- Write your answers clearly. Illegible assignments will not be marked.
- Show all your workings. Correct answers without justification will not receive full marks.
- Wherever possible, answers should be given in exact form.
- This assignment is worth 10% of the total assessment for the course.
- Submit your assignment as a **single PDF** via the Assignment submission link on Blackboard.
- Submit all applications for extensions via the my.UQ portal.
- Marking: The total mark is 25.
  - Marking Scheme for questions worth 1 mark:
    - Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
    - Mark of 1/2: You have the right approach, but need to fine-tune some aspects of your justification/calculations.
    - Mark of 1: You have demonstrated a good understanding of the topic and techniques involved, with clear justification and well-executed calculations.
  - Marking Scheme for questions worth 3 marks:
    - Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
    - Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique. This topic needs more attention!
    - Mark of 2: You have the right approach, but need to fine-tune some aspects of your justification/calculations.
    - Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with clear justification and well-executed calculations.

Attach this page to the front of your submission. Remember to sign the declaration.

I hereby state that the work contained in this assignment has not previously been submitted for assessment, either in whole or in part, by either myself or any other student at either The University of Queensland or at any other tertiary institution except where explicitly acknowledged. To the best of my knowledge and belief, the assignment contains no material that has been previously published or written by another person except where due reference is made. I make this Statement in full knowledge of an understanding that should it be found to be false, I will be subject to disciplinary action under Student Integrity and Misconduct Policy 3.60.04 of the University of Queensland. The University of Queensland's policy on plagiarism can be found at http://ppl.app.uq.edu.au/content/3.60.04-student-integrity-and-misconduct (Reference 3.60.04).

Name .....Student ID .....

Signed .....Date .....

(1) Determine the domains (as subset of $\mathbb{R}$) of the functions (1 mark each)

(a) $f_1(x) = \frac{1}{e^x - e^{-x}}$,

(b) $f_2(x) = \frac{1}{\sqrt{4-x^2}}$.

(c) $f_3(x) = \log \arccos x$.

(2) Given is the function $g(x) = x^2 + 3x$. For a second function f with $f(3) = 0$ we find $(g \circ f)(x) = x^2 - 3x$. What is the function f? Is f unique? (3 mark)

(3) Determine which of the following functions is 1-1? Prove your answer. (1 mark each)

(a) $f_1(x) = e^{-x^2}$

(b) $f_2(x) = 2x^2 - 3x + 1$

(c) $f_3(x) = |x| + 2 \cdot x$

(4) Determine what the following limits are or show that they do not exist: (1 mark each)

(a) $\lim_{n\to\infty} \frac{(n^2+4n-27)(n^3-1)}{(n(n-1))^2}$

(b) $\lim_{n\to\infty} \frac{3n^2-9n+48}{4n^3}$

(c) $\lim_{n\to\infty} \frac{(3n+1)^3-27n^3}{n^2}$

(d) $\lim_{n\to\infty} \frac{2n^2}{2n-1} - n$

(e) $\lim_{n\to\infty} \sqrt{n(n+1)} - n$

Turn over for more questions

(5) Consider the sequence a_n defined by the recursion

$$a_n = a_{n-1} - \frac{1}{4}a_{n-2} \tag{1}$$

for $n = 3, 4, 5, \ldots$. (1 mark each)

(a) Calculate h such that $a_n = h^{n-1}$ fulfills the recursive definition **??**.

(b) What is the limit of the sequence a_n (if it exists)?

(6) Given is a sequence b_n defined in recursive form

$$b_n = \frac{1}{2}\left(b_{n-1} + \frac{A}{b_{n-1}}\right)$$

for a given $A > 0$. You can assume that all values of b_n are non-zero.

(a) For $A = 2$ use your calculator (or MATLAB) to calculate the first four values of the sequence b_n starting from $b_1 = A$ (this is for $n = 1, 2, 3, 4$). Inspecting these values: do you expect the sequence to be convergent or to be divergent? (1 mark)

(b) Assume you know the sequence b_n is converging, what would be its limit (or its limits)? Justify your answer. Is it consistent with your result of part (a)? (3 marks)

(7) Assume you have given a sequence c_n with non-zero values ($c_n \neq 0$ for $n = 1, 2, \ldots$) that fulfils the condition

$$\left|\frac{c_n}{c_{n-1}}\right| \leq q \tag{2}$$

for all $n = 1, 2, 3, \ldots$ for some fixed constant q with $0 < q < 1$.

(a) Show that $c_n \to 0$ for $n \to \infty$. (Hint: use the squeeze theorem) (3 marks)

(b) Use the result from part(a) to show that $\lim_{n\to\infty} \frac{2^n}{n!} = 0$ (1 mark)

(b) Again: use the result from part(a) to show that $\lim_{n\to\infty} \frac{1}{3^n n^3} = 0$ (1 mark)

END OF ASSIGNMENT