Q1: You have been asked to design a Butterworth filter with the following specifications:

$G_{p,dB} = -3$; $\omega_p = 20$; $G_{s,dB} = -25$; $\omega_s = 50$.

Part 1:

Find *n*, the order of the filter. If needed, you should round up the value of *n* to the nearest higher integer.

Part 2:

Find the filter cut-off frequency, ω_c. Use both equations for cut-off frequency (given in the hints) and report both ω_c.

Part 3:

For the *n*-value you obtained in **Part 1**, find s_k values.

Part 4:

Find the ***normalized*** transfer function, $H_n(s)$.

Part 5:

Use the ω_c you obtained in **Part 2** that matches the passband specifications exactly, and find the ***final*** transfer function, H(s).

Part 6:

Use MATLAB and graph the frequency response of the filter, i.e., $|H(s)|$, for ***both*** ω_c you obtained in **Part 2**.

Part 7:

Using the frequency response graphs obtained in **Part 6**, explain the difference between the two designs (i.e., between the two ω_c) from a practical design perspective.

Use the following hints and concepts:

- A *Butterworth Filter* Is A Low-Pass Filter With Amplitude Response Of

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}},$$

where ω_c is the filter *cutoff frequency* and n is the *filter order*.

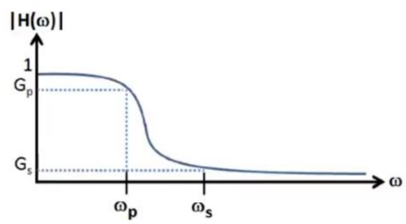
- The Transfer Function of the Normalized Butterworth Filter As

$$\begin{aligned} H_n(s) &= \frac{1}{(s-s_1)(s-s_2)\cdots(s-s_n)} \\ &= \frac{1}{s^n + a_{n-1}s_{n-1} + \cdots + a_1 s + 1} \\ &= \frac{1}{B_n(s)} \end{aligned}$$

where $B_n(s)$ is the nth Order Butterworth Polynomial

$$s_k = e^{\frac{j\pi}{2n}(2k+n-1)} \qquad \text{for } k = 1, \ldots, n.$$

- When Specifying a Low-Pass Filter, Typically Specify 4 Different Numbers
- Passband
  - Passband Gain G_p At Passband Frequency ω_p
- Stopband
  - Stopband Gain G_s At Stopband Frequency ω_s



Filter Order Equation

$$\boxed{n = \frac{\log_{10}\left[\left(10^{-G_{s,dB}/10} - 1\right) / \left(10^{-G_{p,dB}/10} - 1\right)\right]}{2\log_{10}(\omega_s/\omega_p)}}.$$

Filter Cutoff Equation

$$\omega_c = \frac{\omega_p}{\left(10^{-G_{p,dB}/10} - 1\right)^{1/2n}}$$

$$\omega_c = \frac{\omega_s}{\left(10^{-G_{s,dB}/10} - 1\right)^{1/2n}}$$

- We Obtain the Final Transfer Function By Replacing s in $H_n(s)$ With s/ω_c

- The Normalized Amplitude Response Is

$$|H_n(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}.$$

Q2: You have been tasked with investigating the design of a digital second-order lowpass Butterworth filter with a cut-off frequency of 3.4 kHz at a sampling frequency of 8000 Hz.

Part 1:

Use bilinear transformation and compute H(z), the final transfer function.

Part 2:

Draw Direct form II structure of this filter.

Part 3:

Plot frequency response of the filter (both magnitude and phase plot) using MATLAB. Use Normalized Frequency ($\times\pi$ rad/sample) as the x-axis.