Exam Name:	Course Title:	Course Code:	Date:	Duration of Exam:	Number of Questions:	Instructor:	University:	Department:
Midterm	Digital Signal and Image Processing	ELEC 421	Thursday, October 17, 2024	90 minutes	30	Siamak Najarian, Ph.D., P.Eng.	UBC	Electrical and Computer Engineering

Please carefully read the following instructions and guidelines:

1. *This quiz is closed books/notes.*
2. *You will not get a negative mark for choosing the incorrect answer.*
3. *Each question carries 1.17 mark.*

Best of Luck!

Question 1:

Given the discrete system described by the difference equation:

$$y[n] = 4x[n] - 5x[n-1] + 7x[n-2]$$

and the input signal $x[n] = \{\underline{1}, 1, 1\}$, use the convolution sum to determine the output $y[n]$. The convolution sum is defined as:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

What is the output $y[n]$?

A. $\{\underline{4}, 0, -1, 3, 6\}$
B. $\{\underline{4}, -1, 6, 2, 7\}$
C. $\{2, \underline{4}, -1, 5, 6\}$
D. $\{3, -1, \underline{5}, 1, 6\}$

Question 2:

Which of the following more accurately describes the usage of the MATLAB built-in function **symsum(f, k, a, b)**?

A. Computes the numerical approximation of a series for given bounds.

B. Computes the symbolic sum of a series with exact expressions.

C. Requires only the expression defining the terms of the series.

D. Is used to perform operations on matrices rather than summations.

Question 3:

When dealing with convolution, which of the following more accurately describes the Causality Property in digital signal processing? The convolution sum is given by:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

A. The output $y[n]$ depends on future values of the input $x[n+k]$ for $k > 0$.
B. For a causal system, the impulse response $h[k]$ must be zero for values of k greater than 0.
C. For a causal system, the impulse response $h[k]$ must be zero for $k < 0$.
D. A causal system can operate before the impulse response is triggered.

Question 4:

When solving difference equations, which of the following statements is more accurate regarding the relationship between the discrete-time systems and their solutions?

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

A. The solutions consist solely of a homogeneous solution.

B. The equation represents a recursive solution without requiring convolution.

C. The solutions consist of both a homogeneous solution and a particular solution.

D. The system is exclusively a finite impulse response (FIR) system.

Question 5:

Which of the following statements more accurately describes the advantages of using transform methods in solving LTI systems?

A. Transform methods eliminate the need for convolution by changing the system's behavior entirely.

B. The Fourier Transform allows us to decompose signals into sines and cosines, which simplifies the analysis of periodic signals.

C. LTI systems do not respond to sinusoidal inputs in a special way, making Fourier Transforms unnecessary.

D. Sinusoids only occur in artificial systems and are not relevant to natural phenomena.

Question 6:

How do the coefficients a_k in a Fourier series affect the characteristics of a signal, particularly in terms of amplitude and phase?

A. Coefficients a_k only modify the signal's frequency without altering its amplitude or phase.
B. As the value of k increases, the coefficients a_k cause the signal to become static and unchanging.
C. The coefficients a_k fundamentally change the signal's period, regardless of their values.
D. Multiplying the signal by coefficients a_k, such as real numbers or complex numbers, adjusts its amplitude and phase but maintains its period.

Question 7:

How does the nature of the Fourier series coefficients a_k change when dealing with a real input signal $x(t)$?

A. The coefficients a_k will always be real numbers if the input $x(t)$ is real.
B. If $x(t)$ is real, then the coefficients a_k and a_{-k} will be equal.
C. The complex conjugate symmetry shows that $a_k = a^*_{-k}$, indicating that we only need to track positive indices.
D. The coefficients a_k can take on arbitrary values, independent of their a^*_{-k} counterparts.

Question 8:

How does the number of Fourier series coefficients relate to accurately representing different types of signals, including periodic noise?

A. A noisy signal can always be perfectly represented with a limited number of Fourier series coefficients.
B. Increasing the number of Fourier series coefficients allows us to better approximate complex signals like periodic noise.
C. Simple signals, such as triangles or squares, require more Fourier series coefficients than complex noise signals for accurate representation.
D. The more complex the signal, the fewer Fourier series coefficients are needed to represent it accurately.

Question 9:

What does the Gibbs phenomenon describe regarding the behavior of a Fourier series at a discontinuity?

A. The Gibbs phenomenon indicates that there will always be an overshoot of approximately 9% at a discontinuity, regardless of the number of terms used in the Fourier series.
B. The Gibbs phenomenon states that overshoot can be eliminated completely by increasing the number of Fourier series terms.
C. The height of the overshoot can be reduced to below 1% if enough terms are added to the Fourier series.
D. The Gibbs phenomenon shows that the Fourier series will become continuous across the discontinuity without any overshoot.

Question 10:

What does Parseval's Theorem state regarding the relationship between the time domain and the frequency domain?

A. Parseval's Theorem asserts that the average power of a signal in the time domain is equal to the power of its Fourier series coefficients in the frequency domain.
B. Parseval's Theorem indicates that energy is lost when converting between the time domain and the frequency domain.
C. Parseval's Theorem implies that the Fourier series coefficients must be real numbers for the equality to hold.
D. Parseval's Theorem shows that the average power in the time domain is always greater than that in the frequency domain.

Question 11:

What happens to the Fourier series representation when we deal with an aperiodic signal and the period T becomes very large?

A. The Fourier series coefficients become more spaced out in frequency as T increases.
B. As T goes to infinity, the sum of sinusoids becomes more discrete, resembling a set of distinct frequencies.
C. When T becomes very large, the signal is no longer represented by a combination of sinusoids in the frequency domain.
D. As T approaches infinity, the Fourier series sum turns into an integral, and the Fourier transform is obtained.

Question 12:

How are the Fourier series coefficients a_k related to the Fourier transform?

A. The Fourier series coefficients a_k are obtained by continuously integrating the Fourier transform over all frequencies.
B. The Fourier series coefficients a_k are completely independent of the Fourier transform and are computed using a separate process.
C. The Fourier series coefficients a_k are obtained by sampling the Fourier transform at equally spaced values of frequency.
D. The Fourier series coefficients a_k represent the sum of discrete sinusoids, unrelated to the Fourier transform.

Question 13:

What effect does time-shifting a delta function have on its Fourier transform (FT)?

A. Time-shifting a delta function results in a phase shift in its Fourier transform without changing its magnitude.
B. Time-shifting a delta function results in a change in both the magnitude and phase of its Fourier transform.
C. Time-shifting a delta function completely alters the frequency components of its Fourier transform.
D. Time-shifting a delta function has no effect on its Fourier transform.

Question 14:

A rectangular pulse $x(t)$ with a height of 1 and a width of τ is defined as follows:

$$x(t) = \begin{cases} 1, & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$$

What is the Fourier transform $X(\omega)$ of this rectangular pulse, given by the equation:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

A. $X(\omega) = \tau \cdot \text{sinc}\left(\frac{\omega\tau}{2}\right)$

B. $X(\omega) = \frac{1}{\tau} \cdot \text{sinc}(\omega)$

C. $X(\omega) = \frac{\sin(\omega)}{\omega}$

D. $X(\omega) = e^{-j\frac{\omega\tau}{2}}$

Question 15:

Which of the following statements correctly describes the symmetry properties in Fourier transform if the signal, $x(t)$, is real-valued?

A. The magnitude $|X(\omega)|$ is even, meaning it is symmetric about the y-axis.

B. The angle of $X(\omega)$ is even, which implies it is symmetric about the origin.

C. The real part of $X(\omega)$ is odd, leading to asymmetry in the real component.

D. The imaginary part of $X(\omega)$ is even, resulting in symmetry about the y-axis.

Question 16:

Which of the following statements is **true** regarding the convolution and frequency response in an LTI system?

A. Convolution in the time domain is more efficient than multiplication in the frequency domain.
B. For an LTI system, convolution in the time domain is equivalent to multiplication in the frequency domain.
C. The frequency response H(ω) is irrelevant in determining the output of an LTI system.
D. Inverse transformation is not required after multiplication in the frequency domain.

Question 17:

In an LTI system, if the input is $x(t) = e^{j\omega_0 t}$, which of the following expressions correctly represents both $Y(\omega)$ in the frequency domain and $y(t)$ in the time domain? Use the following equations for reference:

- The Fourier transform of $x(t)$:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- The time-domain output $y(t)$ is obtained by taking the inverse Fourier transform of $Y(\omega)$:

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega$$

A. $Y(\omega) = H(\omega) \cdot e^{j\omega t}$, $y(t) = H(\omega) \cdot e^{j\omega t}$
B. $Y(\omega) = 2\pi \cdot \delta(\omega - \omega_0)$, $y(t) = H(\omega_0) \cdot e^{j\omega_0 t}$
C. $Y(\omega) = H(\omega_0) \cdot 2\pi \cdot \delta(\omega - \omega_0)$, $y(t) = H(\omega_0) \cdot e^{j\omega_0 t}$
D. $Y(\omega) = X(\omega) \cdot e^{j\omega_0 t}$, $y(t) = H(\omega) \cdot e^{j\omega t}$

Question 18:

If a system receives an input $\cos(6t)$ and produces an output $\cos(7t)$, what can we conclude about the system?

A. The system is time-invariant but not linear.

B. The system is linear and time-invariant.

C. The system is not an LTI system because the output frequency differs from the input frequency.

D. The system is linear but not time-invariant.

Question 19:

Which of the following is **not** a reason why an ideal rectangular filter in the frequency domain with a cut-off frequency of ω_c is problematic for practical use?

A. The filter is not causal, meaning it requires future signal values to perform filtering.

B. The filter's impulse response in the time domain extends infinitely in both directions, causing long delays in filtering.

C. The oscillations in the time domain create ripple effects in the output signal.

D. The filter is stable and can be implemented using a finite number of past samples.

Question 20:

Consider the filter with the transfer function $H(\omega) = \frac{1}{a+j\omega}$. Which of the following statements is **true** regarding the frequency response of this filter?

A. The filter behaves as a lowpass filter, passing low frequencies and attenuating high frequencies progressively.

B. The filter acts as a bandpass filter by allowing frequencies between a certain range and attenuating others.

C. The filter can be both a lowpass and a stopband filter depending on the value of a.

D. The filter can act as a bandpass filter if a is chosen to be very large.

Question 21:

Given the input signal $x(t) = 3e^{-2t}u(t)$ and the impulse response $h(t) = 5e^{-4t}u(t)$, find the final form of $Y(\omega)$.

A. $\frac{\frac{15}{2}}{2+j\omega} + \frac{\frac{15}{2}}{4+j\omega}$

B. $\frac{\frac{15}{2}}{2+j\omega} - \frac{\frac{15}{2}}{4+j\omega}$

C. $\frac{15}{2(2+j\omega)} + \frac{15}{2(4+j\omega)}$

D. $\frac{15}{4+j\omega} - \frac{15}{2+j\omega}$

Question 22:

Given the discrete signal $x[n] = \{0, 2, -1, 5, 0\}$, where the zero element is $x[0] = -1$, find the discrete-time Fourier transform $X(\omega)$. Use the equation for $X(\omega)$:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

A. $-2e^{j\omega} - 5e^{-j\omega} + 1$

B. $2e^{-j\omega} - 5e^{j\omega} - 1$

C. $5e^{j\omega} + 2e^{-j\omega} - 1$

D. $-1 + 2e^{j\omega} + 5e^{-j\omega}$

Question 23:

Identify the correct statement regarding the DTFT's behavior.

A. The DTFT is only defined for $-\pi \leq \omega \leq +\pi$ and has no periodicity.

B. The DTFT is evaluated for any ω, showing all copies, but only the range $-\pi$ to $+\pi$ is represented.

C. The DTFT can be evaluated for any ω and is not periodic, thus no copies exist outside the main interval.

D. The DTFT displays periodicity with copies occurring every 2π units, but in practice, we may only represent the portion between $-\pi$ and $+\pi$.

Question 24:

The DTFT can be viewed as a Fourier series in reverse. If you take the Fourier series of the DTFT $X(\omega)$, what do the Fourier coefficients a_k represent?

A. The Fourier coefficients a_k represent the continuous-time signal samples.

B. The Fourier coefficients a_k represent the frequency response of the discrete-time system.

C. The Fourier coefficients a_k give the discrete-time samples $x[n]$.

D. The Fourier coefficients a_k indicate the phase shift of the discrete-time signal.

Question 25:

What is the key difference between the absolute summability convergence condition and the mean square convergence condition?

A. Absolute summability convergence requires $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$, ensuring pointwise convergence of the DTFT, while mean square convergence requires $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$ and allows for signals that do not satisfy absolute summability but meet a weaker condition involving the sum of squares.

B. Mean square convergence ensures that the DTFT converges at every point exactly, while absolute summability only guarantees convergence in a probabilistic sense.

C. Both convergence conditions are mathematically equivalent; that is, if $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$, then it follows that $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$, indicating that satisfying mean square convergence implies absolute summability for many signals.

D. Absolute summability convergence allows for unbounded signals, while mean square convergence strictly requires bounded signals to ensure convergence.

Question 26:

Which statement correctly describes the relationship between the DTFT and the nature of digital signals?

A. The DTFT of a finite-length digital signal is always periodic with period 2π. The DTFT of an infinite-length digital signal is not necessarily periodic and may or may not exhibit repetitive behavior, depending on the signal.

B. The DTFT of an infinite-length digital signal is always periodic with a period of 2π, while finite-length digital signals do not exhibit any periodicity.

C. The DTFT of both finite-length and infinite-length digital signals is periodic with a period of 2π, regardless of the nature of the digital signal.

D. The DTFT of a finite-length digital signal may or may not be periodic, while the DTFT of an infinite-length digital signal is always periodic with a period of 2π.

Question 27:

We have a rectangular pulse with a length of $2M$ in the discrete time domain, centered at $n = 0$. The pulse can be defined as follows:

$$x[n] = \begin{cases} 1 & \text{for } -M \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

What is $X(\omega)$ at $\omega = 0$?

The general equation for the DTFT is:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

A. $2M + 1$

B. $2M + 2$

C. $M + 2$

D. $2M - 1$

Question 28:

Given the following:

$$h[n] = \left(\frac{1}{6}\right)^n u[n]$$
$$x[n] = 3e^{j\frac{\pi}{3}n}$$
$$y[n] = A \cdot |H(\omega_0)| e^{j(\arg H(\omega_0) + \omega_0 n)}$$

What is the numerical value of $A \cdot |H(\omega_0)|$?

The frequency response $H(\omega)$ for $a^n u[n]$ is given by:
$H(\omega) = \frac{1}{1-ae^{-j\omega}}$

A. $\frac{5}{2}$

B. $\frac{1}{2}$

C. $\frac{18}{\sqrt{31}}$

D. $\frac{23}{\sqrt{11}}$

Question 29:

Why do we need the z-transform?

A. The DTFT does not always converge because the sum of the signal values may not be finite.
B. The z-transform is more useful because it only applies to periodic signals.
C. The DTFT can only be used for discrete-time signals with zero initial conditions.
D. The z-transform allows us to analyze a broader range of signals, even when the DTFT does not exist.

Question 30:

Based on the pole-zero plots of two signals, identify the correct statement regarding the Region of Convergence (ROC) for left-sided and right-sided signals.

A. If $x[n]$ is a left-sided signal, the ROC is the region outside the outermost pole.
B. If $x[n]$ is a left-sided signal, the ROC is the region inside the innermost pole.
C. If $x[n]$ is a right-sided signal, the ROC is the region inside the innermost pole.
D. If $x[n]$ is a right-sided signal, the ROC is the region outside the outermost pole.

End of Questions