Assignment # 4: MATH1051

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1. **(1 mark each)** This is the main question. Determine the domains (as a subset of $\mathbb{R}$) of the functions

   (a) $f_1(x) = \frac{1}{e^x - e^{-x}}$

   (b) $f_2(x) = \frac{1}{\sqrt{4-x^2}}$

   (c) $f_3(x) = \log \arccos x$

2. **(3 marks)** Given is the function $g(x) = x^2 + 3x$. For a second function f with $f(3) = 0$ we find $(g \circ f)(x) = x^2 - 3x$. What is the function f? Is f unique?

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3. **(1 mark each)** Determine which of the following functions are 1-1? Prove your answer.

   (a)

   (b)

   (c)

4. **(1 mark each)** Determine what the following limits are or show that they do not exist.

(a)

(b)

(c)

(d)

(e)

5. **(1 mark each)** Consider the sequence a_n defined by the recursion

$$a_n = a_{n-1} - \frac{1}{4} a_{n-2} \tag{1}$$

for $n = 3, 4, 5, \ldots$.

(a) Calculate h such that $a_n = h^{n-1}$ fulfils the recursive definition.

(b) What is the limit of the sequence a_n (if it exists)?

6. **(1 mark each)** Given is the sequence b_n defined in recursive form

$$b_n = \frac{1}{2}\left(b_{n-1} + \frac{A}{b_{n-1}}\right)$$

for a given $A > 0$. You can assume that all values of b_n are non-zero.

(a) Use your calculator (or MATLAB) to calculate the first four values of the sequence b_n starting from $b_1 = A$ (this is for $n = 1, 2, 3, 4$). Inspecting these values do you expect the sequence to be convergent or to be divergent?

(b) Assume you know the sequence b_n is converging, what would be its limit (or its limits)? Justify your answer. Is it consistent with your result of part (a)?

7. **(1 mark each)** Assume you have given a sequence c_n with non-zero values ($c_n \neq 0$ for $n = 1, 2, \ldots$) that fulfils the condition

$$\left|\frac{c_n}{c_n - 1}\right| \leq q \tag{2}$$

for all $n = 1, 2, 3, \ldots$ for some fixed constant q with $0 < q < 1$.

(a) Show that $c_n \to 0$ for $n \to \infty$. (Hint: use the squeeze theorem)

(b) Use the result from part(a) to show that $\lim_{n\to\infty} \frac{2^n}{n!} = 0$.

(c) Again: use the result from part(a) to show that $\lim_{n\to\infty} \frac{1}{3^n n^3} = 0$.