Simple Modelling of Synchrotron Emission in High-Energy Astrophysics

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Presentation Outline

Why bother with the origin of radiation?

We see radiation/light emitted or reflected from:

- our sun (a well-known star),
- other stars within our Milky Way Galaxy,
- supernovae,
- interstellar gas and dust,
- other galaxies,
- active centers of galaxies (AGN),
- (possibly) black holes,
- quasars,
- Gamma Ray Bursts (GRBs), etc

We need to know how this happens ... unintended discoveries comes along!

Presentation Outline

Radiative Processes

There are various ways in which radiation is produced ... these include

- Bremsstrahlung or free-free radiation,
- Synchrotron radiation,
- Compton scattering,
- photon-photon absorption and pairproduction,
- photon-hadron interactions.

Here we single out the most dominant high energy astrophysical radiation ... (from relativistic electrons!) ... the synchrotron emission process.

Presentation Outline

Synchrotron radiation

- $\hookrightarrow$ most of the "light" we see is **thermal** radiation: result of electron orbital transition
- $\hookrightarrow$ a tiny amount of "light" we see is **non-thermal** radiation: result of e.g. Synchrotron, inverse Compton, etc.
- $\hookrightarrow$ Synchrotron (Synch) emission: result of accelerating charge particles in a magnetic field
- $\hookrightarrow$ charge particle acceleration most effectively done with electrons (light weight)
- $\hookrightarrow$ charged particles gyrate around magnetic field lines
- $\hookrightarrow$ most exortic objects we observe in the Universe have magnetic fields

The Basics

- ↪ most exortic objects we observe in the Universe have magnetic fields and charged particles
- ↪ charged particles gyrate around magnetic field lines
- ↪ charged particles emit radiation when accelerated in a magnetic field
- ↪ charge particle acceleration most effectively done with electrons (light weight)
- ↪ if particle speed v is comparable to speed of light c then **relativistic** otherwise **non-relativistic**
- ↪ **non-relativistic** motion $\curvearrowright$ **cyclotron** radiation
- ↪ **relativistic** motion $\curvearrowright$ **synchrotron** radiation
- ↪ most of the "light" we see is **thermal** radiation: result of electron orbital transition
- ↪ a tiny amount of "light" we see is **non-thermal** radiation:

Cyclotron radiation — non-relativistic

- Consider an electron of charge $q = e$, mass m_e, moving at velocity v at pitch angle θ to magnetic field $\mathbf{B}$
- define Lorentz factor $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, $\beta = v/c$ and let $v \ll c$
- hence relativistic mass is γm_e and total energy $E = \gamma m_e c^2$
- after balancing centrifugal and Lorentz forces we get (in SI units!), the non-relativistic gyrofrequency and the angular gyrofrequency as

$$\nu_{\rm gyro} = \frac{eB}{2\pi\gamma m_e} \quad \text{and} \quad \omega_{\rm gyro} = \frac{eB}{\gamma m_e}$$

- $\Rightarrow$ angular frequency of the electron around the $\mathbf{B}$-field is

$$\omega = \left(\frac{eB}{m_e}\right)\frac{1}{\gamma} = \frac{\omega_{\rm crit.n}}{\gamma}$$

Synchrotron radiation — ultra-relativistic

- in the limit where $v \longrightarrow c$ we get

$$\nu_{\rm crit.r} = \frac{3}{2}\gamma^2 \nu_{\rm crit.n} \sin\theta = \frac{3eB}{4\pi m_e}\gamma^2 \sin\theta$$

- the observed frequency is boosted by γ such that

$$\nu_{\rm obs} = \gamma^2 \nu_{\rm gyro.n} = \gamma^3 \nu_{\rm gyro.r}$$

- the energy loss of the particle is emitted as radiation
- the energy loss rate of the electrons with pitch angle θ is then

$$\frac{dE}{dt} = -\left(\frac{e^4 B^2}{6\pi\epsilon_0 m_e^2 c}\right)\beta^2\gamma^2 \sin^2\theta$$

Synchrotron radiation

- after averaging over all possible pitch angles and defining $u_B = B^2/2\mu_0$ as **B**-field energy density

$$\frac{dE}{dt} = -\frac{4}{3}\sigma_T c u_B \beta^2 \gamma^2$$

with $\sigma_T = \frac{e^4}{6\pi\epsilon_o^2 c^4 m_e^2} = 6.65 \times 10^{-29}\ \mathrm{m}^2$ the Thomson cross-section

- the **emissivity** of such a single electron is a function of frequency ω:

$$j(\omega) = -\frac{\sqrt{3}e^3 B \sin\theta}{8\pi^2\epsilon_0 m_e c} F(x)$$

where $F(x)$ are integrals of *modified Bessel functions* of $x = \frac{2\omega r}{3c\gamma^3}$

Synchrotron Radiation

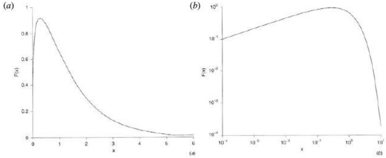

Fig. 8.8 The spectrum of the synchrotron radiation of a single electron shown (a) with linear axes; (b) with logarithmic axes. The function is plotted in terms of $x = \omega/\omega_c = \nu/\nu_c$ where ω_c is the critical angular frequency $\omega_c = 2\pi\nu_c = (3/2)\,(c/v)\,\gamma^2\omega_g \sin\alpha$ where α is the pitch angle of the electron and ω_g is the non-relativistic gyrofrequency, $\omega_g = eB/m_e$.

$$j(\omega) = -\frac{\sqrt{3}e^3 B \sin\theta}{8\pi^2\epsilon_0 c m_e} F(x)$$

the asymptotic behaviour of the emissivity is:

$$j(\nu) \propto \begin{cases} \nu^{1/2}, & \text{for high frequencies, i.e } \nu \gg \nu_{\mathrm{crit}} \\ \nu^{1/3}, & \text{for low frequencies, i.e } \nu \ll \nu_{\mathrm{crit}} \end{cases}$$

Synchrotron Radiation

- we may know the energy distribution of the electrons
- assume a *power law energy distribution* such that the number of elctrons per unit volume with energies in the range from E to $E + dE$ is

$$n(E) = \kappa E^{-p} dE$$

  where p is the **energy index** of the electron distribution (normalized with κ).
- then the emission per unit volume becomes

$$J(\omega) = \frac{\sqrt{3\pi} e^3 B\kappa}{16\pi^2 \epsilon_0 m_e c(p+1)} \left(\frac{\omega m_e^3 c^4}{3eB}\right)^{\frac{-(p-1)}{2}} \frac{\Gamma\left(\frac{p}{4} + \frac{19}{12}\right) \Gamma\left(\frac{p}{4} - \frac{1}{12}\right) \Gamma\left(\frac{p}{4} + \frac{5}{4}\right)}{\Gamma\left(\frac{p}{4} + \frac{7}{4}\right)}$$

Synchrotron Radiation

- which can be summarized as

$$\begin{aligned} J(\nu) &\propto \kappa\, B^{(p+1)/2}\, \nu^{-(p-1)/2} \\ &= \kappa\, B^{\alpha+1}\, \nu^{-\alpha} \end{aligned}$$

where $\alpha = \frac{p-1}{2}$ is the **spectral index**, and $\Gamma(t)$ is the math *gamma-function*, the extension of the *factorial function*.

- note also that the critical frequency is

$$\nu_{\rm crit} = \left(4.2 \times 10^{10} \;\; \gamma^2 \;\; B_{\rm T}\right) \quad {\rm Hz}$$

where $B_{\rm T}$ is the value of the field in Tesla.

Presentation Outline

Getting our hands dirty with a model

Thanks to the philosophy of **free software** that we can use Omar Jamil's code under the GNU General Public License (GPL) ... it has two degrees of freedom in this case, we can edit it *freely as in freedom* and it is *free as in free beer!*.

- get a synchrotron emission code (written in Python) by Omar Jamil from `https://github.com/omarjamil/SimpleSynch`
- identify the various theoretical statements in the code
- compile the code and get spectral energy distribution plots
- do a brief parameter study by varrying various input parameters, such as γs, B, p, etc.

Presentation Outline

Applications of the synchrotron concept

- Synchrotron radiation is one of the well-studied and pioneering mechanism in Astrophysics
- In the medical industry
- The LHC