Q1 :

part 1 :

$$n = \frac{\log_{10}\left(\frac{10^{+25/10}-1}{10^{3/10}-1}\right)}{2\log_{10}\left(\frac{50}{20}\right)} = 3.1420406839 \approx 4$$

round to next highest

part 2 :

using passband $\rightarrow$ $w_c = \dfrac{20}{(10^{+3/10}-1)^{\frac{1}{2(4)}}} = 20.0118758831$ rad s^{-1}

using stopband $\rightarrow$ $w_c = \dfrac{50}{(10^{25/10}-1)^{1/2(4)}} = 24.3580179575$ rad s^{-1}

part 3 :

$$s_k = e^{\frac{j\pi}{2n}(2k+n-1)}$$

$$s_k = e^{\frac{j\pi}{2n}(2k+n-1)}$$

$$s_1 = e^{\frac{j\pi}{2(4)}(2+4-1)} = e^{j\frac{5\pi}{8}}$$

$$s_2 = e^{\frac{j\pi}{2(4)}(2(2)+4-1)} = e^{j\frac{7\pi}{8}}$$

$$s_3 = e^{\frac{j\pi}{2(4)}(2(3)+4-1)} = e^{j\frac{9\pi}{8}}$$

$$s_4 = e^{\frac{j\pi}{2(4)}(2(4)+4-1)} = e^{j\frac{11\pi}{8}}$$

part 4:

$$H_n(s) = \frac{1}{(s-e^{j\frac{5\pi}{8}})(s-e^{j7\pi/8})(s-e^{j9\pi/8})(s-e^{j\frac{11\pi}{8}})}$$

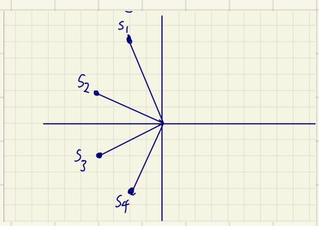
part 5 :

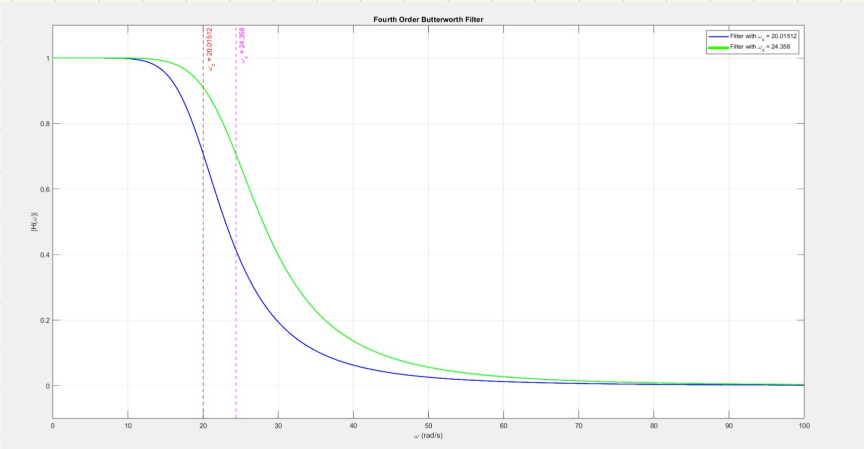
$$H(s) = \frac{1}{(\frac{s}{20.01}-e^{j5\pi/8})(\frac{s}{20.01}-e^{j7\pi/8})(\frac{s}{20.01}-e^{j9\pi/8})(\frac{s}{20.01}-e^{j11\pi/8})}$$

part 6 :

$$|H_n(jw)| = \frac{1}{\sqrt{1+w_{c_s}^{2n}}}$$

$$|H_n(jw)| = \frac{1}{\sqrt{1+w_{cp}^{2n}}}$$





part 7: $\rightarrow$ explain the difference of using passband vs. stopband from a design perspective

From a design perspective, using the passband results in a smaller frequency and therefore a higher gain. This narrower passband in a lower cutoff means that the filter is lower-pass and filters out higher frequencies more aggressively.

Q2: You have been tasked with investigating the design of a digital second-order lowpass Butterworth filter with a cut-off frequency of 3.4 kHz at a sampling frequency of 8000 Hz.

Part 1:

Use bilinear transformation and compute H(z), the final transfer function.

Part 2:

Draw Direct form II structure of this filter.

Part 3:

Plot frequency response of the filter (both magnitude and phase plot) using MATLAB. Use Normalized Frequency (×π rad/sample) as the x-axis.

2. Cutoff frequency: 3.4kHz, Sampling frequency: 8000Hz

Given $f_c = 3.4$ kHz, $w_c = \frac{2\cdot\pi\cdot f_c}{f_s} = 2.6704$ rad

$$\Omega_c = \frac{2}{T}\cdot\tan\left(\frac{w_c}{2}\right) = 2\cdot(8000)\cdot\tan\left(\frac{2.6704}{2}\right) = 66644.7963 \text{ rad/s}$$

$$H_N(s) = \frac{1}{B_N(s)}, \quad B_N(s) = \prod_{k=0}^{N-1}(s-p_k), \quad p_k = e^{j\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2N}\right)}, \quad k=0,1, \quad N=2$$

$$\Rightarrow P_0 = e^{j\left(\frac{\pi}{2}+\frac{\pi}{4}\right)}$$

$$P_1 = e^{j\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)}$$

$$\Rightarrow H_N(s) = \frac{1}{\left(s-e^{j\left(\frac{3\pi}{4}\right)}\right)\left(s-e^{j\left(\frac{5\pi}{4}\right)}\right)}$$

$$\Rightarrow H(s) = \frac{1}{\left(\frac{s}{\Omega_c}-e^{j\left(\frac{3\pi}{4}\right)}\right)\left(\frac{s}{\Omega_c}-e^{j\left(\frac{5\pi}{4}\right)}\right)}$$

Using $s = \frac{2}{T}\cdot\frac{z-1}{z+1}$ for bilinear transformation

$$\Rightarrow H(z) = \frac{1}{\left(\frac{1}{\tan(1.3352)}\frac{z-1}{z+1}-e^{j\left(\frac{3\pi}{4}\right)}\right)\left(\frac{1}{\tan(1.3352)}\frac{z-1}{z+1}-e^{j\left(\frac{5\pi}{4}\right)}\right)}$$

$$\boxed{H(z) = \frac{1}{\left(\frac{1}{\tan(1.3352)}\frac{z-1}{z+1}-\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j\right)\right)\left(\frac{1}{\tan(1.3352)}\frac{z-1}{z+1}-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j\right)\right)}}$$

Part 1

Using Wolfram Alpha:

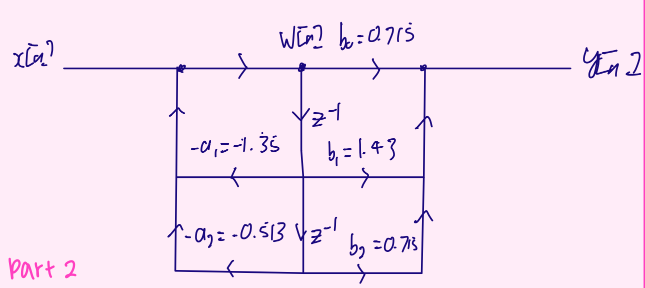
$$H(z) = \frac{0.714735z^2+2(0.714735)z+0.714735}{z^2+1.34637z+0.512573}$$

$$\Rightarrow H(z) = \frac{0.715+1.43z^{-1}+0.715z^{-2}}{1+1.35z^{-1}+0.513z^{-2}}$$

$b_0 = 0.715$ $\quad a_1 = 1.35$

$b_1 = 1.43$ $\quad a_2 = 0.513$

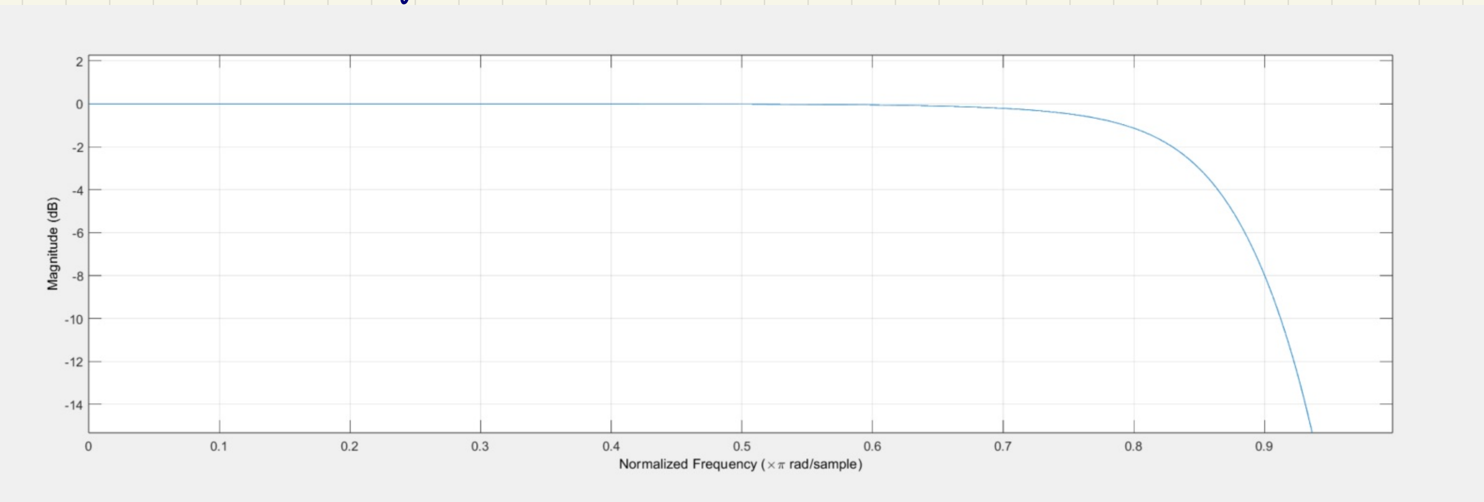
$b_2 = 0.715$



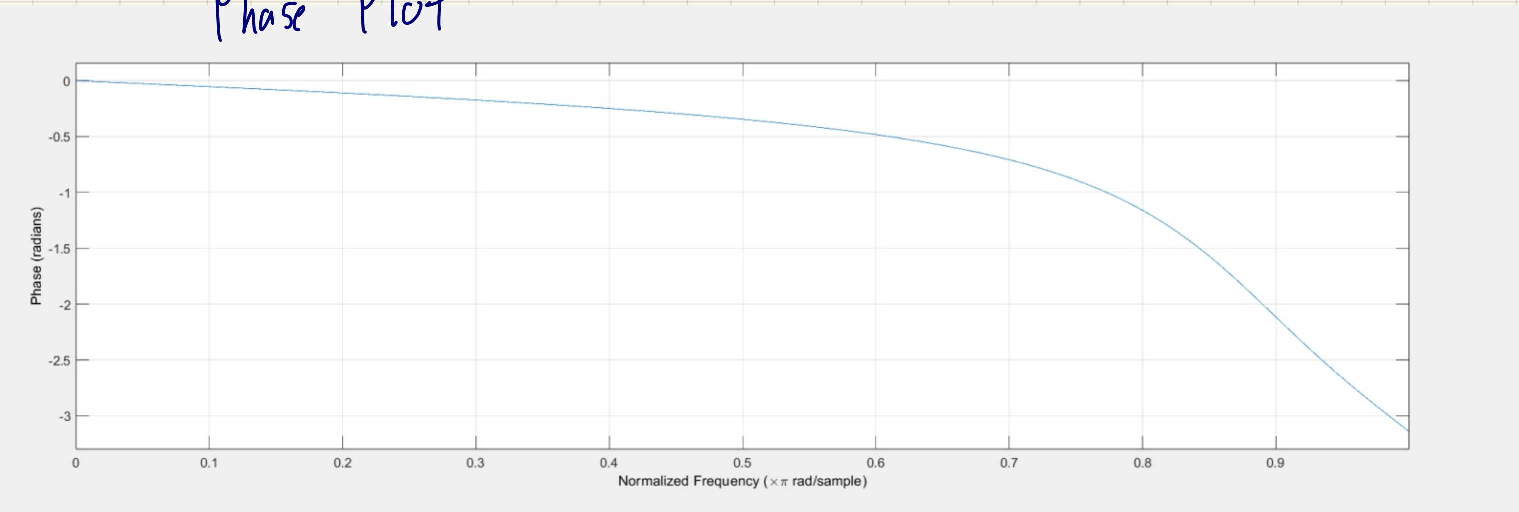
Part 2

Part 3

Magnitude Plot



Phase Plot



Assignment Group Self-Assessment

Group #	7
Assignment #	3
Date	November 27, 2024

Each student should fill in their personal information, the percentage of contribution (out of 100%) given to them by the group, and their signature. By signing this form, the students agree with both their own percentage of contribution and their colleagues percentage of contribution.

Equal between all

First Name	Last Name	Student ID #	Percentage of Contribution	Signature
Athina	Law	68032507	100%	Athina
Sara	Hematy	47109236	100%	
Abhinava Tejas	PRATHIVADHI BHAYANKARAM	72395650	100%	
Fatima	Mirzayeva	20932695	100%	Fatima Mirzayeva
Benjamin	Liu	63031306	100%	
Alex	Sun	67050294	100%	Alex