Quantum Many-Body Simulations of Double Dot System

Alocias Mariadason

Institute of Physics

Contents

Introduction

Quantum-Dot

- Small semiconductor nanostructures
- 2-10 nanometers with $10-50$ particles

Quantum-Dot Model

- Schrödinger equation
  - $\mathscr{H}\left|\psi\right\rangle = E\left|\psi\right\rangle$

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Quantum-Dot Model

- Interaction
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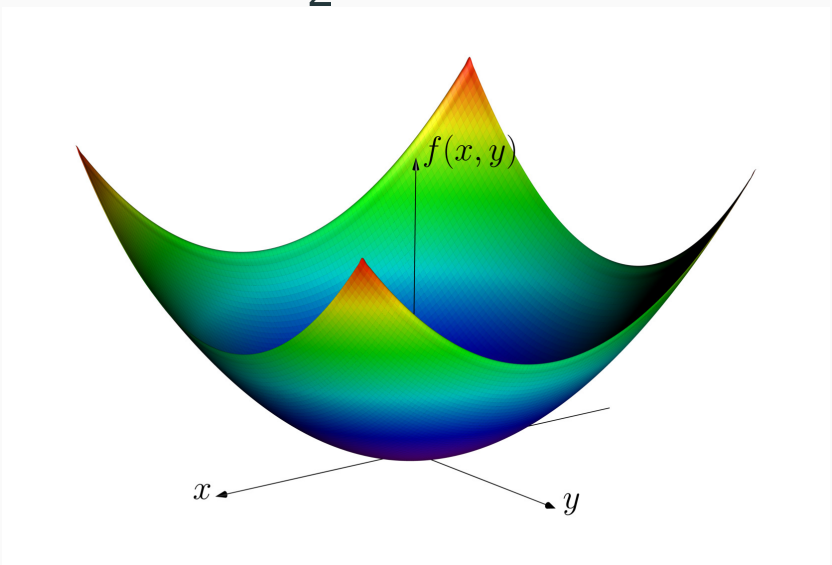
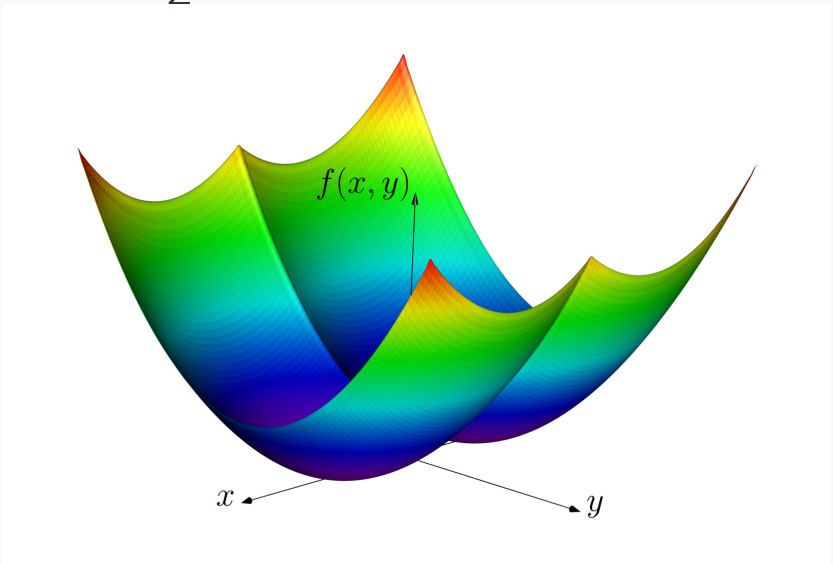
Quantum-Dot Model

- Interaction: Coulomb repulsion
  - $f(\boldsymbol{r}_i, \boldsymbol{r}_j) = \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$
- Confinement: Harmonic Oscillator[1], Double-Well[2]

$$V(\boldsymbol{r}) = \frac{1}{2}\omega m r^2 \qquad V(\boldsymbol{R}, \boldsymbol{r}) = \frac{1}{2} m\omega^2 \left(r^2 - \delta R|x| + R^2\right)$$





[1] S. Kvaal. "Harmonic Oscillator Eigenfunction Expansions, Quantum dots, and Effective Interactions". In: *Phys. Rev. B* 80 (4 2009), p. 045321.

[2] M. J. A. Schuetz et al. "Nuclear Spin Dynamics in Double Quantum Dots: Multistability, Dynamical Polarization, Criticality, and Entanglement". In: *Phys. Rev. B* 89 (19 2014), p. 195310.

Methods

Hartree-Fock

Variational Monte-Carlo

Methods: Variational Principle

$$E_0 \leq \frac{\langle \Psi | \mathscr{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

Methods: Slater Determinant and Energy Functional

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  - $\Psi_T^{\mathsf{AS}} = \frac{1}{\sqrt{N!}} \sum_P (-1)^p \mathscr{P}_P \prod_i \psi_i$
  - $\Psi_T^{\mathsf{S}} = \sqrt{\frac{\prod_{i=1}^{N} n_i!}{N!}} \sum_P \mathscr{P}_P \prod_i \psi_i$

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- $E[\Psi] = \frac{\langle \Psi | \mathscr{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \sum_p \langle p | \mathscr{H}_0 | p \rangle + \frac{1}{2} \sum_{p,q} [\langle pq | f_{12} | pq \rangle \pm \langle pq | f_{12} | qp \rangle]$
- $\mathscr{H}_0 = -\frac{1}{2} \sum_i \nabla_i^2 + V(r)$

Methods: Hartree-Fock

- Assumptions
  - The Born-Oppenheimer approximation holds.
  - All relativistic effects are negligible.
  - The wavefunction can be described by a single *Slater determinant*.
  - The Mean Field Approximation holds.

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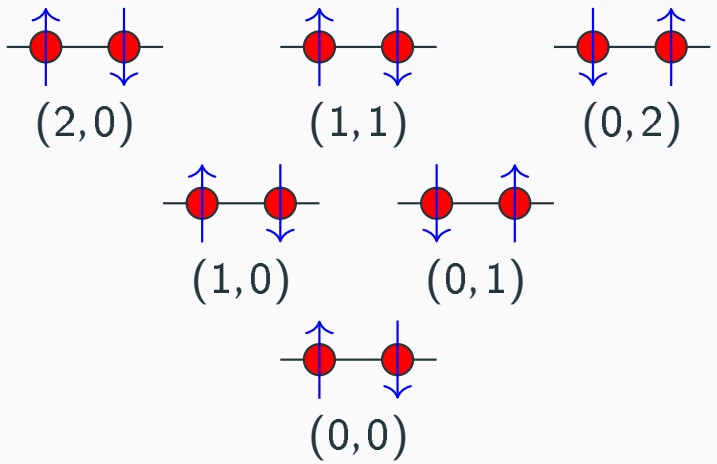

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  - Fock-operator: $\mathscr{F} \equiv \mathscr{H}_0 + \mathscr{J} \pm \mathscr{K}$
    - $\mathscr{J} \equiv \langle \psi_k^* | f_{12} | \psi_k \rangle = \int \psi_k^*(\boldsymbol{r}) f_{12} \psi_k(\boldsymbol{r}) dr$
    - $\mathscr{K} \equiv \langle \psi_k^* | f_{12} | \psi \rangle = \int \psi_k^*(\boldsymbol{r}) f_{12} \psi(\boldsymbol{r}) dr$
    - $\mathscr{F} | \psi \rangle = \boldsymbol{\varepsilon} | \psi \rangle, \boldsymbol{\varepsilon} = (\varepsilon_0, \ldots, \varepsilon_N)$

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    - $\mathscr{F} | \psi \rangle = \boldsymbol{\varepsilon} | \psi \rangle, \boldsymbol{\varepsilon} = (\varepsilon_0, \ldots, \varepsilon_N)$
  - $N+1$ equations to be solved.

Methods: Hartree-Fock

- Integrate out spin
- Pair spins as: $\{\psi_{2l-1}, \psi_{2l}\} = \{\phi_l(\boldsymbol{r})\alpha(s), \phi_l(\boldsymbol{r})\beta(s)\}$

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- Expand: $\phi_i(\boldsymbol{r}) = \sum_{p=1}^{L} C_{pi}\chi_p(\boldsymbol{r})$
- Roothan-Hall: $\boldsymbol{F}\boldsymbol{C}_i = \boldsymbol{\varepsilon} S \boldsymbol{C}_i$
  - $F_{pq} = h_{pq} + \sum_{pq} \rho_{pq}\left(2D_{prqs} \pm D_{prsq}\right)$
  - $h_{pq} \equiv \langle p | h | q \rangle$
  - $\rho_{pq} \equiv \sum_{i=1}^{\frac{N}{2}} C_{pi} C_{qi}^*$
  - $D_{pqrs} \equiv \langle pq | f_{12} | rs \rangle$
  - $S_{pq} \equiv \langle p | q \rangle$
- Poople-Nesbet: $\boldsymbol{F}^+\boldsymbol{C}^+ = \boldsymbol{\varepsilon}\boldsymbol{S}\boldsymbol{C}^+$, $\boldsymbol{F}^-\boldsymbol{C}^- = \boldsymbol{\varepsilon}^-\boldsymbol{S}\boldsymbol{C}^-$
  - $F^{\pm}_{pq} = h_{pq} + \sum_{k_\pm}\sum_{rs} C^{\pm\dagger}_{rk_\pm} C^{\pm\dagger}_{sk_\pm}\left[D_{prqs} - D_{prsq}\right] + \sum_{k_\mp}\sum_{rs} C^{\mp\dagger}_{rk_\mp} C^{\mp\dagger}_{sk_\mp} D_{prqs}$

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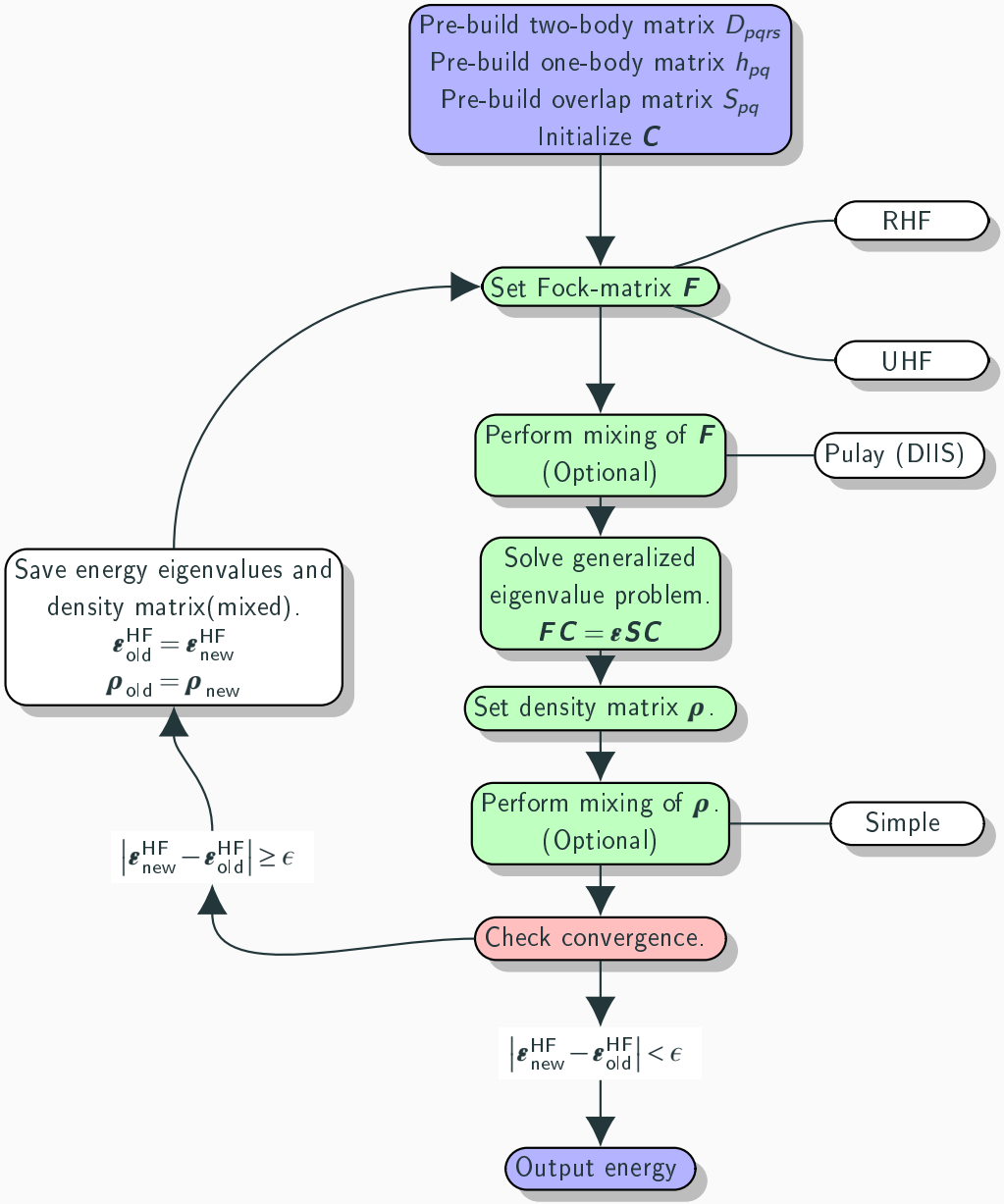

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  - $E_L(\boldsymbol{R};\boldsymbol{\alpha}) \equiv \frac{1}{\Psi}\mathscr{H}\Psi$
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  - $r^{\text{new}} = r^{\text{old}} + \Delta t\xi$
  - $A_{i\to j} = \min\left(\frac{P_{i\to j}}{P_{j\to i}}\frac{T_{i\to j}}{T_{j\to i}}, 1\right)$

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  - Importance Sampling
    - $r^{\text{new}} = r^{\text{old}} + D\Delta t F^{\text{old}} + \sqrt{\Delta t}\xi$
    - $F = \frac{2}{\Psi}\nabla\Psi$
    - $\frac{T(b,a,\Delta t)}{T(a,b,\Delta t)} = \sum_i \exp\left(-\frac{\left(\boldsymbol{r}_i^{(b)} - \boldsymbol{r}_i^{(a)} - D\Delta t\boldsymbol{F}_i^{(a)}\right)^2}{4D\Delta t} + \frac{\left(\boldsymbol{r}_i^{(a)} - \boldsymbol{r}_i^{(b)} - D\Delta t\boldsymbol{F}_i^{(b)}\right)^2}{4D\Delta t}\right)$

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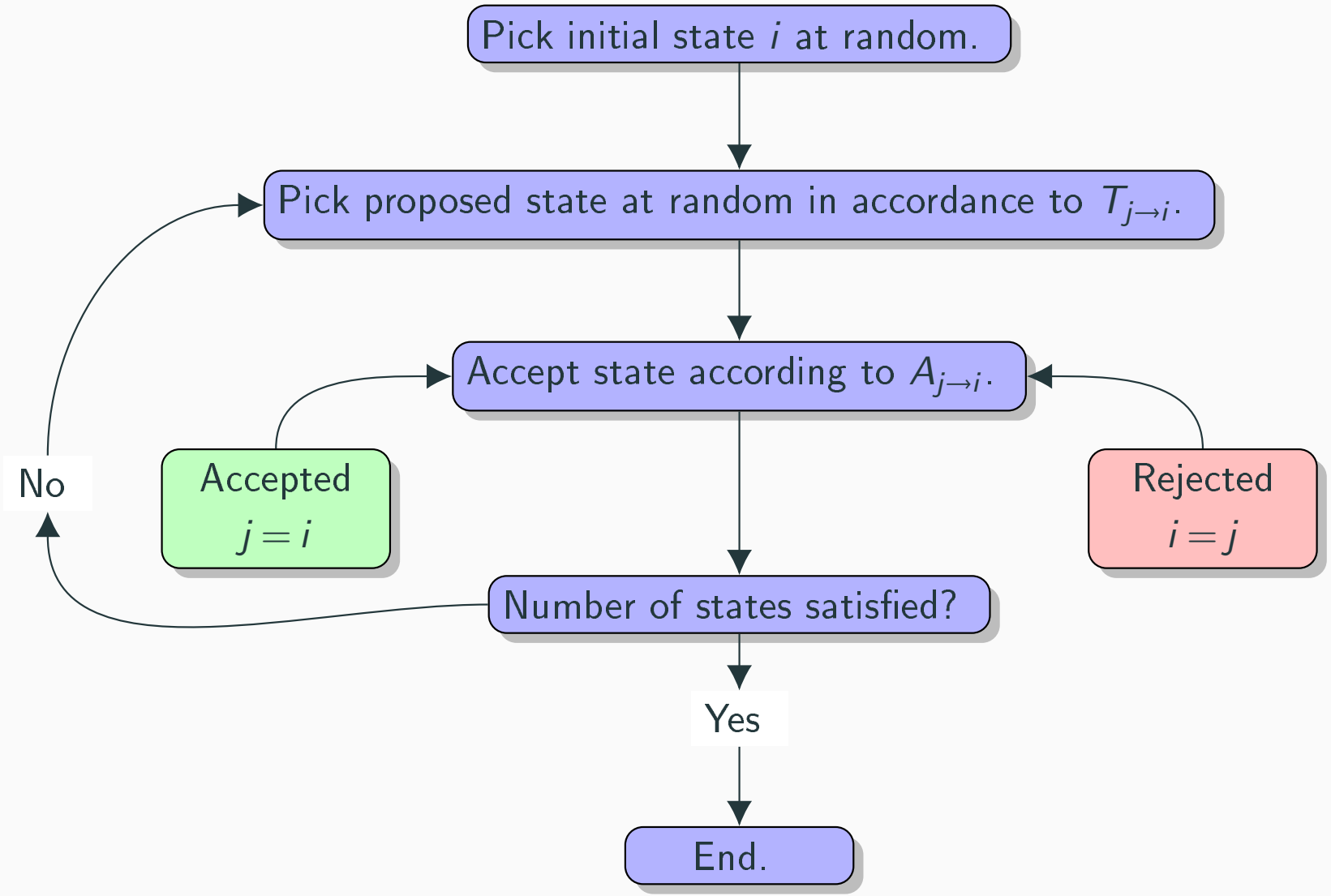

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Wavefunction: Integral Elements

$$\left\langle \phi_i(\boldsymbol{r}) \middle| \phi_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle \phi_i(\boldsymbol{r}) \middle| x_d^k \middle| \phi_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle \phi_i(\boldsymbol{r}) \middle| \nabla^2 \middle| \phi_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle \phi_i(\boldsymbol{r}_1)\phi_j(\boldsymbol{r}_2) \middle| f_{12} \middle| \phi_k(\boldsymbol{r}_1)\phi_l(\boldsymbol{r}_2) \right\rangle$$

Wavefunction: Single-Well

- Hermite Function: $\psi_n(\boldsymbol{r}) \equiv \prod_d N_d H_{n_d}(\sqrt{w} x_d) \exp\left(-\frac{w}{2} x_d^2\right)$

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- Solution in polar[3]

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- $\psi_n(\boldsymbol{r}) = \prod_d N_d \sum_{l=1}^{n_d} C_{n_d l}^{\text{Hermite}} g_l\left(\frac{\omega}{2}, \boldsymbol{r}, \boldsymbol{0}\right)$

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- Solution in polar[5]
- $\psi_n(\boldsymbol{r}) = \prod_d N_d \sum_{l=1}^{n_d} C_{n_d l}^{\text{Hermite}} g_l\left(\frac{\omega}{2}, \boldsymbol{r}, \boldsymbol{0}\right)$
- Solution in Cartesian[6]

$$\left\langle g_i(\boldsymbol{r}) \middle| g_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle g_i(\boldsymbol{r}) \middle| x_d^k \middle| g_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle g_i(\boldsymbol{r}) \middle| \nabla^2 \middle| g_j(\boldsymbol{r}) \right\rangle$$

$$\left\langle g_i(\boldsymbol{r}_1) g_j(\boldsymbol{r}_2) \middle| f_{12} \middle| g_k(\boldsymbol{r}_1) g_l(\boldsymbol{r}_2) \right\rangle$$

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[5] E. Anisimovas and A. Matulis. "Energy spectra of few-electron quantum dots". In: *Journal of Physics: Condensed Matter* (1998).

[6] J. Olsen T. Helgaker P. Jørgensen. *Molecular Electronic-Structure Theory*. Wiley, 2014. isbn: 978-0-47-196755-2. doi: 10.1002/9781119019572.

Wavefunction: Single-Well Integral Elements

$$\left\langle \psi_i^{\mathrm{HO}} \middle| \psi_j^{\mathrm{HO}} \right\rangle = N_i \delta_{ij}$$

$$\left\langle \psi_i^{\mathrm{HO}} \middle| h^{\mathrm{HO}} \middle| \psi_j^{\mathrm{HO}} \right\rangle = N_i \varepsilon_i^{\mathrm{HO}} \delta_{ij}$$

$$\left\langle \psi_i^{\mathrm{HO}} \psi_j^{\mathrm{HO}} \middle| \frac{1}{r_{12}} \middle| \psi_k^{\mathrm{HO}} \psi_l^{\mathrm{HO}} \right\rangle = \frac{aN_{ijkl}}{\sqrt{2\omega}} \sum_{tuvw}^{ijkl} H_{tuvw}^{ijkl} \sum_{pq}^{t+v,u+w} E_p^{tv} E_q^{uw} (-1)^q \xi_{p+q}\left(\tfrac{\omega}{2}, \mathbf{0}\right)$$

$$E_t^{i+1,j} = \frac{1}{2(\alpha+\beta)} E_{t-1}^{ij} - \frac{\beta}{\alpha+\beta}(A_x - B_x) E_t^{ij} + (t+1) E_{t+1}^{ij}$$

$$E_t^{i,j+1} = \frac{1}{2(\alpha+\beta)} E_{t-1}^{ij} - \frac{\alpha}{\alpha+\beta}(A_y - B_y) E_t^{ij} + (t+1) E_{t+1}^{ij}$$

$$E_0^{00} = K_{AB}$$

$$\xi_{t+1,u}^n = t\xi_{t-1,u}^{n+1} + X_{AB}\xi_{t,u}^{n+1}$$

$$\xi_{t,u+1}^n = u\xi_{t,u-1}^{n+1} + Y_{AB}\xi_{t,u}^{n+1}$$

$$\xi_{00}^n = \left(\frac{-2\alpha\beta}{\alpha+\beta}\right)^n \zeta_n\left(\frac{\alpha\beta}{\alpha+\beta} R_{AB}^2\right)$$

$$\zeta_n(x) = \int_{-1}^{1} \frac{u^{2n}}{\sqrt{1-u^2}} e^{-u^2 x} du$$

$$\xi_{t+1,u,v}^n = t\xi_{t-1,u,v}^{n+1} + X_{AB}\xi_{t,u,v}^{n+1}$$

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- Expand in HO-functions: $\left|\psi_p^{\mathrm{DW}}\right\rangle = \sum_l C_{lp}^{\mathrm{DW}} \left|\psi_l^{\mathrm{HO}}\right\rangle$

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- Eigenvalue equation: $\boldsymbol{H}^{\mathrm{DW}} \boldsymbol{C}^{\mathrm{DW}} = \boldsymbol{\varepsilon}^{\mathrm{DW}} \boldsymbol{C}^{\mathrm{DW}}$
  - $H_{ij}^{\mathrm{DW}} = \varepsilon_i^{\mathrm{HO}} \delta_{ij} + \left\langle \psi_i^{\mathrm{HO}} \middle| V_n^{\mathrm{DW}} \middle| \psi_j^{\mathrm{HO}} \right\rangle$

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- Integral-Elements

$$\left\langle \psi_p^{\mathrm{DW}} \middle| \psi_q^{\mathrm{DW}} \right\rangle = \delta_{pq} \varepsilon_p^{\mathrm{DW}} \delta_{pq}$$

$$\left\langle \psi_p^{\mathrm{DW}} \middle| h^{\mathrm{DW}} \middle| \psi_q^{\mathrm{DW}} \right\rangle = \varepsilon_p^{\mathrm{DW}} \delta_{pq}$$

$$\left\langle \psi_p^{\mathrm{DW}} \psi_q^{\mathrm{DW}} \middle| \frac{1}{r_{12}} \middle| \psi_r^{\mathrm{DW}} \psi_s^{\mathrm{DW}} \right\rangle = \sum_{tuvw}^{ijkl} C_{tp}^{\mathrm{DW}} C_{uq}^{\mathrm{DW}} C_{vr}^{\mathrm{DW}} C_{ws}^{\mathrm{DW}} \left\langle \psi_t^{\mathrm{HO}} \psi_u^{\mathrm{HO}} \middle| \frac{1}{r_{12}} \middle| \psi_v^{\mathrm{HO}} \psi_w^{\mathrm{HO}} \right\rangle$$

Implementation

Summary and Conclusion

Questions?

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