Project_on_Machine_Learning

June 27, 2018

- Ising Model
- Metha et al, arXiv 1803.08823 accompanied by a Jupyter notebook.
- Phases
  - Ordered
  - Critical
  - Disordered

Linear Regression

- $\{y_i, \boldsymbol{x}_i\}_{i=1}^n$, $i = 1, \ldots, n$
- $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$
- minimize the L_2-norm $\min_{\boldsymbol{\beta}} \left|\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{y}\right|^2$
- Solution: $\boldsymbol{\beta}_{\mathrm{LS}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left|\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{y}\right|^2 \Rightarrow \boldsymbol{\beta}_{\mathrm{LS}} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1} \boldsymbol{X}^T\boldsymbol{y}$

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Ridge Regression

- L_2-regularization: $\boldsymbol{y} = \boldsymbol{X\beta} + \alpha\sum\limits_{i=1}^{m}\beta_i^2$
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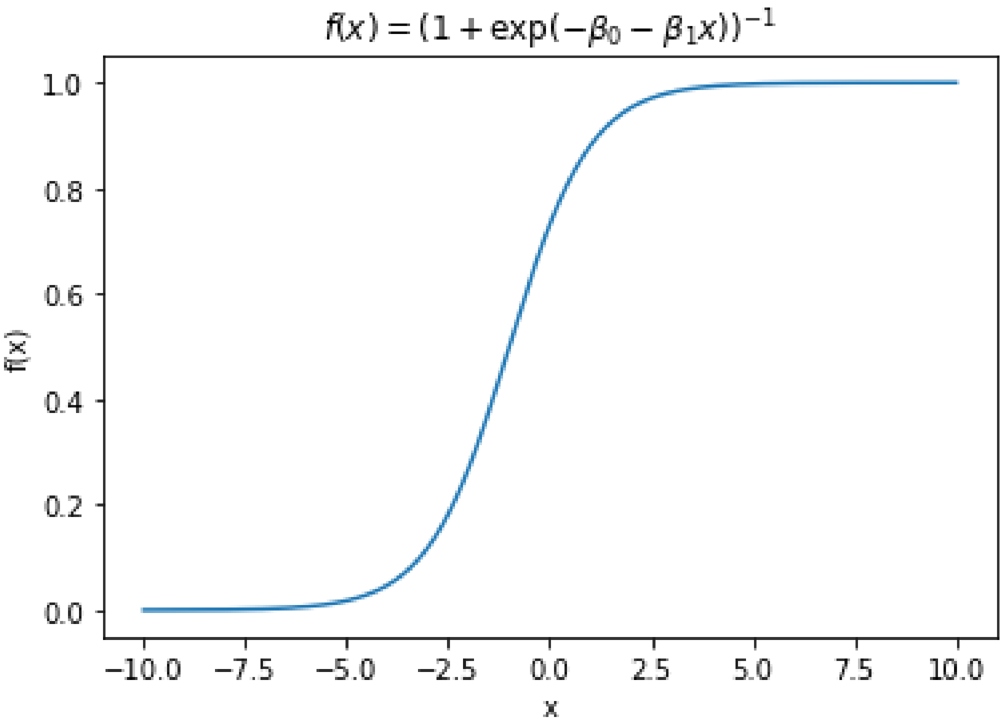
Lasso Regression

- L_1-regularization: $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \alpha \sum_{i=1}^{m} \left|\beta_i\right|$
- Constrained ($|\boldsymbol{\beta}| \leq t$): $\boldsymbol{\beta}_{\mathrm{Lasso}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left(\left|\boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{y}\right|^2 + \alpha\left|\boldsymbol{\beta}\right|\right)$
- Solution: $\beta_j^{\mathrm{Lasso}} = \operatorname{sign}\left(\beta_j^{\mathrm{LS}}\right)\left(\left|\beta_j^{\mathrm{LS}}\right| - \alpha\right)_+$

Logistic Regression

- Fit to sigmoid $f(x) = \frac{1}{1+\exp(-\boldsymbol{x}^T\boldsymbol{w})}$



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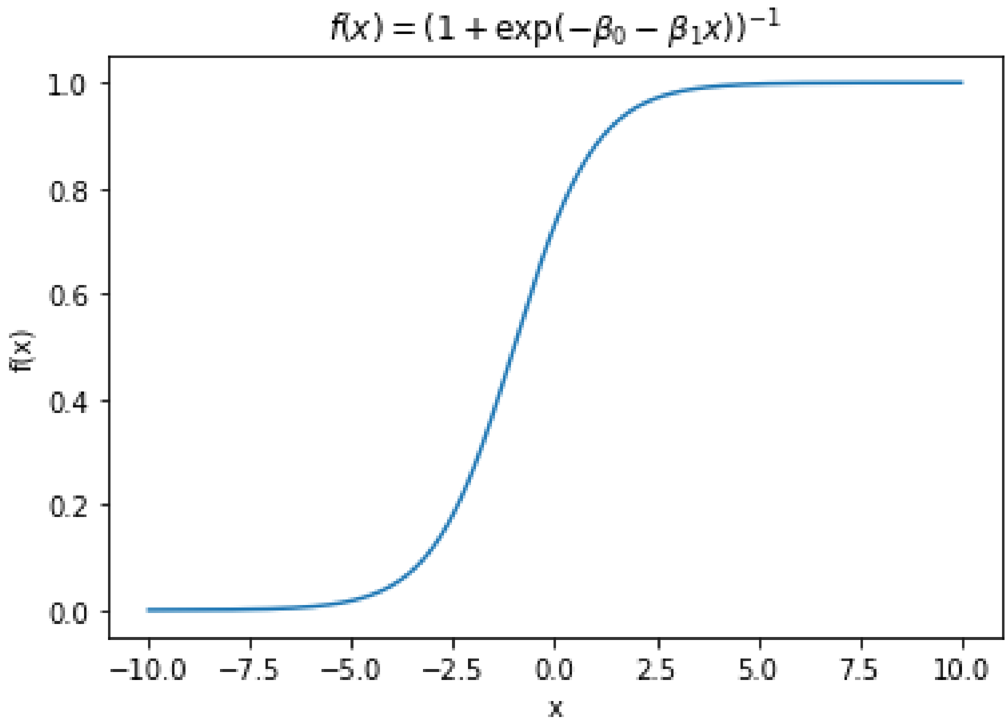

- Categorisation

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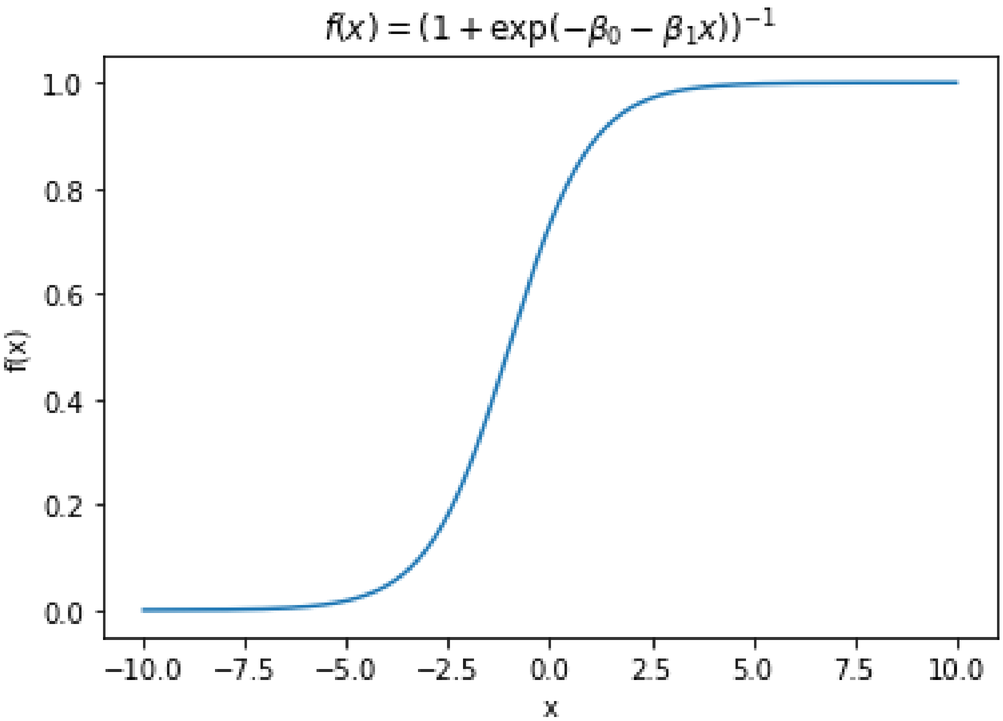

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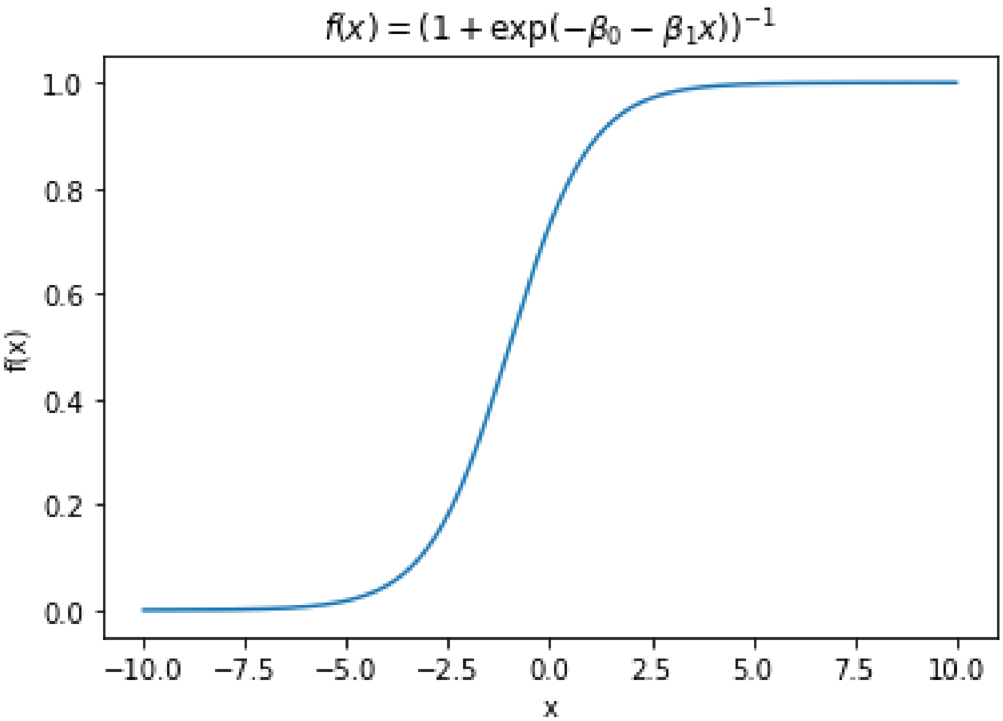

- Categorisation
  - Binary: $P(Y|X)$
- Cost: $C(\boldsymbol{w}) = -\sum_{i=1}^{n} \left[Y_i \log \left(f\left(\boldsymbol{X}_i^T \boldsymbol{w} \right) \right) + (1 - Y_i) \log \left(1 - f\left(\boldsymbol{X}_i^T \boldsymbol{w} \right) \right) \right]$

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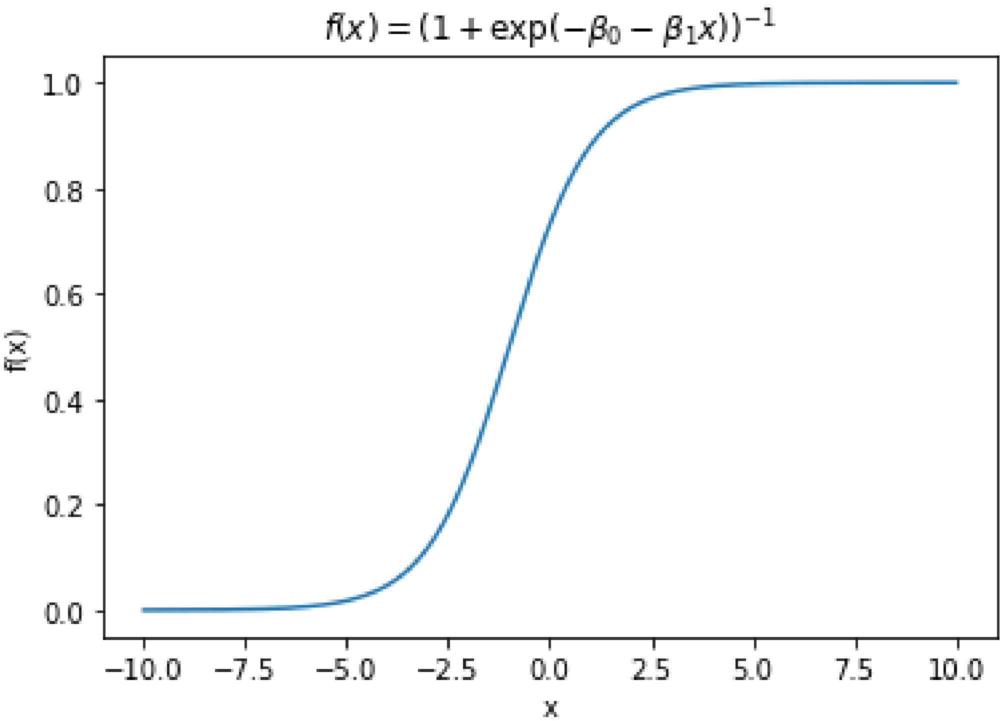

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- Numerical minimization: $\boldsymbol{0} = \nabla C(\boldsymbol{w}) = \sum_{i=1}^{n}\left[f\left(\boldsymbol{X}_i^T\boldsymbol{w}\right) - Y_i\right]\boldsymbol{X}_i$

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- Extremized Random Forest: Split decision trees at random

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- Cost function

Estimating the Coupling Constant of the 1D Ising Model

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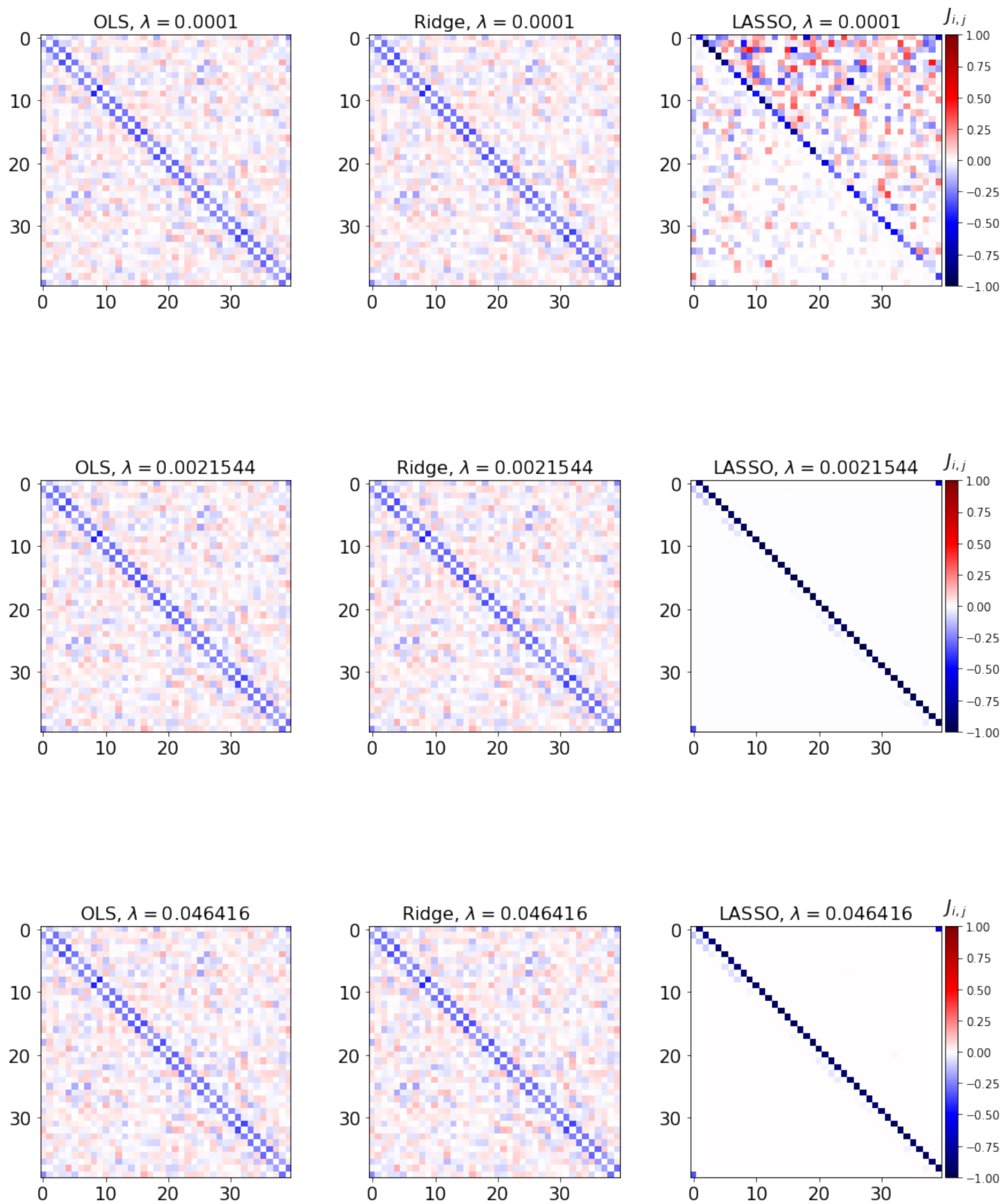

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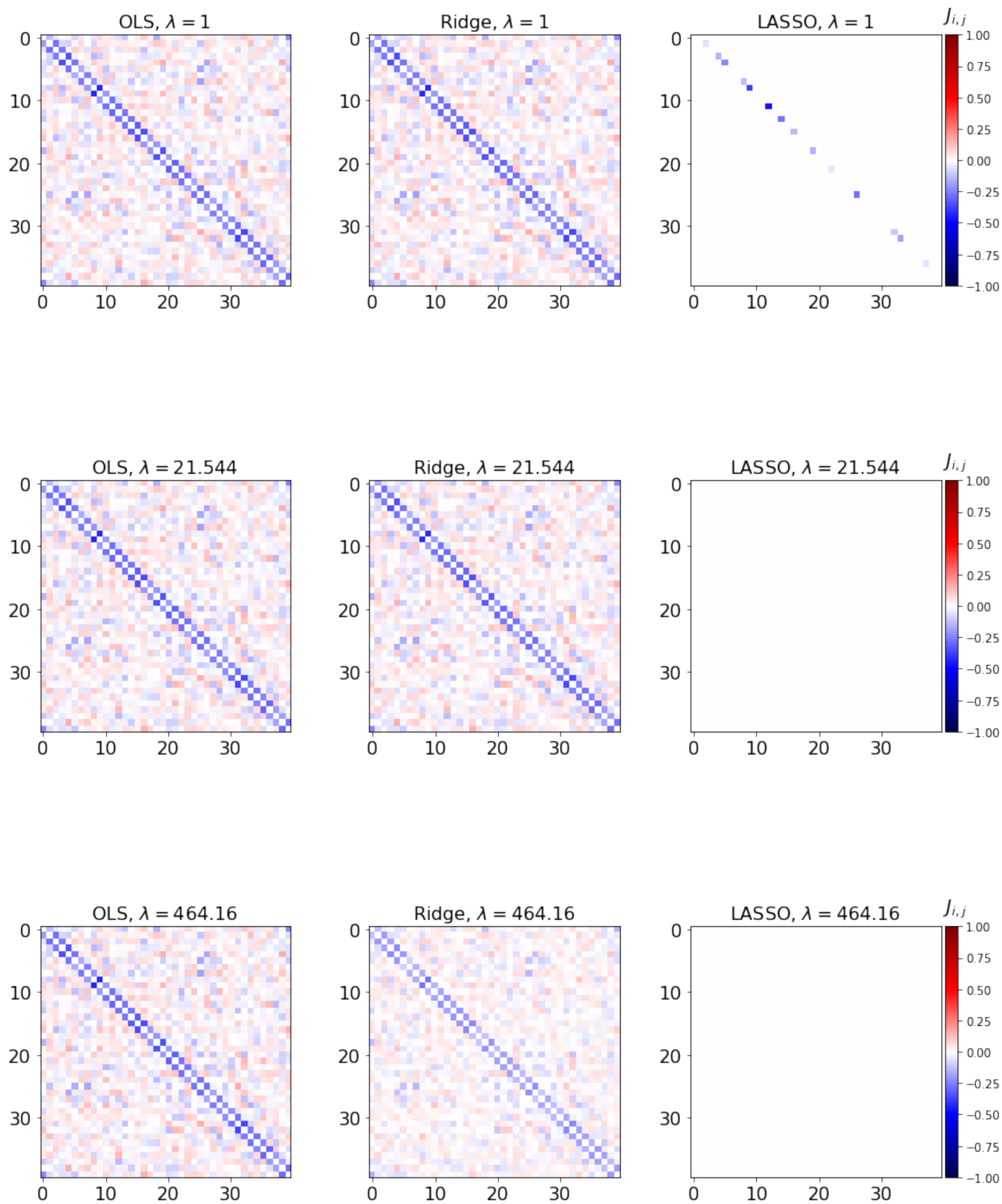

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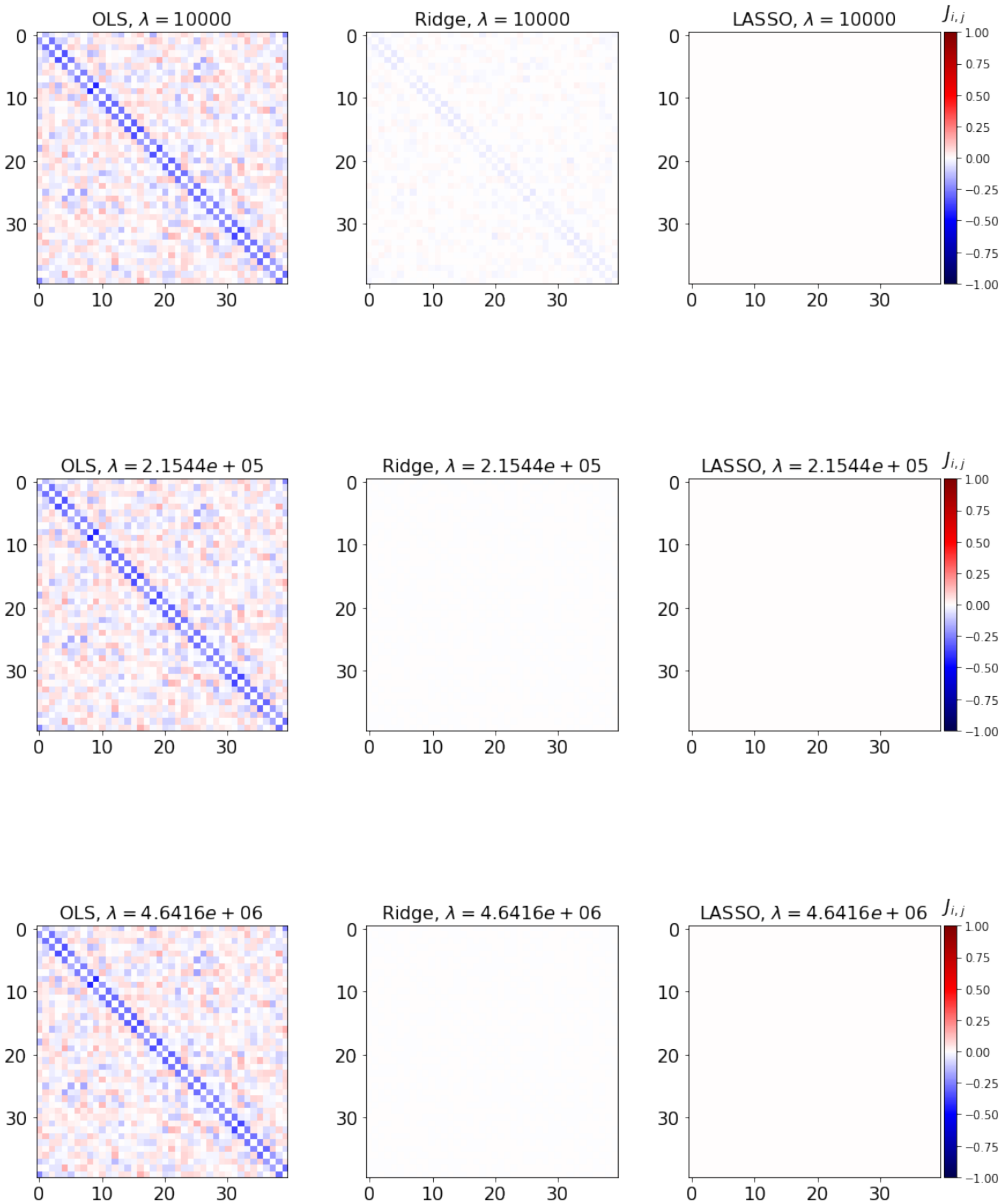

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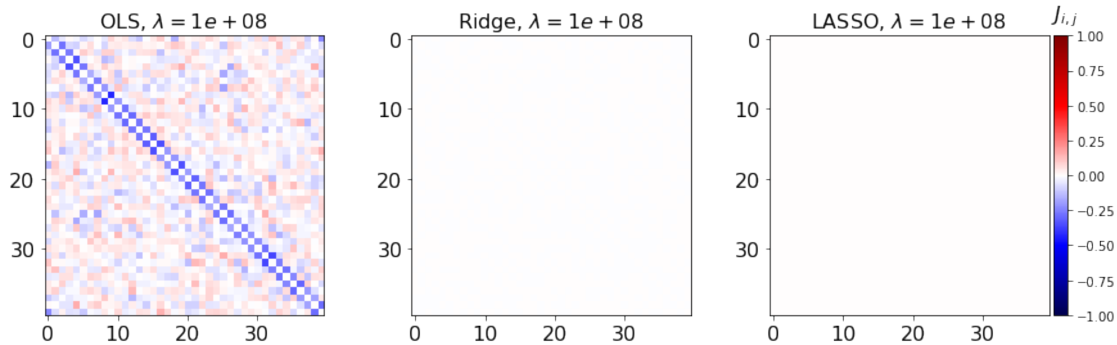

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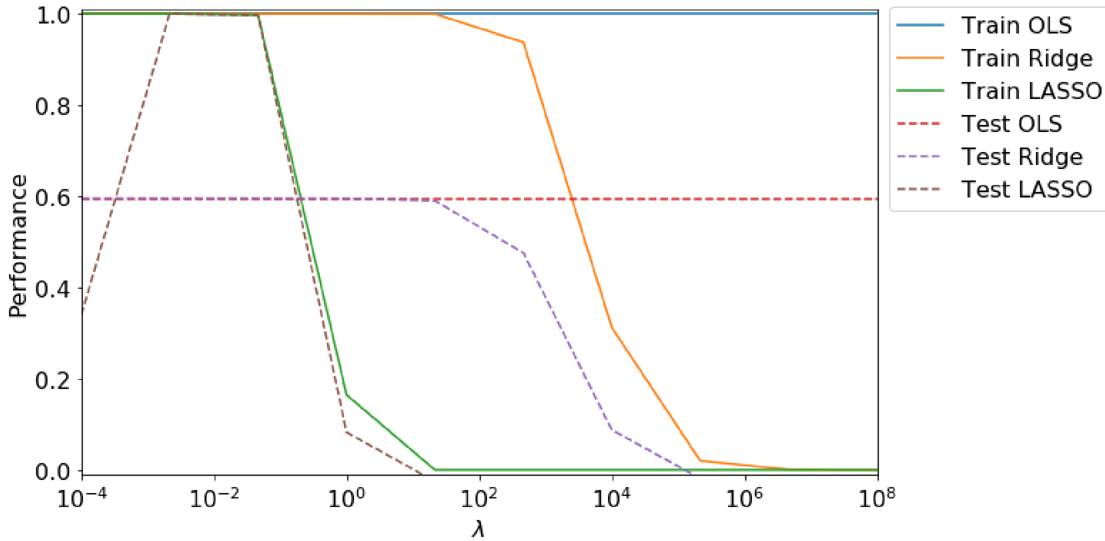
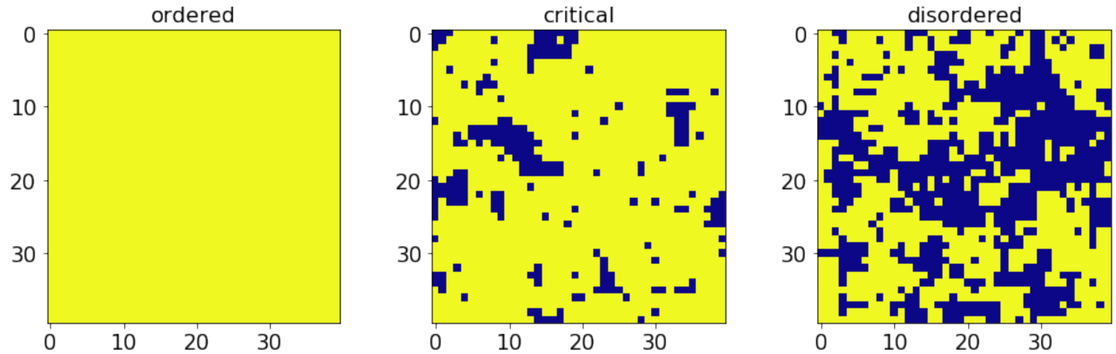


Determine the Phase of the Two-Dimensional Ising Model



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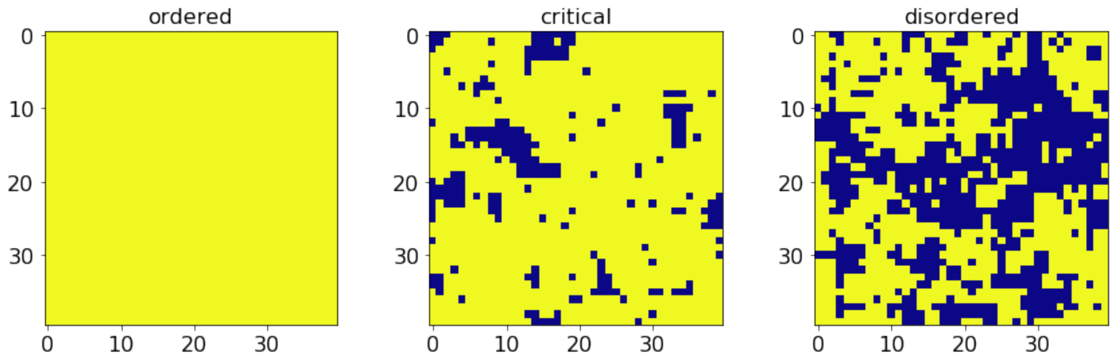

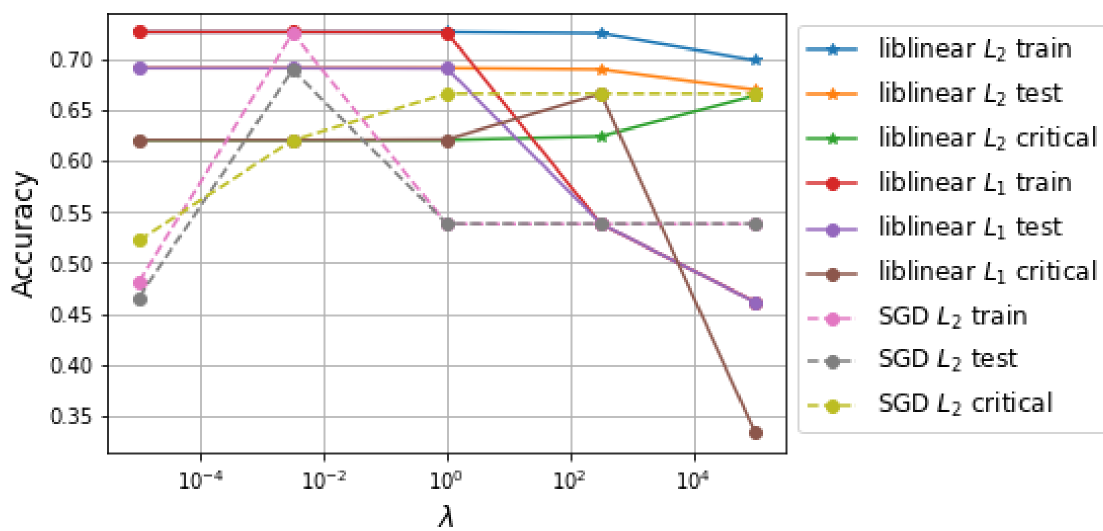

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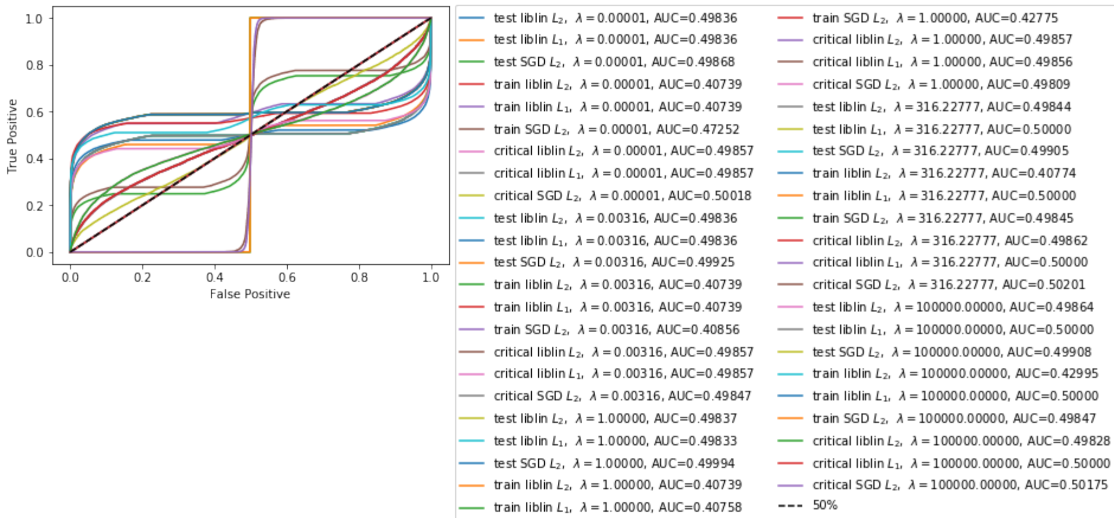
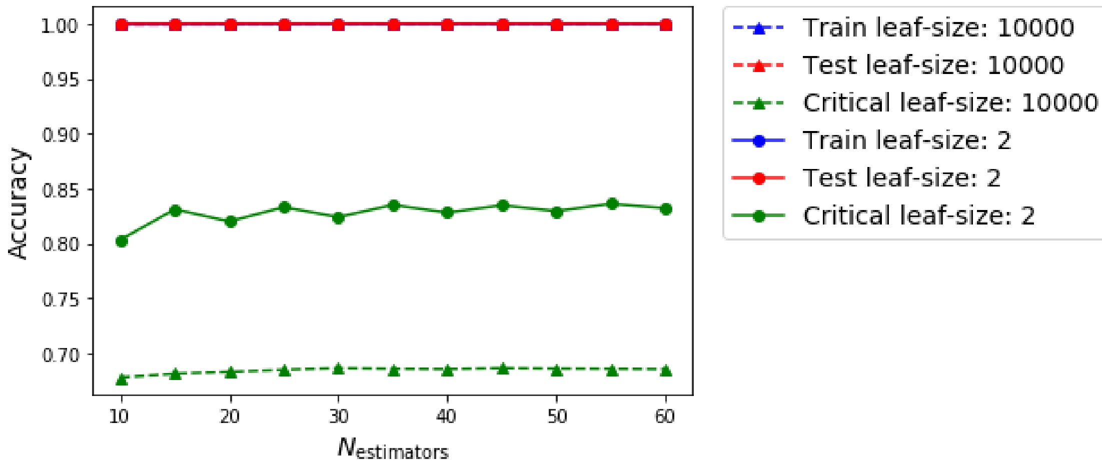
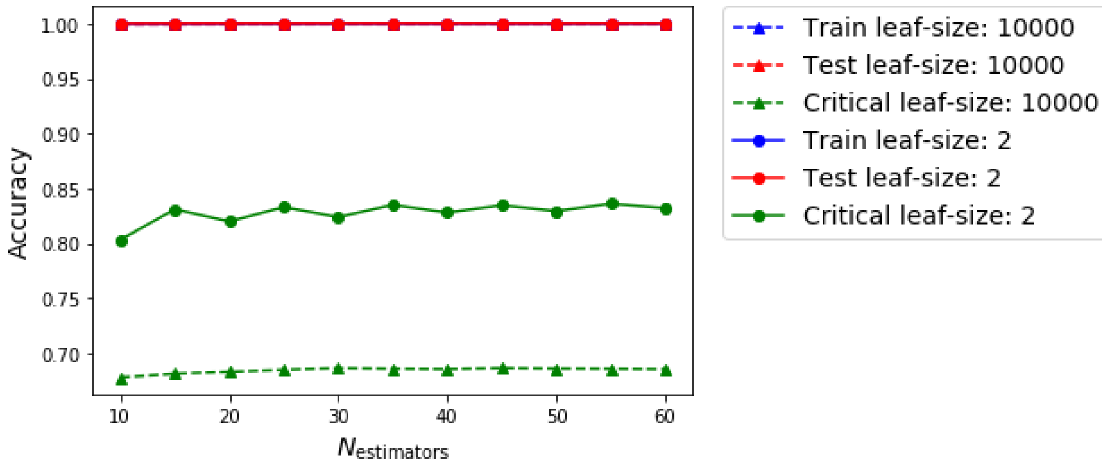
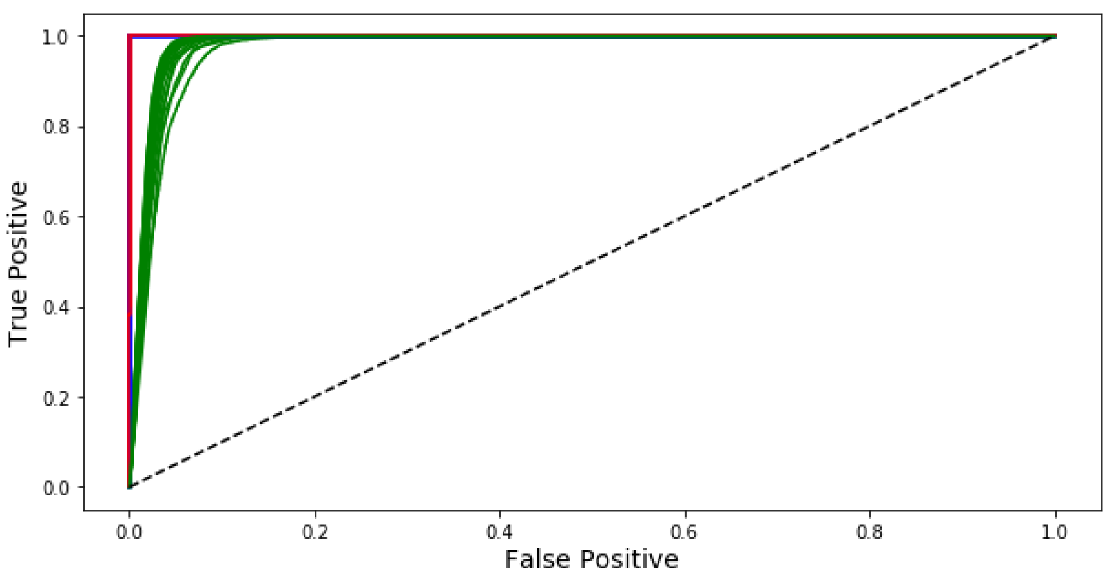


0.0.1 Using Random Forest to Classify Phases in the Ising Model



0.0.2 Using Random Forest to Classify Phases in the Ising Model





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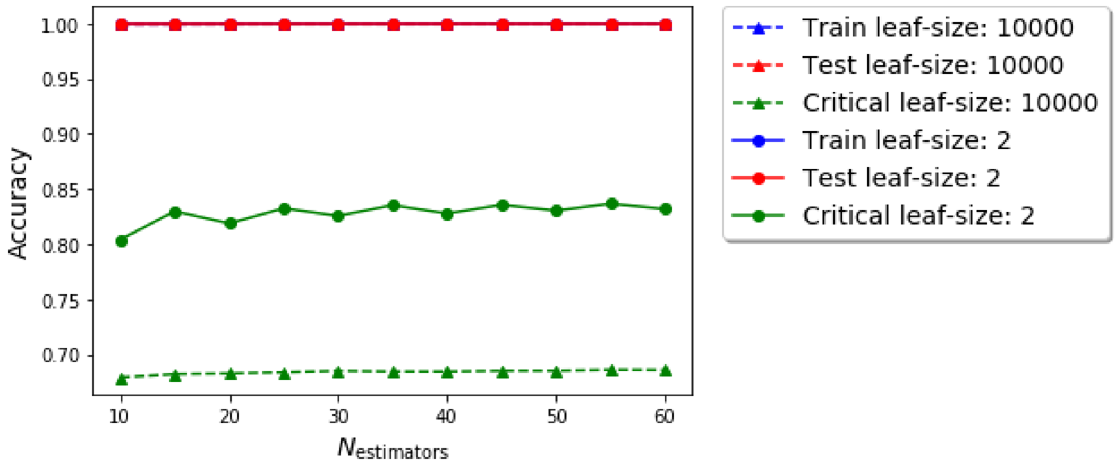

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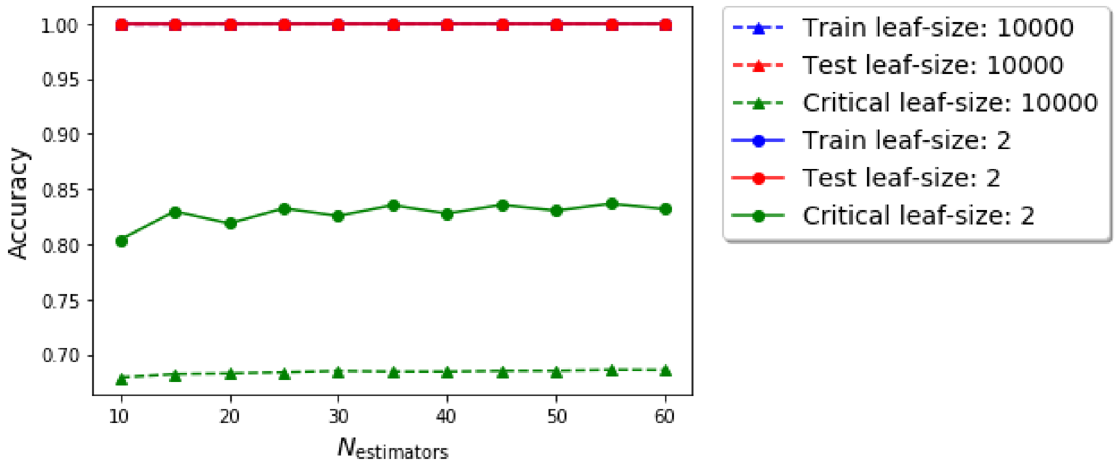
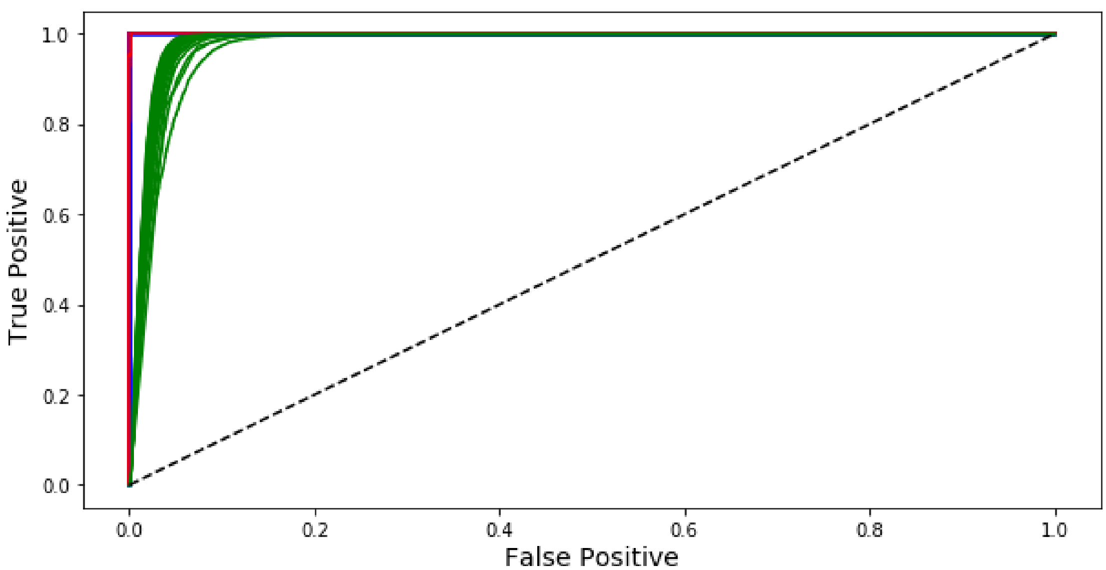
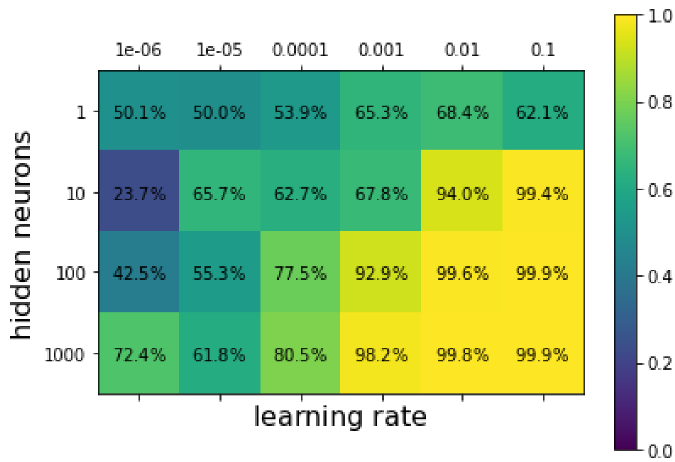
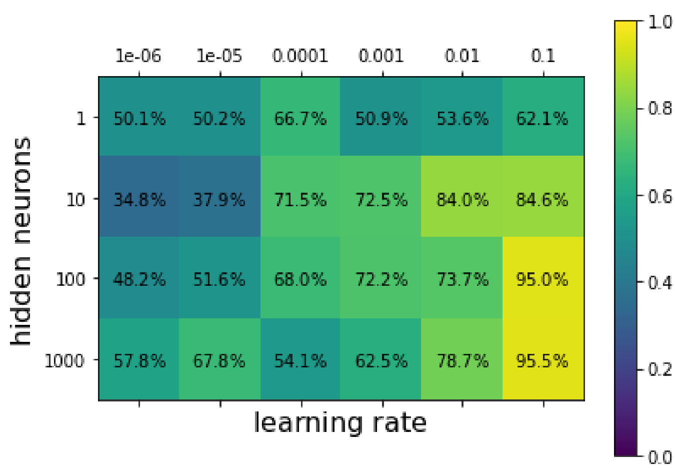




0.0.5 Classifying the Ising Model Phase Using Neural Networks





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