Simple Modelling of Synchrotron Emission in High-Energy Astrophysics

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Presentation Outline

- Background
- 2 Radiative Processes
- Synchrotron radiation
- Student activity
- 6 Applications

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We need to know how this happens ... unintended discoveries comes along!

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Radiative Processes

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- after balancing centrifugal and Lorentz forces we get (in SI units!), the non-relativistic gyrofrequency and the angular gyrofrequency as

$$u_{\mathrm{gyro}} = \frac{eB}{2\pi\gamma m_{\mathrm{e}}} \quad \mathrm{and} \quad \omega_{\mathrm{gyro}} = \frac{eB}{\gamma m_{\mathrm{e}}}$$



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 $\bullet \Rightarrow$ angular frequency of the electron around the **B**-field is

$$\omega = \left(\frac{eB}{m_e}\right) \frac{1}{\gamma} = \frac{\omega_{\text{crit.n}}}{\gamma_{\text{constable for the property of the propert$$

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- ullet the energy loss rate of the electrons with pitch angle heta is then

$$\frac{dE}{dt} = -\left(\frac{e^4 B^2}{6\pi\epsilon_0 m_e^2 c}\right) \beta^2 \gamma^2 \sin^2 \theta$$



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Synchrotron radiation

• after averaging over all possible pitch angles and defining $u_B = B^2/2\mu_0$ as **B**-field energy density

$$\frac{dE}{dt} = -\frac{4}{3}\sigma_T c u_B \beta^2 \gamma^2$$

with $\sigma_T=\frac{e^4}{6\pi\epsilon_o^2c^4m_e^2}=6.65\times 10^{-29}~{\rm m}^2$ the Thomson cross-section

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• the **emissivity** of such a single electron is a function of frequency ω :

$$j(\omega) = -\frac{\sqrt{3}e^3 B \sin \theta}{8\pi^2 \epsilon_0 m_e c} F(x)$$

where F(x) are integrals of modified Bessel functions of $x = \frac{2\omega r}{3co^3}$

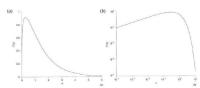


Fig. 8.8 The spectrum of the synchrotron radiation of a single electron shown (a) with linear axes; (b) with logarithmic axes. The function is plotted in terms of $\mathbf{x} = \omega/\alpha_c = \nu/\nu_c$, where α_b is the critical angular frequency $\alpha_c = 2\pi\nu_c = (3/2) (\epsilon/\nu) \gamma^2 \omega_b$ sin α where α is the pitch angle of the electron and ω_b is the non-relativistic conformation, $\alpha_c = \epsilon \delta \ell m_c$.

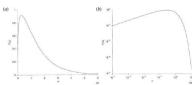
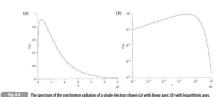


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the asymptotic behaviour of the emissivity is:

$$j(\nu) \propto egin{cases}
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u \gg
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- assume a power law energy distribution such that the number of elctrons per unit volume with energies in the range from E to E + dE is

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then the emission per unit volume becomes

$$J(\omega) = \frac{\sqrt{3\pi}e^3B\kappa}{16\pi^2\epsilon_0m_ec(p+1)} \Big(\frac{\omega m_e^3c^4}{3eB}\Big)^{\frac{-(p-1)}{2}} \frac{\Gamma(\frac{p}{4}+\frac{19}{12})\,\Gamma(\frac{p}{4}-\frac{1}{12})\,\Gamma(\frac{p}{4}+\frac{5}{4})}{\Gamma(\frac{p}{4}+\frac{7}{4})}$$



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• note also that the critical frequency is

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ullet fair approximation: most of the radiated photons are at $u_{
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Thanks to the philosophy of **free software** that we can use Omar Jamil's code under the GNU General Public License (GPL) ... it has two degrees of freedom in this case, we can edit it *freely as in freedom* and it is *free as in free beer!*.

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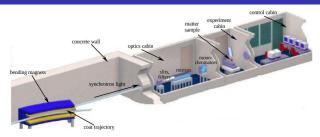
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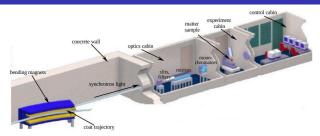
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- do a brief parameter study by varrying various input parameters, such as γ s, B, p, etc.



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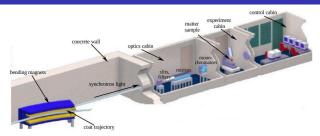
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- man-made circular particle accelerators



The synchrotron facility



Advanced Photon Source at Argonne National Laboratory

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• man-made circular particle accelerators