \end{bmatrix}$.

(b) (5pts) $B = \begin{bmatrix} \cos(bt^2) & 0 & \sin(bt^2) \\ 0 & 1 & 0 \\ -\sin(bt^2) & & \cos(bt^2) \end{bmatrix}$.

(c) (7.5pts) AB, without explicitly calculating AB and taking derivative of it (*Hint:* $(\dot{AB}) = \dot{A}B + A\dot{B}$ *here.*)

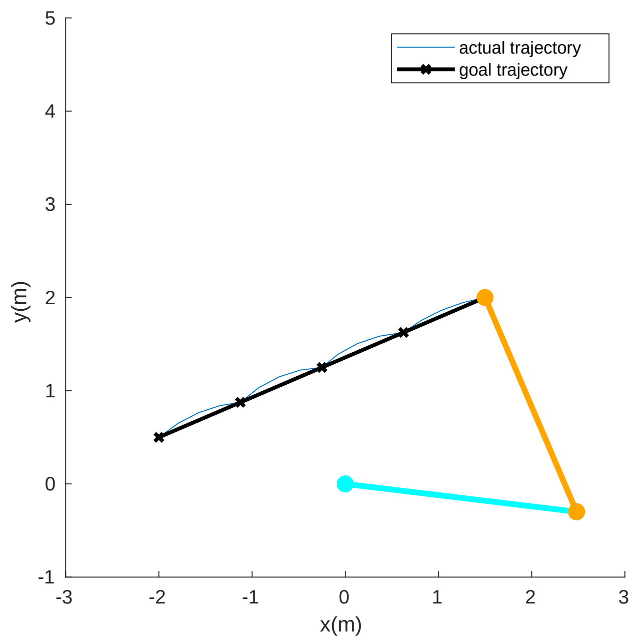

Figure 2: A sneak peek of the programming part.

2 Programming (30pts)

In this part, you will implement the line drawing objective without relying on the toolbox for the robot in Figure 1. Here, the turquoise link is fixed and both links are of the same length $d_1 = d_2 = d$. You can use the values supplied in the example script for the definition of the path and the arm length.

1. (5pts) Implement function `ik_2link_arm`.

2. (10pts) Implement function `line_ik_2link_arm`. Test the behavior of your implementation using the supplied animation function and the example script. Here, it is assumed that the joint variables have constant velocity between IK solutions for the actual trajectory of the robot. The elbow orientation (up or down) should not change as the robot draws the line.

3. (7.5pts) Increase/decrease the number of samples taken from goal trajectory to run IK. Add your observations/claims to your submission and supply them with plots. As the robot fully extends, the error seems to increase, why?

4. (7.5pts) Assume that the end-effector moves at a velocity of 1 m/s. Plot the values of the joint variables for each sample on the goal trajectory w.r.t. time. Assume that joint variables have constant velocity between sampled points. Plot joint *velocities* for each sample on the goal trajectory w.r.t time. Comment on the behavior of the joint velocities with low/high number of samples.

5. You don't need to answer this for this homework, but can you think of a way to calculate which functions are approximated for the joint values/velocities as the number of samples increase? You can plot this exact function for personal glory on top of the approximations.

3 Other Specifications

- All of the rotation matrices are to be assumed right handed.
- In Theory 3, you can define your own variables in the definition of the final set as long as you briefly show how they can be calculated.
- You can complete the programming assignment without relying on any toolbox, including the robotics toolbox. If you require a toolbox utility, please publicly ask in ODTUCLASS. We may or may not allow it.
- Your programming assignment will be reviewed in a glass-box fashion, therefore please be diligent with your comments in your implementation (you do not have to put comments in every line, just put comments to places that you think it is complicated for the reader to understand. Your comments should provide information about *why* rather than *what*. Avoid "`a = a.*b % element-wise multiply a and b`" sort of comments).
- Please ask your questions regarding the assignment publicly in ODTUCLASS, unless your question reveals part of your solution. In that case, send a mail to `onem@ceng.metu.edu.tr`.
- **This is an individual assignment. Using any piece of code that is not your own is strictly forbidden and constitutes as cheating. This includes friends, previous homeworks, or the Internet. The violators will be punished according to the department regulations.**

4 Submission

Submit your solution as a compressed zip file name `hw2_eXXXXXXX.zip` to ODTUCLASS, containing:

- a pdf scan (you can take this scan with a camera as well using Adobe Scan etc.) of your hand-written answers or a typeset pdf document named `theory_hw2_eXXXXXXX.pdf`.
- All of your matlab code regarding your plots and filled-in stubs provided with your homework. All of your work should be reproducable with the code you supplied. Supply a readme file that explains which file contains what if your file collection is too complex.