Q1: Consider the analog signal, $x_a(t) = 3\cos(100\pi t)$.

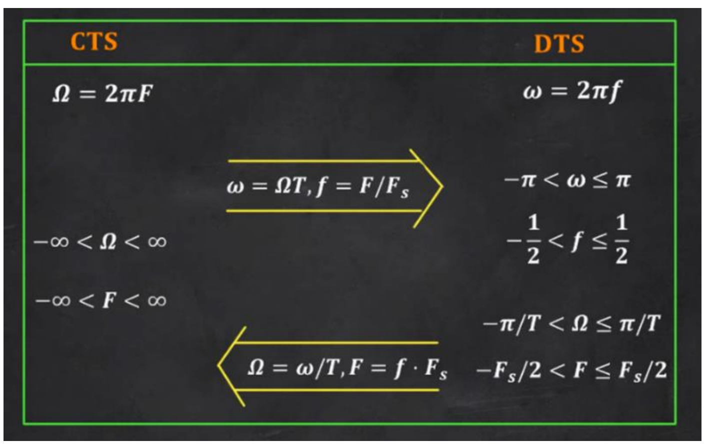
Part 1: Determine the minimum sampling rate required to avoid aliasing.

Part 2: Suppose that the signal is sampled at the rate $F_s = 200$ Hz. What is the discrete-time signal obtained after sampling? Graph the analog signal with the sampling points shown on it.

Part 3: Suppose the signal is sampled at the rate $F_s = 75$ Hz. What is discrete-time signal obtained after sampling? Graph the analog signal with the sampling points shown on it.

Part 4: What is the frequency $0 < F < F_s/2$ of a sinusoid that yields samples identical to those obtained in **Part 3**? Graph the analog signal with the sampling points shown on it and compare it with the results of **Part 3**.

You may use the following conversion table from CTS (Continuous-Time Signal) to DTS (Discrete-Time Signal), and vice versa.



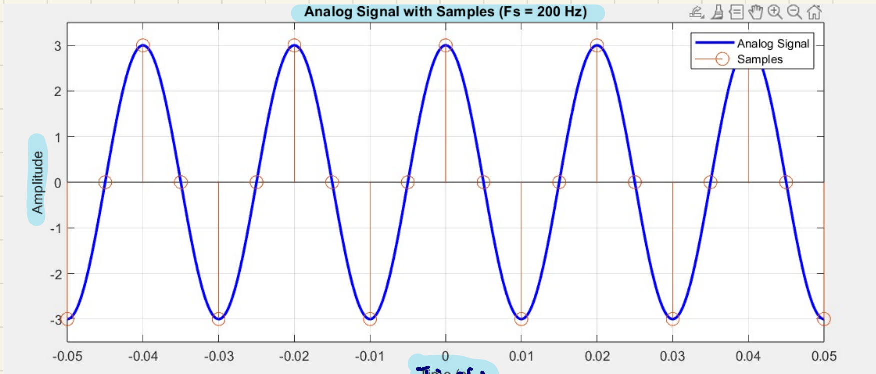
Question 1: $x_a(t) = 3\cos(100\pi t)$

Part 1: $\omega = 100\pi \Rightarrow f = \frac{\omega}{2\pi} = 50Hz$. The minimum sampling rate to avoid aliasing is double our recorded frequency, i.e. $\boxed{f_N = (50Hz \cdot 2) = 100Hz}$

Part 2: Given $f_s = 200Hz$, $T_s = \frac{1}{200}$

Since $x[n] = x(n \cdot T_s) = 3\cos\left(\frac{100\pi}{200} \cdot n\right)$

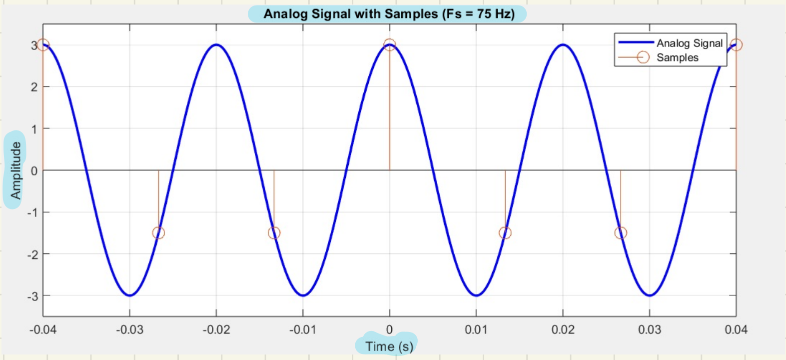
$\Rightarrow \boxed{x[n] = 3\cos\left(\frac{\pi}{2} n\right)}$



Part 3: Given $f_s = 75\ Hz \Rightarrow T_s = \frac{1}{75}$

$x[n] = x(nT_s) = 3\cos\left(\frac{100}{75}\pi \cdot n\right)$

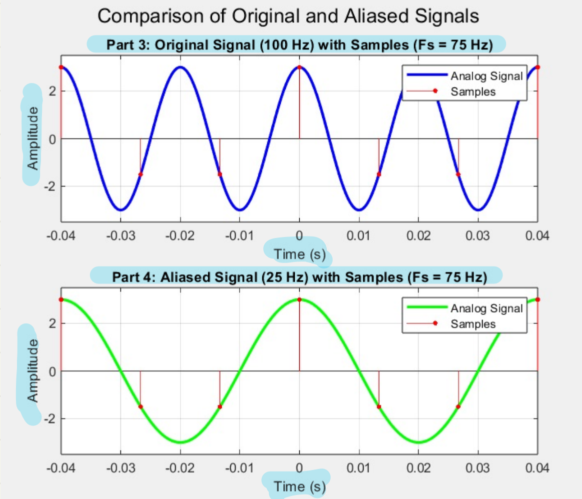
$\boxed{\Rightarrow x[n] = 3\cos\left(\frac{4}{3}\pi \cdot n\right)}$



Part 4: If we sample @ $F_s = 75Hz$, our folding frequency is $\frac{F_s}{2} = 37.5Hz$, so our original 50Hz signal is aliased to 25Hz. So sampling $x_a(t)$ at 75Hz is the same as sampling $x_b(t) = 3 \cdot \cos(50\pi t)$

$x[n] = 3 \cdot \cos\left(50\pi \cdot \frac{1}{75} n\right)$

$\boxed{x[n] = 3\cos\left(\frac{2}{3}\pi \cdot n\right)}$



Q2: Consider the analog signal, $x_a(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$

What is the Nyquist rate for this signal?

$\omega_1 = 50\pi$ $\quad\quad$ $\omega_2 = 300\pi$ $\quad\quad$ $\omega_3 = 100\pi$

$\Rightarrow f_{N_1} = 50\,Hz$ $\quad\quad$ $f_{N2} = 300\,Hz$ $\quad\quad$ $f_{N3} = 100\,Hz$

The nyquist rate is the highest of the three and is thus, 300Hz

Q3: Consider the analog signal, x(t) = 3 cos(2000πt) + 5 sin(6000πt) + 10 cos(12000πt)

Part 1: What is the Nyquist rate for this signal?

Part 2: Assume now that we sample this signal using a sampling rate F_s = 5000 samples/s. What is the discrete-time signal obtained after sampling? Graph this signal.

Part 3: What is the analog signal x(t) that we can reconstruct from the samples if we use ideal interpolation? Graph this signal and compare it with the results of **Part 2**.

Part 1: $\omega_1 = 2000\pi$ $\quad \omega_2 = 6000\pi$ $\quad \omega_3 = 12000\pi$

$\Rightarrow f_{N_1} = 2000\,Hz$ $\quad \Rightarrow f_{N_2} = 6000\,Hz$ $\quad \Rightarrow f_{N_3} = 12000\,Hz$

$\boxed{\text{The Nyquist rate for this signal is } 12000\,Hz}$

Part 2: Sampling rate : $F_s = 5000 \frac{\text{samples}}{s}$

Since $x[n] = x(nT_s)$, $T_s = \frac{1}{F_s} = \frac{1}{5000}$

$$\Rightarrow x[n] = 3\cos\left(2000\pi \cdot \frac{n}{5000}\right) + 5\sin\left(6000\pi \cdot \frac{n}{5000}\right) + 10\cos\left(12000\pi \cdot \frac{n}{5000}\right)$$

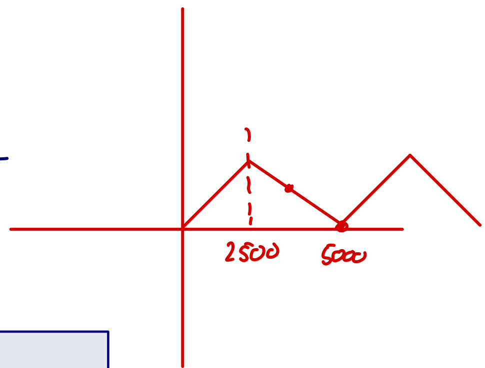
$$\boxed{x[n] = 3\cos\left(\frac{2}{5}\pi \cdot n\right) + 5\sin\left(\frac{6}{5}\pi \cdot n\right) + 10\cos\left(\frac{12}{5}\pi \cdot n\right)}$$

Part 3: Ideal Interpolation

i) $3\cos(2000\pi t)$ Perfectly reconstructed

ii) $5\cdot\cos(6000\pi t)$ Aliased, 3000 Hz to 2000 Hz

iii) $10\cdot\cos(12000\pi t)$ Aliased, 6000 Hz to 1000 Hz

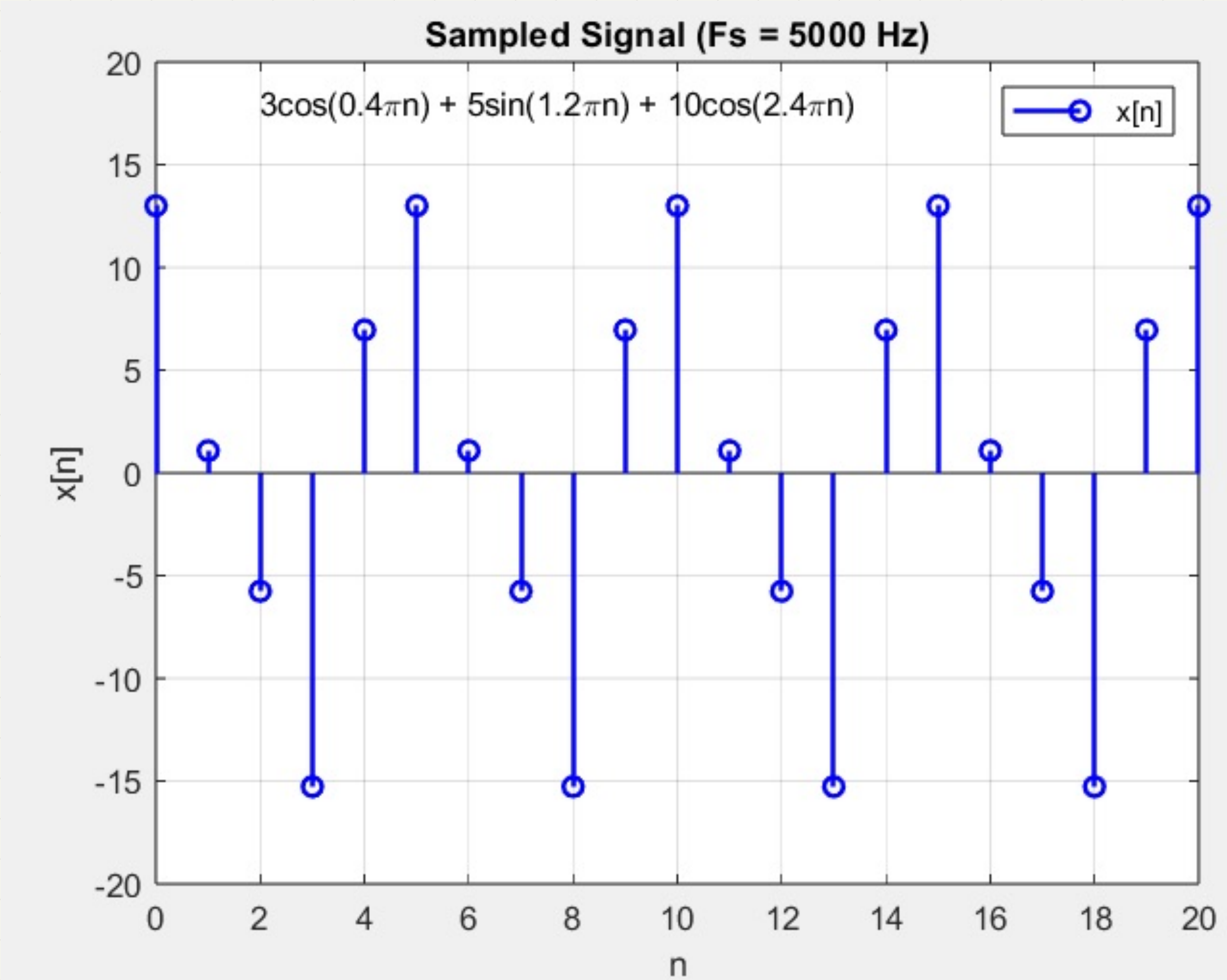


Reconstructed signal:

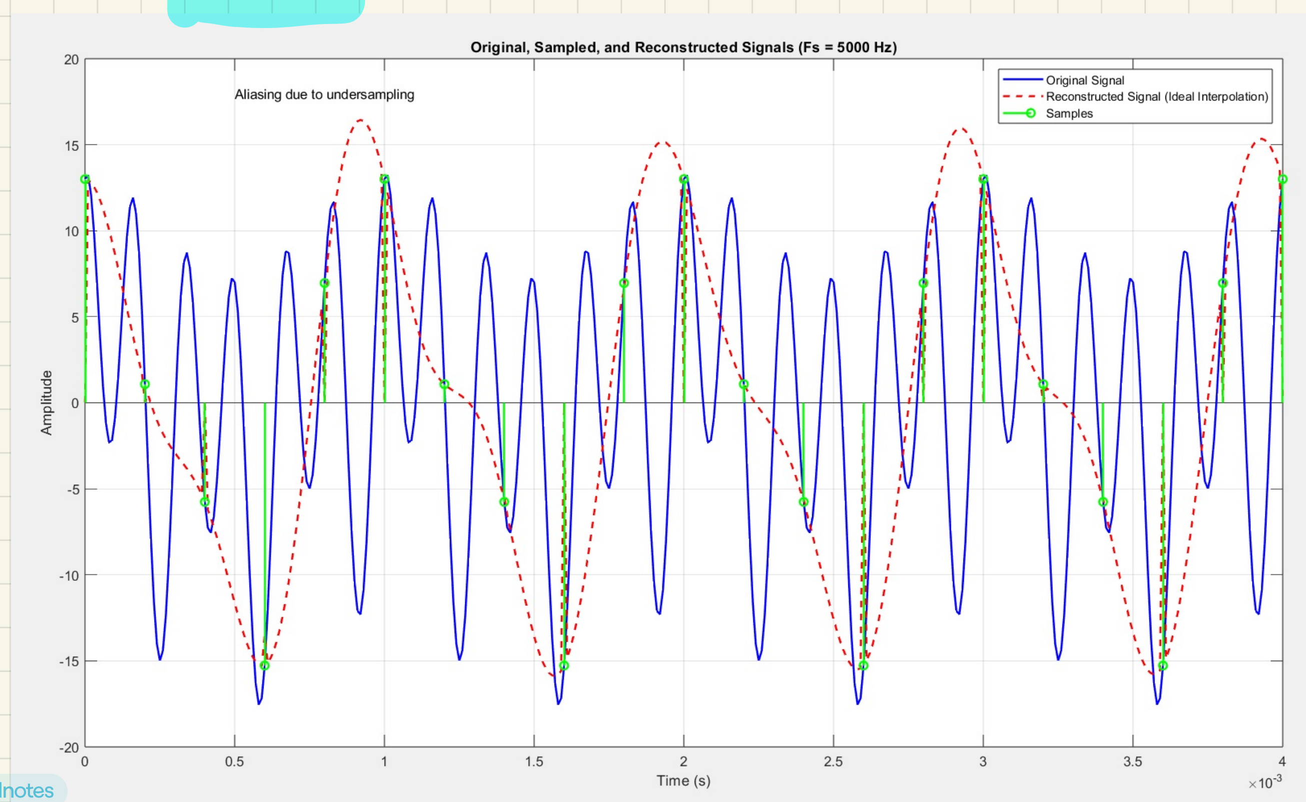
$x(t) = 3\cos(2000\pi t) + 5\cos(4000\pi t) + 10\cos(2000\pi t)$

$x(t) = 13\cos(2000\pi t) + 5\cos(4000\pi t)$

Question 3 Part 2:



Question 3 Part 3:



Q4: Find the DTFS representation of the digital periodic signal x[k] using the inspection method. For inspection method, just use the Euler's formulas. Here, x[k] = sin(Ω_0k), where $\Omega_0 = \frac{N_0}{2\pi}$.

Use the given hints.

Definition

The DTFS representation of the periodic signal $x[k]$ with fundamental period N_0 (fundamental frequency $\Omega_0 = 2\pi/N_0$) is written as

$$x[k] = \sum_{r=0}^{N_0-1} \mathcal{D}_r e^{jr\Omega_0 k}$$

where $\mathcal{D}_r$ are the DTFS coefficients of the signal $x[k]$

$$x[k] = \sin(\Omega_0 k), \quad \Omega_0 = \frac{2\pi}{N_0}$$

$$= \sum_{r=0}^{N_0-1} D_r e^{jr\Omega_0 k}$$

$$x[k] = \frac{e^{j\Omega_0 k} - e^{-j\Omega_0 k}}{2j}$$

$$= \frac{e^{j\Omega_0 k}}{2j} - \frac{e^{-j\Omega_0 k}}{2j}$$

$$\boxed{D_1 = \frac{1}{2j} \qquad D_{-1} = -\frac{1}{2j}}$$

Q5: You have been tasked with investigating the DTFS representation of the given digital signal x[k] using the definition of DTFS. Here, x[k] = sin(0.1πk), where $\Omega_0 = \frac{N_0}{2\pi}$.

Part 1:

Show that the signal, x[k], is periodic with a period of N_0 = 20.

Part 2:

Find the DTFS representation of the digital periodic signal x[k] using the definition. The sigma in D_m equation, the definition of DTFS representation, can have any lower limit and upper limit, as long as the length is 20.

Part 3:

Plot DT sinusoid, x[k], as a function of k using MATLAB.

Part 4:

Plot amplitude spectrum as a function of omega using MATLAB.

Part 5:

Plot phase spectrum as a function of omega using MATLAB.

Use the following hints:

The definition of DTFS representation:

$$\boxed{D_m = \frac{1}{N_0} \sum_{k=0}^{N_0-1} x[k] e^{-jm\Omega_0 k}}$$

Some useful formulas and concepts:

$$\sum_{N_0} e^{j\Omega_0 k(r-m)} = \begin{cases} 0 & r \neq m \\ N_0 & r = m \end{cases}$$

Definition

The DTFS representation of the periodic signal $x[k]$ with fundamental period N_0 (fundamental frequency $\Omega_0 = 2\pi/N_0$) is written as

$$x[k] = \sum_{r=0}^{N_0-1} \mathcal{D}_r e^{jr\Omega_0 k}$$

where $\mathcal{D}_r$ are the DTFS coefficients of the signal $x[k]$

Question 5:

Part 1: $\frac{2\pi}{N_0} = \frac{\pi}{10} \Rightarrow N_0 = \frac{2\pi}{\pi} \cdot 10 = 20$

$$\boxed{\Rightarrow \Omega_0 = \frac{2\pi}{20} = 0.1\pi}$$

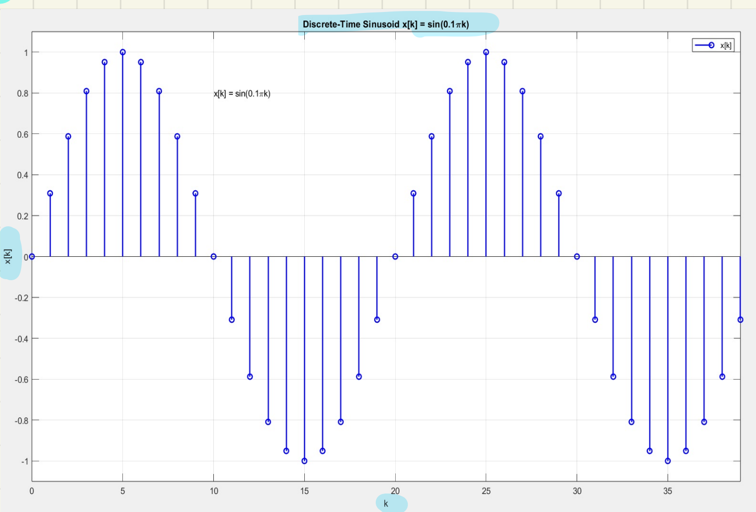
Part 2:

$$D_m = \frac{1}{20} \sum_{k=0}^{19} \sin(0.1\pi k) e^{-jm\Omega_0 k}, \quad \sin(0.1\pi k) = \frac{e^{j0.1\pi k} - e^{-j(0.1)\pi k}}{2j}$$

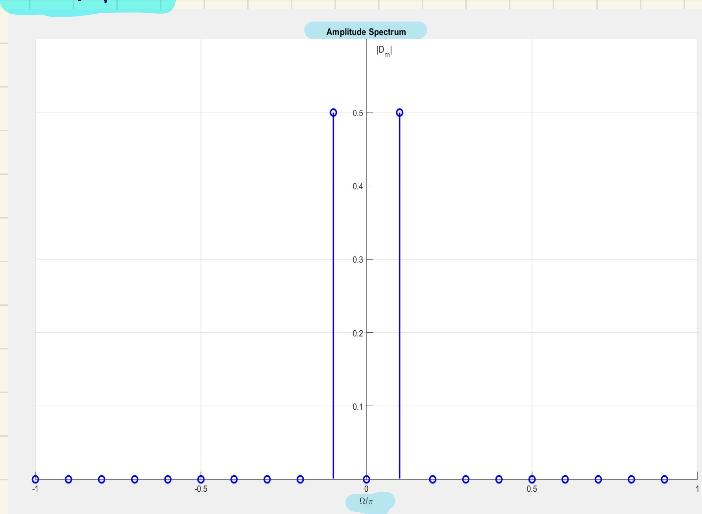
$$\Rightarrow D_m = \frac{1}{40j} \sum_{k=0}^{19} \left(e^{j(0.1)\pi k} - e^{-j(0.1)\pi k}\right) e^{-jm(0.1\pi)k} \Rightarrow D_m = \frac{1}{40j} \sum_{k=0}^{19} \left[e^{j(0.1\pi)k(1-m)} - e^{-j(0.1)\pi k(-1-m)}\right]$$

$$\boxed{\Rightarrow D_1 = \frac{1}{2j}, \quad D_{-1} = -\frac{1}{2j}, \quad \text{Else}, \ D_m = 0}$$

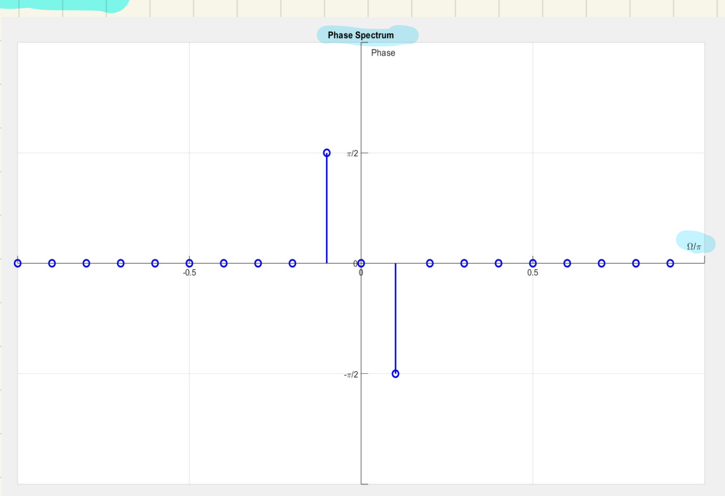
Part 3:



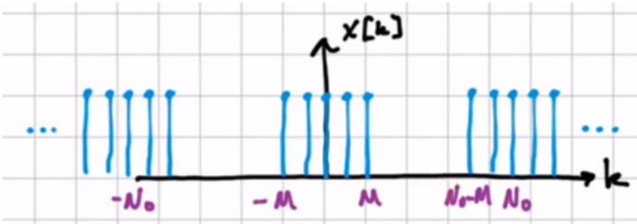
Part 4:



Part 5:



Q6: You have been tasked with investigating the DTFS representation of the given digital signal x[k] using the definition of DTFS. As shown below, x[k] is a square wave signal.



Part 1:

Find D_r values.

Part 2:

For $N_0 = 50$, and $M = 4$, plot x[k] as a function of k, from k = 0 to k = 50. Also, plot amplitude spectrum. Use MATLAB.

Part 3:

For $N_0 = 50$, and $M = 12$, plot x[k] as a function of k, from k = 0 to k = 50. Also, plot amplitude spectrum. Use MATLAB.

Part 4:

Compare the results of **Part 2** with the results of **Part 3**.

Use the following hints:

The definition of DTFS representation:

$$\boxed{D_m = \frac{1}{N_0} \sum_{k=0}^{N_0-1} x[k] e^{-jm\Omega_0 k}}$$

Some useful formulas and concepts:

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

Definition

The DTFS representation of the periodic signal $x[k]$ with fundamental period N_0 (fundamental frequency $\Omega_0 = 2\pi/N_0$) is written as

$$x[k] = \sum_{r=0}^{N_0-1} \mathcal{D}_r e^{jr\Omega_0 k}$$

where $\mathcal{D}_r$ are the DTFS coefficients of the signal $x[k]$

Question 6:

Part 1:

$$D_r = \frac{1}{N_0}\sum_{k=-M}^{M} e^{-jk\Omega_0 r} \Rightarrow \frac{1}{N_0}\sum_{l=0}^{2M} e^{-j(l-M)\Omega_0 r}$$

let $l = k+M$

$$= \frac{1}{N_0}\sum_{l=0}^{2M} e^{-jl\Omega_0 r} e^{jM\Omega_0 r}$$

$$= \frac{1}{N_0} e^{jM\Omega_0 r}\left[\frac{1-e^{-j\Omega_0 r(2M+1)}}{1-e^{-j\Omega_0 r}}\right]$$

$$= \frac{1}{N_0}\cdot e^{+\frac{j\Omega_0 r}{2}}\left[\frac{e^{jM\Omega_0 r}-e^{-j\Omega_0 r(M+1)}}{e^{\frac{j\Omega_0 r}{2}}-e^{-\frac{j\Omega_0 r}{2}}}\right]$$

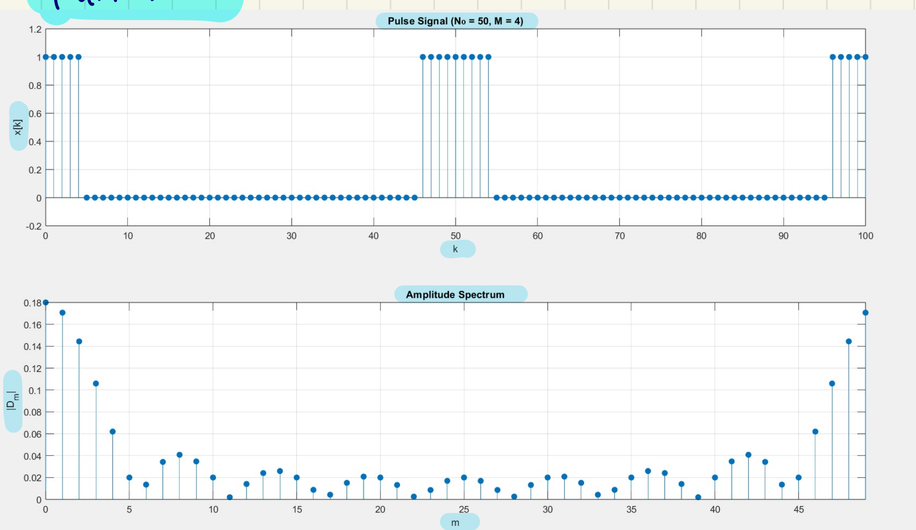
$$\Rightarrow = \frac{1}{N_0}\frac{e^{\frac{j\Omega_0 r}{2}}}{e^{\frac{j\Omega_0 r}{2}}}\left[\frac{e^{j(M+\frac{1}{2})\Omega_0 r}-e^{-j\Omega_0 r(M+\frac{1}{2})}}{e^{\frac{j\Omega_0 r}{2}}-e^{-\frac{j\Omega_0 r}{2}}}\right]$$

$$= \frac{1}{N_0}\frac{\sin\left((M+\frac{1}{2})\Omega_0 r\right)}{\sin\left(\frac{\Omega_0 r}{2}\right)} = \frac{\frac{1}{N_0}\sin\left((M+\frac{1}{2})\frac{2\pi}{N_0}r\right)}{\sin\left(\frac{\pi}{N_0}\cdot r\right)}$$

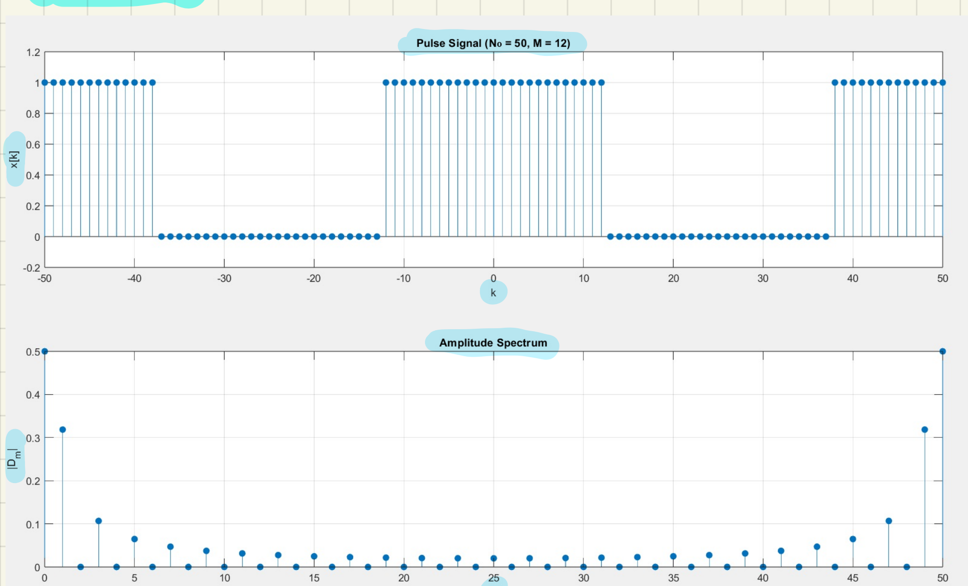
$N_0 = N_0$

$\Rightarrow \Omega_0 = \frac{2\pi}{N_0}$

Part 2:



Part 3:



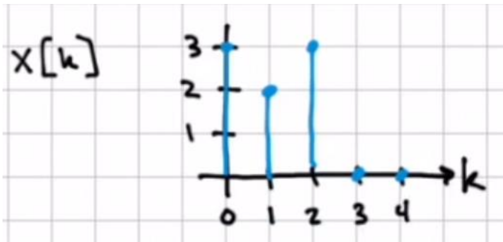
Part 4:

Compared to $M=4$, $M=12$ has many more peaks and troughs in the amplitude spectrum, also, only odd values are nonzero for $M=12$ (excluding the DC component $m=0$).

Part 1:

$$X_r = \sum_{N_0} x[k]\, e^{-jr\Omega_0 k}$$

Q7: This digital signal is given to us:



$$N_0 = 5 \Rightarrow \Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{5}$$

$$X_0 = \sum_{k=0}^{4} x[k]\, e^{-j(0)\cdot\frac{2\pi}{5}k} = \boxed{3+2+3 = 8}$$

Part 1: Find the DFT of x[k]. That is, find X_0, X_1, and X_2.

Part 2: Find the X(Ω) equation.

Part 3: Find the |X(Ω)| equation and plot amplitude spectrum as function of Ω. Show DFT points on the same graph.

$$X_1 = \sum_{k=0}^{4} x[k]\, e^{-j\cdot\frac{2\pi}{5}k} = \boxed{-3 + 2e^{-j\frac{2\pi}{5}} + 3e^{-j\frac{4\pi}{5}}}$$

Use the following hint:

$$X_r = \sum_{N_0} x[k]\, e^{-jr\Omega_0 k}$$

$$X_2 = \sum_{k=0}^{4} x[k]\, e^{-j\cdot\frac{4\pi}{5}k} = \boxed{3 + 2e^{-j\frac{4\pi}{5}} + 3e^{-j\frac{8\pi}{5}}}$$

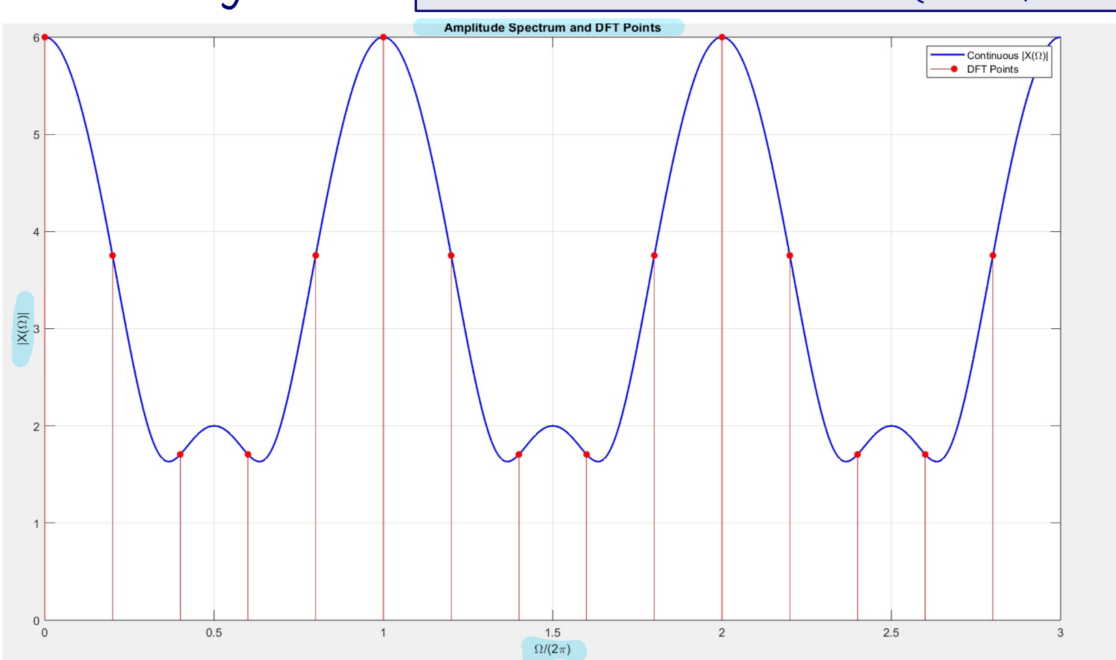
Part 2:

$$\left.\begin{aligned} X(\Omega) &= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \\ X(\Omega) &= 3 + 2e^{-2j\omega} + 3e^{-3j\omega} \end{aligned}\right\} \text{DTFT}$$

Part 3:

$$X(\Omega) = 3 + 2\left[\cos(2\omega) - j\sin(2\omega)\right] + 3\left[\cos(3\omega) - j\sin(3\omega)\right]$$

$$\boxed{|X(\Omega)| = \sqrt{(3 + 2\cos(2\omega) + 3\cos(3\omega))^2 + (-\sin(2\omega) - \sin(3\omega))^2}}$$



Assignment Group Self-Assessment

Group #	7
Assignment #	2
Date	October 24, 2024

Each student should fill in their personal information, the percentage of contribution (out of 100%) given to them by the group, and their signature. By signing this form, the students agree with both their own percentage of contribution and their colleagues percentage of contribution.

Equal between all

First Name	Last Name	Student ID #	Percentage of Contribution	Signature
Athina	Law	68032507	100%	[signature]
Sara	Hematy	47109236	100%	[signature]
Abhinava Tejas	PRATHIVADHI BHAYANKARAM	72395650	100%	[signature]
Fatima	Mirzayeva	20932695	100%	Fatima Mirzayeva
Benjamin	Liu	63031306	100%	[signature]
Alex	Sun	67050294	100%	[signature]