

Project_on_Machine_Learning

June 27, 2018

- Ising Model
- [Metha et al, arXiv 1803.08823](#) accompanied by a [Jupyter notebook](#).
- Phases
 - Ordered
 - Critical
 - Disordered

Linear Regression

- $\{y_i, \mathbf{x}_i\}_{i=1}^n, i = 1, \dots, n$
- $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$
- minimize the L_2 -norm $\min_{\boldsymbol{\beta}} |\mathbf{X}\boldsymbol{\beta} - \mathbf{y}|^2$
- Solution: $\boldsymbol{\beta}_{\text{LS}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} |\mathbf{X}\boldsymbol{\beta} - \mathbf{y}|^2 \Rightarrow \boldsymbol{\beta}_{\text{LS}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$

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Ridge Regression

- L_2 -regularization: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \alpha \sum_{i=1}^m \beta_i^2$
- $\boldsymbol{\beta}_{\text{Ridge}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} (|\mathbf{X}\boldsymbol{\beta} - \mathbf{y}|^2 + \alpha |\boldsymbol{\beta}|^2) \Rightarrow \boldsymbol{\beta}_{\text{Ridge}} = (\mathbf{X}^T \mathbf{X} + \alpha \mathbf{I})^{-1} \mathbf{X}^T \mathbf{y}$

Linear Regression

- $\{y_i, x_i\}_{i=1}^n, i = 1, \dots, n$
- $y = X\beta + \varepsilon$
- minimize the L_2 -norm $\min_{\beta} |X\beta - y|^2$
- Solution: $\beta_{LS} = \operatorname{argmin}_{\beta} |X\beta - y|^2 \Rightarrow \beta_{LS} = (X^T X)^{-1} X^T y$

Ridge Regression

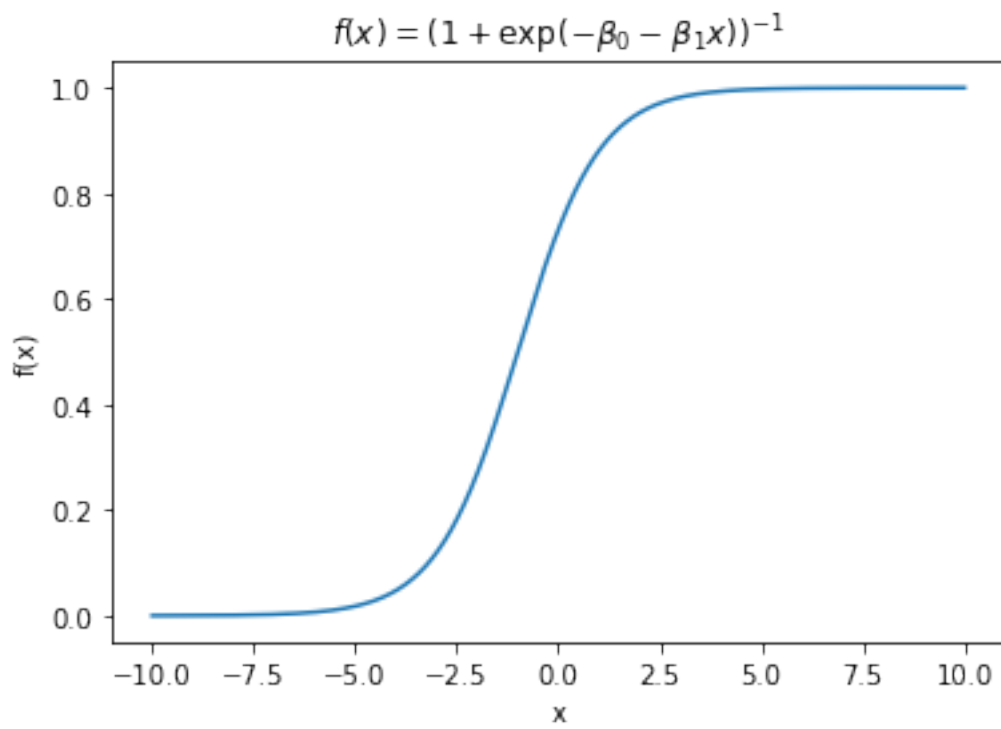
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Lasso Regression

- L_1 -regularization: $y = X\beta + \alpha \sum_{i=1}^m |\beta_i|$
- Constrained ($|\beta| \leq t$): $\beta_{\text{Lasso}} = \operatorname{argmin}_{\beta} (|X\beta - y|^2 + \alpha |\beta|)$
- Solution: $\beta_j^{\text{Lasso}} = \operatorname{sign}(\beta_j^{\text{LS}}) (|\beta_j^{\text{LS}}| - \alpha)_+$

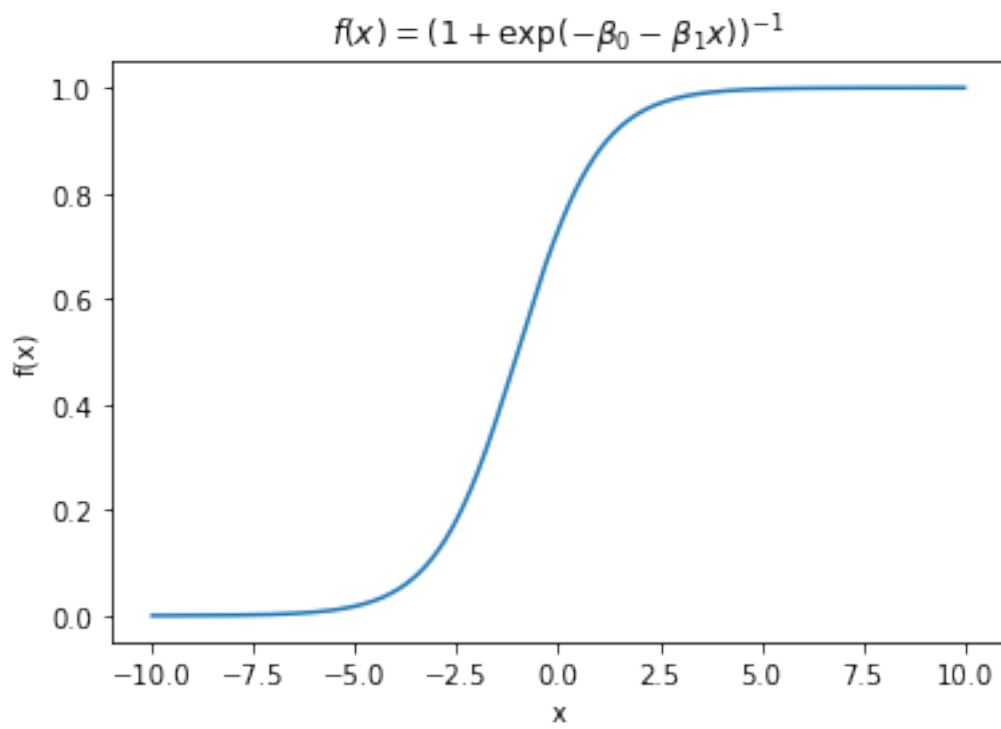
Logistic Regression

- Fit to sigmoid $f(x) = \frac{1}{1+\exp(-x^T w)}$



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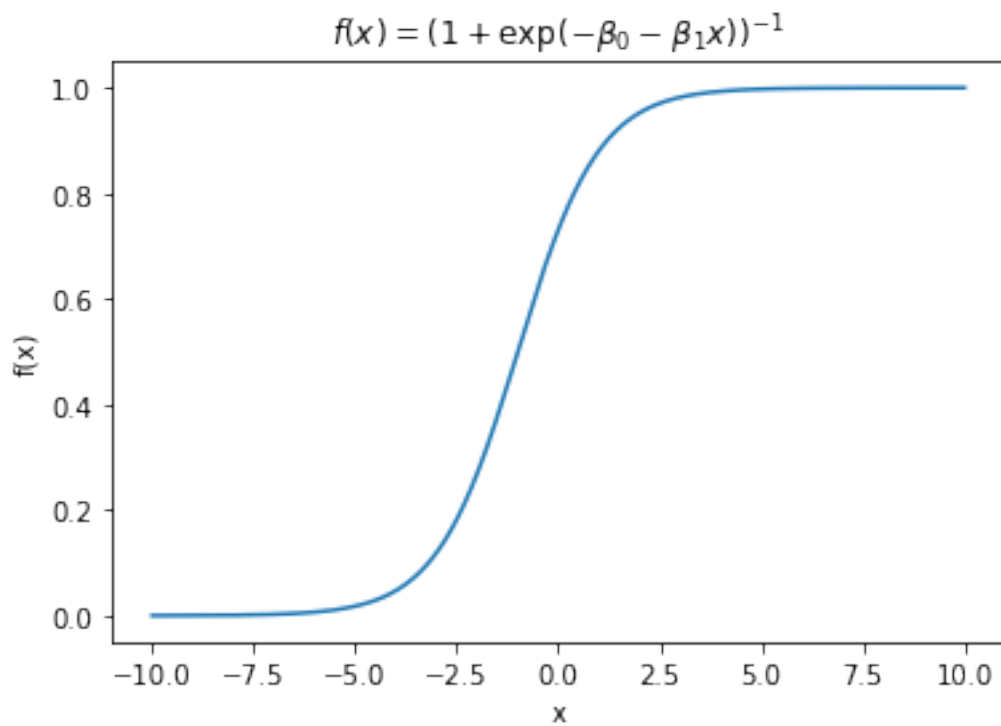
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Logistic Regression

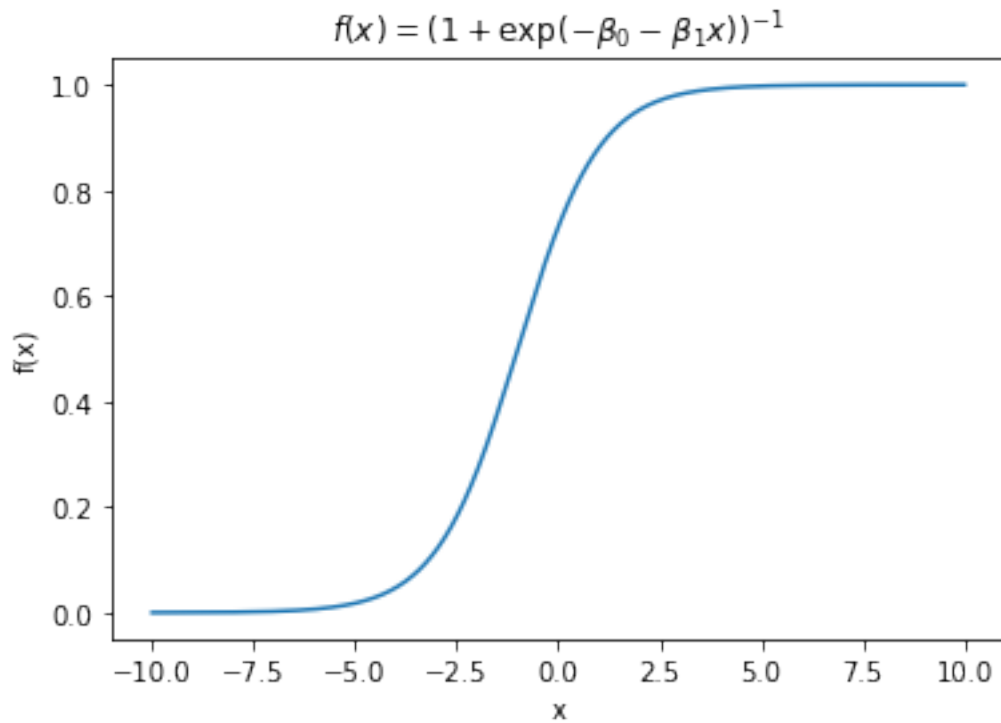
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- Categorisation
 - Binary: $P(Y|X)$

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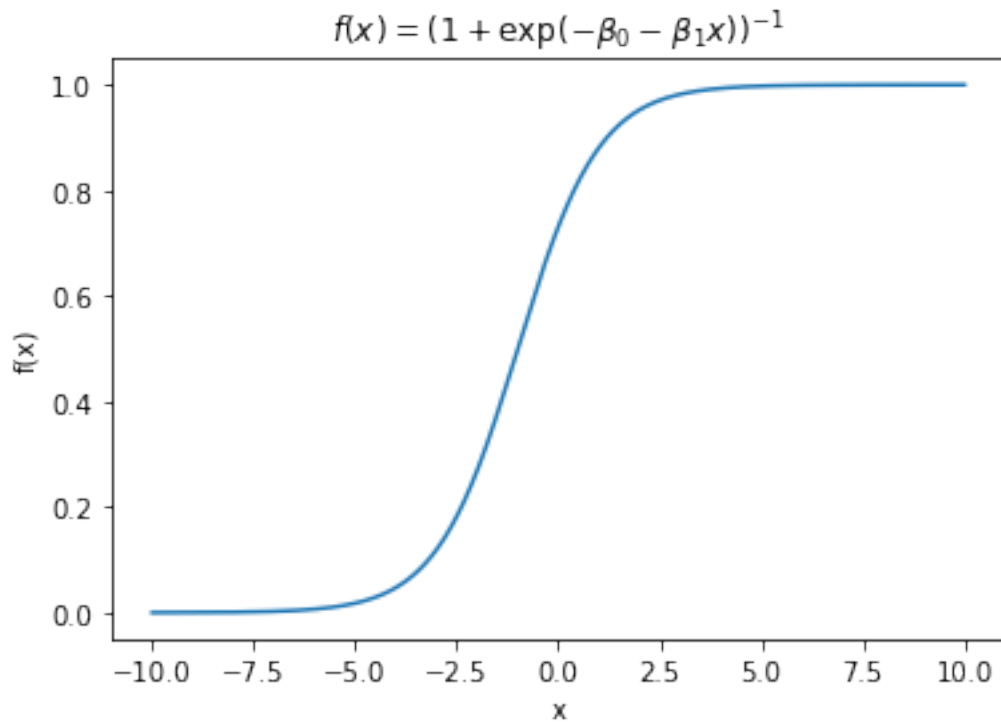
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 - Binary: $P(Y|X)$
- Cost: $C(w) = - \sum_{i=1}^n [Y_i \log (f (X_i^T w)) + (1 - Y_i) \log (1 - f (X_i^T w))]$

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- Cost: $C(w) = - \sum_{i=1}^n [Y_i \log (f(\mathbf{X}_i^T w)) + (1 - Y_i) \log (1 - f(\mathbf{X}_i^T w))]$
- Numerical minimization: $\mathbf{0} = \nabla C(w) = \sum_{i=1}^n [f(\mathbf{X}_i^T w) - Y_i] \mathbf{X}_i$

Random Forest Algorithm

- Consists of Decision Trees

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Random Forest Algorithm

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- Bagging: Create bootstrap ensemble from data
- Feature Bagging: Bagging on data and labels
- Extremized Random Forest: Split decision trees at random

Neural Networks

- input: $x = \begin{pmatrix} x_{11} & \dots & x_{1d} \\ \vdots & \ddots & \vdots \\ x_{d1} & \dots & x_{nd} \end{pmatrix}$

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- Backpropagation:
 - **Activation at input layer:** Calculate activations $a_j^l = \sigma \left(\sum_k w_{kj}^l a_k^{l-1} \right)$
 - **Feedforward:** Compute z^l and a^l for subsequent layers
 - **Error at output:** Calculate error in output
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- Cost function

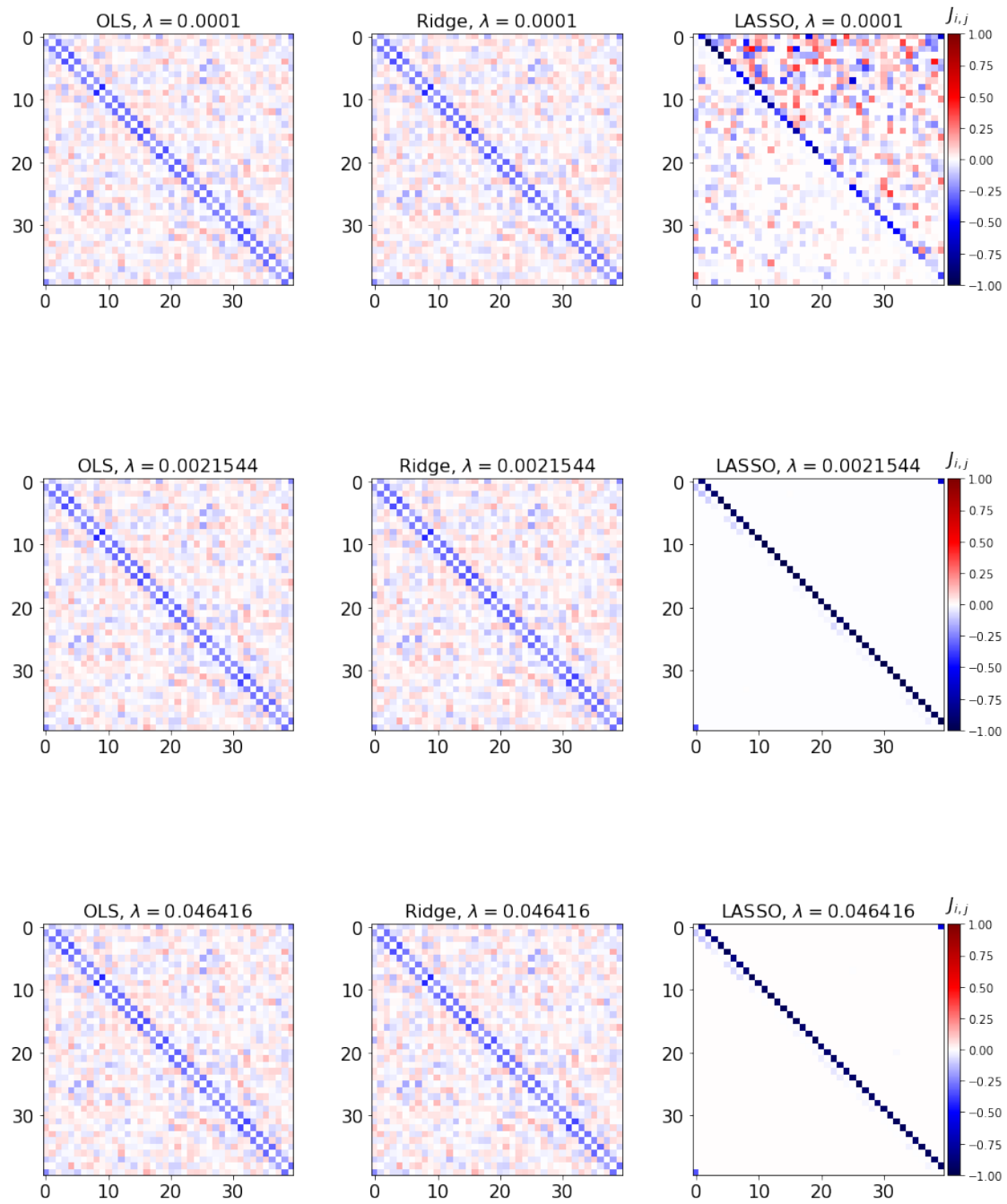
Estimating the Coupling Constant of the 1D Ising Model

- Data with $J = 1$

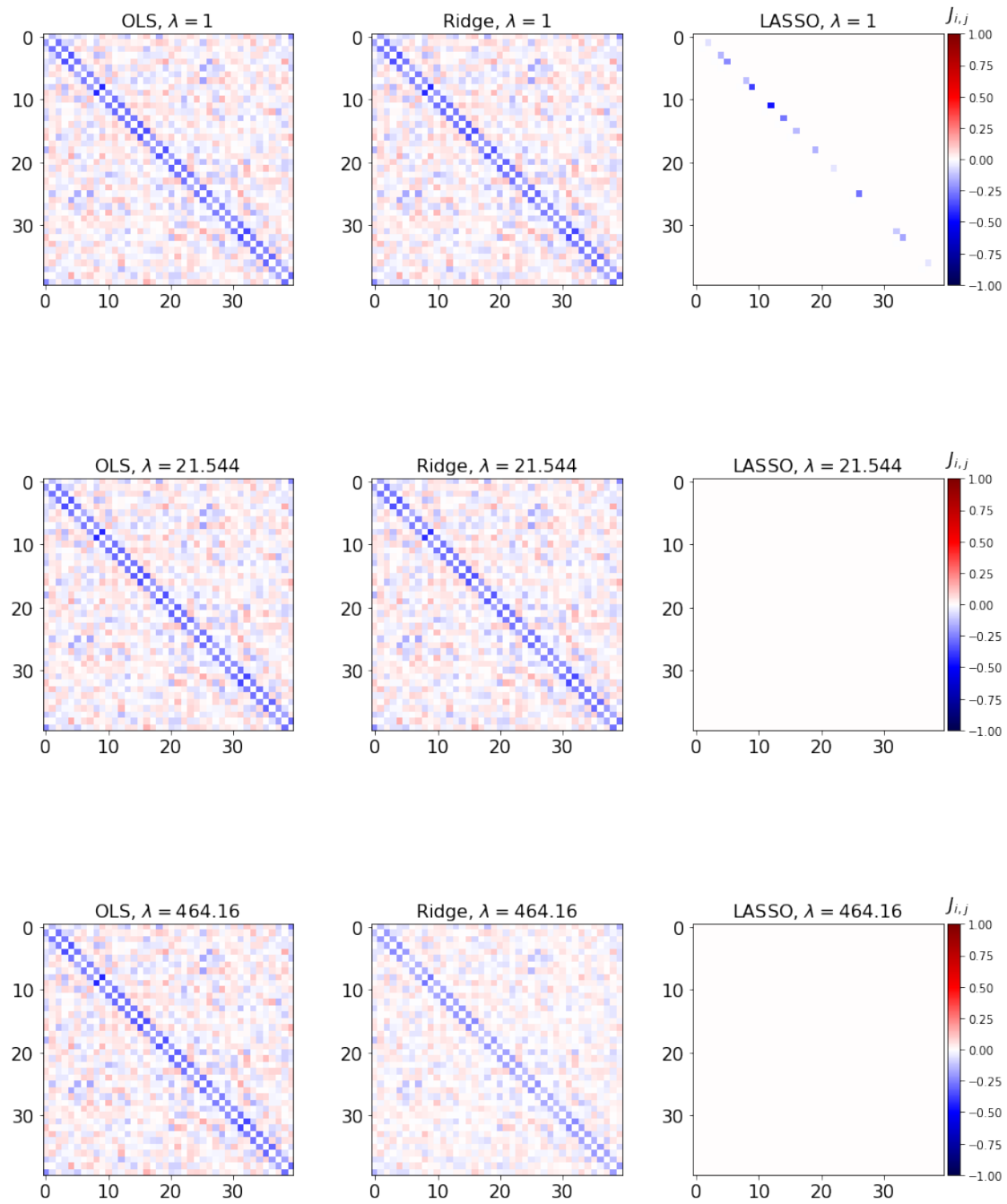
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- Data with $J = 1$
- $E^{(i)} = -\mathbf{X}^{(i)} \cdot \mathbf{J}$

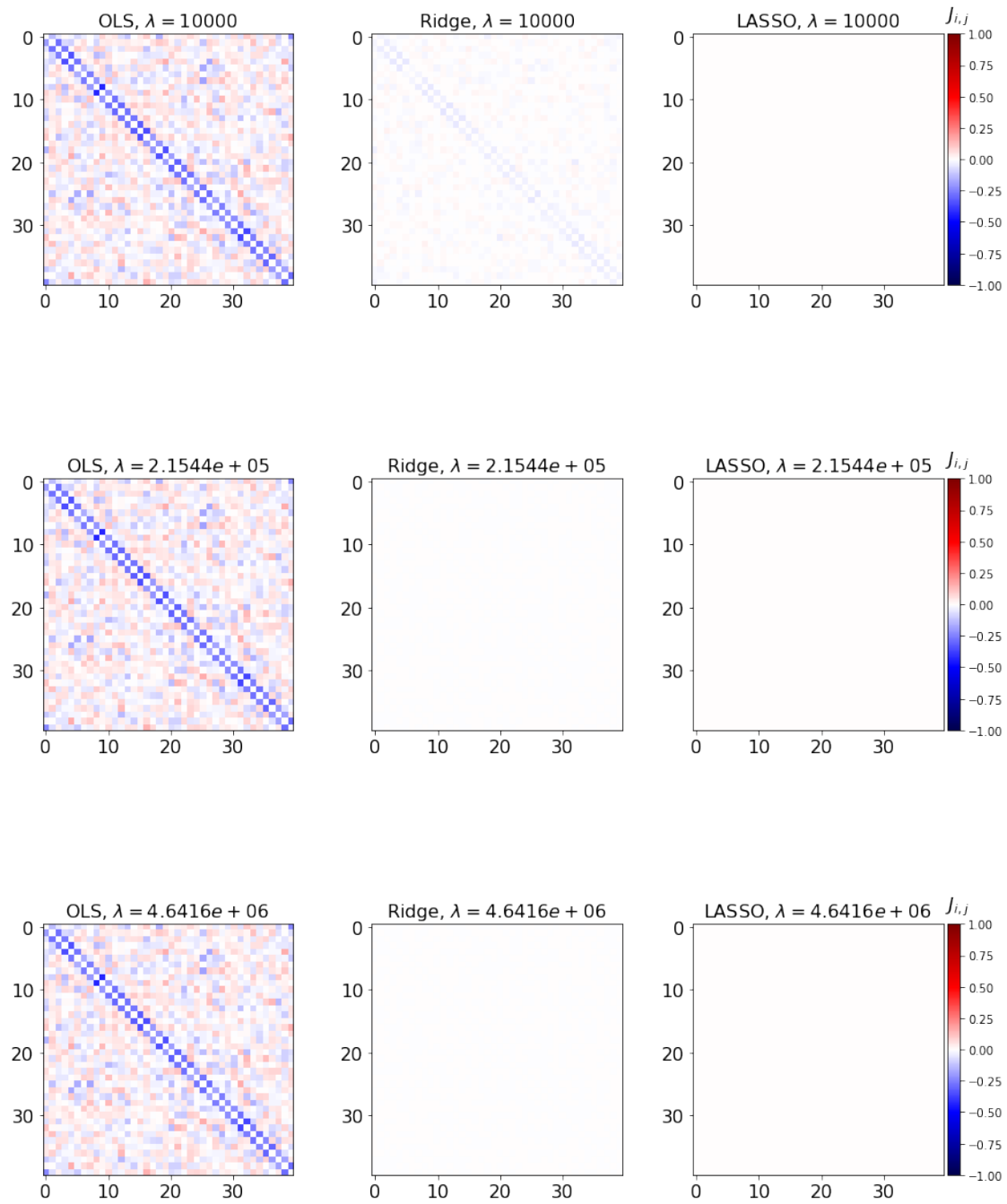
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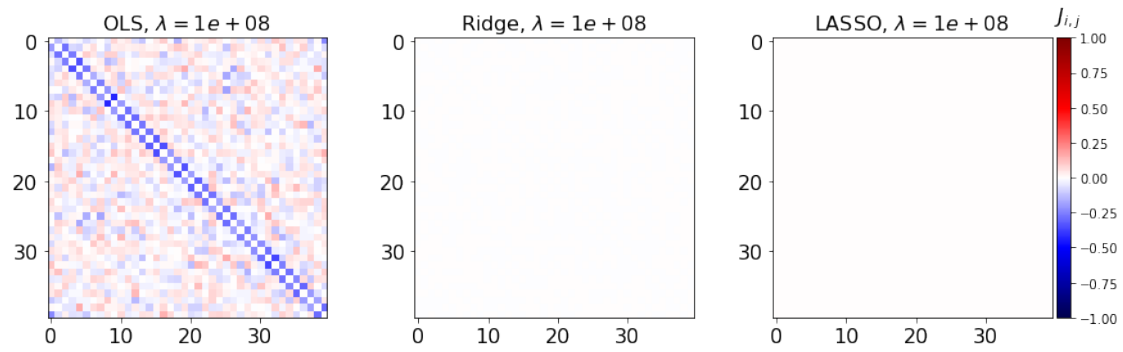
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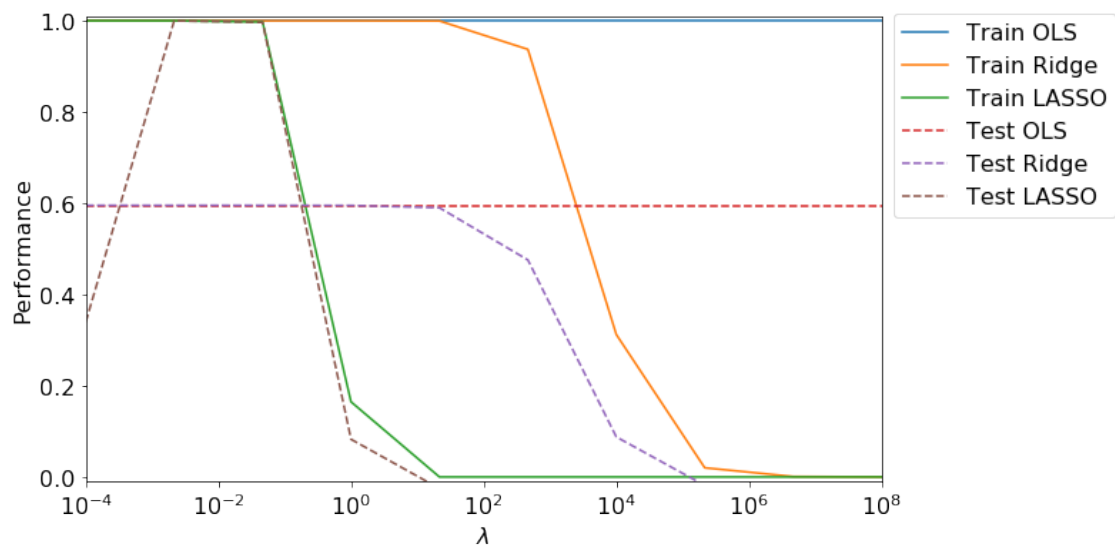
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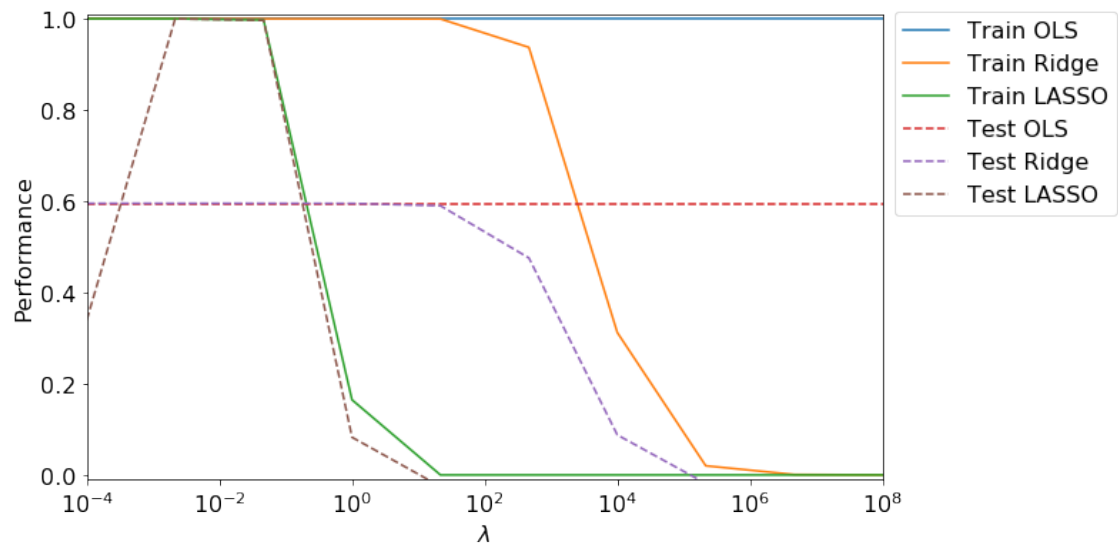
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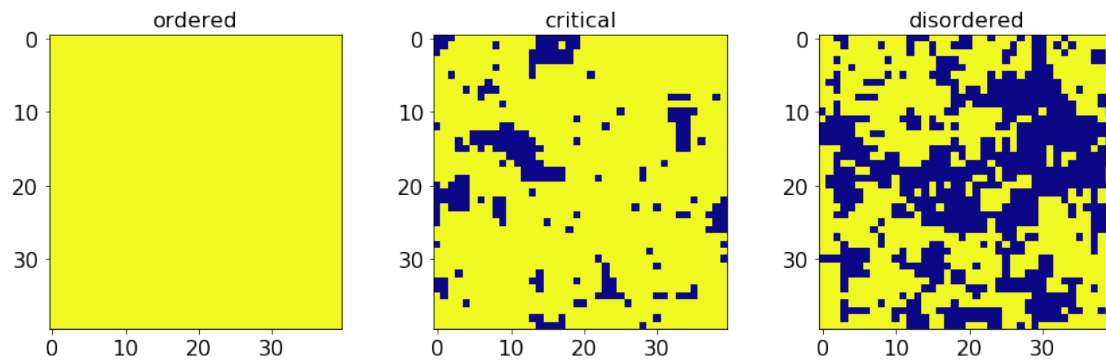
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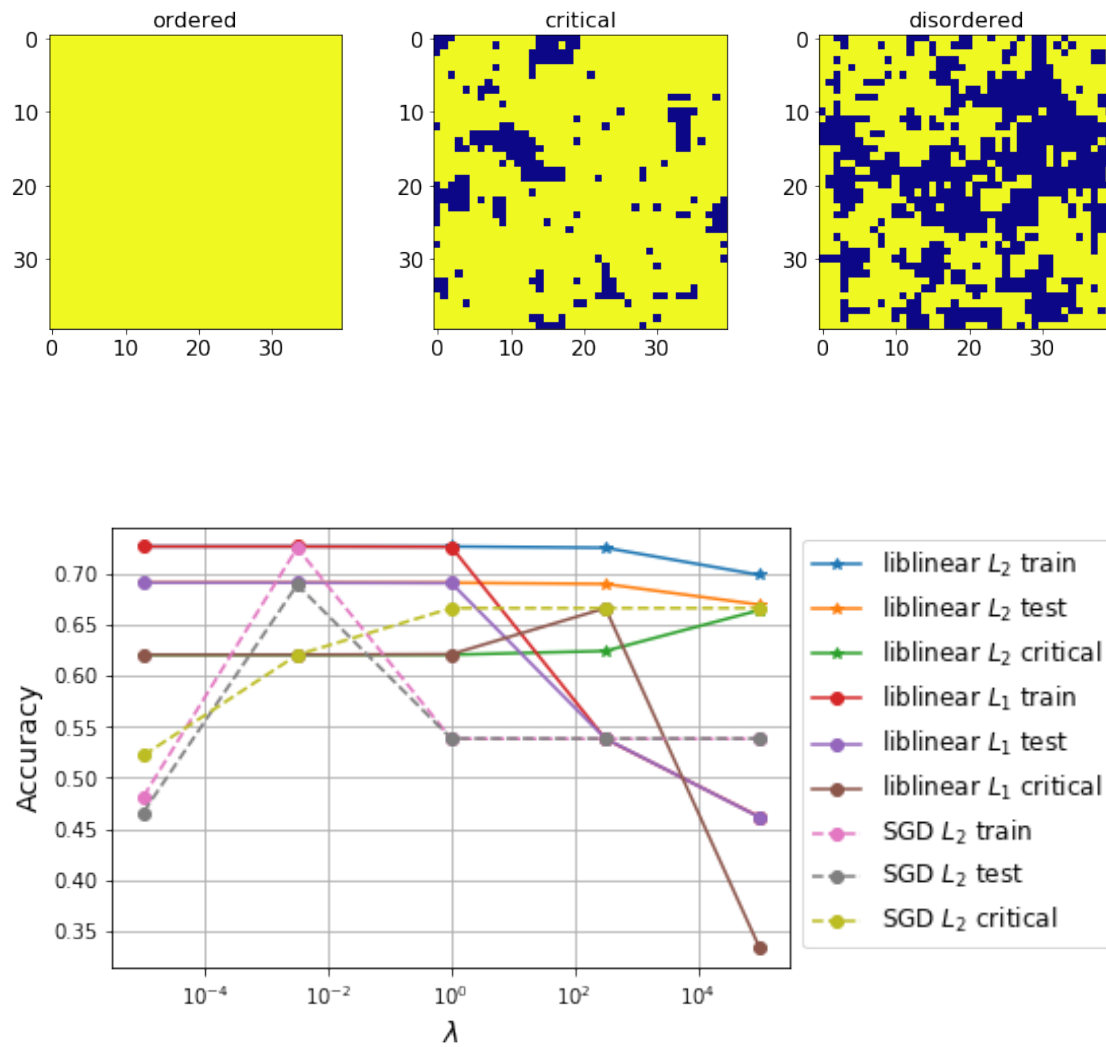
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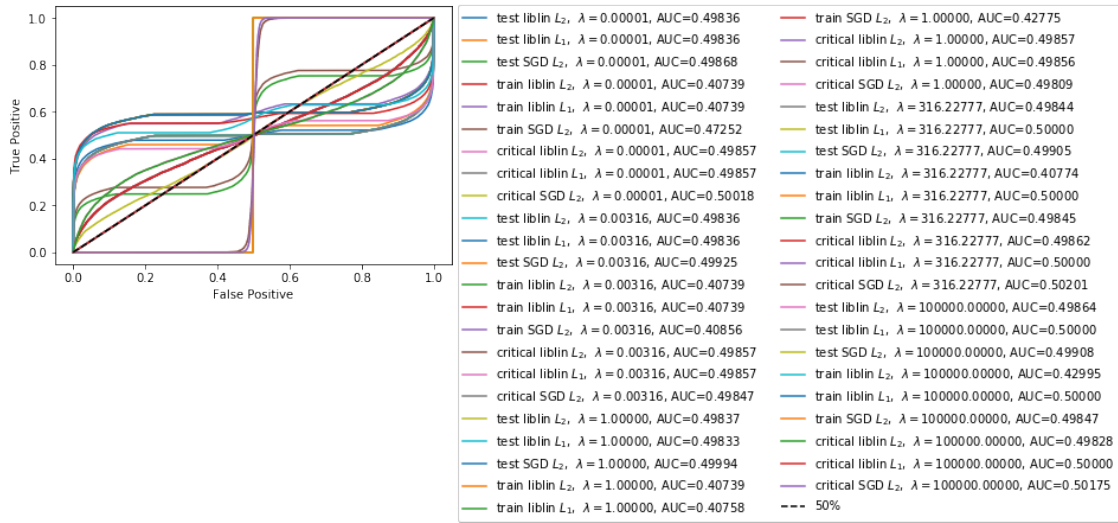
Determine the Phase of the Two-Dimensional Ising Model



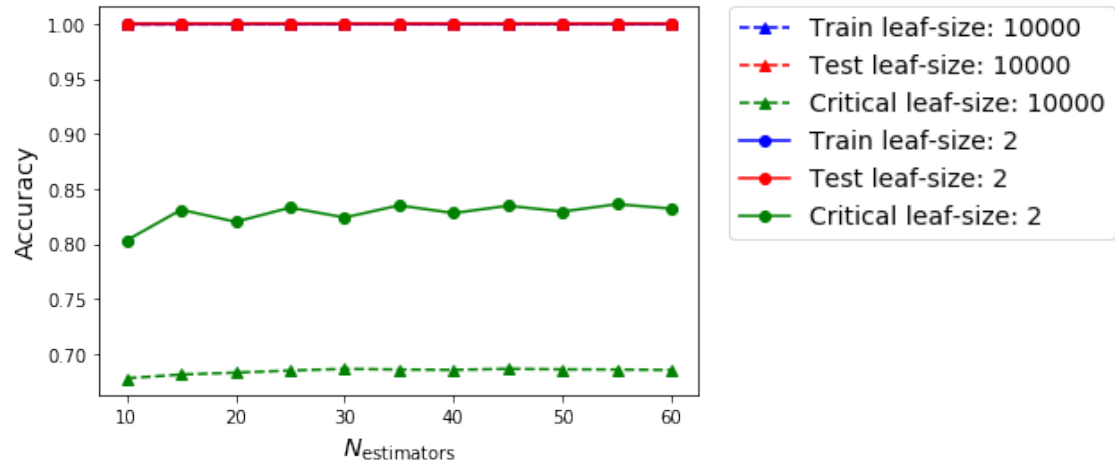
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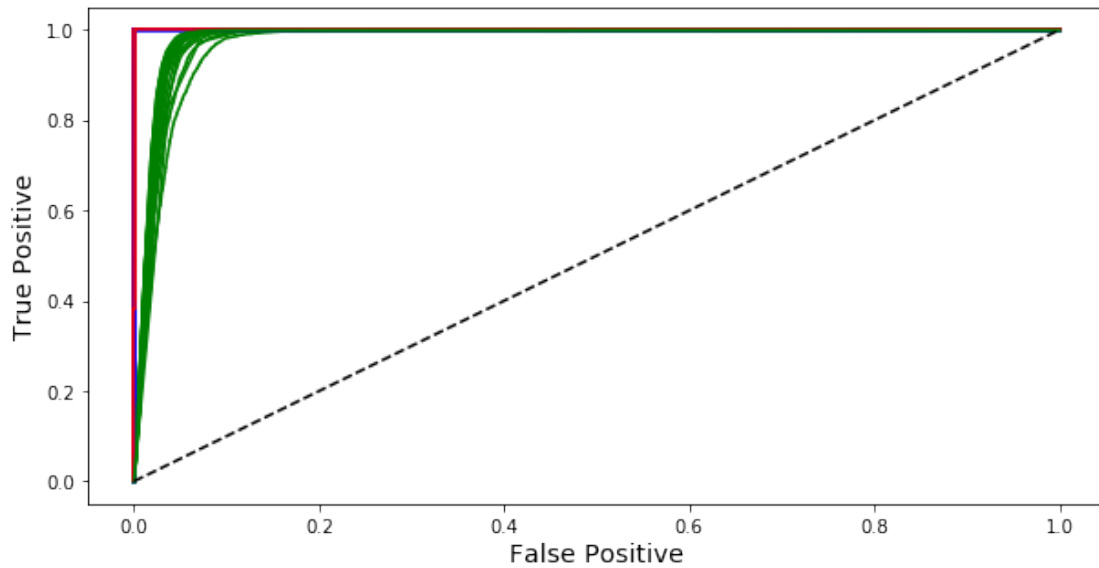
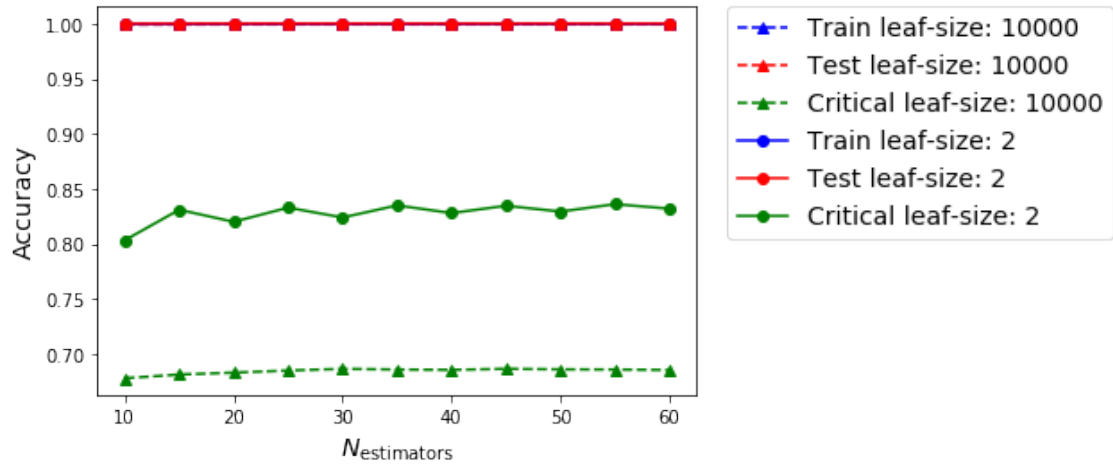
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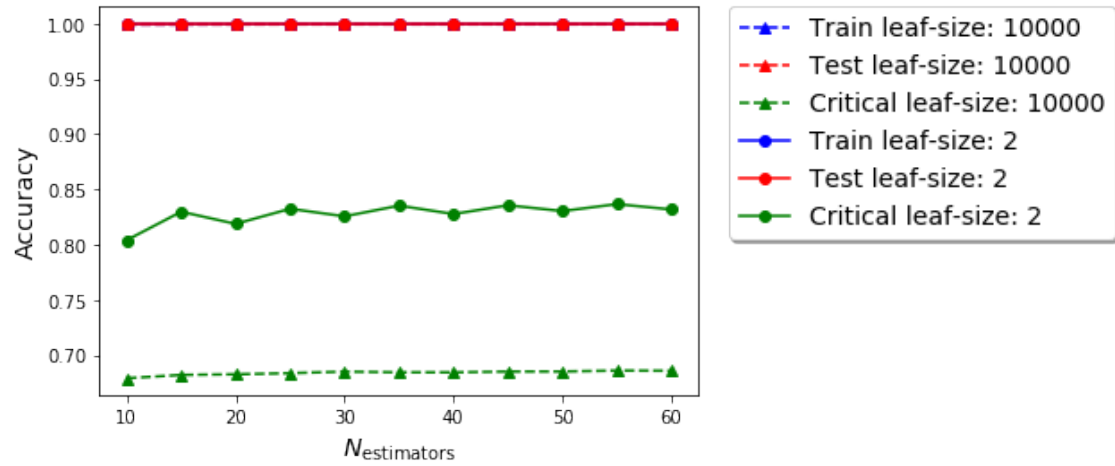
0.0.1 Using Random Forest to Classify Phases in the Ising Model



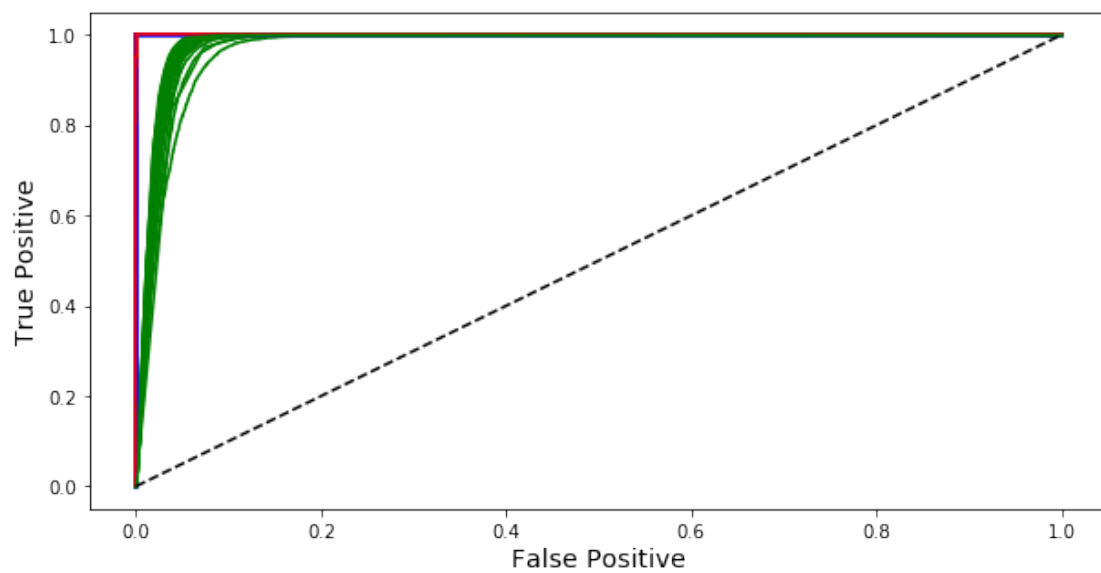
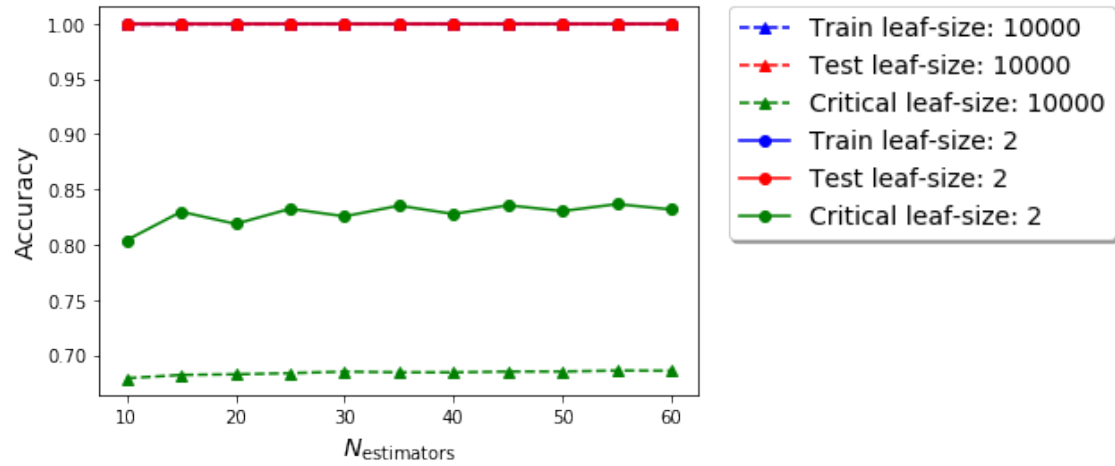
0.0.2 Using Random Forest to Classify Phases in the Ising Model



0.0.3 Using Random Forest to Classify Phases in the Ising Model



0.0.4 Using Random Forest to Classify Phases in the Ising Model



0.0.5 Classifying the Ising Model Phase Using Neural Networks

