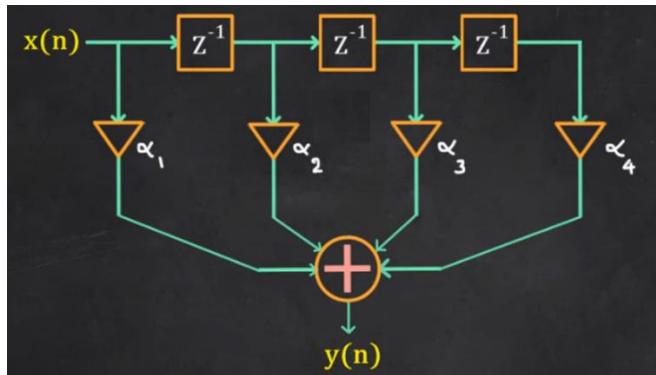
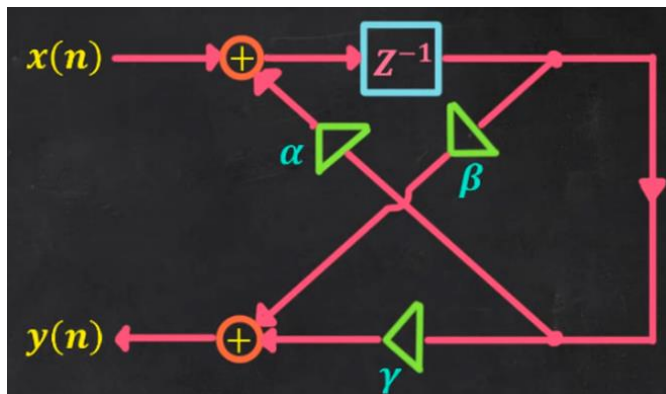


Q1: Find $y(n]$ expression in the following block diagrams, **(a)** and **(b)**. In **(a)**, assume that α 's are scalar values. In **(b)**, assume α, β, γ are all scalar values.

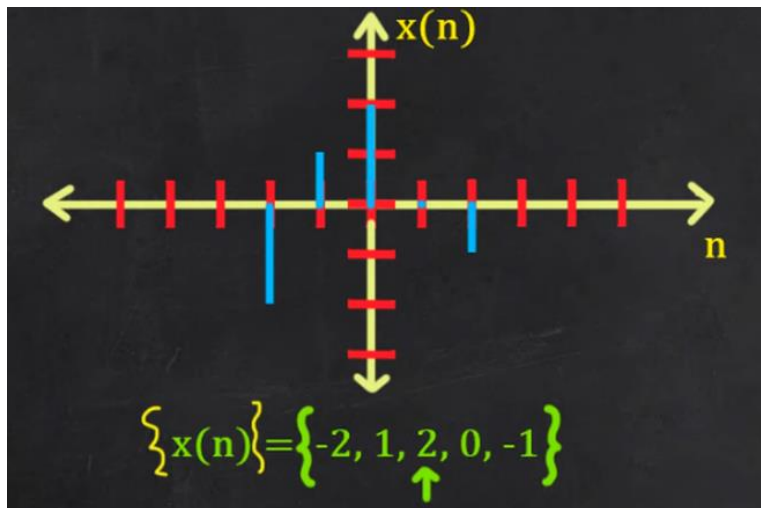
(a)



(b)



Q2: Find the expressions for the even digital sequence/signal, $x_e(n)$, and the odd digital sequence/signal, $x_o(n)$, for the given digital sequence/signal, $x(n)$. Plot each sequence/signal.



Q3: Determine which of the following signals are energy signals or power signals.

$x(n) = 4, n > 0$	$x(n) = \left(\frac{1}{4}\right)^n u(n)$	$x(n) = \begin{cases} 3(-1)^n, n \geq 0 \\ 0, n < 0 \end{cases}$
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Use the following hints:

	CT Signals	DT Signals
Instantaneous Power	$P(t) = x(t) ^2$	$P(n) = x(n) ^2$
Total Energy	$E = \int_{-\infty}^{\infty} x(t) ^2 dt$	$E = \sum_{n=-\infty}^{\infty} x(n) ^2$
Average Power	$P = \lim_{T \rightarrow \infty} \left[\frac{\int_{-T}^T x(t) ^2 dt}{2T} \right]$	$P = \lim_{N \rightarrow \infty} \left[\frac{\sum_{n=-N}^N x(n) ^2}{2N + 1} \right]$

Energy Signal & Power Signal

Energy Signal : $0 < E < \infty$ (Finite Total Energy) and $P = 0$ (Zero Average Power)

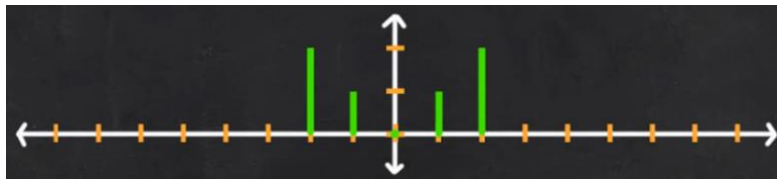
Power Signal : $0 < P < \infty$ (Finite Average Power) and $E = \infty$ (Infinite Total Energy)

A signal cannot be both an energy signal and a power signal

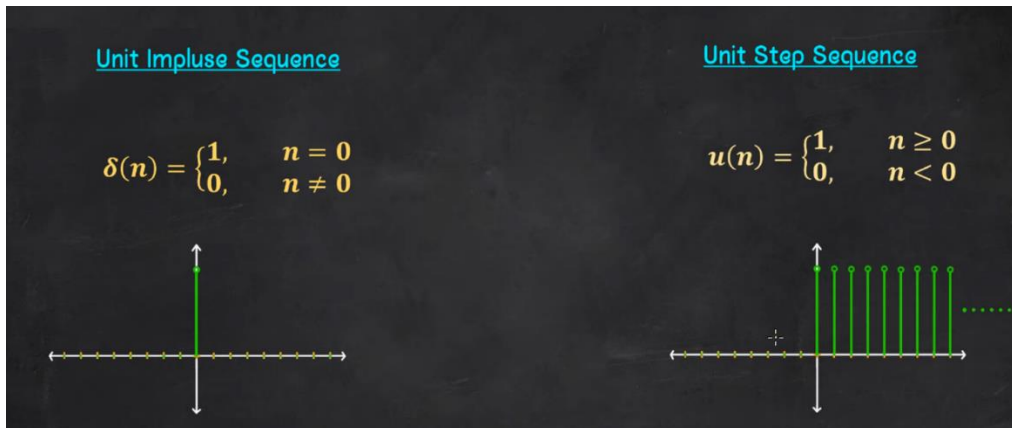
There can be signals that are neither an energy signal nor a power signal

No physical signal can have infinite energy or infinite average power

Q4: Express the following signal in terms of **(a)** unit impulse signal, and **(b)** unit step signal.



Use the mathematical definitions of unit impulse sequence and unit step sequence given below.



Q5: Find the fundamental period for the following digital signals:

$$\bar{x}_1(n) = 4 \cos\left(\frac{2\pi n}{5}\right)$$

$$\bar{x}_2(n) = \sin(0.6\pi n + 0.6\pi)$$

Note that a sinusoidal discrete signal is periodic if $2\pi/\omega$ equals a rational number, as shown below:

$$\omega N = 2\pi r, \quad r \rightarrow +ve \text{ integer}$$
$$\frac{2\pi}{\omega} = \frac{N}{r}$$

In the above, both N and r are positive integers, where N is the period of the digital signal.

Q6: Find the fundamental period for the following digital signal:

$$\bar{x}(n) = \cos(1.2\pi n + 0.65\pi) - 4 \sin(0.9\pi n) + 5 \cos(0.5\pi n)$$

Use the following hint, where LCM is the Least Common Multiple. Note that the LCM is the smallest positive integer that is evenly divisible by two or more given numbers.

$$\begin{array}{ccc} \bar{x}_1(n) & \bar{x}_2(n) & \bar{x}_3(n) \\ N_1 & N_2 & N_3 \\ \bar{x}_4(n) = \alpha \bar{x}_1(n) + \beta \bar{x}_2(n) + \gamma \bar{x}_3(n) \\ N_4 = LCM(N_1, N_2, N_3) \end{array}$$

You need to show the graphs of each digital signal using MATLAB. Also, plot the graph of the overall signal of $x_4(n)$.

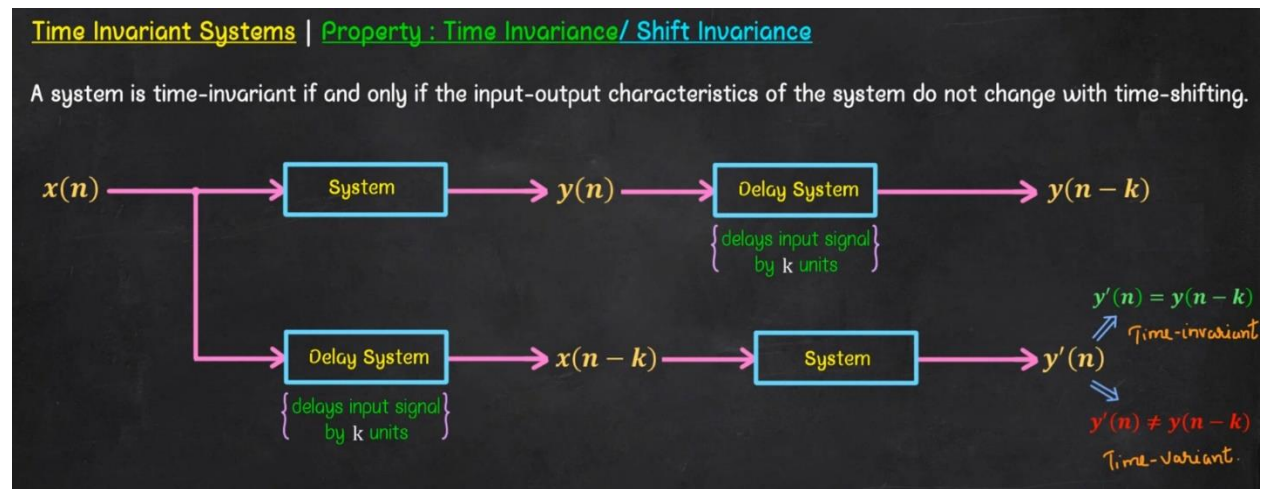
Q7: Show whether the given digital signals are time-variant or time-invariant.

Part 1: $y(n) = x(2n)$.

Part 2: $y(n) = 2n + x(n)$.

Part 3: $y(n) = n \cdot x(n)$.

Use the given hints.



For a system to be time-invariant :

1. It should not have time scaling property
2. Added/subtracted term in the system equation should be constant or zero
3. Coefficient of terms in the system equation should be constant

Q8: Show whether the given digital signals are causal or non-causal.

Part 1: $y(n) = x(n) - x(n-1)$.

Part 2: $y(n) = a \cdot x(n)$.

Part 3: $y(n) = \sum_{k=-\infty}^n x(k)$.

Part 4: $y(n) = x(n) + 3x(n+4)$.

Part 5: $y(n) = x(n^2)$.

Part 6: $y(n) = x(-n)$.

Use the given hints.

Causal Systems | Property : Causality

A system is said to be causal if the output of the system at any time n depends only on the present and past inputs, but does not depend on future inputs.

For a causal system,

$$y(n) = F[x(n), x(n-1), x(n-2), \dots]$$

The output of a **non-causal system** depends not only on the present and past inputs, but also on the future inputs.

Q9: Show whether the given digital systems are static system (memoryless) or dynamic (memory) system.

Part 1: $y(n) = [2 + \sin(n - 1)] \cdot x(n)$

Part 2: $y(n) = \max[x(n), x(n - 1), \dots, x(-\infty)]$

Part 3: $y(n) = x(-n)$.

Use the given hints.

Memoryless Systems | Property : Memory

A discrete-time system is called memoryless if its output at any instant n depends at most on the input sample at the same time, but not on the past or future samples of the input.

Memoryless systems or **Static systems** $y(n) = ax(n)$

$y(n) = nx(n) + bx^3(n)$

Memoryless systems are also called **Static systems**

Memory systems are systems whose outputs also depend on past values of input or output.

$y(n) = \sum_{k=0}^N x(n - k) = x[n - N] + x[n - (N - 1)] + \dots + x[n - 1] + x[n]$ **Memory system**

↓
Dynamic system

If $0 < N < \infty$, the system is said to have finite memory

If $N = \infty$, the system is said to have infinite memory

Q10: Consider the following LTI system, $y[k] - 0.8y[k - 1] = x[k]$.

Part 1:

Find the frequency response of the system, i.e., both the amplitude response and the phase response. Only find the equations and no need to plot them.

Part 2:

Let $x[k] = 1$ be the input with the frequency response obtained in **Part 1**. Find the system response.

Part 3:

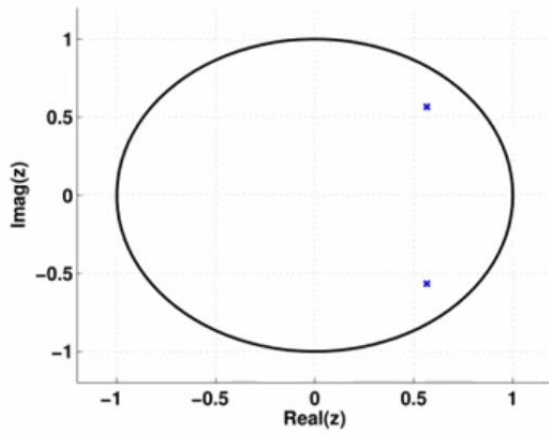
Let $x[k] = \cos\left(\frac{\pi}{6}k - 0.4\right)$ be the input with the frequency response obtained in **Part 1**. Find the system response.

Part 4:

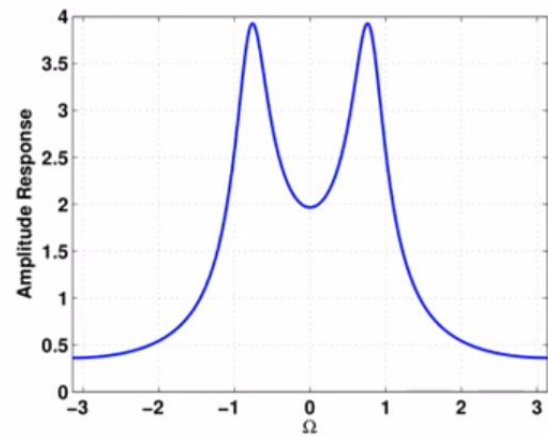
Let $x(t) = \cos(1500t)$ be the *continuous* input with the frequency response obtained in **Part 1**. Find the system response if we sample the continuous signal every 0.001 seconds.

Q11: Explain the general shapes of the amplitude response using the given pole/zero plots.

Part 1:

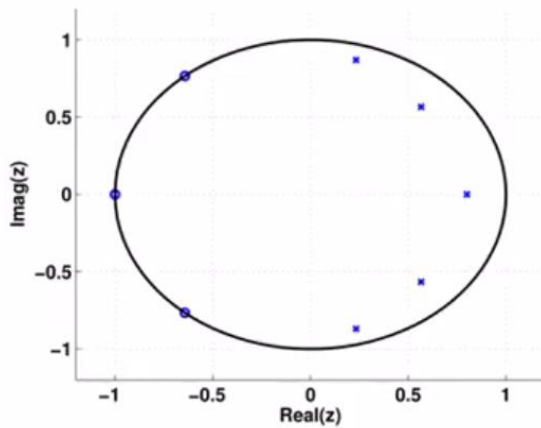


(a) Pole/Zero Plot

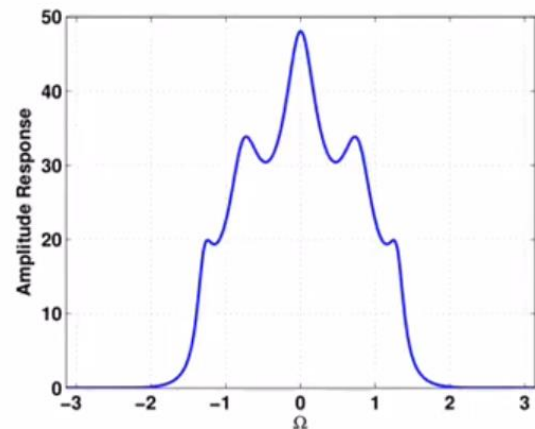


(b) Frequency Response

Part 2:



(c) Pole/Zero Plot



(d) Frequency Response

Use the following hints:

- The Transfer Function Of A DT LTI System Can Be Written As

$$H(z) = K \frac{(z - z_1)(z - z_2) \cdots (z - z_n)}{(z - \gamma_1)(z - \gamma_2) \cdots (z - \gamma_n)}$$

where z_i and γ_i are Zeros and Poles of the System.

- Let $z = e^{j\Omega}$ (i.e evaluate $H(z)$ on the unit circle). This Gives The System Frequency Response

$$H(e^{j\Omega}) = H(\Omega) = K \frac{(e^{j\Omega} - z_1)(e^{j\Omega} - z_2) \cdots (e^{j\Omega} - z_n)}{(e^{j\Omega} - \gamma_1)(e^{j\Omega} - \gamma_2) \cdots (e^{j\Omega} - \gamma_n)}$$

- The Quantity $e^{j\Omega}$ Is A Point On the Unit Circle
- The Quantity $e^{j\Omega} - z_i$ Is A Vector From z_i to $e^{j\Omega}$
- The Quantity $e^{j\Omega} - \gamma_i$ Is A Vector From γ_i to $e^{j\Omega}$
- Use Polar Coordinates To Define

$$r_i e^{j\phi_i} = e^{j\Omega} - z_i$$

and

$$d_i e^{j\theta_i} = e^{j\Omega} - \gamma_i$$

- The Frequency Response Can Now Be Written As

$$\begin{aligned} H(\Omega) &= K \frac{(r_1 e^{j\phi_1})(r_2 e^{j\phi_2}) \cdots (r_n e^{j\phi_n})}{(d_1 e^{j\theta_1})(d_2 e^{j\theta_2}) \cdots (d_n e^{j\theta_n})} \\ &= K \frac{r_1 r_2 \cdots r_n}{d_1 d_2 \cdots d_n} e^{j[(\phi_1 + \phi_2 + \cdots + \phi_n) - (\theta_1 + \theta_2 + \cdots + \theta_n)]} \end{aligned}$$

- The Amplitude Response Is

$$\begin{aligned} |H(\Omega)| &= \left| K \frac{(r_1 e^{j\phi_1})(r_2 e^{j\phi_2}) \cdots (r_n e^{j\phi_n})}{(d_1 e^{j\theta_1})(d_2 e^{j\theta_2}) \cdots (d_n e^{j\theta_n})} \right| \\ &= K \frac{r_1 r_2 \cdots r_n}{d_1 d_2 \cdots d_n} \\ &= K \frac{\text{product of distances of zeros to } e^{j\Omega}}{\text{product of distances of poles to } e^{j\Omega}} \end{aligned}$$

- The Phase Response Is

$$\begin{aligned} \angle H(\Omega) &= \angle \left(K \frac{(r_1 e^{j\phi_1})(r_2 e^{j\phi_2}) \cdots (r_n e^{j\phi_n})}{(d_1 e^{j\theta_1})(d_2 e^{j\theta_2}) \cdots (d_n e^{j\theta_n})} \right) \\ &= (\phi_1 + \phi_2 + \cdots + \phi_n) \\ &\quad - (\theta_1 + \theta_2 + \cdots + \theta_n) \\ &= \text{sum of zero angles to } e^{j\Omega} \\ &\quad - \text{sum of pole angles to } e^{j\Omega} \end{aligned}$$

- The Amplitude and Phase Response Of A Filter Is Controlled By Its Pole and Zero Locations