Machine Elements in Mechanical Design

Fourth Edition

Robert L. Mott

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MACHINE ELEMENTS IN MECHANICAL DESIGN

Fourth Edition

Robert L. Mott, P.E.
University of Dayton

PEARSON
Prentice Hall

Upper Saddle River, New Jersey
Columbus, Ohio

To my wife, Marge
our children, Lynné, Robert, Jr., and Stephen
and my Mother and Father

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Preface

The objective of this book is to provide the concepts, procedures, data, and decision analysis techniques necessary to design machine elements commonly found in mechanical devices and systems. Students completing a course of study using this book should be able to execute original designs for machine elements and integrate the elements into a system composed of several elements.

This process requires a consideration of the performance requirements of an individual element and of the interfaces between elements as they work together to form a system. For example, a gear must be designed to transmit power at a given speed. The design must specify the number of teeth, pitch, tooth form, face width, pitch diameter, material, and method of heat treatment. But the gear design also affects, and is affected by, the mating gear, the shaft carrying the gear, and the environment in which it is to operate. Furthermore, the shaft must be supported by bearings, which must be contained in a housing. Thus, the designer should keep the complete system in mind while designing each individual element. This book will help the student approach design problems in this way.

This text is designed for those interested in practical mechanical design. The emphasis is on the use of readily available materials and processes and appropriate design approaches to achieve a safe, efficient design. It is assumed that the person using the book will be the designer, that is, the person responsible for determining the configuration of a machine or a part of a machine. Where practical, all design equations, data, and procedures needed to make design decisions are specified.

It is expected that students using this book will have a good background in statics, strength of materials, college algebra, and trigonometry. Helpful, but not required, would be knowledge of kinematics, industrial mechanisms, dynamics, materials, and manufacturing processes.

Among the important features of this book are the following:

1. It is designed to be used at the undergraduate level in a first course in machine design.
2. The large list of topics allows the instructor some choice in the design of the course. The format is also appropriate for a two-course sequence and as a reference for mechanical design project courses.
3. Students should be able to extend their efforts into topics not covered in classroom instruction because explanations of principles are straightforward and include many example problems.
4. The practical presentation of the material leads to feasible design decisions and is useful to practicing designers.
5. The text advocates and demonstrates use of computer spreadsheets in cases requiring long, laborious solution procedures. Using spreadsheets allows the designer to make decisions and to modify data at several points within the problem while the computer performs all computations. See Chapter 6 on columns, Chapter 9 on spur gears, Chapter 12 on shafts, Chapter 13 on shrink fits, and Chapter 19 on spring design. Other computer-aided calculation software can also be used.

6. References to other books, standards, and technical papers assist the instructor in presenting alternate approaches or extending the depth of treatment.
7. Lists of Internet sites pertinent to topics in this book are included at the end of most chapters to assist readers in accessing additional information or data about commercial products.
8. In addition to the emphasis on original design of machine elements, much of the discussion covers commercially available machine elements and devices, since many design projects require an optimum combination of new, uniquely designed parts and purchased components.
9. For some topics the focus is on aiding the designer in selecting commercially available components, such as rolling contact bearings, flexible couplings, ball screws, electric motors, belt drives, chain drives, clutches, and brakes.
10. Computations and problem solutions use both the International System of Units (SI) and the U.S. Customary System (inch-pound-second) approximately equally. The basic reference for the usage of SI units is IEEE/ASTM-SI-10 *Standard for Use of the International System of Units (SI): The Modern Metric System*, which has replaced ASTM E380 and ANSI/IEEE Standard 268-1992.
11. Extensive appendices are included along with detailed tables in many chapters to help the reader to make real design decisions, using only this text.

MDESIGN–MECHANICAL DESIGN SOFTWARE INCLUDED IN THE BOOK

The design of machine elements inherently involves extensive procedures, complex calculations, and many design decisions. Data must be found from numerous charts and tables. Furthermore, design is typically iterative, requiring the designer to try several options for any given element, leading to the repetition of design calculations with new data or new design decisions. This is especially true for complete mechanical devices containing several components as the interfaces between components are considered. Changes to one component often require changes to mating elements. Use of computer-aided mechanical design software can facilitate the design process by performing many of the tasks while leaving the major design decisions to the creativity and judgment of the designer or engineer.

We emphasize that users of computer software must have a solid understanding of the principles of design and stress analysis to ensure that design decisions are based on reliable foundations. We recommend that the software be used only after mastering a given design methodology by careful study and using manual techniques.

MDESIGN

Included in this book is the MDESIGN mechanical design software created by the TEDATA Company. Derived from the successful MDESIGN mec software produced for the European market, the U.S. version of MDESIGN employs standards and design methods that are typically in use in North America. Many of the textual aids and design procedures come directly from this book, *Machine Elements in Mechanical Design*.

Topics for which the MDESIGN software can be used as a supplement to this book include:

Beam stress analysis	Beam deflections	Mohr's circle	Columns
Belt drives	Chain drives	Spur gears	Helical gears
Shafts	Keys	Power screws	Springs
Rolling contact bearings	Plain surface bearings	Bolted connections	Fasteners
Clutches	Brakes		

Special icons as shown on the preceding page are placed in the margins at places in this book where use of the software is pertinent. Also, the Solutions Manual, available only to instructors using this book in scheduled classes, includes guidance for use of the software.

FEATURES OF THE FOURTH EDITION

The practical approach to designing machine elements in the context of complete mechanical designs is retained and refined in this edition. An extensive amount of updating has been accomplished through the inclusion of new photographs of commercially available machine components, new design data for some elements, new or revised standards, new end-of-chapter references, listings of Internet sites, and some completely new elements. The following list summarizes the primary features and the updates.

1. The three-part structure that was introduced in the third edition has been maintained.
   - Part I (Chapters 1–6) focuses on reviewing and upgrading readers' understanding of design philosophies, the principles of strength of materials, the design properties of materials, combined stresses, design for different types of loading, and the analysis and design of columns.
   - Part II (Chapters 7–15) is organized around the concept of the design of a complete power transmission system, covering some of the primary machine elements such as belt drives, chain drives, gears, shafts, keys, couplings, seals, and rolling contact bearings. These topics are tied together to emphasize both their interrelationships and their unique characteristics. Chapter 15, **Completion of the Design of a Power Transmission**, is a guide through detailed design decisions such as the overall layout, detail drawings, tolerances, and fits.
   - Part III (Chapters 16–22) presents methods of analysis and design of several important machine elements that were not pertinent to the design of a power transmission. These chapters can be covered in any order or can be used as reference material for general design projects. Covered here are plain surface bearings, linear motion elements, fasteners, springs, machine frames, bolted connections, welded joints, electric motors, controls, clutches, and brakes.
2. **The Big Picture, You Are the Designer**, and **Objectives** features introduced in earlier editions are maintained and refined. Feedback about these features from users, both students and instructors, has been enthusiastically favorable. They help readers to draw on their own experiences and to appreciate what competencies they will acquire from the study of each chapter. Constructivist theories of learning espouse this approach.
3. Some of the new or updated topics from individual chapters are summarized here.
   - In Chapter 1, the discussion of the mechanical design process is refined, and several new photographs are added. Internet sites for general mechanical design are included that are applicable to many later chapters. Some are for standards organizations, stress analysis software, and searchable databases for a wide variety of technical products and services.
   - Chapter 2, **Materials in Mechanical Design**, is refined, notably through added material on creep, austempered ductile iron (ADI), toughness, impact energy, and the special considerations for selecting plastics. An entirely new section on materials selection has been added. The extensive list of Internet sites provides readers access to industry data for virtually all types of materials discussed in the chapter with some tied to new practice problems.

- Chapter 3, a review of **Stress and Deformation Analysis**, has an added review of force analysis and refinement of the concepts of stress elements, combined normal stresses, and beams with concentrated bending moments.
- Chapter 5, **Design for Different Types of Loading**, is extensively upgraded and refined in the topics of endurance strength, design philosophy, design factors, predictions of failure, an overview of statistical approaches to design, finite life, and damage accumulation. The recommended approach to fatigue design has been changed from the *Soderberg criterion* to the *Goodman method*. The *modified Mohr method* is added for members made from brittle materials.
- In Chapter 7, synchronous belt drives are added and new design data for chain power ratings are included.
- Chapter 9, **Spur Gear Design**, is refined with new photographs of gear production machinery, new AGMA standards for gear quality, new discussion of functional measurement of gear quality, enhanced description of the geometry factor *I* for pitting resistance, more gear lubrication information, and a greatly expanded section on plastics gearing.
- In Chapter 11, new information is provided for keyless hub to shaft connections of the Ringfeder® and polygon types, and the Cornay™ universal joint. The extensive listing of Internet sites provides access to data for keys, couplings, universal joints, and seals.
- Critical speeds, other dynamic considerations, and flexible shafts are added to Chapter 12, **Shaft Design**.
- An all-new section, Tribology: Friction, Lubrication, and Wear, is added to Chapter 16, **Plain Surface Bearings**. More data on *pV factors* for boundary lubricated bearings are provided.
- Chapter 17 has been retitled **Linear Motion Elements** and includes power screws, ball screws, and linear actuators.
- Refinements to Chapter 18, **Fasteners**, include the shear strength of threads, components of torque applied to a fastener, and methods of bolt tightening.

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My appreciation is extended to all who provided helpful suggestions for improvements to this book. I thank the editorial staff of Prentice Hall Publishing Company, those who provided illustrations, and the many users of the book, both instructors and students, with whom I have had discussions. Special appreciation goes to my colleagues at the University of Dayton, Professors David Myszka, James Penrod, Joseph Untener, Philip Doepker, and Robert Wolff. I also thank those who provided thoughtful reviews of the prior edition: Marian Barasch, Hudson Valley Community College; Ismail Fidan, Tennessee Tech University; Paul Unangst, Milwaukee School of Engineering; Richard Alexander, Texas A & M University; and Gary Qi, The University of Memphis. I especially thank my students—past and present—for their encouragement and their positive feedback about this book.

Robert L. Mott

Contents

PART I

Principles of Design and Stress Analysis

OBJECTIVES AND CONTENT OF PART I

As you complete the first six chapters of this book, you will gain an understanding of design philosophies, and you will build on earlier-learned principles of strength of materials, materials science, and manufacturing processes. The competencies gained from these chapters are useful throughout the book and in general machine design or product design projects.

Chapter 1: The Nature of Mechanical Design helps you see the big picture of the process of mechanical design. Several examples are shown from different industry sectors: consumer products, manufacturing systems, construction equipment, agricultural equipment, transportation equipment, ships, and space systems. The responsibilities of designers are discussed, along with an illustration of the iterative nature of the design process. Units and conversions complete the chapter.

Chapter 2: Materials in Mechanical Design emphasizes the design properties of materials. Much of this chapter is probably review for you, but it is presented here to emphasize the importance of material selection to the design process and to explain the data for materials presented in the Appendices.

Chapter 3: Stress and Deformation Analysis is a review of the basic principles of stress and deflection analysis. It is essential that you understand the basic concepts summarized here before proceeding with later material. Reviewed are direct tensile, compressive, and shearing stresses; bending stresses; and torsional shear stresses.

Chapter 4: Combined Stresses and Mohr's Circle is important because many general design problems and the design of machine elements covered in later chapters of the book involve combined stresses. You may have covered these topics in a course in strength of materials.

Chapter 5: Design for Different Types of Loading is an in-depth discussion of design factors, fatigue, and many of the details of stress analysis as used in this book.

Chapter 6: Columns discusses the long, slender, axially loaded members that tend to fail by buckling rather than by exceeding the yield, ultimate, or shear stress of the material. Special design and analysis methods are reviewed here.

1

The Nature of Mechanical Design

The Big Picture

The Nature of Mechanical Design

Discussion Map

- ☐ To design mechanical components and devices, you must be competent in the design of individual elements that comprise the system.
- ☐ But you must also be able to integrate several components and devices into a coordinated, robust system that meets your customer's needs.

Discover

Think, now, about the many fields in which you can use mechanical design:

What are some of the products of those fields?

What kinds of materials are used in the products?

What are some of the unique features of the products?

How were the components made?

How were the parts of the products assembled?

Consider consumer products, construction equipment, agricultural machinery, manufacturing systems, and transportation systems on the land, in the air, in space, and on and under water.

In this book, you will find the tools to learn the principles of ***Machine Elements in Mechanical Design***.

Design of machine elements is an integral part of the larger and more general field of mechanical design. Designers and design engineers create devices or systems to satisfy specific needs. Mechanical devices typically involve moving parts that transmit power and accomplish specific patterns of motion. Mechanical systems are composed of several mechanical devices.

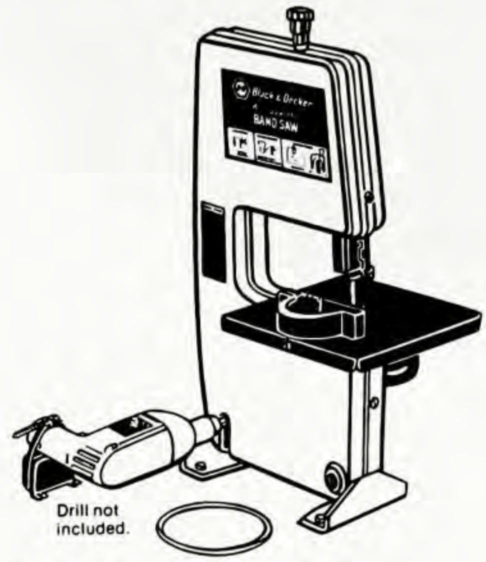
Therefore, to design mechanical devices and systems, you must be competent in the design of individual machine elements that comprise the system. But you must also be able to integrate several components and devices into a coordinated, robust system that meets your customer's needs. From this logic comes the name of this book, *Machine Elements in Mechanical Design*.

Think about the many fields in which you can use mechanical design. Discuss these fields with your instructor and with your colleagues who are studying with you. Talk with people who are doing mechanical design in local industries. Try to visit their companies if possible, or meet designers and design engineers at meetings of professional societies. Consider the following fields where mechanical products are designed and produced.

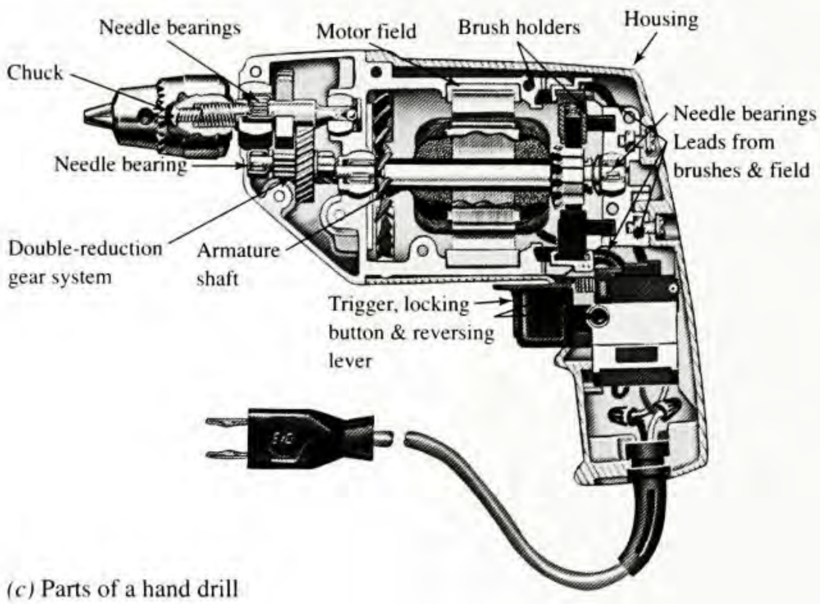
- ***Consumer products:*** Household appliances (can openers, food processors, mixers, toasters, vacuum cleaners, clothes washers), lawn mowers, chain saws, power tools, garage door openers, air conditioning systems, and many others. See Figures 1–1 and 1–2 for a few examples of commercially available products.
- ***Manufacturing systems:*** Material handling devices, conveyors, cranes, transfer devices, industrial robots, machine tools, automated assembly systems, special-purpose processing systems, forklift trucks, and packaging equipment. See Figures 1–3, 1–4, and 1–5.
- ***Construction equipment:*** Tractors with front-end loaders or backhoes, mobile cranes, power shovels, earthmovers, graders, dump trucks, road pavers, concrete mixers, powered nailers and staplers, compressors, and many others. See Figures 1–5 and 1–6.

FIGURE 1–1 Drill-powered band saw [Courtesy of Black & Decker (U.S.) Inc.]

(a) Bandsaw

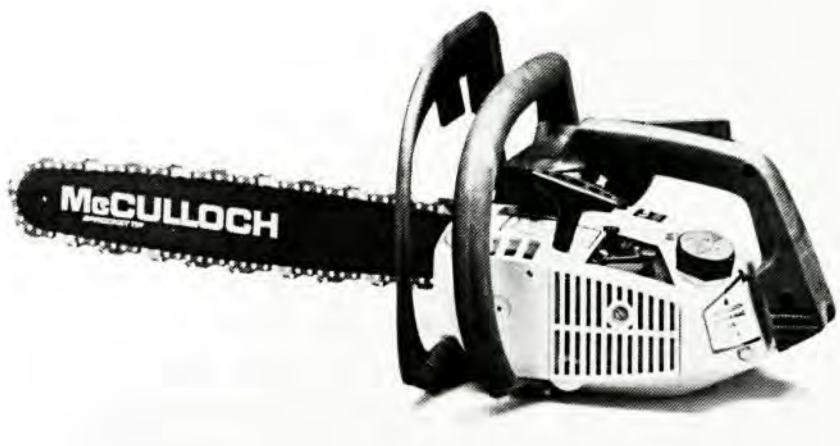


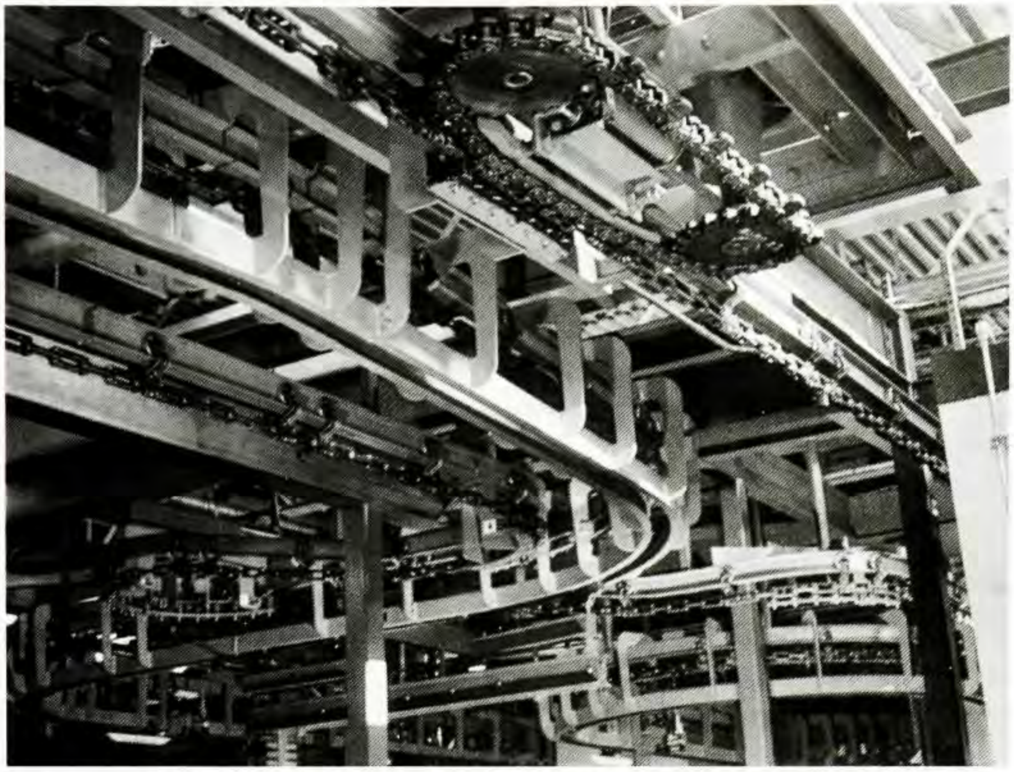
(b) Hand drill powering band saw



(c) Parts of a hand drill

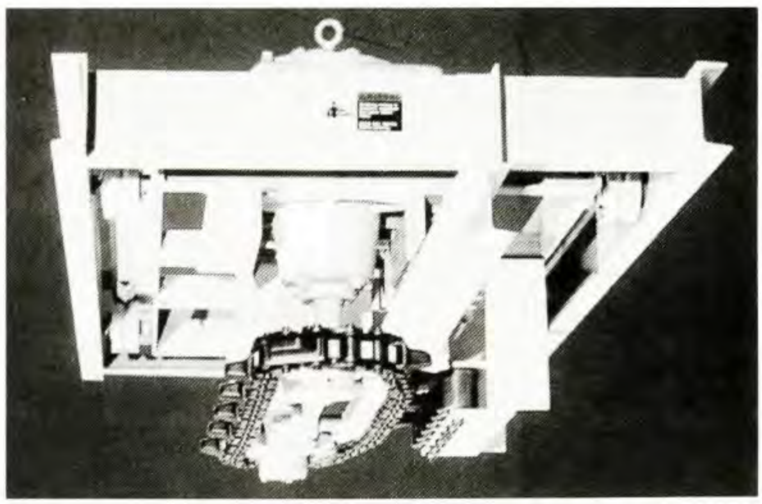
FIGURE 1–2 Chain saw (Copyright McCulloch Corporation, Los Angeles, CA)





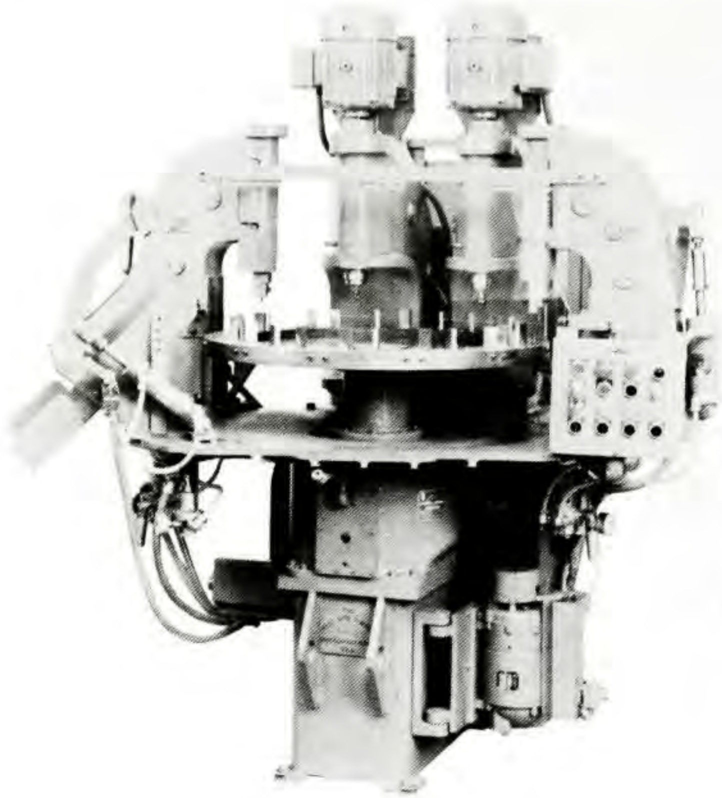
(a) Chain conveyor installation showing the drive system engaging the chain

(b) Chain and roller system supported on an I-beam

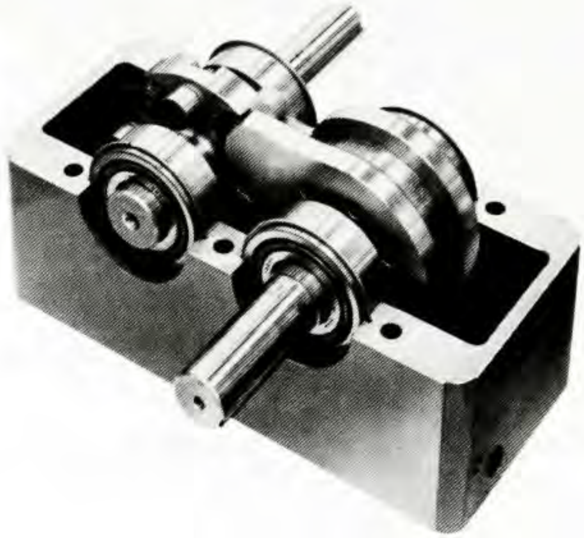


(c) Detail of the drive system and its structure

FIGURE 1–3 Chain conveyor system (Richards-Wilcox, Inc., Aurora, IL)



(a) Automatic assembly machine with indexing table



(b) Indexing drive mechanism

FIGURE 1–4 Machinery to automatically assemble automotive components (Industrial Motion Control, LLC, Wheeling, IL)

FIGURE 1–5 Industrial crane (Air Technical Industries, Mentor, OH)

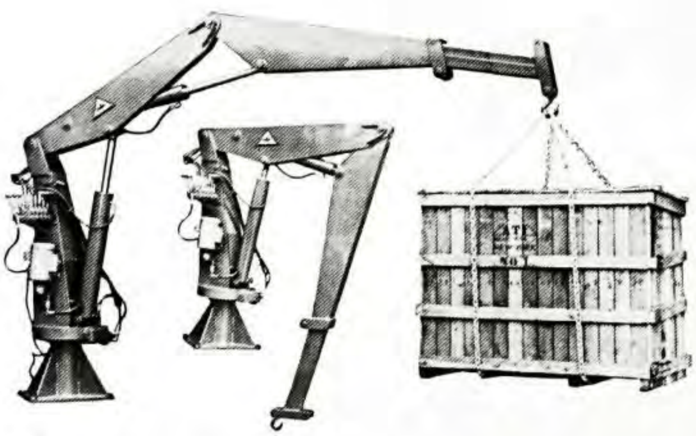


FIGURE 1–6 Tractor with a front-end-loader attachment (Case IH, Racine, WI)

FIGURE 1–7 Tractor pulling an implement (Case IH, Racine, WI)

FIGURE 1–8 Cutaway of a tractor (Case IH, Racine, WI)

- ***Agricultural equipment:*** Tractors, harvesters (for corn, wheat, tomatoes, cotton, fruit, and many other crops), rakes, hay balers, plows, disc harrows, cultivators, and conveyors. See Figures 1–6, 1–7, and 1–8.
- ***Transportation equipment:*** (a) Automobiles, trucks, and buses, which include hundreds of mechanical devices such as suspension components (springs, shock absorbers, and struts); door and window operators; windshield wiper mechanisms; steering systems; hood and trunk latches and hinges; clutch and braking systems; transmissions; drive shafts; seat adjusters; and numerous parts of the engine systems. (b) Aircraft, which include retractable landing gear, flap and rudder actuators, cargo handling devices, seat reclining mechanisms, dozens of latches, structural components, and door operators. See Figures 1–9 and 1–10.
- ***Ships:*** Winches to haul up the anchor, cargo-handling cranes, rotating radar antennas, rudder steering gear, drive gearing and drive shafts, and the numerous sensors and controls for operating on-board systems.
- ***Space systems:*** Satellite systems, the space shuttle, the space station, and launch systems, which contain numerous mechanical systems such as devices to deploy antennas, hatches, docking systems, robotic arms, vibration control devices, devices to secure cargo, positioning devices for instruments, actuators for thrusters, and propulsion systems.

How many examples of mechanical devices and systems can you add to these lists?

What are some of the unique features of the products in these fields?

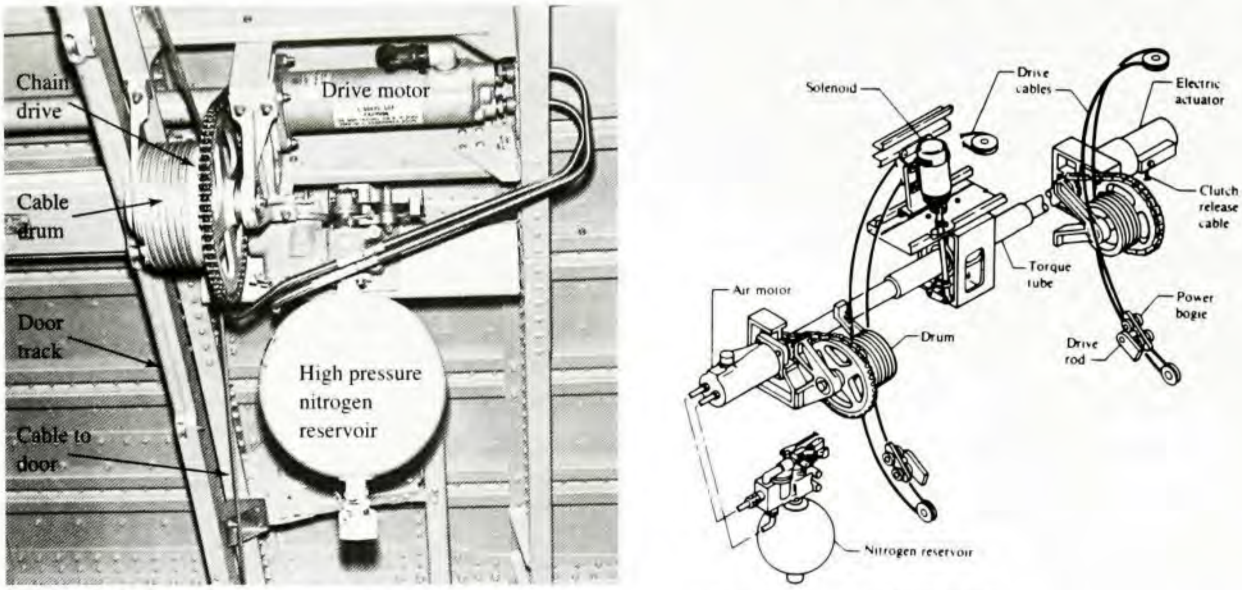
What kinds of mechanisms are included?

What kinds of materials are used in the products?

How were the components made?

How were the parts assembled into the complete products?

In this book, you will find the tools to learn the principles of *Machine Elements in Mechanical Design*. In the introduction to each chapter, we include a brief scenario called *You Are the Designer*. The purpose of these scenarios is to stimulate your thinking about the material presented in the chapter and to show examples of realistic situations in which you may apply it.



(*a*) Photograph of installed mechanism

(*b*) Cabin door drive mechanism

FIGURE 1–9 Aircraft door drive mechanism (The Boeing Company, Seattle, WA)

FIGURE 1–10 Aircraft landing gear assembly (The Boeing Company, Seattle, WA)

You Are the Designer

Consider, now, that you are the designer responsible for the design of a new consumer product, such as the band saw for a home workshop shown in Figure 1–1. What kind of technical preparation would you need to complete the design? What steps would you follow? What information would you need? How would you show, by calculation, that the design is safe and that the product will perform its desired function?

The general answers to these questions are presented in this chapter. As you complete the study of this book, you will learn about many design techniques that will aid in your design of a wide variety of machine elements. You will also learn how to integrate several machine elements into a mechanical system by considering the relationships between and among elements.

1–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Recognize examples of mechanical systems in which the application of the principles discussed in this book is necessary to complete their design.
2. List what design skills are required to perform competent mechanical design.
3. Describe the importance of integrating individual machine elements into a more comprehensive mechanical system.
4. Describe the main elements of the *product realization process*.
5. Write statements of *functions* and *design requirements* for mechanical devices.
6. Establish a set of criteria for evaluating proposed designs.
7. Work with appropriate units in mechanical design calculations both in the U.S. Customary Unit System and in SI metric units.
8. Distinguish between *force* and *mass*, and express them properly in both unit systems.
9. Present design calculations in a professional, neat, and orderly manner that can be understood and evaluated by others knowledgeable in the field of machine design.

1–2 THE MECHANICAL DESIGN PROCESS

The ultimate objective of mechanical design is to produce a useful product that satisfies the needs of a customer and that is safe, efficient, reliable, economical, and practical to manufacture. Think broadly when answering the question, "Who is the customer for the product or system I am about to design?" Consider the following scenarios:

- ***You are designing a can opener for the home market.*** The ultimate customer is the person who will purchase the can opener and use it in the kitchen of a home. Other customers may include the designer of the packaging for the opener, the manufacturing staff who must produce the opener economically, and service personnel who repair the unit.
- ***You are designing a piece of production machinery for a manufacturing operation.*** The customers include the manufacturing engineer who is responsible for the production operation, the operator of the machine, the staff who install the machine, and the maintenance personnel who must service the machine to keep it in good running order.
- ***You are designing a powered system to open a large door on a passenger aircraft.*** The customers include the person who must operate the door in normal service or in emergencies, the people who must pass through the door during use,

the personnel who manufacture the opener, the installers, the aircraft structure designers who must accommodate the loads produced by the opener during flight and during operation, the service technicians who maintain the system, and the interior designers who must shield the opener during use while allowing access for installation and maintenance.

It is essential that you know the desires and expectations of all customers before beginning product design. Marketing professionals are often employed to manage the definition of customer expectations, but designers will likely work with them as a part of a product development team.

Many methods are used to determine what the customer wants. One popular method, called *quality function deployment* or *QFD*, seeks (1) to identify all of the features and performance factors that customers desire and (2) to assess the relative importance of these factors. The result of the QFD process is a detailed set of functions and design requirements for the product. (See Reference 8.)

It is also important to consider how the design process fits with all functions that must happen to deliver a satisfactory product to the customer and to service the product throughout its life cycle. In fact, it is important to consider how the product will be disposed of after it has served its useful life. The total of all such functions that affect the product is sometimes called the *product realization process* or *PRP*. (See References 3, 10.) Some of the factors included in PRP are as follows:

- Marketing functions to assess customer requirements
- Research to determine the available technology that can reasonably be used in the product
- Availability of materials and components that can be incorporated into the product
- Product design and development
- Performance testing
- Documentation of the design
- Vendor relationships and purchasing functions
- Consideration of global sourcing of materials and global marketing
- Work-force skills
- Physical plant and facilities available
- Capability of manufacturing systems
- Production planning and control of production systems
- Production support systems and personnel
- Quality systems requirements
- Operation and maintenance of the physical plant
- Distribution systems to get products to the customer
- Sales operations and time schedules
- Cost targets and other competitive issues
- Customer service requirements
- Environmental concerns during manufacture, operation, and disposal of the product
- Legal requirements
- Availability of financial capital

Can you add to this list?

You should be able to see that the design of a product is but one part of a comprehensive process. In this book, we will focus more carefully on the design process itself, but the producibility of your designs must always be considered. This simultaneous consideration of product design and manufacturing process design is often called *concurrent engineering*. Note that this process is a subset of the larger list given previously for the product realization process. Other major books discussing general approaches to mechanical design are listed as References 6, 7, and 12–16.

1–3 SKILLS NEEDED IN MECHANICAL DESIGN

Product engineers and mechanical designers use a wide range of skills and knowledge in their daily work, including the following:

1. Sketching, technical drawing, and computer-aided design
2. Properties of materials, materials processing, and manufacturing processes
3. Applications of chemistry such as corrosion protection, plating, and painting
4. Statics, dynamics, strength of materials, kinematics, and mechanisms
5. Oral communication, listening, technical writing, and teamwork skills
6. Fluid mechanics, thermodynamics, and heat transfer
7. Fluid power, the fundamentals of electrical phenomena, and industrial controls
8. Experimental design and performance testing of materials and mechanical systems
9. Creativity, problem solving, and project management
10. Stress analysis
11. Specialized knowledge of the behavior of machine elements such as gears, belt drives, chain drives, shafts, bearings, keys, splines, couplings, seals, springs, connections (bolted, riveted, welded, adhesive), electric motors, linear motion devices, clutches, and brakes

It is expected that you will have acquired a high level of competence in items 1–5 in this list prior to beginning the study of this text. The competencies in items 6–8 are typically acquired in other courses of study either before, concurrently, or after the study of design of machine elements. Item 9 represents skills that are developed continuously throughout your academic study and through experience. Studying this book will help you acquire significant knowledge and skills for the topics listed in items 10 and 11.

1–4 FUNCTIONS, DESIGN REQUIREMENTS, AND EVALUATION CRITERIA

Section 1–2 emphasized the importance of carefully identifying the needs and expectations of the customer prior to beginning the design of a mechanical device. You can formulate these by producing clear, complete statements of *functions*, *design requirements*, and *evaluation criteria*:

- ***Functions*** tell what the device must do, using general, nonquantitative statements that employ action phrases such as *to support a load*, *to lift a crate*, *to transmit power*, or *to hold two structural members together*.
- ***Design requirements*** are detailed, usually quantitative statements of *expected performance levels*, *environmental conditions in which the device must operate*, *limitations on space or weight*, or *available materials and components that may be used*.
- ***Evaluation criteria*** are statements of *desirable qualitative characteristics* of a design that assist the designer in deciding which alternative design is optimum—that is, the design that maximizes benefits while minimizing disadvantages.

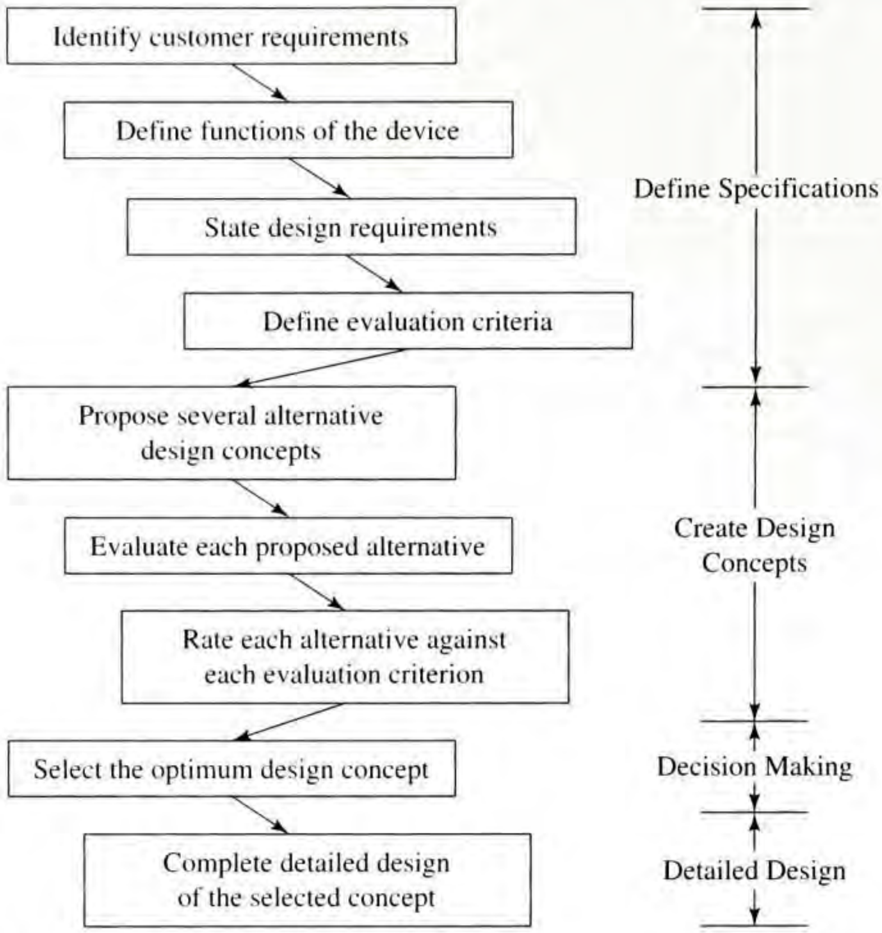


FIGURE 1–11 Steps in the design process

Together these elements can be called the *specifications* for the design.

Most designs progress through a cycle of activities as outlined in Figure 1–11. You should typically propose more than one possible alternative design concept. This is where creativity is exercised to produce truly novel designs. Each design concept must satisfy the functions and design requirements. A critical evaluation of the desirable features, advantages, and disadvantages of each design concept should be completed. Then a rational decision analysis technique should use the evaluation criteria to decide which design concept is the optimum and, therefore, should be produced.

The final block in the design flowchart is the detailed design, and the primary focus of this book is on that part of the overall design process. It is important to recognize that a significant amount of activity precedes the detailed design.

Example of Functions, Design Requirements, and Evaluation Criteria

Consider that you are the designer of a speed reducer that is part of the power transmission for a small tractor. The tractor's engine operates at a fairly high speed, while the drive for the wheels must rotate more slowly and transmit a higher torque than is available at the output of the engine.

To begin the design process, let us list the *functions* of the speed reducer. What is it supposed to do? Some answers to this question are as follows:

Functions

1. To receive power from the tractor's engine through a rotating shaft.
2. To transmit the power through machine elements that reduce the rotational speed to a desired value.
3. To deliver the power at the lower speed to an output shaft that ultimately drives the wheels of the tractor.

Now the *design requirements* should be stated. The following list is hypothetical, but if you were on the design team for the tractor, you would be able to identify such requirements from your own experience and ingenuity and/or by consultation with fellow designers, marketing staff, manufacturing engineers, service personnel, suppliers, and customers.

The product realization process calls for personnel from all of these functions to be involved from the earliest stages of design.

Design Requirements

1. The reducer must transmit 15.0 hp.
2. The input is from a two-cylinder gasoline engine with a rotational speed of 2000 rpm.
3. The output delivers the power at a rotational speed in the range of 290 to 295 rpm.
4. A mechanical efficiency of greater than 95% is desirable.
5. The minimum output torque capacity of the reducer should be 3050 pound-inches (lb·in).
6. The reducer output is connected to the drive shaft for the wheels of a farm tractor. Moderate shock will be encountered.
7. The input and output shafts must be in-line.
8. The reducer is to be fastened to a rigid steel frame of the tractor.
9. Small size is desirable. The reducer must fit in a space no larger than 20 in $\times$ 20 in, with a maximum height of 24 in.
10. The tractor is expected to operate 8 hours (h) per day, 5 days per week, with a design life of 10 years.
11. The reducer must be protected from the weather and must be capable of operating anywhere in the United States at temperatures ranging from 0 to 130°F.
12. Flexible couplings will be used on the input and output shafts to prohibit axial and bending loads from being transmitted to the reducer.
13. The production quantity is 10 000 units per year.
14. A moderate cost is critical to successful marketing.
15. All government and industry safety standards must be met.

Careful preparation of function statements and design requirements will ensure that the design effort is focused on the desired results. Much time and money can be wasted on designs that, although technically sound, do not meet design requirements. Design requirements should include everything that is needed, but at the same time they should offer ample opportunity for innovation.

Evaluation criteria should be developed by all members of a product development team to ensure that the interests of all concerned parties are considered. Often weights are assigned to the criteria to reflect their relative importance.

Safety must always be the paramount criterion. Different design concepts may have varying levels of inherent safety in addition to meeting stated safety requirements as noted in the design requirements list. Designers and engineers are legally liable if a person is injured because of a design error. You must consider any reasonably foreseeable uses of the device and ensure safety of those operating it or those who may be close by.

Achieving a high overall performance should also be a high priority. Certain design concepts may have desirable features not present on others.

The remaining criteria should reflect the special needs of a particular project. The following list gives examples of possible evaluation criteria for the small tractor.

Evaluation Criteria

1. Safety (the relative inherent safety over and above stated requirements)
2. Performance (the degree to which the design concept exceeds requirements)
3. Ease of manufacture
4. Ease of service or replacement of components
5. Ease of operation
6. Low initial cost
7. Low operating and maintenance costs
8. Small size and low weight
9. Low noise and vibration; smooth operation
10. Use of readily available materials and purchased components
11. Prudent use of both uniquely designed parts and commercially available components
12. Appearance that is attractive and appropriate to the application

1–5 EXAMPLE OF THE INTEGRATION OF MACHINE ELEMENTS INTO A MECHANICAL DESIGN

Mechanical design is the process of designing and/or selecting mechanical components and putting them together to accomplish a desired function. Of course, machine elements must be compatible, must fit well together, and must perform safely and efficiently. The designer must consider not only the performance of the element being designed at a given time but also the elements with which it must interface.

To illustrate how the design of machine elements must be integrated with a larger mechanical design, let us consider the design of a speed reducer for the small tractor discussed in Section 1–4. Suppose that, to accomplish the speed reduction, you decide to design a double-reduction, spur gear speed reducer. You specify four gears, three shafts, six bearings, and a housing to hold the individual elements in proper relation to each other, as shown in Figure 1–12.

The primary elements of the speed reducer in Figure 1–12 are:

1. The input shaft (shaft 1) is to be connected to the power source, a gasoline engine whose output shaft rotates at 2000 rpm. A flexible coupling is to be employed to minimize difficulties with alignment.
2. The first pair of gears, A and B, causes a reduction in the speed of the intermediate shaft (shaft 2) proportional to the ratio of the numbers of teeth in the gears. Gears B and C are both mounted to shaft 2 and rotate at the same speed.
3. A key is used at the interface between the hub of each gear and the shaft on which it is mounted to transmit torque between the gear and the shaft.

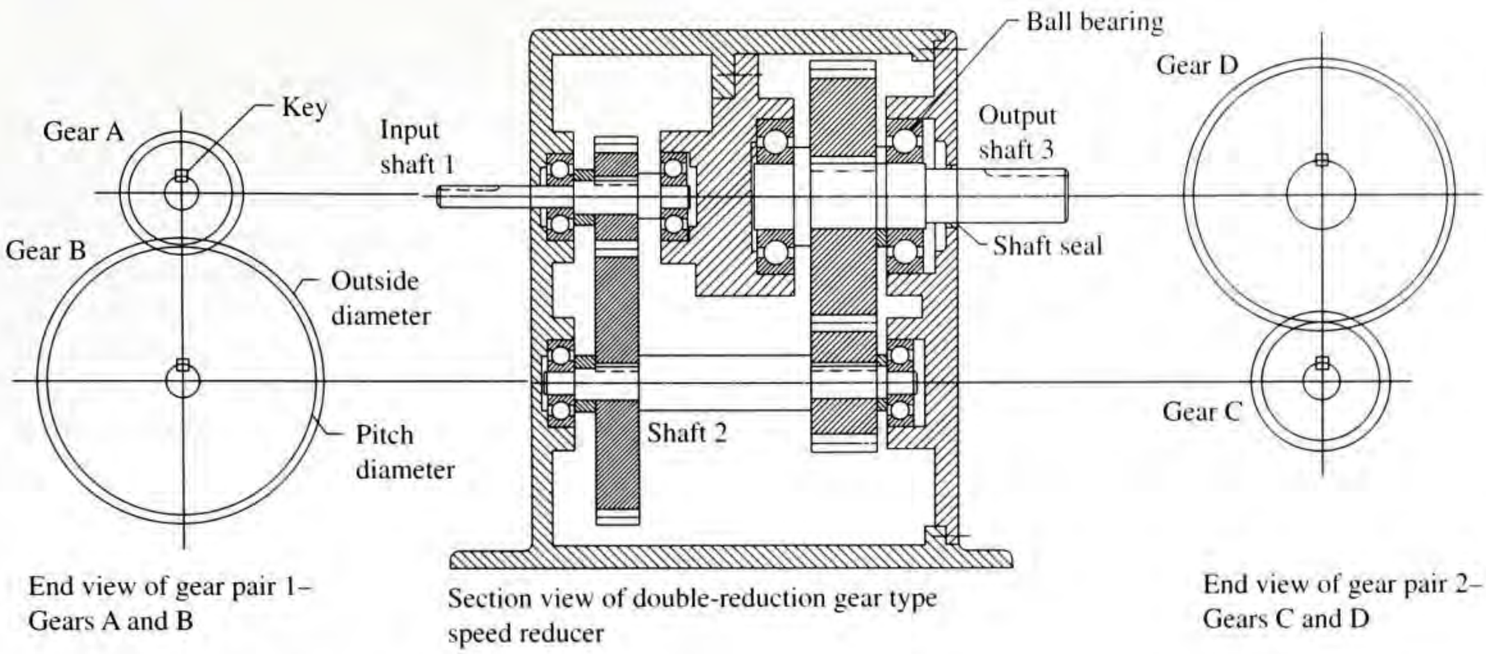


FIGURE 1–12 Conceptual design for a speed reducer

4. The second pair of gears, C and D, further reduces the speed of gear D and the output shaft (shaft 3) to the range of 290 to 295 rpm.
5. The output shaft is to carry a chain sprocket (not shown). The chain drive ultimately is to be connected to the drive wheels of the tractor.
6. Each of the three shafts is supported by two ball bearings, making them statically determinate and allowing the analysis of forces and stresses using standard principles of mechanics.
7. The bearings are held in a housing that is to be attached to the frame of the tractor. Note the manner of holding each bearing so that the inner race rotates with the shaft while the outer race is held stationary.
8. Seals are shown on the input and output shafts to prohibit contaminants from entering the housing.
9. Other parts of the housing are shown schematically. Details of how the active elements are to be installed, lubricated, and aligned are only suggested at this stage of the design process to demonstrate feasibility. One possible assembly process could be as follows:
   - Start by placing the gears, keys, spacers, and bearings on their respective shafts.
   - Then insert shaft 1 into its bearing seat on the left side of the housing.
   - Insert the left end of shaft 2 into its bearing seat while engaging the teeth of gears A and B.
   - Install the center bearing support to provide support for the bearing at the right side of shaft 1.
   - Install shaft 3 by placing its left bearing into the seat on the center bearing support while engaging gears C and D.
   - Install the right side cover for the housing while placing the final two bearings in their seats.
   - Ensure careful alignment of the shafts.
   - Place gear lubricant in the lower part of the housing.

Figures 9–34 to 9–36 in Chapter 9 show three examples of commercially available double-reduction gear reducers where you can see these details.

The arrangement of the gears, the placement of the bearings so that they straddle the gears, and the general configuration of the housing are also design decisions. The design process cannot rationally proceed until these kinds of decisions are made. Notice that the sketch of Figure 1–12 is where *integration* of the elements into a whole design begins. When the overall design is conceptualized, the design of the individual machine elements in the speed reducer can proceed. As each element is discussed, scan the relevant chapters of the book. Part II of this book, including Chapters 7–15, provides details for the elements of the reducer. You should recognize that you have already made many design decisions by rendering such a sketch. First, you chose *spur gears* rather than helical gears, a worm and wormgear, or bevel gears. In fact, other types of speed reduction devices—belt drives, chain drives, or many others—could be appropriate.

Gears

For the gear pairs, you must specify the number of teeth in each gear, the pitch (size) of the teeth, the pitch diameters, the face width, and the material and its heat treatment. These specifications depend on considerations of strength and wear of the gear teeth and the motion requirements (kinematics). You must also recognize that the gears must be mounted on shafts in a manner that ensures proper location of the gears, adequate torque transmitting capability from the gears to the shafts (as through keys), and safe shaft design.

Shafts

Having designed the gear pairs, you next consider the shaft design (Chapter 12). The shaft is loaded in bending and torsion because of the forces acting at the gear teeth. Thus, its design must consider strength and rigidity, and it must permit the mounting of the gears and bearings. Shafts of varying diameters may be used to provide shoulders against which to seat the gears and bearings. There may be keyseats cut into the shaft (Chapter 11). The input and output shafts will extend beyond the housing to permit coupling with the engine and the drive axle. The type of coupling must be considered, as it can have a dramatic effect on the shaft stress analysis (Chapter 11). Seals on the input and output shafts protect internal components (Chapter 11).

Bearings

Design of the bearings (Chapter 14) is next. If rolling contact bearings are to be used, you will probably select commercially available bearings from a manufacturer's catalog, rather than design a unique one. You must first determine the magnitude of the loads on each bearing from the shaft analysis and the gear designs. The rotational speed and reasonable design life of the bearings and their compatibility with the shaft on which they are to be mounted must also be considered. For example, on the basis of the shaft analysis, you could specify the minimum allowable diameter at each bearing seat location to ensure safe stress levels. The bearing selected to support a particular part of the shaft, then, must have a bore (inside diameter) no smaller than the safe diameter of the shaft. Of course, the bearing should not be grossly larger than necessary. When a specific bearing is selected, the diameter of the shaft at the bearing seat location and allowable tolerances must be specified, according to the bearing manufacturer's recommendations, to achieve proper operation and life expectancy of the bearing.

Keys

Now the keys (Chapter 11) and the keyseats can be designed. The diameter of the shaft at the key determines the key's basic size (width and height). The torque that must be transmitted is used in strength calculations to specify key length and material. Once the working components are designed, the housing design can begin.

Housing

The housing design process must be both creative and practical. What provisions should be made to mount the bearings accurately and to transmit the bearing loads safely through the case to the structure on which the speed reducer is mounted? How will the various elements be assembled into the housing? How will the gears and bearings be lubricated? What housing material should be used? Should the housing be a casting, a weldment, or an assembly of machined parts?

The design process as outlined here implies that the design can progress in sequence: from the gears to the shafts, to the bearings, to the keys and couplings, and finally to the housing. It would be rare, however, to follow this logical path only once for a given design. Usually the designer must go back many times to adjust the design of certain components affected by changes in other components. This process, called *iteration,* continues until an acceptable overall design is achieved. Frequently prototypes are developed and tested during iteration.

Chapter 15 shows how all of the machine elements are finally integrated into a unit.

1–6 COMPUTATIONAL AIDS IN THIS BOOK

Because of the usual need for several iterations and because many of the design procedures require long, complex calculations, spreadsheets, mathematical analysis software, computer programs, or programmable calculators are often useful in performing the design analysis. Interactive spreadsheets or programs allow you, the designer, to make design decisions during the design process. In this way, many trials can be made in a short time, and the effects of changing various parameters can be investigated. Spreadsheets using Microsoft Excel are used most frequently as examples in this book for computer-aided design and analysis calculations.

1–7 DESIGN CALCULATIONS

As you study this book and as you progress in your career as a designer, you will make many design calculations. It is important to record the calculations neatly, completely, and in an orderly fashion. You may have to explain to others how you approached the design, which data you used, and which assumptions and judgments you made. In some cases, someone else will actually check your work when you are not there to comment on it or to answer questions. Also, an accurate record of your design calculations is often useful if changes in design are likely. In all of these situations, you are going to be asked to communicate your design to someone else in written and graphic form.

To prepare a careful design record, you will usually take the following steps:

1. Identify the machine element being designed and the nature of the design calculation.
2. Draw a sketch of the element, showing all features that affect performance or stress analysis.
3. Show in a sketch the forces acting on the element (the free-body diagram), and provide other drawings to clarify the actual physical situation.

4. Identify the kind of analysis to be performed, such as stress due to bending, deflection of a beam, buckling of a column, and so on.
5. List all given data and assumptions.
6. Write the formulas to be used in symbol form, and clearly indicate the values and units of the variables involved. If a formula is not well known to a potential reader of your work, give the source. The reader may want to refer to it to evaluate the appropriateness of the formula.
7. Solve each formula for the desired variable.
8. Insert data, check units, and perform computations.
9. Judge the reasonableness of the result.
10. If the result is not reasonable, change the design decisions and recompute. Perhaps a different geometry or material would be more appropriate.
11. When a reasonable, satisfactory result has been achieved, specify the final values for all important design parameters, using standard sizes, convenient dimensions, readily available materials, and so on.

Figure 1–13 shows a sample design calculation. A beam is to be designed to span a 60-in pit to support a large gear weighing 2050 pounds (lb). The design assumes that a rectangular shape is to be used for the cross section of the beam. Other practical shapes could have been used. The objective is to compute the required dimensions of the cross section, considering both stress and deflection. A material for the beam is also chosen. Refer to Chapter 3 for a review of stress due to bending.

1–8 PREFERRED BASIC SIZES, SCREW THREADS, AND STANDARD SHAPES

One responsibility of a designer is to specify the final dimensions for load-carrying members. After completing the analyses for stress and deformation (strain), the designer will know the minimum acceptable values for dimensions that will ensure that the member will meet performance requirements. The designer then typically specifies the final dimensions to be standard or convenient values that will facilitate the purchase of materials and the manufacture of the parts. This section presents some guides to aid in these decisions and specifications.

Preferred Basic Sizes

Table A2–1 lists preferred basic sizes for fractional-inch, decimal-inch, and metric sizes.[1] You should choose one of these preferred sizes as the final part of your design. An example is at the end of the sample design calculation shown in Figure 1–13. You may, of course, specify another size if there is a sound functional reason.

American Standard Screw Threads

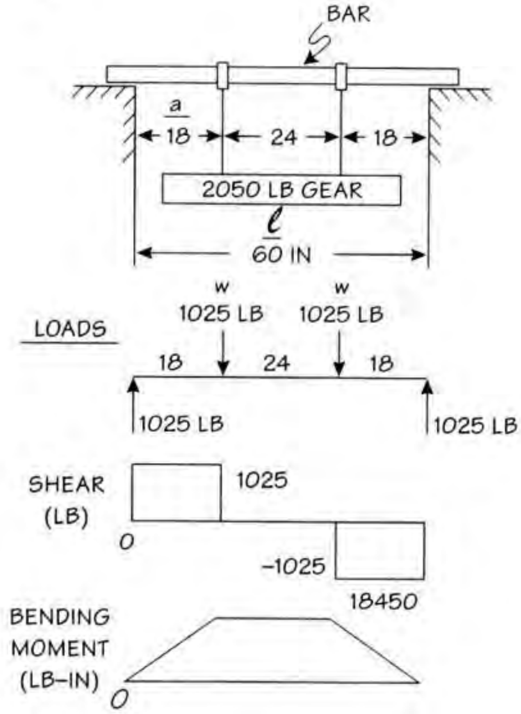
Threaded fasteners and machine elements having threaded connections are manufactured according to standard dimensions to ensure interchangeability of parts and to permit convenient manufacture with standard machines and tooling. Table A2–2 gives the dimensions

[1] Throughout this book, some references to tables and figures have the letter *A* included in their numbers; these tables and figures are in the Appendices in the back of the book. For example, Table A2–1 is the first table in Appendix 2; Figure A15–4 is the fourth figure in Appendix 15. These tables and figures are clearly identified in their captions in the Appendices.

R. L. MOTT

DESIGN OF A BAR TO SUPPORT A GEAR IN A SOAKING PIT

BAR IS TO BE 60 IN LONG BETWEEN SUPPORTS
GEAR WEIGHT 2050 LB
HANGERS TO BE 24 IN APART



BAR IS A BEAM IN BENDING

(1) $\sigma = M/S$

ASSUME A RECTANGULAR SHAPE

S = SECTION MODULUS
$S = th^2/6$
LET $h \approx 3t$
THEN $S = t(3t)^2/6 = 9t^3/6$
$S = 1.5\,t^3$
(2) REQUIRED $t = \sqrt[3]{S/1.5}$

TRY AISI 1040 HR STEEL BAR
$S_y = 42000$ PSi (YIELD STRENGTH)
LET $\sigma = \sigma_d = S_y/N$ = DESIGN STRESS
N = DESIGN FACTOR
LET N = 2 (DEAD LOAD)
$\sigma_d = 42000/2 = 21000$ PSi

THEN FROM (1) : $S = M/\sigma_d$ = REQUIRED SECTION MODULUS

$$S = \frac{18450\ \text{LB·IN}}{21000\ \text{LB/IN}^2} = 0.879\ \text{IN}^3$$

FROM (2)

$$t = \sqrt[3]{S/1.5} = \sqrt[3]{0.879\ \text{IN}^3/1.5} = 0.837\ \text{IN}$$

THEN $h = 3t = 3(0.837\ \text{IN}) = 2.51$ IN

SUPPLIER HAS $3/4 \times 2\,3/4$ AVAILABLE $[h/t = 2.75/0.75 = 3.67$ ok$]$

CHECK $S = th^2/6 = (0.75\ \text{IN})(2.75\ \text{IN})^2/6 = 0.945\ \text{IN}^3 > 0.837\ \text{IN}^3$ ok
$\sigma = M/S = 18450\ \text{LB·IN}/0.945\ \text{IN}^3 = 19500$ PSi
$N = S_y/\sigma = 42000\ \text{PSi}/19500\ \text{PSi} = 2.15$ ok

CHECK DEFLECTION AT CENTER: $y = \dfrac{Wa}{24\,EI}(3\ell^2 - 4a^2)$ (REF MACHINERY'S HANDBOOK 26th ED., P. 238, CASE 4)

$$y = \frac{(1025)(18)[3(60)^2 - 4(18)^2]}{24(30 \times 10^6)(1.30)} = 0.187\ \text{IN}$$
ACCEPTABLE

$$I = th^3/12 = \frac{(0.75)(2.75)^3}{12} = 1.30\ \text{IN}^4$$

SPECIFY: $3/4 \times 2\,3/4$ RECTANGULAR STEEL BAR. AISI 1040 HR

FIGURE 1–13 Sample design calculation

of American Standard Unified threads. Sizes smaller than 1/4 in are given numbers from 0 to 12, while fractional-inch sizes are specified for 1/4 in and larger sizes. Two series are listed: UNC is the designation for coarse threads, and UNF designates fine threads. Standard designations are as follows:

6–32 UNC (number size 6, 32 threads per inch, coarse thread)
12–28 UNF (number size 12, 28 threads per inch, fine thread)
$\frac{1}{2}$–13 UNC (fractional size 1/2 in, 13 threads per inch, coarse thread)
$1\frac{1}{2}$–12 UNF (fractional size $1\frac{1}{2}$ in, 12 threads per inch, fine thread)

Given in the tables are the basic major diameter (D), the number of threads per inch (n), and the tensile stress area (A_t), found from

Tensile Stress Area for Threads

$$A_t = 0.7854\left(D - \frac{0.9743}{n}\right)^2 \quad \textbf{(1–1)}$$

When a threaded member is subjected to direct tension, the tensile stress area is used to compute the average tensile stress. It is based on a circular area computed from the mean of the pitch diameter and the minor diameter of the threaded member.

Metric Screw Threads

Table A2–3 gives similar dimensions for metric threads. Standard metric thread designations are of the form

$$\text{M}10 \times 1.5$$

where M stands for metric

The following number is the basic major diameter, D, in mm

The last number is the pitch, P, between adjacent threads in mm

The tensile stress area for metric threads is computed from the following equation and is based on a slightly different diameter. (See Reference 11, page 1483.)

$$A_t = 0.7854\,(D - 0.9382P)^2 \quad \textbf{(1–2)}$$

Thus, the designation above would denote a metric thread with a basic major diameter of $D = 10.0$ mm and a pitch of $P = 1.5$ mm. Note that pitch $= 1/n$. The tensile stress area for this thread is 58.0 mm^2.

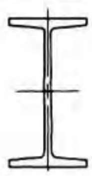
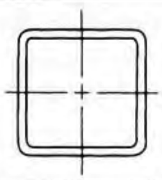
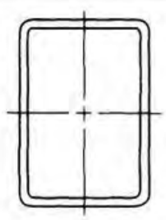
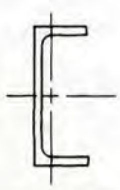
Steel Structural Shapes

Steel manufacturers provide a large array of standard structural shapes that are efficient in the use of material and that are convenient for specification and installation into building structures or machine frames. Included, as shown in Table 1–1, are standard angles (L-shapes), channels (C-shapes), wide-flange beams (W-shapes), American Standard beams (S-shapes), structural tubing, and pipe. Note that the W-shapes and the S-shapes are often referred to in general conversation as "I-beams" because the shape of the cross section looks like the capital letter I.

Appendix 16 gives geometric properties of selected steel structural shapes that cover a fairly wide range of sizes. Note that many more sizes are available as presented in Reference 2. The tables in Appendix 16 give data for the area of the cross section (A), the weight per foot of length, the location of the centroid of the cross section, the moment of inertia (I), the section modulus (S), and the radius of gyration (r). The values of I and S are important in the analysis and design of beams. For column analysis, I and r are needed.

Materials used for structural shapes are typically called *structural steels*, and their characteristics and properties are described more fully in Chapter 2. Refer to Appendix 7 for typical strength data. Rolled W-shapes are most readily available in ASTM A992, A572 Grade 50, or A36. S-shapes and C-shapes are typically made from ASTM A572 Grade 50

TABLE 1–1 Designations for steel and aluminum shapes

Name of shape	Shape	Symbol	Example designation and Appendix table
Angle		L	$L4 \times 3 \times \frac{1}{2}$ Table A16–1
Channel		C	$C15 \times 50$ Table A16–2
Wide-flange beam		W	$W14 \times 43$ Table A16–3
American Standard beam		S	$S10 \times 35$ Table A16–4
Structural tubing—square			$4 \times 4 \times \frac{1}{4}$ Table A16–5
Structural tubing—rectangular			$6 \times 4 \times \frac{1}{4}$ Table A16–5
Pipe			4-inch standard weight 4-inch Schedule 40 Table A16–6
Aluminum Association channel		C	$C4 \times 1.738$ Table A17–1
Aluminum Association I-beam		I	$I8 \times 6.181$ Table A17–2

or A36. ASTM A36 should be specified for steel angles and plates. Hollow structural shapes (HSS) are most readily available in ASTM A500.

Steel Angles (L-Shapes)

Table A16–1 shows sketches of the typical shapes of steel angles having equal or unequal leg lengths. Called *L-shapes* because of the appearance of the cross section, angles are often used as tension members of trusses and towers, framing members for machine structures, lintels over windows and doors in construction, stiffeners for large plates used in housings and beams, brackets, and ledge-type supports for equipment. Some refer to these shapes as "angle iron." The standard designation takes the following form, using one example size:

$$L4 \times 3 \times \tfrac{1}{2}$$

where L refers to the L-shape
4 is the length of the longer leg
3 is the length of the shorter leg
$\frac{1}{2}$ is the thickness of the legs
Dimensions are in inches

American Standard Channels (C-Shapes)

See Table A16–2 for the appearance of channels and their geometric properties. Channels are used in applications similar to those described for angles. The flat web and the two flanges provide a generally stiffer shape than angles.

The sketch at the top of the table shows that channels have tapered flanges and webs with constant thickness. The slope of the flange taper is approximately 2 inches in 12 inches, and this makes it difficult to attach other members to the flanges. Special tapered washers are available to facilitate fastening. Note the designation of the x- and y-axes in the sketch, defined with the web of the channel vertical which gives it the characteristic C-shape. This is most important when using channels as beams or columns. The x-axis is located on the horizontal axis of symmetry, while the dimension x, given in the table, locates the y-axis relative to the back of the web. The centroid is at the intersection of the x- and y-axes.

The form of the standard designation for channels is

$$C15 \times 50$$

where C indicates that it is a standard C-shape
15 is the nominal (and actual) depth in inches with the web vertical
50 is the weight per unit length in lb/ft

Wide-Flange Shapes (W-Shapes)

Refer to Table A16–3, which illustrates the most common shape used for beams. W-shapes have relatively thin webs and somewhat thicker, flat flanges with constant thickness. Most of the area of the cross section is in the flanges, farthest away from the horizontal centroidal axis (x-axis), thus making the moment of inertia very high for a given amount of material.

Note that the properties of moment of inertia and section modulus are very much higher with respect to the x-axis than they are for the y-axis. Therefore, W-shapes are typically used in the orientation shown in the sketch in Table A16–3. Also, these shapes are best when used in pure bending without twisting because they are quite flexible in torsion.

The standard designation for W-shapes carries much information. Consider the following example:

$$W14 \times 43$$

where W indicates that it is a W-shape
14 is the nominal depth in inches
43 is the weight per unit length in lb/ft

The term depth is the standard designation for the vertical height of the cross section when placed in the orientation shown in Table A16–3. Note from the data in the table that the actual depth is often different from the nominal depth. For the W14 $\times$ 43, the actual depth is 13.66 in.

American Standard Beams (S-Shapes)

Table A16–4 shows the properties for S-shapes. Much of the discussion given for W-shapes applies to S-shapes as well. Note that, again, the weight per foot of length is included in the designation such as the S10 $\times$ 35, which weighs 35 lb/ft. For most, but not all, of the S-shapes, the actual depth is the same as the nominal depth. The flanges of the S-shapes are tapered at a slope of approximately 2 inches in 12 inches, similar to the flanges of the C-shapes. The x- and y-axes are defined as shown with the web vertical.

Often wide-flange shapes (W-shapes) are preferred over S-shapes because of their relatively wide flanges, the constant thickness of the flanges, and the generally higher section properties for a given weight and depth.

Hollow Structural Shapes (HSS Square and Rectangular)

See Table A16–5 for the appearance and properties for hollow structural shapes. These shapes are usually formed from flat sheet and welded along the length. The section properties account for the corner radii. Note the sketches showing the x- and y-axes. The standard designation takes the form

$$6 \times 4 \times \tfrac{1}{4}$$

where 6 is the depth of the longer side in inches
4 is the width of the shorter side in inches
$\frac{1}{4}$ is the wall thickness in inches

Square tubing and rectangular tubing are very useful in machine structures because they provide good section properties for members loaded as beams in bending and for torsional loading (twisting) because of the closed cross section. The flat sides often facilitate fastening of members together or the attachment of equipment to the structural members. Some frames are welded into an integral unit that functions as a stiff space-frame. Square tubing makes an efficient section for columns.

Pipe

Hollow circular sections, commonly called *pipe,* are very efficient for use as beams, torsion members, and columns. The placement of the material uniformly away from the center of the pipe enhances the moment of inertia for a given amount of material and gives the pipe uniform properties with respect to all axes through the center of the cross section. The closed cross-sectional shape gives it high strength and stiffness in torsion as well as in bending.

Table A16–6 gives the properties for American National Standard Schedule 40 welded and seamless wrought steel pipe. This type of pipe is often used to transport water and other fluids, but it also performs well in structural applications. Note that the actual inside and outside diameters are somewhat different from the nominal size, except for the very large sizes. Construction pipe is often called *Standard Weight Pipe,* and it has the same dimensions as the Schedule 40 pipe for sizes from 1/2 in to 10 in. Other "schedules" and "weights" of pipe are available with larger and smaller wall thicknesses.

Other hollow circular sections are commonly available that are referred to as *tubing.* These sections are available in carbon steel, alloy steel, stainless steel, aluminum, copper, brass, titanium, and other materials. See References 1, 2, 5, and 9 for a variety of types and sizes of pipe and tubing.

Aluminum Association Standard Channels and I-Beams

Tables A17–1 and A17–2 give the dimensions and section properties of channels and I-beams developed by the Aluminum Association (see Reference 1). These are extruded shapes having uniform thicknesses of the webs and flanges with generous radii where they meet. The proportions of these sections are somewhat different from those of the rolled steel sections described earlier. The extruded form offers advantages in the efficient use of material and in the joining of members. This book will use the following forms for the designation of aluminum sections:

$$C4 \times 1.738 \quad \text{or} \quad I8 \times 6.181$$

where C or I indicates the basic section shape
4 or 8 indicates the depth of the shape when in the orientation shown
1.738 or 6.181 indicates the weight per unit length in lb/ft

1–9 UNIT SYSTEMS

We will perform computations in this book by using either the U.S. Customary Unit System (inch-pound-second) or the International System (SI). Table 1–2 lists the typical units used in the study of machine design. *SI,* the abbreviation for "Le Système International d'Unités," is the standard for metric units throughout the world. (See Reference 4.) For convenience, the term *SI units* will be used instead of *metric units.*

Prefixes applied to the basic units indicate order of magnitude. Only those prefixes listed in Table 1–3, which differ by a factor of 1000, should be used in technical calculations. The final result for a quantity should be reported as a number between 0.1 and 10 000, times some multiple of 1000. Then the unit with the appropriate prefix should be specified. Table 1–4 lists examples of proper SI notation.

Sometimes you have to convert a unit from one system to another. Appendix 18 provides tables of conversion factors. Also, you should be familiar with the typical order of magnitude of the quantities encountered in machine design so that you can judge the reasonableness of design calculations (see Table 1–5 for several examples).

TABLE 1–2 Typical units used in machine design

Quantity	U.S. Customary unit	SI unit
Length or distance	inch (in) foot (ft)	meter (m) millimeter (mm)
Area	square inch (in^2)	square meter (m^2) or square millimeter (mm^2)
Force	pound (lb) kip (K) (1000 lb)	newton (N) ($1\ N = 1\ kg{\cdot}m/s^2$)
Mass	slug ($lb{\cdot}s^2/ft$)	kilogram (kg)
Time	second (s)	second (s)
Angle	degree (°)	radian (rad) or degree (°)
Temperature	degrees Fahrenheit (°F)	degrees Celsius (°C)
Torque or moment	pound-inch (lb·in) or pound-foot (lb·ft)	newton-meter (N·m)
Energy or work	pound-inch (lb·in)	joule (J) (1 J = 1 N·m)
Power	horsepower (hp) (1 hp = 550 lb·ft/s)	watt (W) or kilowatts (kW) (1 W = 1 J/s = 1 N·m/s)
Stress, pressure, or modulus of elasticity	pounds per square inch (lb/in^2, or psi) kips per square inch (K/in^2, or ksi)	pascal (Pa) ($1\ Pa = 1\ N/m^2$) kilopascal (kPa) ($1\ kPa = 10^3\ Pa$) megapascal (MPa) ($1\ MPa = 10^6\ Pa$) gigapascal (GPa) ($1\ GPa = 10^9\ Pa$)
Section modulus	inches cubed (in^3)	meters cubed (m^3) or millimeters cubed (mm^3)
Moment of inertia	inches to the fourth power (in^4)	meters to the fourth power (m^4) or millimeters to the fourth power (mm^4)
Rotational speed	revolutions per min (rpm)	radians per second (rad/s)

TABLE 1–3 Prefixes used with SI units

Prefix	SI symbol	Factor
micro-	μ	$10^{-6} = 0.000\ 001$
milli-	m	$10^{-3} = 0.001$
kilo-	k	$10^3 = 1000$
mega-	M	$10^6 = 1\ 000\ 000$
giga-	G	$10^9 = 1\ 000\ 000\ 000$

TABLE 1–4 Quantities expressed in SI units

Computed result	Reported Result
0.001 65 m	1.65×10^{-3} m, or 1.65 mm
32 540 N	32.54×10^3 N, or 32.54 kN
1.583×10^5 W	158.3×10^3 W, or 158.3 kW; or $0.158\ 3 \times 10^6$ W; or 0.158 3 MW
2.07×10^{11} Pa	207×10^9 Pa, or 207 GPa

TABLE 1–5 Typical order of magnitude for commonly encountered quantities

Quantity	U.S. Customary unit	SI unit
Dimensions of a wood standard 2 × 4	1.50 in × 3.50 in	38 mm × 89 mm
Moment of inertia of a 2 × 4 (3.50-in side vertical)	5.36 in^4	$2.23 \times 10^6\ mm^4$, or $2.23 \times 10^{-6}\ m^4$
Section modulus of a 2 × 4 (3.50-in side vertical)	3.06 in^3	$5.02 \times 10^4\ mm^3$, or $5.02 \times 10^{-5}\ m^3$
Force required to lift 1.0 gal of gasoline	6.01 lb	26.7 N
Density of water	1.94 $slugs/ft^3$	1000 kg/m^3, or 1.0 Mg/m^3
Compressed air pressure in a factory	100 psi	690 kPa
Yield point of AISI 1040 hot-rolled steel	42 000 psi, or 42 ksi	290 MPa
Modulus of elasticity of steel	30 000 000 psi, or 30×10^6 psi	207 GPa

Example Problem 1–1 Express the diameter of a shaft in millimeters if it is measured to be 2.755 in.

Solution Table A18 gives the conversion factor for length to be 1.00 in = 25.4 mm. Then

$$\text{Diameter} = 2.755\ \text{in}\frac{25.4\ \text{mm}}{1.00\ \text{in}} = 69.98\ \text{mm}$$

Example Problem 1–2 An electric motor is rotating at 1750 revolutions per minute (rpm). Express the speed in radians per second (rad/s).

Solution A series of conversions is required.

$$\text{Rotational speed} = \frac{1750\ \text{rev}}{\text{min}}\ \frac{2\pi\ \text{rad}}{\text{rev}}\ \frac{1\ \text{min}}{60\ \text{s}} = 183.3\ \text{rad/s}$$

1–10 DISTINCTION AMONG WEIGHT, FORCE, AND MASS

Distinction must be made among the terms *force*, *mass*, and *weight*. *Mass* is the quantity of matter in a body. A *force* is a push or pull applied to a body that results in a change in the body's motion or in some deformation of the body. Clearly these are two different physical phenomena, but the distinction is not always understood. The units for force and mass used in this text are listed in Table 1–2.

The term *weight*, as used in this book, refers to the amount of *force* required to support a body against the influence of gravity. Thus, in response to "What is the weight of 75 kg of steel?" we would use the relationship between force and mass from physics:

Weight/Mass Relationship

$$F = ma \quad \text{or} \quad w = mg$$

where F = force
m = mass
a = acceleration
w = weight
g = acceleration due to gravity

We will use

$$g = 32.2\ \text{ft/s}^2 \quad \text{or} \quad g = 9.81\ \text{m/s}^2$$

Then, to compute the weight,

$$w = mg = 75\ \text{kg}(9.81\ \text{m/s}^2)$$
$$w = 736\ \text{kg}\cdot\text{m/s}^2 = 736\ \text{N}$$

Remember that, as shown in Table 1-2, the newton (N) is equivalent to $1.0\ \text{kg}\cdot\text{m/s}^2$. In fact, the newton is defined as the force required to give a mass of 1.0 kg an acceleration of $1.0\ \text{m/s}^2$. In our example, then, we would say that the 75-kg mass of steel has a weight of 736 N.

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INTERNET SITES FOR GENERAL MECHANICAL DESIGN

Included here are Internet sites that can be used in many of the chapters of this book and in general design practice to identify commercial suppliers of machine elements and standards for design or to perform stress analyses. Later chapters include sites specific to the topics covered there.

1. **American National Standards Institute (ANSI)** *www.ansi.org* A private, nonprofit organization that administers and coordinates the U.S. voluntary standardization and conformity assessment system.
2. **Global Engineering Documents** *http://global.ihs.com* A searchable database of standards and publications offered by many standards-developing organizations such as ASME, ASTM, and ISO.
3. **GlobalSpec** *www.globalspec.com* A searchable database of a wide variety of technical products and services that provides for searching by technical specifications, access to supplier information, and comparison of suppliers for a given product. The Mechanical Components category includes many of the topics addressed in this book.
4. **MDSOLIDS** *www.mdsolids.com* Educational software for strength of materials topics, including beams, flexure, torsion members, columns, axial structures, statically indeterminate structures, trusses, section properties, and Mohr's circle analysis. This software may serve as a review tool for the prerequisite knowledge needed in this book.
5. **StressAlyzer** *www.me.cmu.edu* A highly interactive problem-solving package for topics in strength of materials, including axial loading, torsional loading, shear force and bending moment diagrams, beam deflections, Mohr's circle (stress transformations), and load and stress calculations in three dimensions.
6. **Orand Systems–Beam 2D** *www.orandsystems.com* Stress and deflection analysis software package providing solutions for beams under static loading. Numerous beam cross sections, materials, loading patterns, and support conditions can be input. Output includes bending stress, shear stress, deflection, and slope for the beam.
7. **Power Transmission Home Page** *www.powertransmission.com* Clearinghouse on the Internet for buyers, users, and sellers of power transmission products and services. Included are gears, gear drives, belt drives, chain drives, bearings, clutches, brakes, and many other machine elements covered in this book.

PROBLEMS

Functions and Design Requirements

For the devices described in Problems 1–14, write a set of functions and design requirements in a similar manner to those in Section 1–4. You or your instructor may add more specific information to the general descriptions given.

1. The hood latch for an automobile
2. A hydraulic jack used for car repair
3. A portable crane to be used in small garages and homes
4. A machine to crush soft-drink or beer cans
5. An automatic transfer device for a production line
6. A device to raise a 55-gallon (gal) drum of bulk materials and dump the contents into a hopper
7. A paper feed device for a copier
8. A conveyor to elevate and load gravel into a truck
9. A crane to lift building materials from the ground to the top of a building during construction
10. A machine to insert toothpaste tubes into cartons
11. A machine to insert 24 cartons of toothpaste into a shipping container
12. A gripper for a robot to grasp a spare tire assembly and insert it into the trunk of an automobile on an assembly line
13. A table for positioning a weldment in relation to a robotic welder
14. A garage door opener

Units and Conversions

For Problems 15–28, perform the indicated conversion of units. (Refer to Appendix 18 for conversion factors.) Express the results with the appropriate prefix as illustrated in Tables 1–3 and 1–4.

15. Convert a shaft diameter of 1.75 in to mm.
16. Convert the length of a conveyor from 46 ft to meters.
17. Convert the torque developed by a motor of 12 550 lb · in to N · m.
18. A wide-flange steel-beam shape, W12 × 14, has a cross-sectional area of 4.12 in^2. Convert the area to mm^2.
19. The W12 × 14 beam shape has a section modulus of 14.8 in^3. Convert it to mm^3.
20. The W12 × 14 beam shape has a moment of inertia of 88.0 in^4. Convert it to mm^4.
21. What standard steel equal leg angle would have a cross-sectional area closest to (but greater than) 750 mm^2? See Table A16–1.
22. An electric motor is rated at 7.5 hp. What is its rating in watts (W)?
23. A vendor lists the ultimate tensile strength of a steel to be 127 000 psi. Compute the strength in MPa.
24. Compute the weight of a steel shaft, 35.0 mm in diameter and 675 mm long. (See Appendix 3 for the density of steel.)
25. A torsional spring requires a torque of 180 lb · in to rotate it 35°. Convert the torque to N · m and the rotation to radians. If the *scale of the spring* is defined as the applied torque per unit of angular rotation, compute the spring scale in both unit systems.
26. To compute the energy used by a motor, multiply the power that it draws by the time of operation. Consider a motor that draws 12.5 hp for 16 h/day, five days per week. Compute the energy used by the motor for one year. Express the result in ft · lb and W · h.
27. One unit used for fluid viscosity in Chapter 16 of this book is the *reyn*, defined as 1.0 lb · s/in^2. If a lubricating oil has a viscosity of 3.75 reyn, convert the viscosity to the standard units in the U.S. Customary System (lb · s/ft^2) and in the SI (N · s/m^2).
28. The life of a bearing supporting a rotating shaft is expressed in number of revolutions. Compute the life of a bearing that rotates 1750 rpm continuously for 24 h/day for five years.

2

Materials in Mechanical Design

The Big Picture

Materials in Mechanical Design

Discussion Map

- ☐ You must understand the behavior of materials to make good design decisions and to communicate with suppliers and manufacturing staff.

Discover

Examine consumer products, industrial machinery, automobiles, and construction machinery.

What materials are used for the various parts?

Why do you think those materials were specified?

How were they processed?

What material properties were important to the decisions to use particular materials?

Examine the Appendices tables, and refer to them later as you read about specific materials.

This chapter summarizes the design properties of a variety of materials. The Appendices include data for many examples of these materials in many conditions.

It is the designer's responsibility to specify suitable materials for each component of a mechanical device. Your initial efforts in specifying a material for a particular component of a mechanical design should be directed to the basic kind of material to be used. Keep an open mind until you have specified the functions of the component, the kinds and magnitudes of loads it will carry, and the environment in which it must operate. Your selection of a material must consider its physical and mechanical properties and match them to the expectations placed on it. First consider the following classes of materials:

Metals and their alloys	Plastics	Composites
Elastomers	Woods	Ceramics and glasses

Each of these classes contains a large number of specific materials covering a wide range of actual properties. However, you probably know from your experience the general behavior of each kind and have some feel for the applications in which each is typically used. Most of the applications considered in the study of design of machine elements in this book use metal alloys, plastics, and composites.

Satisfactory performance of machine components and systems depends greatly on the materials that the designer specifies. As a designer, you must understand how materials behave, what properties of the material affect the performance of the parts, and how you should interpret the large amounts of data available on material properties. Your ability to effectively communicate your specifications for materials with suppliers, purchasing agents, metallurgists, manufacturing process personnel, heat treatment personnel, plastics molders, machinists, and quality assurance specialists often has a strong influence on the success of a design.

Explore what kinds of materials are used in consumer products, industrial machinery, automobiles, construction machinery, and other devices and systems that you come into contact with each day. Make judgments about why each material was specified for a particular application. Where do you see steel being used? Contrast that usage with where aluminum or other nonferrous materials are used. How are the products produced? Can you find different parts that are machined, cast, forged, roll-formed, and welded? Why do you think those processes were specified for those particular products?

Document several applications for plastics and describe the different forms that are available and that have been made by different manufacturing processes. Which are made by plastic molding processes, vacuum forming, blow molding, and others? Can you identify parts made from composite materials that have a significant amount of high-strength fibers embedded in a plastic matrix? Check out sporting goods and parts of cars, trucks, and airplanes.

From the products that you found from the exploration outlined previously, identify the basic properties of the materials that were important to the designers: strength, rigidity (stiffness), weight (density), corrosion resistance, appearance, machinability, weldability, ease of forming, cost, and others.

This chapter focuses on material selection and the use of material property data in design decisions, rather than on the metallurgy or chemistry of the materials. One of the uses of the information in this chapter is as a glossary of terms that you can use throughout the book; important terms are given in *italic* type. Also, there are numerous references to Appendices 3 through 13, where tables of data for material properties are given. Go there now and see what kinds of data are provided. Then you can study the tables in more depth as you read the text. Note that many of the problems that you will solve in this book and the design projects that you complete will use data from these tables.

Now apply some of what you have gained from **The Big Picture** exploration to a specific design situation as outlined in **You Are the Designer**, which follows.

You Are the Designer

You are part of a team responsible for the design of an electric lawn mower for the household market. One of your tasks is to specify suitable materials for the various components. Consider your own experience with such lawn mowers and think what materials would be used for these key components: *wheels, axles, housing*, and *blade*. What are their functions? What conditions of service will each encounter? What is one reasonable type of material for each component and what general properties should it have? How could they be manufactured? Possible answers to these questions follow.

Wheels

Function: Support the weight of the mower. Permit easy, rolling movement. Provide for mounting on an axle. Ensure safe operation on flat or sloped lawn surfaces.

Conditions of service: Must operate on grass, hard surfaces, and soft earth. Exposed to water, lawn fertilizers, and general outdoor conditions. Will carry moderate loads. Requires an attractive appearance.

One reasonable material: One-piece plastic wheel incorporating the tire, rim, and hub. Must have good strength, stiffness, toughness, and wear resistance.

Manufacturing method: Plastic injection molding

Axles

Function: Transfer the weight of mower from the housing to the wheels. Allow rotation of the wheels. Maintain location of the wheels relative to the housing.

Conditions of service: Exposure to general outdoor conditions. Moderate loads.

One possible material: Steel rod with provisions for mounting wheels and attaching to housing. Requires moderate strength, stiffness, and corrosion resistance.

Manufacturing method: Commercially available cylindrical rod. Possibly machining.

Housing

Function: Support, safely enclose, and protect operating components, including the blade and motor. Accommodate the attachment of two axles and a handle. Permit cut grass to exit the cutting area.

Conditions of service: Moderate loads and vibration due to motor. Possible shock loads from wheels. Multiple attachment points for axles, handle, and motor. Exposed to wet grass and general outdoor conditions. Requires attractive appearance.

One possible material: Heavy-duty plastic with good strength, stiffness, impact resistance, toughness, and weather resistance.

Manufacturing method: Plastic injection molding. May require machining for holes and mounting points for the motor.

Blade

Function: Cut blades of grass and weeds while rotating at high speed. Facilitate connection to motor shaft. Operate safely when foreign objects are encountered, such as stones, sticks, or metal pieces.

Conditions of service: Normally moderate loads. Occasional shock and impact loads. Must be capable of sharpening a portion of the blade to ensure clean cutting of grass. Maintain sharpness for reasonable time during use.

One possible material: Steel with high strength, stiffness, impact resistance, toughness, and corrosion resistance.

Manufacturing method: Stamping from flat steel strip. Machining and/or grinding for cutting edge.

This simplified example of the material selection process should help you to understand the importance of the information provided in this chapter about the behavior of materials commonly used in the design of machine elements. A more comprehensive discussion of material selection occurs at the end of the chapter.

2–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. State the types of material properties that are important to the design of mechanical devices and systems.
2. Define the following terms: *tensile strength, yield strength, proportional limit, elastic limit, modulus of elasticity in tension, ductility and percent elongation, shear strength, Poisson's ratio, modulus of elasticity in shear, hardness, machinability, impact strength, density, coefficient of thermal expansion, thermal conductivity*, and *electrical resistivity*.
3. Describe the nature of *carbon and alloy steels*, the number-designation system for steels, and the effect of several kinds of alloying elements on the properties of steels.
4. Describe the manner of designating the condition and heat treatment of steels, including *hot rolling*, *cold drawing*, *annealing*, *normalizing*, *through-hardening*, *tempering*, and *case hardening by flame hardening*, *induction hardening*, and *carburizing*.
5. Describe *stainless steels* and recognize many of the types that are commercially available.
6. Describe *structural steels* and recognize many of their designations and uses.
7. Describe *cast irons* and several kinds of *gray iron*, *ductile iron*, and *malleable iron*.
8. Describe *powdered metals* and their properties and uses.
9. Describe several types of *tool steels* and *carbides* and their typical uses.
10. Describe *aluminum alloys* and their conditions, such as *strain hardening* and *heat treatment*.
11. Describe the nature and typical properties of *zinc*, *titanium*, and *bronze*.
12. Describe several types of *plastics*, both *thermosetting* and *thermoplastic*, and their typical properties and uses.
13. Describe several kinds of *composite materials* and their typical properties and uses.
14. Implement a rational material selection process.

2–2 PROPERTIES OF MATERIALS

Machine elements are very often made from one of the metals or metal alloys such as steel, aluminum, cast iron, zinc, titanium, or bronze. This section describes the important properties of materials as they affect mechanical design.

Strength, elastic, and ductility properties for metals, plastics, and other types of materials are usually determined from a *tensile test* in which a sample of the material, typically in the form of a round or flat bar, is clamped between jaws and pulled slowly until it breaks in tension. The magnitude of the force on the bar and the corresponding change in length (strain) are monitored and recorded continuously during the test. Because the stress in the bar is equal to the applied force divided by the area, stress is proportional to the applied force. The data from such tensile tests are often shown on *stress-strain diagrams*, such as those shown in Figures 2–1 and 2–2. In the following paragraphs, several strength, elastic, and ductility properties of metals are defined.

Tensile Strength, s_u

The peak of the stress-strain curve is considered the *ultimate tensile strength* (s_u), sometimes called the *ultimate strength* or simply the *tensile strength*. At this point during the test, the highest *apparent stress* on a test bar of the material is measured. As shown in Figures 2–1 and 2–2, the curve appears to drop off after the peak. However, notice that the instrumentation used to create the diagrams is actually plotting *load versus deflection* rather than *true*

FIGURE 2–1 Typical stress-strain diagram for steel

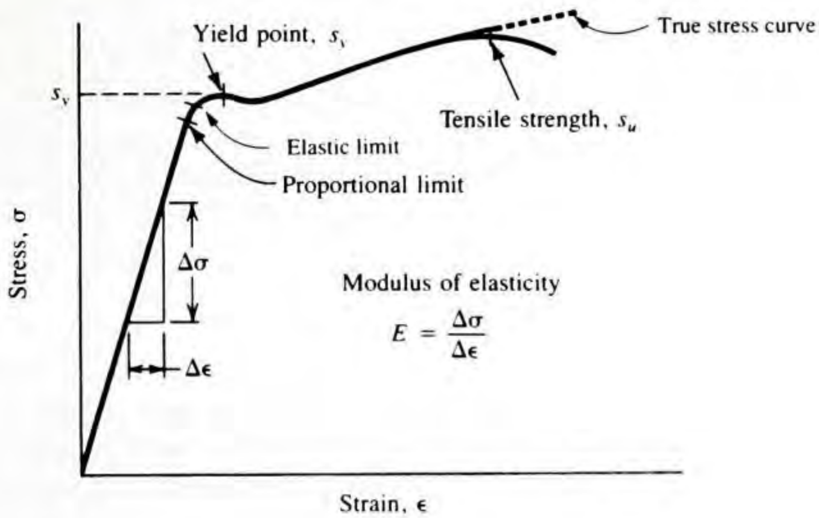
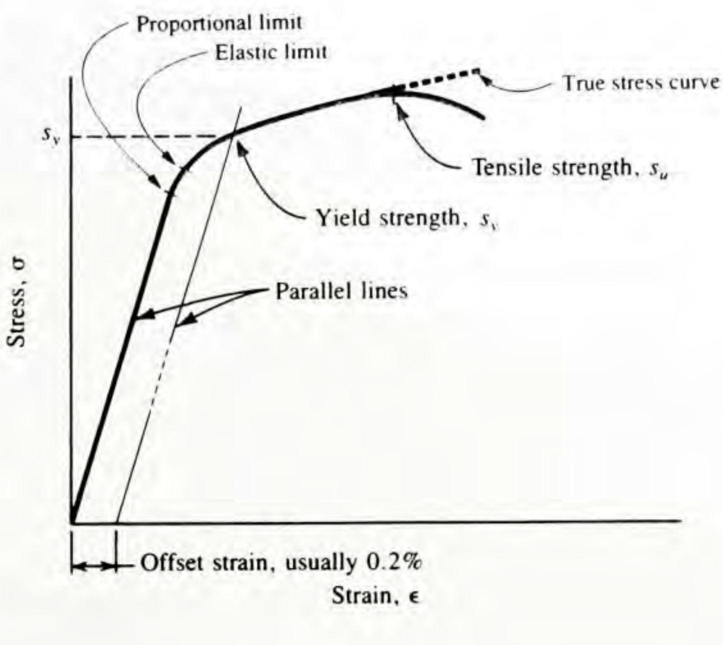


FIGURE 2–2 Typical stress-strain diagram for aluminum and other metals having no yield point



stress versus strain. The apparent stress is computed by dividing the load by the original cross-sectional area of the test bar. After the peak of the curve is reached, there is a pronounced decrease in the bar's diameter, referred to as *necking down*. Thus, the load acts over a smaller area, and the *actual stress* continues to increase until failure. It is very difficult to follow the reduction in diameter during the necking-down process, so it has become customary to use the peak of the curve as the tensile strength, although it is a more conservative value.

Yield Strength, s_y

That portion of the stress-strain diagram where there is a large increase in strain with little or no increase in stress is called the *yield strength* (s_y). This property indicates that the material has, in fact, yielded or elongated plastically, permanently, and to a large degree. If the point of yielding is quite noticeable, as it is in Figure 2–1, the property is called the *yield point* rather than the yield strength. This is typical of a plain carbon hot rolled steel.

Figure 2–2 shows the stress-strain diagram form that is typical of a nonferrous metal such as aluminum or titanium or of certain high-strength steels. Notice that there is no pronounced yield point, but the material has actually yielded at or near the stress level indicated as s_y. That point is determined by the *offset method*, in which a line is drawn parallel to the straight-line portion of the curve and is offset to the right by a set amount, usually 0.20% strain (0.002 in/in). The intersection of this line and the stress-strain curve defines the material's yield strength. In this book, the term *yield strength* will be used for s_y, regardless of whether the material exhibits a true yield point or whether the offset method is used.

Proportional Limit

That point on the stress-strain curve where it deviates from a straight line is called the *proportional limit*. That is, at or above that stress value, stress is no longer proportional to strain. Below the proportional limit, Hooke's law applies: Stress is proportional to strain. In mechanical design, materials are rarely used at stresses above the proportional limit.

Elastic Limit

At some point, called the *elastic limit*, a material experiences some amount of plastic strain and thus will not return to its original shape after release of the load. Below that level, the material behaves completely elastically. The proportional limit and the elastic limit lie quite close to the yield strength. Because they are difficult to determine, they are rarely reported.

Modulus of Elasticity in Tension, E

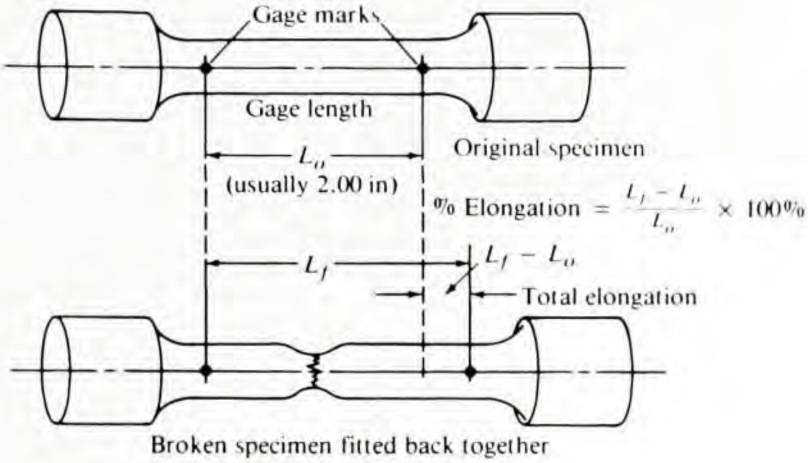
For the part of the stress-strain diagram that is straight, stress is proportional to strain, and the value of *E, the modulus of elasticity*, is the constant of proportionality. That is,

Modulus of Elasticity in Tension

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\epsilon} \tag{2–1}$$

This is the slope of the straight-line portion of the diagram. The modulus of elasticity indicates the stiffness of the material, or its resistance to deformation.

FIGURE 2–3
Measurement of percent elongation



Ductility and Percent Elongation

Ductility is the degree to which a material will deform before ultimate fracture. The opposite of ductility is *brittleness*. When ductile materials are used in machine members, impending failure is detected easily, and sudden failure is unlikely. Also, ductile materials normally resist the repeated loads on machine elements better than brittle materials.

The usual measure of ductility is the *percent elongation* of the material after fracture in a standard tensile test. Figure 2–3 shows a typical standard tensile specimen before and after the test. Before the test, gage marks are placed on the bar, usually 2.00 in apart. Then, after the bar is broken, the two parts are fitted back together, and the final length between the gage marks is measured. The percent elongation is the difference between the final length and the original length divided by the original length, converted to a percentage. That is,

Percent Elongation

$$\text{percent elongation} = \frac{L_f - L_o}{L_o} \times 100\% \tag{2–2}$$

The percent elongation is assumed to be based on a gage length of 2.00 in unless some other gage length is specifically indicated. Tests of structural steels often use a gage length of 8.00 in.

Theoretically, a material is considered ductile if its percent elongation is greater than 5% (lower values indicate brittleness). For practical reasons, it is advisable to use a material with a value of 12% or higher for machine members subject to repeated loads or shock or impact.

Percent reduction in area is another indication of ductility. To find this value, compare the original cross-sectional area with the final area at the break for the tensile test specimen.

Shear Strength, s_{ys} and s_{us}

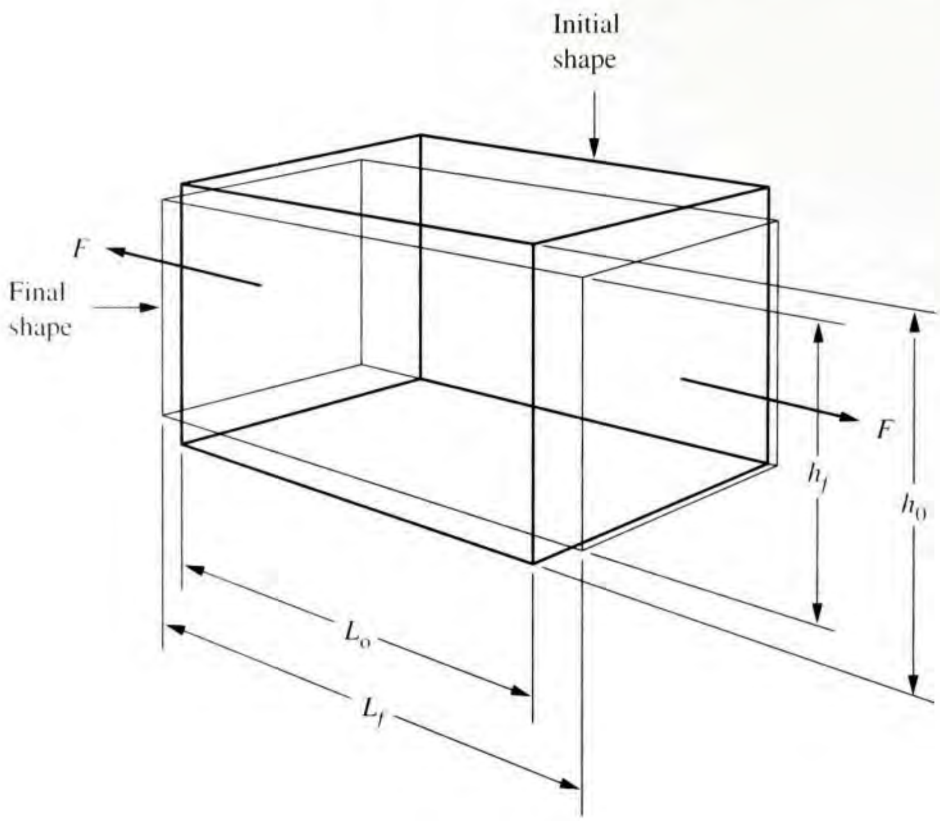
Both the yield strength and the ultimate strength in shear (s_{ys} and s_{us}, respectively) are important properties of materials. Unfortunately, these values are seldom reported. We will use the following estimates:

Estimates for s_{ys} and s_{us}

$$s_{ys} = s_y/2 = 0.50\, s_y = \text{yield strength in shear} \tag{2–3}$$

$$s_{us} = 0.75 s_u = \text{ultimate strength in shear} \tag{2–4}$$

FIGURE 2–4
Illustration of Poisson's ratio for an element in tension



$$\text{Axial strain} = \frac{L_f - L_o}{L_o} = \epsilon_a$$

$$\text{Lateral strain} = \frac{h_f - h_o}{h_o} = \epsilon_L$$

$$\text{Poisson's ratio} = \frac{-\epsilon_L}{\epsilon_a} = \nu$$

Poisson's Ratio, ν

When a material is subjected to a tensile strain, there is a simultaneous shortening of the cross-sectional dimensions perpendicular to the direction of the tensile strain. The ratio of the shortening strain to the tensile strain is called *Poisson's ratio*, usually denoted by ν, the Greek letter nu. (The Greek letter mu, μ, is sometimes used for this ratio.) Poisson's ratio is illustrated in Figure 2–4. Typical ranges of values for Poisson's ratio are 0.25–0.27 for cast iron, 0.27–0.30 for steel, and 0.30–0.33 for aluminum and titanium.

Modulus of Elasticity in Shear, *G*

The *modulus of elasticity in shear* (G) is the ratio of shearing stress to shearing strain. This property indicates a material's stiffness under shear loading—that is, the resistance to shear deformation. There is a simple relationship between E, G, and Poisson's ratio:

Modulus of Elasticity in Shear

$$G = \frac{E}{2(1 + \nu)} \tag{2–5}$$

This equation is valid within the elastic range of the material.

Flexural Modulus

Another stiffness measure often reported, particularly for plastics, is called the *flexural modulus*, or *modulus of elasticity in flexure*. As the name implies, a specimen of the material is loaded as a beam in flexure (bending) with data taken and plotted for load versus deflection. From these data and from knowledge of the geometry of the specimen, stress and strain can be computed. The ratio of stress to strain is a measure of the flexural modulus. ASTM standard D 790[1] defines the complete method. Note that the values are significantly different from the tensile modulus because the stress pattern in the specimen is a combination of tension and compression. The data are useful for comparing the stiffness of different materials when a load-carrying part is subjected to bending in service.

Hardness

The resistance of a material to indentation by a penetrator is an indication of its *hardness*. Several types of devices, procedures, and penetrators measure hardness; the Brinell hardness tester and the Rockwell hardness tester are most frequently used for machine elements. For steels, the Brinell hardness tester employs a hardened steel ball 10 mm in diameter as the penetrator under a load of 3000-kg force. The load causes a permanent indentation in the test material, and the diameter of the indentation is related to the Brinell hardness number, which is abbreviated BHN or HB. The actual quantity being measured is the load divided by the contact area of the indentation. For steels, the value of HB ranges from approximately 100 for an annealed, low-carbon steel to more than 700 for high-strength, high-alloy steels in the as-quenched condition. In the high ranges, above HB 500, the penetrator is sometimes made of tungsten carbide rather than steel. For softer metals, a 500-kg load is used.

The Rockwell hardness tester uses a hardened steel ball with a 1/16-in diameter under a load of 100-kg force for softer metals, and the resulting hardness is listed as Rockwell B, R_B, or HRB. For harder metals, such as heat-treated alloy steels, the Rockwell C scale is used. A load of 150-kg force is placed on a diamond penetrator (a *brale* penetrator) made in a sphero-conical shape. Rockwell C hardness is sometimes referred to as R_C or HRC. Many other Rockwell scales are used.

The Brinell and Rockwell methods are based on different parameters and lead to quite different numbers. However, since they both measure hardness, there is a correlation between them, as noted in Appendix 19. It is also important to note that, especially for highly hardenable alloy steels, there is a nearly linear relationship between the Brinell hardness number and the tensile strength of the steel, according to the equation

Approximate Relationship between Hardness and Strength for Steel

$$0.50(\text{HB}) = \text{approximate tensile strength (ksi)} \tag{2–6}$$

This relationship is shown in Figure 2–5.

To compare the hardness scales with the tensile strength, consider Table 2–1. Note that there is some overlap between the HRB and HRC scales. Normally, HRB is used for the softer metals and ranges from approximately 60 to 100, whereas HRC is used for harder metals and ranges from 20 to 65. Using HRB numbers above 100 or HRC numbers below 20 is not recommended. Those shown in Table 2–1 are for comparison purposes only.

Hardness in a steel indicates wear resistance as well as strength. Wear resistance will be discussed in later chapters, particularly with regard to gear teeth.

[1] ASTM International. *Standard Test Method for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials, Standard D790*. West Conshohocken, PA: ASTM International, 2003.

FIGURE 2–5
Hardness conversions

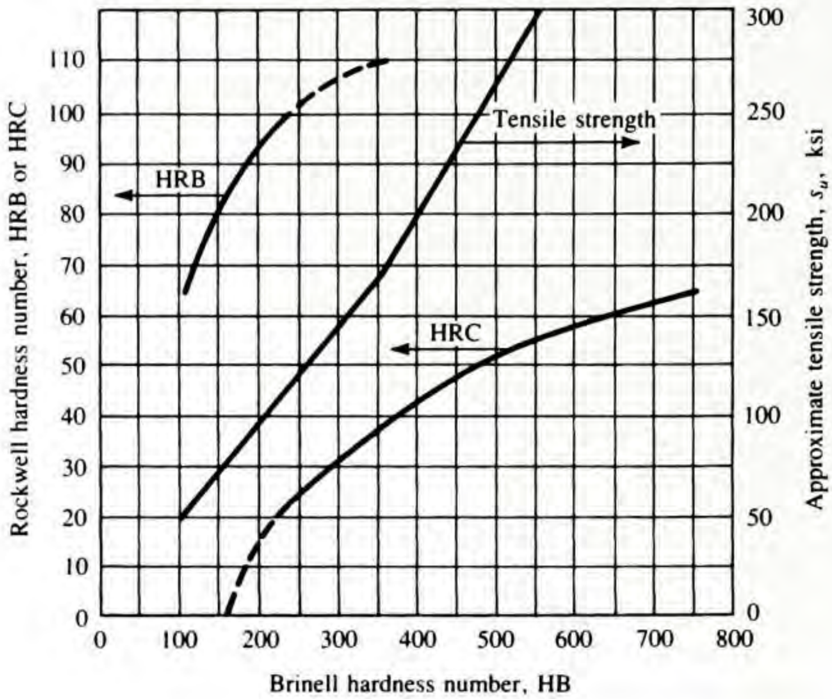


TABLE 2–1 Comparison of hardness scales with tensile strength

Material and condition	Hardness			Tensile strength	
	HB	HRB	HRC	ksi	MPa
1020 annealed	121	70		60	414
1040 hot-rolled	144	79		72	496
4140 annealed	197	93	13	95	655
4140 OQT 1000	341	109	37	168	1160
4140 OQT 700	461		49	231	1590

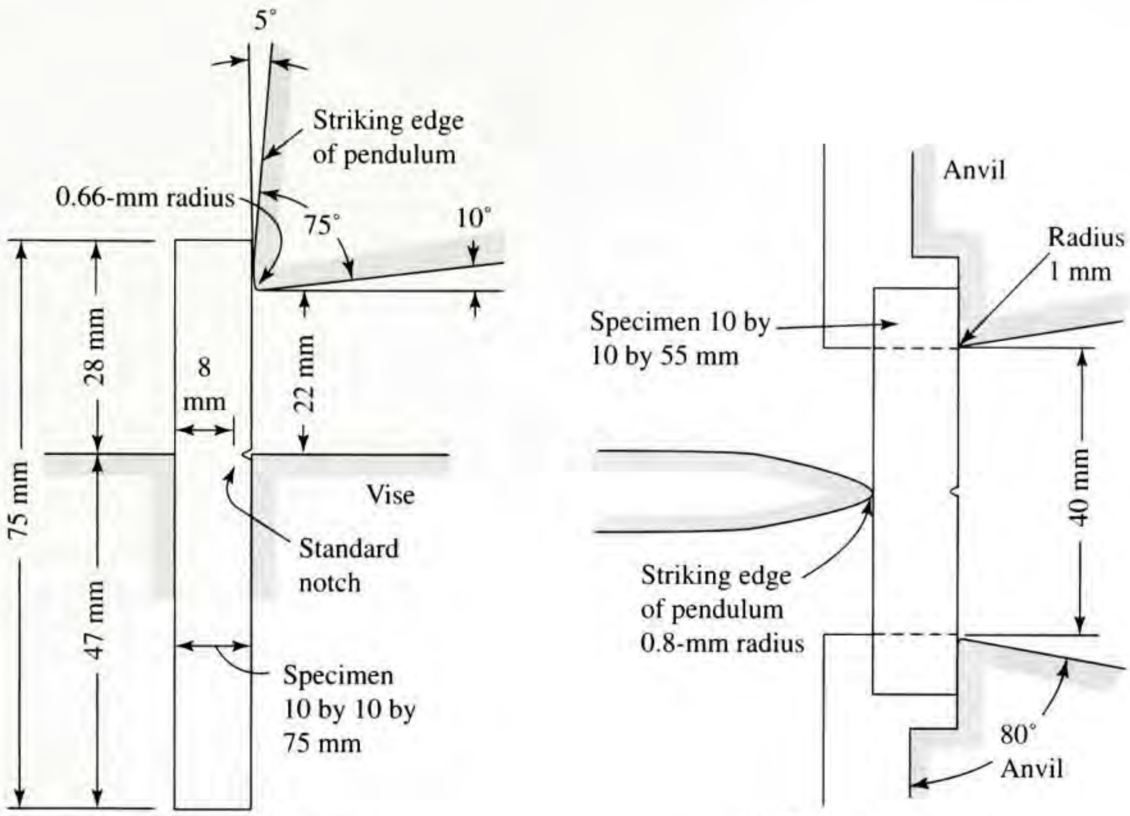
Machinability

Machinability is related to the ease with which a material can be machined to a good surface finish with reasonable tool life. Production rates are directly affected by machinability. It is difficult to define measurable properties related to machinability, so machinability is usually reported in comparative terms, relating the performance of a given material with some standard.

Toughness, Impact Energy

Toughness is the ability of a material to absorb applied energy without failure. Parts subjected to suddenly applied loads, shock, or impact need a high level of toughness. Several methods are used to measure the amount of energy required to break a particular specimen made from a material of interest. The energy absorption value from such tests is often called *impact energy* or *impact resistance*. However, it is important to note that the actual value is highly dependent on the nature of the test sample, particularly its geometry. It is not possible to use the test results in a quantitative way when making design calculations. Rather, the impact energy for several candidate materials for a particular application can

FIGURE 2–6 Impact testing using Charpy and Izod methods



(*a*) Izod (side view) (*b*) Charpy (top view)

be compared with each other as a qualitative indication of their toughness. The final design should be tested under real service conditions to verify its ability to survive safely during expected use.

For metals and plastics, two methods of determining impact energy, *Izod* and *Charpy*, are popular, with data often reported in the literature from vendors of the material. Figure 2–6 shows sketches of the dimensions of standard specimens and the manner of loading. In each method, a pendulum with a heavy mass carrying a specially designed striker is allowed to fall from a known height. The striker contacts the specimen with a high velocity at the bottom of the pendulum's arc; therefore, the pendulum possesses a known amount of kinetic energy. The specimen is typically broken during the test, taking some of the energy from the pendulum but allowing it to pass through the test area. The testing machine is configured to measure the final height to which the pendulum swings and to indicate the amount of energy removed. That value is reported in energy units of J (Joules or N · m) or ft · lb. Some highly ductile metals and many plastics do not break during the test, and the result is then reported as *No Break*.

The standard *Izod* test employs a square specimen with a V-shaped notch carefully machined 2.0 mm (0.079 in) deep according to specifications in ASTM standard D 256.[2] The specimen is clamped in a special vise with the notch aligned with the top edge of the vise. The striker contacts the specimen at a height of 22 mm above the notch, loading it as a cantilever in bending. When used for plastics, the width dimension can be different from that shown in Figure 2–6. This obviously changes the total amount of energy that the specimen will absorb during fracture. Therefore, the data for impact energy are divided by the

[2] ASTM International. *Standard Test Methods for Determining the Izod Pendulum Impact Resistance of Plastics, Standard D256.* West Conshohocken, PA: ASTM International, 2003.

actual width of the specimen, and the results are reported in units of N·m/m or ft·lb/in. Also, some vendors and customers may agree to test the material with the notch facing away from the striker rather than toward it as shown in Figure 2–6. This gives a measure of the material's impact energy with less influence from the notch.

The *Charpy* test also uses a square specimen with a 2.0 mm (0.079 in) deep notch, but it is centered along the length. The specimen is placed against a rigid anvil without being clamped. See ASTM standard A 370[3] for the specific geometry and testing procedure. The notch faces away from the place where the striker contacts the specimen. The loading can be described as the bending of a simply supported beam. The Charpy test is most often used for testing metals.

Another impact testing method used for some plastics, composites, and completed products is the *drop-weight* tester. Here a known mass is elevated vertically above the test specimen to a specified height. Thus, it has a known amount of potential energy. Allowing the mass to fall freely imparts a predictable amount of kinetic energy to the specimen clamped to a rigid base. The initial energy, the manner of support, the specimen geometry, and the shape of the striker (called a *tup*) are critical to the results found. One standard method, described in ASTM D 3763[4], employs a spherical tup with a diameter of 12.7 mm (0.50 in). The tup usually pierces the specimen. The apparatus is typically equipped with sensors that measure and plot the load versus deflection characteristics dynamically, giving the designer much information about how the material behaves during an impact event. Summary data reported typically include maximum load, deflection of the specimen at the point of maximum load, and the energy dissipated up to the maximum load point. The energy is calculated by determining the area under the load-deflection diagram. The appearance of the test specimen is also described, indicating whether fracture occurred and whether it was a ductile or brittle fracture.

Fatigue Strength or Endurance Strength

Parts subjected to repeated applications of loads or to stress conditions that vary with time over several thousands or millions of cycles fail because of the phenomenon of *fatigue*. Materials are tested under controlled cyclic loading to determine their ability to resist such repeated loads. The resulting data are reported as the *fatigue strength*, also called the *endurance strength* of the material. (See Chapter 5.)

Creep

When materials are subjected to high loads continuously, they may experience progressive elongation over time. This phenomenon, called *creep*, should be considered for metals operating at high temperatures. You should check for creep when the operating temperature of a loaded metal member exceeds approximately 0.3 (T_m) where T_m is the melting temperature expressed as an absolute temperature. (See Reference 22.) Creep can be important for critical members in internal combustion engines, furnaces, steam turbines, gas turbines, nuclear reactors, or rocket engines. The stress can be tension, compression, flexure, or shear. (See Reference 8.)

Figure 2–7 shows the typical behavior of metals that creep. The vertical axis is the creep strain, in units such as in/in or mm/mm, over that which occurs initially as the load

[3] ASTM International. *Standard Test Methods and Definitions for Mechanical Testing of Steel Products, Standard A370*. West Conshohocken, PA: ASTM International, 2003.

[4] ASTM International. *Standard Test Methods for High Speed Puncture of Plastics Using Load and Displacement Sensors, Standard D3763*. West Conshohocken, PA: ASTM International, 2003.

FIGURE 2–7
Typical creep behavior

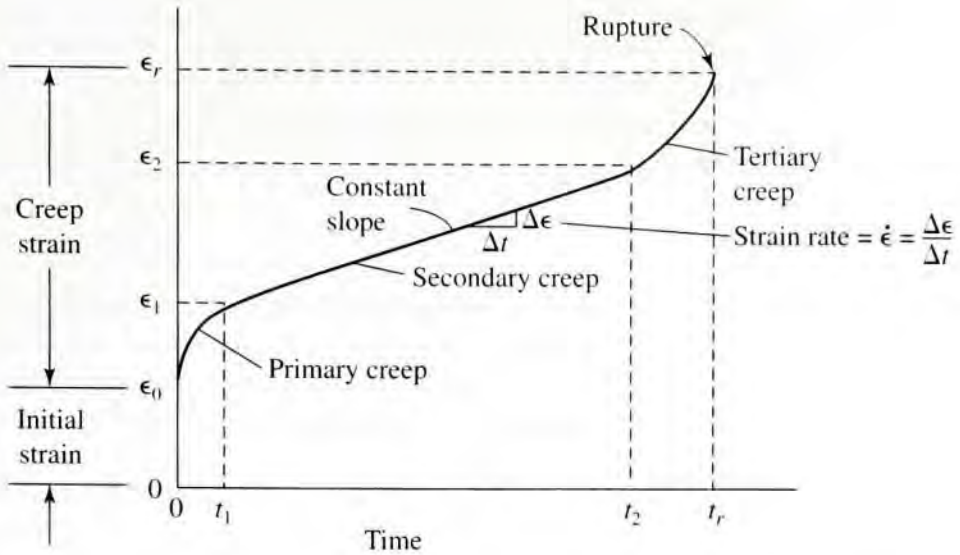
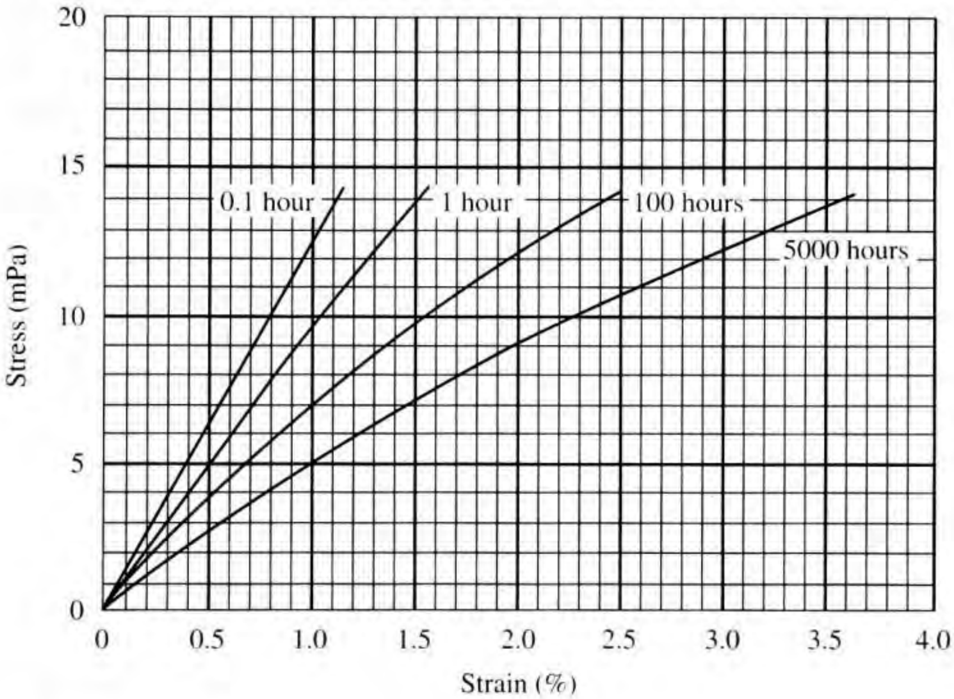


FIGURE 2–8
Example of stress versus strain as a function of time for nylon 66 plastic at 23°C (73°F) (DuPont Polymers, Wilmington, DE)



is applied. The horizontal axis is time, typically measured in hours because creep develops slowly over a long term. During the primary portion of the creep strain versus time curve, the rate of increase in strain initially rises with a rather steep slope that then decreases. The slope is constant (straight line) during the secondary portion of the curve. Then the slope increases in the tertiary portion that precedes the ultimate fracture of the material.

Creep is measured by subjecting a specimen to a known steady load, possibly through application of a dead weight, while the specimen is heated and maintained at a uniform temperature. Data for strain versus time are taken at least into the secondary creep stage and possibly all the way to fracture to determine the creep rupture strain. Testing over a range of temperatures gives a family of curves that are useful for design.

Creep can occur for many plastics even at or near room temperature. Figure 2–8 shows one way that creep data are displayed for plastic materials. (See Reference 8.) It is a graph of applied stress versus strain in the member with data shown for a specific temperature of the specimen. The curves show the amount of strain that would be developed within the specified times at increasing stress levels. For example, if this material were subjected to a constant stress of 5.0 MPa for 5000 hours, the total strain would be 1.0%. That is, the specimen would elongate by an amount 0.01 times the original length. If the stress were 10.0 MPa for 5000 hours, the total strain would be approximately 2.25%. The designer must take this creep strain into account to ensure that the product performs satisfactorily over time.

Example Problem 2–1 A solid circular bar has a diameter of 5.0 mm and a length of 250 mm. It is made from nylon 66 plastic and subjected to a steady tensile load of 240 N. Compute the elongation of the bar immediately after the load is applied and after 5000 hr (approximately seven months). See Appendix 13 and Figure 2–8 for properties of the nylon.

Solution The stress and deflection immediately after loading will first be computed using fundamental equations of strength of materials:

$$\sigma = F/A \text{ and } \delta = FL/EA$$

See Chapter 3 for a review of strength of materials.

Then creep data from Figure 2–8 will be applied to determine the elongation after 5000 hr.

Results *Stress:*

The cross-sectional area of the bar is

$$A = \pi D^2/4 = \pi(5.0 \text{ mm})^2/4 = 19.63 \text{ mm}^2$$

$$\sigma = \frac{F}{A} = \frac{240 \text{ N}}{19.63 \text{ mm}^2} = 12.2 \text{ N/mm}^2 = 12.2 \text{ MPa}$$

Appendix 13 lists the tensile strength for nylon 66 to be 83 MPa. Therefore, the rod is safe from fracture.

Elongation:

The tensile modulus of elasticity for nylon 66 is found from Appendix 13 to be $E = 2900$ MPa. Then the initial elongation is,

$$\delta = \frac{FL}{EA} = \frac{(240 \text{ N})(250 \text{ mm})}{(2900 \text{ N/mm}^2)(19.63 \text{ mm}^2)} = 1.054 \text{ mm}$$

Creep:

Referring to Figure 2–8 we find that when a tensile stress of 12.2 MPa is applied to the nylon 66 plastic for 5000 hr, a total strain of approximately 2.95% occurs. This can be expressed as

$$\epsilon = 2.95\% = 0.0295 \text{ mm/mm} = \delta/L$$

Then,

$$\delta = \epsilon L = (0.0295 \text{ mm/mm})(250\text{mm}) = 7.375 \text{ mm}$$

Comment This is approximately seven times as much deformation as originally experienced when the load was applied. So designing with the reported value of modulus of elasticity is not appropriate when stresses are applied continuously for a long time. We can now compute an apparent modulus of elasticity, E_{app}, for this material at the 5000 hr service life.

$$E_{app} = \sigma/\epsilon = (12.2 \text{ MPa})/(0.0295 \text{ mm/mm}) = 414 \text{ MPa}$$

Relaxation

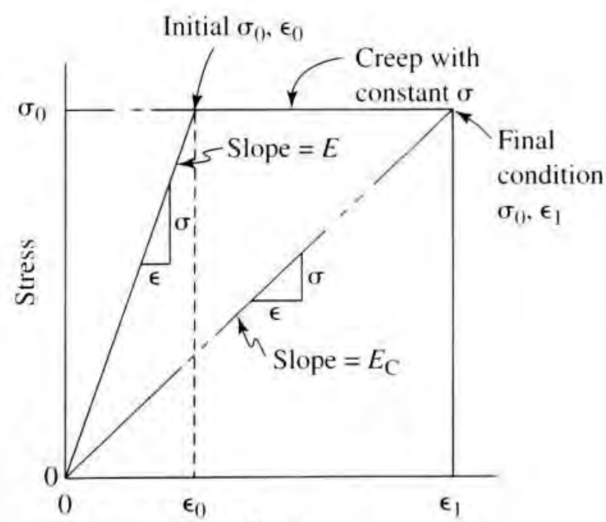
A phenomenon related to creep occurs when a member under stress is captured under load, giving it a certain fixed length and a fixed strain. Over time, the stress in the member would decrease, exhibiting a behavior called *relaxation*. This is important in such applications as clamped joints, press-fit parts, and springs installed with a fixed deflection. Figure 2–9 shows the comparison between creep and relaxation. For stresses below approximately 1/3 of the ultimate tensile strength of the material at any temperature, the apparent modulus in either creep or relaxation at any time of loading may be considered similar for engineering purposes. Furthermore, values for apparent modulus are the same for tension, compression, or flexure. (See Reference 8.) Analysis of relaxation is complicated by the fact that as the stress decreases, the rate of creep also decreases. Additional material data beyond that typically reported would be required to accurately predict the amount of relaxation at any given time. Testing under realistic conditions is recommended.

Physical Properties

Here we will discuss density, coefficient of thermal expansion, thermal conductivity, and electrical resistivity.

Density. *Density* is defined as the mass per unit volume of a material. Its usual units are kg/m^3 in the SI and lb/in^3 in the U.S. Customary Unit System, where the pound unit is taken to be pounds-mass. The Greek letter rho (ρ) is the symbol for density.

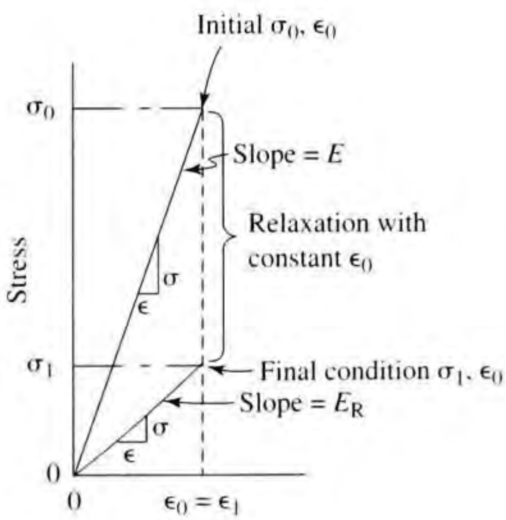
FIGURE 2–9
Comparison of creep and relaxation (DuPont Polymers, Wilmington, DE)



$E = \dfrac{\sigma_0}{\epsilon_0}$ = Tensile modulus

$E_C = \dfrac{\sigma_0}{\epsilon_1}$ = Creep modulus

(*a*) Creep behavior



$E = \dfrac{\sigma_0}{\epsilon_0}$ = Tensile modulus

$E_R = \dfrac{\sigma_1}{\epsilon_0}$ = Relaxation modulus

(*b*) Relaxation behavior

In some applications, the term *specific weight* or *weight density* is used to indicate the weight per unit volume of a material. Typical units are N/m^3 in the SI and lb/in^3 in the U.S. Customary Unit System, where the pound is taken to be pounds-force. The Greek letter gamma (γ) is the symbol for specific weight.

Coefficient of Thermal Expansion. The *coefficient of thermal expansion* is a measure of the change in length of a material subjected to a change in temperature. It is defined by the relation

Coefficient of Thermal Expansion

$$\alpha = \frac{\text{change in length}}{L_o\,(\Delta T)} = \frac{\text{strain}}{(\Delta T)} = \frac{\epsilon}{(\Delta T)} \tag{2–7}$$

where L_o = original length
ΔT = change in temperature

Virtually all metals and plastics expand with increasing temperature, but different materials expand at different rates. For machines and structures containing parts of more than one material, the different rates can have a significant effect on the performance of the assembly and on the stresses produced.

Thermal Conductivity. *Thermal conductivity* is the property of a material that indicates its ability to transfer heat. Where machine elements operate in hot environments or where significant internal heat is generated, the ability of the elements or of the machine's housing to transfer heat away can affect machine performance. For example, wormgear speed reducers typically generate frictional heat due to the rubbing contact between the worm and the wormgear teeth. If not adequately transferred, heat causes the lubricant to lose its effectiveness, allowing rapid gear-tooth wear.

Electrical Resistivity. For machine elements that conduct electricity while carrying loads, the electrical resistivity of the material is as important as its strength. *Electrical resistivity* is a measure of the resistance offered by a given thickness of a material; it is measured in ohm-centimeters ($\Omega \cdot \text{cm}$). *Electrical conductivity*, a measure of the capacity of a material to conduct electric current, is sometimes used instead of resistivity. It is often reported as a percentage of the conductivity of a reference material, usually the International Annealed Copper Standard.

2–3 CLASSIFICATION OF METALS AND ALLOYS

Various industry associations take responsibility for setting standards for the classification of metals and alloys. Each has its own numbering system, convenient to the particular metal covered by the standard. But this leads to confusion at times when there is overlap between two or more standards and when widely different schemes are used to denote the metals. Order has been brought to the classification of metals by the use of the Unified Numbering Systems (UNS) as defined in the Standard E 527-83 (Reapproved 1997), *Standard Practice for Numbering Metals and Alloys (UNS)*, by the American Society for Testing and Materials, or ASTM. (See References 12, 13) Besides listing materials under the control of ASTM itself, the UNS coordinates designations of the following:

The Aluminum Association (AA)

The American Iron and Steel Institute (AISI)

The Copper Development Association (CDA)

The Society of Automotive Engineers (SAE)

TABLE 2–2 Unified numbering system (UNS)

Number series	Types of metals and alloys	Responsible organization
Nonferrous metals and alloys		
A00001–A99999	Aluminum and aluminum alloys	AA
C00001–C99999	Copper and copper alloys	CDA
E00001–E99999	Rare earth metals and alloys	ASTM
L00001–L99999	Low-melting metals and alloys	ASTM
M00001–M99999	Miscellaneous nonferrous metals and alloys	ASTM
N00001–N99999	Nickel and nickel alloys	SAE
P00001–P99999	Precious metals and alloys	ASTM
R00001–R99999	Reactive and refractory metals and alloys	SAE
Z00001–Z99999	Zinc and zinc alloys	ASTM
Ferrous metals and alloys		
D00001–D99999	Steels; mechanical properties specified	SAE
F00001–F99999	Cast irons and cast steels	ASTM
G00001–G99999	Carbon and alloy steels (includes former SAE carbon and alloy steels)	AISI
H00001–H99999	H-steels; specified hardenability	AISI
J00001–J99999	Cast steels (except tool steels)	ASTM
K00001–K99999	Miscellaneous steels and ferrous alloys	ASTM
S00001–S99999	Heat- and corrosion-resistant (stainless) steels	ASTM
T00001–T99999	Tool steels	AISI

The primary series of numbers within UNS are listed in Table 2–2, along with the organization having responsibility for assigning numbers within each series.

Many alloys within the UNS retain the familiar numbers from the systems used for many years by the various associations as a *part* of the UNS number. Examples are shown in Section 2–5 for carbon and alloy steel. Also, the former designations remain widely used. For these reasons, this book will use the four-digit designation system of the AISI as described in Section 2–5 for most machine steels. Many of the designations of the SAE use the same four numbers. We will also use the designation systems of the ASTM when referring to structural steels and cast irons.

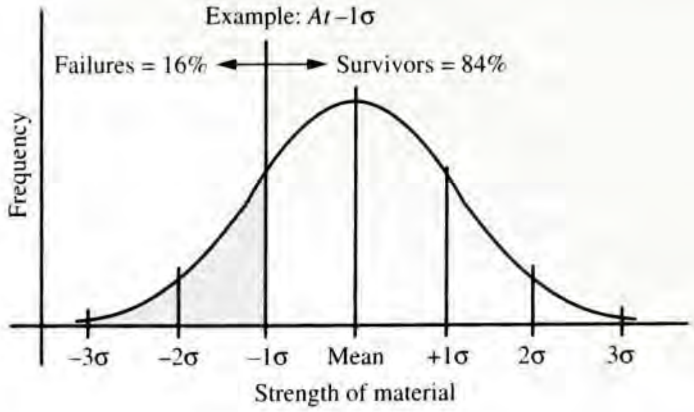
2–4 VARIABILITY OF MATERIAL PROPERTIES DATA

Tables of data such as those shown in Appendices 3 through 13 normally report single values for the strength, the modulus of elasticity (stiffness), or the percent elongation (ductility) of a particular material at a particular condition created by heat treatment or by the manner in which it was formed. It is important for you to understand the limitations of such data in making design decisions. You should seek information about the bases for the reported data.

Some tables of data report *guaranteed minimum values* for tensile strength, yield strength, and other values. This might be the case when you are using data obtained from a particular supplier. With such data, you should feel confident that the material that actually goes into your product has at least the reported strength. The supplier should be able to provide actual test data and statistical analyses used to determine the reported minimum strengths. Alternatively, you could arrange to have the actual materials to be used in a project tested to determine their minimum strength values. Such tests are costly, but they may be justified in critical designs.

Other tables of data report *typical values* for material properties. Thus, most batches of material (greater than 50%) delivered will have the stated values or greater. However,

FIGURE 2–10 Normal statistical distribution of material strength



Assuming normal distribution of strength:

Stress level	% Surviving
Mean	50%
-1σ	84%
-2σ	97%
-3σ	99.8%
-4σ	99.99%

about 50% will have lower values, and this fact will affect your confidence in specifying a particular material and heat treatment if strength is critical. In such cases, you are advised to use higher than average design factors in your calculations of allowable (design) strength. (See Chapter 5.)

Using the guaranteed minimum values for strength in design decisions would be the safest approach. However, it is very conservative because most of the material actually delivered would have strengths significantly greater than the listed values.

One way to make the design more favorable is to acquire data for the statistical distribution of strength values taken for many samples. Then applications of probability theories can be used to specify suitable conditions for the material with a reasonable degree of confidence that the parts will perform according to specifications. Figure 2–10 illustrates some of the basic concepts of statistical distribution. The variation of strength over the entire population of samples is often assumed to have a normal distribution around some mean or average value. If you used a strength value that is one standard deviation (1σ) below the mean, 84% of the products would survive. At two standard deviations, greater than 97% would survive; at three standard deviations, more than 99.8%; and at four standard deviations, more than 99.99%.

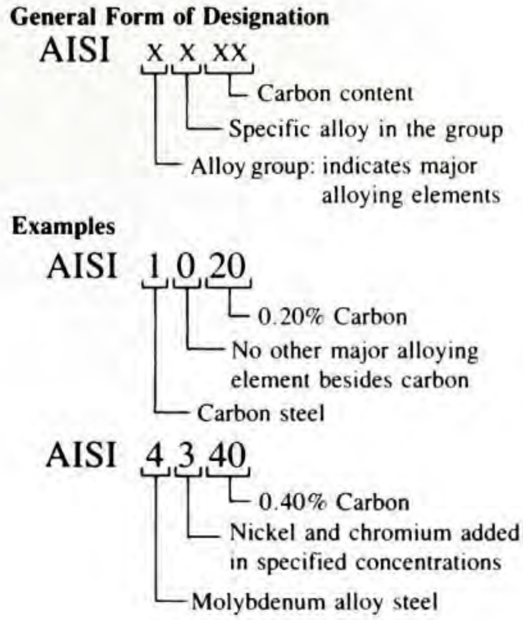
As the designer, you must carefully judge the reliability of the data that you use. Ultimately, you should evaluate the reliability of the final product by considering the actual variations in material properties, the manufacturing considerations that may affect performance, and the interactions of various components with each other. There is more discussion on this point in Chapter 5.

2–5 CARBON AND ALLOY STEEL

Steel is possibly the most widely used material for machine elements because of its properties of high strength, high stiffness, durability, and relative ease of fabrication. Many types of steels are available. This section will discuss the methods used for designating steels and will describe the most frequently used types.

The term *steel* refers to an alloy of iron, carbon, manganese, and one or more other significant elements. Carbon has a very strong effect on the strength, hardness, and ductility of any steel alloy. The other elements affect hardenability, toughness, corrosion resistance, machinability, and strength retention at high temperatures. The primary alloying

FIGURE 2–11 Steel designation system



elements present in the various alloy steels are sulfur, phosphorus, silicon, nickel, chromium, molybdenum, and vanadium.

Designation Systems

The AISI uses a four-digit designation system for carbon and alloy steel as shown in Figure 2–11. The first two digits indicate the specific alloy group that identifies the primary alloying elements other than carbon in the steel. (See Table 2–3.) The last two digits indicate the amount of carbon in the steel as described next.

Importance of Carbon

Although most steel alloys contain less than 1.0% carbon, it is included in the designation because of its effect on the properties of steel. As Figure 2–11 illustrates, the last two digits indicate carbon content in hundredths of a percent. For example, when the last two digits are 20, the alloy includes approximately 0.20% carbon. Some variation is allowed. The carbon content in a steel with *20 points of carbon* ranges from 0.18% to 0.23%.

As carbon content increases, strength and hardness also increase under the same conditions of processing and heat treatment. Since ductility decreases with increasing carbon content, selecting a suitable steel involves some compromise between strength and ductility.

As a rough classification scheme, a *low-carbon steel* is one having fewer than 30 points of carbon (0.30%). These steels have relatively low strength but good formability. In machine element applications where high strength is not required, low-carbon steels are frequently specified. If wear is a potential problem, low-carbon steels can be carburized (as discussed in Section 2–6) to increase the carbon content in the very outer surface of the part and to improve the combination of properties.

Medium-carbon steels contain 30 to 50 points of carbon (0.30%–0.50%). Most machine elements having moderate to high strength requirements with fairly good ductility and moderate hardness requirements come from this group.

High-carbon steels have 50 to 95 points of carbon (0.50%–0.95%). The high carbon content provides better wear properties suitable for applications requiring durable cutting edges and for applications where surfaces are subjected to constant abrasion. Tools, knives, chisels, and many agricultural implement components are among these uses.

TABLE 2–3 Alloy groups in the AISI numbering system

10xx	Plain carbon steel: No significant alloying element except carbon and manganese; less than 1.0% manganese. Also called *nonresulfurized.*
11xx	Free-cutting steel: Resulfurized. Sulfur content (typically 0.10%) improves machinability.
12xx	Free-cutting steel: Resulfurized and rephosphorized. Presence of increased sulfur and phosphorus improves machinability and surface finish.
12Lxx	Free-cutting steel: Lead added to 12xx steel further improves machinability.
13xx	Manganese steel: Nonresulfurized. Presence of approximately 1.75% manganese increases hardenability.
15xx	Carbon steel: Nonresulfurized; greater than 1.0% manganese.
23xx	Nickel steel: Nominally 3.5% nickel.
25xx	Nickel steel: Nominally 5.0% nickel.
31xx	Nickel-chromium steel: Nominally 1.25% Ni; 0.65% Cr.
33xx	Nickel-chromium steel: Nominally 3.5% Ni; 1.5% Cr.
40xx	Molybdenum steel: 0.25% Mo.
41xx	Chromium-molybdenum steel: 0.95% Cr; 0.2% Mo.
43xx	Nickel-chromium-molybdenum steel: 1.8% Ni; 0.5% or 0.8% Cr; 0.25% Mo.
44xx	Molybdenum steel: 0.5% Mo.
46xx	Nickel-molybdenum steel: 1.8% Ni; 0.25% Mo.
48xx	Nickel-molybdenum steel: 3.5% Ni; 0.25% Mo.
5xxx	Chromium steel: 0.4% Cr.
51xx	Chromium steel: Nominally 0.8% Cr.
51100	Chromium steel: Nominally 1.0% Cr; bearing steel, 1.0% C.
52100	Chromium steel: Nominally 1.45% Cr; bearing steel, 1.0% C.
61xx	Chromium-vanadium steel: 0.50%–1.10% Cr; 0.15% V.
86xx	Nickel-chromium-molybdenum steel: 0.55% Ni; 0.5% Cr; 0.20% Mo.
87xx	Nickel-chromium-molybdenum steel: 0.55% Ni; 0.5% Cr; 0.25% Mo.
92xx	Silicon steel: 2.0% silicon.
93xx	Nickel-chromium-molybdenum steel: 3.25% Ni; 1.2% Cr; 0.12% Mo.

A *bearing steel* nominally contains 1.0% carbon. Common grades are 50100, 51100, and 52100; the usual four-digit designation is replaced by five digits, indicating 100 points of carbon.

Alloy Groups

As indicated in Table 2–3, sulfur, phosphorus, and lead improve the machinability of steels and are added in significant amounts to the 11xx, 12xx, and 12Lxx grades. These grades are used for screw machine parts requiring high production rates where the resulting parts are not subjected to high stresses or wear conditions. In the other alloys, these elements are controlled to a very low level because of their adverse effects, such as increased brittleness.

Nickel improves the toughness, hardenability, and corrosion resistance of steel and is included in most of the alloy steels. Chromium improves hardenability, wear and abrasion resistance, and strength at elevated temperatures. In high concentrations, chromium provides significant corrosion resistance, as discussed in the section on stainless steels. Molybdenum also improves hardenability and high-temperature strength.

The steel selected for a particular application must be economical and must provide optimum properties of strength, ductility, toughness, machinability, and formability. Frequently, metallurgists, manufacturing engineers, and heat treatment specialists are consulted. (See also References 4, 14, 16, and 24.)

Table 2–4 lists some common steels used for machine parts, with typical applications listed for the alloys. You should benefit from the decisions of experienced designers when specifying materials.

TABLE 2–4 Uses of some steels

UNS number	AISI number	Applications
G10150	1015	Formed sheet-metal parts; machined parts (may be carburized)
G10300	1030	General-purpose, bar-shaped parts, levers, links, keys
G10400	1040	Shafts, gears
G10800	1080	Springs; agricultural equipment parts subjected to abrasion (rake teeth, disks, plowshares, mower teeth)
G11120	1112	Screw machine parts
G12144	12L14	Parts requiring good machinability
G41400	4140	Gears, shafts, forgings
G43400	4340	Gears, shafts, parts requiring good through-hardening
G46400	4640	Gears, shafts, cams
G51500	5150	Heavy-duty shafts, springs, gears
G51601	51B60	Shafts, springs, gears with improved hardenability
G52986	E52100	Bearing races, balls, rollers (bearing steel)
G61500	6150	Gears, forgings, shafts, springs
G86500	8650	Gears, shafts
G92600	9260	Springs

Examples of the Relationships between AISI and UNS Numbering Systems

Table 2–4 presents both the AISI and the UNS designations for the listed steels. Notice that for most carbon and alloy steels, the four-digit AISI number becomes the first four digits of the UNS number. The final digit in the UNS number is typically zero.

There are some exceptions, however. For high-carbon-bearing steels made in an electric furnace, such as AISI E52100, the UNS designation is G52986. Leaded steels have extra lead to improve machinability, and they have the letter L added between the second and third digits of the AISI number such as AISI 12L14, which becomes UNS G12144. Extra boron is added to some special alloys to improve hardenability. For example, alloy AISI 5160 is a chromium steel that carries the UNS designation G51600. But a similar alloy with added boron is AISI 51B60, and it carries the UNS designation G51601.

2–6 CONDITIONS FOR STEELS AND HEAT TREATMENT

The final properties of steels are dramatically affected by the way the steels are produced. Some processes involve mechanical working, such as rolling to a particular shape or drawing through dies. In machine design, many bar-shaped parts, shafts, wire, and structural members are produced in these ways. But most machine parts, particularly those carrying heavy loads, are heat-treated to produce high strength with acceptable toughness and ductility.

Carbon steel bar and sheet forms are usually delivered in the *as-rolled condition;* that is, they are rolled at an elevated temperature that eases the rolling process. The rolling can also be done cold to improve strength and surface finish. Cold-drawn bar and wire have the highest strength of the worked forms, along with a very good surface finish. However, when a material is designated to be *as-rolled*, it should be assumed that it was hot-rolled.

Heat Treating

Heat treating is any process in which steel is subjected to elevated temperatures to modify its properties. Of the several processes available, those most used for machine steels are annealing, normalizing, through-hardening (quench and temper), and case hardening. (See References 3 and 15.)

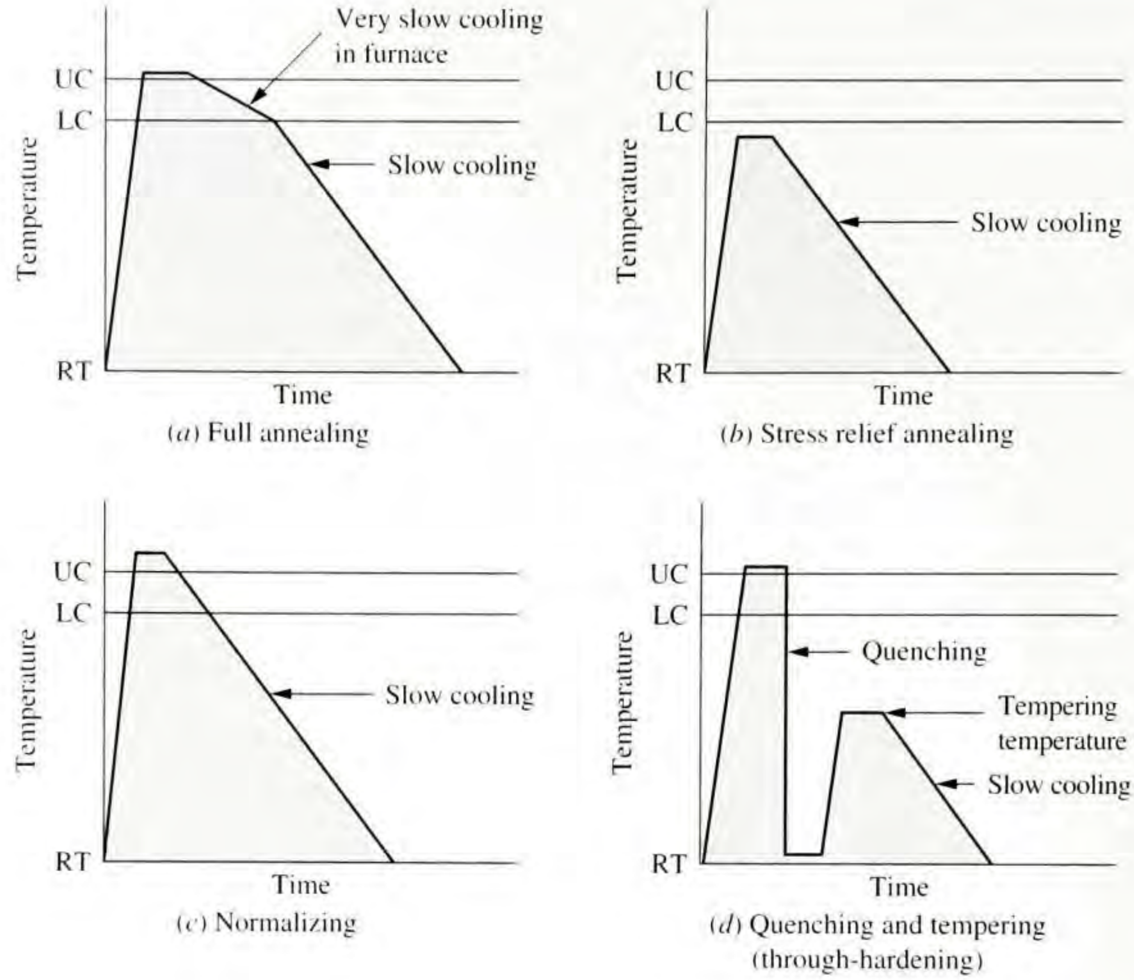


FIGURE 2–12 Heat treatments for steel

Note:
RT = room temperature
LC = lower critical temperature
UC = upper critical temperature

Figure 2–12 shows the temperature–time cycles for these heat treatment processes. The symbol RT indicates normal room temperature, and LC refers to the lower critical temperature at which the transformation of ferrite to austenite begins during the heating of the steel. At the upper critical temperature (UC), the transformation is complete. These temperatures vary with the composition of the steel. For most medium-carbon (0.30%–0.50% carbon) steels, UC is approximately 1500°F (822°C). References giving detailed heat treatment process data should be consulted.

Annealing. *Full annealing* [Figure 2–12(a)] is performed by heating the steel above the upper critical temperature and holding it until the composition is uniform. Then the steel is cooled very slowly in the furnace to below the lower critical temperature. Slow cooling to room temperature outside the furnace completes the process. This treatment produces a soft, low-strength form of the material, free of significant internal stresses. Parts are frequently cold-formed or machined in the annealed condition.

Stress relief annealing [Figure 2–12(b)] is often used following welding, machining, or cold forming to relieve residual stresses and thereby minimize subsequent distortion. The steel is heated to approximately 1000°F to 1200°F (540°C–650°C), held to achieve uniformity, and then slowly cooled in still air to room temperature.

Normalizing. *Normalizing* [Figure 2–12(c)] is performed in a similar manner to annealing, but at a higher temperature, above the transformation range where austenite is formed, approximately 1600°F (870°C). The result is a uniform internal structure in the steel and somewhat higher strength than annealing produces. Machinability and toughness are usually improved over the as-rolled condition.

Through-hardening and Quenching and Tempering. *Through-hardening* [Figure 2–12(d)] is accomplished by heating the steel to above the transformation range where austenite forms and then rapidly cooling it in a *quenching* medium. The rapid cooling causes the formation of martensite, the hard, strong form of steel. The degree to which martensite forms depends on the alloy's composition. An alloy containing a minimum of 80% of its structure in the martensite form over the entire cross section has *high hardenability*. This is an important property to look for when selecting a steel requiring high strength and hardness. The common quenching media are water, brine, and special mineral oils. The selection of a quenching medium depends on the rate at which cooling should proceed. Most machine steels use either oil or water quenching.

Tempering is usually performed immediately after quenching and involves reheating the steel to a temperature of 400°F to 1300°F (200°C–700°C) and then slowly cooling it in air back to room temperature. This process modifies the steel's properties: Tensile strength and yield strength decrease with increasing tempering temperature, whereas ductility improves, as indicated by an increase in the percent elongation. Thus, the designer can tailor the properties of the steel to meet specific requirements. Furthermore, the steel in its as-quenched condition has high internal stresses and is usually quite brittle. Machine parts should normally be tempered at 700°F (370°C) or higher after quenching.

To illustrate the effects of tempering on the properties of steels, several charts in Appendix 4 show graphs of strength versus tempering temperature. Included in these charts are tensile strength, yield point, percent elongation, percent reduction of area, and hardness number HB, all plotted in relation to tempering temperature. Note the difference in the shape of the curves and the absolute values of the strength and hardness when comparing the plain carbon AISI 1040 steel with the alloy steel AISI 4340. Although both have the same nominal carbon content, the alloy steel reaches a much higher strength and hardness. Note also the as-quenched hardness in the upper right part of the heading of the charts; it indicates the degree to which a given alloy can be hardened. When the case-hardening processes (described next) are used, the as-quenched hardness becomes very important.

Appendix 3 lists the range of properties that can be expected for several grades of carbon and alloy steels. The alloys are listed with their AISI numbers and conditions. For the heat-treated conditions, the designation reads, for example, AISI 4340 OQT 1000, which indicates that the alloy was oil-quenched and tempered at 1000°F. Expressing the properties at the 400°F and 1300°F tempering temperatures indicates the end-points of the possible range of properties that can be expected for that alloy. To specify a strength between these limits, you could refer to graphs such as those shown in Appendix 4, or you could determine the required heat treatment process from a specialist. For the purposes of material specification in this book, a rough interpolation between given values will be satisfactory. As noted before, you should seek more specific data for critical designs.

Case Hardening. In many cases, the bulk of the part requires only moderate strength although the surface must have a very high hardness. In gear teeth, for example, high surface hardness is necessary to resist wear as the mating teeth come into contact several million times during the expected life of the gears. At each contact, a high stress develops at the surface of the teeth. For applications such as this, *case hardening* is used; the surface

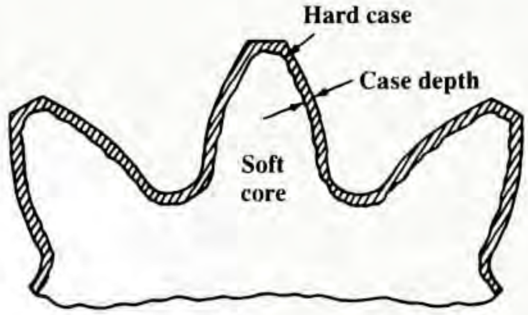


FIGURE 2–13
Typical case-hardened gear-tooth section

(or *case*) of the part is given a high hardness to a depth of perhaps 0.010 to 0.040 in (0.25–1.00 mm), although the interior of the part (the *core*) is affected only slightly, if at all. The advantage of surface hardening is that as the surface receives the required wear-resisting hardness, the core of the part remains in a more ductile form, resistant to impact and fatigue. The processes used most often for case hardening are flame hardening, induction hardening, carburizing, nitriding, cyaniding, and carbo-nitriding. (See Reference 17.)

Figure 2–13 shows a drawing of a typical case-hardened gear-tooth section, clearly showing the hard case surrounding the softer, more ductile core. Case hardening is used in applications requiring high wear and abrasion resistance in normal service (gear teeth, crane wheels, wire-rope sheaves, and heavy-duty shafts).

The most commonly used processes for case hardening are described in the following list.

1. ***Flame hardening and induction hardening:*** The processes of flame hardening and induction hardening involve the rapid heating of the surface of the part for a limited time so that a small, controlled depth of the material reaches the transformation range. Upon immediate quenching, only that part above the transformation range produces the high level of martensite required for high hardness.

   Flame hardening uses a concentrated flame impinging on a localized area for a controlled amount of time to heat the part, followed by quenching in a bath or by a stream of water or oil. *Induction hardening* is a process in which the part is surrounded by a coil through which high-frequency electric current is passed. Because of the electrical conductivity of the steel, current is *induced* primarily near the surface of the part. The resistance of the material to the flow of current results in a heating effect. Controlling the electrical power and the frequency of the induction system, and the time of exposure, determines the depth to which the material reaches the transformation temperature. Rapid quenching after heating hardens the surface. (See Reference 26.)

   Note that for flame or induction hardening to be effective, the material must have a good hardenability. Usually the goal of case hardening is to produce a case hardness in the range of Rockwell C hardness HRC 55 to 60 (Brinell hardness approximately HB 550 to 650). Therefore, the material must be capable of being hardened to the desired level. Carbon and alloy steels with fewer than 30 points of carbon typically cannot meet this requirement. Thus, the alloy steels with 40 points or more of carbon are the usual types given flame- or induction-hardening treatments.

2. ***Carburizing, nitriding, cyaniding, and carbo-nitriding:*** The remaining case-hardening processes—carburizing, nitriding, cyaniding, and carbo-nitriding—actually alter the composition of the surface of the material by exposing it to

carbon-bearing gases, liquids, or solids at high temperatures that produce carbon and diffuse it into the surface of the part. The concentration and the depth of penetration of carbon depend on the nature of the carbon-bearing material and the time of exposure. Nitriding and cyaniding typically result in very hard, thin cases that are good for general wear resistance. Where high load-carrying capability in addition to wear resistance is required, as with gear teeth, carburizing is preferred because of the thicker case.

Several steels are produced as carburizing grades. Among these are 1015, 1020, 1022, 1117, 1118, 4118, 4320, 4620, 4820, and 8620. Appendix 5 lists the expected properties of these carburized steels. Note when evaluating a material for use that the core properties determine its ability to withstand prevailing stresses, and the case hardness indicates its wear resistance. Carburizing, properly done, will virtually always produce a case hardness from HRC 55 to 64 (Rockwell C hardness) or from HB 550 to 700 (Brinell hardness).

Carburizing has several variations that allow the designer to tailor the properties to meet specific requirements. The exposure to the carbon atmosphere takes place at a temperature of approximately 1700°F (920°C) and usually takes 8 h. Immediate quenching achieves the highest strength, although the case is somewhat brittle. Normally, a part is allowed to cool slowly after carburizing. It is then reheated to approximately 1500°F (815°C) and then quenched. A tempering at the relatively low temperature of either 300°F or 450°F (150°C or 230°C) follows, to relieve stresses induced by quenching. As shown in Appendix 5, the higher tempering temperature lowers the core strength and the case hardness by a small amount, but in general it improves the part's toughness. The process just described is *single quenching and tempering*.

When a part is quenched in oil and tempered at 450°F, for example, the condition is *case hardening by carburizing, SOQT 450*. Reheating after the first quench and quenching again further refines the case and core properties; this process is *case hardening by carburizing, DOQT 450*. These conditions are listed in Appendix 5.

2–7 STAINLESS STEELS

The term *stainless steel* characterizes the high level of corrosion resistance offered by alloys in this group. To be classified as a stainless steel, the alloy must have a chromium content of at least 10%. Most have 12% to 18% chromium. (See Reference 5.)

The AISI designates most stainless steels by its 200, 300, and 400 series. As mentioned previously (Section 2–3), another designation system is the unified numbering system (UNS) developed by the SAE and the ASTM. Appendix 6 lists the properties of several grades, giving both designations.

The three main groups of stainless steels are austenitic, ferritic, and martensitic. *Austenitic* stainless steels fall into the AISI 200 and 300 series. They are general-purpose grades with moderate strength. Most are not heat-treatable, and their final properties are determined by the amount of working, with the resulting temper referred to as 1/4 hard, 1/2 hard, 3/4 hard, and full hard. These alloys are nonmagnetic and are typically used in food processing equipment.

Ferritic stainless steels belong to the AISI 400 series, designated as 405, 409, 430, 446, and so on. They are magnetic and perform well at elevated temperatures, from 1300°F to 1900°F (700°C–1040°C), depending on the alloy. They are not heat-treatable, but they can be cold-worked to improve properties. Typical applications include heat exchanger tubing, petroleum refining equipment, automotive trim, furnace parts, and chemical equipment.

Martensitic stainless steels are also members of the AISI 400 series, including 403, 410, 414, 416, 420, 431, and 440 types. They are magnetic, can be heat-treated, and have higher strength than the 200 and 300 series, while retaining good toughness. Typical uses include turbine engine parts, cutlery, scissors, pump parts, valve parts, surgical instruments, aircraft fittings, and marine hardware.

There are many other grades of stainless steels, many of which are proprietary to particular manufacturers. A group used for high-strength applications in aerospace, marine, and vehicular applications is of the precipitation-hardening type. They develop very high strengths with heat treatments at relatively low temperatures, from 900°F to 1150°F (480°C–620°C). This characteristic helps to minimize distortion during treatment. Some examples are 17-4PH, 15-5PH, 17-7PH, PH15-7Mo, and AMS362 stainless steels.

2–8 STRUCTURAL STEEL

Most structural steels are designated by ASTM numbers established by the American Society for Testing and Materials. One common grade is ASTM A36, which has a minimum yield point of 36 000 psi (248 MPa) and is very ductile. It is basically a low-carbon, hot-rolled steel available in sheet, plate, bar, and structural shapes such as some wide-flange beams, American Standard beams, channels, and angles. The geometric properties of some of each of these sections are listed in Appendix 16.

Most wide-flange beams (W-shapes) are currently made using ASTM A992 structural steel, which has a yield point of 50 to 65 ksi (345 to 448 MPa) and a minimum tensile strength of 65 ksi (448 MPa). An additional requirement is that the maximum ratio of the yield point to the tensile strength is 0.85. This is a highly ductile steel, having a minimum of 21% elongation in a 2.00-inch gage length. Using this steel instead of the lower strength ASTM A36 steel typically allows smaller, lighter structural members at little or no additional cost.

Hollow structural sections (HSS) are typically made from ASTM A500 steel that is cold-formed and either welded or made seamless. Included are round tubes and square and rectangular shapes. Note in Appendix 7 that there are different strength values for round tubes as compared with the shaped forms. Also, several strength grades can be specified. Some of these HSS products are made from ASTM A501 hot-formed steel having properties similar to the ASTM A36 hot-rolled steel shapes.

Many higher-strength grades of structural steel are available for use in construction, vehicular, and machine applications. They provide yield points in the range from 42 000 to 100 000 psi (290–700 MPa). Some of these grades, referred to as *high-strength, low-alloy (HSLA) steels*, are ASTM A242, A440, A514, A572, and A588.

Appendix 7 lists the properties of several structural steels.

2–9 TOOL STEELS

The term *tool steels* refers to a group of steels typically used for cutting tools, punches, dies, shearing blades, chisels, and similar uses. The numerous varieties of tool steel materials have been classified into seven general types as shown in Table 2–5. Whereas most uses of tool steels are related to the field of manufacturing engineering, they are also pertinent to machine design where the ability to maintain a keen edge under abrasive conditions is required (Type H and F). Also, some tool steels have rather high shock resistance which may be desirable in machine components such as parts for mechanical clutches, pawls, blades, guides for moving materials, and clamps (Types S, L, F, and W). (See Reference 6 for a more extensive discussion of tool steels.)

2–10 CAST IRON

Large gears, machine structures, brackets, linkage parts, and other important machine parts are made from cast iron. The several types of grades available span wide ranges of strength, ductility, machinability, wear resistance, and cost. These features are attractive in many applications. The three most commonly used types of cast iron are gray iron,

TABLE 2–5 Examples of tool steel types

General type	Type symbol	Specific types: Major alloying elements	Examples: AISI No.	Examples: UNS No.	Typical uses (and other common alloys)
High-speed	M	Molybdenum	M2 M10 M42	T11302 T11310 T11342	General-purpose tool steels for cutting tools and dies for forging, extrusion, bending, drawing, and piercing (M1, M3, M4–M7, M30, M34, M36, M41–M47)
	T	Tungsten	T1 T15	T12001 T12015	Similar to uses for M-types (T2, T4, T5, T6, T8)
Hot-worked	H	Chromium	H10	T20810	Cold-heading dies, shearing knives, aircraft parts, low-temperature extrusion and die-casting dies (H1–H19)
		Tungsten	H21	T20821	Higher-temperature dies, hot shearing knives (H20–H39)
		Molybdenum	H42	T20842	Applications that tend to produce high wear (H40–H59)
Cold-worked	D	High-carbon, high-chromium	D2	T30402	Stamping dies, punches, gages (D3–D5, D7)
	A	Medium-alloy, air-hardening	A2	T30102	Punches, thread-rolling dies, die-casting dies (A3–A10)
	O	Oil-hardening	O1	T31501	Taps, reamers, broaches, gages, jigs and fixtures, bushings, machine tool arbors, tool shanks (O2, O6, O7)
Shock-resisting	S		S1	T41901	Chisels, pneumatic tools, heavy-duty punches, machine parts subject to shocks (S2, S4–S7)
Molded steels	P		P2	T51602	Plastic molding dies, zinc die-casting dies (P3–P6, P20, P21)
Special-purpose	L	Low-alloy types	L2	T61202	Tooling and machine parts requiring high toughness (L3, L6)
	F	Carbon-tungsten types	F1	T60601	Similar to L-types but with higher abrasion resistance (F2)
Water-hardened	W		W1	T72301	General-purpose tool and die uses, vise and chuck jaws, hand tools, jigs and fixtures, punches (W2, W5)

ductile iron, and malleable iron. Appendix 8 lists the properties of several cast irons. (See also Reference 9.)

Gray iron is available in grades having tensile strengths ranging from 20 000 to 60 000 psi (138–414 MPa). Its ultimate compressive strength is much higher, three to five times as high as the tensile strength. One disadvantage of gray iron is that it is brittle and therefore should not be used in applications where impact loading is likely. But it has excellent wear resistance, is relatively easy to machine, has good vibration damping ability, and can be surface-hardened. Applications include engine blocks, gears, brake parts, and machine bases. The gray irons are rated by the ASTM specification A48-94a in classes 20, 25, 30, 40, 50, and 60, where the number refers to the minimum tensile strength in kips/in^2(ksi). For example, class 40 gray iron has a minimum tensile strength of 40 ksi or 40 000 psi (276 MPa). Because it is brittle, gray iron does not exhibit the property of yield strength.

Malleable iron is a group of heat-treatable cast irons with moderate to high strength, high modulus of elasticity (stiffness), good machinability, and good wear resistance. The five-digit designation roughly indicates the yield strength and the expected percent elongation of the iron. For example, Grade 40010 has a yield strength of 40 ksi (276 MPa) and a 10% elongation. The strength properties listed in Appendix 8 are for the non-heat-treated condition. Higher strengths would result from heat treating. See ASTM specifications A 47-99 and A 220-99.

Ductile irons have higher strengths than the gray irons and, as the name implies, are more ductile. However, their ductility is still much lower than that of typical steels. A three-part grade designation is used for ductile iron in the ASTM A536-84 specification. The first number refers to the tensile strength in ksi, the second is the yield strength in ksi, and the third is the approximate percent elongation. For example, the grade 80-55-06 has a tensile strength of 80 ksi (552 MPa), a yield strength of 55 ksi (379 MPa), and a 6% elongation in 2.00 in. Higher-strength cast parts, such as crankshafts and gears, are made from ductile iron.

Austempered ductile iron (ADI) is an alloyed and heat-treated ductile iron. (See Reference 9.) It has attractive properties that lead to its use in transportation equipment, industrial machinery, and other applications where the low cost, good machinability, high damping characteristics, good wear resistance, and near-net-shape advantages of casting offer special benefits. Examples are drive train gears, parts for constant velocity joints, and suspension components. ASTM Standard 897-90 lists five grades of ADI ranging in tensile strength from 125 ksi (850 MPa) to 230 ksi (1600 MPa). Yield strengths range from 80 ksi (550 MPa) to 185 ksi (1300 MPa). Ductility decreases with increasing strength and hardness with percent elongation values in the range from approximately 10% to less than 1%. ADI begins as a conventional ductile iron with careful control of composition and the casting process to produce a sound, void-free casting. Small amounts of copper, nickel, and molybdenum are added to enhance the metal's response to the special heat treatment cycle shown in Figure 2–14. It is heated to the austenitizing temperature (1550° to 1750°F, or 843° to 954°C) depending on the composition. It is held at that temperature for one to three hours as the material becomes fully austenitic. A rapid quench follows in a medium at 460° to 750°F (238° to 400°C), and the casting is held at this temperature for one-half to four hours. This is the *austempering* part of the cycle during which all of the material is converted to a mixture of mostly austenite and ferrite, sometimes called *ausferrite*. It is important that neither pearlite nor bainite form during this cycle. The casting is then allowed to cool to room temperature.

2–11 POWDERED METALS

Making parts with intricate shapes by powder metallurgy can sometimes eliminate the need for extensive machining. Metal powders are available in many formulations whose properties approach those of the wrought form of the metal. The processing involves preparing a preform by compacting the powder in a die under high pressure. Sintering at a high temperature to fuse the powder into a uniform mass is the next step. Re-pressing is sometimes done to improve properties or dimensional accuracy of the part. Typical parts made by the powder metallurgy (PM) process are gears, gear segments, cams, eccentrics, and various machine parts having oddly shaped holes or projections. Dimensional tolerances of 0.001 to 0.005 in (0.025–0.125 mm) are typical.

One disadvantage of PM parts is that they are usually brittle and should not be used in applications where high-impact loading is expected. Another important application is in sintered bearings, which are made to a relatively low density with consequent high porosity. The bearing is impregnated with a lubricant that may be sufficient for the life of the part. This type of material is discussed further in Chapter 16.

Manufacturers of metal powders have many proprietary formulations and grades. However, the Metal Powder Industries Federation (MPIF) is promoting standardization of materials. Figure 2–15 shows photographs of some powder metal parts. (See Reference 3.)

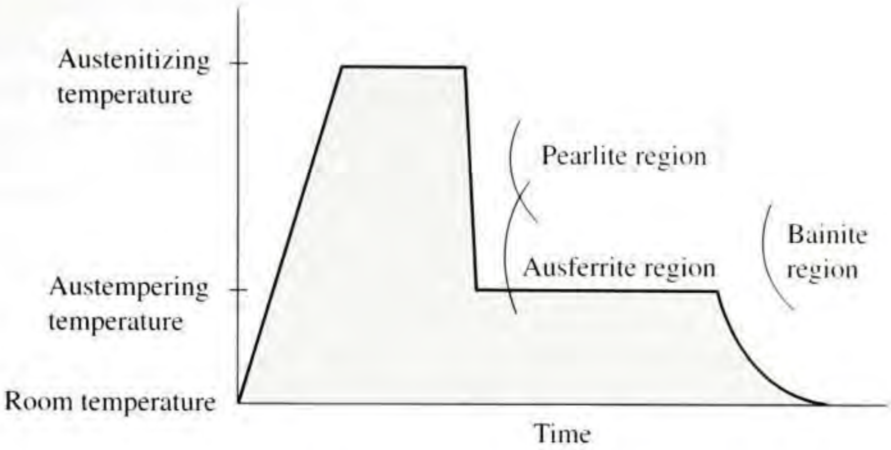


FIGURE 2–14 Heat treatment cycle for austempered ductile iron (ADI)

FIGURE 2–15 Examples of powder metal components (GKN Sinter Metals Auburn Hills, MI)

2–12 ALUMINUM

Aluminum is widely used for structural and mechanical applications. Chief among its attractive properties are light weight, good corrosion resistance, relative ease of forming and machining, and pleasing appearance. Its density is approximately one-third that of steel. However, its strength is somewhat lower, also. (See References 1, 8, and 12.) Table 2–6 lists the commonly used alloy groups.

The standard designations for aluminum alloys listed by the Aluminum Association use a four-digit system. The first digit indicates the alloy type according to the major alloying element. The second digit, if it is other than zero, indicates modifications of another alloy or limits placed on impurities in the alloy. The presence of impurities is particularly important for electrical conductors. Within each group are several specific alloys, indicated by the last two digits in the designation.

Table 2–7 lists several common alloys, along with the forms in which they are typically produced and some of their major applications. The table also lists several of the 50 or more available alloys that span the range of typical applications. The table should aid you in selecting a suitable alloy for a particular application.

TABLE 2–6 Aluminum alloy groups

Alloy designations (by major alloying element)	
1xxx	99.00% or greater aluminum content
2xxx	Copper
3xxx	Manganese
4xxx	Silicon
5xxx	Magnesium
6xxx	Magnesium and silicon
7xxx	Zinc

TABLE 2–7 Common aluminum alloys and their uses

Alloy	Applications	Forms
1060	Chemical equipment and tanks	Sheet, plate, tube
1350	Electrical conductors	Sheet, plate, tube, rod, bar, wire, pipe, shapes
2014	Aircraft structures and vehicle frames	Sheet, plate, tube, rod, bar, wire, shapes, forgings
2024	Aircraft structures, wheels, machine parts	Sheet, plate, tube, rod, bar, wire, shapes, rivets
2219	Parts subjected to high temperatures (to 600°F)	Sheet, plate, tube, rod, bar, shapes, forgings
3003	Chemical equipment, tanks, cooking utensils, architectural parts	Sheet, plate, tube, rod, bar, wire, shapes, pipe, rivets, forgings
5052	Hydraulic tubes, appliances, sheet-metal fabrications	Sheet, plate, tube, rod, bar, wire, rivets
6061	Structures, vehicle frames and parts, marine uses	All forms
6063	Furniture, architectural hardware	Tube, pipe, extruded shapes
7001	High-strength structures	Tube, extruded shapes
7075	Aircraft and heavy-duty structures	All forms except pipe

The mechanical properties of the aluminum alloys are highly dependent on their condition. For this reason, the specification of an alloy is incomplete without a reference to its *temper*. The following list describes the usual tempers given to aluminum alloys. Note that some alloys respond to heat treating, and others are processed by strain hardening. *Strain hardening* is controlled cold working of the alloy, in which increased working increases hardness and strength while reducing ductility. Commonly available tempers are described next.

F (as-fabricated): No special control of properties is provided. Actual limits are unknown. This temper should be accepted only when the part can be thoroughly tested prior to service.

O (annealed): A thermal treatment that results in the softest and lowest strength condition. Sometimes specified to obtain the most workable form of the alloy. The resulting part can be heat-treated for improved properties if it is made from alloys in the 2xxx, 4xxx, 6xxx, or 7xxx series. Also, the working itself may provide some improvement in properties similar to that produced by strain hardening for alloys in the 1xxx, 3xxx, and 5xxx series.

H (strain-hardened): A process of cold working under controlled conditions that produces improved, predictable properties for alloys in the 1xxx, 3xxx, and 5xxx groups. The greater the amount of cold work, the higher the strength and hardness, although the ductility is decreased. The *H* designation is followed by two or more digits (usually 12, 14, 16, or 18) that indicate progressively higher strength. However, several other designations are used.

T (heat-treated): A series of controlled heating and cooling processes applied to alloys in the 2xxx, 4xxx, 6xxx, and 7xxx groups. The letter *T* is followed by one or more numbers to indicate specific processes. The more common designations for mechanical and structural products are T4 and T6.

Property data for aluminum alloys are included in Appendix 9. Because these data are typical values, not guaranteed values, the supplier should be consulted for data at the time of purchase.

For mechanical design applications, alloy 6061 is one of the most versatile types. Note that it is available in virtually all forms, has good strength and corrosion resistance, and is heat-treatable to obtain a wide variety of properties. It also has good weldability. In its softer forms, it is easily formed and worked. Then, if higher strength is required, it can be heat-treated after forming. However, it has low machinability.

2–13 ZINC ALLOYS

Zinc is the fourth most commonly used metal in the world. Much of it is in the form of zinc galvanizing used as a corrosion inhibitor for steels, but very large quantities of zinc alloys are used in castings and for bearing materials. Figure 2–16 shows examples of cast zinc parts. (See Reference 19.)

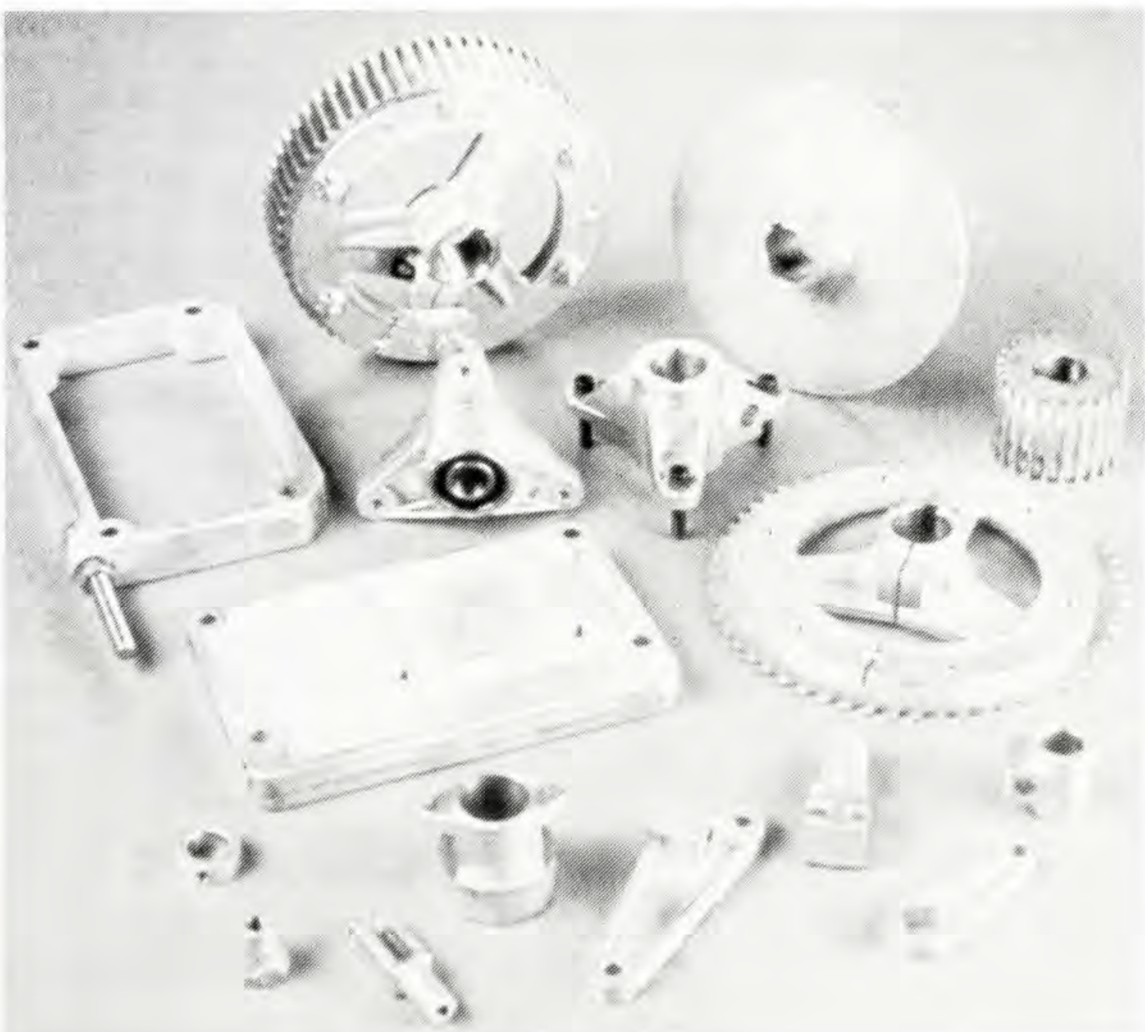
High production quantities are made using zinc pressure die casting, which results in very smooth surfaces and excellent dimensional accuracy. A variety of coating processes can be used to produce desirable finish appearance and to inhibit corrosion. Although the as-cast parts have inherently good corrosion resistance, the performance in some environments can be enhanced with chromate or phosphate treatments or anodizing. Painting and chrome plating are also used to produce a wide variety of attractive surface finishes.

In addition to die casting, zinc products are often made by permanent mold casting, graphite permanent mold casting, sand casting, and shell-mold casting. Other, less frequently used processes are investment casting, low-pressure permanent mold casting, centrifugal casting, continuous casting, and rubber-mold casting. Plaster-mold casting is often used for prototyping. Continuous casting is used to produce standard shapes (rod, bar, tube, and slabs). Prototypes or finished products can then be machined from these shapes.

Zinc alloys typically contain aluminum and a small amount of magnesium. Some alloys include copper or nickel. The performance of the final products can be very sensitive to small amounts of other elements, and maximum limits are placed on the content of iron, lead, cadmium, and tin in some alloys.

The most widely used zinc casting alloy is called *alloy No. 3*, sometimes referred to as *Zamak 3*. It has 4% aluminum and 0.035% magnesium. Another is called *Zamak 5*, and it

FIGURE 2–16 Cast zinc parts (INTERZINC, Washington, D.C.)



also contains 4% aluminum with 0.055% magnesium and 1.0% copper. A group of alloys having higher aluminum content are the ZA-alloys, with ZA-8, ZA-12, and ZA-27 being the most popular. Appendix 10 gives a summary of the composition and the typical properties of these alloys. As with most cast materials, some variations are to be expected with the size of the cast sections, the thermal treatment of the casting, the operating temperature of the product, and the quality assurance during the casting process.

2–14 TITANIUM

The applications of titanium include aerospace structures and components, chemical tanks and processing equipment, fluids-handling devices, and marine hardware. Titanium has very good corrosion resistance and a high strength-to-weight ratio. Its stiffness and density are between those of steel and aluminum; its modulus of elasticity is approximately 16×10^6 psi (110 GPa), and its density 0.160 lb/in^3 (4.429 kg/m^3). Typical yield strengths range from 25 to 175 ksi (172–1210 MPa). Disadvantages of titanium include relatively high cost and difficult machining.

The classification of titanium alloys usually falls into four types: commercially pure alpha titanium, alpha alloys, alpha-beta alloys, and beta alloys. Appendix 11 shows the properties of some of these grades. The term *alpha* refers to the hexagonal, close-packed, metallurgical structure that forms at low temperatures, and *beta* refers to the high-temperature, body-centered, cubic structure.

The grades of commercially pure titanium indicate the approximate expected yield strength of the material. For example, Ti-50A has an expected yield strength of 50 000 psi (345 MPa). As a class, these alloys exhibit only moderate strength but good ductility.

One popular grade of alpha alloy is titanium alloyed with 0.20% palladium (Pd), called *Ti-0.2Pd*. Its properties are listed in Appendix 11 for one heat-treat condition. Some alpha alloys have improved high-temperature strength and weldability.

Generally speaking, the alpha-beta alloys and the beta alloys are stronger forms of titanium. They are heat-treatable for close control of their properties. Since several alloys are available, a designer can tailor the properties to meet special needs for formability, machinability, forgeability, corrosion resistance, high-temperature strength, weldability, and creep resistance, as well as basic room-temperature strength and ductility. Alloy Ti-6Al-4V contains 6% aluminum and 4% vanadium and is used in a variety of aerospace applications.

2–15 COPPER, BRASS, AND BRONZE

Copper is widely used in its nearly pure form for electrical and plumbing applications because of its high electrical conductivity and good corrosion resistance. It is rarely used for machine parts because of its relatively low strength compared with that of its alloys, *brass* and *bronze*. (See Reference 3.)

Brass is a family of alloys of copper and zinc, with the content of zinc ranging from about 5% to 40%. Brass is often used in marine applications because of its resistance to corrosion in salt water. Many brass alloys also have excellent machinability and are used as connectors, fittings, and other parts made on screw machines. *Yellow brass* contains about 30% or more of zinc and often contains a significant amount of lead to improve machinability. *Red brass* contains 5% to 15% zinc. Some alloys also contain tin, lead, nickel, or aluminum.

Bronze is a class of alloys of copper with several different elements, one of which is usually tin. They are useful in gears, bearings, and other applications where good strength and high wear resistance are desirable.

Wrought bronze alloys are available in four types:

Phosphor bronze: Copper-tin-phosphorus alloy

Leaded phosphor bronze: Copper-tin-lead-phosphorus alloy

Aluminum bronze: Copper-aluminum alloy

Silicon bronze: Copper-silicon alloy

Cast bronze alloys have four main types:

Tin bronze: Copper-tin alloy

Leaded tin bronze: Copper-tin-lead alloy

Nickel tin bronze: Copper-tin-nickel alloy

Aluminum bronze: Copper-aluminum alloy

The cast alloy called *manganese bronze* is actually a high-strength form of brass because it contains zinc, the characteristic alloying element of the brass family. Manganese bronze contains copper, zinc, tin, and manganese.

In the UNS, copper alloys are designated by the letter *C*, followed by a five-digit number. Numbers from 10000 to 79900 refer to wrought alloys; 80000 to 99900 refer to casting alloys. See Appendix 12 for typical properties.

2–16 NICKEL-BASED ALLOYS

Nickel alloys are often used in place of steel where operation at high temperatures and in certain corrosive environments is required. Examples are turbine engine components, furnace parts, chemical processing systems, and critical marine system components. (See Reference 7.) Some nickel alloys are called *superalloys*, and many of the commonly used alloys are proprietary. The following list gives some of the commercially available alloy types:

Inconel (International Nickel Co.): Nickel-chromium alloys

Monel (International Nickel Co.): Nickel-copper alloys

Ni-Resist (International Nickel Co.): Nickel-iron alloys

Hastelloy (Haynes International): Nickel-molybdenum alloys, sometimes with chromium, iron, or copper

2–17 PLASTICS

Plastics include a wide variety of materials formed of large molecules called *polymers*. The thousands of different plastics are created by combining different chemicals to form long molecular chains.

One method of classifying plastics is by the terms *thermoplastic* and *thermosetting*. In general, the *thermoplastic* materials can be formed repeatedly by heating or molding because their basic chemical structure is unchanged from its initial linear form. *Thermosetting* plastics do undergo some change during forming and result in a structure in which the molecules are cross-linked and form a network of interconnected molecules. Some designers recommend the terms *linear* and *cross-linked* in place of the more familiar *thermoplastic* and *thermosetting*.

Listed next are several thermoplastics and several thermosets that are used for load-carrying parts and that are therefore of interest to the designer of machine elements. These listings show the main advantages and uses of a sample of the many plastics available. Appendix 13 lists typical properties.

Thermoplastics

- *Nylon:* Good strength, wear resistance, and toughness; wide range of possible properties depending on fillers and formulations. Used for structural parts, mechanical devices such as gears and bearings, and parts needing wear resistance.
- *Acrylonitrile-butadiene-styrene (ABS):* Good impact resistance, rigidity, moderate strength. Used for housings, helmets, cases, appliance parts, pipe, and pipe fittings.

- *Polycarbonate:* Excellent toughness, impact resistance, and dimensional stability. Used for cams, gears, housings, electrical connectors, food processing products, helmets, and pump and meter parts.
- *Acrylic:* Good weather resistance and impact resistance; can be made with excellent transparency or translucent or opaque with color. Used for glazing, lenses, signs, and housings.
- *Polyvinyl chloride (PVC):* Good strength, weather resistance, and rigidity. Used for pipe, electrical conduit, small housings, ductwork, and moldings.
- *Polyimide:* Good strength and wear resistance; very good retention of properties at elevated temperatures up to 500°F. Used for bearings, seals, rotating vanes, and electrical parts.
- *Acetal:* High strength, stiffness, hardness, and wear resistance; low friction; good weather resistance and chemical resistance. Used for gears, bushings, sprockets, conveyor parts, and plumbing products.
- *Polyurethane elastomer:* A rubberlike material with exceptional toughness and abrasion resistance; good heat resistance and resistance to oils. Used for wheels, rollers, gears, sprockets, conveyor parts, and tubing.
- *Thermoplastic polyester resin (PET):* Polyethylene terephthalate (PET) resin with fibers of glass and/or mineral. Very high strength and stiffness, excellent resistance to chemicals and heat, excellent dimensional stability, and good electrical properties. Used for pump parts, housings, electrical parts, motor parts, auto parts, oven handles, gears, sprockets, and sporting goods.
- *Polyether-ester elastomer:* Flexible plastic with excellent toughness and resilience, high resistance to creep, impact, and fatigue under flexure, good chemical resistance. Remains flexible at low temperatures and retains good properties at moderately elevated temperatures. Used for seals, belts, pump diaphragms, protective boots, tubing, springs, and impact absorbing devices. High modulus grades can be used for gears and sprockets.

Thermosets

- *Phenolic:* High rigidity, good moldability and dimensional stability, very good electrical properties. Used for load-carrying parts in electrical equipment, switchgear, terminal strips, small housings, handles for appliances and cooking utensils, gears, and structural and mechanical parts. Alkyd, allyl, and amino thermosets have properties and uses similar to those of the phenolics.
- *Polyester:* Known as *fiber glass* when reinforced with glass fibers; high strength and stiffness, good weather resistance. Used for housings, structural shapes, and panels.

Special Considerations for Selecting Plastics

A particular plastic is often selected for a combination of properties, such as light weight, flexibility, color, strength, stiffness, chemical resistance, low friction characteristics, or transparency. Table 2–8 lists the primary plastic materials used for six different types of applications. References 11 and 23 provide an extensive comparative study of the design properties of plastics.

While most of the same definitions of design properties described in Section 2–2 of this chapter can be used for plastics as well as metals, a significant amount of additional information is typically needed to specify a suitable plastic material. Some of the special

TABLE 2–8 Applications of plastic materials

Applications	Desired properties	Suitable plastics
Housings, containers, ducts	High impact strength, stiffness, low cost, formability, environmental resistance, dimensional stability	ABS, polystyrene, polypropylene, PET, polyethylene, cellulose acetate, acrylics
Low friction—bearings, slides	Low coefficient of friction; resistance to abrasion, heat, corrosion	TFE fluorocarbons, nylon, acetals
High-strength components, gears, cams, rollers	High tensile and impact strength, stability at high temperatures, machinable	Nylon, phenolics, TFE-filled acetals, PET, polycarbonate
Chemical and thermal equipment	Chemical and thermal resistance, good strength, low moisture absorption	Fluorocarbons, polypropylene, polyethylene, epoxies, polyesters, phenolics
Electrostructural parts	Electrical resistance, heat resistance, high impact strength, dimensional stability, stiffness	Allyls, alkyds, aminos, epoxies, phenolics, polyesters, silicones, PET
Light-transmission components	Good light transmission in transparent and translucent colors, formability, shatter resistance	Acrylics, polystyrene, cellulose acetate, vinyls

characteristics of plastics follow. The charts shown in Figures 2–17 to 2–20 are examples only and are not meant to indicate the general nature of the performance of the given type of material. There is a wide range of properties among the many formulations of plastics even within a given class. Consult the extensive amount of design guidance available from vendors of the plastic materials.

1. Most properties of plastics are highly sensitive to temperature. In general, tensile strength, compressive strength, elastic modulus, and impact failure energy decrease significantly as the temperature increases. Figure 2–17 shows the tensile strength of nylon 66 at four temperatures. Note also the rather different shapes of the stress-strain curves. The slope of the curve at any point indicates the elastic modulus, and you can see a large variation for each curve.
2. Many plastics absorb a considerable amount of moisture from the environment and exhibit dimensional changes and degradation of strength and stiffness properties as a result. See Figure 2–18 that shows the flexural modulus versus temperature for a nylon in dry air, 50% relative humidity (RH), and 100% RH. A consumer product may well experience a major part of this range. At a temperature of 20°C (68°F, near room temperature), the flexural modulus would decrease dramatically from approximately 2900 MPa to about 500 MPa as humidity changes from dry air to 100% RH. The product may also see a temperature range from 0°C (32°F, freezing point of water) to 40°C (104°F). Over this range, the flexural modulus for the nylon at 50% RH would decrease from approximately 2300 MPa to 800 MPa.
3. Components that carry loads continuously must be designed to accommodate creep or relaxation. See Figures 2–17 to 2–19 and Example Problem 2–1.
4. Fatigue resistance data of a plastic must be acquired for the specific formulation used and at a representative temperature. Chapter 5 gives more information about fatigue. Figure 2–19 shows the fatigue stress versus number of cycles to failure for an acetal resin plastic. Curve 1 is at 23°C (73°F, near room temperature) with cyclic loading in tension only as when a tensile load is applied and

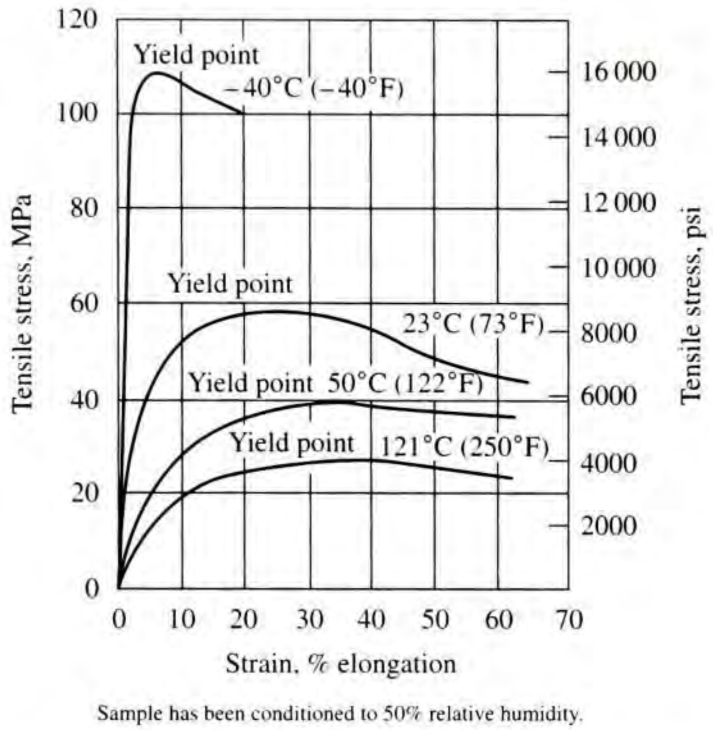


FIGURE 2–17 Stress-strain curves for nylon 66 at four temperatures (DuPont Polymers, Wilmington, DE)

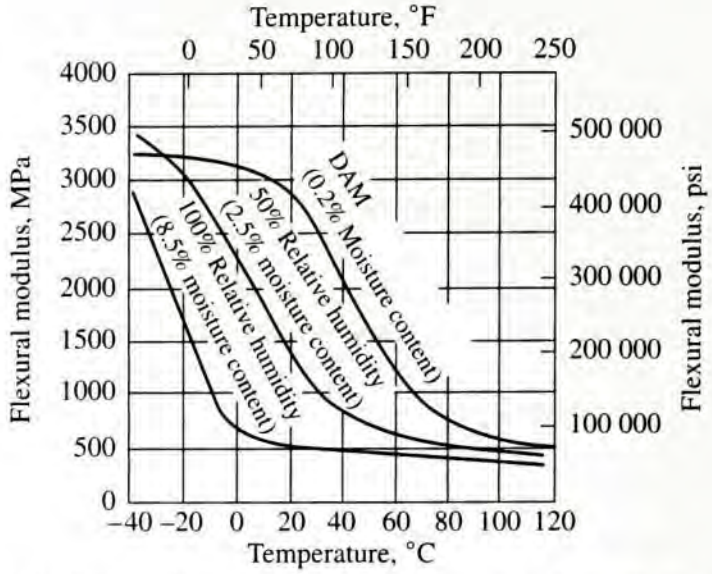


FIGURE 2–18 Effect of temperature and humidity on the flexural modulus of nylon 66 (DuPont Polymers, Wilmington, DE)

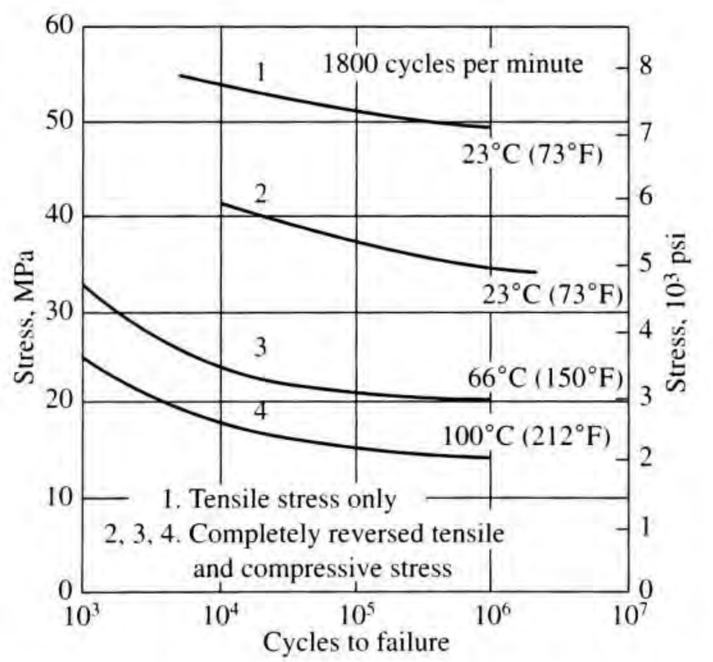


FIGURE 2–19 Fatigue stress vs. number of cycles to failure for an acetal resin plastic (DuPont Polymers, Wilmington, DE)

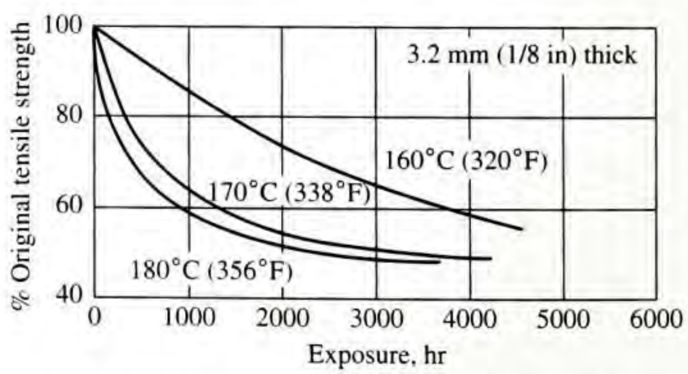


FIGURE 2–20 Effect of exposure to elevated temperature on a thermoplastic polyester resin (PET) (DuPont Polymers, Wilmington, DE)

removed many times. Curve 2 is at the same temperature, but the loading is completely reversed tension and compression as would be experienced with a rotating beam or shaft loaded in bending. Curve 3 is the reversed bending load at 66°C (150°F), and Curve 4 is the same loading at 100°C (212°F) to show the effect of temperature on fatigue data.

5. Processing methods can have large effects on the final dimensions and properties of parts made from plastics. Molded plastics shrink significantly during solidification and curing. Parting lines produced where mold halves meet may affect strength. The rate of solidification may be widely different in a given part depending on the section thicknesses, the complexity of the shape and the location of sprues that deliver molten plastic into the mold. The same material can produce different properties depending on whether it is processed by injection molding, extrusion, blow molding, or machining from a solid block or bar.
6. Resistance to chemicals, weather, and other environmental conditions must be checked.
7. Plastics may exhibit a change in properties as they age, particularly when subjected to elevated temperatures. Figure 2–20 shows the reduction in tensile strength for a thermoplastic polyester resin when subjected to temperatures from 160°C (320°F) to 180°C (356°F) for a given number of hours of exposure. The reduction can be as much as 50% in as little time as 2000 hours (12 weeks).
8. Flammability and electrical characteristics must be considered. Some plastics are specially formulated for high flammability ratings as called for by Underwriters Laboratory and other agencies.
9. Plastics used for food storage or processing must meet U.S. Food and Drug Administration standards.

2–18 COMPOSITE MATERIALS

Composite materials are composed of two or more different materials that act together to produce properties that are different from, and generally superior to, those of the individual components. Typical composites include a polymeric resin matrix material with fibrous reinforcing material dispersed within it. Some advanced composites have a metal matrix. (See References 10 and 20.)

Designers can tailor the properties of composite materials to meet the specific needs of a particular application by the selection of each of several variables that determine the performance of the final product. Among the factors under the designer's control are the following:

1. Matrix resin or metal
2. Type of reinforcing fibers
3. Amount of fiber contained in the composite
4. Orientation of the fibers
5. Number of individual layers used
6. Overall thickness of the material
7. Orientation of the layers relative to each other
8. Combination of two or more types of composites or other materials into a composite structure

Typically, the filler is a strong, stiff material, whereas the matrix has a relatively low density. When the two materials bond together, much of the load-carrying ability of the composite is produced by the filler material. The matrix serves to hold the filler in a favorable orientation relative to the manner of loading and to distribute the loads to the filler. The result is a somewhat optimized composite that has high strength and high stiffness with low weight. Table 2–9 lists some of the composites formed by combinations of resins and fibers and their general characteristics and uses.

A virtually unlimited variety of composite materials can be produced by combining different matrix materials with different fillers in different forms and in different orientations. Some typical materials are listed below.

Matrix Materials

The following are among the more frequently used matrix materials:

- Thermoplastic polymers: Polyethylene, nylon, polypropylene, polystyrene, polyamides
- Thermosetting polymers: Polyester, epoxy, phenolic polyimide
- Ceramics and glass
- Carbon and graphite
- Metals: Aluminum, magnesium, titanium

Forms of Filler Materials

Many forms of filler materials are used:

- Continuous fiber strand consisting of many individual filaments bound together
- Chopped strands in short lengths (0.75 to 50 mm or 0.03 to 2.00 in)
- Chopped strands randomly spread in the form of a mat
- Roving: A group of parallel strands
- Woven fabric made from roving or strands
- Metal filaments or wires
- Solid or hollow microspheres
- Metal, glass, or mica flakes
- Single-crystal whiskers of materials such as graphite, silicon carbide, and copper

TABLE 2–9 Examples of composite materials and their uses

Type of composite	Typical applications
Glass/epoxy	Automotive and aircraft parts, tanks, sporting goods, printed wiring boards
Boron/epoxy	Aircraft structures and stabilizers, sporting goods
Graphite/epoxy	Aircraft and spacecraft structures, sporting goods, agricultural equipment, material handling devices, medical devices
Aramid/epoxy	Filament-wound pressure vessels, aerospace structures and equipment, protective clothing, automotive components
Glass/polyester	Sheet-molding compound (SMC), body panels for trucks and cars, large housings

Types of Filler Materials

Fillers, also called *fibers*, come in many types based on both organic and inorganic materials. The following are some of the more popular fillers:

- Glass fibers in five different types:
  A-glass: Good chemical resistance because it contains alkalis such as sodium oxide
  C-glass: Special formulations for even higher chemical resistance than A-glass
  E-glass: Widely used glass with good electrical insulating ability and good strength
  S-glass: High-strength, high-temperature glass
  D-glass: Better electrical properties than E-glass
- Quartz fibers and high-silica glass: Good properties at high temperatures up to 2000°F (1095°C)
- Carbon fibers made from PAN-base carbon (PAN is polyacrylonitrile): Approximately 95% carbon with very high modulus of elasticity
- Graphite fibers: Greater than 99% carbon and an even higher modulus of elasticity than carbon; the stiffest fibers typically used in composites
- Boron coated onto tungsten fibers: Good strength and a higher modulus of elasticity than glass
- Silicon carbide coated onto tungsten fibers: Strength and stiffness similar to those of boron/tungsten, but with higher temperature capability
- Aramid fibers: A member of the polyamide family of polymers; higher strength and stiffness with lower density as compared with glass; very flexible. (Aramid fibers produced by the DuPont Company carry the name *Kevlar*™.)

Processing of Composites

One method that is frequently used to produce composite products is first to place layers of sheet-formed fabrics on a form having the desired shape and then to impregnate the fabric with wet resin. Each layer of fabric can be adjusted in its orientation to produce special properties of the finished article. After the lay-up and resin impregnation are completed, the entire system is subjected to heat and pressure while a curing agent reacts with the base resin to produce cross-linking that binds all of the elements into a three-dimensional, unified structure. The polymer binds to the fibers and holds them in their preferred position and orientation during use.

An alternative method of fabricating composite products starts with a process of preimpregnating the fibers with the resin material to produce strands, tape, braids, or sheets. The resulting form, called a *prepreg*, can then be stacked into layers or wound onto a form to produce the desired shape and thickness. The final step is the curing cycle as described for the wet process.

Polyester-based composites are often produced as *sheet-molding compounds (SMC)* in which preimpregnated fabric sheets are placed into a mold and shaped and cured simultaneously under heat and pressure. Large body panels for automotive applications can be produced in this manner.

Pultrusion is a process in which the fiber reinforcement is coated with resin as it is pulled through a heated die to produce a continuous form in the desired shape. This process is used to produce rod, tubing, structural shapes (I-beams, channels, angles, and so on), tees, and hat sections used as stiffeners in aircraft structures.

Filament winding is used to make pipe, pressure vessels, rocket motor cases, instrument enclosures, and odd-shaped containers. The continuous filament can be placed in a variety of patterns, including helical, axial, and circumferential, to produce desired strength and stiffness characteristics.

Advantages of Composites

Designers typically seek to produce products that are safe, strong, stiff, lightweight, and highly tolerant of the environment in which the product will operate. Composites often excel in meeting these objectives when compared with alternative materials such as metals, wood, and unfilled plastics. Two parameters that are used to compare materials are *specific strength* and *specific modulus*, defined as follows:

> **Specific strength** ***is the ratio of the tensile strength of a material to its specific weight.***
>
> **Specific modulus** ***is the ratio of the modulus of elasticity of a material to its specific weight.***

Because the modulus of elasticity is a measure of the stiffness of a material, the specific modulus is sometimes called *specific stiffness*.

Although obviously not a length, both of these quantities have the *unit* of length, derived from the ratio of the units for strength or modulus of elasticity and the units for specific weight. In the U.S. Customary System, the units for tensile strength and modulus of elasticity are lb/in^2, whereas specific weight (weight per unit volume) is in lb/in^3. Thus, the unit for specific strength or specific modulus is inches. In the SI, strength and modulus are expressed in N/m^2 (pascals), whereas specific weight is in N/m^3. Then the unit for specific strength or specific modulus is meters.

Table 2–10 gives comparisons of the specific strength and specific stiffness of selected composite materials with certain steel, aluminum, and titanium alloys. Figure 2–21 shows a comparison of these materials using bar charts. Figure 2–22 is a plot of these data with specific strength on the vertical axis and specific modulus on the horizontal axis. When weight is critical, the ideal material will lie in the upper-right part of this chart. Note that data in these charts and figures are for composites having the filler materials aligned in the most favorable direction to withstand the applied loads.

Advantages of composites can be summarized as follows:

1. Specific strengths for composite materials can range as high as five times those of high-strength steel alloys. See Table 2–10 and Figures 2–21 and 2–22.
2. Specific modulus values for composite materials can be as high as eight times those for steel, aluminum, or titanium alloys. See Table 2–10 and Figures 2–21 and 2–22.
3. Composite materials typically perform better than steel or aluminum in applications where cyclic loads can lead to the potential for fatigue failure.
4. Where impact loads and vibrations are expected, composites can be specially formulated with materials that provide high toughness and a high level of damping.
5. Some composites have much higher wear resistance than metals.
6. Careful selection of the matrix and filler materials can provide superior corrosion resistance.
7. Dimensional changes due to changes in temperature are typically much less for composites than for metals.

TABLE 2–10 Comparison of specific strength and specific modulus for selected materials

Material	Tensile strength, s_u (ksi)	Specific weight, γ (lb/in^3)	Specific strength (in)	Specific modulus (in)
Metals				
Steel ($E = 30 \times 10^6$ psi)				
AISI 1020 HR	55	0.283	0.194×10^6	1.06×10^8
AISI 5160 OQT 700	263	0.283	0.929×10^6	1.06×10^8
Aluminum ($E = 10.0 \times 10^6$ psi)				
6061-T6	45	0.098	0.459×10^6	1.02×10^8
7075-T6	83	0.101	0.822×10^6	0.99×10^8
Titanium ($E = 16.5 \times 10^6$ psi)				
Ti-6Al-4V, quenched and aged at 1000°F	160	0.160	1.00×10^6	1.03×10^8
Composites				
Glass/epoxy composite ($E = 4.0 \times 10^6$ psi)				
34% fiber content	114	0.061	1.87×10^6	0.66×10^8
Aramid/epoxy composite ($E = 11.0 \times 10^6$ psi)				
60% fiber content	200	0.050	4.0×10^6	2.20×10^8
Boron/epoxy composite ($E = 30.0 \times 10^6$ psi)				
60% fiber content	270	0.075	3.60×10^6	4.00×10^8
Graphite/epoxy composite ($E = 19.7 \times 10^6$ psi)				
62% fiber content	278	0.057	4.86×10^6	3.45×10^8
Graphite/epoxy composite ($E = 48 \times 10^6$ psi)				
Ultrahigh modulus	160	0.058	2.76×10^6	8.28×10^8

8. Because composite materials have properties that are highly directional, designers can tailor the placement of reinforcing fibers in directions that provide the required strength and stiffness under the specific loading conditions to be encountered.
9. Composite structures can often be made in complex shapes in one piece, thus reducing the number of parts in a product and the number of fastening operations required. The elimination of joints typically improves the reliability of such structures as well.
10. Composite structures are typically made in their final form directly or in a near-net shape, thus reducing the number of secondary operations required.

Limitations of Composites

Designers must balance many properties of materials in their designs while simultaneously considering manufacturing operations, costs, safety, life, and service of the product. Listed next are some of the major concerns when using composites.

1. Material costs for composites are typically higher than for many alternative materials.
2. Fabrication techniques are quite different from those used to shape metals. New manufacturing equipment may be required, along with additional training for production operators.

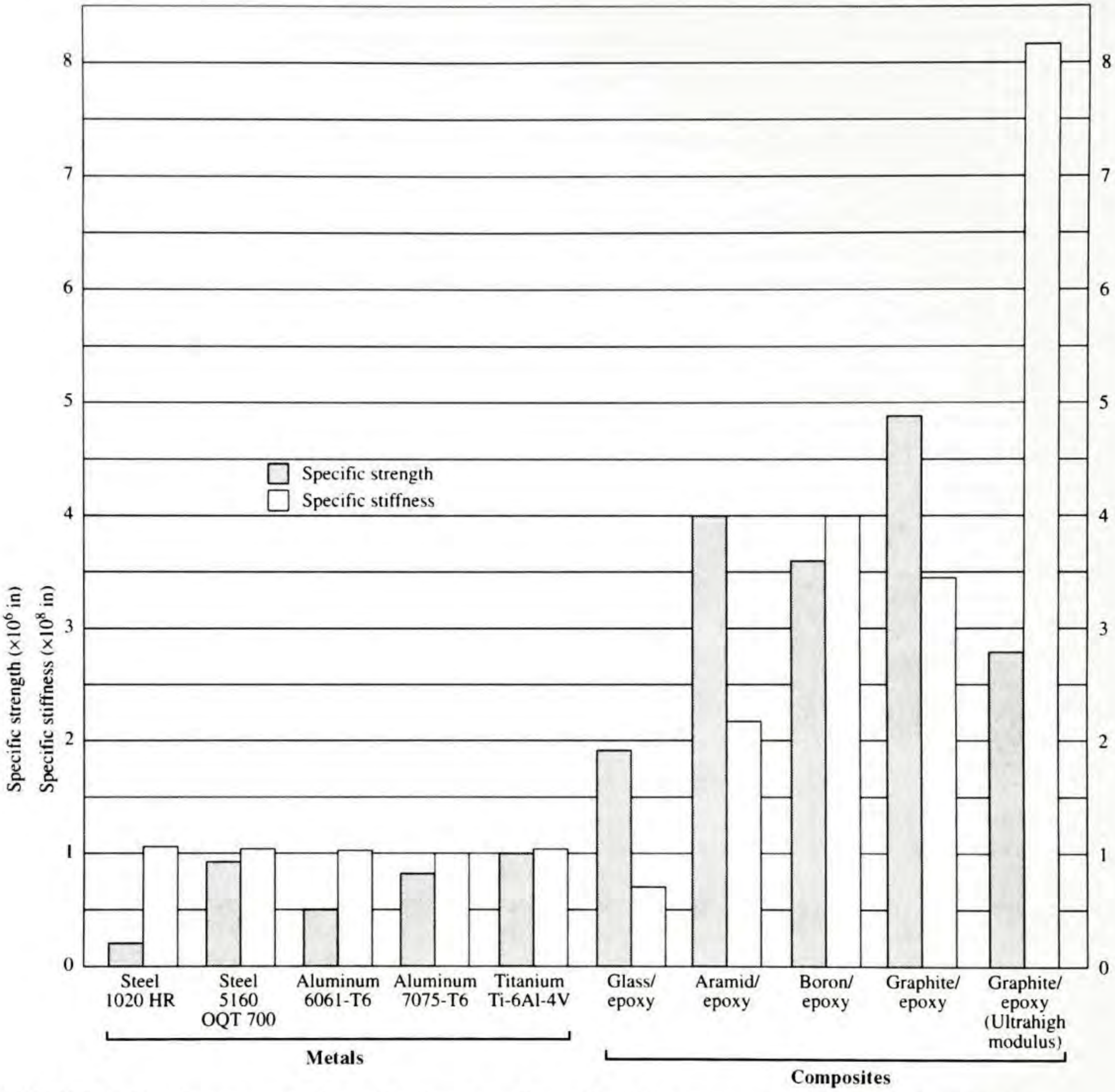
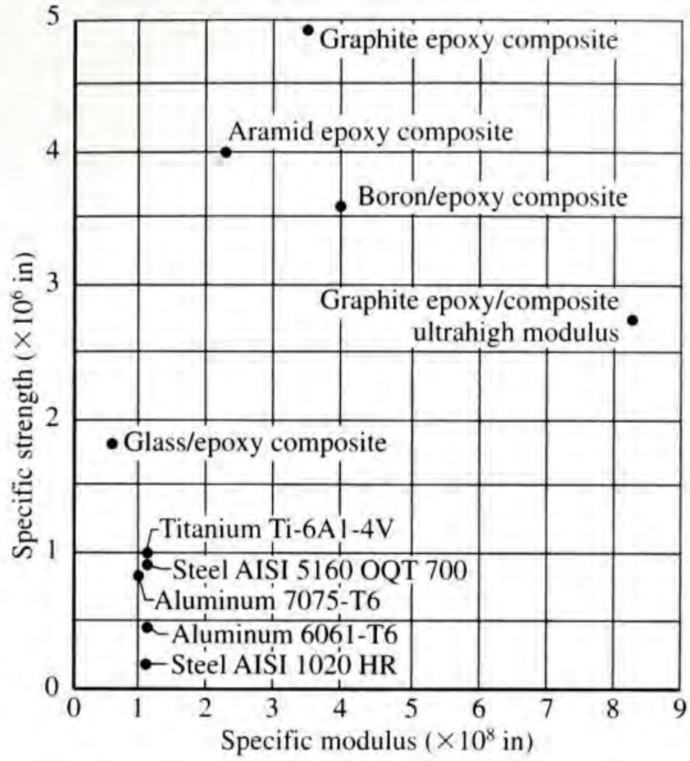


FIGURE 2–21 Comparison of specific strength and specific stiffness for selected metals and composites

3. The performance of products made from some composite production techniques is subject to a wider range of variability than the performance of products made from most metal fabrication techniques.
4. The operating temperature limits for composites having a polymeric matrix are typically 500°F (260°C). (But ceramic or metal matrix composites can be used at higher temperatures such as those found in engines.)
5. The properties of composite materials are not isotropic: Properties vary dramatically with the direction of the applied loads. Designers must account for these variations to ensure safety and satisfactory operation under all expected types of loading.

FIGURE 2–22
Specific strength versus specific modulus for selected metals and composites



6. At this time, many designers lack understanding of the behavior of composite materials and the details of predicting failure modes. Whereas major advancements have been made in certain industries such as the aerospace and recreational equipment fields, there is a need for more general understanding about designing with composite materials.
7. The analysis of composite structures requires detailed knowledge of more properties of the materials than would be required for metals.
8. Inspection and testing of composite structures are typically more complicated and less precise than for metal structures. Special nondestructive techniques may be required to ensure that there are no major voids in the final product that could seriously weaken the structure. Testing of the complete structure may be required rather than testing of simply a sample of the material because of the interaction of different parts on each other and because of the directionality of the material properties.
9. Repair and maintenance of composite structures are serious concerns. Some of the initial production techniques require special environments of temperature and pressure that may be difficult to reproduce in the field when damage repair is required. Bonding of a repaired area to the parent structure may also be difficult.

Laminated Composite Construction

Many structures made from composite materials are made from several layers of the basic material containing both the matrix and the reinforcing fibers. The manner in which the layers are oriented relative to one another affects the final properties of the completed structure.

As an illustration, consider that each layer is made from a set of parallel strands of the reinforcing filler material, such as E-glass fibers, embedded in the resin matrix, such as polyester. As mentioned previously, in this form, the material is sometimes called a *prepreg*, indicating that the filler has been preimpregnated with the matrix prior to the forming of the structure and the curing of the assembly. To produce the maximum strength and stiffness in a particular direction, several layers or plies of the prepreg could be laid on top of one another with all of the fibers aligned in the direction of the expected tensile load. This is called a *unidirectional laminate*. After curing, the laminate would have a very high strength and stiffness when loaded in the direction of the strands, called the *longitudinal* direction. However, the resulting product would have a very low strength and stiffness in the direction perpendicular to the fiber direction, called the *transverse* direction. If any off-axis loads are encountered, the part may fail or deform significantly. Table 2–11 gives sample data for a unidirectional laminated, carbon/epoxy composite.

To overcome the lack of off-axis strength and stiffness, laminated structures should be made with a variety of orientations of the layers. One popular arrangement is shown in Figure 2–23. Naming the longitudinal direction of the surface layer the *0° ply*, this structure is referred to as

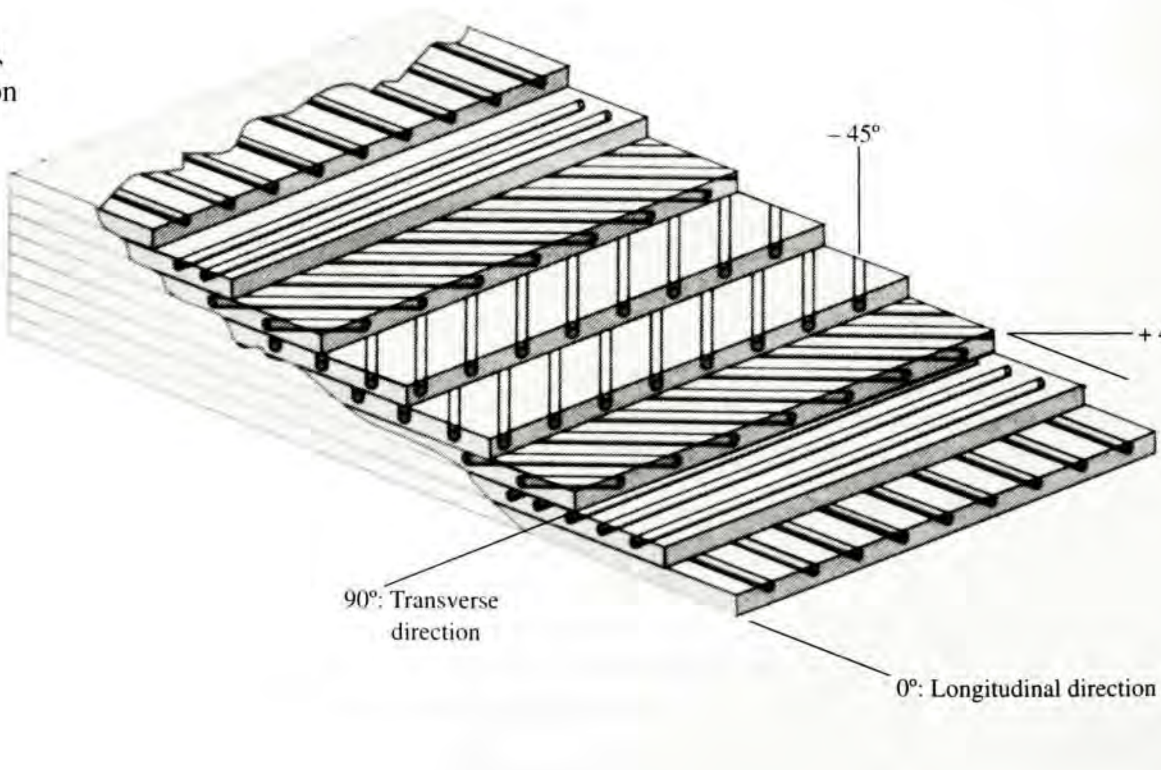
$$0^\circ, 90^\circ, +45^\circ, -45^\circ, -45^\circ, +45^\circ, 90^\circ, 0^\circ$$

The symmetry and the balance of this type of layering technique result in more nearly uniform properties in two directions. The term *quasi-isotropic* is sometimes used to describe such a structure. Note that the properties perpendicular to the faces of the layered

TABLE 2–11 Examples of the effect of laminate construction on strength and stiffness

	Tensile strength				Modulus of elasticity			
	Longitudinal		Transverse		Longitudinal		Transverse	
Laminate type	ksi	MPa	ksi	MPa	10^6 psi	GPa	10^6 psi	GPa
Unidirectional	200	1380	5	34	21	145	1.6	11
Quasi-isotropic	80	552	80	552	8	55	8	55

FIGURE 2–23 Multilayer, laminated, composite construction designed to produce quasi-isotropic properties



structure (through the thickness) are still quite low because fibers do not extend in that direction. Also, the strength and the stiffness in the primary directions are somewhat lower than if the plies were aligned in the same direction. Table 2–11 also shows sample data for a quasi-isotropic laminate compared with one having unidirectional fibers in the same matrix.

Predicting Composite Properties

The following discussion summarizes some of the important variables needed to define the properties of a composite. The subscript c refers to the composite, m refers to the matrix, and f refers to the fibers. The strength and the stiffness of a composite material depend on the elastic properties of the fiber and matrix components. But another parameter is the relative volume of the composite composed of fibers, V_f, and that composed of the matrix material, V_m. That is,

$$V_f = \text{volume fraction of fiber in the composite}$$

$$V_m = \text{volume fraction of matrix in the composite}$$

Note that for a unit volume, $V_f + V_m = 1$; thus, $V_m = 1 - V_f$.

We will use an ideal case to illustrate the way in which the strength and the stiffness of a composite can be predicted. Consider a composite with unidirectional, continuous fibers aligned in the direction of the applied load. The fibers are typically much stronger and stiffer than the matrix material. Furthermore, the matrix will be able to undergo a larger strain before fracture than the fibers can. Figure 2–24 shows these phenomena on a plot of stress versus strain for the fibers and the matrix. We will use the following notation for key parameters from Figure 2–24:

$$s_{uf} = \text{ultimate strength of fiber}$$

$$\epsilon_{uf} = \text{strain in the fiber corresponding to its ultimate strength}$$

$$\sigma'_m = \text{stress in the matrix at the same strain as } \epsilon_{uf}$$

FIGURE 2–24
Stress versus strain for fiber and matrix materials

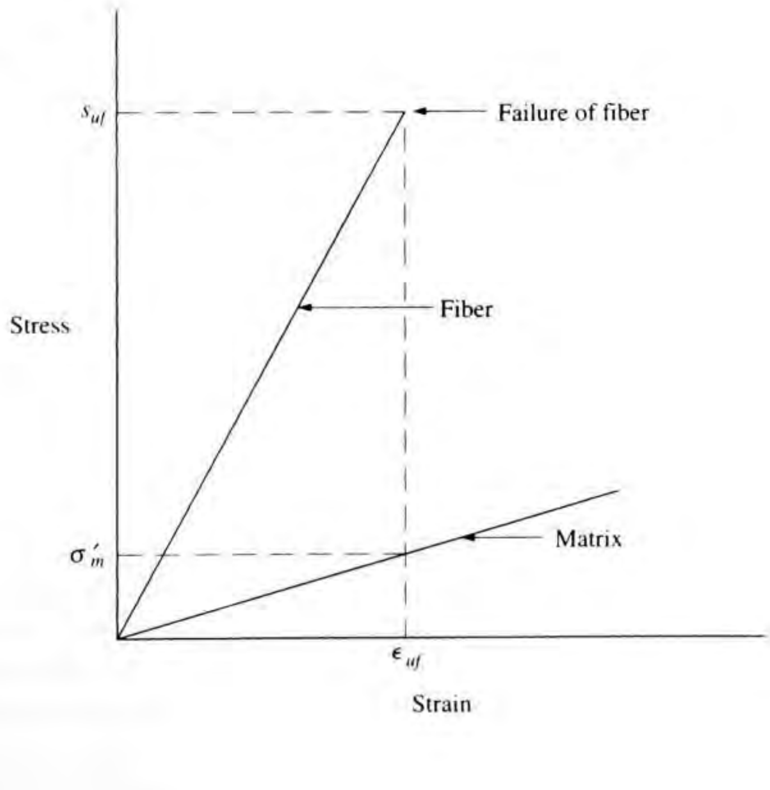
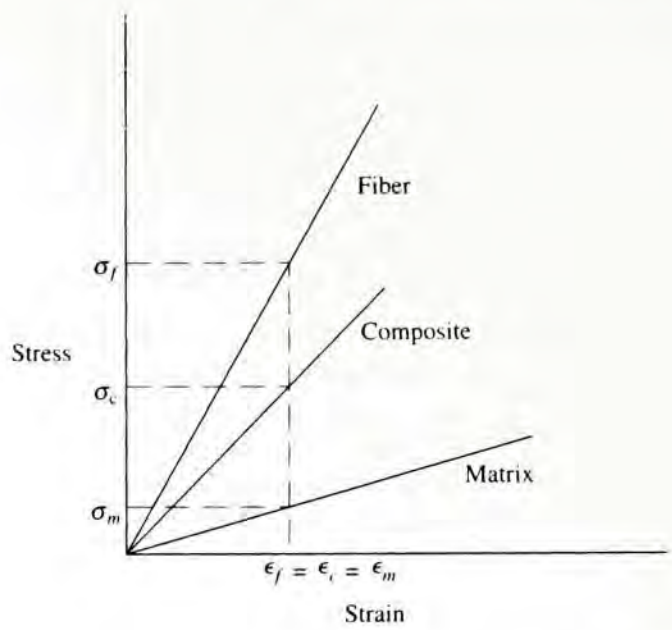


FIGURE 2–25
Relationship among stresses and strains for a composite and its fiber and matrix materials



The ultimate strength of the composite, s_{uc}, is at some intermediate value between s_{uf} and σ'_m, depending on the volume fraction of fiber and matrix in the composite. That is,

Rule of Mixtures for Ultimate Strength

$$s_{uc} = s_{uf} V_f + \sigma'_m V_m \tag{2–8}$$

At any lower level of stress, the relationship among the overall stress in the composite, the stress in the fibers, and the stress in the matrix follows a similar pattern:

Rule of Mixtures for Stress in a Composite

$$\sigma_c = \sigma_f V_f + \sigma_m V_m \tag{2–9}$$

Figure 2–25 illustrates this relationship on a stress-strain diagram.

Both sides of Equation (2–9) can be divided by the strain at which these stresses occur. Since for each material, $\sigma/\epsilon = E$, the modulus of elasticity for the composite can be shown as

Rule of Mixtures for Modulus of Elasticity

$$E_c = E_f V_f + E_m V_m \tag{2–10}$$

The density of a composite can be computed in a similar fashion:

Rule of Mixtures for Density of a Composite

$$\rho_c = \rho_f V_f + \rho_m V_m \tag{2–11}$$

As mentioned previously (Section 2–2), density is defined as mass per unit volume. A related property, *specific weight*, is defined as weight per unit volume and is denoted by the symbol γ (the Greek letter gamma). The relationship between density and specific weight is simply $\gamma = \rho g$, where g is the acceleration due to gravity. Multiplying each term in Equation (2–11) by g gives the formula for the specific weight of a composite:

Rule of Mixtures for Specific Weight of a Composite

$$\gamma_c = \gamma_f V_f + \gamma_m V_m \tag{2–12}$$

The forms of Equations (2–8) through (2–12) are examples of *the rules of mixtures*.

Table 2–12 lists example values for the properties of some matrix and filler materials. Remember that wide variations can occur in such properties, depending on the exact formulation and the condition of the materials.

TABLE 2–12 Example properties of matrix and filler materials

	Tensile strength		Tensile modulus		Specific weight	
	ksi	MPa	10^6 psi	GPa	lb/in^3	kN/m^3
Matrix materials:						
Polyester	10	69	0.40	2.76	0.047	12.7
Epoxy	18	124	0.56	3.86	0.047	12.7
Aluminum	45	310	10.0	69	0.100	27.1
Titanium	170	1170	16.5	114	0.160	43.4
Filler materials:						
S-glass	600	4140	12.5	86.2	0.09	24.4
Carbon-PAN	470	3240	33.5	231	0.064	17.4
Carbon-PAN (high-strength)	820	5650	40	276	0.065	17.7
Carbon (high-modulus)	325	2200	100	690	0.078	21.2
Aramid	500	3450	19.0	131	0.052	14.1

Example Problem 2–2 Compute the expected properties of ultimate tensile strength, modulus of elasticity, and specific weight of a composite made from unidirectional strands of carbon-PAN fibers in an epoxy matrix. The volume fraction of fibers is 30%. Use data from Table 2–12.

Solution Objective Compute the expected values of s_{uc}, E_c, and γ_c for the composite.

Given Matrix-epoxy: $s_{um} = 18$ ksi; $E_m = 0.56 \times 10^6$ psi; $\gamma_m = 0.047\ lb/in^3$.
Fiber-carbon-PAN: $s_{uf} = 470$ ksi; $E_f = 33.5 \times 10^6$ psi; $\gamma_f = 0.064\ lb/in^3$.
Volume fraction of fiber: $V_f = 0.30$, and $V_m = 1.0 - 0.30 = 0.70$.

Analysis and Results The ultimate tensile strength, s_{uc}, is computed from Equation (2–8):

$$s_{uc} = s_{uf}V_f + \sigma'_m V_m$$

To find σ'_m, we first find the strain at which the fibers would fail at s_{uf}. Assume that the fibers are linearly elastic to failure. Then

$$\epsilon_f = s_{uf}/E_f = (470 \times 10^3\ \text{psi})/(33.5 \times 10^6\ \text{psi}) = 0.014$$

At this same strain, the stress in the matrix is

$$\sigma'_m = E_m\epsilon = (0.56 \times 10^6\ \text{psi})(0.014) = 7840\ \text{psi}$$

Then, in Equation (2–8),

$$s_{uc} = (470\,000\ \text{psi})(0.30) + (7840\ \text{psi})(0.70) = 146\,500\ \text{psi}$$

The modulus of elasticity computed from Equation (2–10):

$$E_c = E_f V_f + E_m V_m = (33.5 \times 10^6)(0.30) + (0.56 \times 10^6)(0.70)$$
$$E_c = 10.4 \times 10^6\ \text{psi}$$

The specific weight is computed from Equation (2–12):

$$\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.064)(0.30) + (0.047)(0.70) = 0.052 \text{ lb/in}^3$$

Summary of Results

$$s_{uc} = 146\,500 \text{ psi}$$
$$E_c = 10.4 \times 10^6 \text{ psi}$$
$$\gamma_c = 0.052 \text{ lb/in}^3$$

Comment Note that the resulting properties for the composite are intermediate between those for the fibers and the matrix.

Design Guidelines for Members Made from Composites

The most important difference between designing with metals and designing with composites is that metals are typically taken to be homogeneous with isotropic strength and stiffness properties, whereas composites are decidedly *not* homogeneous or isotropic.

The failure modes of composite materials are complex. Tensile failure when the load is in-line with continuous fibers occurs when the individual fibers break. If the composite is made with shorter, chopped fibers, failure occurs when the fibers are pulled free from the matrix. Tensile failure when the load is perpendicular to continuous fibers occurs when the matrix itself fractures. If the fibers are in a woven form, or if a mat having shorter, randomly oriented fibers is used, other failure modes, such as fiber breakage or pullout, prevail. Such composites would have more nearly equal properties in any direction, or, as shown in Figure 2–23, multilayer laminate construction can be used.

Thus, an important design guideline to produce optimum strength is as follows:

Align the fibers with the direction of the load.

Another important failure mode is *interlaminar shear*, in which the plies of a multilayer composite separate under the action of shearing forces. The following is another design guideline:

Avoid shear loading, if possible.

Connections to composite materials are sometimes difficult to accomplish and provide places where fractures or fatigue failure could initiate. The manner of forming composites often allows the integration of several components into one part. Brackets, ribs, flanges, and the like, can be molded in along with the basic form of the part. The design guideline, then, is the following:

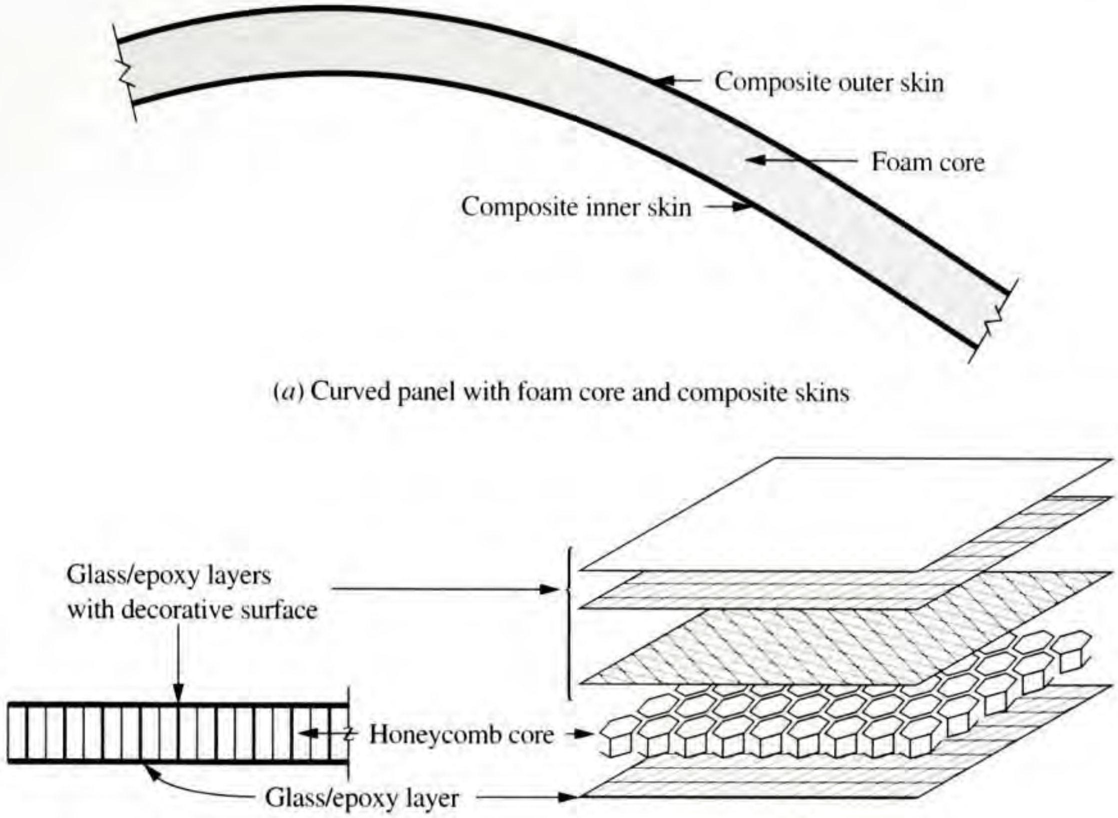
Combine several components into an integral structure.

When high panel stiffness is desired to resist flexure, as in beams or in broad panels such as floors, the designer can take advantage of the fact that the most effective material is near the outside surfaces of the panel or beam shape. Placing the high-strength fibers on these outer layers while filling the core of the shape with a light, yet rigid, material produces an efficient design in terms of weight for a given strength and stiffness. Figure 2–26 illustrates some examples of such designs. Another design guideline follows:

Use light core material covered with strong composite layers.

Because most composites use a polymeric material for the matrix, the temperatures that they can withstand are limited. Both strength and stiffness decrease as temperature in-

FIGURE 2–26
Laminated panels with lightweight cores



(*a*) Curved panel with foam core and composite skins

(*b*) Flat panel with honeycomb core and composite skins

creases. The polyimides provide better high-temperature properties [up to 600ºF (318ºC)] than most other polymer matrix materials. Epoxies are typically limited to from 250°F to 350°F (122°C–178°C). Any application above room temperature should be checked with material suppliers. The following is a design guideline:

Avoid high temperatures.

As described earlier in this section, many different fabrication techniques are used for composite materials. The shape can dictate a part's manufacturing technique. This is a good reason to implement the principles of concurrent engineering and adopt another design guideline:

Involve manufacturing considerations early in the design.

2–19 MATERIALS SELECTION

One of the most important tasks for a designer is the specification of the material from which any individual component of a product is to be made. The decision must consider a huge number of factors, many of which have been discussed in this chapter.

The process of material selection must commence with a clear understanding of the functions and design requirements for the product and the individual component. Refer to Section 1–4 in Chapter 1 for a discussion of these concepts. Then, the designer should consider the interrelationships among the following:

- The functions of the component
- The component's shape
- The material from which the component is to be made
- The manufacturing process used to produce the component

Overall requirements for the performance of the component must be detailed. This includes, for example:

- The nature of the forces applied to the component
- The types and magnitudes of stresses created by the applied forces
- The allowable deformation of the component at critical points
- Interfaces with other components of the product
- The environment in which the component is to operate
- Physical size and weight of the component
- Aesthetics expected for the component and the overall product
- Cost targets for the product as a whole and this component in particular
- Anticipated manufacturing processes available

A much more detailed list may be made with more knowledge of specific conditions.

From the results of the exercises described previously, you should develop a list of key material properties that are important. Examples often include:

1. Strength as indicated by ultimate tensile strength, yield strength, compressive strength, fatigue strength, shear strength, and others
2. Stiffness as indicated by the tensile modulus of elasticity, shear modulus of elasticity, or flexural modulus
3. Weight and mass as indicated by specific weight or density
4. Ductility as indicated by the percent elongation
5. Toughness as indicated by the impact energy (Izod, Charpy, etc.)
6. Creep performance data
7. Corrosion resistance and compatibility with the environment
8. Cost for the material
9. Cost to process the material

A list of candidate materials should then be created using your knowledge of the behavior of several material types, successful similar applications, and emerging materials technologies. A rational decision analysis should be applied to determine the most suitable types of materials from the list of candidates. This could take the form of a matrix in which data for the properties just listed for each candidate material are entered and ranked. An analysis of the complete set of data will aid in making the final decision.

More comprehensive materials selection processes are described in References 2, 21, 22, and 25.

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INTERNET SITES RELATED TO DESIGN PROPERTIES OF MATERIALS

1. **AZoM.com (The A to Z of Materials)** *www.azom.com* Materials information resource for the design community. No cost, searchable databases for metals, ceramics, polymers, and composites. Can also search by keyword, application, or industry type.
2. **Matweb** *www.matweb.com* Database of material properties for many metals, plastics, ceramics, and other engineering materials.
3. **ASM International** *www.asm-intl.org* The society for materials engineers and scientists, a worldwide network dedicated to advancing industry, technology, and applications of metals and other materials.
4. **TECHstreet** *www.techstreet.com* A store for purchasing standards for the metals industry.
5. **SAE International** *www.sae.org* The Society of Automotive Engineers, the engineering society for advancing mobility on land or sea, in air or space. A resource for technical information used in designing self-propelled vehicles. Offers standards on metals, plastics, and other materials along with components and subsystems of vehicles.
6. **ASTM International** *www.astm.org* Formerly known as the American Society for Testing and Materials. Develops and sells standards for material properties, testing procedures, and numerous other technical standards.
7. **American Iron and Steel Institute** *www.steel.org* AISI develops industry standards for steel materials and

products made from steel. Steel product manuals and industry standards are made available through the Iron & Steel Society (ISS), listed separately.

8. **Iron & Steel Society** *www.iss.org* Provides industry standards and other publications for advancing knowledge exchange in the global iron and steel industry.
9. **Aluminum Association** *www.aluminum.org* The association of the aluminum industry. Provides numerous publications that can be purchased.
10. **Alcoa, Inc.** *www.alcoa.com* A producer of aluminum and fabricated products. Website can be searched for properties of specific alloys.
11. **Copper Development Association** *www.copper.org* Provides a large searchable database of properties of wrought and cast copper, copper alloys, brasses, and bronzes. Allows searching for appropriate alloys for typical industrial uses based on several performance characteristics.
12. **Metal Powder Industries Federation** *www.mpif.org* The international trade association representing the powder metal producers. Standards and publications related to the design and production of products using powder metals.
13. **INTERZINC** *www.interzinc.com* A market development and technology transfer group dedicated to increasing awareness of zinc casting alloys. Provides design assistance, alloy selection guide, alloy properties, and descriptions of casting alloys.
14. **RAPRA Technology Limited** *www.rapra.net* Comprehensive information source for the plastics and rubber industries. Formerly Rubber and Plastics Research Association. This site also hosts the Cambridge Engineering Selector, a computerized resource using the materials selection methodology of M. F. Ashby. See Reference 2.
15. **DuPont Plastics** *www.plastics.dupont.com* Information and data on DuPont plastics and their properties. Searchable database by type of plastic or application.
16. **PolymerPlace.com** *www.polymerplace.com* Information resource for the polymer industry.
17. **Plastics Technology Online** *www.plasticstechnology.com* Online resource of Plastics Technology magazine.
18. **PLASPEC Materials Selection Database** *www.plaspec.com* Affiliated with Plastics Technology Online. Provides current articles and information about plastics injection molding, extrusion, blow molding, materials, tooling, and auxiliary equipment.
19. **Society of Plastics Engineers** *www.4spe.org* SPE promotes scientific and engineering knowledge and education about plastics and polymers worldwide.

PROBLEMS

1. Define *ultimate tensile strength.*
2. Define *yield point.*
3. Define *yield strength* and tell how it is measured.
4. What types of materials would have a yield point?
5. What is the difference between proportional limit and elastic limit?
6. Define *Hooke's law.*
7. What property of a material is a measure of its stiffness?
8. What property of a material is a measure of its ductility?
9. If a material is reported to have a percent elongation in a 2.00-in gage length of 2%, is it ductile?
10. Define *Poisson's ratio.*
11. If a material has a tensile modulus of elasticity of 114 GPa and a Poisson's ratio of 0.33, what is its modulus of elasticity in shear?
12. A material is reported to have a Brinell hardness of 525. What is its approximate hardness on the Rockwell C scale?
13. A steel is reported to have a Brinell hardness of 450. What is its approximate tensile strength?

For Problems 14–17, describe what is wrong with each statement.

14. "After annealing, the steel bracket had a Brinell hardness of 750."
15. "The hardness of that steel shaft is HRB 120."
16. "The hardness of that bronze casting is HRC 12."
17. "Based on the fact that this aluminum plate has a hardness of HB 150, its approximate tensile strength is 75 ksi."
18. Name two tests used to measure impact energy.
19. What are the principal constituents in steels?
20. What are the principal alloying elements in AISI 4340 steel?
21. How much carbon is in AISI 4340 steel?
22. What is the typical carbon content of a low-carbon steel? Of a medium-carbon steel? Of a high-carbon steel?
23. How much carbon does a bearing steel typically contain?
24. What is the main difference between AISI 1213 steel and AISI 12L13 steel?
25. Name four materials that are commonly used for shafts.
26. Name four materials that are typically used for gears.

27. Describe the properties desirable for the auger blades of a post hole digger, and suggest a suitable material.
28. Appendix 3 lists AISI 5160 OQT 1000. Describe the basic composition of this material, how it was processed, and its properties in relation to other steels listed in that table.
29. If a shovel blade is made from AISI 1040 steel, would you recommend flame hardening to give its edge a surface hardness of HRC 40? Explain.
30. Describe the differences between through-hardening and carburizing.
31. Describe the process of induction hardening.
32. Name 10 steels used for carburizing. What is their approximate carbon content prior to carburizing?
33. What types of stainless steels are nonmagnetic?
34. What is the principal alloying element that gives a stainless steel corrosion resistance?
35. Of what material is a typical wide-flange beam made?
36. With regard to structural steels, what does the term *HSLA* mean? What strengths are available in HSLA steel?
37. Name three types of cast iron.
38. Describe the following cast iron materials according to type, tensile strength, yield strength, ductility, and stiffness:

    ASTM A48-83, Grade 30

    ASTM A536-84, Grade 100-70-03

    ASTM A47-84, Grade 35018

    ASTM A220-88, Grade 70003
39. Describe the process of making parts from powdered metals.
40. What properties are typical for parts made from Zamak 3 zinc casting alloy?
41. What are the typical uses for Group D tool steels?
42. What does the suffix *O* in aluminum 6061-O represent?
43. What does the suffix *H* in aluminum 3003-H14 represent?
44. What does the suffix *T* in aluminum 6061-T6 represent?
45. Name the aluminum alloy and condition that has the highest strength of those listed in Appendix 9.
46. Which is one of the most versatile aluminum alloys for mechanical and structural uses?
47. Name three typical uses for titanium alloys.
48. What is the principal constituent of bronze?
49. Describe the bronze having the UNS designation C86200.
50. Name two typical uses for bronze in machine design.
51. Describe the difference between thermosetting plastics and thermoplastics.
52. Suggest a suitable plastic material for each of the following uses:
    - **(a)** Gears
    - **(b)** Football helmets
    - **(c)** Transparent shield
    - **(d)** Structural housing
    - **(e)** Pipe
    - **(f)** Wheels
    - **(g)** Electrical switch-gear, structural part
53. Name eight factors over which the designer has control when specifying a composite material.
54. Define the term *composite*.
55. Name four base resins often used for composite materials.
56. Name four types of reinforcement fibers used for composite materials.
57. Name three types of composite materials used for sporting equipment, such as tennis rackets, golf clubs, and skis.
58. Name three types of composite materials used for aircraft and aerospace structures.
59. What base resin and reinforcement are typically used for sheet-molding compound (SMC)?
60. For what applications are sheet-molding compounds used?
61. Describe six forms in which reinforcing fibers are produced.
62. Describe *wet processing* of composite materials.
63. Describe *preimpregnated materials*.
64. Describe the production processing of sheet-molding compounds.
65. Describe *pultrusion*, and list four shapes produced by this process.
66. Describe *filament winding* and four types of products made by this process.
67. Define the term *specific strength* as it is applied to structural materials.
68. Define the term *specific stiffness* as it is applied to structural materials.
69. Discuss the advantages of composite materials relative to metals with regard to specific strength and specific stiffness.
70. Compare the specific strength of AISI 1020 hot-rolled steel with that of AISI 5160 OQT 700 steel, the two aluminum alloys 6061-T6 and 7075-T6, and titanium Ti-6Al-4V.
71. Compare the specific stiffness of AISI 1020 hot-rolled steel with that of AISI 5160 OQT 700 steel, the two aluminum alloys 6061-T6 and 7075-T6, and titanium Ti-6Al-4V.
72. Compare the specific strengths of each of the five composite materials shown in Figure 2–21 with that of AISI 1020 hot-rolled steel.
73. Compare the specific stiffness of each of the five composite materials shown in Figure 2–21 with that of AISI 1020 hot-rolled steel.
74. Describe the general construction of a composite material identified as $[0/+30/-30/90]$.

75. List and discuss six design guidelines for the application of composite materials.

76. Why is it desirable to form a composite material in layers or plies with the angle of orientation of the different plies in different directions?

77. Why is it desirable to form a composite structural element with relatively thin skins of the stronger composite material over a core of light foam?

78. Describe why concurrent engineering and early manufacturing involvement are important when you are designing parts made from composite materials.

Internet-Based Assignments

79. Use the Matweb website to determine at least three appropriate materials for a shaft design. An alloy steel is preferred with a minimum yield strength of 150 ksi (1035 MPa) and a good ductility as represented by an elongation of 10% or greater.

80. Use the Matweb website to determine at least three appropriate plastic materials for use as a cam. The materials should have good strength properties and a high toughness.

81. Use the DuPont Plastics website to determine at least three appropriate plastic materials for use as a cam. The materials should have good strength properties and a high toughness.

82. Use the DuPont Plastics website to determine at least three appropriate plastic materials for use as a housing for an industrial product. Moderate strength, high rigidity, and high toughness are required.

83. Use the Alcoa website to determine at least three appropriate aluminum alloys for a mechanical component that requires moderate strength, good machinability, and good corrosion resistance.

84. Use the INTERZINC website to determine at least three appropriate zinc casting alloys for a structural component that requires good strength and that is recommended for die casting.

85. Use the Copper Development Association website to recommend at least three copper alloys for a wormgear. Good strength and ductility are desirable along with good wear properties.

86. Use the Copper Development Association website to recommend at least three copper alloys for a bearing application. Moderate strength and good friction and wear properties are required.

87. Locate the description of the ASTM Standard A992 structural steel that is commonly used for rolled steel beam shapes. Determine how to acquire a copy of the standard.

3

Stress and Deformation Analysis

The Big Picture

Stress and Deformation Analysis

Discussion Map

- ☐ As a designer you are responsible for ensuring the safety of the components and systems you design.
- ☐ You must apply your prior knowledge of the principles of strength of materials.

Discover

How could consumer products and machines fail?

Describe some product failures you have seen.

This chapter presents a brief review of the fundamentals of stress analysis. It will help you design products that do not fail, and it will prepare you for other topics later in this book.

A designer is responsible for ensuring the safety of the components and systems that he or she designs. Many factors affect safety, but one of the most critical aspects of design safety is that the level of stress to which a machine component is subjected must be safe under reasonably foreseeable conditions. This principle implies, of course, that nothing actually breaks. Safety may also be compromised if components are permitted to deflect excessively, even though nothing breaks.

You have already studied the principles of strength of materials to learn the fundamentals of stress analysis. Thus, at this point, you should be competent to analyze load-carrying members for stress and deflection due to direct tensile and compressive loads, direct shear, torsional shear, and bending.

Think, now, about consumer products and machines with which you are familiar, and try to explain how they *could fail*. Of course, we do not expect them to fail, because most such products are well designed. But some do fail. Can you recall any? How did they fail? What were the operating conditions when they failed? What was the material of the components that failed? Can you visualize and describe the kinds of loads that were placed on the components that failed? Were they subjected to bending, tension, compression, shear, or torsion? Could there have been more than one type of stress acting at the same time? Are there evidences of accidental overloads? Should such loads have been anticipated by the designer? Could the failure be due to the manufacture of the product rather than its design?

Talk about product and machine failures with your associates and your instructor. Consider parts of your car, home appliances, lawn maintenance equipment, or equipment where you have worked. If possible, bring failed components to the meetings with your associates, and discuss the components and their failure.

Most of this book emphasizes developing special methods to analyze and design machine elements. These methods are all based on the fundamentals of stress analysis, and it is assumed that you have completed a course in strength of materials. This chapter presents a brief review of the fundamentals. (See References 1, 3, 4, and 6.)

You Are the Designer

You are the designer of a utility crane that might be used in an automotive repair facility, in a manufacturing plant, or on a mobile unit such as a truck bed. Its function is to raise heavy loads. A schematic layout of one possible configuration of the crane is shown in Figure 3–1. It is comprised of four primary load-carrying members, labeled 1, 2, 3, and 4. These members are connected to each other with pin-type joints at A, B, C, D, E, and F. The load is applied to the end of the horizontal boom, member 4. Anchor points for the crane are provided at joints A and B that carry the loads from the crane to a rigid structure. Note that this is a simplified view of the crane showing only the primary structural components and the forces in the plane of the applied load. The crane would also need stabilizing members in the plane perpendicular to the drawing.

You will need to analyze the kinds of forces that are exerted on each of the load-carrying members before you can design them. This calls for the use of the principles of statics in which you should have already gained competence. The following discussion provides a review of some of the key principles you will need in this course.

Your work as a designer proceeds as follows:

1. Analyze the forces that are exerted on each load-carrying member using the principles of statics.
2. Identify the kinds of stresses that each member is subjected to by the applied forces.
3. Propose the general shape of each load-carrying member and the material from which each is to be made.
4. Complete the stress analysis for each member to determine its final dimensions.

Let's work through steps 1 and 2 now as a review of statics. You will improve your ability to do steps 3 and 4 as you perform several practice problems in this chapter and in Chapters 4 and 5 by reviewing strength of materials and adding competencies that build on that foundation.

Force Analysis:

One approach to the force analysis is outlined here.

1. Consider the entire crane structure as a free-body with the applied force acting at point G and the reactions acting at support points A and B. See Figure 3–2, which shows these forces and important dimensions of the crane structure.

FIGURE 3–1 Schematic layout of a crane

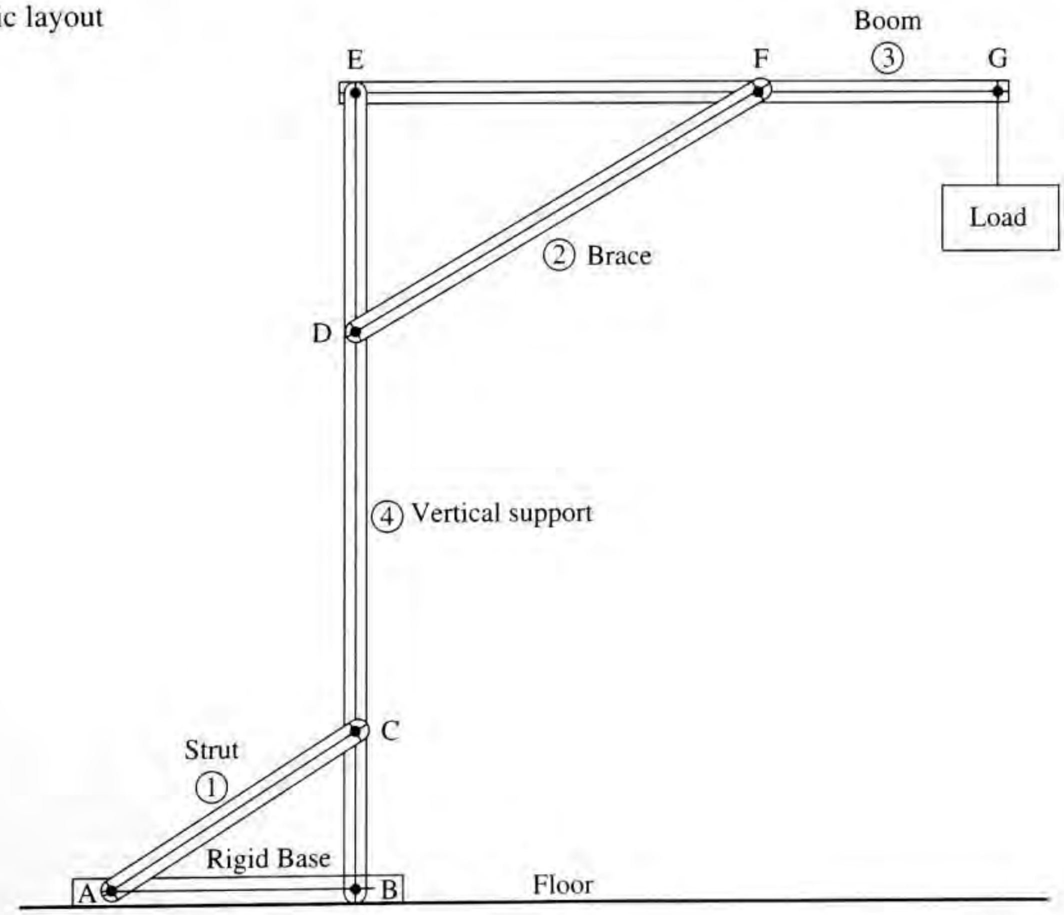
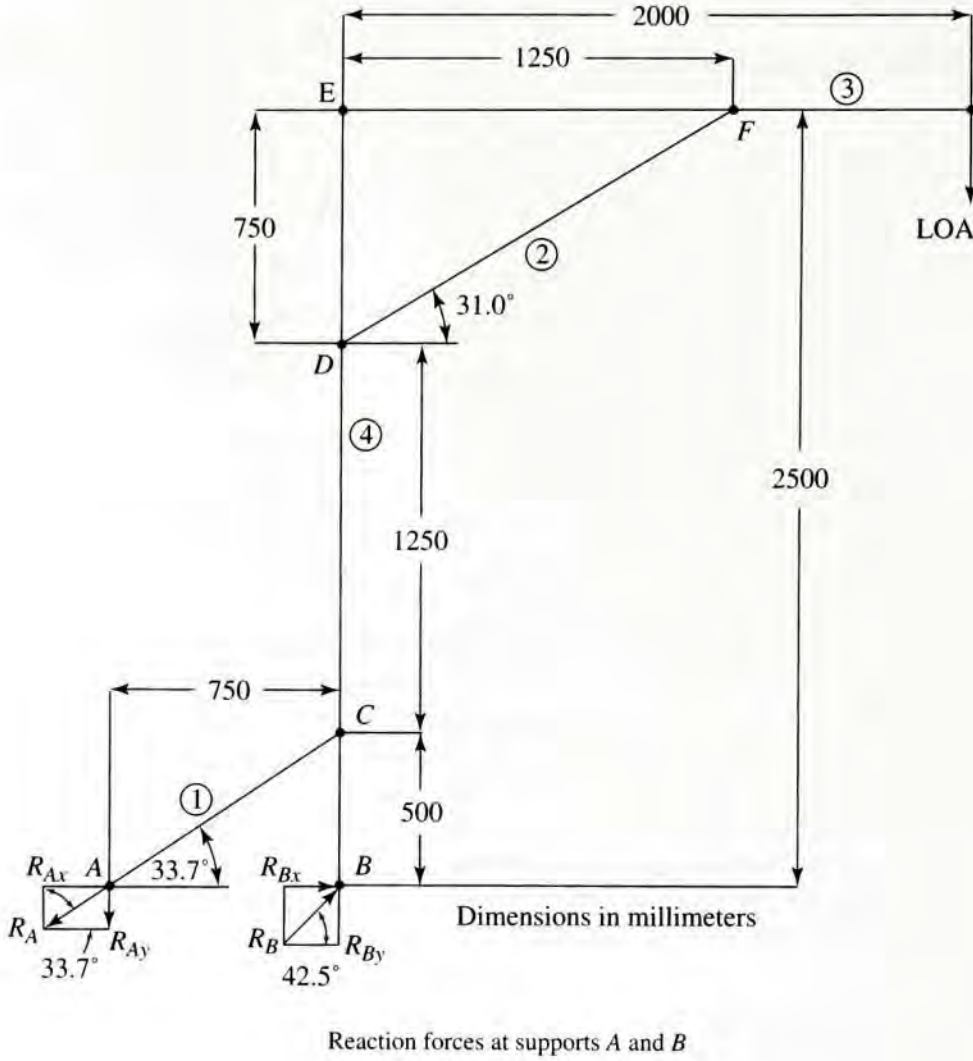


FIGURE 3–2 Free-body diagram of complete crane structure



2. Break the structure apart so that each member is represented as a free-body diagram, showing all forces acting at each joint. See the result in Figure 3–3.
3. Analyze the magnitudes and directions of all forces.

Comments are given here to summarize the methods used in the static analysis and to report results. You should work through the details of the analysis yourself or with colleagues to ensure that you can perform such calculations. All of the forces are directly proportional to the applied force F. We will show the results with an assumed value of $F = 10.0$ kN (approximately 2250 lb).

Step 1: The pin joints at A and B can provide support in any direction. We show the x and y components of the reactions in Figure 3–2. Then, proceed as follows:

1. Sum moments about B to find $R_{Ay} = 2.667\ F = 26.67$ kN
2. Sum forces in the vertical direction to find $R_{By} = 3.667\ F = 36.67$ kN.

At this point we need to recognize that the strut AC is pin-connected at each end and carries loads only at its ends. Therefore, it is a *two-force member,* and the direction of the total force, R_A, acts along the member itself. Then R_{Ay} and R_{Ax} are the rectangular components of R_A as shown in the lower left of Figure 3–2. We can then say that

$$\tan(33.7^\circ) = R_{Ay}/R_{Ax}$$

and then

$$R_{Ax} = R_{Ay}/\tan(33.7^\circ) = 26.67\text{ kN}/\tan(33.7^\circ) = 40.0\text{ kN}$$

The total force, R_A, can be computed from the Pythagorean theorem,

$$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{(40.0)^2 + (26.67)^2} = 48.07\text{ kN}$$

This force acts along the strut AC, at an angle of 33.7° above the horizontal, and it is the force that tends to shear the pin in joint A. The force at C on the strut AC is also 48.07 kN acting upward to the right to balance R_A on the two-force member as shown in Figure 3–3. Member AC is therefore in pure tension.

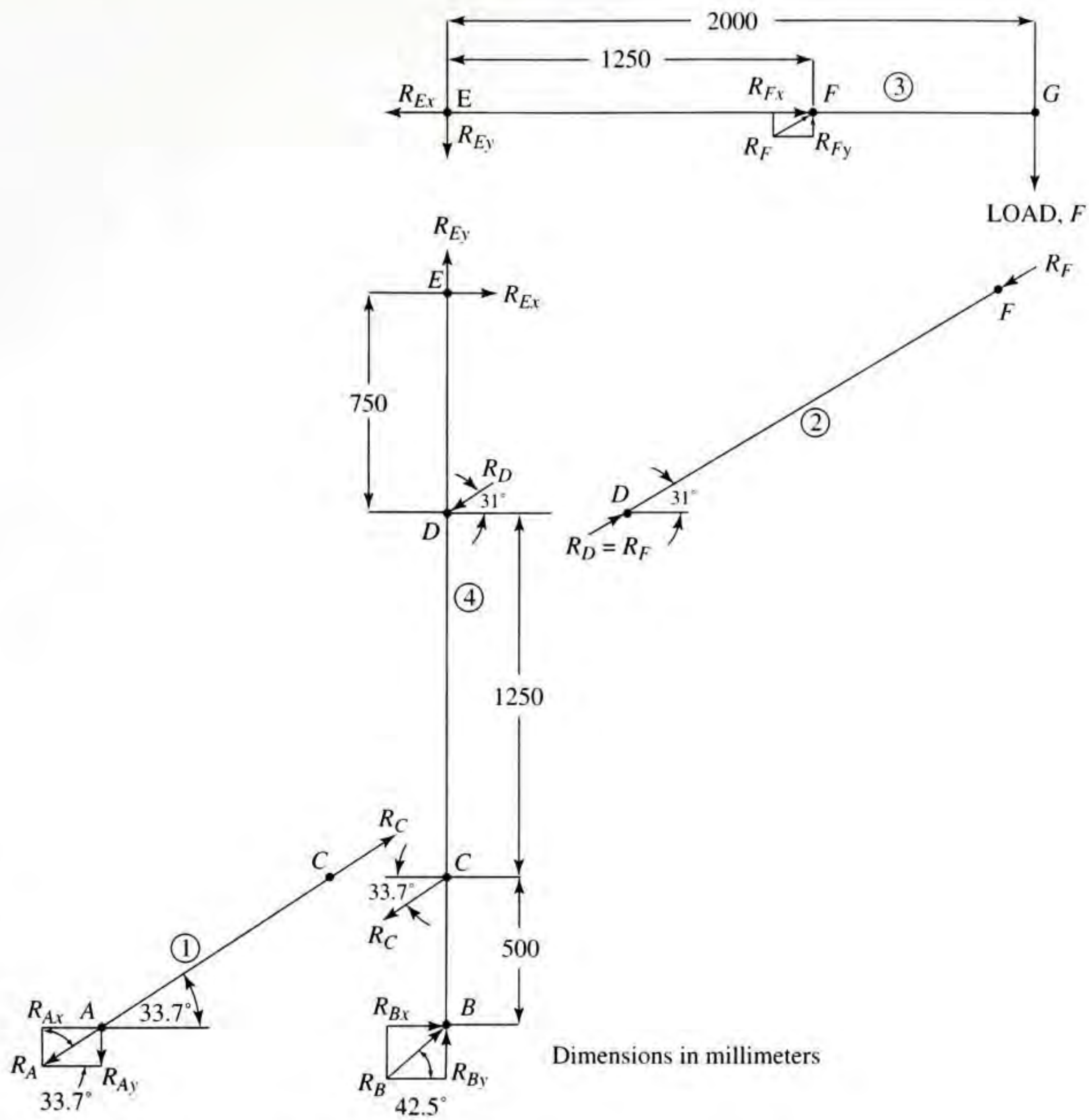


FIGURE 3–3 Free-body diagrams of each component of the crane

We can now use the sum of the forces in the horizontal direction on the entire structure to show that $R_{Ax} = R_{Bx} = 40.0$ kN. The resultant of R_{Bx} and R_{By} is 54.3 kN acting at an angle of 42.5° above the horizontal, and it is the total shearing force on the pin in joint B. See the diagram in the lower right of Figure 3–2.

Step 2: The set of free-body diagrams is shown in Figure 3–3.

Step 3: Now consider the free-body diagrams of all of the members in Figure 3–3. We have already discussed member 1, recognizing it as a two-force member in tension carrying forces R_A and R_C equal to 48.07 kN. The reaction to R_C acts on the vertical member 4.

Now note that member 2 is also a two-force member, but it is in compression rather than tension. Therefore we know that the forces on points D and F are equal and that they act in line with member 2, 31.0° with respect to the horizontal. The reactions to these forces, then, act at point D on the vertical support, member 4, and at point F on the horizontal boom, member 3. We can find the value of R_F by considering the free-body diagram of member 3. You should be able to verify the following results using the methods already demonstrated.

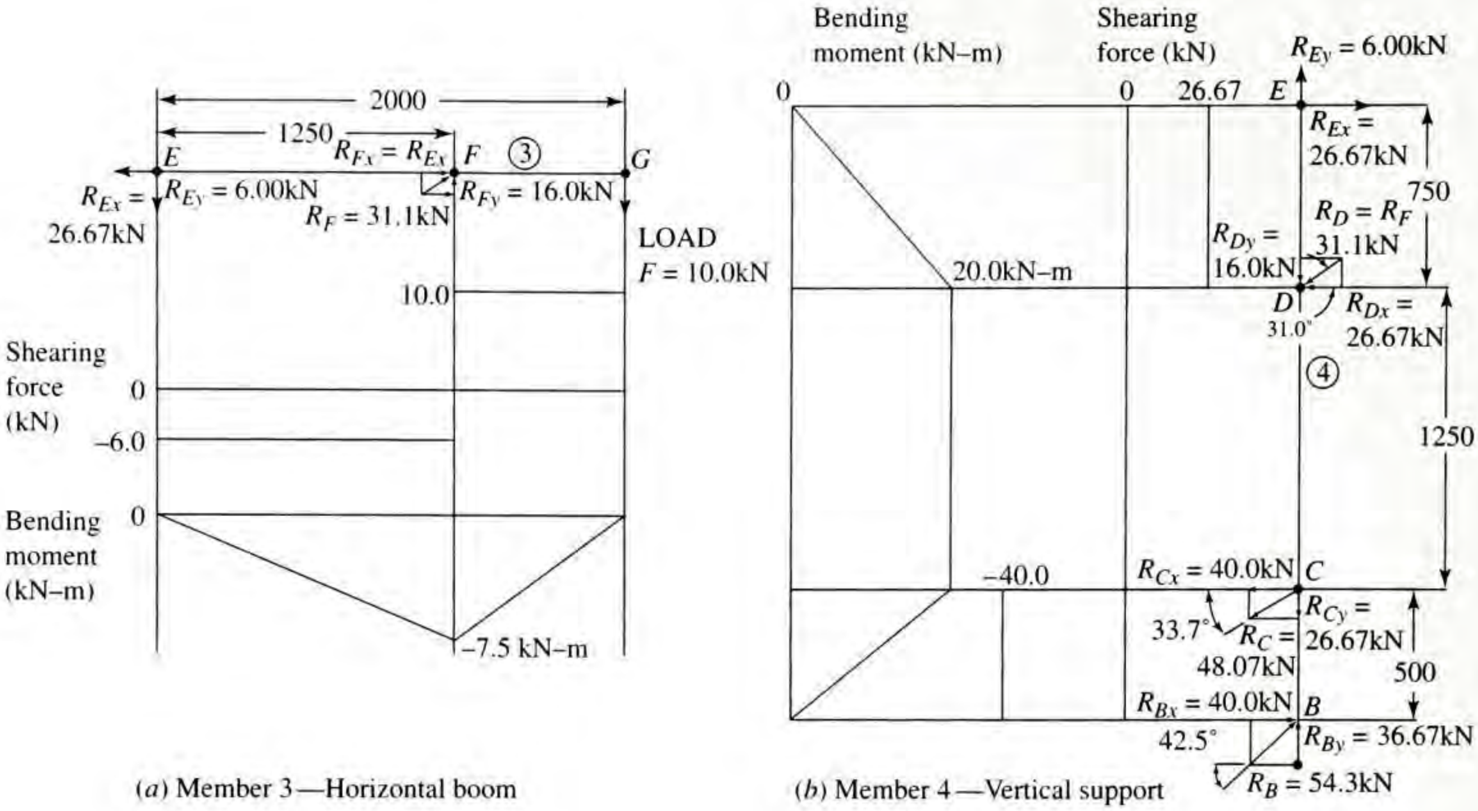


FIGURE 3–4 Shearing force and bending moment diagrams for members 3 and 4

$$R_{Fy} = 1.600\ F = (1.600)(10.0\ \text{kN}) = 16.00\ \text{kN}$$
$$R_{Fx} = 2.667\ F = (2.667)(10.0\ \text{kN}) = 26.67\ \text{kN}$$
$$R_F = 3.110\ F = (3.110)(10.0\ \text{kN}) = 31.10\ \text{kN}$$
$$R_{Ey} = 0.600\ F = (0.600)(10.0\ \text{kN}) = 6.00\ \text{kN}$$
$$R_{Ex} = 2.667\ F = (2.667)(10.0\ \text{kN}) = 26.67\ \text{kN}$$
$$R_E = 2.733\ F = (2.733)(10.0\ \text{kN}) = 27.33\ \text{kN}$$

Now all forces on the vertical member 4 are known from earlier analyses using the principle of action-reaction at each joint.

Types of Stresses on Each Member:

Consider again the free-body diagrams in Figure 3–3 to visualize the kinds of stresses that are created in each member. This will lead to the use of particular kinds of stress analysis as the design process is completed. Members 3 and 4 carry forces perpendicular to their long axes and, therefore, they act as beams in bending. Figure 3–4 shows these members with the additional shearing force and bending moment diagrams. You should have learned to prepare such diagrams in the prerequisite study of strength of materials. The following is a summary of the kinds of stresses in each member.

Member 1: The strut is in pure tension.

Member 2: The brace is in pure compression. Column buckling should be checked.

Member 3: The boom acts as a beam in bending. The right end between *F* and *G* is subjected to bending stress and vertical shear stress. Between *E* and *F* there is bending and shear combined with an axial tensile stress.

Member 4: The vertical support experiences a complex set of stresses depending on the segment being considered as described here.

Between *E* and *D*: Combined bending stress, vertical shear stress, and axial tension.

Between *D* and *C*: Combined bending stress and axial compression.

Between *C* and *B*: Combined bending stress, vertical shear stress, and axial compression.

Pin Joints: The connections between members at each joint must be designed to resist the total reaction force acting at each, computed in the earlier analysis. In general, each connection will likely include a cylindrical pin connecting two parts. The pin will typically be in direct shear.

3–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will:

1. Have reviewed the principles of stress and deformation analysis for several kinds of stresses, including the following:

   Direct tension and compression

   Direct shear

   Torsional shear for both circular and noncircular sections

   Vertical shearing stresses in beams

   Bending

2. Be able to interpret the nature of the stress at a point by drawing the *stress element* at any point in a load-carrying member for a variety of types of loads.
3. Have reviewed the importance of the *flexural center* of a beam cross section with regard to the alignment of loads on beams.
4. Have reviewed beam-deflection formulas.
5. Be able to analyze beam-loading patterns that produce abrupt changes in the magnitude of the bending moment in the beam.
6. Be able to use the principle of superposition to analyze machine elements that are subjected to loading patterns that produce combined stresses.
7. Be able to properly apply stress concentration factors in stress analyses.

3–2 PHILOSOPHY OF A SAFE DESIGN

In this book, every design approach will ensure that the stress level is below yield in ductile materials, automatically ensuring that the part will not break under a static load. For brittle materials, we will ensure that the stress levels are well below the ultimate tensile strength. We will also analyze deflection where it is critical to safety or performance of a part.

Two other failure modes that apply to machine members are fatigue and wear. *Fatigue* is the response of a part subjected to repeated loads (see Chapter 5). *Wear* is discussed within the chapters devoted to the machine elements, such as gears, bearings, and chains, for which it is a major concern.

3–3 REPRESENTING STRESSES ON A STRESS ELEMENT

One major goal of stress analysis is to determine *the point* within a load-carrying member that is subjected to the highest stress level. You should develop the ability to visualize a *stress element*, a single, infinitesimally small cube from the member in a highly stressed area, and to show vectors that represent the kind of stresses that exist on that element. The orientation of the stress element is critical, and it must be aligned with specified axes on the member, typically called x, y, and z.

Figure 3–5 shows three examples of stress elements with three basic fundamental kinds of stress: tensile, compressive, and shear. Both the complete three-dimensional cube and the simplified, two-dimensional square forms for the stress elements are shown. The square is one face of the cube in a selected plane. The sides of the square represent the projections of the faces of the cube that are perpendicular to the selected plane. It is recommended that you visualize the cube form first and then represent a square stress element showing stresses on a particular plane of interest in a given problem. In some problems with more general states of stress, two or three square stress elements may be required to depict the complete stress condition.

Tensile and compressive stresses, called *normal stresses*, are shown acting perpendicular to opposite faces of the stress element. Tensile stresses tend to pull on the element, whereas compressive stresses tend to crush it.

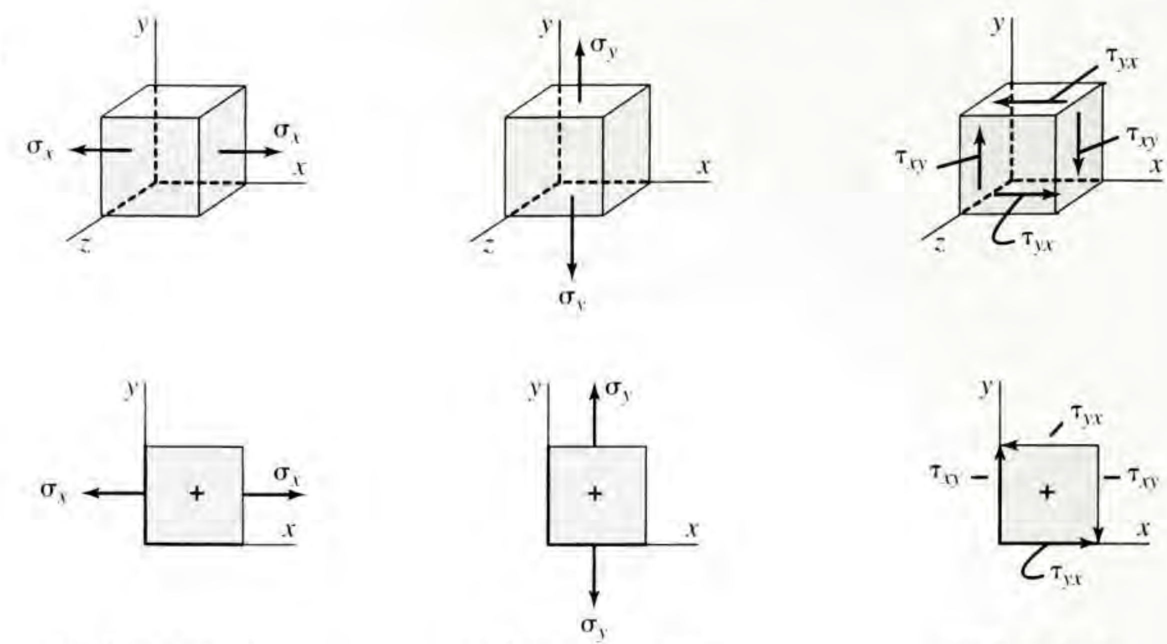


FIGURE 3–5 Stress elements for three types of stresses

(*a*) Direct tension (*b*) Direct compression (*c*) Pure shear

Shear stresses are created by direct shear, vertical shear in beams, or torsion. In each case, the action on an element subjected to shear is a tendency to *cut* the element by exerting a stress downward on one face while simultaneously exerting a stress upward on the opposite, parallel face. This action is that of a simple pair of shears or scissors. But note that if only one pair of shear stresses acts on a stress element, it will not be in equilibrium. Rather, it will tend to spin because the pair of shear stresses forms a couple. To produce equilibrium, a second pair of shear stresses on the other two faces of the element must exist, acting in a direction that opposes the first pair.

In summary, shear stresses on an element will always be shown as two pairs of equal stresses acting on (parallel to) the four sides of the element. Figure 3–5(c) shows an example.

Sign Convention for Shear Stresses

This book adopts the following convention:

Positive shear stresses tend to rotate the element in a clockwise direction.

Negative shear stresses tend to rotate the element in a counterclockwise direction.

A double subscript notation is used to denote shear stresses in a plane. For example, in Figure 3–5(c), drawn for the x-y plane, the pair of shear stresses, τ_{xy}, indicates a shear stress acting on the element face that is perpendicular to the x-axis and parallel to the y-axis. Then τ_{yx} acts on the face that is perpendicular to the y-axis and parallel to the x-axis. In this example, τ_{xy} is positive and τ_{yx} is negative.

3–4 DIRECT STRESSES: TENSION AND COMPRESSION

Stress can be defined as the internal resistance offered by a unit area of a material to an externally applied load. *Normal stresses* (σ) are either *tensile* (positive) or *compressive* (negative).

For a load-carrying member in which the external load is uniformly distributed across the cross-sectional area of the member, the magnitude of the stress can be calculated from the direct stress formula:

Direct Tensile or Compressive Stress

$$\sigma = \text{force/area} = F/A \qquad \textbf{(3–1)}$$

The units for stress are always *force per unit area*, as is evident from Equation 3–1. Common units in the U.S. Customary system and the SI metric system follow.

U.S. Customary Units	*SI Metric Units*
lb/in^2 = psi	N/m^2 = pascal = Pa
$kips/in^2$ = ksi	N/mm^2 = megapascal = 10^6 Pa = MPa
Note: 1.0 kip = 1000 lb	
1.0 ksi = 1000 psi	

Example Problem 3–1 A tensile force of 9500 N is applied to a 12-mm-diameter round bar, as shown in Figure 3–6. Compute the direct tensile stress in the bar.

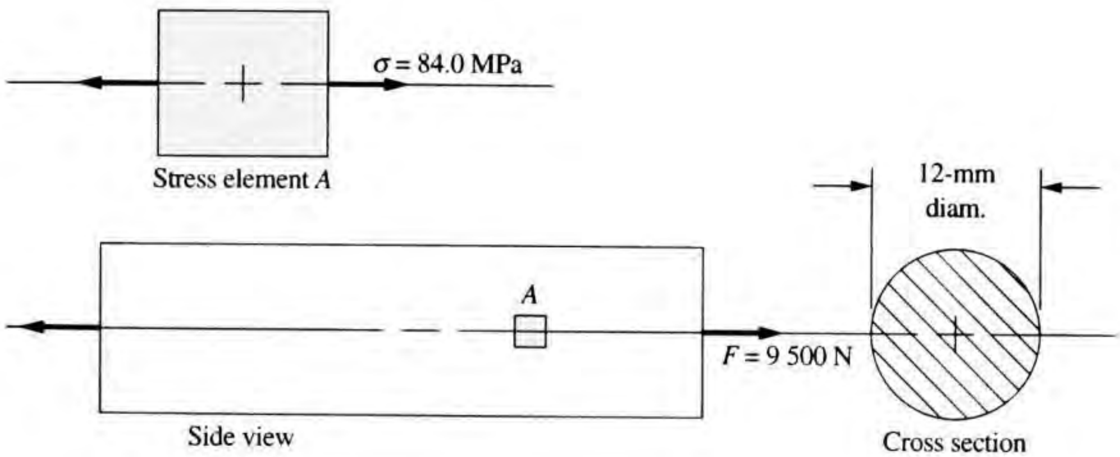


FIGURE 3–6 Tensile stress in a round bar

Solution Objective Compute the tensile stress in the round bar.

Given Force = F = 9500 N; diameter = D = 12 mm.

Analysis Use the direct tensile stress formula, Equation (3–1): $\sigma = F/A$. Compute the cross-sectional area from $A = \pi D^2/4$.

Results $A = \pi D^2/4 = \pi(12\text{ mm})^2/4 = 113\text{ mm}^2$

$\sigma = F/A = (9500\text{ N})/(113\text{ mm}^2) = 84.0\text{ N/mm}^2 = 84.0\text{ MPa}$

Comment The results are shown on stress element A in Figure 3–6, which can be taken to be anywhere within the bar because, ideally, the stress is uniform on any cross section. The cube form of the element is as shown in Figure 3–5 (a).

The conditions on the use of Equation (3–1) are as follows:

1. The load-carrying member must be straight.
2. The line of action of the load must pass through the centroid of the cross section of the member.

3. The member must be of uniform cross section near where the stress is being computed.
4. The material must be homogeneous and isotropic.
5. In the case of compression members, the member must be short to prevent buckling. The conditions under which buckling is expected are discussed in Chapter 6.

3–5 DEFORMATION UNDER DIRECT AXIAL LOADING

The following formula computes the stretch due to a direct axial tensile load or the shortening due to a direct axial compressive load:

Deformation Due to Direct Axial Load

$$\delta = FL/EA \qquad \textbf{(3–2)}$$

where δ = total deformation of the member carrying the axial load
F = direct axial load
L = original total length of the member
E = modulus of elasticity of the material
A = cross-sectional area of the member

Noting that $\sigma = F/A$, we can also compute the deformation from

$$\delta = \sigma L/E \qquad \textbf{(3–3)}$$

Example Problem 3–2 For the round bar subjected to the tensile load shown in Figure 3–6, compute the total deformation if the original length of the bar is 3600 mm. The bar is made from a steel having a modulus of elasticity of 207 GPa.

Solution Objective: Compute the deformation of the bar.

Given: Force = F = 9500 N; diameter = D = 12 mm.
Length = L = 3600 mm; E = 207 GPa

Analysis: From Example Problem 3–1, we found that σ = 84.0 MPa. Use Equation (3–3).

Results:

$$\delta = \frac{\sigma L}{E} = \frac{(84.0 \times 10^6\,\text{N/m}^2)\,(3600\text{ mm})}{(207 \times 10^9\,\text{N/m}^2)} = 1.46\text{ mm}$$

3–6 DIRECT SHEAR STRESS

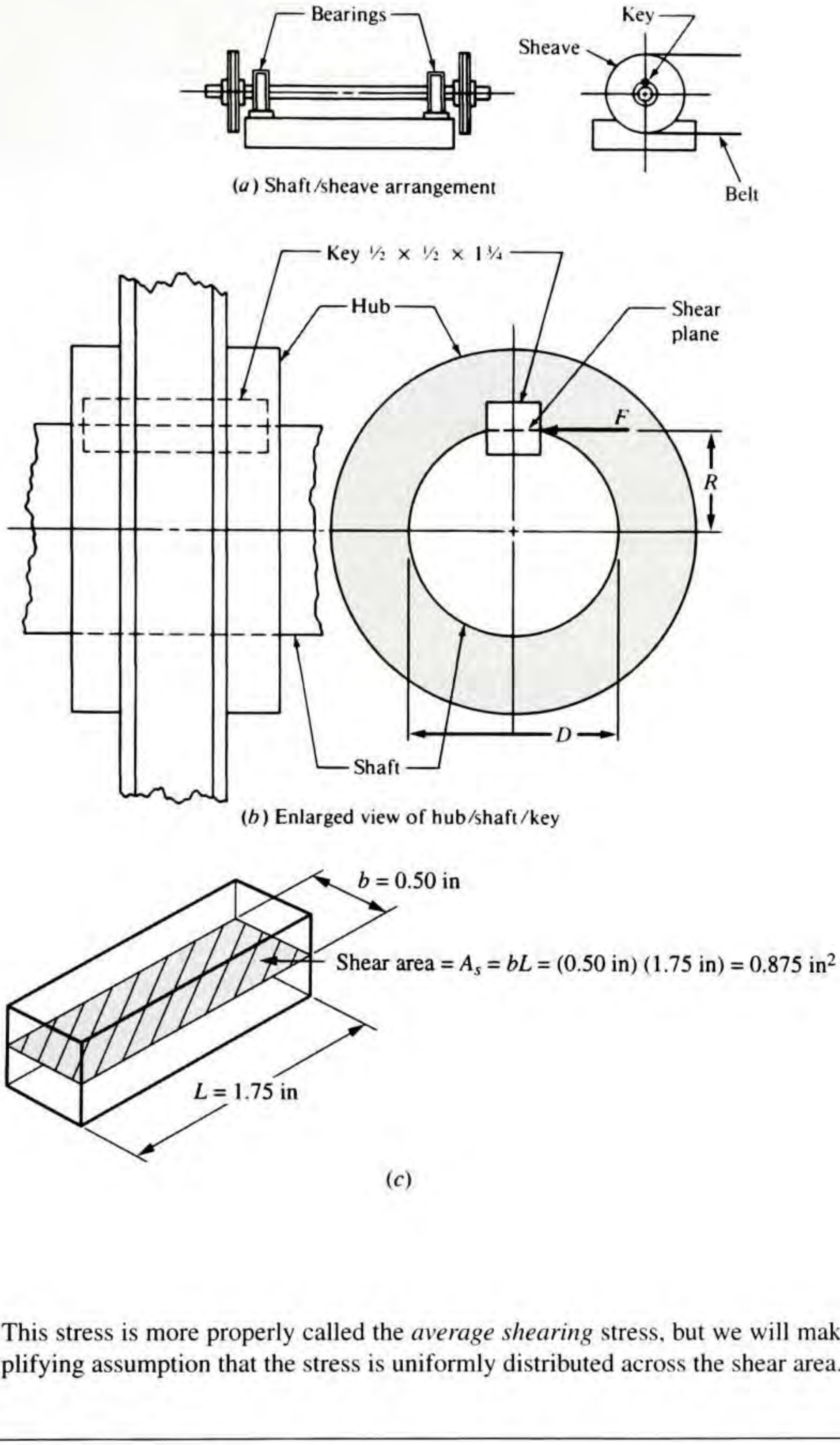
Direct shear stress occurs when the applied force tends to cut through the member as scissors or shears do or when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine design is the tendency for a key to be sheared off at the section between the shaft and the hub of a machine element when transmitting torque. Figure 3–7 shows the action.

The method of computing direct shear stress is similar to that used for computing direct tensile stress because the applied force is assumed to be uniformly distributed across the cross section of the part that is resisting the force. But the kind of stress is *shear stress* rather than *normal stress*. The symbol used for shear stress is the Greek letter tau (τ). The formula for direct shear stress can thus be written

Direct Shear Stress

$$\tau = \text{shearing force/area in shear} = F/A_s \qquad \textbf{(3–4)}$$

FIGURE 3–7 Direct shear on a key



This stress is more properly called the *average shearing* stress, but we will make the simplifying assumption that the stress is uniformly distributed across the shear area.

Example Problem 3–3 Figure 3–7 shows a shaft carrying two sheaves that are keyed to the shaft. Part (b) shows that a force F is transmitted from the shaft to the hub of the sheave through a square key. The shaft is 2.25 inches in diameter and transmits a torque of 14 063 lb.in. The key has a square cross section, 0.50 in on a side, and a length of 1.75 in. Compute the force on the key and the shear stress caused by this force.

Solution Objective Compute the force on the key and the shear stress.

Given Layout of shaft, key, and hub shown in Figure 3–7.

Torque $= T = 14\,063$ lb·in; key dimensions $= 0.5 \times 0.5 \times 1.75$ in.

Shaft diameter $= D = 2.25$ in; radius $= R = D/2 = 1.125$ in.

Analysis Torque T = force $F \times$ radius R. Then $F = T/R$.

Use equation (3–4) to compute shearing stress: $\tau = F/A_s$.

Shear area is the cross section of the key at the interface between the shaft and the hub: $A_s = bL$.

Results $F = T/R = (14\,063\ \text{lb}\cdot\text{in})/(1.125\ \text{in}) = 12\,500\ \text{lb}$

$A_s = bL = (0.50\ \text{in})(1.75\ \text{in}) = 0.875\ \text{in}^2$

$\tau = F/A = (12\,500\ \text{lb})/(0.875\ \text{in}^2) = 14\,300\ \text{lb/in}^2$

Comment This level of shearing stress will be uniform on all parts of the cross section of the key.

3–7 RELATIONSHIP AMONG TORQUE, POWER, AND ROTATIONAL SPEED

The relationship among the power *(P)*, the rotational speed *(n)*, and the torque *(T)* in a shaft is described by the equation

Power-Torque-Speed Relationship

$$T = P/n \tag{3–5}$$

In SI units, power is expressed in the unit of *watt* (W) or its equivalent, *newton meter per second* (N·m/s), and the rotational speed is in *radians per second* (rad/s).

Example Problem 3–4 Compute the amount of torque in a shaft transmitting 750 W of power while rotating at 183 rad/s. (*Note:* This is equivalent to the output of a 1.0-hp, 4-pole electric motor, operating at its rated speed of 1750 rpm. See Chapter 21.)

Solution Objective Compute the torque T in the shaft.

Given Power $= P = 750$ W $= 750$ N·m/s.

Rotational speed $= n = 183$ rad/s.

Analysis Use Equation (3–5).

Results $T = P/n = (750\ \text{N}\cdot\text{m/s})/(183\ \text{rad/s})$

$T = 4.10\ \text{N}\cdot\text{m/rad} = 4.10\ \text{N}\cdot\text{m}$

Comments In such calculations, the unit of N·m/rad is dimensionally correct, and some advocate its use. Most, however, consider the radian to be dimensionless, and thus torque is expressed in N·m or other familiar units of force times distance.

In the U.S. Customary Unit System, power is typically expressed as *horsepower*, equal to 550 ft · lb/s. The typical unit for rotational speed is rpm, or revolutions per minute. But the most convenient unit for torque is the pound-inch (lb · in). Considering all of these quantities and making the necessary conversions of units, we use the following formula to compute the torque (in lb · in) in a shaft carrying a certain power P (in hp) while rotating at a speed of n rpm.

P-T-n Relationship for U.S. Customary Units

$$T = 63\,000\,P/n \tag{3–6}$$

The resulting torque will be in pound-inches. You should verify the value of the constant, 63 000.

Example Problem 3–5 Compute the torque on a shaft transmitting 1.0 hp while rotating at 1750 rpm. Note that these conditions are approximately the same as those for which the torque was computed in Example Problem 3–4 using SI units.

Solution Objective: Compute the torque in the shaft.

Given: $P = 1.0$ hp; $n = 1750$ rpm.

Analysis: Use Equation (3–6).

Results: $T = 63\,000\,P/n = [63\,000(1.0)]/1750 = 36.0\ \text{lb}\cdot\text{in}$

3–8 TORSIONAL SHEAR STRESS

When a *torque*, or twisting moment, is applied to a member, it tends to deform by twisting, causing a rotation of one part of the member relative to another. Such twisting causes a shear stress in the member. For a small element of the member, the nature of the stress is the same as that experienced under direct shear stress. However, in *torsional shear*, the distribution of stress is not uniform across the cross section.

The most frequent case of torsional shear in machine design is that of a round circular shaft transmitting power. Chapter 12 covers shaft design.

Torsional Shear Stress Formula

When subjected to a torque, the outer surface of a solid round shaft experiences the greatest shearing strain and therefore the largest torsional shear stress. See Figure 3–8. The value of the maximum torsional shear stress is found from

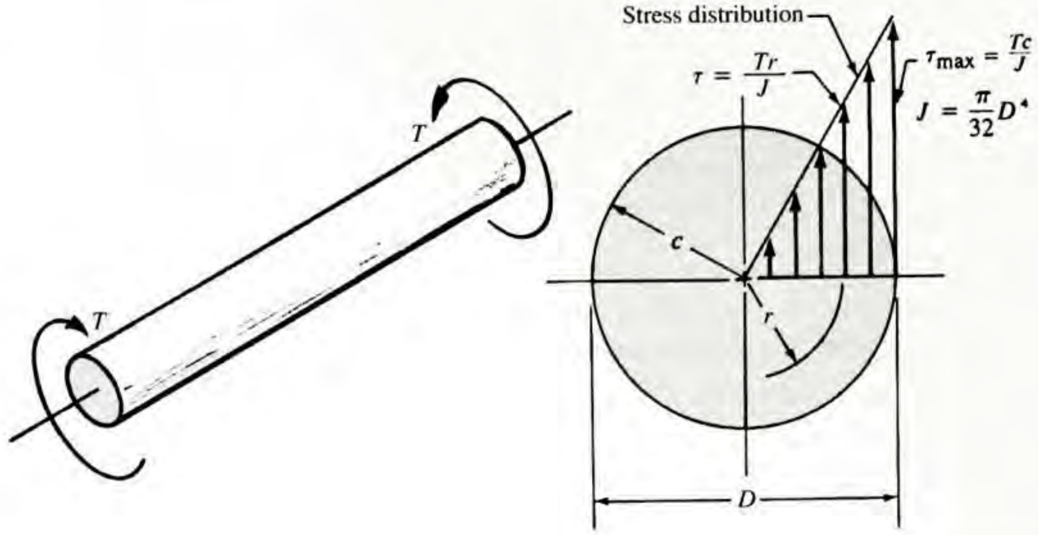
Maximum Torsional Shear Stress in a Circular Shaft

$$\tau_{max} = Tc/J \tag{3–7}$$

where c = radius of the shaft to its outside surface
J = polar moment of inertia

See Appendix 1 for formulas for J.

FIGURE 3–8 Stress distribution in a solid shaft



Example Problem 3–6 Compute the maximum torsional shear stress in a shaft having a diameter of 10 mm when it carries a torque of 4.10 N·m.

Solution Objective Compute the torsional shear stress in the shaft.

Given Torque $= T = 4.10$ N·m; shaft diameter $= D = 10$ mm.
c = radius of the shaft $= D/2 = 5.0$ mm.

Analysis Use Equation (3–7) to compute the torsional shear stress: $\tau_{max} = Tc/J$.
J is the polar moment of inertia for the shaft: $J = \pi D^4/32$ (see Appendix 1).

Results $J = \pi D^4/32 = [(\pi)(10\text{ mm})^4]/32 = 982\text{ mm}^4$

$$\tau_{max} = \frac{(4.10\text{ N}\cdot\text{m})(5.0\text{ mm})}{982\text{ mm}^4}\frac{10^3\text{ mm}}{\text{m}} = 20.9\text{ N/mm}^2 = 20.9\text{ MPa}$$

Comment The maximum torsional shear stress occurs at the outside surface of the shaft around its entire circumference.

If it is desired to compute the torsional shear stress at some point inside the shaft, the more general formula is used:

General Formula for Torsional Shear Stress

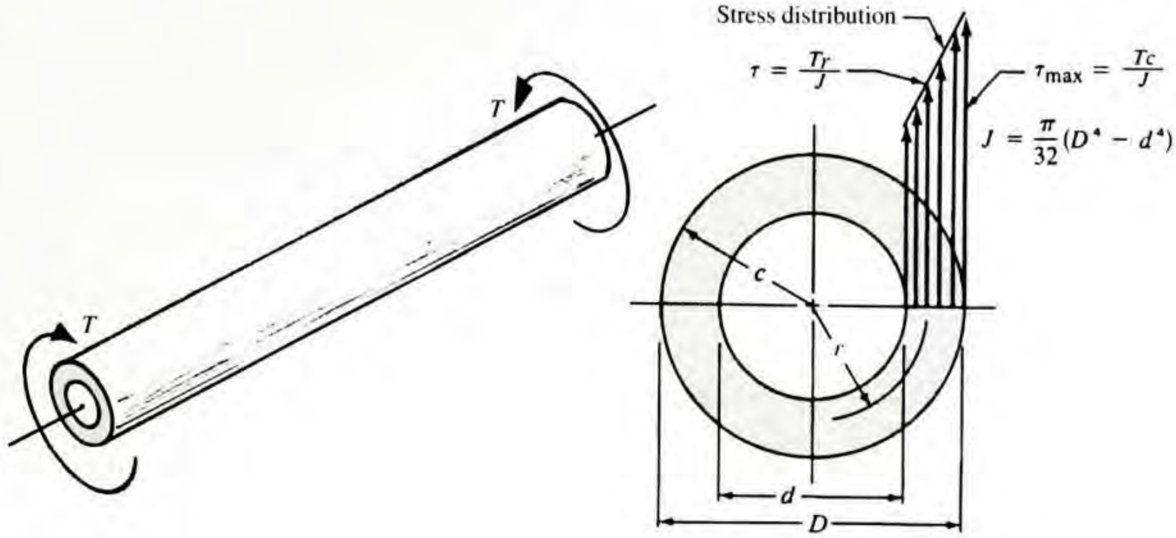
$$\tau = Tr/J \tag{3–8}$$

where r = radial distance from the center of the shaft to the point of interest

Figure 3–8 shows graphically that this equation is based on the linear variation of the torsional shear stress from zero at the center of the shaft to the maximum value at the outer surface.

Equations (3–7) and (3–8) apply also to hollow shafts (Figure 3–9 shows the distribution of shear stress). Again note that the maximum shear stress occurs at the outer surface. Also note that the entire cross section carries a relatively high stress level. As a result, the hollow shaft is more efficient. Notice that the material near the center of the solid shaft is not highly stressed.

FIGURE 3–9 Stress distribution in a hollow shaft



For design, it is convenient to define the *polar section modulus*, Z_p:

Polar Section Modulus

$$Z_p = J/c \tag{3–9}$$

Then the equation for the maximum torsional shear stress is

$$\tau_{max} = T/Z_p \tag{3–10}$$

Formulas for the polar section modulus are also given in Appendix 1. This form of the torsional shear stress equation is useful for design problems because the polar section modulus is the only term related to the geometry of the cross section.

3–9 TORSIONAL DEFORMATION

When a shaft is subjected to a torque, it undergoes a twisting in which one cross section is rotated relative to other cross sections in the shaft. The angle of twist is computed from

Torsional Deformation

$$\theta = TL/GJ \tag{3–11}$$

where θ = angle of twist (radians)
L = length of the shaft over which the angle of twist is being computed
G = modulus of elasticity of the shaft material in *shear*

Example Problem 3–7 Compute the angle of twist of a 10-mm-diameter shaft carrying 4.10 N·m of torque if it is 250 mm long and made of steel with G = 80 GPa. Express the result in both radians and degrees.

Solution Objective Compute the angle of twist in the shaft.

Given Torque = T = 4.10 N·m; length = L = 250 mm.
Shaft diameter = D = 10 mm; G = 80 GPa.

Analysis Use Equation (3–11). For consistency, let $T = 4.10 \times 10^3$ N·mm and $G = 80 \times 10^3$ N/mm^2. From Example Problem 3–6, J = 982 mm^4.

Results
$$\theta = \frac{TL}{GJ} = \frac{(4.10 \times 10^3\ \text{N}\cdot\text{mm})(250\ \text{mm})}{(80 \times 10^3\ \text{N/mm}^2)(982\ \text{mm}^4)} = 0.013\ \text{rad}$$

Using π rad = 180°,

$$\theta = (0.013\ \text{rad})(180\ \text{deg}/\pi\ \text{rad}) = 0.75\ \text{deg}$$

Comment: Over the length of 250 mm, the shaft twists 0.75 deg.

3–10 TORSION IN MEMBERS HAVING NONCIRCULAR CROSS SECTIONS

The behavior of members having noncircular cross sections when subjected to torsion is radically different from that for members having circular cross sections. However, the factors of most use in machine design are the maximum stress and the total angle of twist for such members. The formulas for these factors can be expressed in similar forms to the formulas used for members of circular cross section (solid and hollow round shafts).

The following two formulas can be used:

Torsional Shear Stress

$$\tau_{max} = T/Q \tag{3–12}$$

Deflection for Noncircular Sections

$$\theta = TL/GK \tag{3–13}$$

Note that Equations (3–12) and (3–13) are similar to Equations (3–10) and (3–11), with the substitution of Q for Z_p and K for J. Refer to Figure 3–10 for the methods of determining the values for K and Q for several types of cross sections useful in machine design. These values are appropriate only if the ends of the member are free to deform. If either end is fixed, as by welding to a solid structure, the resulting stress and angular twist are quite different. (See References 2, 4, and 6.)

Example Problem 3–8 A 2.50-in-diameter shaft carrying a chain sprocket has one end milled in the form of a square to permit the use of a hand crank. The square is 1.75 in on a side. Compute the maximum shear stress on the square part of the shaft when a torque of 15 000 lb·in is applied.

Also, if the length of the square part is 8.00 in, compute the angle of twist over this part. The shaft material is steel with $G = 11.5 \times 10^6$ psi.

Solution

Objective: Compute the maximum shear stress and the angle of twist in the shaft.

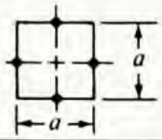
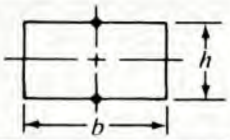
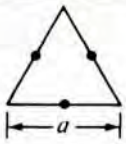
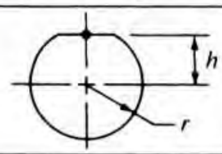
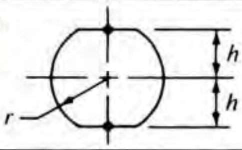
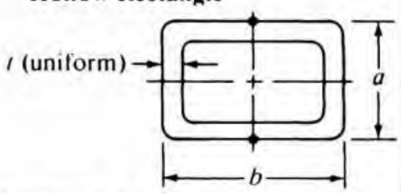
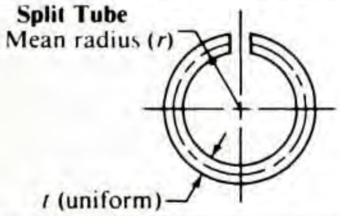
Given: Torque = T = 15 000 lb·in; length = L = 8.00 in.
The shaft is square; thus, a = 1.75 in.
$G = 11.5 \times 10^6$ psi.

Analysis: Figure 3–10 shows the methods for calculating the values for Q and K for use in Equations (3–12) and (3–13).

Results
$$Q = 0.208a^3 = (0.208)(1.75\ \text{in})^3 = 1.115\ \text{in}^3$$
$$K = 0.141a^4 = (0.141)(1.75\ \text{in})^4 = 1.322\ \text{in}^4$$

Now the stress and the deflection can be computed.

FIGURE 3–10 Methods for determining values for K and Q for several types of cross sections

Cross-sectional shape	K = for use in $\theta = TL/GK$; Q = for use in $\tau = T/Q$	Black dot (•) denotes location of τ_{max}
Square 	$K = 0.141a^4$ $Q = 0.208a^3$	τ_{max} at midpoint of each side
Rectangle 	$K = bh^3\left[\frac{1}{3} - 0.21\frac{h}{b}\left(1 - \frac{(h/b)^4}{12}\right)\right]$ $Q = \frac{bh^2}{[3 + 1.8(h/b)]}$	(Approximate; within ≈ 5%) τ_{max} at midpoint of long sides
Triangle (equilateral) 	$K = 0.0217a^4$ $Q = 0.050a^3$	
Shaft with One Flat 	$K = C_1r^4$ $Q = C_2r^3$	see table below
Shaft with Two Flats 	$K = C_3r^4$ $Q = C_4r^3$	see table below
Hollow Rectangle 	$K = \frac{2t(a-t)^2(b-t)^2}{(a+b-2t)}$ $Q = 2t(a-t)(b-t)$ Gives average stress; good approximation of maximum stress if t is small—thin-walled tube Inner corners should have generous fillets	
Split Tube 	$K = 2\pi rt^3/3$ $Q = \frac{4\pi^2r^2t^2}{(6\pi r + 1.8t)}$ t must be small—thin-walled tube	

h/r	0	0.2	0.4	0.6	0.8	1.0
C_1	0.30	0.51	0.78	1.06	1.37	1.57
C_2	0.35	0.51	0.70	0.92	1.18	1.57

h/r	0.5	0.6	0.7	0.8	0.9	1.0
C_3	0.44	0.67	0.93	1.19	1.39	1.57
C_4	0.47	0.60	0.81	1.02	1.25	1.57

$$\tau_{max} = \frac{T}{Q} = \frac{15\,000\text{ lb}\cdot\text{in}}{(1.115\text{ in}^3)} = 13\,460\text{ psi}$$

$$\theta = \frac{TL}{GK} = \frac{(15\,000\text{ lb}\cdot\text{in})(8.00\text{ in})}{(11.5\times10^6\text{ lb/in}^2)(1.322\text{ in}^4)} = 0.0079\text{ rad}$$

Convert the angle of twist to degrees:

$$\theta = (0.0079\text{ rad})(180\text{deg}/\pi\text{ rad}) = 0.452\text{ deg}$$

Comments Over the length of 8.00 in, the square part of the shaft twists 0.452 deg. The maximum shear stress is 13 460 psi, and it occurs at the midpoint of each side as shown in Figure 3–10.

3–11 TORSION IN CLOSED, THIN-WALLED TUBES

A general approach for closed, thin-walled tubes of virtually any shape uses Equations (3–12) and (3–13) with special methods of evaluating K and Q. Figure 3–11 shows such a tube having a constant wall thickness. The values of K and Q are

$$K = 4A^2t/U \tag{3–14}$$

$$Q = 2tA \tag{3–15}$$

where A = area enclosed by the median boundary (indicated by the dashed line in Figure 3–11)
t = wall thickness (which must be uniform and thin)
U = length of the median boundary

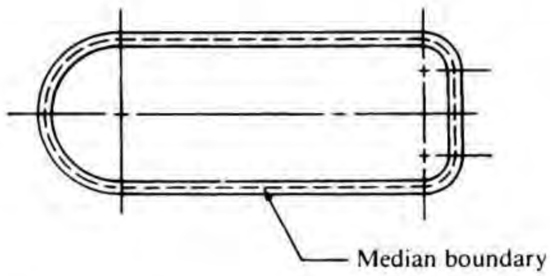


FIGURE 3–11
Closed, thin-walled tube with a constant wall thicknes

The shear stress computed by this approach is the *average stress* in the tube wall. However, if the wall thickness t is small (a thin wall), the stress is nearly uniform throughout the wall, and this approach will yield a close approximation of the maximum stress. For the analysis of tubular sections having nonuniform wall thickness, see References 2, 4, and 7.

To design a member to resist torsion only, or torsion and bending combined, it is advisable to select hollow tubes, either round or rectangular, or some other closed shape. They possess good efficiency both in bending and in torsion.

3–12 OPEN TUBES AND A COMPARISON WITH CLOSED TUBES

The term *open tube* refers to a shape that appears to be tubular but is not completely closed. For example, some tubing is manufactured by starting with a thin, flat strip of steel that is roll-formed into the desired shape (circular, rectangular, square, and so on). Then the seam is welded along the entire length of the tube. It is interesting to compare the properties of the cross section of such a tube before and after it is welded. The following example problem illustrates the comparison for a particular size of circular tubing.

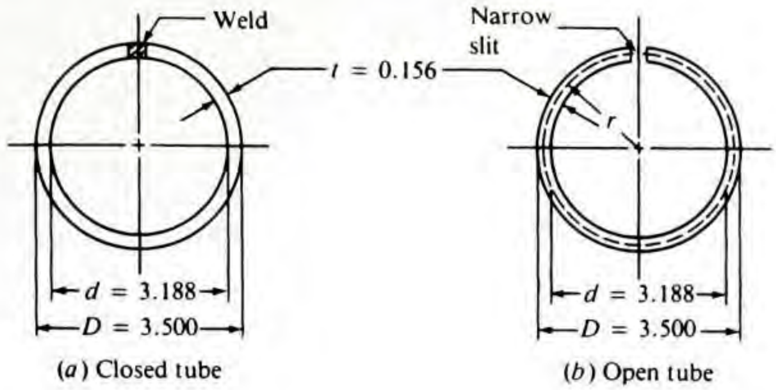
Example Problem 3–9 Figure 3–12 shows a tube before [Part (b)] and after [Part (a)] the seam is welded. Compare the stiffness and the strength of each shape.

Solution Objective: Compare the torsional stiffness and the strength of the closed tube of Figure 3–12(a) with those of the open-seam (split) tube shown in Figure 3–12(b).

Given: The tube shapes are shown in Figure 3–12. Both have the same length, diameter, and wall thickness, and both are made from the same material.

Analysis: Equation (3–13) gives the angle of twist for a noncircular member and shows that the angle is inversely proportional to the value of K. Similarly, Equation (3–11) shows that the an-

FIGURE 3–12
Comparison of closed and open tubes



gle of twist for a hollow circular tube is inversely proportional to the polar moment of inertia J. All other terms in the two equations are the same for each design. Therefore, the ratio of θ_{open} to θ_{closed} is equal to the ratio J/K. From Appendix 1, we find

$$J = \pi(D^4 - d^4)/32$$

From Figure 3–10, we find

$$K = 2\pi r t^3/3$$

Using similar logic, Equations (3–12) and (3–8) show that the maximum torsional shear stress is inversely proportional to Q and Z_p for the open and closed tubes, respectively. Then we can compare the strengths of the two forms by computing the ratio Z_p/Q. By Equation (3–9), we find that

$$Z_p = J/c = J/(D/2)$$

The equation for Q for the split tube is listed in Figure 3–10.

Results We make the comparison of torsional stiffness by computing the ratio J/K. For the closed, hollow tube,

$$J = \pi(D^4 - d^4)/32$$
$$J = \pi(3.500^4 - 3.188^4)/32 = 4.592 \text{ in}^4$$

For the open tube before the slit is welded, from Figure 3–10,

$$K = 2\pi r t^3/3$$
$$K = [(2)(\pi)(1.672)(0.156)^3]/3 = 0.0133 \text{ in}^4$$
$$\text{Ratio} = J/K = 4.592/0.0133 = 345$$

Then we make the comparison of the strengths of the two forms by computing the ratio Z_p/Q.

The value of J has already been computed to be 4.592 in^4. Then

$$Z_p = J/c = J/(D/2) = (4.592 \text{ in}^4)/[(3.500 \text{ in})/2] = 2.624 \text{ in}^3$$

For the open tube,

$$Q = \frac{4\pi^2 r^2 t^2}{(6\pi r + 1.8t)} = \frac{4\pi^2(1.672 \text{ in})^2(0.156 \text{ in})^2}{[6\pi(1.672 \text{ in}) + 1.8(0.156 \text{ in})]} = 0.0845 \text{ in}^3$$

Then the strength comparison is

$$\text{Ratio} = Z_p/Q = 2.624/0.0845 = 31.1$$

Comments Thus, for a given applied torque, the slit tube would twist 345 times as much as the closed tube. The stress in the slit tube would be 31.1 times higher than in the closed tube. Also note that if the material for the tube is thin, it will likely buckle at a relatively low stress level, and the tube will collapse suddenly. This comparison shows the dramatic superiority of the closed form of a hollow section to an open form. A similar comparison could be made for shapes other than circular.

3–13 VERTICAL SHEARING STRESS

A beam carrying loads transverse to its axis will experience shearing forces, denoted by V. In the analysis of beams, it is usual to compute the variation in shearing force across the entire length of the beam and to draw the *shearing force diagram*. Then the resulting vertical shearing stress can be computed from

Vertical Shearing Stress in Beams

$$\tau = VQ/It \tag{3–16}$$

where I = rectangular moment of inertia of the cross section of the beam
t = thickness of the section at the place where the shearing stress is to be computed
Q = *first moment,* with respect to the overall centroidal axis, *of the area* of that part of the cross section that lies away from the axis where the shearing stress is to be computed.

To calculate the value of Q, we define it by the following equation,

First Moment of the Area

$$Q = A_p\bar{y} \tag{3–17}$$

where A_p = that part of the area of the section above the place where the stress is to be computed
$\bar{y}$ = distance from the neutral axis of the section to the centroid of the area A_p

In some books or references, and in earlier editions of this book, Q was called the *statical moment*. Here we will use the term, *first moment of the area*.

For most section shapes, the maximum vertical shearing stress occurs at the centroidal axis. Specifically, if the thickness is not less at a place away from the centroidal axis, then it is assured that the maximum vertical shearing stress occurs at the centroidal axis.

Figure 3–13 shows three examples of how Q is computed in typical beam cross sections. In each, the maximum vertical shearing stress occurs at the neutral axis.

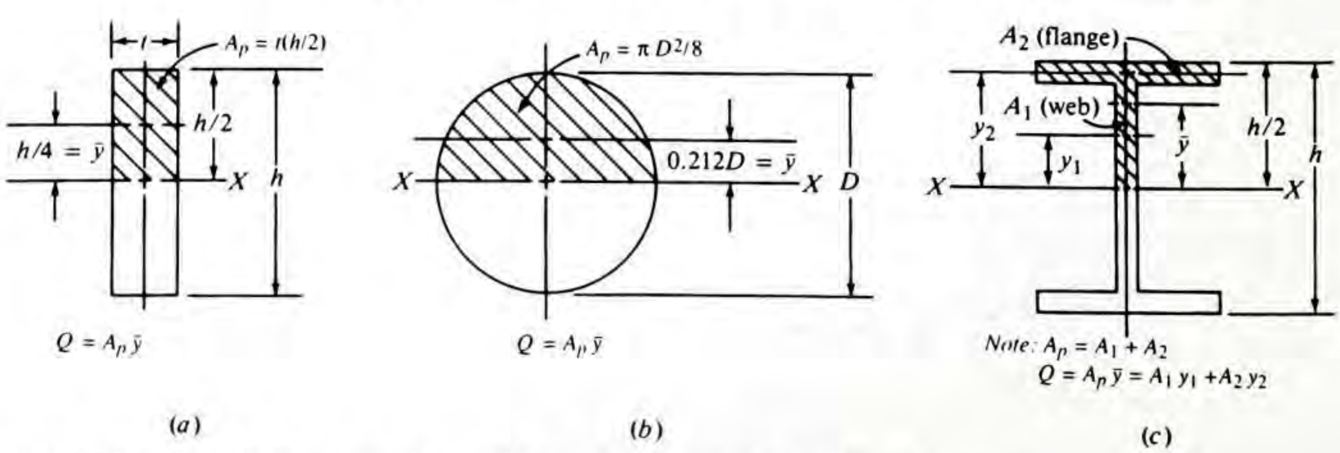
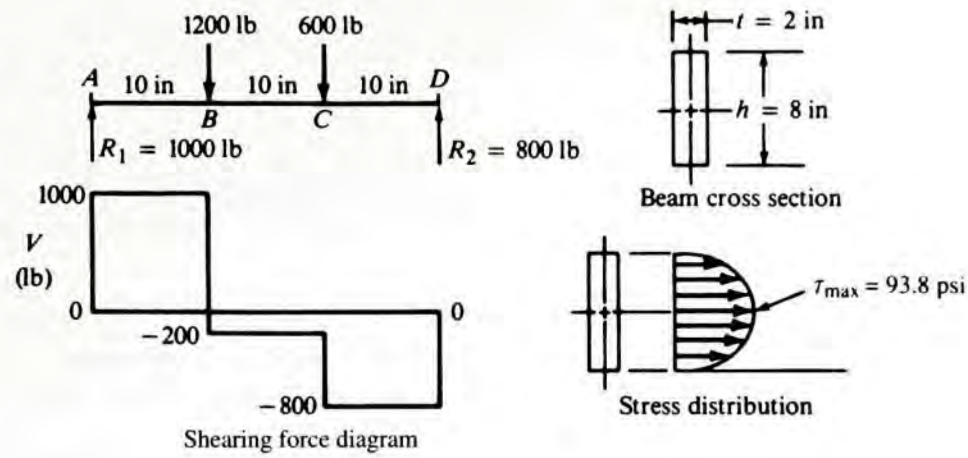


FIGURE 3–13 Illustrations of A_p and $\bar{y}$ used to compute Q for three shapes

FIGURE 3–14
Shearing force diagram and vertical shearing stress for beam



Example Problem 3–10 Figure 3–14 shows a simply supported beam carrying two concentrated loads. The shearing force diagram is shown, along with the rectangular shape and size of the cross section of the beam. The stress distribution is parabolic, with the maximum stress occurring at the neutral axis. Use Equation (3–16) to compute the maximum shearing stress in the beam.

Solution

Objective: Compute the maximum shearing stress τ in the beam in Figure 3–14.

Given: The beam shape is rectangular: $h = 8.00$ in; $t = 2.00$ in.
Maximum shearing force = V = 1000 lb at all points between A and B.

Analysis: Use Equation (3–16) to compute τ. V and t are given. From Appendix 1,

$$I = th^3/12$$

The value of the first moment of the area Q can be computed from Equation (3–17). For the rectangular cross section shown in Figure 3–13(a), $A_p = t(h/2)$ and $\bar{y} = h/4$. Then

$$Q = A_p\bar{y} = (th/2)(h/4) = th^2/8$$

Results:

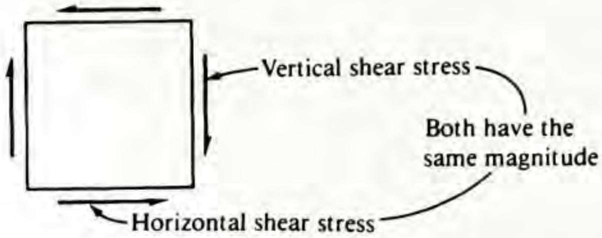
$$I = th^3/12 = (2.0\text{ in})(8.0\text{ in})^3/12 = 85.3\text{ in}^4$$
$$Q = A_p\bar{y} = th^2/8 = (2.0\text{ in})(8.0\text{ in})^2/8 = 16.0\text{ in}^3$$

Then the maximum shearing stress is

$$\tau = \frac{VQ}{It} = \frac{(1000\text{ lb})(16.0\text{ in}^3)}{(85.3\text{ in}^4)(2.0\text{ in})} = 93.8\text{ lb/in}^2 = 93.8\text{ psi}$$

Comments: The maximum shearing stress of 93.8 psi occurs at the neutral axis of the rectangular section as shown in Figure 3–14. The stress distribution within the cross section is generally parabolic, ending with zero shearing stress at the top and bottom surfaces. This is the nature of the shearing stress everywhere between the left support at A and the point of application of the 1200-lb load at B. The maximum shearing stress at any other point in the beam is proportional to the magnitude of the vertical shearing force at the point of interest.

FIGURE 3–15 Shear stresses on an element



Note that the vertical shearing stress is equal to the *horizontal shearing stress* because any element of material subjected to a shear stress on one face must have a shear stress of the same magnitude on the adjacent face for the element to be in equilibrium. Figure 3–15 shows this phenomenon.

In most beams, the magnitude of the vertical shearing stress is quite small compared with the bending stress (see the following section). For this reason it is frequently not computed at all. Those cases where it is of importance include the following:

1. When the material of the beam has a relatively low shear strength (such as wood).
2. When the bending moment is zero or small (and thus the bending stress is small), for example, at the ends of simply supported beams and for short beams.
3. When the thickness of the section carrying the shearing force is small, as in sections made from rolled sheet, some extruded shapes, and the web of rolled structural shapes such as wide-flange beams.

3–14 SPECIAL SHEARING STRESS FORMULAS

Equation (3–16) can be cumbersome because of the need to evaluate the first moment of the area Q. Several commonly used cross sections have special, easy-to-use formulas for the maximum vertical shearing stress:

τ_{max} for Rectangle

$$\tau_{max} = 3V/2A \text{ (exact)} \tag{3–18}$$

where A = total cross-sectional area of the beam

τ_{max} for Circle

$$\tau_{max} = 4V/3A \text{ (exact)} \tag{3–19}$$

τ_{max} for I-Shape

$$\tau_{max} \simeq V/th \text{ (approximate: about 15\% low)} \tag{3–20}$$

where t = web thickness
h = height of the web (for example, a wide-flange beam)

τ_{max} for Thin-walled Tube

$$\tau_{max} \simeq 2V/A \text{ (approximate: a little high)} \tag{3–21}$$

In all of these cases, the maximum shearing stress occurs at the neutral axis.

Example Problem 3–11 Compute the maximum shearing stress in the beam described in Example Problem 3–10 using the special shearing stress formula for a rectangular section.

Solution Objective Compute the maximum shearing stress τ in the beam in Figure 3–14.

Given The data are the same as stated in Example Problem 3–10 and as shown in Figure 3–14.

Analysis Use Equation (3–18) to compute $\tau = 3V/2A$. For the rectangle, $A = th$.

Results

$$\tau_{max} = \frac{3V}{2A} = \frac{3(1000\text{ lb})}{2[(2.0\text{ in})(8.0\text{ in})]} = 93.8\text{ psi}$$

Comment This result is the same as that obtained for Example Problem 3–10, as expected.

3–15 STRESS DUE TO BENDING

A *beam* is a member that carries loads transverse to its axis. Such loads produce bending moments in the beam, which result in the development of bending stresses. Bending stresses are *normal stresses*, that is, either tensile or compressive. The maximum bending stress in a beam cross section will occur in the part farthest from the neutral axis of the section. At that point, the *flexure formula* gives the stress:

Flexure Formula for Maximum Bending Stress

$$\sigma = Mc/I \tag{3–22}$$

where M = magnitude of the bending moment at the section
I = moment of inertia of the cross section with respect to its neutral axis
c = distance from the neutral axis to the outermost fiber of the beam cross section

MDESIGN

The magnitude of the bending stress varies linearly within the cross section from a value of zero at the neutral axis, to the maximum tensile stress on one side of the neutral axis, and to the maximum compressive stress on the other side. Figure 3–16 shows a typical stress distribution in a beam cross section. Note that the stress distribution is independent of the shape of the cross section.

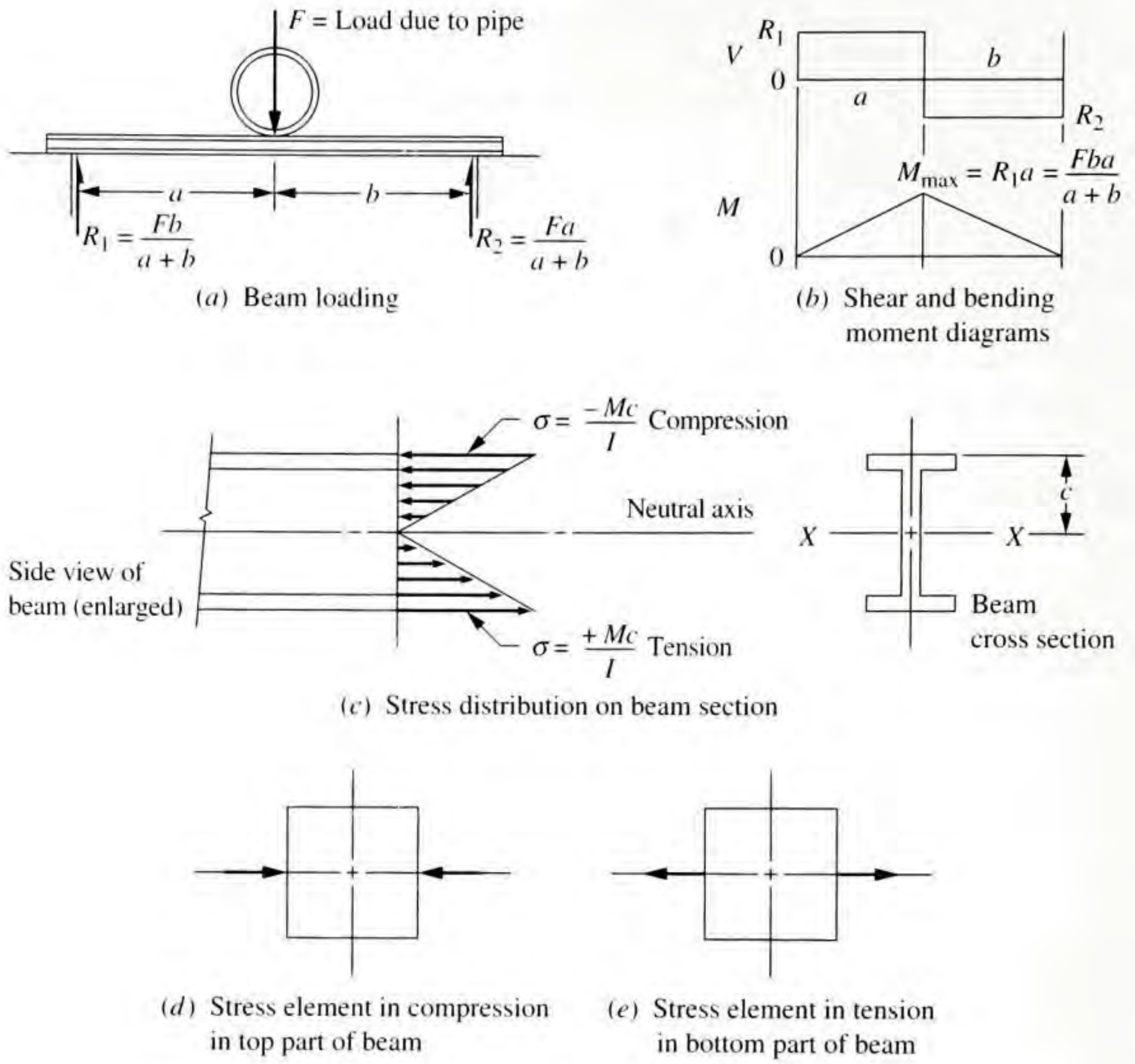
Note that *positive bending* occurs when the deflected shape of the beam is concave upward, resulting in compression on the upper part of the cross section and tension on the lower part. Conversely, *negative bending* causes the beam to be concave downward.

The flexure formula was developed subject to the following conditions:

1. The beam must be in pure bending. Shearing stresses must be zero or negligible. No axial loads are present.
2. The beam must not twist or be subjected to a torsional load.
3. The material of the beam must obey Hooke's law.
4. The modulus of elasticity of the material must be the same in both tension and compression.
5. The beam is initially straight and has a constant cross section.
6. Any plane cross section of the beam remains plane during bending.
7. No part of the beam shape fails because of local buckling or wrinkling.

If condition 1 is not strictly met, you can continue the analysis by using the method of combined stresses presented in Chapter 4. In most practical beams, which are long relative to their height, shear stresses are sufficiently small as to be negligible. Furthermore, the maximum bending stress occurs at the outermost fibers of the beam section, where the shear stress is in fact zero. A beam with varying cross section, which would violate

FIGURE 3–16
Typical bending stress distribution in a beam cross section



(*a*) Beam loading

(*b*) Shear and bending moment diagrams

(*c*) Stress distribution on beam section

(*d*) Stress element in compression in top part of beam

(*e*) Stress element in tension in bottom part of beam

condition 5, can be analyzed by the use of stress concentration factors discussed later in this chapter.

For design, it is convenient to define the term *section modulus*, S, as

$$S = I/c \qquad \textbf{(3–23)}$$

The flexure formula then becomes

Flexure Formula

$$\sigma = M/S \qquad \textbf{(3–24)}$$

Since I and c are geometrical properties of the cross section of the beam, S is also. Then, in design, it is usual to define a design stress, σ_d, and, with the bending moment known, solve for S:

Required Section Modulus

$$S = M/\sigma_d \qquad \textbf{(3–25)}$$

This results in the required value of the section modulus. From it, the required dimensions of the beam cross section can be determined.

Example Problem 3–12 For the beam shown in Figure 3–16, the load F due to the pipe is 12 000 lb. The distances are $a = 4$ ft and $b = 6$ ft. Determine the required section modulus for the beam to limit the stress due to bending to 30 000 psi, the recommended design stress for a typical structural steel in static bending.

Solution Objective Compute the required section modulus S for the beam in Figure 3–16.

Given The layout and the loading pattern are shown in Figure 3–16.

Lengths: Overall length $= L = 10$ ft; $a = 4$ ft; $b = 6$ ft.

Load $= F = 12\,000$ lb.

Design stress $= \sigma_d = 30\,000$ psi.

Analysis Use Equation (3–25) to compute the required section modulus S. Compute the maximum bending moment that occurs at the point of application of the load using the formula shown in Part (b) of Figure 3–16.

Results

$$M_{max} = R_1\,a = \frac{Fba}{a+b} = \frac{(12\,000\text{ lb})(6\text{ ft})(4\text{ ft})}{(6\text{ ft} + 4\text{ ft})} = 28\,800\text{ lb}\cdot\text{ft}$$

$$S = \frac{M}{\sigma_d} = \frac{28\,800\text{ lb}\cdot\text{ft}}{30\,000\text{ lb/in}^2}\,\frac{12\text{ in}}{\text{ft}} = 11.5\text{ in}^3$$

Comments A beam section can now be selected from Tables A16–3 and A16–4 that has at least this value for the section modulus. The lightest section, typically preferred, is the W8×15 wide-flange shape with $S = 11.8$ in^3.

3–16 FLEXURAL CENTER FOR BEAMS

A beam section must be loaded in a way that ensures symmetrical bending; that is, there must be no tendency for the section to twist under the load. Figure 3–17 shows several shapes that are typically used for beams having a vertical axis of symmetry. If the line of

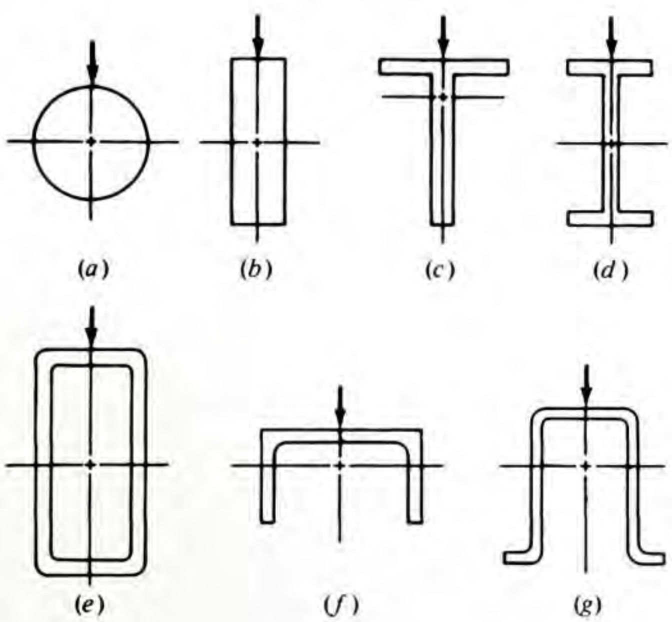


FIGURE 3–17 Symmetrical sections. A load applied through the axis of symmetry results in pure bending in the beam.

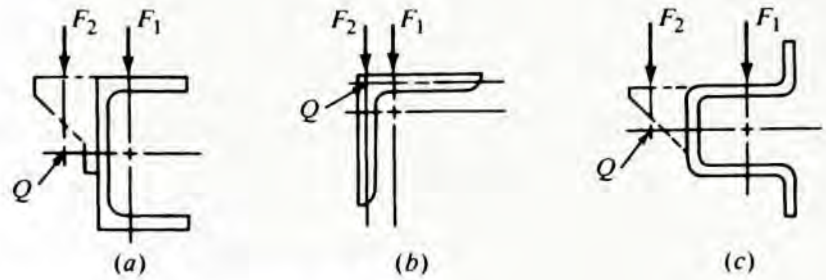


FIGURE 3–18 Nonsymmetrical sections. A load applied as at F_1 would cause twisting; loads applied as at F_2 through the flexural center Q would cause pure bending.

action of the loads on such sections passes through the axis of symmetry, then there is no tendency for the section to twist, and the flexure formula applies.

When there is no vertical axis of symmetry, as with the sections shown in Figure 3–18, care must be exercised in placement of the loads. If the line of action of the loads were shown as F_1 in the figure, the beam would twist and bend, so the flexure formula would not give accurate results for the stress in the section. For such sections, the load must be placed in line with the *flexural center,* sometimes called the *shear center.* Figure 3–18 shows the approximate location of the flexural center for these shapes (indicated by the symbol Q). Applying the load in line with Q, as shown with the forces labeled F_2, would result in pure bending. A table of formulas for the location of the flexural center is available (see Reference 7).

3–17 BEAM DEFLECTIONS

The bending loads applied to a beam cause it to deflect in a direction perpendicular to its axis. A beam that was originally straight will deform to a slightly curved shape. In most cases, the critical factor is either the maximum deflection of the beam or its deflection at specific locations.

Consider the double-reduction speed reducer shown in Figure 3–19. The four gears (A, B, C, and D) are mounted on three shafts, each of which is supported by two bearings. The action of the gears in transmitting power creates a set of forces that in turn act on the shafts to cause bending. One component of the total force on the gear teeth acts in a direction that tends to separate the two gears. Thus, gear A is forced upward while gear B is forced downward. For good gear performance, the net deflection of one gear relative to the other should not exceed 0.005 in (0.13 mm) for medium-sized industrial gearing.

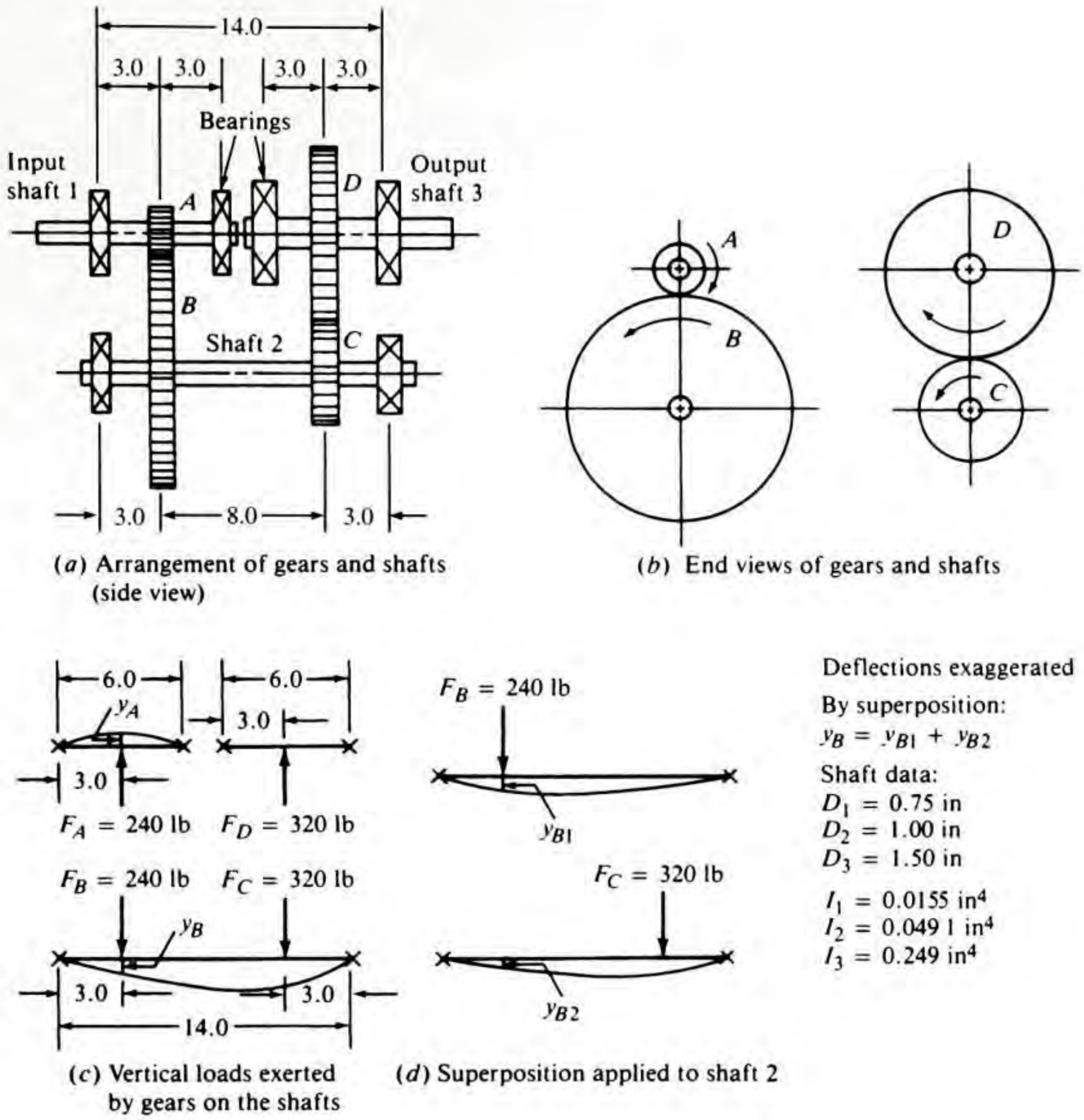
To evaluate the design, there are many methods of computing shaft deflections. We will review briefly those methods using deflection formulas, superposition, and a general analytical approach.

A set of formulas for computing the deflection of beams at any point or at selected points is useful in many practical problems. Appendix 14 includes several cases.

For many additional cases, superposition is useful if the actual loading can be divided into parts that can be computed by available formulas. The deflection for each loading is computed separately, and then the individual deflections are summed at the points of interest.

Many commercially available computer software programs allow the modeling of beams having rather complex loading patterns and varying geometry. The results include reaction forces, shearing force and bending moment diagrams, and deflections at any point. It is important that you understand the principles of beam deflection, studied in strength of materials and reviewed here, so that you can apply such programs accurately and interpret the results carefully.

FIGURE 3–19 Shaft deflection analysis for a double-reduction speed reducer



Example Problem 3–13 For the two gears, A and B, in Figure 3–19, compute the relative deflection between them in the plane of the paper that is due to the forces shown in Part (c). These *separating forces*, or *normal forces*, are discussed in Chapters 9 and 10. It is customary to consider the loads at the gears and the reactions at the bearings to be concentrated. The shafts carrying the gears are steel and have uniform diameters as listed in the figure.

Solution Objective Compute the relative deflection between gears A and B in Figure 3–19.

Given The layout and loading pattern are shown in Figure 3–19. The separating force between gears A and B is 240 lb. Gear A pushes downward on gear B, and the reaction force of gear B pushes upward on gear A. Shaft 1 has a diameter of 0.75 in and a moment of inertia of 0.0155 in^4. Shaft 2 has a diameter of 1.00 in and a moment of inertia of 0.0491 in^4. Both shafts are steel. Use $E = 30 \times 10^6$ psi.

Analysis Use the deflection formulas from Appendix 14 to compute the upward deflection of shaft 1 at gear A and the downward deflection of shaft 2 at gear B. The sum of the two deflections is the total deflection of gear A with respect to gear B.

Case (a) from Table A14–1 applies to shaft 1 because there is a single concentrated force acting at the midpoint of the shaft between the supporting bearings. We will call that deflection y_A.

Shaft 2 is a simply supported beam carrying two nonsymmetrical loads. No single formula from Appendix 14 matches that loading pattern. But we can use superposition to compute the deflection of the shaft at gear B by considering the two forces separately as shown in Part (d) of Figure 3–19. Case (b) from Table A14–1 is used for each load.

We first compute the deflection at B due only to the 240-lb force, calling it y_{B1}. Then we compute the deflection at B due to the 320-lb force, calling it y_{B2}. The total deflection at B is $y_B = y_{B1} + y_{B2}$.

Results The deflection of shaft 1 at gear A is

$$y_a = \frac{F_A L_1^3}{48\,EI} = \frac{(240)(6.0)^3}{48(30 \times 10^6)(0.0155)} = 0.0023 \text{ in}$$

The deflection of shaft 2 at B due only to the 240-lb force is

$$y_{B1} = -\frac{F_B a^2 b^2}{3\,EI_2 L_2} = -\frac{(240)(3.0)^2(11.0)^2}{3(30 \times 10^6)(0.0491)(14)} = -0.0042 \text{ in}$$

The deflection of shaft 2 at B due only to the 320-lb force at C is

$$y_{B2} = -\frac{F_c bx}{6\,EI_2 L_2}(L_2^2 - b^2 - x^2)$$

$$y_{B2} = -\frac{(320)(3.0)(3.0)}{6(30 \times 10^6)(0.0491)(14)}[(14)^2 - (3.0)^2 - (3.0)^2]$$

$$y_{B2} = -0.0041 \text{ in}$$

Then the total deflection at gear B is

$$y_B = y_{B1} + y_{B2} = -0.0042 - 0.0041 = -0.0083 \text{ in}$$

Because shaft 1 deflects upward and shaft 2 deflects downward, the total relative deflection is the sum of y_A and y_B:

$$y_{\text{total}} = y_A + y_B = 0.0023 + 0.0083 = 0.0106 \text{ in}$$

Comment This deflection is very large for this application. How could the deflection be reduced?

3–18 EQUATIONS FOR DEFLECTED BEAM SHAPE

The general principles relating the deflection of a beam to the loading on the beam and its manner of support are presented here. The result will be a set of relationships among the load, the vertical shearing force, the bending moment, the slope of the deflected beam shape, and the actual deflection curve for the beam. Figure 3–20 shows diagrams for these five factors, with θ as the slope and y indicating deflection of the beam from its initial straight position. The product of modulus of elasticity and the moment of inertia, EI, for the beam is a measure of its stiffness or resistance to bending deflection. It is convenient to

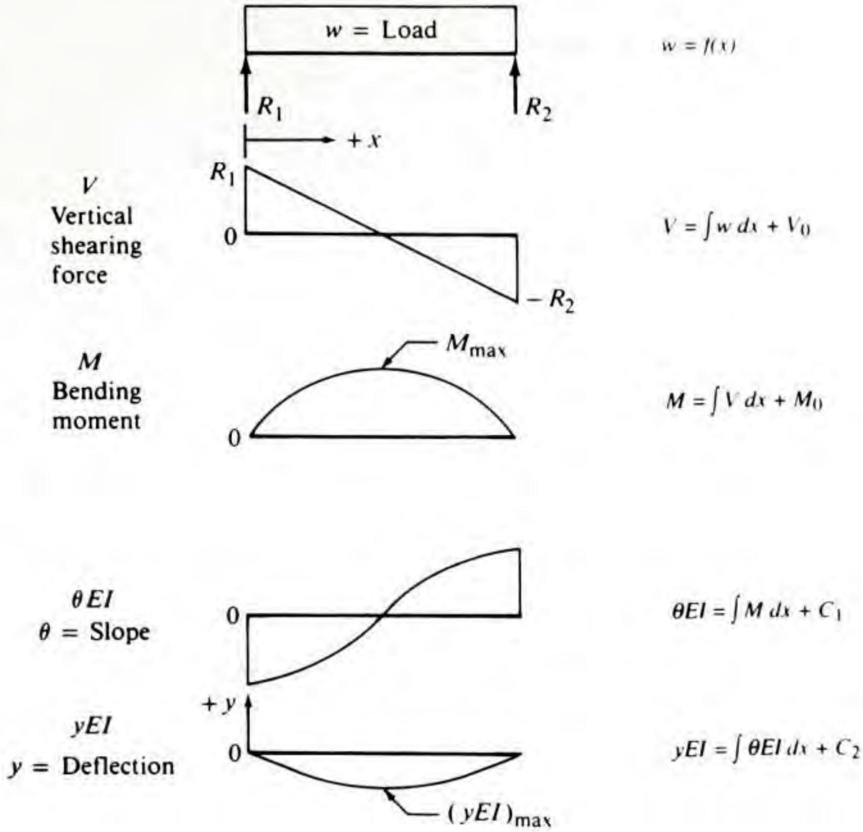


FIGURE 3–20
Relationships of load, vertical shearing force, bending moment, slope of deflected beam shape, and actual deflection curve of a beam

combine EI with the slope and deflection values to maintain a proper relationship, as discussed next.

One fundamental concept for beams in bending is

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

where M = bending moment
x = position on the beam measured along its length
y = deflection

Thus, if it is desired to create an equation of the form $y = f(x)$ (that is, y as a function of x), it would be related to the other factors as follows:

$$y = f(x)$$
$$\theta = \frac{dy}{dx}$$
$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$
$$\frac{V}{EI} = \frac{d^3y}{dx^3}$$
$$\frac{w}{EI} = \frac{d^4y}{dx^4}$$

where w = general term for the load distribution on the beam

The last two equations follow from the observation that there is a derivative (slope) relationship between shear and bending moment and between load and shear.

In practice, the fundamental equations just given are used in reverse. That is, the load distribution as a function of x is known, and the equations for the other factors are derived by successive integrations. The results are

$$
\begin{aligned}
w &= f(x) \\
V &= \int w\,dx + V_0 \\
M &= \int V\,dx + M_0
\end{aligned}
$$

where V_0 and M_0 = constants of integration evaluated from the boundary conditions

In many cases, the load, shear, and bending moment diagrams can be drawn in the conventional manner, and the equations for shear or bending moment can be created directly by the principles of analytic geometry. With M as a function of x, the slope and deflection relations can be found:

$$
\begin{aligned}
\theta EI &= \int M\,dx + C_1 \\
yEI &= \int \theta EI\,dx + C_2
\end{aligned}
$$

The constants of integration must be evaluated from boundary conditions. Texts on strength of materials show the details. (See Reference 3.)

3–19 BEAMS WITH CONCENTRATED BENDING MOMENTS

MDESIGN

Figures 3–16 and 3–20 show beams loaded only with concentrated forces or distributed loads. For such loading in any combination, the moment diagram is continuous. That is, there are no points of abrupt change in the value of the bending moment. Many machine elements such as cranks, levers, helical gears, and brackets carry loads whose line of action is offset from the centroidal axis of the beam in such a way that a concentrated moment is exerted on the beam.

Figures 3–21, 3–22, and 3–23 show three different examples where concentrated moments are created on machine elements. The bell crank in Figure 3–21 pivots around point O and is used to transfer an applied force to a different line of action. Each arm behaves similar to a cantilever beam, bending with respect to an axis through the pivot. For analysis, we can isolate an arm by making an imaginary cut through the pivot and showing the reaction force at the pivot pin and the internal moment in the arm. The shearing force and bending moment diagrams included in Figure 3–21 show the results, and Example Problem 3–14 gives the details of the analysis. Note the similarity to a cantilever beam with the internal concentrated moment at the pivot reacting to the force, F_2, acting at the end of the arm.

Figure 3–22 shows a print head for an impact-type printer in which the applied force, F, is offset from the neutral axis of the print head itself. Thus the force creates a concentrated bending moment at the right end where the vertical lever arm attaches to the horizontal part. The freebody diagram shows the vertical arm cut off and an internal axial force and moment replacing the effect of the extended arm. The concentrated moment causes the abrupt change in the value of the bending moment at the right end of the arm as shown in the bending moment diagram. Example Problem 3–15 gives the details of the analysis.

Figure 3–23 shows an isometric view of a crankshaft that is actuated by the vertical force acting at the end of the crank. One result is an applied torque that tends to rotate the shaft ABC clockwise about its x-axis. The reaction torque is shown acting at the forward end of the crank. A second result is that the vertical force acting at the end of the crank cre-

FIGURE 3–21
Bending moment in a bell crank

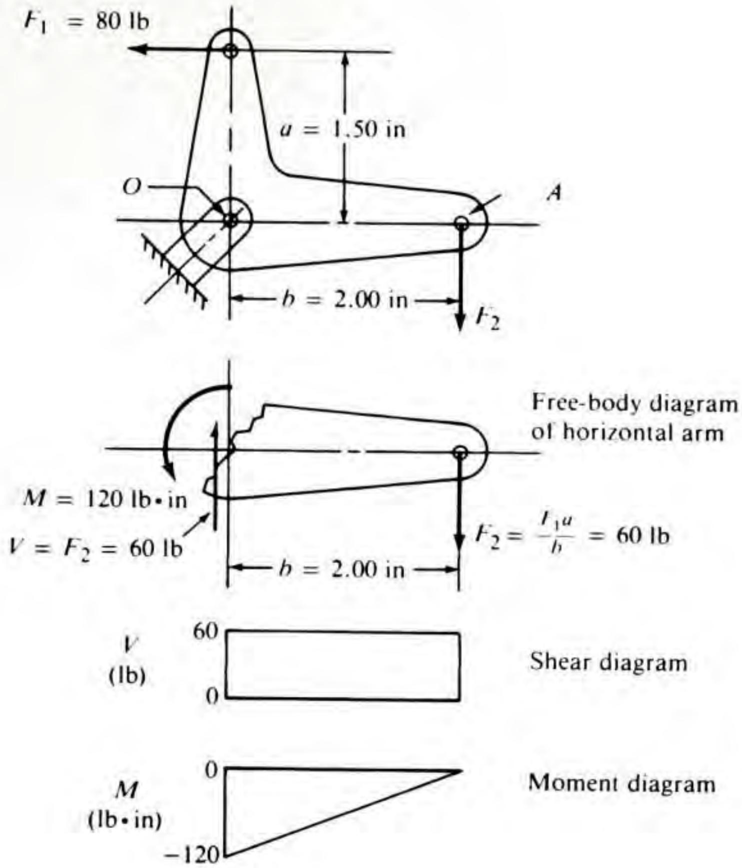
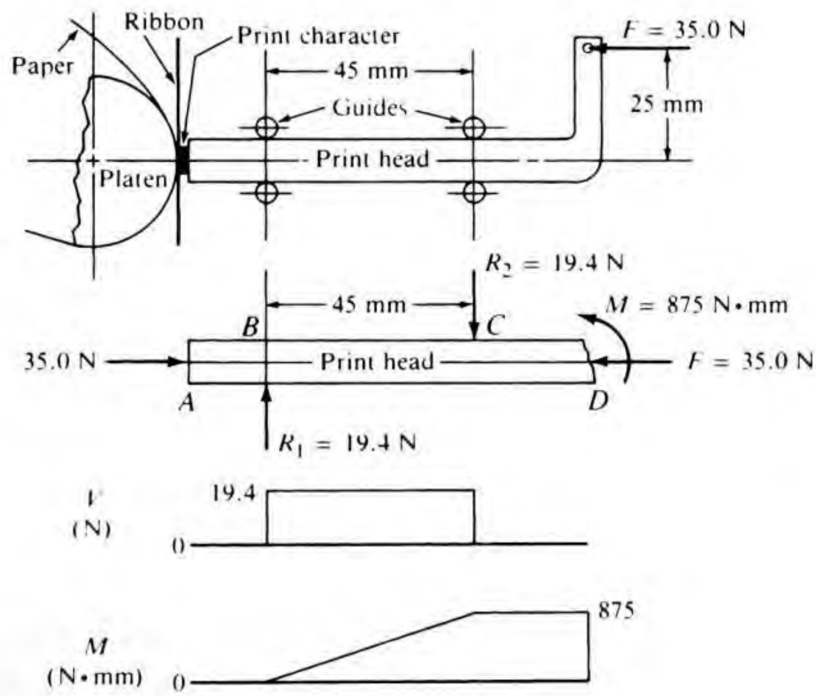


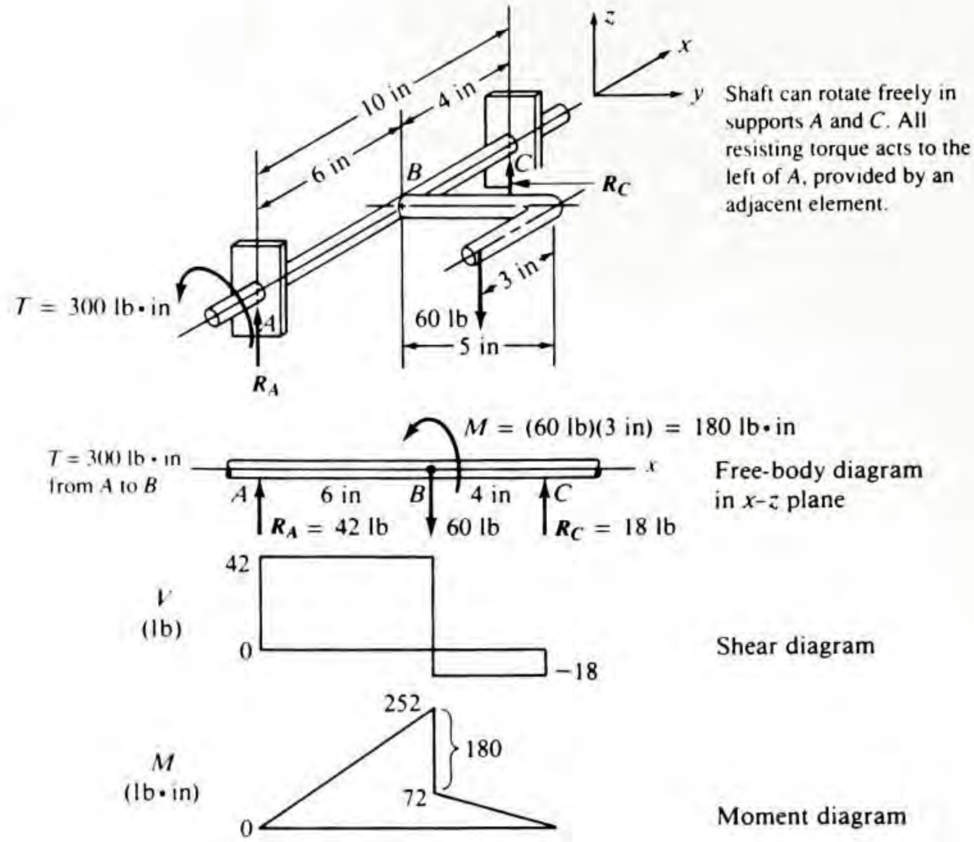
FIGURE 3–22
Bending moment on a print head



ates a twisting moment in the rod attached at B and thus tends to bend the shaft ABC in the x-z plane. The twisting moment is treated as a concentrated moment acting at B with the resulting abrupt change in the bending moment at that location as can be seen in the bending moment diagram. Example Problem 3–16 gives the details of the analysis.

When drawing the bending moment diagram for a member to which a concentrated moment is applied, the following sign convention will be used.

FIGURE 3–23
Bending moment on a shaft carrying a crank



When a concentrated bending moment acts on a beam in a counterclockwise direction, the moment diagram drops; when a clockwise concentrated moment acts, the moment diagram rises.

Example Problem 3–14 The bell crank shown in Figure 3–21 is part of a linkage in which the 80-lb horizontal force is transferred to F_2 acting vertically. The crank can pivot about the pin at O. Draw a free-body diagram of the horizontal part of the crank from O to A. Then draw the shearing force and bending moment diagrams that are necessary to complete the design of the horizontal arm of the crank.

Solution Objective: Draw the free-body diagram of the horizontal part of the crank in Figure 3–21. Draw the shearing force and bending moment diagrams for that part.

Given: The layout from Figure 3–21.

Analysis: Use the entire crank first as a free body to determine the downward force F_2 that reacts to the applied horizontal force F_1 of 80 lb by summing moments about the pin at O.

Then create the free-body diagram for the horizontal part by breaking it through the pin and replacing the removed part with the internal force and moment acting at the break.

Results: We can first find the value of F_2 by summing moments about the pin at O using the entire crank:

$$F_1 \cdot a = F_2 \cdot b$$

$$F_2 = F_1(a/b) = 80\text{ lb}(1.50/2.00) = 60\text{ lb}$$

Below the drawing of the complete crank, we have drawn a sketch of the horizontal part, isolating it from the vertical part. The internal force and moment at the cut section are shown. The externally applied downward force F_2 is reacted by the upward reaction at the pin. Also, because F_2 causes a moment with respect to the section at the pin, an internal reaction moment exists, where

$$M = F_2 \cdot b = (60\text{ lb})(2.00\text{ in}) = 120\text{ lb}\cdot\text{in}$$

The shear and moment diagrams can then be shown in the conventional manner. The result looks much like a cantilever that is built into a rigid support. The difference here is that the reaction moment at the section through the pin is developed in the vertical arm of the crank.

Comments Note that the shape of the moment diagram for the horizontal part shows that the maximum moment occurs at the section through the pin and that the moment decreases linearly as we move out toward point A. As a result, the shape of the crank is optimized, having its largest cross section (and section modulus) at the section of highest bending moment. You could complete the design of the crank using the techniques reviewed in Section 3–15.

Example Problem 3–15 Figure 3–22 represents a print head for a computer printer. The force F moves the print head toward the left against the ribbon, imprinting the character on the paper that is backed up by the platen. Draw the free-body diagram for the horizontal portion of the print head, along with the shearing force and bending moment diagrams.

Solution Objective Draw the free-body diagram of the horizontal part of the print head in Figure 3–22. Draw the shearing force and bending moment diagrams for that part.

Given The layout from Figure 3–22.

Analysis The horizontal force of 35 N acting to the left is reacted by an equal 35 N horizontal force produced by the platen pushing back to the right on the print head. The guides provide simple supports in the vertical direction. The applied force also produces a moment at the base of the vertical arm where it joins the horizontal part of the print head.

We create the free-body diagram for the horizontal part by breaking it at its right end and replacing the removed part with the internal force and moment acting at the break. The shearing force and bending moment diagrams can then be drawn.

Results The free-body diagram for the horizontal portion is shown below the complete sketch. Note that at the right end (section D) of the print head, the vertical arm has been removed and replaced with the internal horizontal force of 35.0 N and a moment of 875 N · mm caused by the 35.0 N force acting 25 mm above it. Also note that the 25 mm-moment arm for the force is taken from the line of action of the force *to the neutral axis of the horizontal part.* The 35.0 N reaction of the platen on the print head tends to place the head in compression over the entire length. The rotational tendency of the moment is reacted by the couple created by R_1 and R_2 acting 45 mm apart at B and C.

Below the free-body diagram is the vertical shearing force diagram in which a constant shear of 19.4 N occurs only between the two supports.

The bending moment diagram can be derived from either the left end or the right end. If we choose to start at the left end at A, there is no shearing force from A to B, and therefore there is no change in bending moment. From B to C, the positive shear causes an increase in bending moment from zero to 875 N · mm. Because there is no shear from C to D, there is no change in bending moment, and the value remains at 875 N·mm. The counterclockwise-directed concentrated moment at D causes the moment diagram to drop abruptly, closing the diagram.

Example Problem 3–16 Figure 3–23 shows a crank in which it is necessary to visualize the three-dimensional arrangement. The 60-lb downward force tends to rotate the shaft ABC around the x-axis. The reaction torque acts only at the end of the shaft outboard of the bearing support at A. Bearings A and C provide simple supports. Draw the complete free-body diagram for the shaft ABC, along with the shearing force and bending moment diagrams.

Solution Objective Draw the free-body diagram of the shaft ABC in Figure 3–23. Draw the shearing force and bending moment diagrams for that part.

Given The layout from Figure 3–23.

Analysis The analysis will take the following steps:

1. Determine the magnitude of the torque in the shaft between the left end and point B where the crank arm is attached.
2. Analyze the connection of the crank at point B to determine the force and moment transferred to the shaft ABC by the crank.
3. Compute the vertical reactions at supports A and C.
4. Draw the shearing force and bending moment diagrams considering the concentrated moment applied at point B, along with the familiar relationships between shearing force and bending moments.

Results The free-body diagram is shown as viewed looking at the x-z plane. Note that the free body must be in equilibrium in all force and moment directions. Considering first the torque (rotating moment) about the x-axis, note that the crank force of 60 lb acts 5.0 in from the axis. The torque, then, is

$$T = (60\text{ lb})(5.0\text{ in}) = 300\text{ lb}\cdot\text{in}$$

This level of torque acts from the left end of the shaft to section B, where the crank is attached to the shaft.

Now the loading at B should be described. One way to do so is to visualize that the crank itself is separated from the shaft and is replaced with a force and moment caused by the crank. First, the downward force of 60 lb pulls down at B. Also, because the 60-lb applied force acts 3.0 in to the left of B, it causes a concentrated moment in the *x-z plane* of 180 lb·in to be applied at B.

Both the downward force and the moment at B affect the magnitude and direction of the reaction forces at A and C. First, summing moments about A,

$$(60\text{ lb})(6.0\text{ in}) - 180\text{ lb}\cdot\text{in} - R_C(10.0\text{ in}) = 0$$
$$R_C = [(360 - 180)\text{lb}\cdot\text{in}]/(10.0\text{ in}) = 18.0\text{ lb upward}$$

Now, summing moments about C,

$$(60\ \text{lb})(4.0\ \text{in}) + 180\ \text{lb}\cdot\text{in} - R_A(10.0\ \text{in}) = 0$$
$$R_A = [(240 + 180)\ \text{lb}\cdot\text{in}]/(10.0\ \text{in}) = 42.0\ \text{lb upward}$$

Now the shear and bending moment diagrams can be completed. The moment starts at zero at the simple support at A, rises to 252 lb·in at B under the influence of the 42-lb shear force, then drops by 180 lb·in due to the counterclockwise concentrated moment at B, and finally returns to zero at the simple support at C.

Comments In summary, shaft ABC carries a torque of 300 lb·in from point B to its left end. The maximum bending moment of 252 lb·in occurs at point B where the crank is attached. The bending moment then suddenly drops to 72 lb·in under the influence of the concentrated moment of 180 lb·in applied by the crank.

3–20 COMBINED NORMAL STRESSES: SUPERPOSITION PRINCIPLE

When the same cross section of a load-carrying member is subjected to both a direct tensile or compressive stress and a stress due to bending, the resulting normal stress can be computed by the method of superposition. The formula is

$$\sigma = \pm\ Mc/I \pm F/A \tag{3–26}$$

where tensile stresses are positive and compressive stresses are negative

An example of a load-carrying member subjected to combined bending and axial tension is shown in Figure 3–24. It shows a beam subjected to a load applied downward and to the right through a bracket below the beam. Resolving the load into horizontal and vertical components shows that its effect can be broken into three parts:

1. The vertical component tends to place the beam in bending with tension on the top and compression on the bottom.
2. The horizontal component, because it acts away from the neutral axis of the beam, causes bending with tension on the bottom and compression on the top.
3. The horizontal component causes direct tensile stress across the entire cross section.

We can proceed with the stress analysis by using the techniques from the previous section to prepare the shearing force and bending moment diagrams and then using Equation 3–26 to combine the effects of the bending stress and the direct tensile stress at any point. The details are shown within Example Problem 3–17.

Example Problem 3–17 The cantilever beam in Figure 3–24 is a steel American Standard beam, S6 × 12.5. The force F is 10 000 lb, and it acts at an angle of 30° below the horizontal, as shown. Use $a = 24$ in and $e = 6.0$ in. Draw the free-body diagram and the shearing force and bending moment diagrams for the beam. Then compute the maximum tensile and maximum compressive stresses in the beam and show where they occur.

Solution Objective Determine the maximum tensile and compressive stresses in the beam.

Given The layout from Figure 3–24(a). Force = F = 10 000 lb; angle $\theta = 30°$.
The beam shape: S6×12.5; length = a = 24 in.

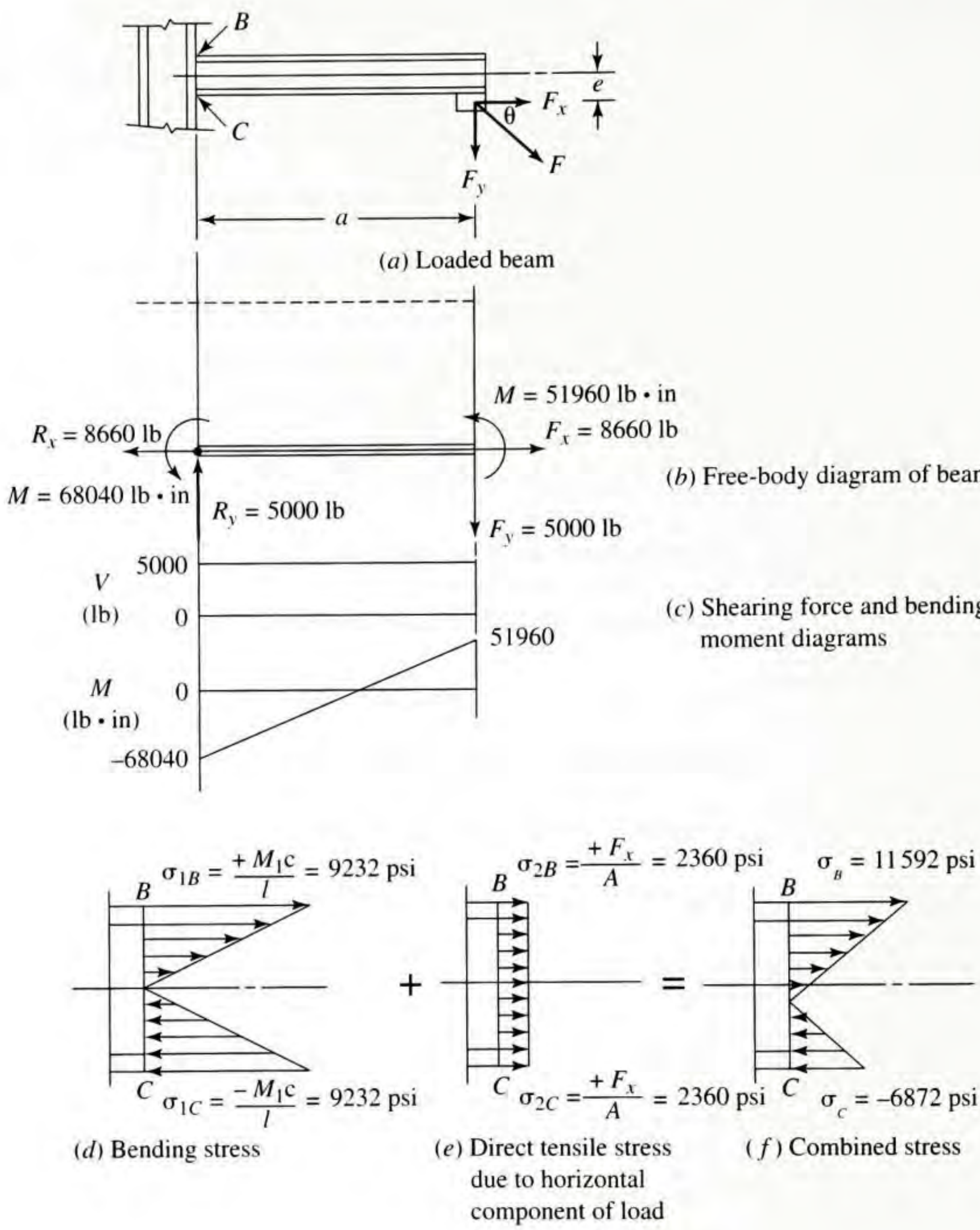


FIGURE 3–24 Beam subjected to combined stresses

Section modulus = S = 7.37 in^3; area = A = 3.67 in^2 (Table A16–4).
Eccentricity of the load = e = 6.0 in from the neutral axis of the beam to the line of action of the horizontal component of the applied load.

Analysis The analysis takes the following steps:

1. Resolve the applied force into its vertical and horizontal components.
2. Transfer the horizontal component to an equivalent loading at the neutral axis having a direct tensile force and a moment due to the eccentric placement of the force.
3. Prepare the free-body diagram using the techniques from Section 3–19.

4. Draw the shearing force and bending moment diagrams and determine where the maximum bending moment occurs.
5. Complete the stress analysis at that section, computing both the maximum tensile and maximum compressive stresses.

Results The components of the applied force are:

$$F_x = F\cos(30^\circ) = (10\,000\text{ lb})[\cos(30^\circ)] = 8660\text{ lb acting to the right}$$
$$F_y = F\sin(30^\circ) = (10\,000\text{ lb})[\sin(30^\circ)] = 5000\text{ lb acting downward}$$

The horizontal force produces a counterclockwise concentrated moment at the right end of the beam with a magnitude of:

$$M_1 = F_x(6.0\text{ in}) = (8660\text{ lb})(6.0\text{ in}) = 51\,960\text{ lb}\cdot\text{in}$$

The free-body diagram of the beam is shown in Figure 3–24(*b*).

Figure 3–24(*c*) shows the shearing force and bending moment diagrams.

The maximum bending moment, 68 040 lb in, occurs at the left end of the beam where it is attached firmly to a column.

The bending moment, taken alone, produces a tensile stress (+) on the top surface at point *B* and a compressive stress (−) on the bottom surface at *C*. The magnitudes of these stresses are:

$$\sigma_1 = \pm\, M/S = \pm\, (68\,040\text{ lb in}) \,/\, (7.37\text{ in}^3) = \pm\, 9232\text{ psi}$$

Figure 3–24(*d*) shows the stress distribution due only to the bending stress.

Now we compute the tensile stress due to the axial force of 8660 lb.

$$\sigma_2 = F_x/A = (8660\text{ lb})/(3.67\text{ in}^2) = 2360\text{ psi}$$

Figure 3–24(*e*) shows this stress distribution, uniform across the entire section.

Next, let's compute the combined stress at *B* on the top of the beam.

$$\sigma_B = +\,\sigma_1 + \sigma_2 = 9232\text{ psi} + 2360\text{ psi} = 11\,592\text{ psi Tensile}$$

At *C* on the bottom of the beam, the stress is:

$$\sigma_C = -\sigma_1 + \sigma_2 = -9232\text{ psi} + 2360\text{ psi} = -6872\text{ psi Compressive}$$

Figure 3–24(*f*) shows the combined stress condition that exists on the cross section of the beam at its left end at the support. It is a superposition of the component stresses shown in Figure 3–24(*d*) and (*e*).

3–21 STRESS CONCENTRATIONS

The formulas reviewed earlier for computing simple stresses due to direct tensile and compressive forces, bending moments, and torsional moments are applicable under certain conditions. One condition is that the geometry of the member is uniform throughout the section of interest.

In many typical machine design situations, inherent geometric discontinuities are necessary for the parts to perform their desired functions. For example, as shown in Figure 12–2 in Chapter 12, shafts carrying gears, chain sprockets, or belt sheaves usually have several diameters that create a series of shoulders that seat the power transmission members and support bearings. Grooves in the shaft allow the installation of retaining rings. Keyseats milled into the shaft enable keys to drive the elements. Similarly, tension members in linkages may be designed with retaining ring grooves, radial holes for pins, screw threads, or reduced sections.

Any of these geometric discontinuities will cause the actual maximum stress in the part to be higher than the simple formulas predict. Defining *stress concentration factors* as the factors by which the actual maximum stress exceeds the nominal stress, σ_{nom} or τ_{nom}, predicted from the simple equations allows the designer to analyze these situations. The symbol for these factors is K_t. In general, the K_t factors are used as follows:

$$\sigma_{max} = K_t\sigma_{nom} \quad \text{or} \quad \tau_{max} = K_t\tau_{nom} \tag{3–27}$$

depending on the kind of stress produced for the particular loading. The value of K_t depends on the shape of the discontinuity, the specific geometry, and the type of stress. Appendix 15 includes several charts for stress concentration factors. (See Reference 5.) Note that the charts indicate the method of computing the nominal stress. Usually, we compute the nominal stress by using the net section in the vicinity of the discontinuity. For example, for a flat plate with a hole in it subjected to a tensile force, the nominal stress is computed as the force divided by the minimum cross-sectional area through the location of the hole.

But there are other cases in which the gross area is used in calculating the nominal stress. For example, we analyze keyseats by applying the stress concentration factor to the computed stress in the full-diameter portion of the shaft.

Figure 3–25 shows an experimental device that demonstrates the phenomenon of stress concentrations. A model of a beam having several different cross-sectional heights is made from a special plastic that reacts to the presence of varying stresses at different points. When the model is viewed through a polarizing filter, several black *fringes* appear. Where there are many closely spaced fringes, the stress is changing rapidly. We can compute the actual magnitude of stress by knowing the optical characteristics of the plastic.

The beam in Figure 3–25 is simply supported near each end and is loaded vertically at its middle. The largest stress occurs to the left of the middle where the height of the cross section is reduced. Note that the fringes are very close together in the vicinity of the fillet connecting the smaller section to the larger part where the load is applied. This indicates

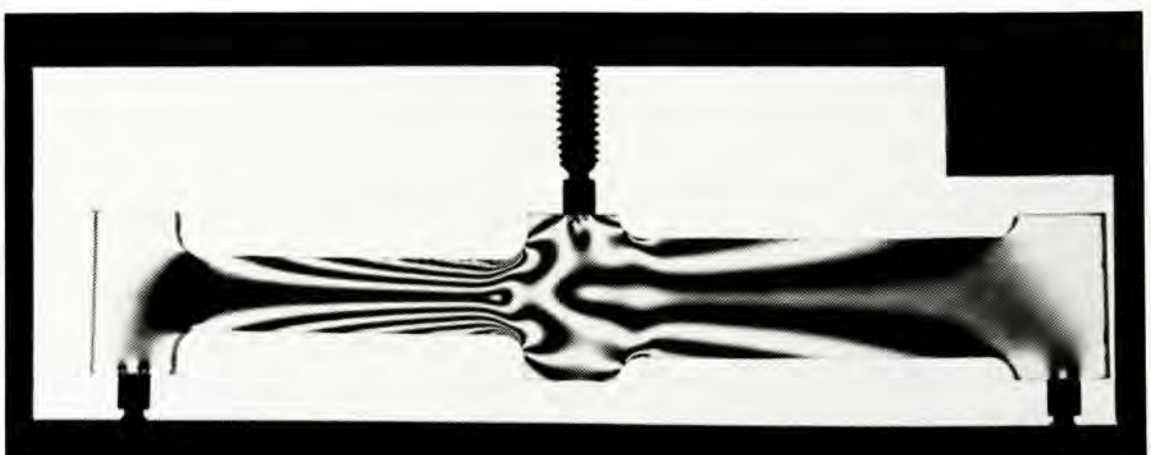


FIGURE 3–25
Illustration of stress concentrations (Source: Measurements Group, Inc., Raleigh, North Carolina, U.S.A.)

that the highest stress occurs in the fillet. Figure A15–2 gives data for the values of the stress concentration factor, K_t. The nominal stress, σ_{nom}, is computed from the classic flexure formula, and the section modulus is based on the smaller cross section near the fillet. These formulas are listed near the stress concentration chart.

An interesting observation can be made from Figure A15-3 showing the stress concentration factors for a flat plate with a central hole. Curves *A* and *B* refer to tension loading, while curve *C* is for bending. The nominal stress for each case is computed on the basis of the net cross section accounting for the material removed by the hole. Curve *C* indicates that the stress concentration factor is taken to be 1.0 for smaller holes, with the ratio of the hole diameter to the width of the plate <0.50.

Figure A15–4 covers the case of a round shaft that has a circular hole completely through it. The three curves cover tension, bending, and torsional loading, and each is based on the stress in the gross section, that is, the geometry of the shaft without the hole. Therefore, the value of K_t includes the effects of both the material removal and the discontinuity created by the presence of the hole. The values of K_t are relatively high, even for the smaller holes, indicating that you should be cautious when applying shafts with holes to ensure that the local stresses are small.

The following are guidelines on the use of stress concentration factors:

1. The worst case occurs for those areas in tension.
2. Always use stress concentration factors in analyzing members under fatigue loading because fatigue cracks usually initiate near points of high local tensile stress.
3. Stress concentrations can be ignored for static loading of ductile materials because if the local maximum stress exceeds the yield strength of the material, the load is redistributed. The resulting member is actually stronger after the local yielding occurs.
4. The stress concentration factors in Appendix 15 are empirical values based only on the geometry of the member and the manner of loading.
5. Use stress concentration factors when analyzing brittle materials under either static or fatigue loading. Because the material does not yield, the stress redistribution described in item 3 cannot occur.
6. Even scratches, nicks, corrosion, excessive surface roughness, and plating can cause stress concentrations. Chapter 5 discusses the care essential to manufacturing, handling, and assembling components subjected to fatigue loading.

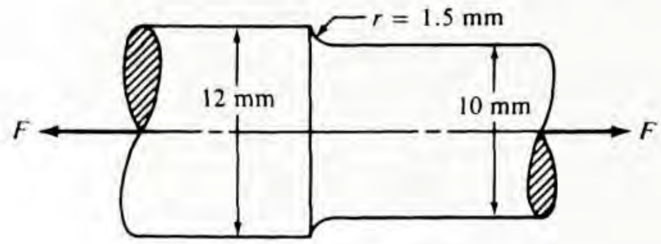
Example Problem 3–18 Compute the maximum stress in a round bar subjected to an axial tensile force of 9800 N. The geometry is shown in Figure 3–26.

Solution Objective: Compute the maximum stress in the stepped bar shown in Figure 3–26.

Given: The layout from Figure 3–26. Force $= F = 9800$ N.
The shaft has two diameters joined by a fillet with a radius of 1.5 mm.
Larger diameter $= D = 12$ mm; smaller diameter $= d = 10$ mm.

Analysis: The presence of the change in diameter at the step causes a stress concentration to occur. The general situation is a round bar subjected to an axial tensile load. We will use the top

FIGURE 3–26
Stepped round bar subjected to axial tensile force



graph of Figure A15–1 to determine the stress concentration factor. That value is used in Equation (3–27) to determine the maximum stress.

Results Figure A15–1 indicates that the nominal stress is computed for the smaller of the two diameters of the bar. The stress concentration factor depends on the ratio of the two diameters and the ratio of the fillet radius to the smaller diameter.

$$D/d = 12\ \text{mm}/10\ \text{mm} = 1.20$$
$$r/d = 1.5\ \text{mm}/10\ \text{mm} = 0.15$$

From these values, we can find that $K_t = 1.60$. The stress is

$$\sigma_{\text{nom}} = F/A = (9800\ \text{N})/[\pi(10\ \text{mm})^2/4] = 124.8\ \text{MPa}$$
$$\sigma_{\text{max}} = K_t\sigma_{\text{nom}} = (1.60)(124.8\ \text{MPa}) = 199.6\ \text{MPa}$$

Comments The maximum tensile stress of 199.6 MPa occurs in the fillet near the smaller diameter. This value is 1.60 times higher than the nominal stress that occurs in the 10-mm-diameter shaft. To the left of the shoulder, the stress reduces dramatically as the effect of the stress concentration diminishes and because the area is larger.

3–22 NOTCH SENSITIVITY AND STRENGTH REDUCTION FACTOR

The amount by which a load-carrying member is weakened by the presence of a stress concentration (notch), considering both the material and the sharpness of the notch, is defined as

$$K_f = \text{fatigue strength reduction factor}$$
$$K_f = \frac{\text{endurance limit of a notch-free specimen}}{\text{endurance limit of a notched specimen}}$$

This factor could be determined by actual test. However, it is typically found by combining the stress concentration factor, K_t, defined in the previous section, and a material factor called the *notch sensitivity*, q. We define

$$q = (K_f - 1)/(K_t - 1) \qquad \textbf{(3–28)}$$

When q is known, K_f can be computed from

$$K_f = 1 + q\,(K_t - 1) \qquad \textbf{(3–29)}$$

Values of q range from zero to 1.0, and therefore K_f varies from 1.0 to K_t. Under repeated bending loads, very ductile steels typically exhibit values of q from 0.5 to 0.7. High-strength steels with hardness approximately HB 400 ($s_u \cong 200$ ksi or 1400 MPa) have values of q from 0.90 to 0.95. (See Reference 2 for further discussion of values of q.)

Because reliable values of q are difficult to obtain, the problems in this book will assume that $q = 1.0$ and $K_f = K_t$, the safest, most conservative value.

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INTERNET SITES RELATED TO STRESS AND DEFORMATION ANALYSIS

1. **BEAM 2D-Stress Analysis 3.1** *www.orandsystems.com* Software for mechanical, structural, civil, and architectural designers providing detailed analysis of statically indeterminate and determinate beams.
2. **MDSolids** *www.mdsolids.com* Educational software devoted to introductory mechanics of materials. Includes modules on basic stress and strain; beam and strut axial problems; trusses; statically indeterminate axial structures; torsion; determinate beams; section properties; general analysis of axial, torsion, and beam members; column buckling; pressure vessels; and Mohr's circle transformations.
3. **StressAlyzer** *http://hpme16.me.cmu.edu/stressalyzer* Interactive courseware for mechanics of materials including modules on axial loading, torsion loading, shear force and bending moment diagrams, load and stress calculations in 3D, beam deflections, and stress transformations.

PROBLEMS

Direct Tension and Compression

1. A tensile member in a machine structure is subjected to a steady load of 4.50 kN. It has a length of 750 mm and is made from a steel tube having an outside diameter of 18 mm and an inside diameter of 12 mm. Compute the tensile stress in the tube and the axial deformation.
2. Compute the stress in a round bar having a diameter of 10.0 mm and subjected to a direct tensile force of 3500 N.
3. Compute the stress in a rectangular bar having cross-sectional dimensions of 10.0 mm by 30.0 mm when a direct tensile force of 20.0 kN is applied.
4. A link in a packaging machine mechanism has a square cross section 0.40 in on a side. It is subjected to a tensile force of 860 lb. Compute the stress in the link.
5. Two circular rods support the 3800 lb weight of a space heater in a warehouse. Each rod has a diameter of 0.375 in and carries 1/2 of the total load. Compute the stress in the rods.
6. A tensile load of 5.00 kN is applied to a square bar, 12 mm on a side and having a length of 1.65 m. Compute the stress and the axial deformation in the bar if it is made from (a) AISI 1020 hot-rolled steel, (b) AISI 8650 OQT 1000 steel, (c) ductile iron A536-88 (60-40-18), (d) aluminum 6061-T6, (e) titanium Ti-6Al-4V, (f) rigid PVC plastic, and (g) phenolic plastic.
7. An aluminum rod is made in the form of a hollow square tube, 2.25 in outside, with a wall thickness of 0.120 in. Its length is 16.0 in. What axial compressive force would cause the tube to shorten by 0.004 in? Compute the resulting compressive stress in the aluminum.

8. Compute the stress in the middle portion of rod AC in Figure P3–8 if the vertical force on the boom is 2500 lb. The rod is rectangular, 1.50 in by 3.50 in.

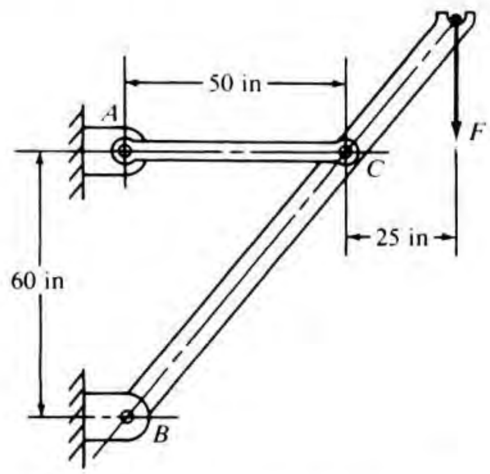


FIGURE P3–8 (Problems 8, 16, and 56)

9. Compute the forces in the two angled rods in Figure P3–9 for an applied force, $F = 1500$ lb, if the angle θ is 45°.

10. If the rods from Problem 9 are circular, determine their required diameter if the load is static and the allowable stress is 18 000 psi.

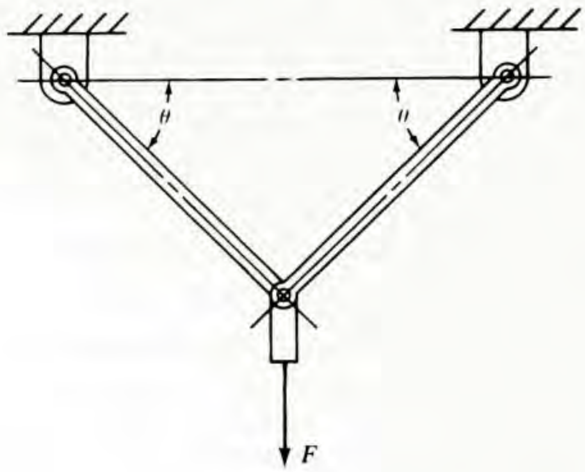


FIGURE P3–9 (Problems 9, 10, 11, 17, and 18)

11. Repeat Problems 9 and 10 if the angle θ is 15°.

12. Figure P3–12 shows a small truss spanning between solid supports and suspending a 10.5 kN load. The cross sections for the three main types of truss members are shown. Compute the stresses in all of the members of the truss near their midpoints away from the connections. Consider all joints to be pinned.

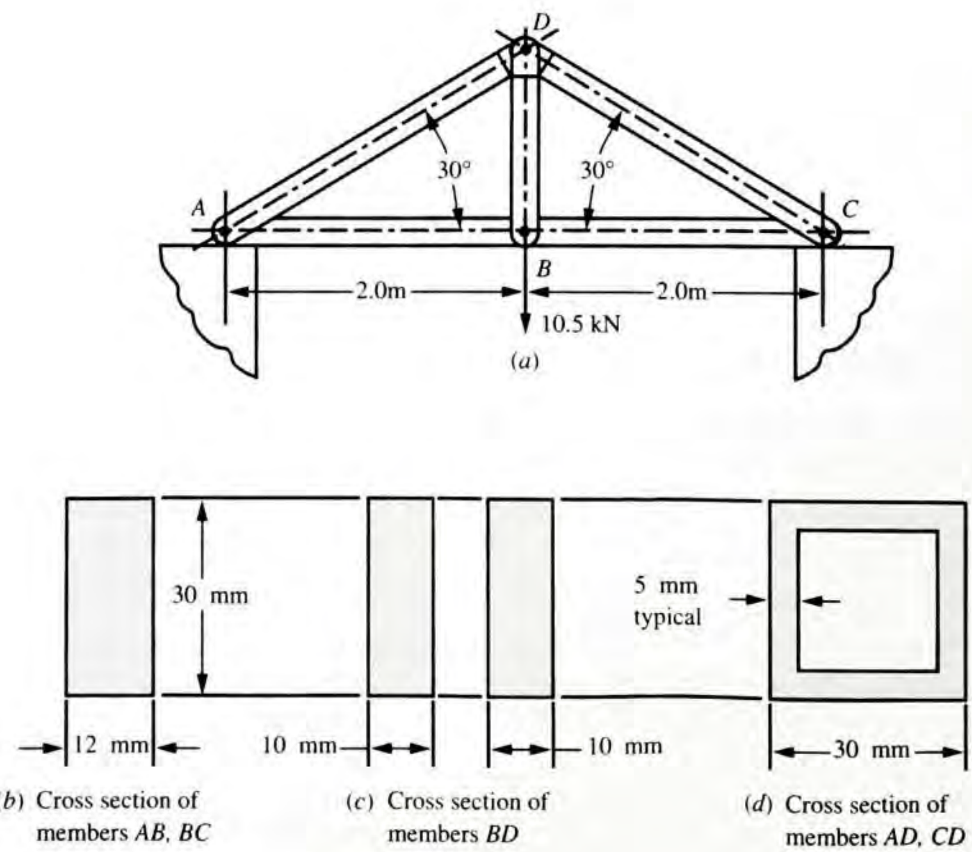
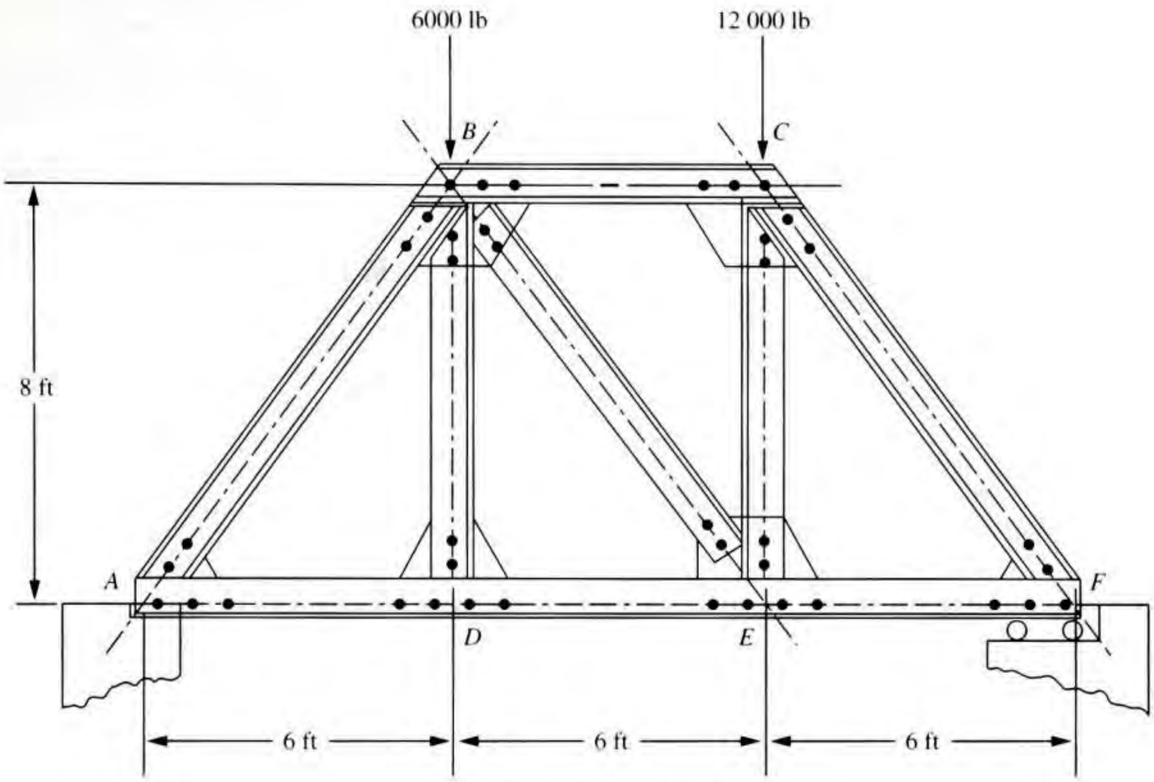


FIGURE P3–12 (Problem 12)

13. The truss shown in Figure P3–13 spans a total space of 18.0 ft and carries two concentrated loads on its top chord. The members are made from standard steel angle and channel shapes as indicated in the figure. Consider all joints to be pinned. Compute the stresses in all members near their midpoints away from the connections.

14. Figure P3–14 shows a short leg for a machine that carries a direct compression load. Compute the compressive



Member specifications:

AD, DE, EF L2 × 2 × 1/8 – doubled —— ⅃L
BD, CE, BE L2 × 2 × 1/8 – single —— ⅃
AB, BC, CF C3 × 4.1 – doubled —— ⊐⊏

FIGURE P3–13 (Problem 13)

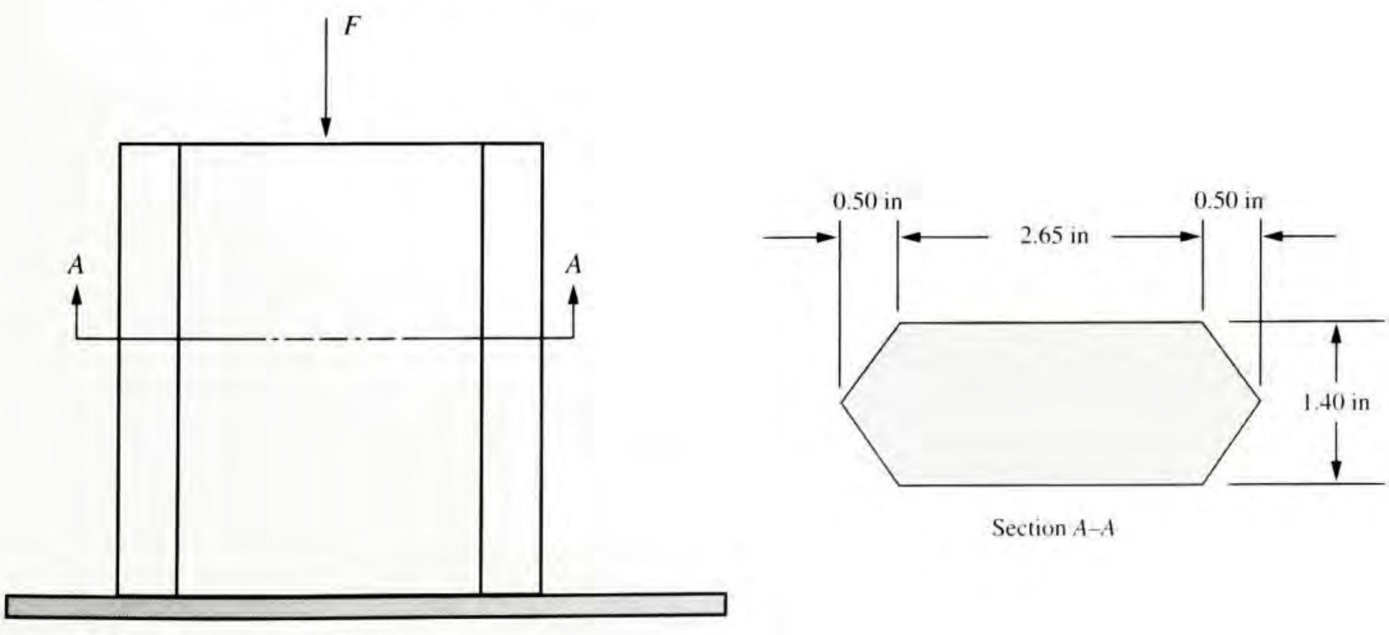


FIGURE P3–14 (Problem 14)

stress if the cross section has the shape shown and the applied force is $F = 52\ 000$ lb.

15. Consider the short compression member shown in Figure P3–15. Compute the compressive stress if the cross section has the shape shown and the applied load is 640 kN.

Direct Shear Stress

16. Refer to Figure P3–8. Each of the pins at A, B, and C has a diameter of 0.50 in and is loaded in double shear. Compute the shear stress in each pin.

17. Compute the shear stress in the pins connecting the rods shown in Figure P3–9 when a load of $F = 1500$ lb is carried. The pins have a diameter of 0.75 in. The angle $\theta = 40°$.

18. Repeat Problem 17, but change the angle to $\theta = 15°$.

19. Refer to Figure 3–7. Compute the shear stress in the key if the shaft transmits a torque of 1600 N · m. The shaft diameter is 60 mm. The key is square with $b = 12$ mm, and it has a length of 45 mm.

20. A punch is attempting to cut a slug having the shape shown in Figure P3–20 from a sheet of aluminum having a thickness of 0.060 in. Compute the shearing stress in the aluminum when a force of 52 000 lb is applied by the punch.

21. Figure P3–21 shows the shape of a slug that is to be cut from a sheet of steel having a thickness of 2.0 mm. If the punch exerts a force of 225 kN, compute the shearing stress in the steel.

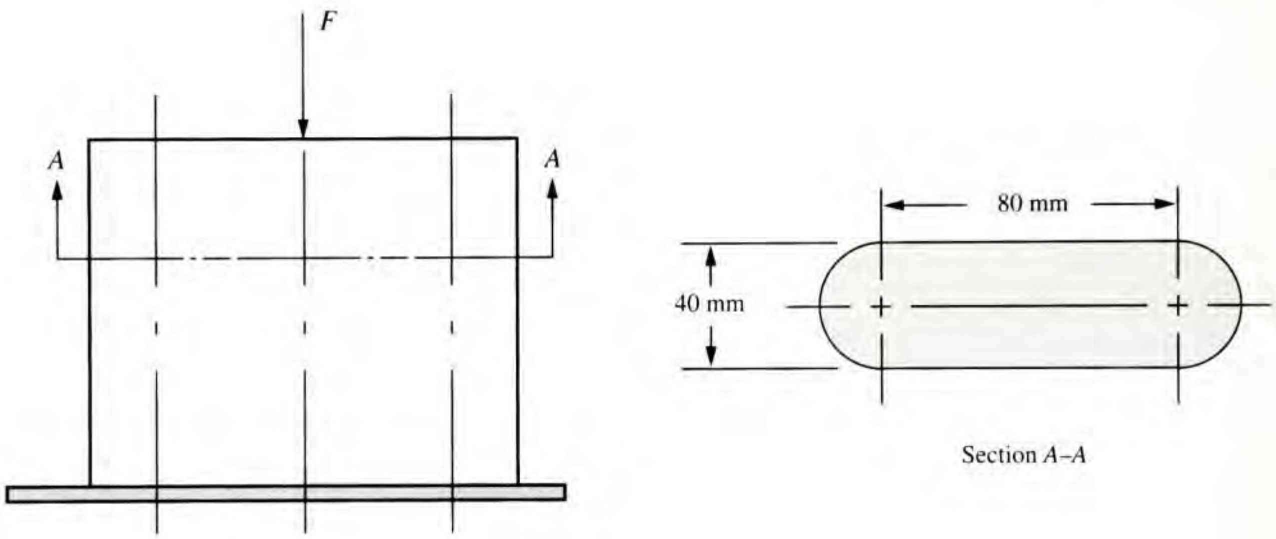


FIGURE P3–15 (Problem 15)

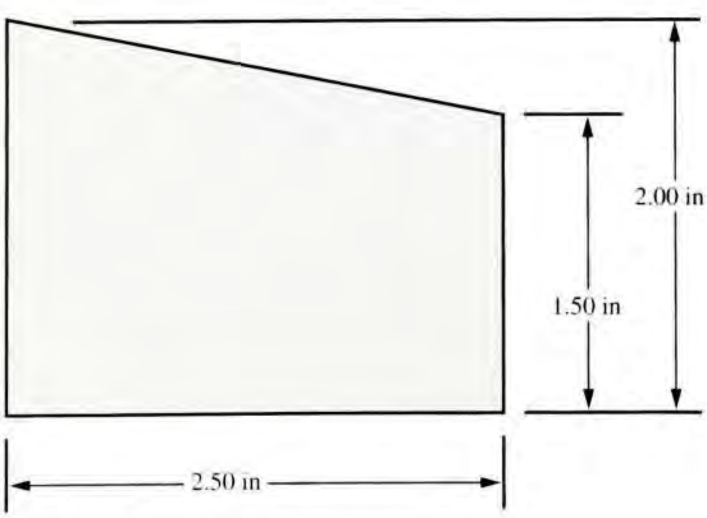


FIGURE P3–20 (Problem 20)

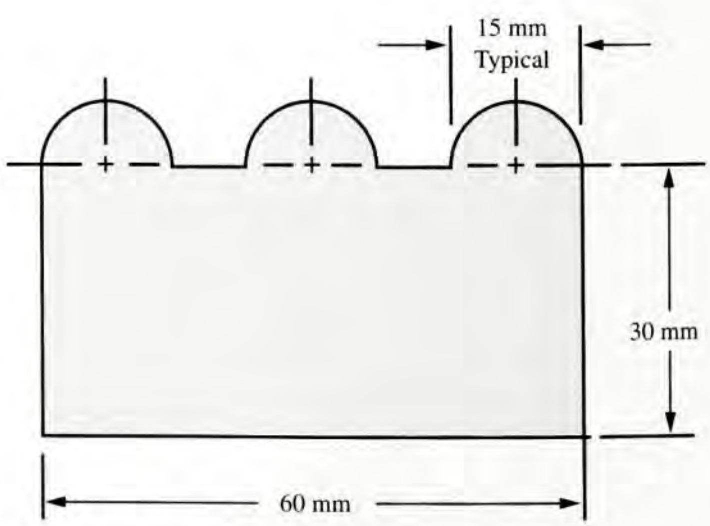


FIGURE P3–21 (Problem 21)

Torsion

22. Compute the torsional shear stress in a circular shaft with a diameter of 50 mm that is subjected to a torque of 800 N·m.

23. If the shaft of Problem 22 is 850 mm long and is made of steel, compute the angle of twist of one end in relation to the other.

24. Compute the torsional shear stress due to a torque of 88.0 lb·in in a circular shaft having a 0.40-in diameter.

25. Compute the torsional shear stress in a solid circular shaft having a diameter of 1.25 in that is transmitting 110 hp at a speed of 560 rpm.

26. Compute the torsional shear stress in a hollow shaft with an outside diameter of 40 mm and an inside diameter of 30 mm when transmitting 28 kilowatts (kW) of power at a speed of 45 rad/s.

27. Compute the angle of twist for the hollow shaft of Problem 26 over a length of 400 mm. The shaft is steel.

Noncircular Members in Torsion

28. A square steel bar, 25 mm on a side and 650 mm long, is subjected to a torque of 230 N·m. Compute the shear stress and the angle of twist for the bar.

29. A 3.00 in-diameter steel bar has a flat milled on one side, as shown in Figure P3–29. If the shaft is 44.0 in long and carries a torque of 10 600 lb·in, compute the stress and the angle of twist.

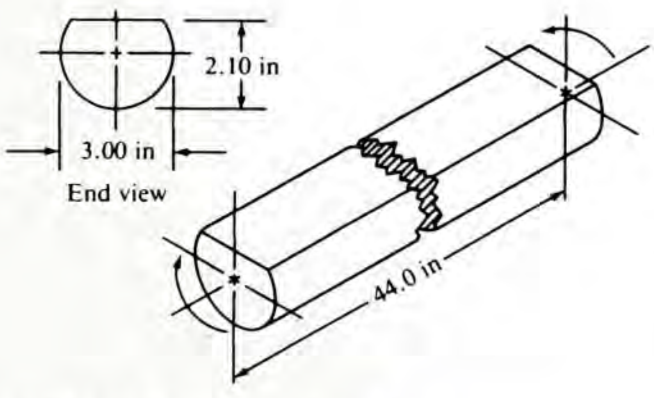


FIGURE P3–29 (Problem 29)

30. A commercial steel supplier lists rectangular steel tubing having outside dimensions of 4.00 by 2.00 in and a wall thickness of 0.109 in. Compute the maximum torque that can be applied to such a tube if the shear stress is to be limited to 6000 psi. For this torque, compute the angle of twist of the tube over a length of 6.5 ft.

Beams

31. A beam is simply supported and carries the load shown in Figure P3–31. Specify suitable dimensions for the beam if it is steel and the stress is limited to 18 000 psi, for the following shapes:

(a) Square

(b) Rectangle with height three times the width

(c) Rectangle with height one-third the width

(d) Solid circular section

(e) American Standard beam section

(f) American Standard channel with the legs down

(g) Standard steel pipe

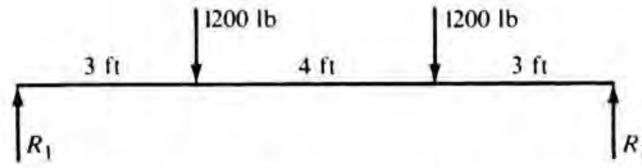


FIGURE P3–31 (Problems 31, 32, and 33)

32. For each beam of Problem 31, compute its weight if the steel weighs 0.283 lb/in^3.

33. For each beam of Problem 31, compute the maximum deflection and the deflection at the loads.

34. For the beam loading of Figure P3–34, draw the complete shearing force and bending moment diagrams, and determine the bending moments at points *A*, *B*, and *C*.

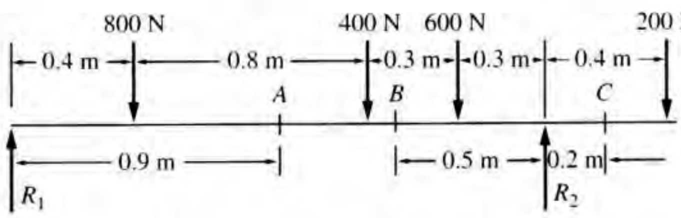


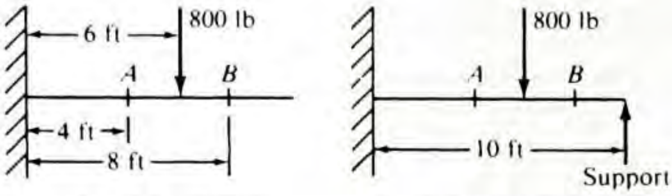
FIGURE P3–34 (Problems 34 and 35)

35. For the beam loading of Figure P3–34, design the beam, choosing a shape that will be reasonably efficient and will limit the stress to 100 MPa.

36. Figure P3–36 shows a beam made from 4 in steel pipe. Compute the deflection at points *A* and *B* for two cases: (a) the simple cantilever and (b) the supported cantilever.

37. Select an aluminum I-beam shape to carry the load shown in Figure P3–37 with a maximum stress of 12 000 psi. Then compute the deflection at each load.

38. Figure P3–38 represents a wood joist for a platform, carrying a uniformly distributed load of 120 lb/ft and two concentrated loads applied by some machinery. Compute



(*a*) Simple cantilever (*b*) Supported cantilever

FIGURE P3–36 (Problem 36)

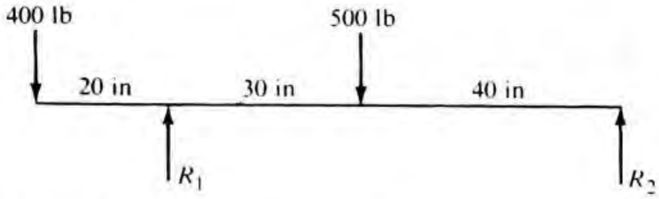


FIGURE P3–37 (Problem 37)

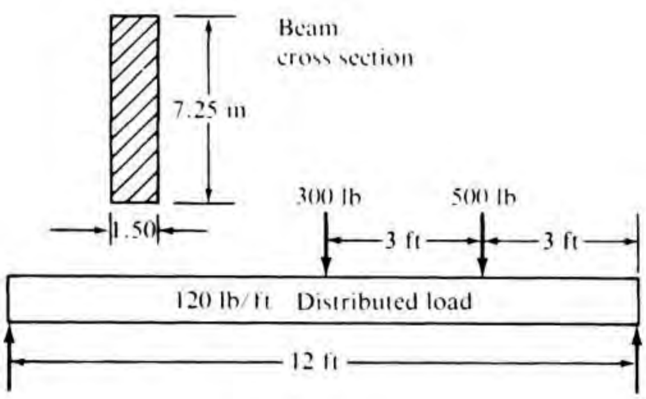


FIGURE P3–38 (Problem 38)

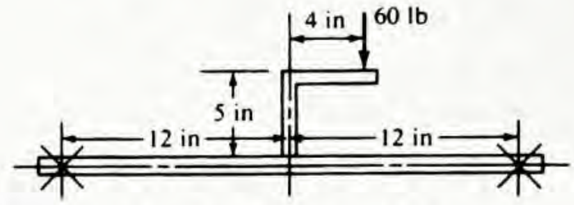


FIGURE P3–39 (Problems 39 and 57)

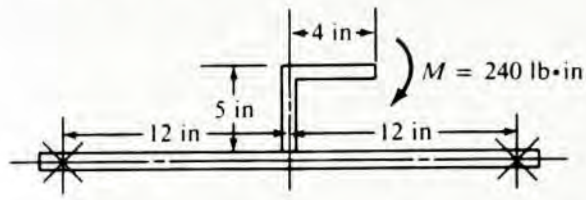


FIGURE P3–40 (Problem 40)

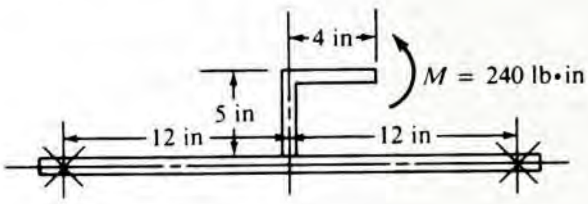


FIGURE P3–41 (Problem 41)

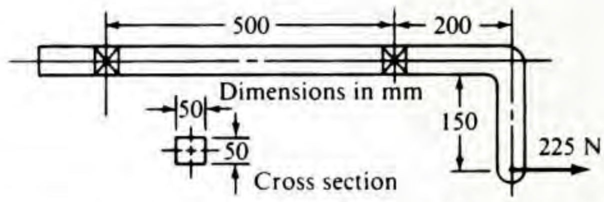


FIGURE P3–42 (Problems 42 and 58)

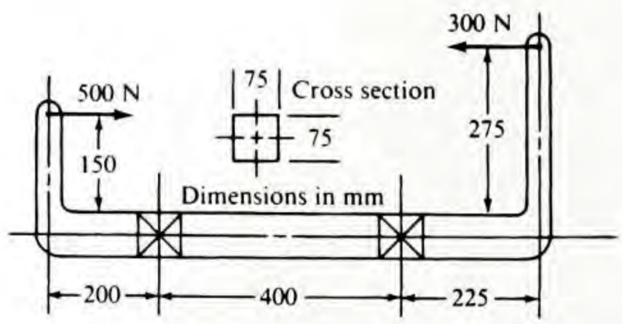


FIGURE P3–43 (Problems 43 and 59)

the maximum stress due to bending in the joist and the maximum vertical shear stress.

Beams with Concentrated Bending Moments

For Problems 39 through 50, draw the free-body diagram of only the horizontal beam portion of the given figures. Then draw the complete shear and bending moment diagrams. Where used, the symbol X indicates a simple support capable of exerting a reaction force in any direction but having no moment resistance. For beams having unbalanced axial loads, you may specify which support offers the reaction.

39. Use Figure P3–39.

40. Use Figure P3–40.

41. Use Figure P3–41.

42. Use Figure P3–42.

43. Use Figure P3–43.

44. Use Figure P3–44.

45. Use Figure P3–45.

46. Use Figure P3–46.

47. Use Figure P3–47.

48. Use Figure P3–48.

49. Use Figure P3–49.

50. Use Figure P3–50.

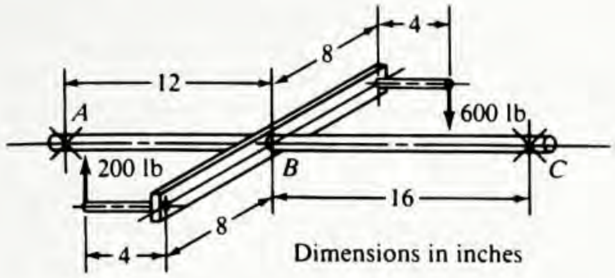


FIGURE P3–44 (Problem 44)

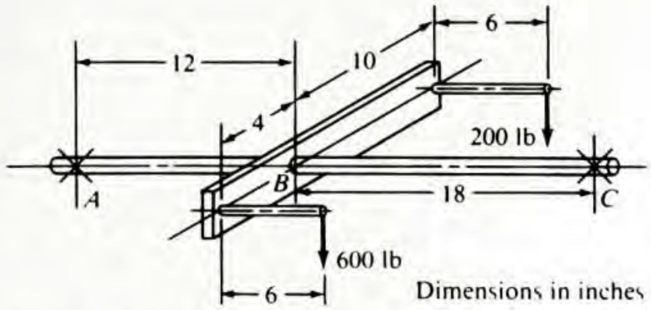


FIGURE P3–45 (Problem 45)

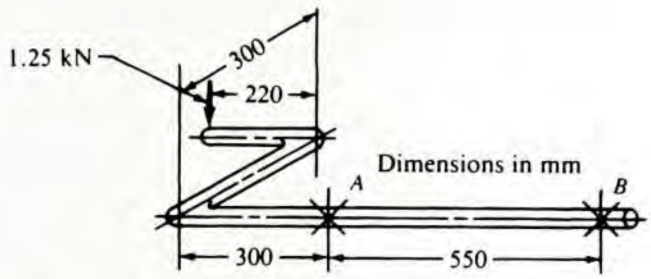


FIGURE P3–46 (Problem 46)

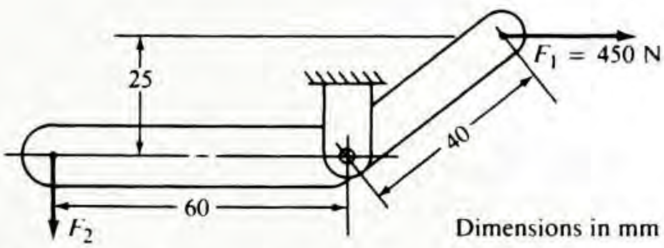


FIGURE P3–47 (Problem 47)

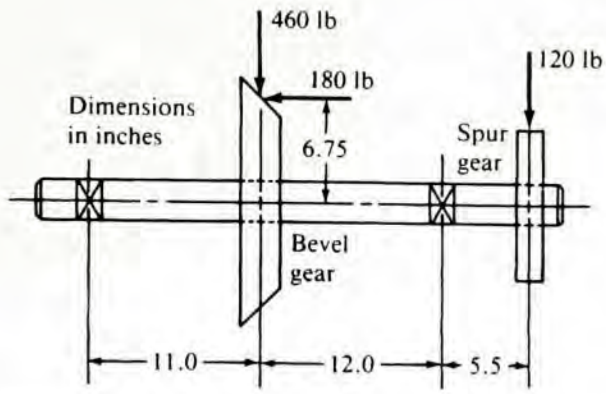


FIGURE P3–48 (Problem 48)

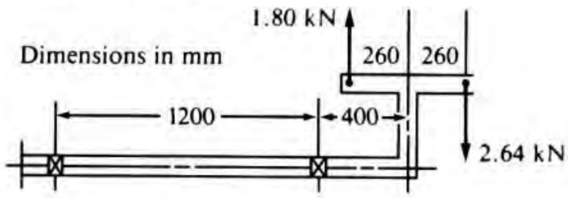


FIGURE P3–49 (Problem 49)

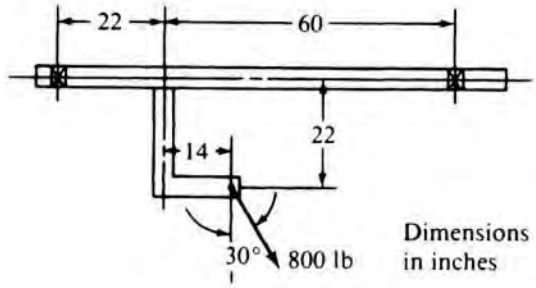


FIGURE P3–50 (Problems 50 and 60)

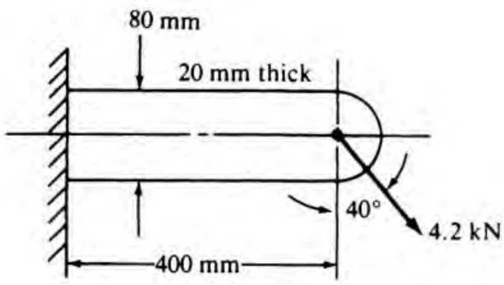


FIGURE P3–51 (Problem 51)

Combined Normal Stresses

51. Compute the maximum tensile stress in the bracket shown in Figure P3–51.

52. Compute the maximum tensile and compressive stresses in the horizontal beam shown in Figure P3–52.

53. For the lever shown in Figure P3–53 (a), compute the stress at section A near the fixed end. Then redesign the lever to the tapered form shown in Part (b) of the figure by adjusting only the height of the cross section at sections *B* and *C* so that they have no greater stress than section *A*.

54. Compute the maximum tensile stress at sections *A* and *B* on the crane boom shown in Figure P3–54.

55. Refer to Figure 3–22. Compute the maximum tensile stress in the print head just to the right of the right guide. The head has a rectangular cross section, 5.0 mm high in the plane of the paper and 2.4 mm thick.

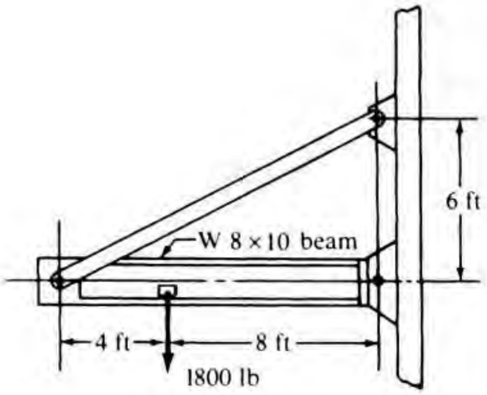
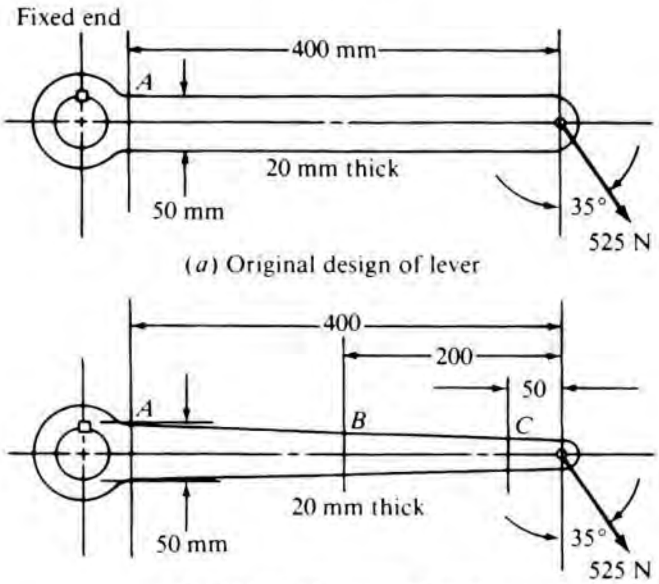


FIGURE P3–52 (Problem 52)



(*a*) Original design of lever

(*b*) Modified design of lever

FIGURE P3–53 (Problem 53)

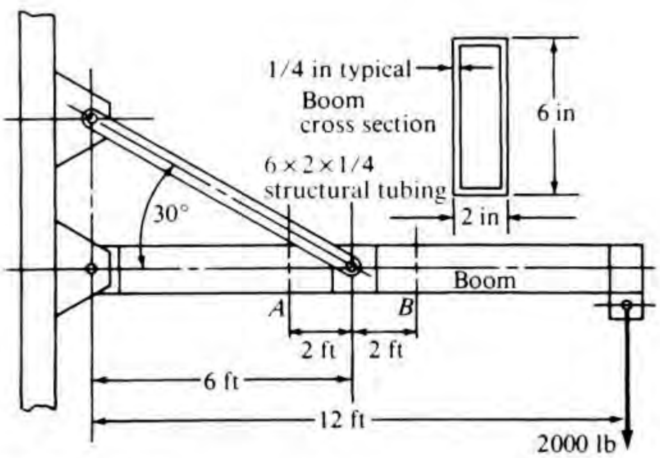


FIGURE P3–54 (Problem 54)

56. Refer to Figure P3–8. Compute the maximum tensile and compressive stresses in the member *B-C* if the load *F* is 1800 lb. The cross section of *B-C* is a 6 × 4 × 1/4 rectangular tube.

57. Refer to P3–39. The vertical member is to be made from steel with a maximum allowable stress of 12 000 psi. Specify the required size of a standard square cross section if sizes are available in increments of 1/16 in.

58. Refer to P3–42. Compute the maximum stress in the horizontal portion of the bar, and tell where it occurs on the cross section. The left support resists the axial force.

59. Refer to P3–43. Compute the maximum stress in the horizontal portion of the bar, and indicate where it occurs on the cross section. The right support resists the unbalanced axial force.

60. Refer to P3–50. Specify a suitable diameter for a solid circular bar to be used for the top horizontal member, which is supported in the bearings. The left bearing resists the axial load. The allowable normal stress is 25 000 psi.

Stress Concentrations

61. Figure P3–61 shows a valve stem from an engine subjected to an axial tensile load applied by the valve spring. For a force of 1.25 kN, compute the maximum stress at the fillet under the shoulder.

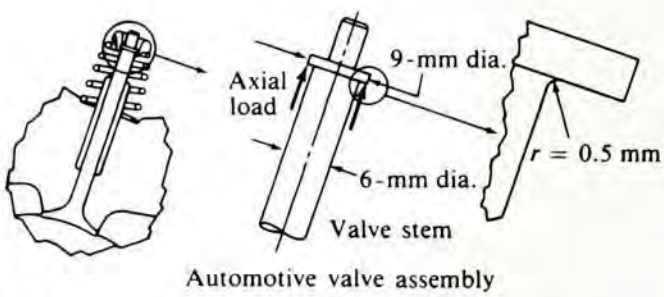


FIGURE P3–61 (Problem 61)

62. The conveyor fixture shown in Figure P3–62 carries three heavy assemblies (1200 lb each). Compute the maximum stress in the fixture, considering stress concentrations at the fillets and assuming that the load acts axially.

63. For the flat plate in tension in Figure P3–63, compute the stress at each hole, assuming that the holes are sufficiently far apart that their effects do not interact.

For Problems 64 through 68, compute the maximum stress in the member, considering stress concentrations.

64. Use Figure P3–64.

65. Use Figure P3–65.

66. Use Figure P3–66.

67. Use Figure P3–67.

68. Use Figure P3–68.

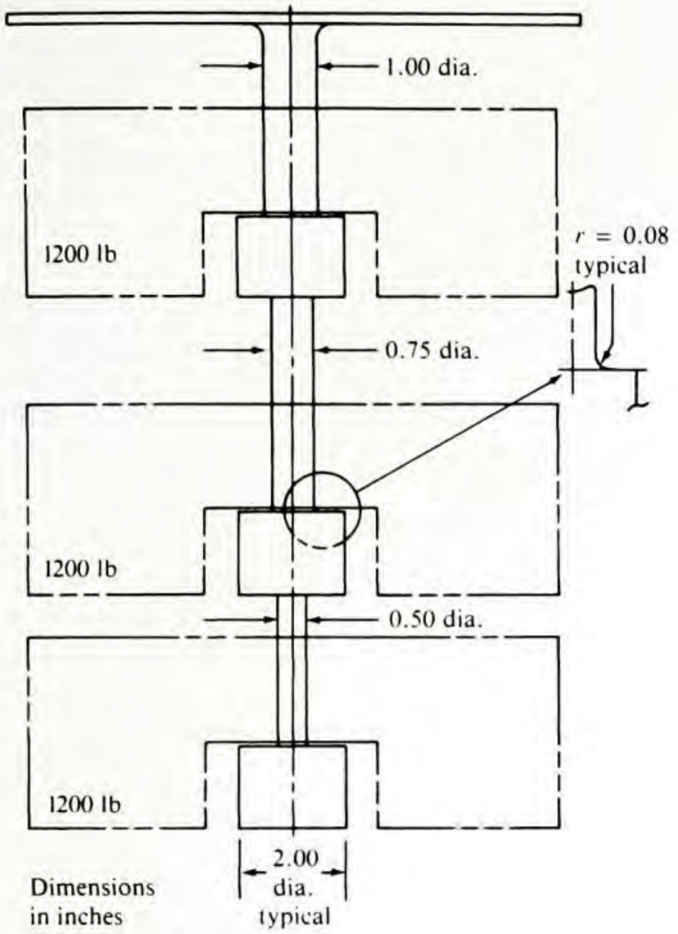


FIGURE P3–62 (Problem 62)

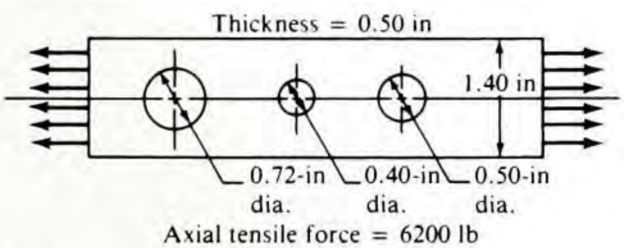


FIGURE P3–63 (Problem 63)

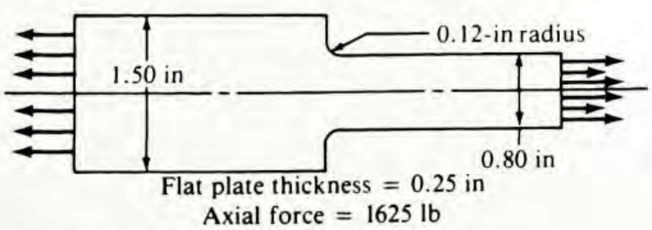


FIGURE P3–64 (Problem 64)

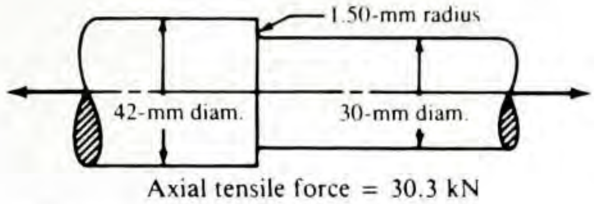


FIGURE P3–65 (Problem 65)

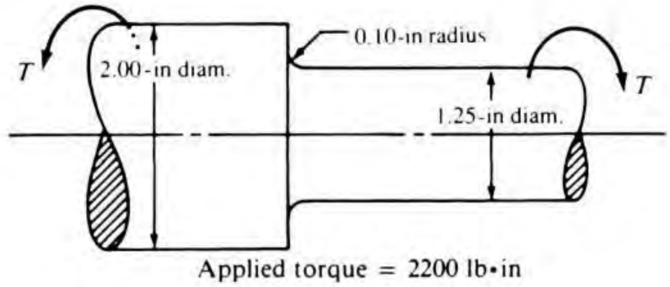


FIGURE P3–66 (Problem 66)

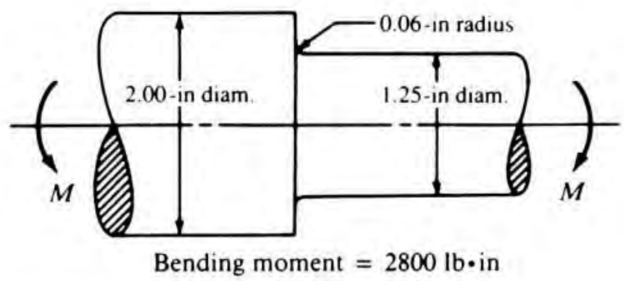


FIGURE P3–67 (Problem 67)

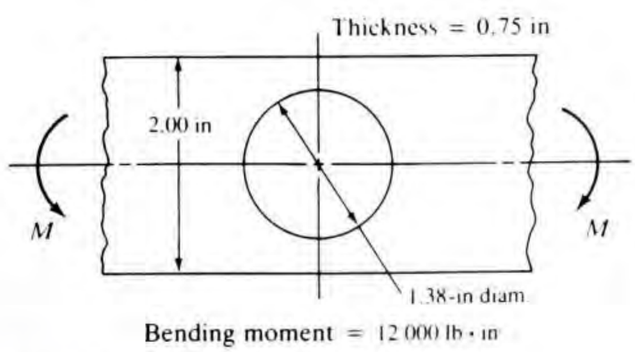


FIGURE P3–68 (Problem 68)

Problems of a General Nature

69. Figure P3–69 shows a horizontal beam supported by a vertical tension link. The cross sections of both the beam and the link are 20 mm square. All connections use 8.00 mm-diameter cylindrical pins in double shear. Compute the tensile stress in member *A-B*, the stress due to bending in *C-D*, and the shearing stress in the pins *A* and *C*.

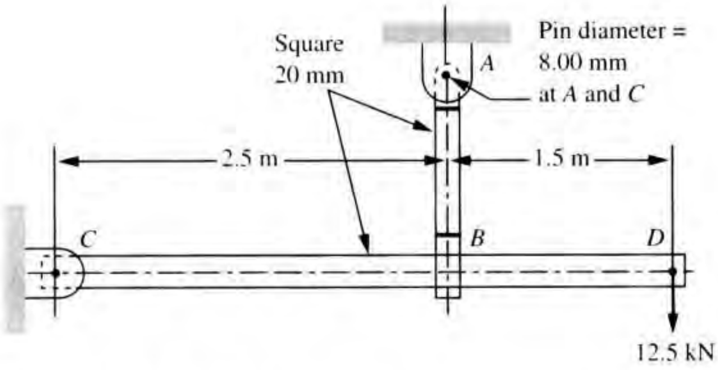


FIGURE P3–69 (Problem 69)

70. Figure P3–70 shows a tapered flat bar that has a uniform thickness of 20 mm. The depth tapers from h_1 = 40 mm near the load to h_2 = 20 mm at each support. Compute the stress due to bending in the bar at points spaced 40 mm apart from the support to the load. Let the load P = 5.0 kN.

71. For the flat bar shown in Figure P3–70, compute the stress in the middle of the bar if a hole of 25 mm diameter is drilled directly under the load on the horizontal centerline. The load is P = 5.0 kN. See data in Problem 70.

72. The beam shown in Figure P3–72 is a stepped, flat bar having a constant thickness of 1.20 in. It carries a single concentrated load at *C* of 1500 lb. Compare the stresses at the following locations:
   - **(a)** In the vicinity of the load
   - **(b)** At the section through the smaller hole to the right of section *C*
   - **(c)** At the section through the larger hole to the right of section *C*
   - **(d)** Near section *B* where the bar changes height

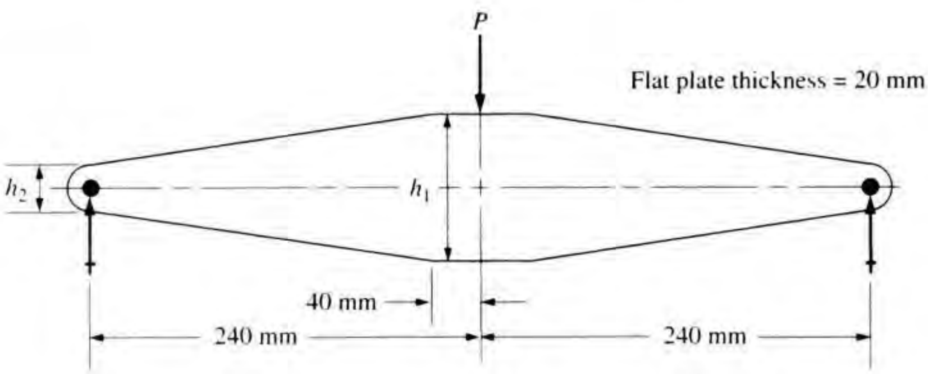


FIGURE P3–70 Tapered flat bar for Problems 70 and 71

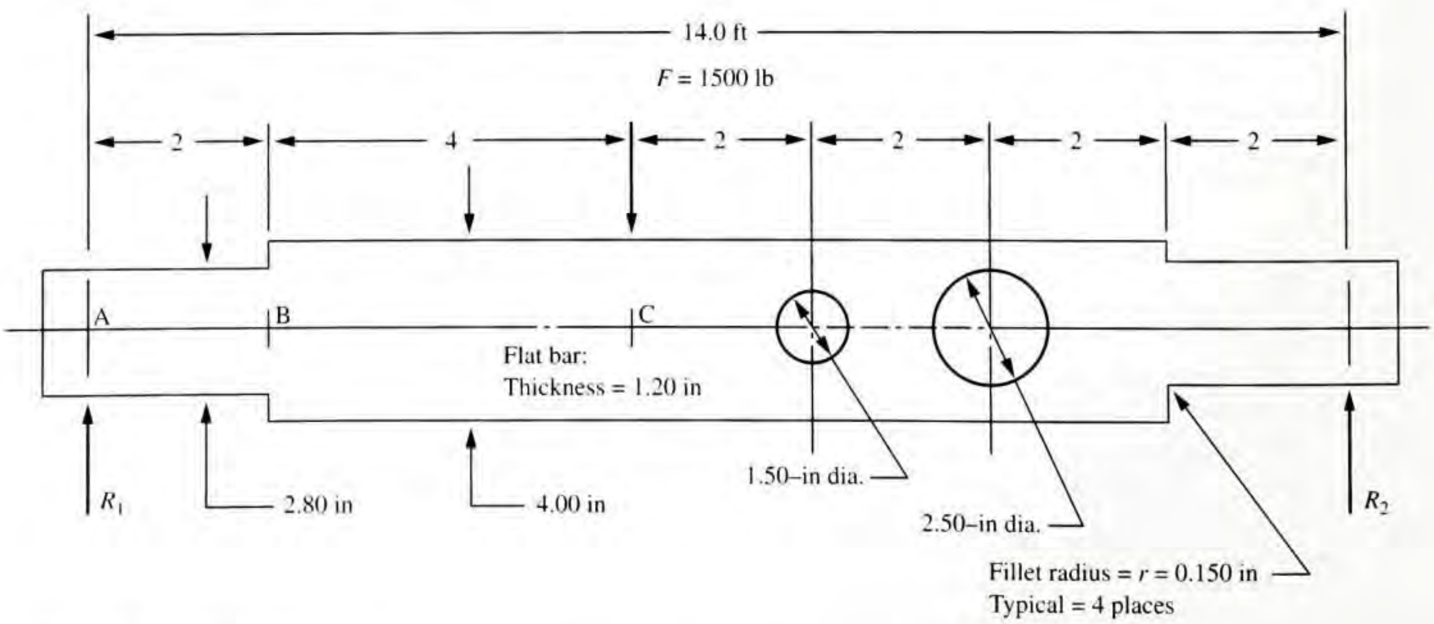


FIGURE P3–72 (Problem 72)

73. Figure P3–73 shows a stepped, flat bar having a constant thickness of 8.0 mm. It carries three concentrated loads as shown. Let $P = 200$ N, $L_1 = 180$ mm, $L_2 = 80$ mm, and $L_3 = 40$ mm. Compute the maximum stress due to bending, and state where it occurs. The bar is braced against lateral bending and twisting. Note that the dimensions in the figure are not drawn to scale.

74. Figure P3–74 shows a bracket carrying opposing forces of $F = 2500$ N. Compute the stress in the upper horizontal part through one of the holes as at B. Use $d = 15.0$ mm for the diameter of the holes.

75. Repeat Problem 74, but use a hole diameter of $d = 12.0$ mm.

76. Figure P3–76 shows a lever made from a rectangular bar of steel. Compute the stress due to bending at the fulcrum (20 in from the pivot) and at the section through the bottom hole. The diameter of each hole is 1.25 in.

77. For the lever in P3–76, determine the maximum stress if the attachment point is moved to each of the other two holes.

78. Figure P3–78 shows a shaft that is loaded only in bending. Bearings are located at points B and D to allow the shaft to rotate. Pulleys at A, C, and E carry cables that support loads from below while allowing the shaft to rotate. Compute the maximum stress due to bending in the shaft considering stress concentrations.

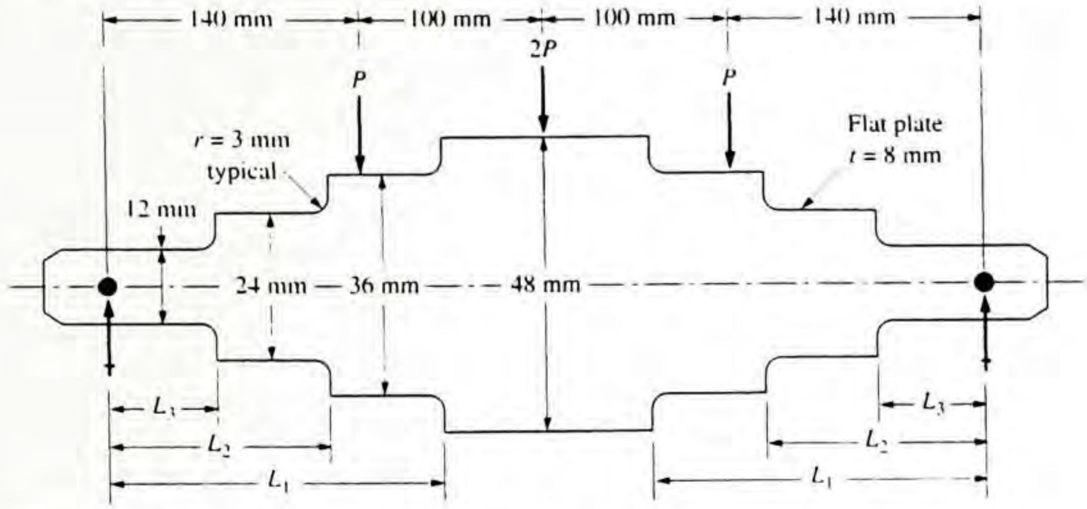


FIGURE P3–73 Stepped flat bar for Problem 73

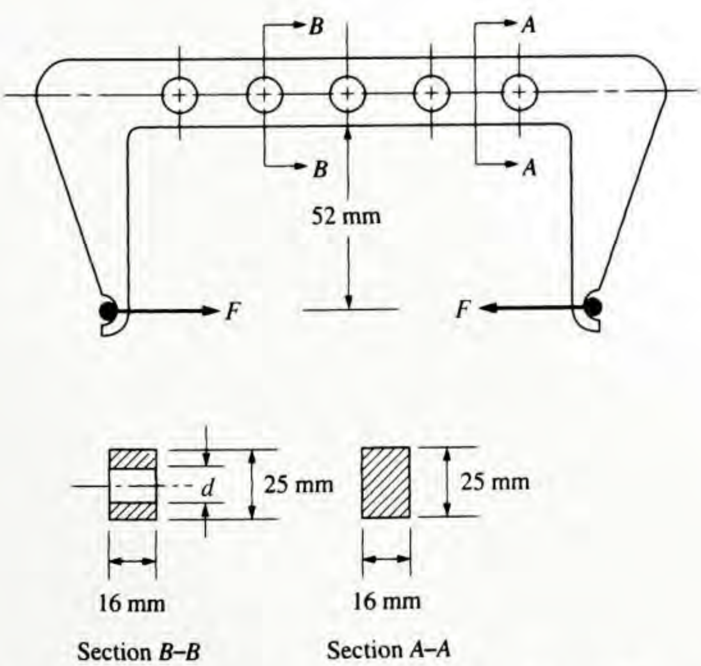


FIGURE P3–74 Bracket for Problems 74 and 75

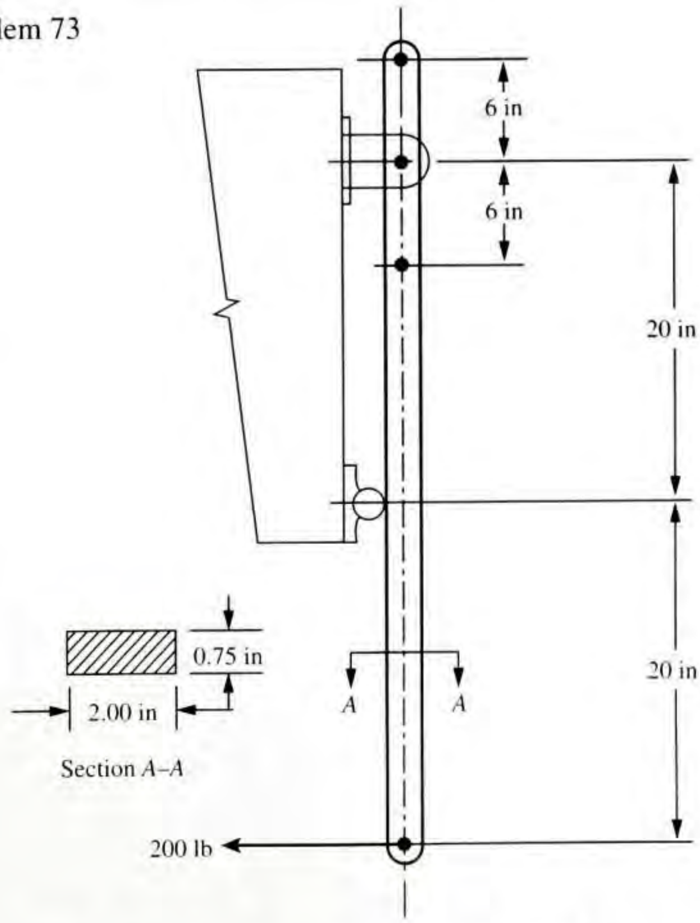


FIGURE P3–76 Lever for Problems 76 and 77

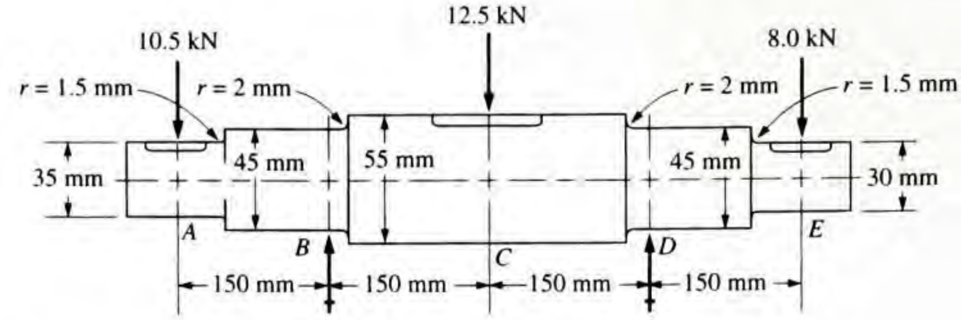


FIGURE P3–78 Data for Problem 78

Internet-Based Assignments

79. Use the MDSolids software to analyze the forces in all members of the truss shown in P3–12.

80. Use the MDSolids software to analyze the forces in all members of the truss shown in P3–13.

81. With the results from Problem 79, use the MDSolids software to analyze the axial tensile or compressive stresses in all members of the truss shown in P3–12.

82. With the results from Problem 80, use the MDSolids software to analyze the axial tensile or compressive stresses in all members of the truss shown in P3–13.

83. Use either the BEAM 2D, MDSolids, or StressAlyzer software to solve Problem 3–34.

84. For the beam shown in P3–37, use either the BEAM 2D, MDSolids, or StressAlyzer software to produce the shearing force and bending moment diagrams.

85. For the beam shown in P3–38, use either the BEAM 2D, MDSolids, or StressAlyzer software to produce the shearing force and bending moment diagrams.

86. For the shaft shown in P3–78, use either the BEAM 2D, MDSolids, or StressAlyzer software to produce the shearing force and bending moment diagrams. Determine the bending moment at point C and at each step in the diameter of the shaft.

4

Combined Stresses and Mohr's Circle

The Big Picture

Combined Stresses and Mohr's Circle

Discussion Map

- ☐ You must build your ability to analyze more complex parts and loading patterns.

Discover

Find products around you that have complex geometries or loading patterns.

Discuss these products with your colleagues.

This chapter helps you analyze complex objects to determine maximum stresses. We will use *Mohr's circle*, a graphical tool for stress analysis, as an aid in understanding how stresses vary within a load-carrying member.

In Chapter 3, you reviewed the basic principles of stress and deformation analysis, practiced the application of those principles to machine design problems, and solved some problems by superposition when two or more types of loads caused normal, either tensile or compressive, stresses.

But what happens when the loading pattern is more complex?

Many practical machine components experience combinations of normal and shear stresses. Sometimes the pattern of loading or the geometry of the component causes the analysis to be very difficult to solve directly using the methods of basic stress analysis.

Look around you and identify products, parts of structures, or machine components that have a more complex loading or geometry. Perhaps some of those identified in **The Big Picture** for Chapter 3 have this characteristic.

Discuss how the selected items are loaded, where the maximum stresses are likely to occur, and how the loads and the geometry are related. Did the designer tailor the shape of the object to be able to carry the applied loads in an efficient manner? How are the shape and the size of critical parts of the item related to the expected stresses?

When we move on to **Chapter 5: Design for Different Types of Loading,** we will need tools to determine the magnitude and the direction of maximum shear stresses or maximum principal (normal) stresses.

Completing this chapter will help you develop a clear understanding of the distribution of stress in a load-carrying member, and it will help you determine the maximum stresses, either normal or shear, so that you can complete a reliable design or analysis.

Some of the techniques of combining stresses require the application of fairly involved equations. A graphical tool, *Mohr's circle*, can be used as an aid in completion of the analysis. Applied properly, the method is precise and should aid you in understanding how the stresses vary within a complex load-carrying member. It should also help you correctly use commercially available stress analysis software.

You Are the Designer

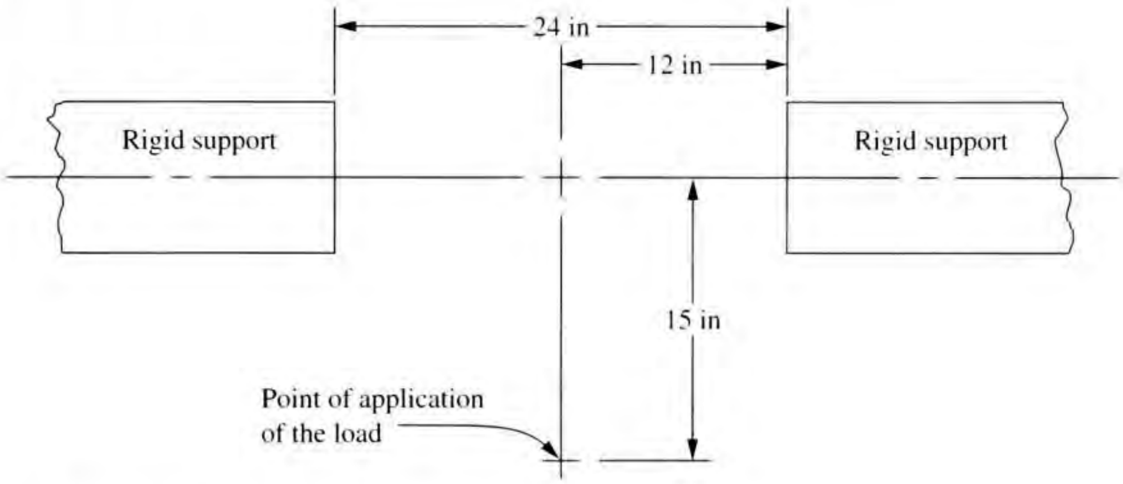
Your company is designing a special machine to test a high-strength fabric under prolonged exposure to a static load to determine whether it continues to deform a greater amount with time. The tests will be run at a variety of temperatures requiring a controlled environment around the test specimen. Figure 4–1 shows the general layout of one proposed design. Two rigid supports are available at the rear of the machine with a 24-in gap between them. The line of action of the load on the test fabric is centered on this gap and 15.0 in out from the middle of the supports. You are asked to design a bracket to hold the upper end of the load frame.

Assume that one of your design concepts uses the arrangement shown in Figure 4–2. Two circular bars are bent 90°. One end of each bar is securely welded to the vertical support surface. A flat bar is attached across the outboard end of each bar so that the load is shared evenly by the two bars.

One of your design problems is to determine the maximum stress that exists in the bent bars to ensure that they are safe. What kinds of stress are developed in the bars? Where are the stresses likely to be the greatest? How can the magnitude of the stresses be computed? Note that the part of the bar near its attachment to the support has a combination of stresses exerted on it.

Consider the element on the top surface of the bar, labeled element A in Figure 4–2. The moment caused by the force acting at an extension of 6.0 in from the support places element A in tension due to the bending action. The torque caused by the force acting 15.0 in out from the axis of the bar at its point of support creates a torsional

FIGURE 4–1 Layout of the load frame supports—top view



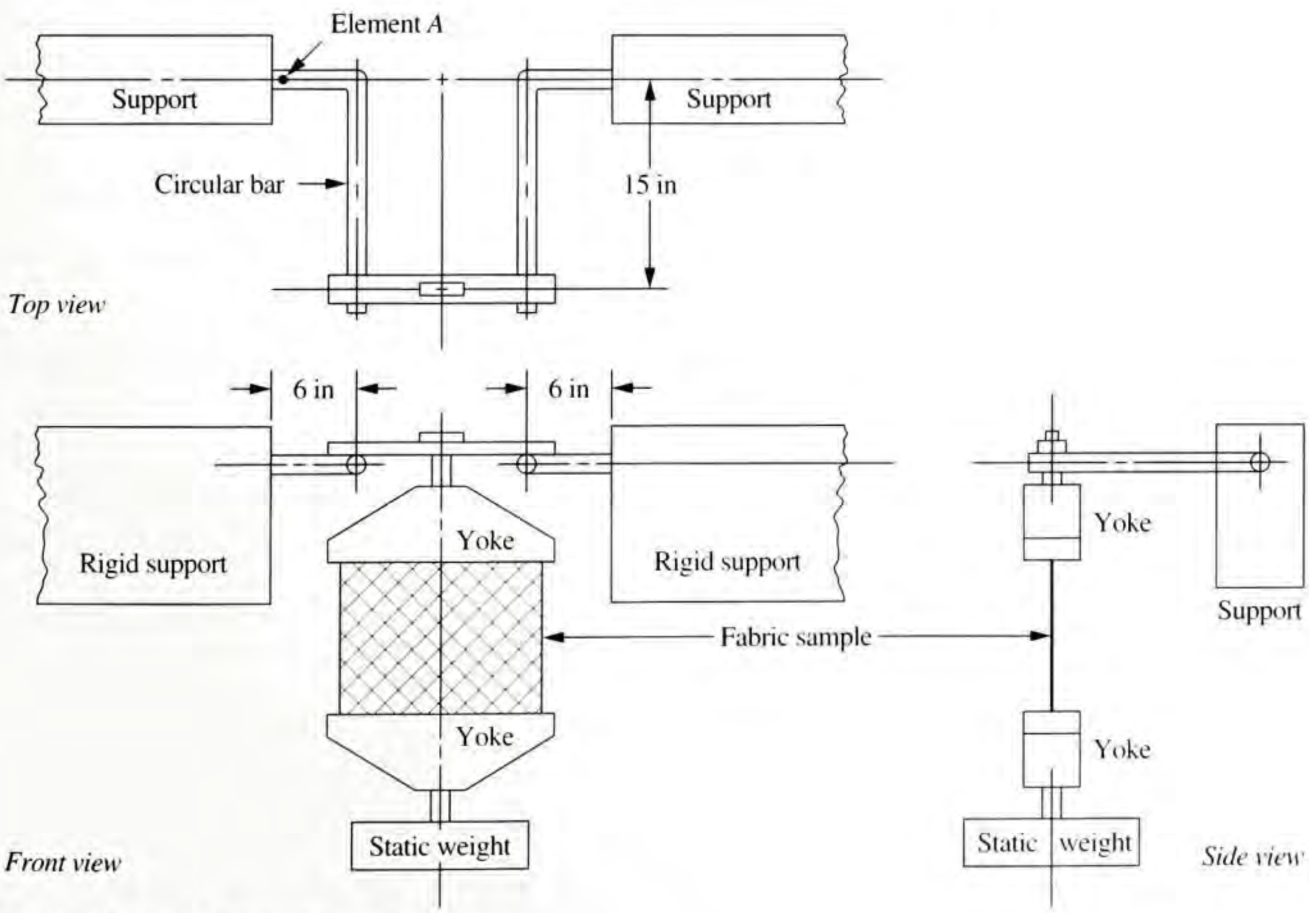


FIGURE 4–2 Proposed bracket design

shear stress on element *A*. Both of these stresses act in the *x-y* plane, subjecting element *A* to a combined normal and shear stress. How do you analyze such a stress condition? How do the tensile and shear stresses act together? What are the maximum normal stress and the maximum shear stress on element *A*, and where do they occur?

You would need such answers to complete the design of the bars. The material in this chapter will enable you to complete the necessary analyses.

4–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Illustrate a variety of combined stresses on stress elements.
2. Analyze a load-carrying member subjected to combined stress to determine the maximum normal stress and the maximum shear stress on any given element.
3. Determine the directions in which the maximum stresses are aligned.
4. Determine the state of stress on an element in any specified direction.
5. Draw the complete Mohr's circle as an aid in completing the analyses for the maximum stresses.

4–2 GENERAL CASE OF COMBINED STRESS

To visualize the general case of combined stress, it is helpful to consider a small element of the load-carrying member on which combined normal and shear stresses act. For this discussion we will consider a two-dimensional stress condition, as illustrated in Figure 4–3. The x- and y-axes are aligned with corresponding axes on the member being analyzed.

The normal stresses, σ_x and σ_y, could be due to a direct tensile force or to bending. If the normal stresses were compressive (negative), the vectors would be pointing in the opposite sense, into the stress element.

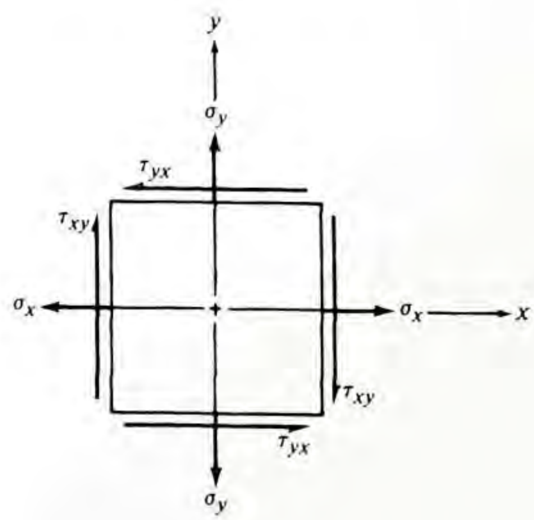
The shear stress could be due to direct shear, torsional shear, or vertical shear stress. The double-subscript notation helps to orient the direction of shear stresses. For example, τ_{xy} indicates the shear stress acting on the element face that is perpendicular to the x-axis and parallel to the y-axis.

A positive shear stress is one that tends to rotate the stress element clockwise.

In Figure 4–3, τ_{xy} is positive, and τ_{yx} is negative. Their magnitudes must be equal to maintain the element in equilibrium.

It is necessary to determine the magnitudes and the signs of each of these stresses in order to show them properly on the stress element. Example Problem 4–1, which follows the definition of principal stresses, illustrates the process.

FIGURE 4–3
General two-dimensional stress element



With the stress element defined, the objectives of the remaining analysis are to determine the maximum normal stress, the maximum shear stress, and the planes on which these stresses occur. The governing formulas follow. (See Reference 1 for the derivations.)

Maximum Normal Stresses: Principal Stresses

The combination of the applied normal and shear stresses that produces the maximum normal stress is called the *maximum principal stress*, σ_1. The magnitude of σ_1 can be computed from the following equation:

Maximum Principal Stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \textbf{(4–1)}$$

The combination of the applied stresses that produces the minimum normal stress is called the *minimum principal stress*, σ_2. Its magnitude can be computed from

Minimum Principal Stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \textbf{(4–2)}$$

Particularly in experimental stress analysis, it is important to know the orientation of the principal stresses. The angle of inclination of the planes on which the principal stresses act, called the *principal planes*, can be found from

Angle for Principal Stress Element

$$\phi_\sigma = \frac{1}{2}\arctan\left[2\tau_{xy}/(\sigma_x - \sigma_y)\right] \quad \textbf{(4–3)}$$

The angle ϕ_σ is measured from the positive x-axis of the original stress element to the maximum principal stress, σ_1. Then the minimum principal stress, σ_2, is on the plane 90° from σ_1.

When the stress element is oriented as discussed so that the principal stresses are acting on it, the shear stress is zero. The resulting stress element is shown in Figure 4–4.

Maximum Shear Stress

On a different orientation of the stress element, the maximum shear stress will occur. Its magnitude can be computed from

Maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \textbf{(4–4)}$$

FIGURE 4–4
Principal stress element

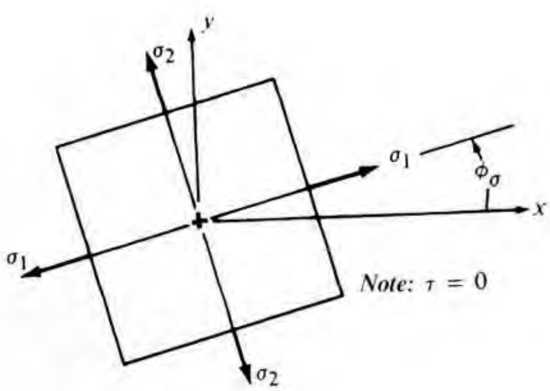
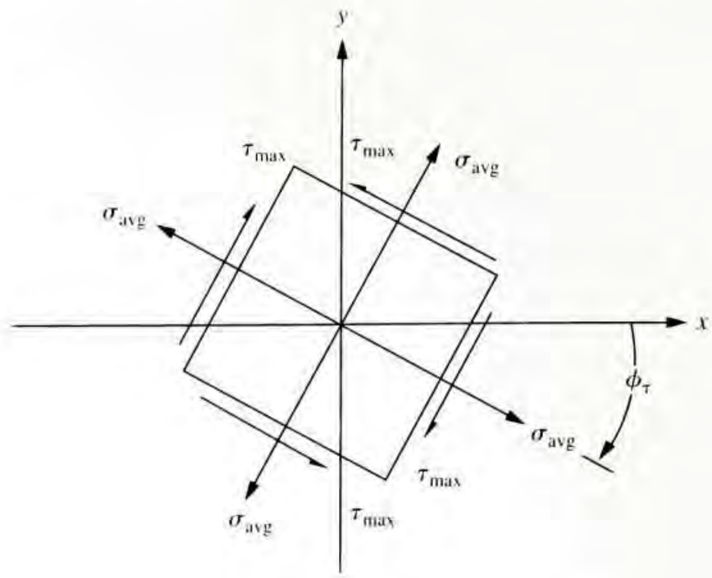


FIGURE 4–5
Maximum shear stress element



The angle of inclination of the element on which the maximum shear stress occurs is computed as follows:

Angle for Maximum Shear Stress Element

$$\phi_\tau = \tfrac{1}{2}\arctan\left[-(\sigma_x - \sigma_y)/2\tau_{xy}\right] \qquad \textbf{(4–5)}$$

The angle between the principal stress element and the maximum shear stress element is always 45°.

On the maximum shear stress element, there will be normal stresses of equal magnitude acting perpendicular to the planes on which the maximum shear stresses are acting. These normal stresses have the value

Average Normal Stress

$$\sigma_{avg} = (\sigma_x + \sigma_y)/2 \qquad \textbf{(4–6)}$$

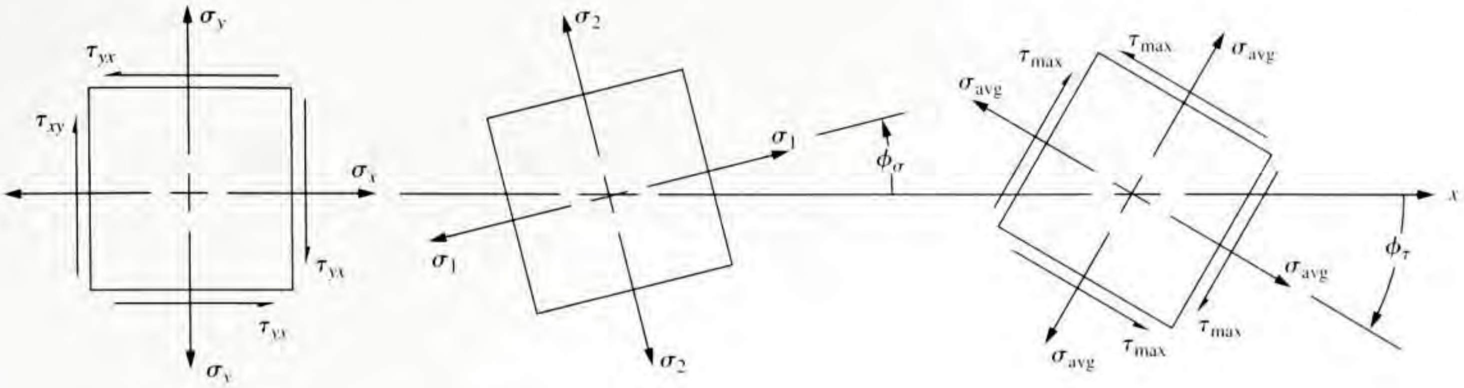
Note that this is the *average* of the two applied normal stresses. The resulting maximum shear stress element is shown in Figure 4–5. Note, as stated above, that the angle between the principal stress element and the maximum shear stress element is always 45°.

Summary and General Procedure for Analyzing Combined Stresses

The following list gives a summary of the techniques presented in this section; it also outlines the general procedure for applying the techniques to a given stress analysis problem.

General Procedure for Computing Principal Stresses and Maximum Shear Stresses

1. Decide for which point you want to compute the stresses.
2. Clearly specify the coordinate system for the object, the free-body diagram, and the magnitude and direction of forces.
3. Compute the stresses on the selected point due to the applied forces, and show the stresses acting on a stress element at the desired point with careful attention to directions. Figure 4–3 is a model for how to show these stresses.
4. Compute the principal stresses on the point and the directions in which they act. Use Equations (4–1), (4–2), and (4–3).



(*a*) Original stress element (*b*) Principal stress element (*c*) Maximum shear stress element

FIGURE 4–6 Relationships among original stress element, principal stress element, and maximum shear stress element for a given loading

5. Draw the stress element on which the principal stresses act, and show its orientation relative to the original x-axis. It is recommended that the principal stress element be drawn beside the original stress element to illustrate the relationship between them.
6. Compute the maximum shear stress on the element and the orientation of the plane on which it acts. Also, compute the normal stress that acts on the maximum shear stress element. Use Equations (4–4), (4–5), and (4–6).
7. Draw the stress element on which the maximum shear stress acts, and show its orientation to the original x-axis. It is recommended that the maximum shear stress element be drawn beside the maximum principal stress element to illustrate the relationship between them.
8. The resulting set of three stress elements will appear as shown in Figure 4–6.

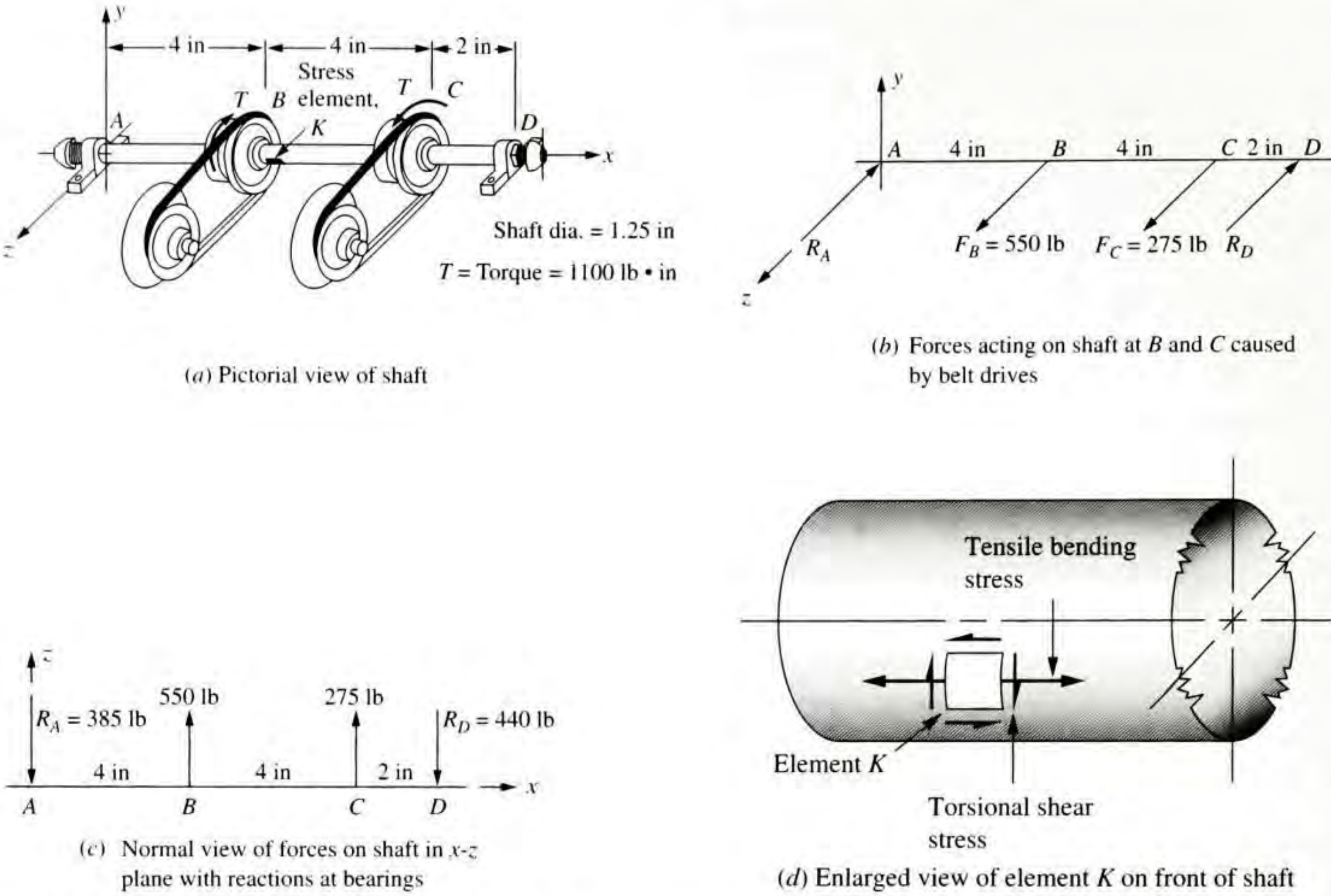
The following example problem illustrates the use of this procedure.

Example Problem 4–1 The shaft shown in Figure 4–7 is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the x-z plane. Sheave B exerts a clockwise torque on the shaft when viewed toward the origin of the coordinate system along the x-axis. Sheave C exerts an equal but opposite torque on the shaft. For the loading condition shown, determine the principal stresses and the maximum shear stress on element K on the front surface of the shaft (on the positive z-side) just to the right of sheave B. Follow the general procedure for analyzing combined stresses given in this section.

Solution Objective Compute the principal stresses and the maximum shear stresses on element K.

Given Shaft and loading pattern shown in Figure 4–7.

Analysis Use the general procedure for analyzing combined stresses.



(*a*) Pictorial view of shaft

(*b*) Forces acting on shaft at *B* and *C* caused by belt drives

(*c*) Normal view of forces on shaft in *x*-*z* plane with reactions at bearings

(*d*) Enlarged view of element *K* on front of shaft

FIGURE 4–7 Shaft supported by two bearings and carrying two V-belt sheaves

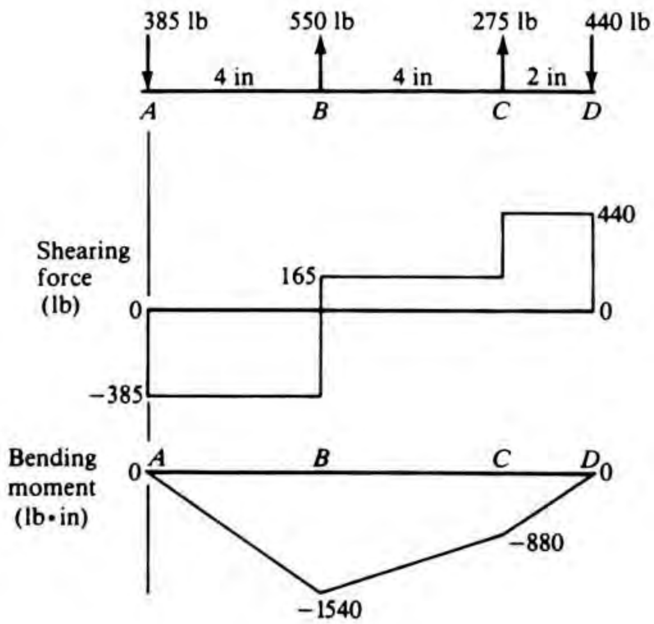


FIGURE 4–8
Shearing force and bending moment diagrams for the shaft

Results Element K is subjected to bending that produces a tensile stress acting in the x-direction. Also, there is a torsional shear stress acting at K. Figure 4–8 shows the shearing force and bending moment diagrams for the shaft and indicates that the bending moment at K is 1540 lb · in. The bending stress is therefore

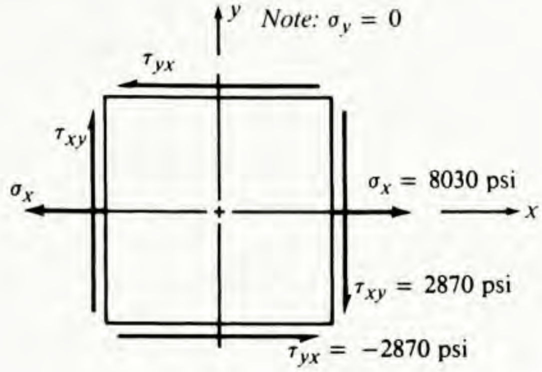


FIGURE 4–9
Stresses on element K

$$\sigma_x = M/S$$
$$S = \pi D^3/32 = [\pi(1.25\text{ in})^3]/32 = 0.192\text{ in}^3$$
$$\sigma_x = (1540\text{ lb}\cdot\text{in})/(0.192\text{ in}^3) = 8030\text{ psi}$$

The torsional shear stress acts on element K in a way that causes a downward shear stress on the right side of the element and an upward shear stress on the left side. This action results in a tendency to rotate the element in a *clockwise* direction, which is the *positive* direction for shear stresses according to the standard convention. Also, the notation for shear stresses uses double subscripts. For example, τ_{xy} indicates the shear stress acting on the face of an element that is perpendicular to the x-axis and parallel to the y-axis. Thus, for element K,

$$\tau_{xy} = T/Z_p$$
$$Z_p = \pi D^3/16 = \pi(1.25\text{ in})^3/16 = 0.383\text{ in}^3$$
$$\tau_{xy} = (1100\text{ lb}\cdot\text{in})/(0.383\text{ in}^3) = 2870\text{ psi}$$

The values of the normal stress, σ_x, and the shear stress, τ_{xy}, are shown on the stress element K in Figure 4–9. Note that the stress in the y-direction is zero for this loading. Also, the value of the shear stress, τ_{yx}, must be equal to τ_{xy}, and it must act as shown in order for the element to be in equilibrium.

We can now compute the principal stresses on the element, using Equations (4–1) through (4–3). The maximum principal stress is

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4–1}$$
$$\sigma_1 = (8030/2) + \sqrt{(8030/2)^2 + (2870)^2}$$
$$\sigma_1 = 4015 + 4935 = 8950\text{ psi}$$

The minimum principal stress is

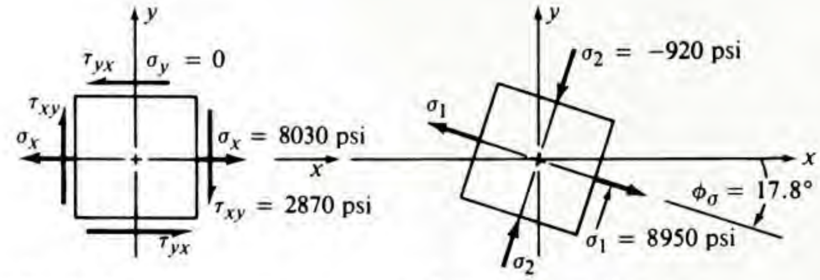
$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4–2}$$
$$\sigma_2 = (8030/2) - \sqrt{(8030/2)^2 + (2870)^2}$$
$$\sigma_2 = 4015 - 4935 = -920\text{ psi (compression)}$$

The direction in which the maximum principal stress acts is

$$\phi_\sigma = \tfrac{1}{2}\arctan\,[2\tau_{xy}/(\sigma_x - \sigma_y)] \tag{4–3}$$
$$\phi_\sigma = \tfrac{1}{2}\arctan\,[(2)(2870)/(8030)] = 17.8^\circ$$

The positive sign calls for a *clockwise* rotation of the element.

FIGURE 4–10
Principal stress element



(*a*) Original stress element (*b*) Principal stress element

The principal stresses can be shown on a stress element as illustrated in Figure 4–10. Note that the element is shown in relation to the original element to emphasize the direction of the principal stresses in relation to the original x-axis. The positive sign for ϕ_σ indicates that the principal stress element is rotated *clockwise* from its original position.

Now the maximum shear stress element can be defined, using Equations (4–4) through (4–6):

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \textbf{(4–4)}$$

$$\tau_{\max} = \sqrt{(8030/2)^2 + (2870)^2}$$

$$\tau_{\max} = \pm\ 4935 \text{ psi}$$

The two pairs of shear stresses, $+\tau_{\max}$ and $-\tau_{\max}$, are equal in magnitude but opposite in direction.

The orientation of the element on which the maximum shear stress acts is found from Equation (4–5):

$$\phi_\tau = \tfrac{1}{2}\arctan\left[-(\sigma_x - \sigma_y)/2\tau_{xy}\right] \quad \textbf{(4–5)}$$

$$\phi_\tau = \tfrac{1}{2}\arctan\left(-8030/[(2)(2870)]\right) = -27.2^\circ$$

The negative sign calls for a *counterclockwise* rotation of the element.

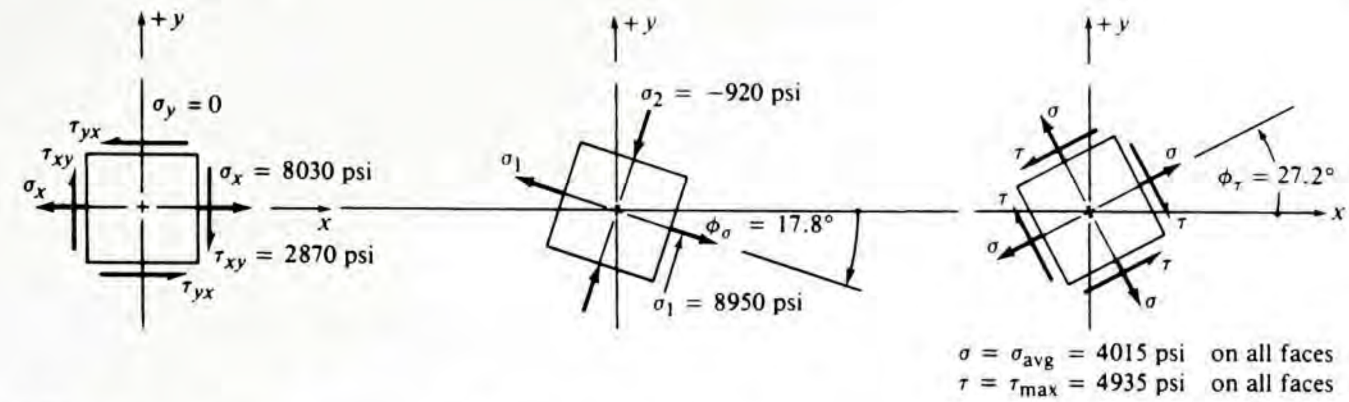
There are equal normal stresses acting on the faces of this stress element, which have the value of

$$\sigma_{\text{avg}} = (\sigma_x + \sigma_y)/2 \quad \textbf{(4–6)}$$

$$\sigma_{\text{avg}} = 8030/2 = 4015 \text{ psi}$$

Comments Figure 4–11 shows the stress element on which the maximum shear stress acts in relation to the original stress element. Note that the angle between this element and the principal stress element is 45°.

Examine the results of Example Problem 4–1. The maximum principal stress, σ_1 = 8950 psi, is 11 percent greater than the value of σ_x = 8030 psi computed for the bending stress in the shaft acting in the x-direction. The maximum shear stress, $\tau_{\max}$ = 4935 psi, is 72 percent greater than the computed applied torsional shear stress of τ_{xy} = 2870 psi. You will see in Chapter 5 that either the maximum normal stress or the maximum shear stress is often required for accurate failure prediction and for safe design decisions. The angles



(*a*) Original stress element, *K* (*b*) Principal stress element (*c*) Maximum shear stress element

FIGURE 4–11 Relation of maximum shear stress element to the original stress element and the principal stress element

of the final stress elements also predict the alignment of the most damaging stresses that can be an aid in experimental stress analysis and the analysis of actual failed components.

Another concept, called the *von Mises stress*, is used in the distortion energy theory of failure described in Chapter 5. The von Mises stress is a unique combination of the maximum principal stress, σ_1, and the minimum principal stress, σ_2, which can be compared directly with the yield strength of the material to predict failure by yielding.

The process of computing the principal stresses and the maximum shear stress shown in Example Problem 4–1 may seem somewhat abstract. These same results can be obtained using a method called *Mohr's circle*, which is discussed next. This method uses a combination of a graphical aid and simple calculations. With practice, the use of Mohr's circle should provide you with a more intuitive feel for the variations in stress that exist at a point in relation to the angle of orientation of the stress element. In addition, it provides a streamlined approach to determining the stress condition on any plane of interest.

4–3 MOHR'S CIRCLE

DESIGN

Because of the many terms and signs involved, and the many calculations required in the computation of the principal stresses and the maximum shear stress, there is a rather high probability of error. Using the graphic aid Mohr's circle helps to minimize errors and gives a better "feel" for the stress condition at the point of interest.

After Mohr's circle is constructed, it can be used for the following:

1. Finding the maximum and minimum principal stresses and the directions in which they act.
2. Finding the maximum shear stresses and the orientation of the planes on which they act.
3. Finding the value of the normal stresses that act on the planes where the maximum shear stresses act.
4. Finding the values of the normal and shear stresses that act on an element with any orientation.

The data needed to construct Mohr's circle are, of course, the same as those needed to compute the preceding values, because the graphical approach is an exact analogy to the computations.

If the normal and shear stresses that act on any two mutually perpendicular planes of an element are known, the circle can be constructed and any of items 1 through 4 can be found.

Mohr's circle is actually a plot of the combinations of normal and shearing stresses that exist on a stress element for all possible angles of orientation of the element. This method is particularly valuable in experimental stress analysis work because the results obtained from many types of standard strain gage instrumentation techniques give the necessary inputs for the creation of Mohr's circle. (See Reference 1.) When the principal stresses and the maximum shear stress are known, the complete design and analysis can be done, using the various theories of failure discussed in Chapter 5.

Procedure for Constructing Mohr's Circle

1. Perform the stress analysis to determine the magnitudes and directions of the normal and shear stresses acting at the point of interest.
2. Draw the stress element at the point of interest as shown in Figure 4–12(a). Normal stresses on any two mutually perpendicular planes are drawn with tensile stresses positive—projecting outward from the element. Compressive stresses are negative—directed inward on the face. Note that the *resultants* of all normal stresses acting in the chosen directions are plotted. Shear stresses are considered to be positive if they tend to rotate the element in a *clockwise* (cw) direction, and negative otherwise.

   Note that on the stress element illustrated, σ_x is positive, σ_y is negative, τ_{xy} is positive, and τ_{yx} is negative. This assignment is arbitrary for the purpose of illustration. In general, any combination of positive and negative values could exist.
3. Refer to Figure 4–12(b). Set up a rectangular coordinate system in which the positive horizontal axis represents positive (tensile) normal stresses, and the positive vertical axis represents positive (clockwise) shear stresses. Thus, the plane created will be referred to as the σ-τ *plane*.
4. Plot points on the σ-τ plane corresponding to the stresses acting on the faces of the stress element. If the element is drawn in the x-y plane, the two points to be plotted are σ_x, τ_{xy} and σ_y, τ_{yx}.
5. Draw the line connecting the two points.
6. The resulting line crosses the σ-axis at the center of Mohr's circle at the average of the two applied normal stresses, where

   $$\sigma_{avg} = (\sigma_x + \sigma_y)/2$$

   The center of Mohr's circle is called O in Figure 4–12.
7. Note in Figure 4–12 that a right triangle has been formed, having the sides a, b, and R, where

   $$R = \sqrt{a^2 + b^2}$$

   By inspection, we can see that

   $$a = (\sigma_x - \sigma_y)/2$$
   $$b = \tau_{xy}$$

   The point labeled O is at a distance of $\sigma_x - a$ from the origin of the coordinate system. We can now proceed with the construction of the circle.
8. Draw the complete circle with the center at O and a radius of R, as shown in Figure 4–13.
9. The point where the circle crosses the σ-axis at the right gives the value of the maximum principal stress, σ_1. Note that $\sigma_1 = \sigma_{avg} + R$.
10. The point where the circle crosses the σ-axis at the left gives the minimum principal stress, σ_2. Note that $\sigma_2 = \sigma_{avg} - R$.

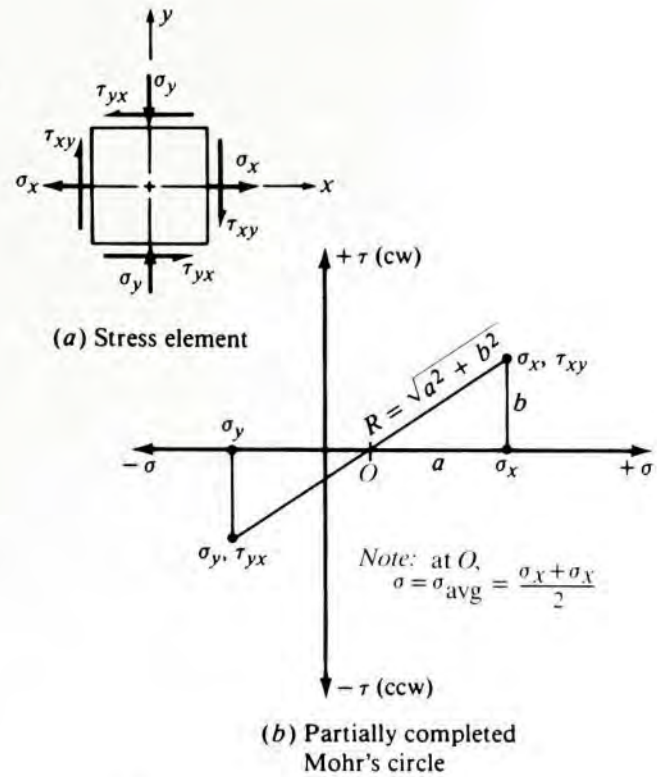


FIGURE 4–12
Partially completed Mohr's circle, Steps 1–7

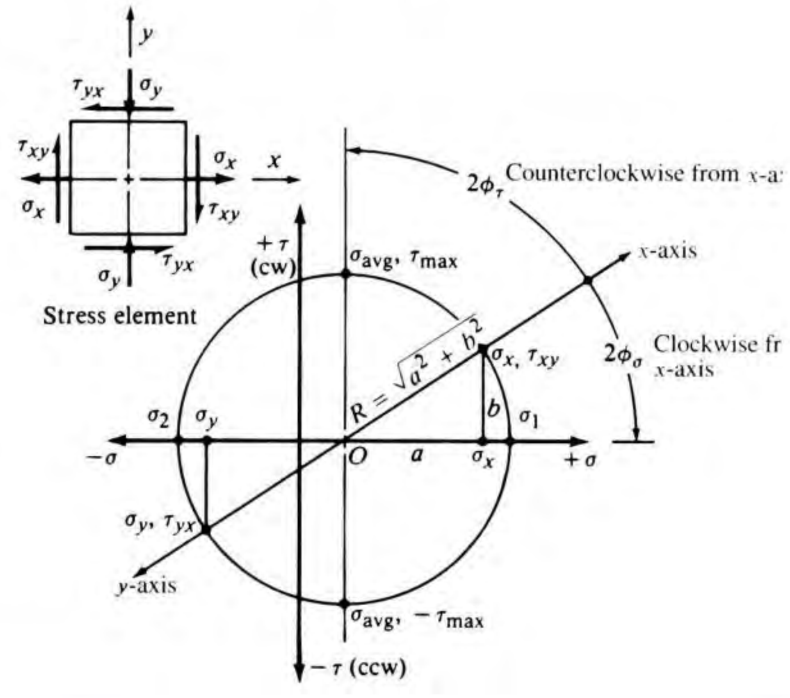


FIGURE 4–13
Completed Mohr's circle, Steps 8–14

11. The coordinates of the top of the circle give the maximum shear stress and the average normal stress that act on the element having the maximum shear stress. Note that $\tau_{max} = R$.

Note: The following steps relate to determining the angles of inclination of the principal stress element and the maximum shear stress element in relation to the original x-axis. It is important to realize that angles on Mohr's circle are actually *double* the true angles. Refer to Figure 4–13; the line from O through the first point plotted, σ_x, τ_{xy}, represents the original x-axis, as noted in the figure. The line from O through the point σ_y, τ_{yx} represents the original y-axis. Of course, on the original element, these axes are 90° apart, not 180°, illustrating the double-angle feature of Mohr's circle. Having made this observation, we can continue with the development of the process.

12. The angle $2\phi_\sigma$ is measured from the x-axis on the circle to the σ-axis. Note that

$$2\phi_\sigma = \arctan(b/a)$$

It is also important to note the direction *from the x-axis to the* σ*-axis* (clockwise or counterclockwise). This is necessary for representing the relation of the principal stress element to the original stress element properly.

13. The angle from the x-axis on the circle to the vertical line through τ_{max} gives $2\phi_\tau$. From the geometry of the circle, in the example shown, we can see that

$$2\phi_\tau = 90^\circ - 2\phi_\sigma$$

Other combinations of the initial stresses will result in different relationships between $2\phi_\sigma$ and $2\phi_\tau$. The specific geometry on the circle should be used each time. See Example Problems 4–3 to 4–8 that follow this section.

Again it is important to note the direction *from the x-axis to the* τ_{max}*-axis* for use in orienting the maximum shear stress element. You should also note that the σ-axis and the τ_{max}-axis are always 90° apart on the circle and therefore 45° apart on the actual element.

14. The final step in the process of using Mohr's circle is to draw the resulting stress elements in their proper relation to the original element, as shown in Figure 4–14.

We will now illustrate the construction of Mohr's circle by using the same data as in Example Problem 4–1, in which the principal stresses and the maximum shear stress were computed directly from the equations.

Example Problem 4–2 The shaft shown in Figure 4–7 is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the x-z plane. Sheave B exerts a clockwise torque on the shaft when viewed toward the origin of the coordinate system along the x-axis. Sheave C exerts an equal but opposite torque on the shaft. For the loading condition shown, determine the principal stresses and the maximum shear stress on element K on the front surface of the shaft (on the positive z-side) just to the right of sheave B. Use the procedure for constructing Mohr's circle in this section.

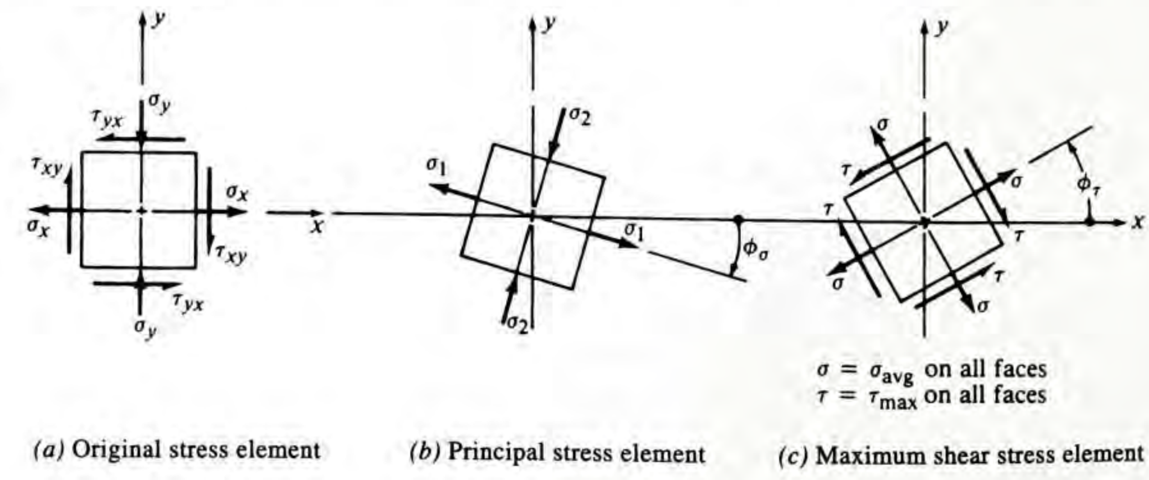


FIGURE 4–14 Display of results from Mohr's circle

Solution Objective Determine the principal stresses and the maximum shear stresses on element K.

Given Shaft and loading pattern shown in Figure 4–7.

Analysis Use the *Procedure for Constructing Mohr's Circle*. Some intermediate results will be taken from the solution to Example Problem 4–1 and from Figures 4–7, 4–8, and 4–9.

Results ***Steps 1 and 2.*** The stress analysis for the given loading was completed in Example Problem 4–1. Figure 4–15 is identical to Figure 4–9 and represents the results of Step 2 of the Mohr's circle procedure.

Steps 3–6. Figure 4–16 shows the results. The first point plotted was

$$\sigma_x = 8030 \text{ psi}, \tau_{xy} = 2870 \text{ psi}$$

The second point was plotted at

$$\sigma_y = 0 \text{ psi}, \tau_{yx} = -2870 \text{ psi}$$

Then a line was drawn between them, crossing the σ-axis at O. The value of the stress at O is

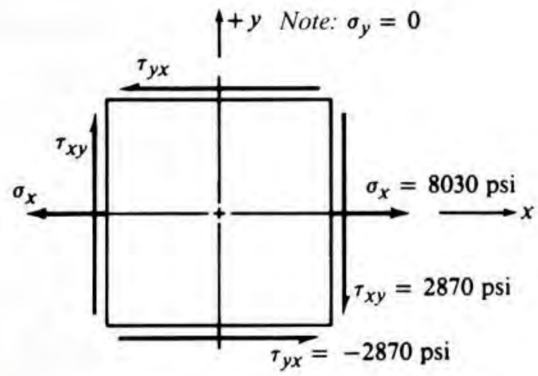


FIGURE 4–15
Stresses on element K

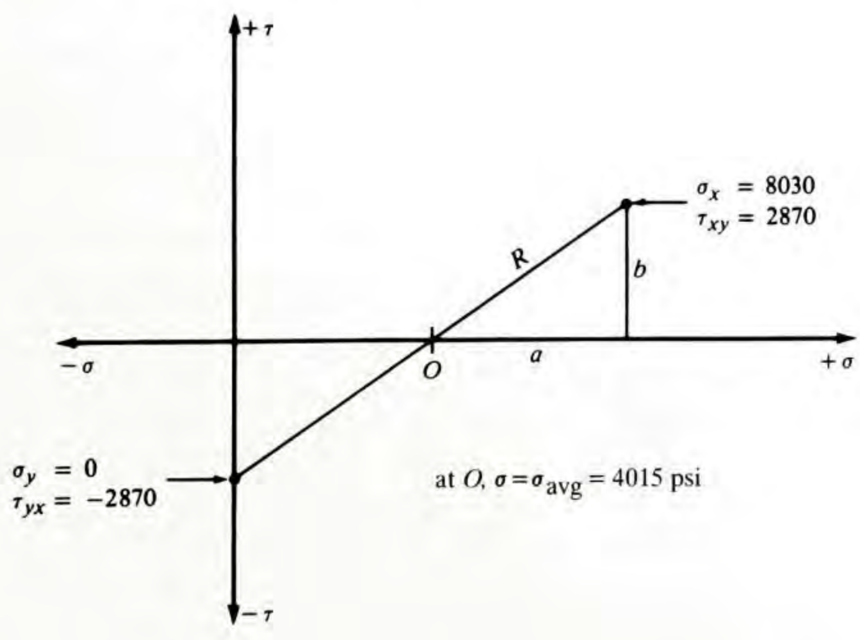


FIGURE 4–16
Partially completed Mohr's circle

$$\sigma_{avg} = (\sigma_x + \sigma_y)/2 = (8030 + 0)/2 = 4015 \text{ psi}$$

Step 7. We compute the values for a, b, and R from

$$a = (\sigma_x - \sigma_y)/2 = (8030 - 0)/2 = 4015 \text{ psi}$$

$$b = \tau_{xy} = 2870 \text{ psi}$$

$$R = \sqrt{a^2 + b^2} = \sqrt{(4015)^2 + (2870)^2} = 4935 \text{ psi}$$

Step 8. Figure 4–17 shows the completed Mohr's circle. The circle has its center at O and the radius R. Note that the circle passes through the two points originally plotted. It must do so because the circle represents all possible states of stress on the element K.

Step 9. The maximum principal stress is at the right side of the circle.

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_1 = 4015 + 4935 = 8950 \text{ psi}$$

Step 10. The minimum principal stress is at the left side of the circle.

$$\sigma_2 = \sigma_{avg} - R$$

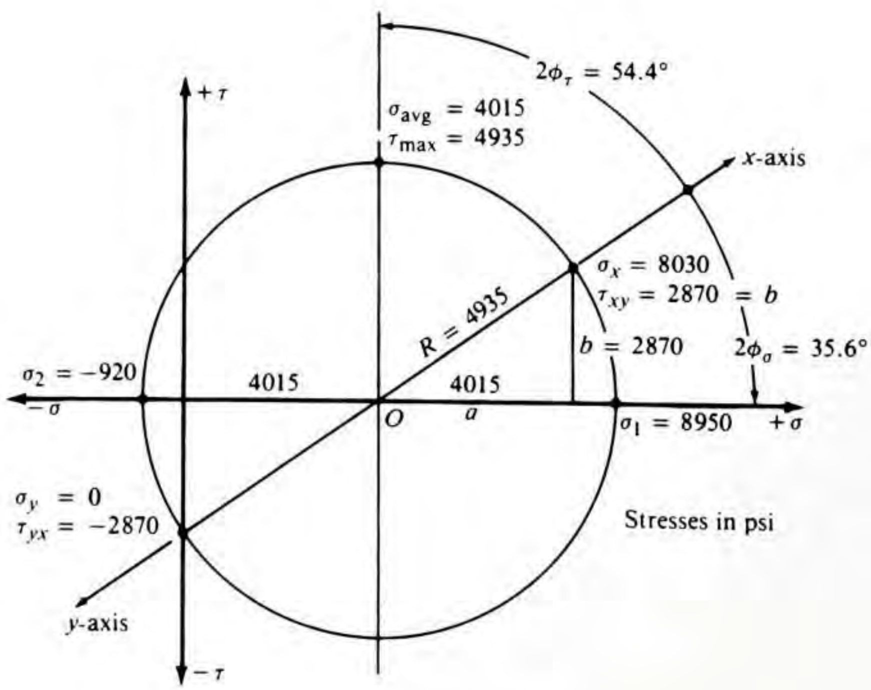
$$\sigma_2 = 4015 - 4935 = -920 \text{ psi}$$

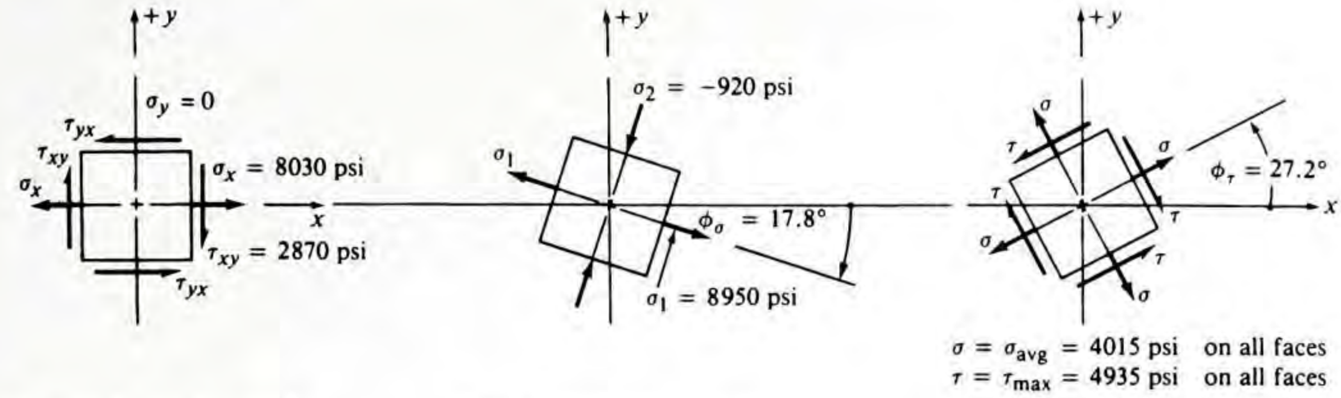
Step 11. At the top of the circle,

$$\sigma = \sigma_{avg} = 4015 \text{ psi}$$

$$\tau = \tau_{max} = R = 4935 \text{ psi}$$

FIGURE 4–17
Completed Mohr's circle





(*a*) Original stress element, *K*, from Figure 4–7 (*b*) Principal stress element (*c*) Maximum shear stress element

FIGURE 4–18 Results from Mohr's circle analysis

The value of the normal stress on the element that carries the maximum shear stress is the same as the coordinate of *O*, the center of the circle.

Step 12. Compute the angle $2\phi_\sigma$ and then ϕ_σ. Use the circle as a guide.

$$2\phi_\sigma = \arctan(b/a) = \arctan(2870/4015) = 35.6^\circ$$
$$\phi_\sigma = 35.6^\circ/2 = 17.8^\circ$$

Note that ϕ_σ must be measured *clockwise* from the original x-axis to the direction of the line of action of σ_1 for this set of data. The principal stress element will be rotated in the same direction as part of step 14.

Step 13. Compute the angle $2\phi_\tau$ and then ϕ_τ. From the circle we see that

$$2\phi_\tau = 90^\circ - 2\phi_\sigma = 90^\circ - 35.6^\circ = 54.4^\circ$$
$$\phi_\tau = 54.4^\circ/2 = 27.2^\circ$$

Note that the stress element on which the maximum shear stress acts must be rotated *counterclockwise* from the orientation of the original element for this set of data.

Step 14. Figure 4–18 shows the required stress elements. They are identical to those shown in Figure 4–11.

4–4 MOHR'S CIRCLE PRACTICE PROBLEMS

DESIGN

To a person seeing Mohr's circle for the first time, it may seem long and involved. But with practice under a variety of combinations of normal and shear stresses, you should be able to execute the 14 steps quickly and accurately.

Table 4–1 gives six sets of data (Example Problems 4–3 through 4–8) for normal and shear stresses in the x-y plane. You are advised to complete the Mohr's circle for each before looking at the solutions in Figures 4–19 through 4–24. From the circle, determine the two principal stresses, the maximum shear stress, and the planes on which these stresses act. Then draw the given stress element, the principal stress element, and the maximum shear stress element, all oriented properly with respect to the x- and y-directions.

TABLE 4–1 Practice problems for Mohr's circle

Example Problem	σ_x	σ_y	τ_{xy}	Fig. No.
4–3	+10.0 ksi	−4.0 ksi	+5.0 ksi	4–19
4–4	+10.0 ksi	−2.0 ksi	−4.0 ksi	4–20
4–5	+4.0 ksi	−10.0 ksi	+4.0 ksi	4–21
4–6	+120 MPa	−30 MPa	+60 MPa	4–22
4–7	−80 MPa	+20 MPa	−50 MPa	4–23
4–8	−80 MPa	+20 MPa	+50 Mpa	4–24

Example Problem 4–3

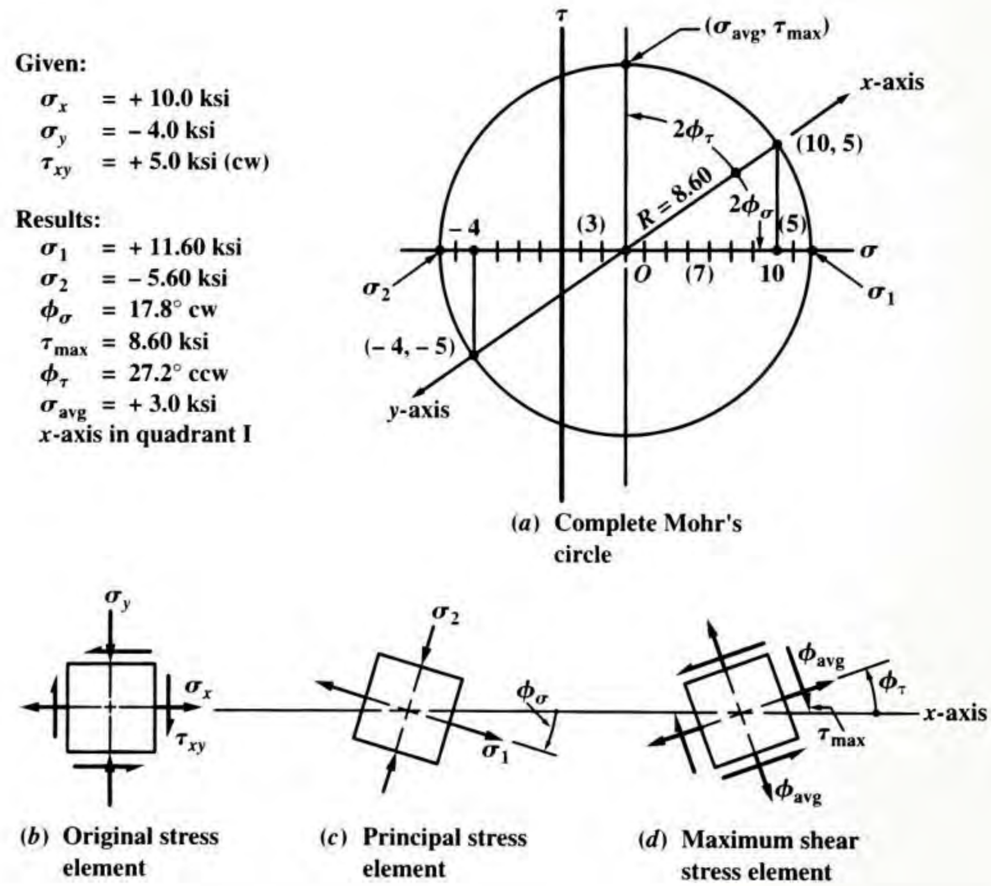


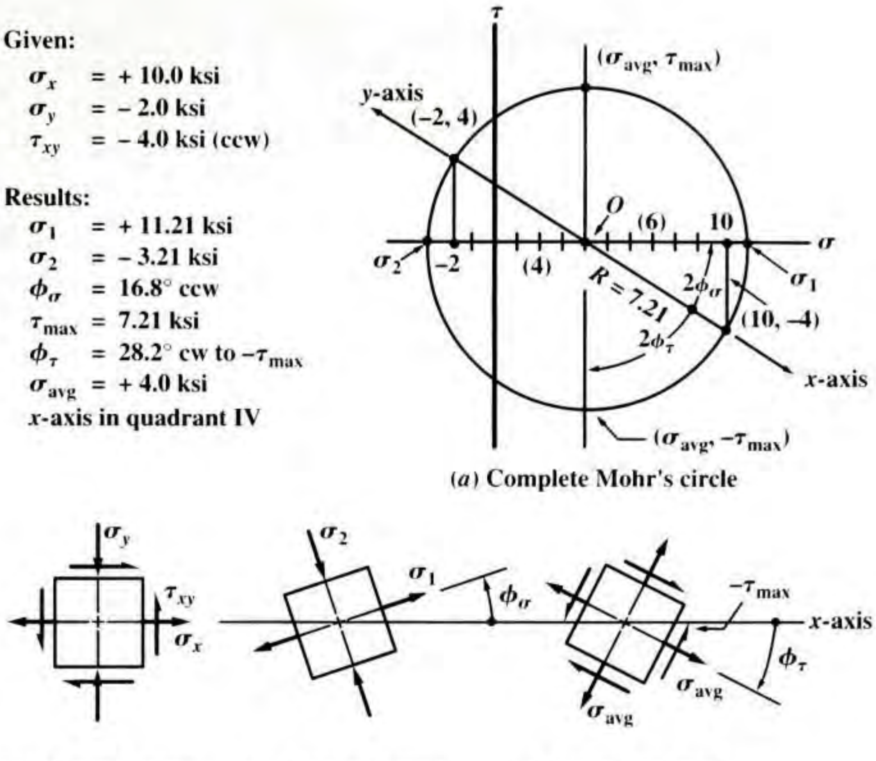
FIGURE 4–19 Solution for Example Problem 4–3

Example Problem 4–4

FIGURE 4–20
Solution for Example Problem 4–4

Given:
σ_x = + 10.0 ksi
σ_y = – 2.0 ksi
τ_{xy} = – 4.0 ksi (ccw)

Results:
σ_1 = + 11.21 ksi
σ_2 = – 3.21 ksi
ϕ_σ = 16.8° ccw
τ_{max} = 7.21 ksi
ϕ_τ = 28.2° cw to $-\tau_{max}$
σ_{avg} = + 4.0 ksi
x-axis in quadrant IV



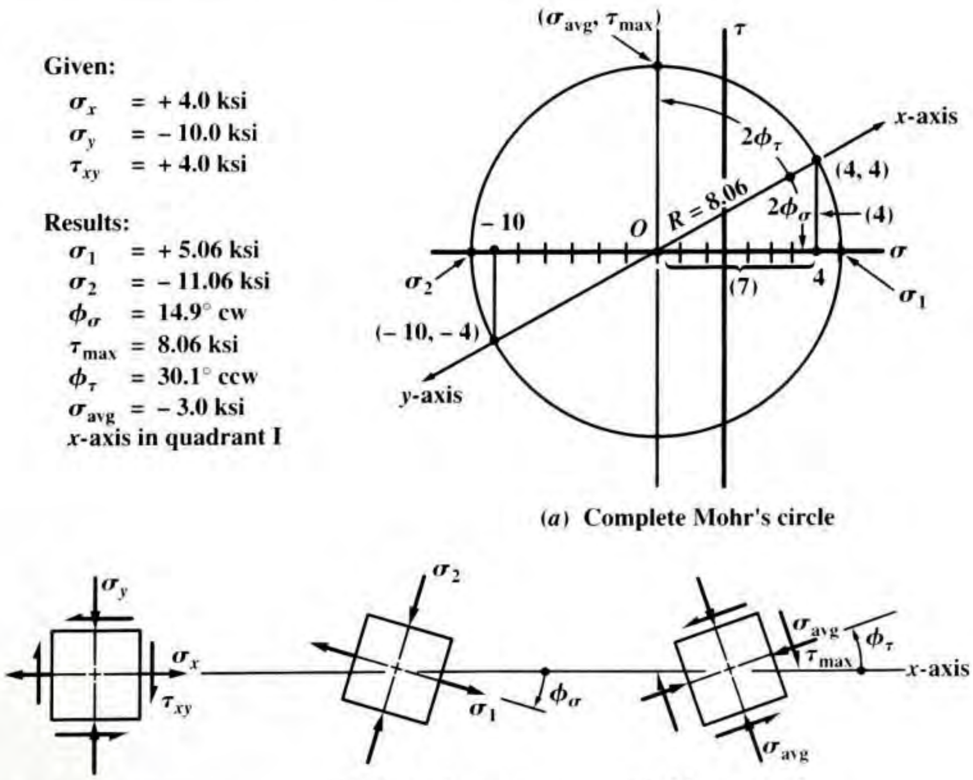
(*a*) Complete Mohr's circle
(*b*) Original stress element
(*c*) Principal stress element
(*d*) Maximum shear stress element

Example Problem 4–5

FIGURE 4–21
Solution for Example Problem 4–5

Given:
σ_x = + 4.0 ksi
σ_y = – 10.0 ksi
τ_{xy} = + 4.0 ksi

Results:
σ_1 = + 5.06 ksi
σ_2 = – 11.06 ksi
ϕ_σ = 14.9° cw
τ_{max} = 8.06 ksi
ϕ_τ = 30.1° ccw
σ_{avg} = – 3.0 ksi
x-axis in quadrant I



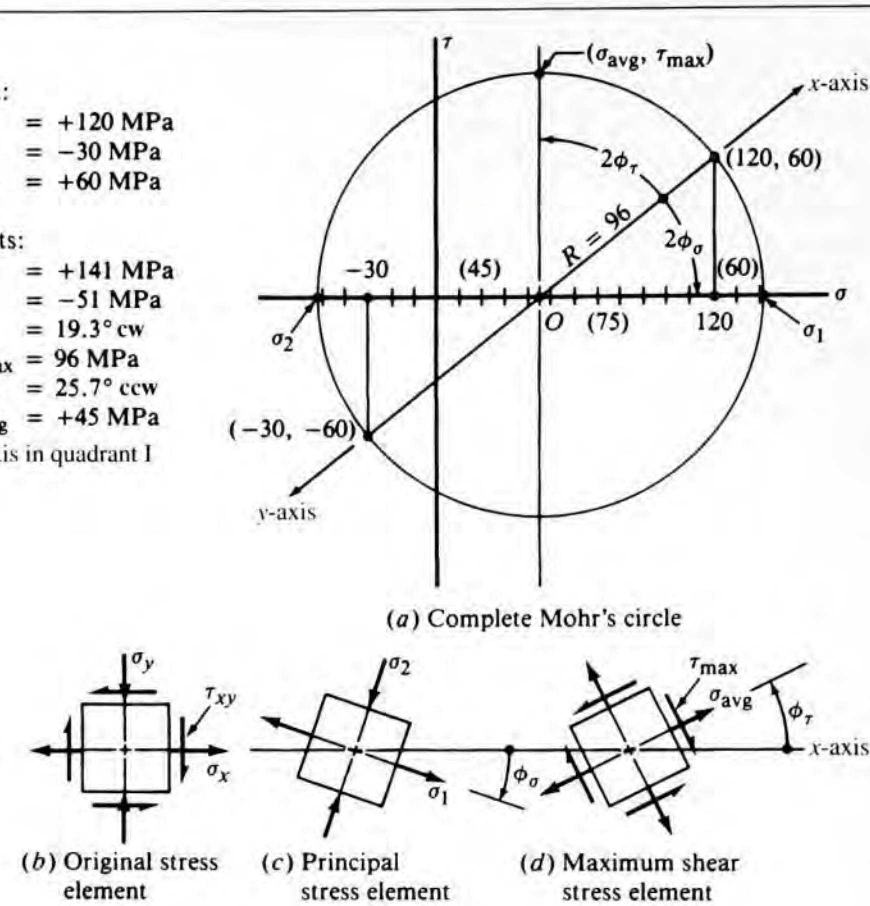
(*a*) Complete Mohr's circle
(*b*) Original stress element
(*c*) Principal stress element
(*d*) Maximum shear stress element

Example Problem 4–6

FIGURE 4–22
Solution for Example Problem 4–6

Given:
$\sigma_x = +120$ MPa
$\sigma_y = -30$ MPa
$\tau_{xy} = +60$ MPa

Results:
$\sigma_1 = +141$ MPa
$\sigma_2 = -51$ MPa
$\phi_\sigma = 19.3°$ cw
$\tau_{max} = 96$ MPa
$\phi_\tau = 25.7°$ ccw
$\sigma_{avg} = +45$ MPa
x-axis in quadrant I

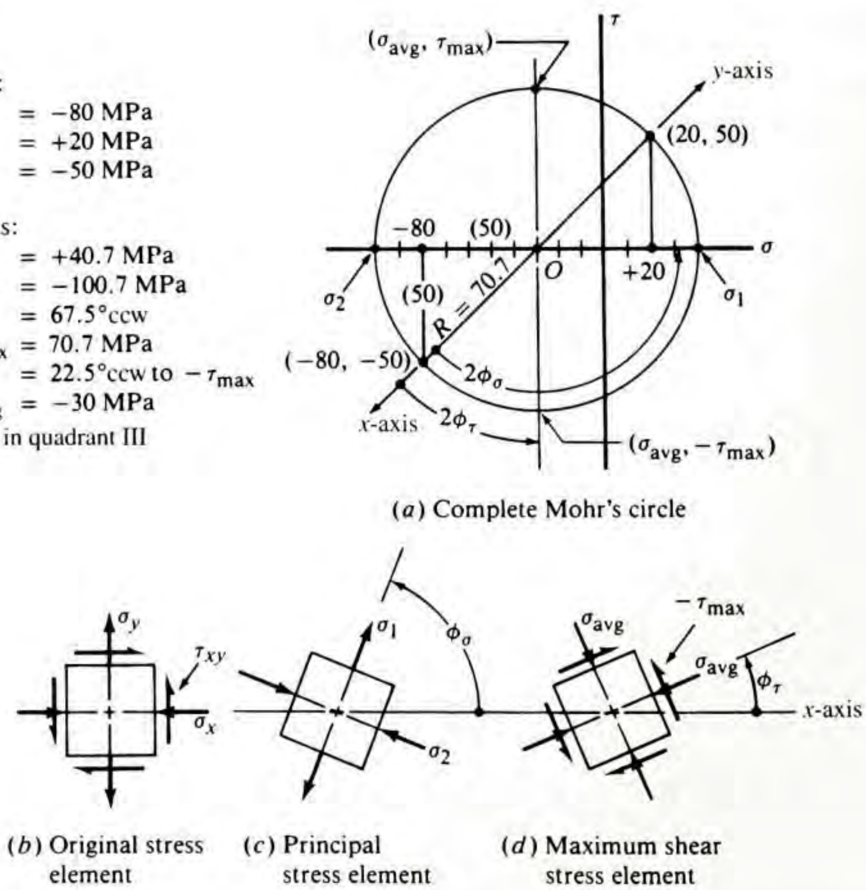


Example Problem 4–7

FIGURE 4–23
Solution for Example Problem 4–7

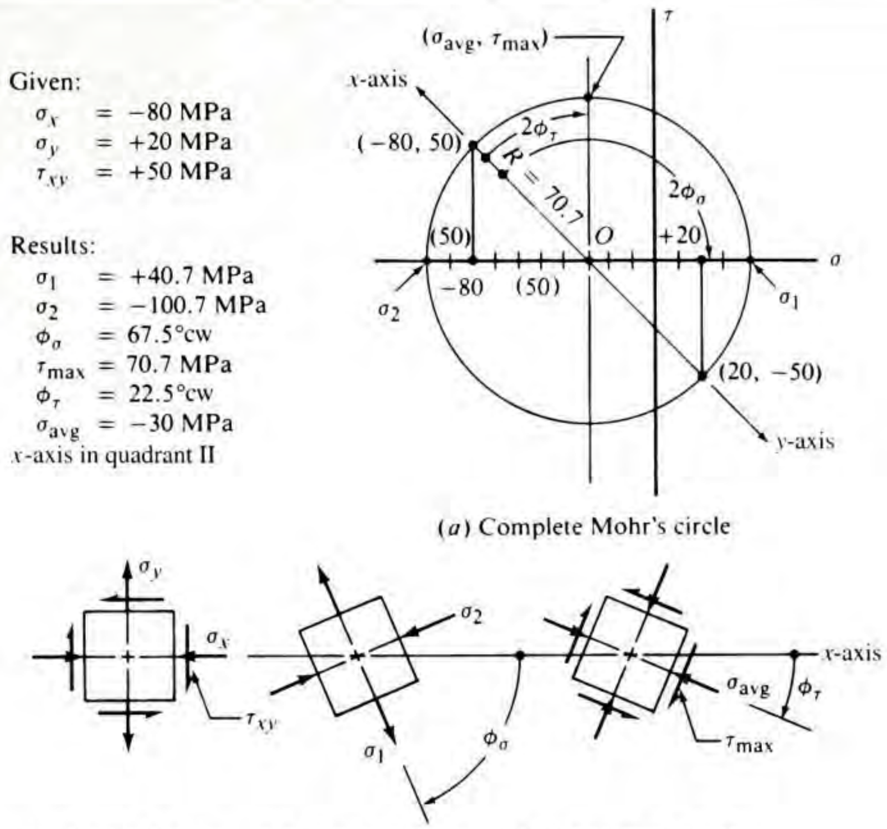
Given:
$\sigma_x = -80$ MPa
$\sigma_y = +20$ MPa
$\tau_{xy} = -50$ MPa

Results:
$\sigma_1 = +40.7$ MPa
$\sigma_2 = -100.7$ MPa
$\phi_\sigma = 67.5°$ccw
$\tau_{max} = 70.7$ MPa
$\phi_\tau = 22.5°$ccw to $-\tau_{max}$
$\sigma_{avg} = -30$ MPa
x-axis in quadrant III



Example Problem 4–8

FIGURE 4–24
Solution for Example Problem 4–8



(*a*) Complete Mohr's circle

(*b*) Original stress element (*c*) Principal stress element (*d*) Maximum shear stress element

4–5 CASE WHEN BOTH PRINCIPAL STRESSES HAVE THE SAME SIGN

MDESIGN

Remember that all of the problems presented thus far have been plane stress problems, also called *biaxial stress* problems because stresses are acting in only two directions within one plane. Obviously, real load-carrying members are three-dimensional objects. The assumption here is that if no stress is given for the third direction, it is zero. In most cases, the solutions as given will produce the true maximum shear stress, along with the two principal stresses for the given plane. This will always be true if the two principal stresses have opposite signs—that is, if one is tensile and the other is compressive.

But the true maximum shear stress on the element will not be found if the two principal stresses are of the same sign. In such cases, you must consider the three-dimensional case.

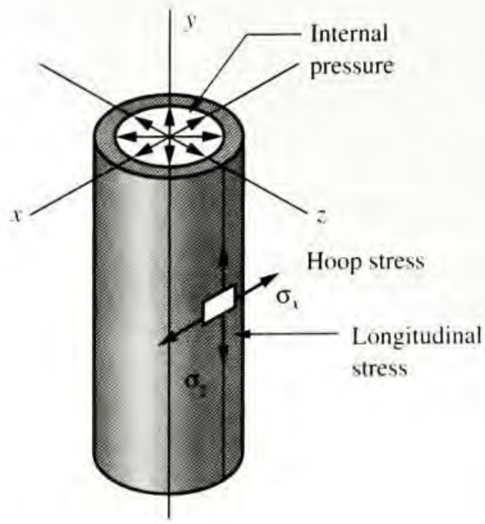
Familiar examples of real products in which two principal stresses have the same sign are various forms of pressure vessels. A hydraulic cylinder with closed ends contains fluids under high pressure that tend to burst the walls of the cylinder. In strength of materials, you learned that the outer surfaces of the walls of such cylinders are subjected to tensile stresses in two directions: (1) tangent to its circumference and (2) axially, parallel to the axis of the cylinder. The stress perpendicular to the wall at the outer surface is zero.

Figure 4–25 shows the stress condition on an element of the surface of the cylinder. The tangential stress, also called *hoop stress*, is aligned with the x-direction and is labeled σ_x. The axially directed stress, also called *longitudinal stress*, acts in line with the y-direction and is labeled σ_y.

In strength of materials, you learned that if the wall of the cylinder is relatively thin, the maximum hoop stress is

$$\sigma_x = pD/2t$$

FIGURE 4–25
Thin-walled cylinder subjected to pressure with its ends closed



where p = internal pressure in the cylinder
D = diameter of the cylinder
t = thickness of the cylinder wall

Also, the longitudinal stress is

$$\sigma_y = pD/4t$$

Both stresses are tensile, and the hoop stress is twice as large as the longitudinal stress.

The analysis would be similar for any kind of thin-walled cylindrical vessel carrying an internal pressure. Examples are storage tanks for compressed gases, pipes carrying moving fluids under pressure, and the familiar beverage can that releases internal pressure when the top is popped open.

Let's use the hydraulic cylinder as an example for illustrating the special use of Mohr's circle when both principal stresses have the same sign. Consider that Figure 4–25 shows a cylinder with closed ends carrying an internal pressure of 500 psi. The wall thickness is $t = 0.080$ in, and the diameter of the cylinder is $D = 4.00$ in. The ratio of $D/t = 50$ indicates that the cylinder can be considered thin-walled. Any ratio over 20 is typically considered to be thin-walled.

The computed hoop and longitudinal stresses in the wall are

Hoop Stress

$$\sigma_x = \frac{pD}{2t} = \frac{(500\ \text{psi})(4.0\ \text{in})}{(2)(0.080\ \text{in})} = 12\,500\ \text{psi (tension)}$$

Longitudinal Stress

$$\sigma_y = \frac{pD}{4t} = \frac{(500\ \text{psi})(4.0\ \text{in})}{(4)(0.080\ \text{in})} = 6250\ \text{psi (tension)}$$

There are no shear stresses applied in the x- and y-directions.

Figure 4–26(a) shows the stress element for the x-y plane, and Part (b) shows the corresponding Mohr's circle. Because there are no applied shear stresses, σ_x and σ_y are the principal stresses for the plane. The circle would predict the maximum shear stress to be equal to the radius of the circle, 3125 psi.

But notice Part (c) of the figure. We could have chosen the x-z plane for analysis, instead of the x-y plane. The stress in the z-direction is zero because it is perpendicular to the

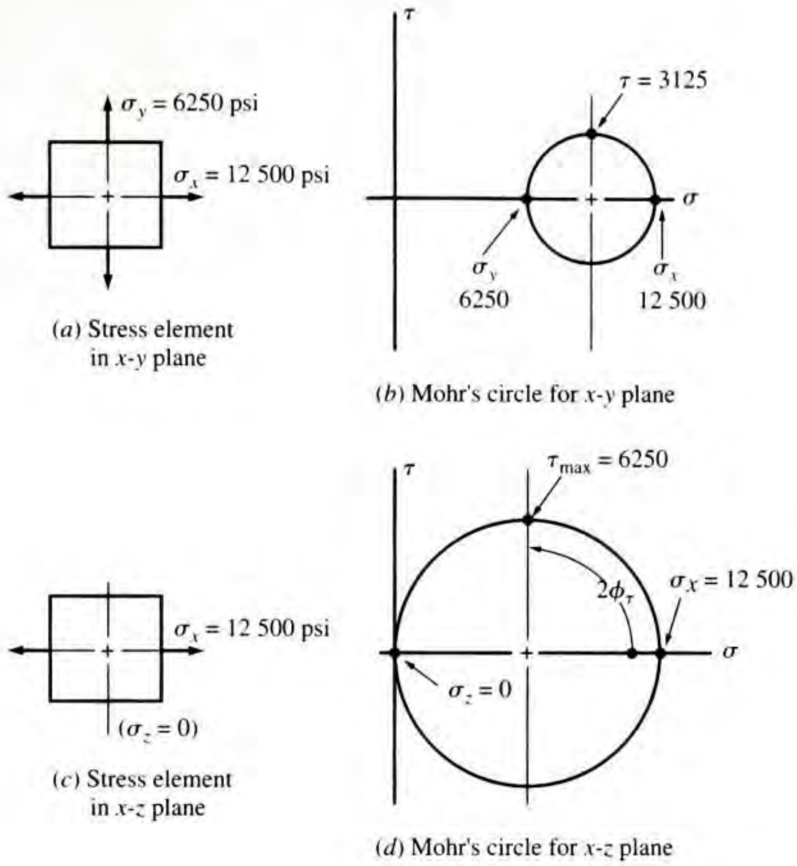


FIGURE 4–26 Stress analysis for a thin-walled cylinder

free face of the element. Likewise, there are no shear stresses on this face. The Mohr's circle for this plane is shown in Part (d) of the figure. The maximum shear stress is equal to the radius of the circle, 6250 psi, or *twice* as much as would be predicted from the x-y plane. This approach should be used any time the two principal stresses in a biaxial stress problem have the same sign.

In summary, on a general three-dimensional stress element, there will be one orientation of the element in which there are no shear stresses acting. The normal stresses on the three perpendicular faces are then the three principal stresses. If we call these stresses σ_1, σ_2, and σ_3, taking care to order them such that $\sigma_1 > \sigma_2 > \sigma_3$, then the maximum shear stress on the element will always be

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Figure 4–27 shows the three-dimensional element.

For the cylinder of Figure 4–25, we can conclude that

$$\begin{aligned}
\sigma_1 &= \sigma_x = 12\,500 \text{ psi} \\
\sigma_2 &= \sigma_y = 6250 \text{ psi} \\
\sigma_3 &= \sigma_z = 0 \\
\tau_{max} &= (\sigma_1 - \sigma_3)/2 = (12\,500 - 0)/2 = 6250 \text{ psi}
\end{aligned}$$

Figure 4–28 shows two additional examples in which the two principal stresses in the given plane have the same sign. Then the zero stress in the third direction is added to the diagram, and the new Mohr's circle is superimposed on the original one. This serves to illustrate that the maximum shear stress will occur on the Mohr's circle having the largest radius.

FIGURE 4–27
Three-dimensional stress element

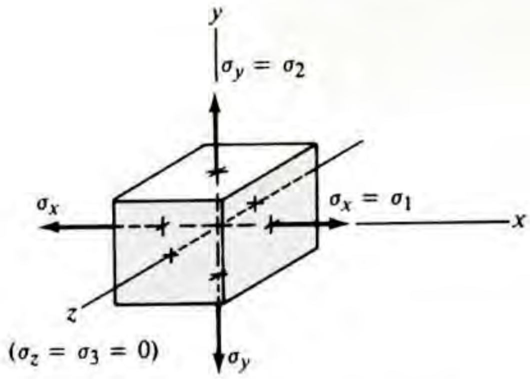


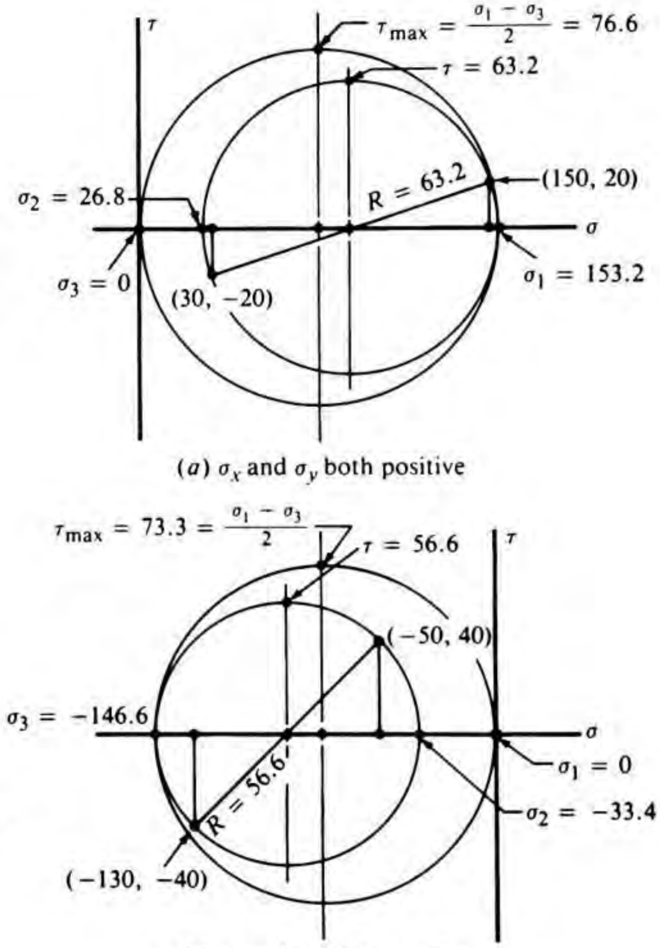
FIGURE 4–28
Mohr's circle for cases in which two principal stresses have the same sign

Given:
σ_x = +150 MPa
σ_y = +30 MPa
σ_z = 0
τ_{xy} = +20 MPa

Results:
σ_1 = 153.2 MPa
σ_2 = 26.8 MPa
σ_3 = 0
τ_{max} = 76.6 MPa

Given:
σ_x = −50 MPa
σ_y = −130 MPa
σ_z = 0
τ_{xy} = 40 MPa

Results:
σ_1 = 0
σ_2 = −33.4 MPa
σ_3 = −146.6 MPa
τ_{max} = 73.3 MPa



(a) σ_x and σ_y both positive

(b) σ_x and σ_y both negative

4–6 MOHR'S CIRCLE FOR SPECIAL STRESS CONDITIONS

MDESIGN

Mohr's circle is used here to demonstrate the relationship among the applied stresses, the principal stresses, and the maximum shear stress for the following special cases:

Pure uniaxial tension

Pure uniaxial compression

Pure torsional shear

Uniaxial tension combined with torsional shear

These are important, frequently encountered stress conditions, and they will be used in later chapters to illustrate failure theories and design methods. These failure theories are based on the values of the principal stresses and the maximum shear stress.

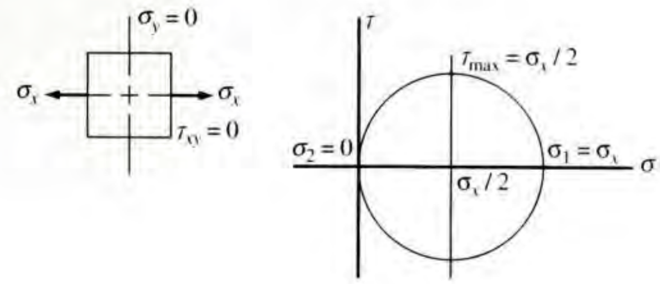


FIGURE 4–29
Mohr's circle for pure uniaxial tension

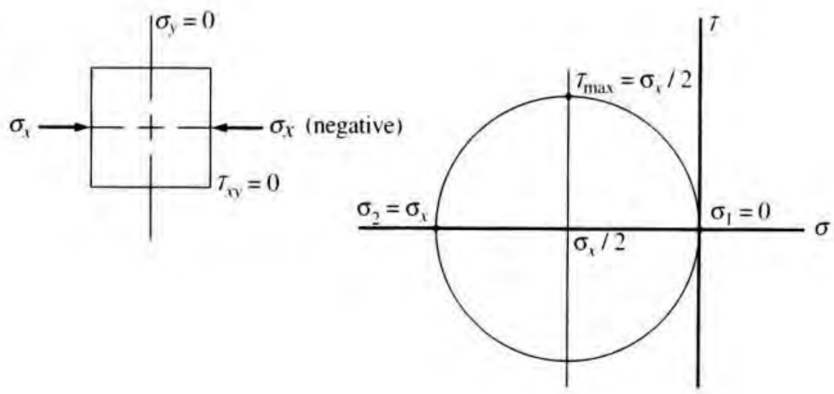


FIGURE 4–30
Mohr's circle for pure uniaxial compression

Pure Uniaxial Tension

The stress condition produced in all parts of a standard tensile test specimen is pure uniaxial tension. Figure 4–29 shows the stress element and the corresponding Mohr's circle. Note that the maximum principal stress, σ_1, is equal to the applied stress, σ_x; the minimum principal stress, σ_2, is zero; and the maximum shear stress, τ_{max}, is equal to $\sigma_x/2$.

Pure Uniaxial Compression

Figure 4–30 shows pure uniaxial compression as it would be produced by a standard compression test. Mohr's circle shows that $\sigma_1 = 0$; $\sigma_2 = \sigma_x$ (a negative value); and the magnitude of the maximum shear stress is $\tau_{max} = \sigma_x/2$.

Pure Torsion

Figure 4–31 shows that the Mohr's circle for this special case has its center at the origin of the σ-τ axes and that the radius of the circle is equal to the value of the applied shear stress, τ_{xy}. Therefore, $\tau_{max} = \tau_{xy}$; $\sigma_1 = \tau_{xy}$; and $\sigma_2 = -\tau_{xy}$.

Uniaxial Tension Combined with Torsional Shear

This is an important special case because it describes the stress condition in a rotating shaft carrying bending loads while simultaneously transmitting torque. This is the type of stress condition on which the procedure for designing shafts, presented in Chapter 12, is based. If the applied stresses are called σ_x and τ_{xy}, the Mohr's circle in Figure 4–32 shows that

$$\tau_{max} = R = \text{radius of circle} = \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} \qquad \textbf{(4–7)}$$

$$\sigma_1 = \sigma_x/2 + R = \sigma_x/2 + \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} \qquad \textbf{(4–8)}$$

$$\sigma_2 = \sigma_x/2 - R = \sigma_x/2 - \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} \qquad \textbf{(4–9)}$$

FIGURE 4–31
Mohr's circle for pure torsional shear

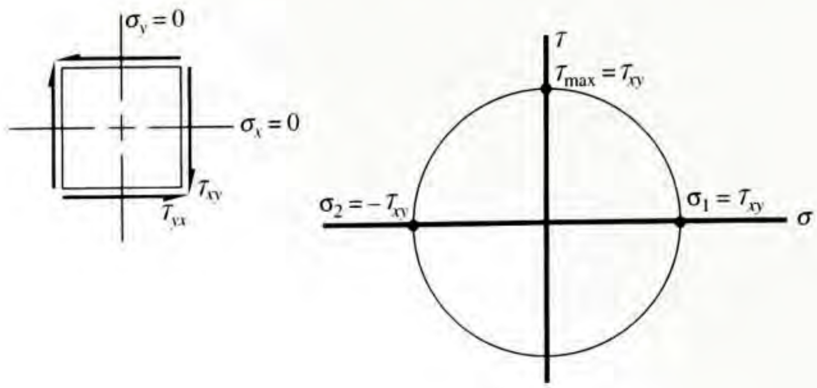


FIGURE 4–32
Mohr's circle for uniaxial tension combined with torsional shear

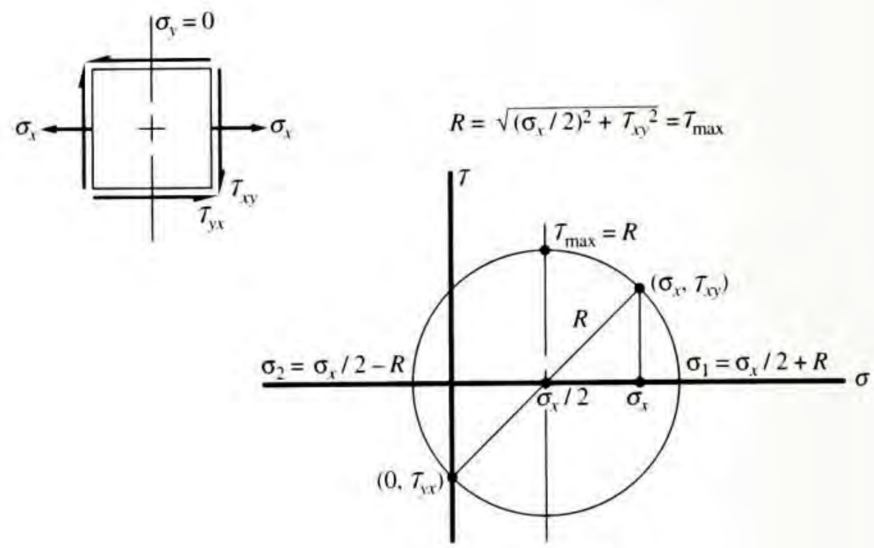
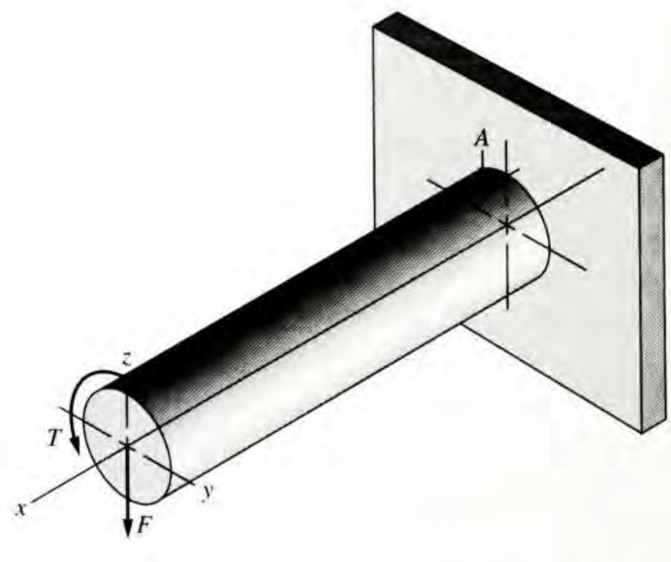


FIGURE 4–33
Circular bar in bending and torsion



A convenient and useful concept called *equivalent torque* can be developed from Equation (4–7) for the special case of a body subjected to only bending and torsion.

An example is shown in Figure 4–33, where a circular bar is loaded at one end by a downward force and a torsional moment. The force causes bending in the bar with the maximum moment at the point where the bar is attached to the support. The moment causes a tensile stress on the top of the bar in the x-direction at the point called A, where the magnitude of the stress is

$$\sigma_x = M/S \tag{4–10}$$

where S = section modulus of the round bar

Now the torsional moment causes torsional shear stress in the x-y plane at point A having a magnitude of

$$\tau_{xy} = T/Z_p \tag{4–11}$$

where Z_p = polar section modulus of the bar

Point A then is subjected to a tensile stress combined with shear, the special case shown in the Mohr's circle of Figure 4–32. The maximum shear stress can be computed from Equation (4–7). If we substitute Equations (4–10) and (4–11) in Equation (4–7), we get

$$\tau_{max} = \sqrt{(M/2S)^2 + (T/Z_p)^2} \tag{4–12}$$

Note from Appendix 1 that $Z_p = 2S$. Equation (4–12) can then be written as

$$\tau_{max} = \frac{\sqrt{M^2 + T^2}}{Z_p} \tag{4–13}$$

It is convenient to define the quantity in the numerator of this equation to be the *equivalent torque*, T_e. Then the equation becomes

$$\tau_{max} = T_e/Z_p \tag{4–14}$$

4–7 ANALYSIS OF COMPLEX LOADING CONDITIONS

The examples shown in this chapter involved relatively simple part geometries and loading conditions for which the necessary stress analysis can be performed using familiar methods of statics and strength of materials. If more complex geometries or loading conditions are involved, you may not be able to complete the required analysis to create the original stress element from which the Mohr's circle is derived.

Consider, for example, a cast wheel for a high-performance racing car. The geometry would likely involve webs or spokes of a unique design connecting the hub to the rim to create a lightweight wheel. The loading would be a complex combination of torsion, bending, and compression generated by the cornering action of the wheel.

One method of analysis of such a load-carrying member would be accomplished by experimental stress analysis using strain gages or photoelastic techniques. The results would identify the stress levels at selected points in certain specified directions that could be used as the input to the construction of the Mohr's circle for critical points in the structure.

Another method of analysis would involve the modeling of the geometry of the wheel as a *finite-element model*. The three-dimensional model would be divided into several hundred small-volume elements. Points of support and restraint would be defined on the model, and then external loads would be applied at appropriate points. The complete data set would be input to a special type of computer analysis program called *finite-element analysis*. The output from the program lists the stresses and the deflection for each of the elements. These data can be plotted on the computer model so that the designer can visualize the stress distribution within the model. Most such programs list the principal stresses and the maximum shear stress for each element, eliminating the need to actually draw the Mohr's circle. A special form of stress, called the *von Mises stress*, is often computed by combining the principal stresses. (See Section 5–8 for a more complete discussion of the von Mises stress and its use.) Several different finite-element analysis programs are commercially available for use on personal computers, on engineering work stations, or on mainframe computers.

REFERENCE

Mott, Robert L. *Applied Strength of Materials*. 4th ed. Upper Saddle River, NJ: Prentice Hall, 2002.

INTERNET SITE RELATED TO MOHR'S CIRCLE ANALYSIS

1. **MDSolids** *www.mdsolids.com* Educational software devoted to introductory mechanics of materials. Includes modules on basic stress and strain, beam and strut axial problems, trusses, statically indeterminate axial structures, torsion, determinate beams, section properties, general analysis of axial, torsion, and beam members, column buckling, pressure vessels, and Mohr's circle transformations.

PROBLEMS

For the sets of given stresses on an element given in Table 4–2, draw a complete Mohr's circle, find the principal stresses and the maximum shear stress, and draw the principal stress element and the maximum shear stress element. Any stress components not shown are assumed to be zero.

TABLE 4–2 Stresses for Problems 1–30

Problem	σ_x	σ_y	τ_{xy}
1.	20 ksi	0	10 ksi
2.	85 ksi	−40 ksi	30 ksi
3.	40 ksi	−40 ksi	−30 ksi
4.	−80 ksi	−40 ksi	−30 ksi
5.	120 ksi	40 ksi	20 ksi
6.	20 ksi	140 ksi	20 ksi
7.	20 ksi	−40 ksi	0
8.	120 ksi	−40 ksi	100 ksi
9.	100 MPa	0	80 MPa
10.	250 MPa	−80 MPa	110 MPa
11.	50 MPa	−80 MPa	40 MPa
12.	−150 MPa	−80 MPa	−40 MPa
13.	150 MPa	80 MPa	−40 MPa
14.	50 MPa	180 MPa	40 MPa
15.	250 MPa	−80 MPa	0
16.	50 MPa	−80 MPa	−30 MPa
17.	400 MPa	−300 MPa	200 MPa
18.	−120 MPa	180 MPa	−80 MPa
19.	−30 MPa	20 MPa	40 MPa
20.	220 MPa	−120 MPa	0 MPa
21.	40 ksi	0 ksi	0 ksi
22.	0 ksi	0 ksi	40 ksi
23.	38 ksi	−25 ksi	−18 ksi
24.	55 ksi	15 ksi	−40 ksi
25.	22 ksi	0 ksi	6.8 ksi
26.	−4250 psi	3250 psi	2800 psi
27.	300 MPa	100 MPa	80 MPa
28.	250 MPa	150 MPa	40 MPa
29.	−840 kPa	−335 kPa	−120 kPa
30.	−325 kPa	−50 kPa	−60 kPa

31. Refer to Figure 3–23 in Chapter 3. For the shaft aligned with the x-axis, create a stress element on the bottom of the shaft just to the left of section B. Then draw the Mohr's circle for that element. Use $D = 0.50$ in.

32. Refer to Figure P3–44 in Chapter 3. For the shaft ABC, create a stress element on the bottom of the shaft just to the right of section B. The torque applied to the shaft at B is resisted at support C only. Draw the Mohr's circle for the stress element. Use $D = 1.50$ in.

33. Repeat Problem 32 for the shaft in Figure P3–45 $D = 0.50$ in.

34. Refer to Figure P3–46 in Chapter 3. For the shaft AB, create a stress element on the bottom of the shaft just to the right of section A. The torque applied to the shaft by the crank is resisted at support B only. Draw the Mohr's circle for the stress element. Use $D = 50$ mm.

35. A short cylindrical bar having a diameter of 4.00 in is subjected to an axial compressive force of 75 000 lb and a torsional moment of 20 000 lb · in. Draw a stress element on the surface of the bar. Then draw the Mohr's circle for the element.

36. A torsion bar is used as a suspension element for a vehicle. The bar has a diameter of 20 mm. It is subjected to a torsional moment of 450 N · m and an axial tensile force of 36.0 kN. Draw a stress element on the surface of the bar, and then draw the Mohr's circle for the element.

37. Use the Mohr's Circle module from the MDSolids software to complete any of Problems 1 to 30 in this chapter.

5

Design for Different Types of Loading

The Big Picture

Design for Different Types of Loading

Discussion Map

- ☐ This chapter provides additional tools you can use to design load-carrying components that are safe and reasonably efficient in their use of materials.
- ☐ You must learn how to classify the kind of loading the component is subjected to: *static, repeated and reversed, fluctuating, shock,* and *impact.*
- ☐ You will learn to identify the appropriate analysis techniques based on the type of load and the type of material.

Discover

Identify components of real products or structures that are subjected to static loads.

Identify components that are subjected to equal, repeated loads that reverse directions.

Identify components that experience fluctuating loads that vary with time.

Identify components that are loaded with shock or impact, such as being struck by a hammer or dropped onto a hard surface.

Using the techniques you learn in this chapter will help you to complete a wide variety of design tasks

For the concepts considered in this chapter, the big picture encompasses a huge array of examples in which you will build on the principles of strength of materials that you reviewed in Chapters 3 and 4 and extend them from the analysis mode to the design mode. Several steps are involved, and you must learn to make rational judgments about the appropriate method to apply to complete the design.

In this chapter you will learn how to do the following:

1. Recognize the manner of loading for a part: Is it static, repeated and reversed, fluctuating, shock, or impact?
2. Select the appropriate method to analyze the stresses produced.
3. Determine the strength property for the material that is appropriate to the kind of loading and to the kind of material: Is the material a metal or a nonmetal? Is it brittle or ductile? Should the design be based on the yield strength, ultimate tensile strength, compressive strength, endurance strength, or some other material property?
4. Specify a suitable *design factor*, often called a *factor of safety*.
5. Design a wide variety of load-carrying members to be safe under their particular expected loading patterns.

The following paragraphs show by example some of the situations to be studied in this chapter.

An ideal *static load* is one that is applied slowly and is never removed. Some loads that are applied slowly and removed and replaced very infrequently can also be considered to be static. What examples can you think of for products or their components that are subjected to static loads? Consider load-carrying members of structures, parts of furniture pieces, and brackets or support rods holding equipment in your home or in a business or factory. Try to identify specific examples, and describe them to your colleagues. Discuss how the load is applied and which parts of the load-carrying member are subjected to the higher stress levels. Some of the examples that you discovered during **The Big Picture** discussion for Chapter 3 could be used again here.

Fluctuating loads are those that vary during the normal service of the product. They typically are applied for quite a long time so the part experiences many thousands or millions of cycles of stress during its expected life. There are many examples in consumer products around your home, in your car, in commercial buildings, and in manufacturing facilities. Consider virtually anything that has moving parts. Again, try to identify specific examples, and describe them to your colleagues. How does the load fluctuate? Is it applied and then completely removed each cycle? Or is there always some level of mean or average load with an alternating load superimposed on it? Does the load swing from a positive maximum value to a negative minimum value of equal magnitude during each cycle of loading? Consider parts with rotating shafts, such as engines or agricultural, production, and construction machinery.

Consider products that have failed. You may have identified some from **The Big Picture** discussion for Chapter 3. Did they fail the first time they were used? Or did they fail after some fairly long service? Why do you think they were able to operate for some time before failure?

Can you find components that failed suddenly because the material was brittle, such as cast iron, some ceramics, or some plastics? Can you find others that failed only after some considerable deformation? Such failures are called *ductile fractures*.

What were the consequences of the failures that you have found? Was anyone hurt? Was there damage to some other valuable component or property? Or was the failure simply an inconvenience? What was the order of magnitude of cost related to the failure? The answer to some of these questions can help you make rational decisions about design factors to be used in your designs.

It is the designer's responsibility to ensure that a machine part is safe for operation under reasonably foreseeable conditions. This requires that a stress analysis be performed in which the predicted stress levels in the part are compared with the *design stress*, or that level of stress permitted under the operating conditions.

The stress analysis can be performed either analytically or experimentally, depending on the degree of complexity of the part, the knowledge about the loading conditions, and the material properties. The designer must be able to verify that the stress to which a part is subjected is safe.

The manner of computing the design stress depends on the manner of loading and on the type of material. Loading types include the following:

Static

Repeated and reversed

Fluctuating

Shock or impact

Random

Material types are many and varied. Among the metallic materials, the chief classification is between *ductile* and *brittle* materials. Other considerations include the manner of forming the material (casting, forging, rolling, machining, and so on), the type of heat treatment, the surface finish, the physical size, the environment in which it is to operate, and the geometry of the part. Different factors must be considered for plastics, composites, ceramics, wood, and others.

This chapter outlines methods of analyzing load-carrying machine parts to ensure that they are safe. Several different cases are described in which knowledge of the combinations of material types and loading patterns leads to the determination of the appropriate method of analysis. It will then be your job to apply these tools correctly and judiciously as you continue your career.

You Are the Designer

Recall the task presented at the start of Chapter 4, in which you were the designer of a bracket to hold a fabric sample during a test to determine its long-term stretch characteristics. Figure 4–2 showed a proposed design.

Now you are asked to continue this design exercise by selecting a material from which to make the two bent circular bars that are welded to the rigid support. Also, you must specify a suitable diameter for the bars when a certain load is applied to the test material.

5–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Identify various kinds of loading commonly encountered by machine parts, including *static*, *repeated and reversed*, *fluctuating*, *shock or impact*, and *random*.
2. Define the term *stress ratio* and compute its value for the various kinds of loading.
3. Define the concept of *fatigue*.
4. Define the material property of *endurance strength* and determine estimates of its magnitude for different materials.
5. Recognize the factors that affect the magnitude of endurance strength.
6. Define the term *design factor*.
7. Specify a suitable value for the design factor.
8. Define the *maximum normal stress theory of failure* and the *modified Mohr method* for design with brittle materials.
9. Define the *maximum shear stress theory of failure*.
10. Define the *distortion energy theory*, also called the *von Mises theory* or the *Mises-Hencky theory*.
11. Describe the *Goodman method* and apply it to the design of parts subjected to fluctuating stresses.
12. Consider *statistical approaches*, *finite life*, and *damage accumulation methods* for design.

5–2 TYPES OF LOADING AND STRESS RATIO

The primary factors to consider when specifying the type of loading to which a machine part is subjected are the manner of variation of the load and the resulting variation of stress with time. Stress variations are characterized by four key values:

1. Maximum stress, σ_{max}
2. Minimum stress, σ_{min}
3. Mean (average) stress, σ_m
4. Alternating stress, σ_a (*stress amplitude*)

The maximum and minimum stresses are usually computed from known information by stress analysis or finite-element methods, or they are measured using experimental stress analysis techniques. Then the mean and alternating stresses can be computed from

$$\sigma_m = (\sigma_{max} + \sigma_{min})/2 \tag{5–1}$$

$$\sigma_a = (\sigma_{max} - \sigma_{min})/2 \tag{5–2}$$

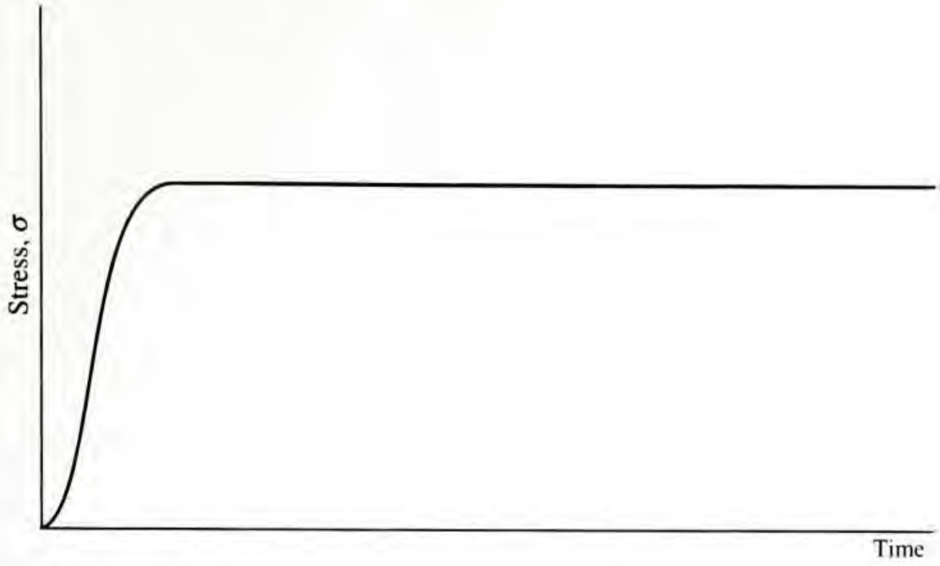


FIGURE 5–1 Static stress

The behavior of a material under varying stresses is dependent on the manner of the variation. One method used to characterize the variation is called *stress ratio*. Two types of stress ratios are commonly used, defined as

$$\begin{aligned} \text{Stress ratio } R &= \frac{\text{minimum stress}}{\text{maximum stress}} = \frac{\sigma_{min}}{\sigma_{max}} \\ \text{Stress ratio } A &= \frac{\text{alternating stress}}{\text{mean stress}} = \frac{\sigma_a}{\sigma_m} \end{aligned} \tag{5–3}$$

Static Stress

When a part is subjected to a load that is applied slowly, without shock, and is held at a constant value, the resulting stress in the part is called *static stress*. An example is the load on a structure due to the dead weight of the building materials. Figure 5–1 shows a diagram of stress versus time for static loading. Because $\sigma_{max} = \sigma_{min}$, the stress ratio for static stress is $R = 1.0$.

Static loading can also be assumed when a load is applied and is removed slowly and then reapplied, if the number of load applications is small, that is, under a few thousand cycles of loading.

Repeated and Reversed Stress

A stress reversal occurs when a given element of a load-carrying member is subjected to a certain level of tensile stress followed by the *same level* of compressive stress. If this stress cycle is repeated many thousands of times, the stress is called *repeated and reversed*. Figure 5–2 shows the diagram of stress versus time for repeated and reversed stress. Because $\sigma_{min} = -\sigma_{max}$, the stress ratio is $R = -1.0$, and the mean stress is zero.

An important example in machine design is a rotating circular shaft loaded in bending such as that shown in Figure 5–3. In the position shown, an element on the bottom of the shaft experiences tensile stress while an element on the top of the shaft sees a compressive stress of equal magnitude. As the shaft is rotated 180° from the given position, these two elements experience a complete reversal of stress. Now if the shaft continues to rotate, all parts of the shaft that are in bending see repeated, reversed stress. This is a description of the classical loading case of *reversed bending*.

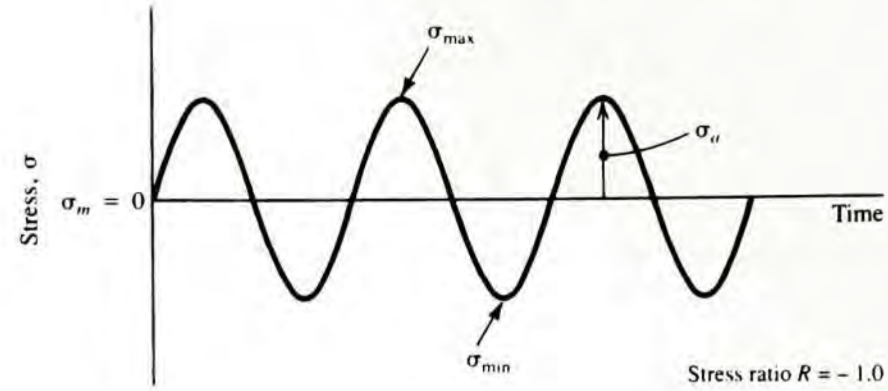


FIGURE 5–2
Repeated, reversed stress

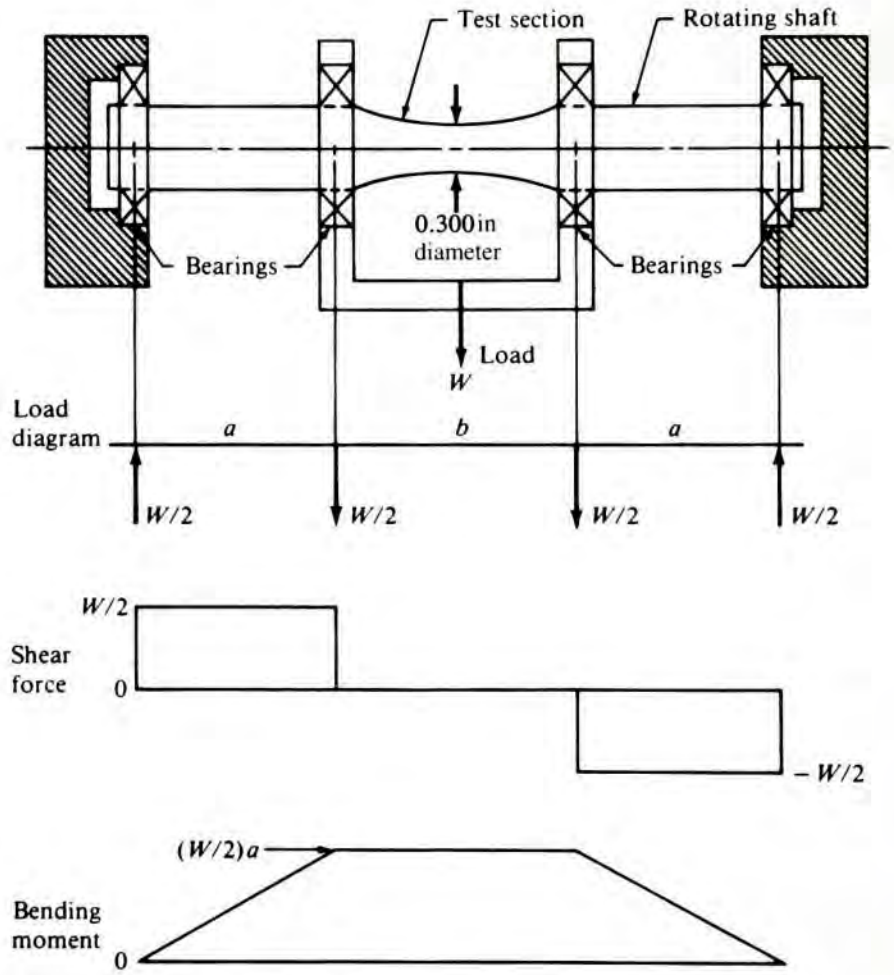


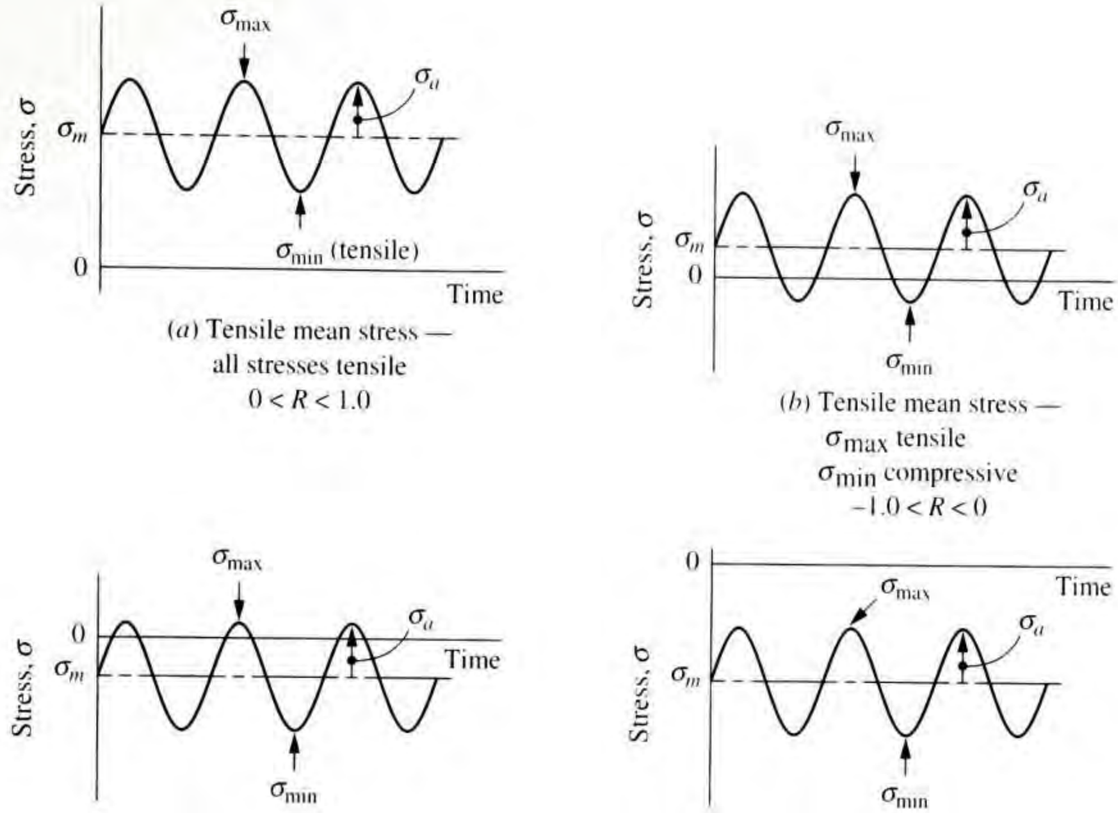
FIGURE 5–3
R. R. Moore fatigue test device

This type of loading is often called *fatigue loading*, and a machine of the type shown in Figure 5–3 is called a *standard R. R. Moore fatigue test device*. Such machines are used to test materials for their ability to resist repeated loads. The material property called *endurance strength* is measured in this manner. More is said later in this chapter about endurance strength. Actually, reversed bending is only a special case of fatigue loading, since any stress that varies with time can lead to fatigue failure of a part.

Fluctuating Stress

When a load-carrying member is subjected to an alternating stress with a nonzero mean, the loading produces *fluctuating stress*. Figure 5–4 shows four diagrams of stress versus time for this type of stress. Differences among the four diagrams occur in whether the various stress levels are positive (tensile) or negative (compressive). *Any varying stress with a*

FIGURE 5–4
Fluctuating stresses



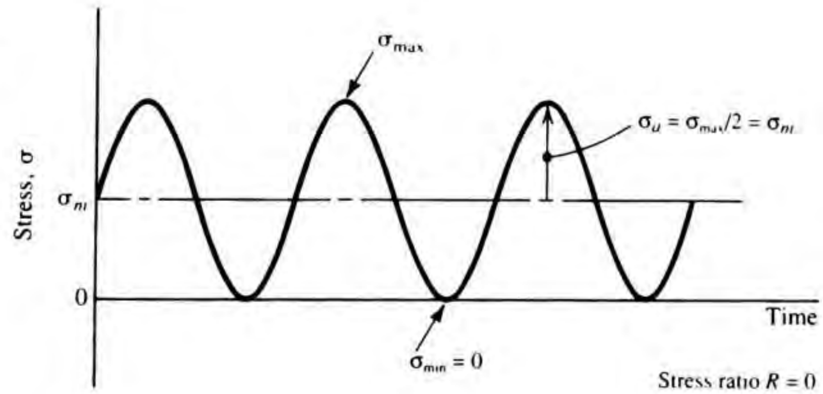
(*a*) Tensile mean stress — all stresses tensile $0 < R < 1.0$

(*b*) Tensile mean stress — σ_{max} tensile σ_{min} compressive $-1.0 < R < 0$

(*c*) Compressive mean stress — σ_{max} tensile σ_{min} compressive $-\infty < R < -1.0$

(*d*) Compressive mean stress — all stresses compressive $1.0 < R < \infty$

FIGURE 5–5
Repeated, one-direction stress, a special case of fluctuating stress



nonzero mean is considered a fluctuating stress. Figure 5–4 also shows the possible ranges of values for the stress ratio R for the given loading patterns.

A special, frequently encountered case of fluctuating stress is *repeated, one-direction stress*, in which the load is applied and removed many times. As shown in Figure 5–5, the stress varies from zero to a maximum with each cycle. Then, by observation,

$$\sigma_{min} = 0$$
$$\sigma_m = \sigma_a = \sigma_{max}/2$$
$$R = \sigma_{min}/\sigma_{max} = 0$$

An example of a machine part subjected to fluctuating stress of the type shown in Figure 5–4(a) is shown in Figure 5–6, in which a reciprocating cam follower feeds

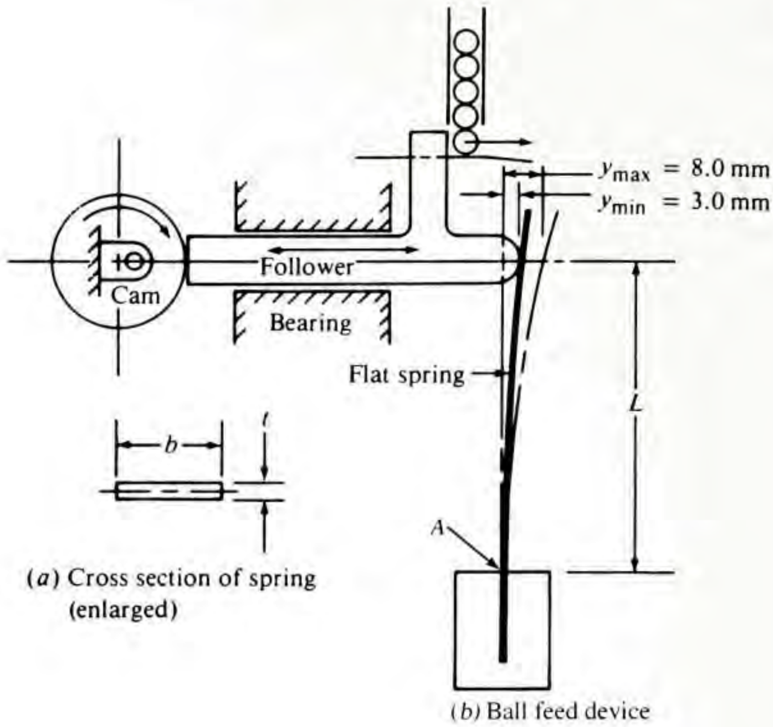


FIGURE 5–6 Example of cyclic loading in which the flat spring is subjected to fluctuating stress

spherical balls one at a time from a chute. The follower is held against the eccentric cam by a flat spring loaded as a cantilever. When the follower is farthest to the left, the spring is deflected from its free (straight) position by an amount y_{min} = 3.0 mm. When the follower is farthest to the right, the spring is deflected to y_{max} = 8.0 mm. Then, as the cam continues to rotate, the spring sees the cyclic loading between the minimum and maximum values. Point A at the base of the spring on the convex side experiences the varying tensile stresses of the type shown in Figure 5–4(a). Example Problem 5–1 completes the analysis of the stress in the spring at point A.

Example Problem 5–1 For the flat steel spring shown in Figure 5–6, compute the maximum stress, the minimum stress, the mean stress, and the alternating stress. Also compute the stress ratio, R. The length L is 65 mm. The dimensions of the spring cross section are t = 0.80 mm and b = 6.0 mm.

Solution Objective Compute the maximum, minimum, mean, and alternating tensile stresses in the flat spring. Compute the stress ratio, R.

Given Layout shown in Figure 5–6. The spring is steel: L = 65 mm.

Spring cross section dimensions: t = 0.80 mm and b = 6.0 mm.

Maximum deflection of the spring at the follower = 8.0 mm.

Minimum deflection of the spring at the follower = 3.0 mm.

Analysis Point A at the base of the spring experiences the maximum tensile stress. Determine the force exerted on the spring by the follower for each level of deflection using the formulas from Table A14–2, Case (a). Compute the bending moment at the base of the spring for each deflection. Then compute the stresses at point A using the bending stress formula, $\sigma = Mc/I$. Use Equations (5–1), (5–2), and (5–3) for the mean and alternating stresses and R.

Results Case (a) of Table A14–2 gives the following formula for the amount of deflection of a cantilever for a given applied force:

$$y = PL^3/3EI$$

Solve for the force as a function of deflection:

$$P = 3EIy/L^3$$

Appendix 3 gives the modulus of elasticity for steel to be $E = 207$ GPa. The moment of inertia, I, for the spring cross section is found from

$$I = bt^3/12 = (6.00\text{mm})(0.80\text{mm})^3/12 = 0.256 \text{ mm}^4$$

Then the force on the spring when the deflection y is 3.0 mm is

$$P = \frac{3(207 \times 10^9 \text{ N/m}^2)(0.256 \text{ mm}^4)(3.0 \text{ mm})}{(65 \text{ mm})^3} \frac{(1.0 \text{ m}^2)}{(10^6 \text{ mm}^2)} = 1.74 \text{ N}$$

The bending moment at the support is

$$M = P \cdot L = (1.74 \text{ N})(65 \text{ mm}) = 113 \text{ N} \cdot \text{mm}$$

The bending stress at point A caused by this moment is

$$\sigma = \frac{Mc}{I} = \frac{(113 \text{ N} \cdot \text{mm})(0.40 \text{ mm})}{0.256 \text{ mm}^4} = 176 \text{ N/mm}^2 = 176 \text{ MPa}$$

This is the lowest stress that the spring sees in service, and therefore $\sigma_{min} = 176$ MPa.

Because the force on the spring is proportional to the deflection, the force exerted when the deflection is 8.0 mm is

$$P = (1.74 \text{ N})(8.0 \text{ mm})/(3.0 \text{ mm}) = 4.63 \text{ N}$$

The bending moment is

$$M = P \cdot L = (4.63 \text{ N})(65 \text{ mm}) = 301 \text{ N} \cdot \text{mm}$$

The bending stress at point A is

$$\sigma = \frac{Mc}{I} = \frac{(301 \text{ N} \cdot \text{mm})(0.40 \text{ mm})}{0.256 \text{ mm}^4} = 470 \text{ N/mm}^2 = 470 \text{ MPa}$$

This is the maximum stress that the spring sees, and therefore $\sigma_{max} = 470$ MPa.

Now the mean stress can be computed:

$$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (470 + 176)/2 = 323 \text{ MPa}$$

Finally, the alternating stress is

$$\sigma_a = (\sigma_{\max} - \sigma_{\min})/2 = (470 - 176)/2 = 147 \text{ MPa}$$

The stress ratio is found using Equation (5–3):

$$\text{Stress ratio } R = \frac{\text{minimum stress}}{\text{maximum stress}} = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{176 \text{ MPa}}{470 \text{ MPa}} = 0.37$$

Comments The sketch of stress versus time shown in Figure 5–4(a) illustrates the form of the fluctuating stress on the spring. In Section 5–9, you will see how to design parts subjected to this kind of stress.

Shock or Impact Loading

Loads applied suddenly and rapidly cause shock or impact. Examples include a hammer blow, a weight falling onto a structure, and the action inside a rock crusher. The design of machine members to withstand shock or impact involves an analysis of their energy-absorption capability, a topic not considered in this book. (See References 8 to 13).

Random Loading

When varying loads are applied that are not regular in their amplitude, the loading is called *random*. Statistical analysis is used to characterize random loading for purposes of design and analysis. This topic is not covered in this book. See Reference 14.

5–3 ENDURANCE STRENGTH

The *endurance strength* of a material is its ability to withstand fatigue loads. In general, it is the stress level that a material can survive for a given number of cycles of loading. If the number of cycles is infinite, the stress level is called the *endurance limit*.

Endurance strengths are usually charted on a graph like that shown in Figure 5–7, called an *S-N diagram*. Curves *A*, *B*, and *D* are representative of a material that does exhibit an endurance limit, such as a plain carbon steel. Curve *C* is typical of most nonferrous metals, such as aluminum, which do not exhibit an endurance limit. For such materials, the number of cycles to failure should be reported for the given endurance strength.

Data for the endurance strength of the specific material for a part should be used whenever it is available, either from test results or from reliable published data. However, such data are not always readily available. Reference 13 suggests the following approximations for the basic endurance strength for wrought steel:

$$\text{Endurance strength} = 0.50(\text{ultimate tensile strength}) = 0.50(s_u)$$

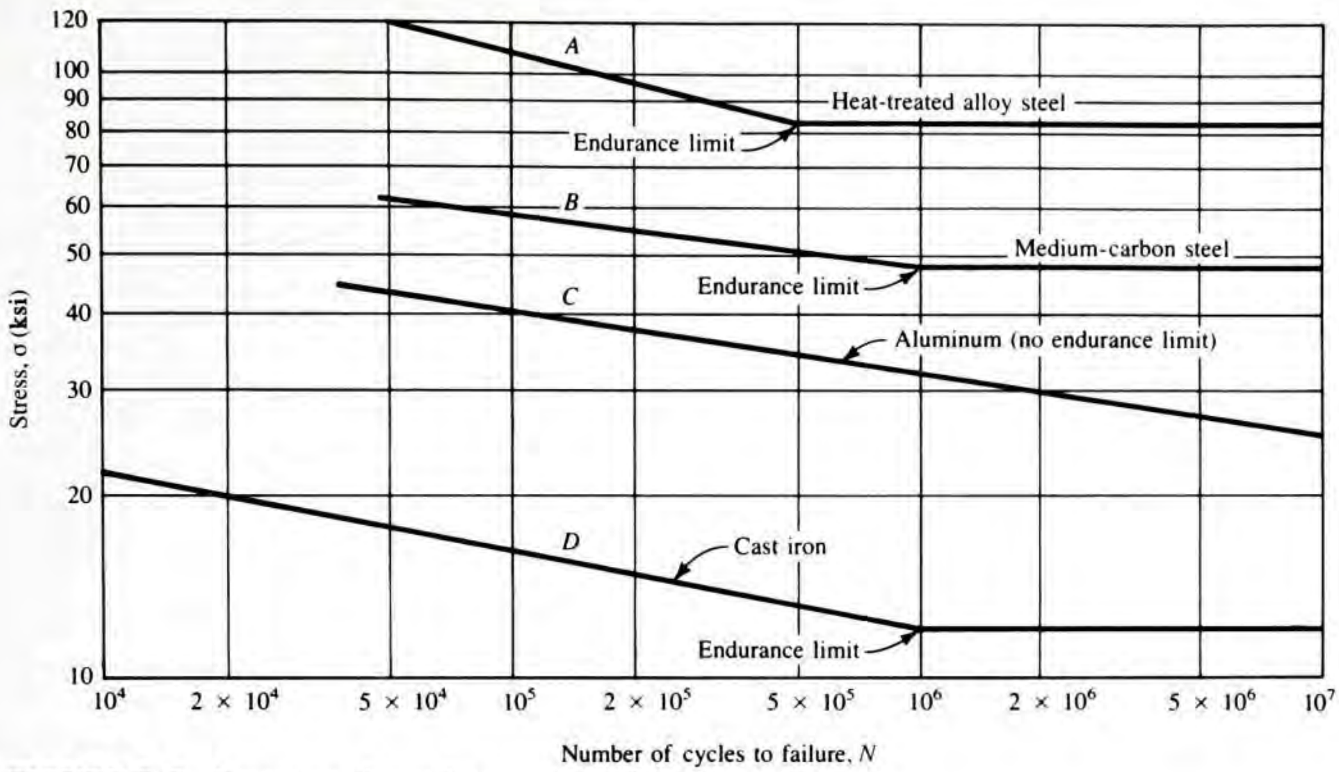


FIGURE 5–7 Representative endurance strengths

This approximation, along with published data, refers to the special case of repeated and reversed bending stress in a polished steel specimen having a diameter of 0.300 in (7.62 mm) as used in the R. R. Moore fatigue testing device shown in Figure 5–3. The next section discusses adjustments required when other, more realistic conditions exist.

5–4 ESTIMATED ACTUAL ENDURANCE STRENGTH, s'_n

If the actual material characteristics or operating conditions for a machine part are different from those for which the basic endurance strength was determined, the fatigue strength must be reduced from the reported value. Some of the factors that decrease the endurance strength are discussed in this section. The discussion relates only to the endurance strength for materials subjected to normal tensile stresses such as bending and direct axial tensile stress. Cases involving endurance strength in shear are discussed separately in Section 5–9.

We begin by presenting a procedure for estimating the *actual endurance strength*, s'_n, for the material for the part being designed. It involves applying several factors to the basic endurance strength for the material. Additional elaboration on the factors follows.

Procedure for Estimating Actual Endurance Strength, s'_n

1. Specify the material for the part and determine its ultimate tensile strength, s_u, considering its condition, as it will be used in service.
2. Specify the manufacturing process used to produce the part with special attention to the condition of the surface in the most highly stressed area.

3. Use Figure 5–8 to estimate the modified endurance strength, s_n.
4. Apply a material factor, C_m, from the following list.

Wrought steel:	$C_m = 1.00$	Malleable cast iron:	$C_m = 0.80$
Cast steel:	$C_m = 0.80$	Gray cast iron:	$C_m = 0.70$
Powdered steel:	$C_m = 0.76$	Ductile cast iron:	$C_m = 0.66$

5. Apply a type-of-stress factor: $C_{st} = 1.0$ for bending stress; $C_{st} = 0.80$ for axial tension.
6. Apply a reliability factor, C_R, from Table 5–1.
7. Apply a size factor, C_s, using Figure 5–9 and Table 5–2 as guides.
8. Compute the estimated actual endurance strength, s_n', from

$$s_n' = s_n\,(C_m)(C_{st})(C_R)(C_s) \qquad \textbf{(5–4)}$$

These are the only factors that will be used consistently in this book. If data for other factors can be determined from additional research, they should be multiplied as additional terms in Equation 5–4. In most cases, we suggest accounting for other factors for which reasonable data cannot be found by adjusting the value of the design factor as discussed in Section 5–8.

Stress concentrations caused by sudden changes in geometry are, indeed, likely places for fatigue failures to occur. Care should be taken in the design and manufacture of cyclically loaded parts to keep stress concentration factors to a low value. We will apply stress concentration factors to the computed stress rather than to the endurance strength. See Section 5–9.

While 12 factors affecting endurance strength are discussed in the following section, note that the procedure just given includes only the first five. They are *surface finish*, *material factor*, *type-of-stress factor*, *reliability factor*, and *size factor*. The others are mentioned to alert you to the variety of conditions you should investigate as you complete a design. However, generalized data are difficult to acquire for all factors. Special testing or additional literature searching should be done when conditions exist for which no data are provided in this book. The end-of-chapter references contain a huge amount of such information.

Surface Finish

Any deviation from a polished surface reduces endurance strength because the rougher surface provides sites where locally increased stresses or irregularities in the material structure promote the initiation of microscopic cracks that can progress to fatigue failures. Manufacturing processes, corrosion, and careless handling produce detrimental surface roughening.

Figure 5–8, adapted from data in Reference 11, shows estimates for the endurance strength s_n compared with the ultimate tensile strength of steels for several practical surface conditions. The data first estimate the endurance strength for the polished specimen to be 0.50 times the ultimate strength and then apply a factor related to the surface condition. U.S. Customary units are used for the bottom and left axes while SI units are shown on the top and right axes. Project vertically from the s_u axis to the appropriate curve and then horizontally to the endurance strength axis.

The data from Figure 5–8 should not be extrapolated for su > 220 ksi (1520 MPa) without specific testing as empirical data reported in Reference 6 are inconsistent at higher strength levels.

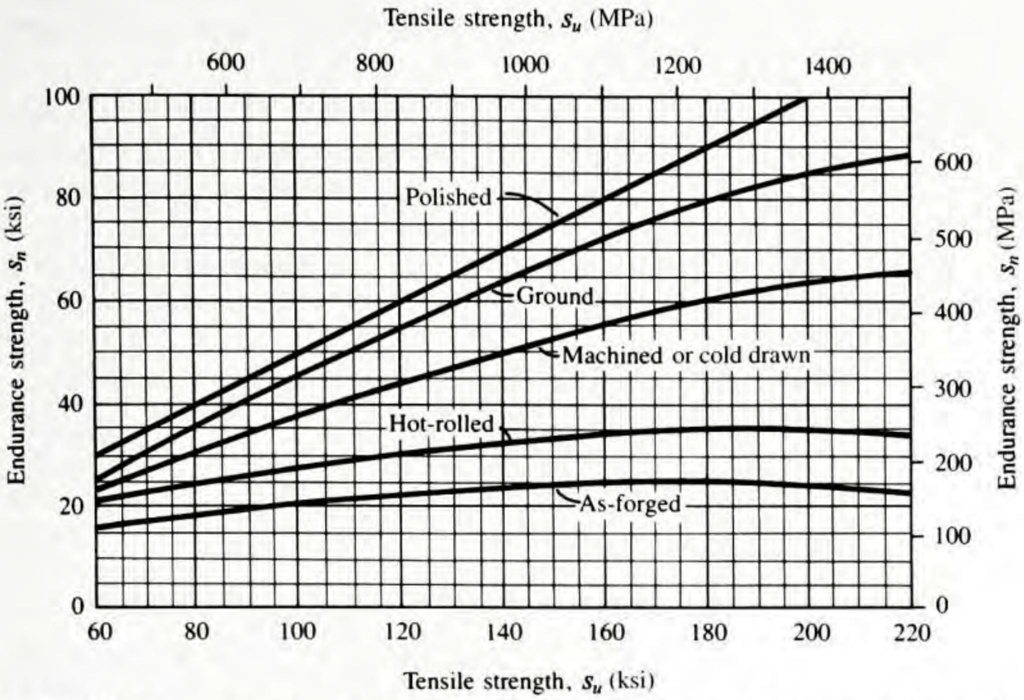


FIGURE 5–8 Endurance strength s_n versus tensile strength for wrought steel for various surface conditions

TABLE 5–1 Approximate reliability factors, C_R

Desired reliability	C_R
0.50	1.0
0.90	0.90
0.99	0.81
0.999	0.75

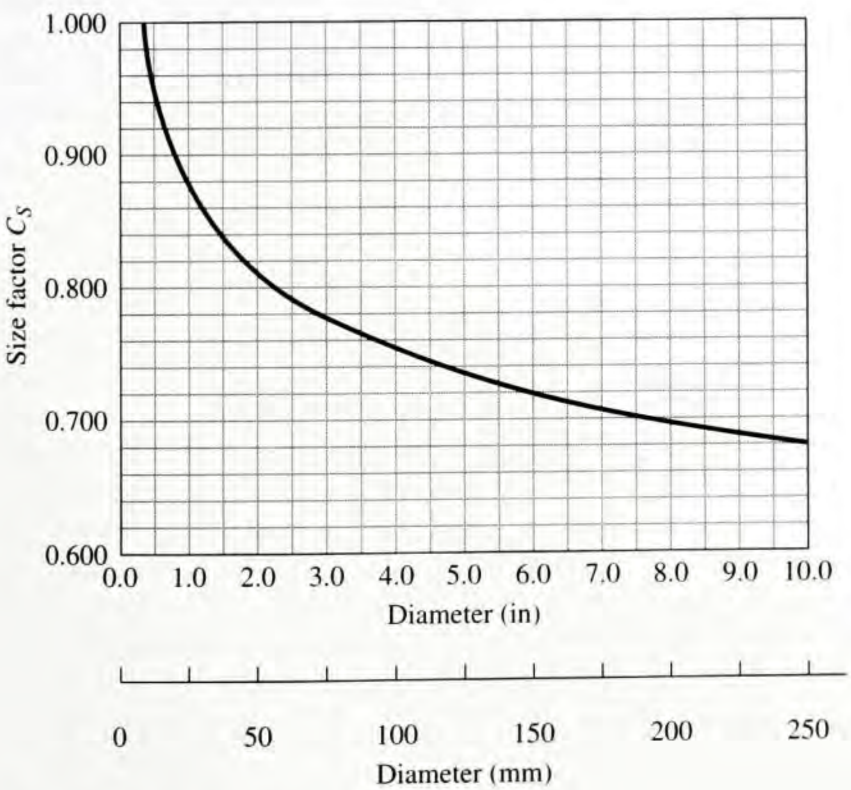


FIGURE 5–9 Size factor

TABLE 5–2 Size factors

U.S. customary units	
Size Range	For D in inches
$D \leq 0.30$	$C_S = 1.0$
$0.30 < D \leq 2.0$	$C_S = (D/0.3)^{-0.11}$
$2.0 < D < 10.0$	$C_S = 0.859\text{-}0.02125D$
SI units	
Size Range	For D in mm
$D \leq 7.62$	$C_S = 1.0$
$7.62 < D \leq 50$	$C_S = (D/7.62)^{-0.11}$
$50 < D < 250$	$C_S = 0.859\text{-}0.000837D$

Ground surfaces are fairly smooth and reduce the endurance strength by a factor of approximately 0.90 for $s_u < 160$ ksi (1 100 MPa), decreasing to about 0.80 for $s_u = 220$ ksi (1520 MPa). Machining or cold drawing produce a somewhat rougher surface because of tooling marks resulting in a reduction factor in the range of 0.80 to 0.60 over the range of strengths shown. The outer part of a hot rolled steel has a roughened oxidized scale that produces a reduction factor from 0.72 to 0.30. If a part is forged and not subsequently machined, the reduction factor ranges from 0.57 to 0.20.

From these data it should be obvious that you must give special attention to surface finish for critical surfaces exposed to fatigue loading in order to benefit from the steel's basic strength. Also, critical surfaces of fatigue-loaded parts must be protected from nicks, scratches, and corrosion because they drastically reduce fatigue strength.

Material Factors

Metal alloys having similar chemical composition can be wrought, cast, or made by powder metallurgy to produce the final form. Wrought materials are usually rolled or drawn, and they typically have higher endurance strength than cast materials. The grain structure of many cast materials or powder metals and the likelihood of internal flaws and inclusions tend to reduce their endurance strength. Reference 13 provides data from which the *material factors* listed in step 4 of the procedure outlined previously are taken.

Type-of-Stress Factor

Most endurance strength data are obtained from tests using a rotating cylindrical bar subjected to repeated and reversed bending in which the outer part experiences the highest stress. Stress levels decrease linearly to zero at the center of the bar. Because fatigue cracks usually initiate in regions of high tensile stress, a relatively small proportion of the material experiences such stresses. Contrast this with the case of a bar subjected to direct axial tensile stress for which *all* of the material experiences the maximum stress. There is a higher statistical probability that local flaws anywhere in the bar may start fatigue cracks. The result is that the endurance strength of a material subjected to repeated and reversed axial stress is approximately 80% of that from repeated and reversed bending. Therefore, we recommend that a factor $C_{st} = 1.0$ be applied for bending stress and $C_{st} = 0.80$ for axial loading.

Reliability Factor

The data for endurance strength for steel shown in Figure 5–8 represent average values derived from many tests of specimens having the appropriate ultimate strength and surface conditions. Naturally, there is variation among the data points; that is, half are higher and half are lower than the reported values on the given curve. The curve, then, represents a reliability of 50%, indicating that half of the parts would fail. Obviously, it is advisable to design for a higher reliability, say, 90%, 99%, or 99.9%. A factor can be used to estimate a lower endurance strength that can be used for design to produce the higher reliability values. Ideally, a statistical analysis of actual data for the material to be used in the design should be obtained. By making certain assumptions about the form of the distribution of strength data, Reference 11 reports the values in Table 5–1 as approximate reliability factors, C_R.

Size Factor—Circular Sections in Rotating Bending

Recall that the basic endurance strength data were taken for a specimen with a circular cross section that has a diameter of 0.30 in (7.6 mm) and that it was subjected to repeated and reversed bending while rotating. Therefore, each part of the surface is subjected to the maximum tensile bending stress with each revolution. Furthermore, the most likely place for fatigue failure to initiate is in the zone of maximum tensile stress within a small distance of the outer surface.

Data from References 2, 11, and 13 show that as the diameter of a rotating circular bending specimen increases, the endurance strength decreases because the stress gradient (change in stress as a function of radius) places a greater proportion of the material in the highly stressed region. Figure 5–9 and Table 5–2 show the size factor to be used in this book, adapted from Reference 13. These data can be used for either solid or hollow circular sections.

Size Factor—Other Conditions

We need different approaches to determining the size factor when a part with a circular section is subjected to repeated and reversed bending but it is *not rotating*, or if the part has a noncircular cross section. Here we show a procedure adapted from Reference 13 that focuses on the volume of the part that experiences 95% or more of the maximum stress. It is in this volume that fatigue failure is most likely to be initiated. Furthermore, in order to relate the physical size of such sections to the size factor data in Figure 5–9, we develop an equivalent diameter, D_e.

When the parts in question have a uniform geometry over the length of interest, the volume is the product of the length and the cross sectional area. We can compare different shapes by considering a unit length for each and looking only at the areas. As a base, let's begin by determining an expression for that part of a circular section subjected to 95% or more of the maximum bending stress, calling this area, A_{95}. Because the stress is directly proportional to the radius, we need the area of the thin ring between the outside surface with the full diameter D and a circle whose diameter is $0.95D$, as shown in Figure 5–10(a). Then,

$$A_{95} = (\pi/4)[D^2 - (0.95D)^2] = 0.0766D^2 \tag{5–5}$$

You should demonstrate that this same equation applies to a hollow circular section as shown in Figure 5–10(b). This verifies that the data for size factor shown in Figure 5–9 and Table 5–2 apply directly to either the solid or hollow circular sections when they experience rotating bending.

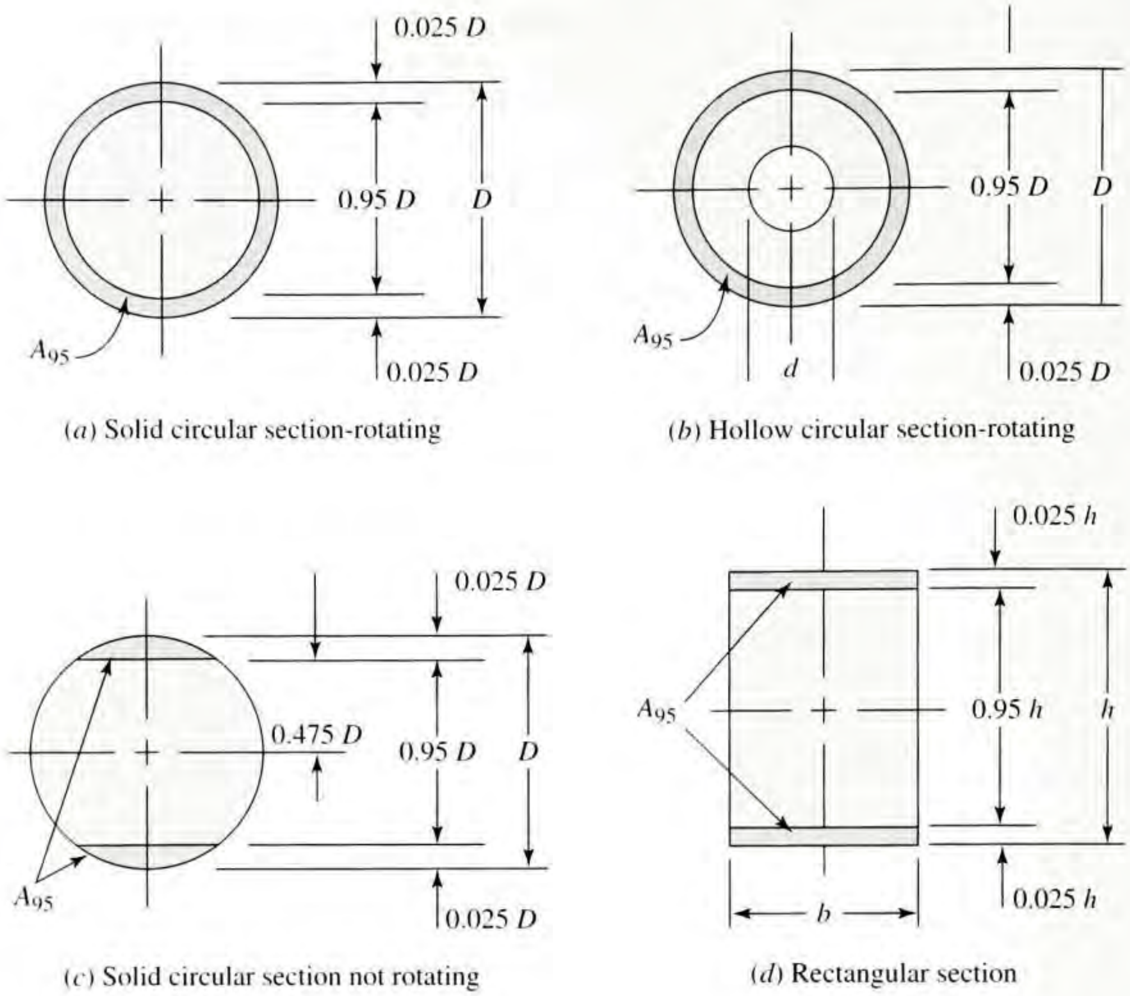
Nonrotating Circular Section in Repeated and Reversed Flexure. Now consider a solid circular section that does not rotate but that is flexed back and forth in repeated and reversed bending. Only the top and bottom segments beyond a radius of $0.475D$ experience 95% or higher of the maximum bending stress as shown in Figure 5–10(c). By using properties of a segment of a circle, it can be shown that

$$A_{95} = 0.0105D^2 \tag{5–6}$$

Now we can determine the *equivalent diameter*, D_e, for this area by equating equations (5–5) and (5–6) while designating the diameter in equation (5–5) as D_e and then solving for D_e.

$$\begin{aligned} 0.0766D_e^2 &= 0.0105D^2 \\ D_e &= 0.370D \end{aligned} \tag{5–7}$$

FIGURE 5–10
Geometry of sections for computing A_{95} area



This same equation applies to a hollow circular section. The diameter, D_e, can be used in Figure 5–9 or in Table 5–2 to find the size factor.

Rectangular Section in Repeated and Reversed Flexure. The A_{95} area is shown in Figure 5–10(d) as the two strips having a thickness of $0.025h$ at the top and bottom of the section. Therefore,

$$A_{95} = 0.05\ hb$$

Equating this to A_{95} for a circular section gives,

$$\begin{aligned} 0.0766D_e^2 &= 0.05\ hb \\ D_e &= 0.808\sqrt{hb} \end{aligned} \tag{5–8}$$

This diameter can be used in Figure 5–9 or in Table 5–2 to find the size factor.

Other shapes can be analyzed in a similar manner.

Other Factors

The following factors are not included quantitatively in problem solutions in this book because of the difficulty of finding generalized data. However, you should consider each one as you engage in future designs and seek additional data as appropriate.

Flaws. Internal flaws of the material, especially likely in cast parts, are places in which fatigue cracks initiate. Critical parts can be inspected by x-ray techniques for internal flaws. If they are not inspected, a higher-than-average design factor should be specified for cast parts, and a lower endurance strength should be used.

Temperature. Most materials have a lower endurance strength at high temperatures. The reported values are typically for room temperatures. Operation above 500°F (260°C) will reduce the endurance strength of most steels. See Reference 13.

Nonuniform Material Properties. Many materials have different strength properties in different directions because of the manner in which the material was processed. Rolled sheet or bar products are typically stronger in the direction of rolling than they are in the transverse direction. Fatigue tests are likely to have been run on test bars oriented in the stronger direction. Stressing of such materials in the transverse direction may result in lower endurance strength.

Nonuniform properties are also likely to exist in the vicinity of welds because of incomplete weld penetration, slag inclusions, and variations in the geometry of the part at the weld. Also, welding of heat-treated materials may alter the strength of the material because of local annealing near the weld. Some welding processes may result in the production of residual tensile stresses that decrease the effective endurance strength of the material. Annealing or normalizing after welding is often used to relieve these stresses, but the effect of such treatments on the strength of the base material must be considered.

Residual Stresses. Fatigue failures typically initiate at locations of relatively high tensile stress. Any manufacturing process that tends to produce residual tensile stress will decrease the endurance strength of the component. Welding has already been mentioned as a process that may produce residual tensile stress. Grinding and machining, especially with high material removal rates, also cause undesirable residual tensile stresses. Critical areas of cyclically loaded components should be machined or ground in a gentle fashion.

Processes that produce residual *compressive* stresses can prove to be beneficial. Shot blasting and peening are two such methods. *Shot blasting* is performed by directing a high-velocity stream of hardened balls or pellets at the surface to be treated. *Peening* uses a series of hammer blows on the surface. Crankshafts, springs, and other cyclically loaded machine parts can benefit from these methods.

Corrosion and Environmental Factors. Endurance strength data are typically measured with the specimen in air. Operating conditions that expose a component to water, salt solutions, or other corrosive environments can significantly reduce the effective endurance strength. Corrosion may cause harmful local surface roughness and may also alter the internal grain structure and chemistry of the material. Steels exposed to hydrogen are especially affected adversely.

Nitriding. Nitriding is a surface-hardening process for alloy steels in which the material is heated to 950°F (514°C) in a nitrogen atmosphere, typically ammonia gas, followed by slow cooling. Improvement of endurance strength of 50% or more can be achieved with nitriding.

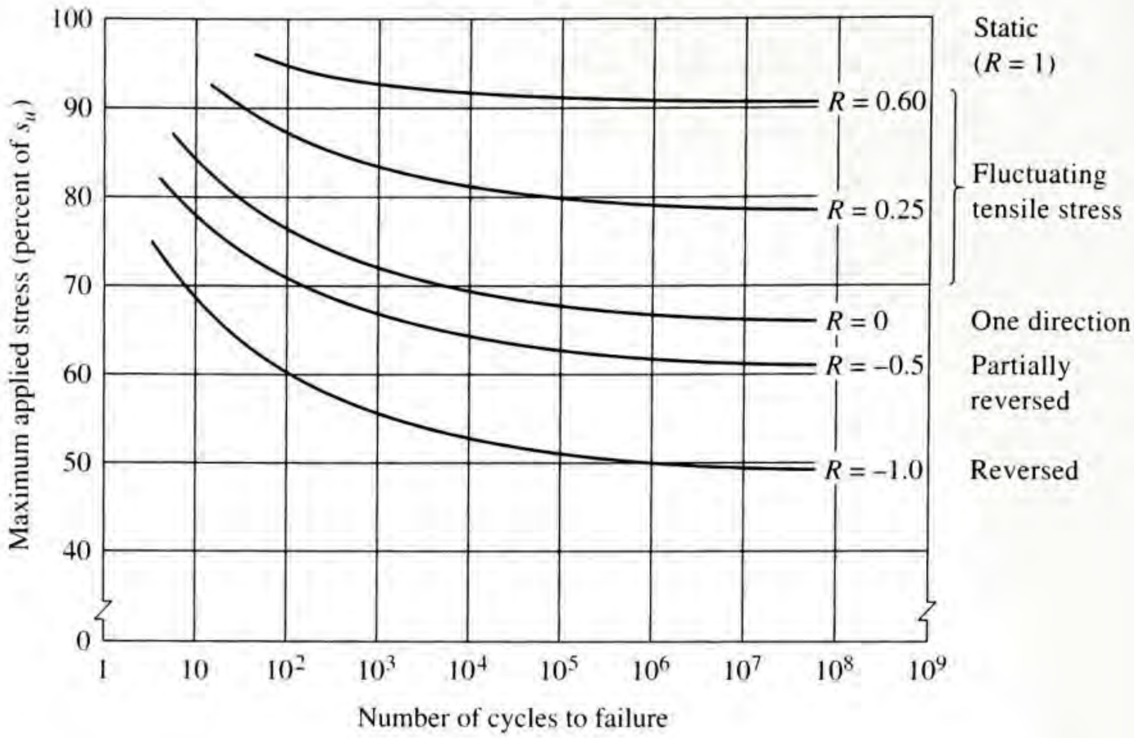


FIGURE 5–11 Effect of stress ratio R on endurance strength of a material

Effect of Stress Ratio on Endurance Strength. Figure 5–11 shows the general variation of endurance-strength data for a given material when the stress ratio R varies from -1.0 to $+1.0$, covering the range of cases including the following:

- Repeated, reversed stress (Figure 5–3); $R = -1.0$
- Partially reversed fluctuating stress with a tensile mean stress [Figure 5–4(b)]; $-1.0 < R < 0$
- Repeated, one-direction tensile stress (Figure 5–6); $R = 0$
- Fluctuating tensile stress [Figure 5–4(a)]; $0 < R < 1.0$
- Static stress (Figure 5–1); $R = 1$

Note that Figure 5–11 is only an example, and it should not be used to determine actual data points. If such data are desired for a particular material, specific data for that material must be found either experimentally or in published literature.

The most damaging kind of stress among those listed is the repeated, reversed stress with $R = -1$. (See Reference 6.) Recall that the rotating shaft in bending as shown in Figure 5–3 is an example of a load-carrying member subjected to a stress ratio $R = -1$.

Fluctuating stresses with a compressive mean stress as shown in Parts (c) and (d) of Figure 5–4 do not significantly affect the endurance strength of the material because fatigue failures tend to originate in regions of tensile stress.

Note that the curves of Figure 5–11 show estimates of the endurance strength, s_n, as a function of the ultimate tensile strength for steel. These data apply to ideal polished specimens and do not include any of the other factors discussed in this section. For example, the curve for $R = -1.0$ (reversed bending) shows that the endurance strength for steel is approximately 0.5 times the ultimate strength ($0.50 \times s_u$) for large numbers of cycles of loading (approximately 10^5 or higher). This is a good general estimate for steels. The chart also shows that types of loads producing R greater than -1.0 but less than 1.0 have less of an effect on the endurance strength. This illustrates that using data from the reversed bending test is the most conservative.

We will not use Figure 5–11 directly for problems in this book because our procedure for estimating the actual endurance strength starts with the use of Figure 5–8, which presents data from reversed bending tests. Therefore, the effect of stress ratio is already included. Section 5–9 includes methods of analysis for loading cases in which the fluctuating stress produces a stress ratio different from $R = -1.0$.

5–5 EXAMPLE PROBLEMS FOR ESTIMATING ACTUAL ENDURANCE STRENGTH

This section shows two examples that demonstrate the application of the *Procedure for Estimating Actual Endurance Strength*, s_n' that is presented in the previous section.

Example Problem 5–2 Estimate the actual endurance strength of AISI 1050 cold-drawn steel when used in a circular shaft subjected to rotating bending only. The shaft will be machined to a diameter of approximately 1.75 in.

Solution

Objective: Compute the estimated actual endurance strength of the shaft material.

Given:

AISI 1050 cold-drawn steel, machined.

Size of section: $D = 1.75$ in.

Type of stress: Reversed, repeated bending.

Analysis: Use the Procedure for Estimating Actual Endurance Strength, s_n'

Step 1: The ultimate tensile strength: $s_u = 100$ ksi from Appendix 3.

Step 2: Diameter is machined.

Step 3: From Figure 5–8, $s_n = 38$ ksi

Step 4: Material factor for wrought steel: $C_m = 1.0$

Step 5: Type-of-stress factor for reversed bending: $C_{st} = 1.0$

Step 6: Specify a desired reliability of 0.99. Then $C_R = 0.81$ (Design decision)

Step 7: Size factor for circular section with $D = 1.75$ in.
From Figure 5–9, $C_s = 0.83$.

Step 8: Use Equation 5–4 to compute the estimated actual endurance strength.

$$s_n' = s_n(C_m)(C_{st})(C_R)(C_s) = 38\text{ ksi}(1.0)(1.0)(0.81)(0.83) = 25.5\text{ ksi}$$

Comments: This is the level of stress that would be expected to produce fatigue failure in a rotating shaft due to the action of reversed bending. It accounts for the basic endurance strength of the wrought AISI 1050 cold-drawn material, the effect of the machined surface, the size of the section, and the desired reliability.

Example Problem 5–3 Estimate the actual endurance strength of cast steel having an ultimate strength of 120 ksi when used in a bar subjected to a reversed, repeated, bending load. The bar will be machined to a rectangular cross section, 1.50 in wide × 2.00 in high.

Solution Objective Compute the estimated actual endurance strength of the bar material.

Given Cast steel, machined: $s_u = 120$ ksi.
Size of section: $b = 1.50$ in., $h = 2.00$ in rectangular
Type of stress: Repeated, reversed bending.

Analysis Use the Procedure for Estimating Actual Endurance Strength s_n'.

Step 1: The ultimate tensile strength is given to be $s_u = 120$ ksi.

Step 2: Surfaces are machined.

Step 3: From Figure 5–8, $s_n = 44$ ksi

Step 4: Material factor for cast steel: $C_m = 0.80$

Step 5: Type-of-stress factor for bending: $C_{st} = 1.00$

Step 6: Specify a desired reliability of 0.99. Then $C_R = 0.81$ (Design decision)

Step 7: Size factor for rectangular section: First use Equation 5–8 to determine the equivalent diameter,

$$D_e = 0.808\sqrt{hb} = 0.808\sqrt{(2.00\text{ in})(1.50\text{ in})} = 1.40\text{ in}$$

Then from Figure 5–9, $C_s = 0.85$.

Step 8: Use Equation 5–4 to compute the estimated actual endurance strength.

$$s_n' = s_n(C_m)(C_{st})(C_R)(C_s) = 44\text{ ksi}(0.80)(1.00)(0.81)(0.85) = 24.2\text{ ksi}$$

5–6 DESIGN PHILOSOPHY

It is the designer's responsibility to ensure that a machine part is safe for operation under reasonably foreseeable conditions. You should evaluate carefully the application in which the component is to be used, the environment in which it will operate, the nature of applied loads, the types of stresses to which the component will be exposed, the type of material to be used, and the degree of confidence you have in your knowledge about the application. Some general considerations are:

1. ***Application.*** Is the component to be produced in large or small quantities? What manufacturing techniques will be used to make the component? What are the consequences of failure in terms of danger to people and economic cost? How cost-sensitive is the design? Are small physical size or low weight important? With what other parts or devices will the component interface? For what life is the component being designed? Will the component be inspected and serviced periodically? How much time and expense for the design effort can be justified?
2. ***Environment.*** To what temperature range will the component be exposed? Will the component be exposed to electrical voltage or current? What is the potential for corrosion? Will the component be inside a housing? Will guarding protect access to the component? Is low noise important? What is the vibration environment?
3. ***Loads.*** Identify the nature of loads applied to the component being designed in as much detail as practical. Consider all modes of operation, including startup, shut down, normal operation, and foreseeable overloads. The loads should be characterized as *static*, *repeated and reversed*, *fluctuating*, *shock*, or *impact* as discussed in Section 5–2. Key magnitudes of loads are the *maximum*, *minimum*, and *mean*.

Variations of loads over time should be documented as completely as practical. Will high mean loads be applied for extended periods of time, particularly at high temperatures, for which creep must be considered? This information will influence the details of the design process.

4. ***Types of Stresses.*** Considering the nature of the loads and the manner of supporting the component, what kinds of stresses will be created: direct tension, direct compression, direct shear, bending, or torsional shear? Will two or more kinds of stresses be applied simultaneously? Are stresses developed in one direction (*uniaxially*), two directions (*biaxially*), or three directions (*triaxially*)? Is buckling likely to occur?
5. ***Material.*** Consider the required material properties of yield strength, ultimate tensile strength, ultimate compressive strength, endurance strength, stiffness, ductility, toughness, creep resistance, corrosion resistance, and others in relation to the application, loads, stresses, and the environment. Will the component be made from a ferrous metal such as plain carbon, alloy, stainless, or structural steel, or cast iron? Or will a nonferrous metal such as aluminum, brass, bronze, titanium, magnesium, or zinc be used? Is the material brittle (percent elongation $< 5\%$) or ductile (percent elongation $> 5\%$)? Ductile materials are highly preferred for components subjected to fatigue, shock, or impact loads. Will plastics be used? Is the application suitable for a composite material? Should you consider other nonmetals such as ceramics or wood? Are thermal or electrical properties of the material important?
6. ***Confidence.*** How reliable are the data for loads, material properties, and stress calculations? Are controls for manufacturing processes adequate to ensure that the component will be produced as designed with regard to dimensional accuracy, surface finish, and final as-made material properties? Will subsequent handling, use, or environmental exposure create damage that can affect the safety or life of the component? These considerations will affect your decision for the design factor, N, to be discussed in the next section.

All design approaches must define the relationship between the applied stresses on a component and the strength of the material from which it is to be made, considering the conditions of service. The strength basis for design can be yield strength in tension, compression, or shear; ultimate strength in tension, compression, or shear; endurance strength; or some combination of these. The goal of the design process is to achieve a suitable *design factor*, N, (sometimes called a *factor of safety*) that ensures the component is safe. That is, the strength of the material must be greater than the applied stresses. Design factors are discussed in the next section.

The sequence of design analysis will be different depending on what has already been specified and what is left to be determined. For example,

1. ***Geometry of the component and the loading are known:*** We apply the desired design factor, N, to the actual expected stress to determine the required strength of the material. Then a suitable material can be specified.
2. ***Loading is known and the material for the component has been specified:*** We compute a *design stress* by applying the desired design factor, N, to the appropriate strength of the material. This is the maximum allowable stress to which any part of the component can be exposed. We can then complete the stress analysis to determine what shape and size of the component will ensure that stresses are safe.
3. ***Loading is known, and the material and the complete geometry of the component have been specified:*** We compute both the expected maximum applied stress and the design stress. By comparing these stresses, we can determine the resulting

design factor, N, for the proposed design and judge its acceptability. A redesign may be called for if the design factor is either too low (unsafe) or too high (over designed).

Practical Considerations. While ensuring that a component is safe, the designer is expected to also make the design practical to produce, considering several factors.

- Each design decision should be tested against the cost of achieving it.
- Material availability must be checked.
- Manufacturing considerations may affect final specifications for overall geometry, dimensions, tolerances, or surface finish.
- In general, components should be as small as practical unless operating conditions call for larger size or weight.
- After computing the minimum acceptable dimension for a feature of a component, standard or preferred sizes should be specified using normal company practice or tables of preferred sizes such as those listed in Appendix 2.
- Before a design is committed to production, tolerances on all dimensions and acceptable surface finishes must be specified so the manufacturing engineer and the production technician can specify suitable manufacturing processes.
- Surface finishes should only be as smooth as required for the function of a particular area of a component, considering appearance, effects on fatigue strength, and whether or not the area mates with another component. Producing smoother surfaces increases cost dramatically. See Chapter 13.
- Tolerances should be as large as possible while maintaining acceptable performance of the component. The cost to produce smaller tolerances rises dramatically. See Chapter 13.
- The final dimensions and tolerances for some features may be affected by the need to mate with other components. Proper clearances and fits must be defined, as discussed in Chapter 13. Another example is the mounting of a commercially available bearing on a shaft for which the bearing manufacturer specifies the nominal size and tolerances for the bearing seat on the shaft. Chapter 16 gives guidelines for clearances between the moving and stationary parts where either boundary or hydrodynamic lubrication is used.
- Will any feature of the component be subsequently painted or plated, affecting the final dimensions?

Deformations. Machine elements can also fail because of excessive deformation or vibration. From your study of strength of materials, you should be able to compute deformations due to axial tensile or compressive loads, bending, torsion, or changes in temperature. Some of the basic concepts are reviewed in Chapter 3. For more complex shapes or loading patterns, computer-based analysis techniques such as finite element analysis (FEA) or beam analysis software are important aids.

Criteria for failure due to deformation are often highly dependent on the machine's use. Will excessive deformation cause two or more members to touch when they should not? Will the desired precision of the machine be compromised? Will the part look or feel too flexible (flimsy)? Will parts vibrate excessively or resonate at the frequencies experienced during operation? Will rotating shafts exhibit a critical speed during operation, resulting in wild oscillations of parts carried by the shaft?

This chapter will not pursue the quantitative analysis of deformation, leaving that to be your responsibility as the design of a machine evolves. Later chapters do address some critical cases such as the interference fit between two mating parts (Chapter 13), the position of the teeth of one gear relative to its mating gear (Chapter 9), the radial clearance between a journal bearing and the shaft rotating within it (Chapter 16), and the deformation of springs (Chapter 19). Also, Section 5–10, as a part of the general design procedure, suggests some guidelines for limiting deflections.

5–7 DESIGN FACTORS

The term *design factor*, N, is a measure of the relative safety of a load-carrying component. In most cases, the strength of the material from which the component is to be made is divided by the design factor to determine a *design stress*, σ_d, sometimes called the *allowable stress*. Then the actual stress to which the component is subjected should be less than the design stress. For some kinds of loading, it is more convenient to set up a relationship from which the design factor, N, can be computed from the actual applied stresses and the strength of the material. Still in other cases, particularly for the case of the buckling of columns, as discussed in Chapter 6, the design factor is applied to the *load* on the column rather than the strength of the material.

Section 5–9 presents methods for computing the design stress or design factor for several different kinds of loading and materials.

The designer must determine what a reasonable value for the design factor should be in any given situation. Often the value of the design factor or the design stress is governed by codes established by standards-setting organizations such as the American Society of Mechanical Engineers, the American Gear Manufacturers Association, the U.S. Department of Defense, the Aluminum Association, or the American Institute of Steel Construction. For structures, local or state building codes often prescribe design factors or design stresses. Some companies have adopted their own policies specifying design factors based on past experience with similar conditions.

In the absence of codes or standards, the designer must use judgment to specify the desired design factor. Part of the design philosophy, discussed in Section 5–6, discussed issues such as the nature of the application, environment, nature of the loads on the component to be designed, stress analysis, material properties, and the degree of confidence in data used in the design processes. All of these considerations affect the decision about what value for the design factor is appropriate. This book will use the following guidelines.

Ductile Materials

1. **N = 1.25 to 2.0.** Design of structures under static loads for which there is a high level of confidence in all design data.
2. **N = 2.0 to 2.5.** Design of machine elements under dynamic loading with average confidence in all design data. (Typically used in problem solutions in this book.)
3. **N = 2.5 to 4.0.** Design of static structures or machine elements under dynamic loading with uncertainty about loads, material properties, stress analysis, or the environment.
4. **N = 4.0 or higher.** Design of static structures or machine elements under dynamic loading with uncertainty about some combination of loads, material properties, stress analysis, or the environment. The desire to provide extra safety to critical components may also justify these values.

Brittle Materials

5. **N = 3.0 to 4.0.** Design of structures under static loads for which there is a high level of confidence in all design data.
6. **N = 4.0 to 8.0.** Design of static structures or machine elements under dynamic loading with uncertainty about loads, material properties, stress analysis, or the environment.

The following Sections 5–8 and 5–9 provide guidance on the introduction of the design factor into the design process with particular attention to the selection of the strength basis for the design and the computation of the design stress. In general, design for static loading involves applying the design factor to the yield strength or ultimate strength of the material. Dynamic loading requires the application of the design factor to the endurance strength using the methods described in Section 5–5 to estimate the actual endurance strength for the conditions under which the component is operating.

5–8 PREDICTIONS OF FAILURE

Designers should understand the various ways that load-carrying components can fail in order to complete a design that ensures that failure *does not occur*. Several different methods of predicting failure are available, and it is the designer's responsibility to select the one most appropriate to the conditions of the project. In this section we describe the methods that have found a high level of use in the field and discuss the situations in which each is applicable. The factors involved are the nature of the load (static, repeated and reversed, or fluctuating), the type of material involved (ductile or brittle), and the amount of design effort and analysis that can be justified by the nature of the component or product being designed.

The design analysis methods described in the following Section 5–9 define the relationship between the applied stresses on a component and the strength of the material from which it is to be made that is most relevant to the conditions of service. The strength basis for design can be yield strength, ultimate strength, endurance strength, or some combination of these. The goal of the design process is to achieve a suitable design factor, N, that ensures that the component is safe. That is, the strength of the material must be greater than the applied stresses.

The following types of failure prediction are described in this section. Reference 12 gives an excellent historical review of failure prediction and complete derivations of the fundamentals underlying the methods discussed here.

Failure Prediction Method	*Uses*
1. Maximum normal stress	Uniaxial static stress on brittle materials
2. Modified Mohr	Biaxial static stress on brittle materials
3. Yield strength	Uniaxial static stress on ductile materials
4. Maximum shear stress	Biaxial static stress on ductile materials [Moderately conservative]
5. Distortion energy	Biaxial or triaxial stress on ductile materials [Good predictor]
6. Goodman	Fluctuating stress on ductile materials [Slightly conservative]
7. Gerber	Fluctuating stress on ductile materials [Good predictor]
8. Soderberg	Fluctuating stress on ductile materials [Moderately conservative]

Maximum Normal Stress Method for Uniaxial Static Stress on Brittle Materials

The maximum normal stress theory states that a material will fracture when the maximum normal stress (either tension or compression) exceeds the ultimate strength of the material as obtained from a standard tensile or compressive test. Its use is limited, namely for brittle materials under pure uniaxial static tension or compression. When applying this theory, any stress concentration factor at the region of interest should be applied to the computed stress because brittle materials do not yield and therefore cannot redistribute the increased stress.

The following equations apply the maximum normal stress theory to design.

For tensile stress: $K_t\sigma < \sigma_d = s_{ut}/N$ **(5–9)**

For compressive stress: $K_t\sigma < \sigma_d = s_{uc}/N$ **(5–10)**

Note that many brittle materials such as gray cast iron have a significantly higher compressive strength than tensile strength.

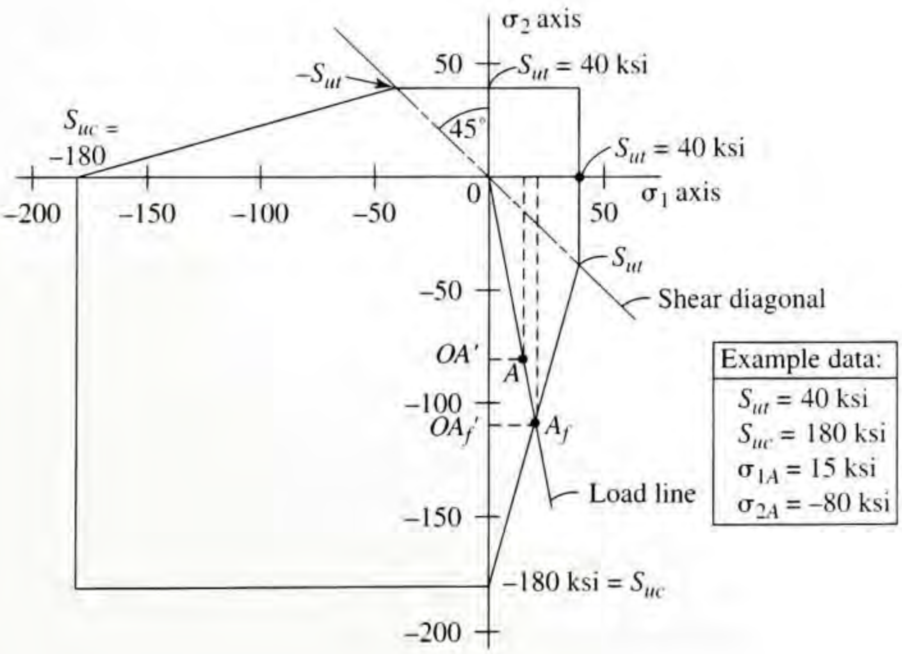
Modified Mohr Method for Biaxial Static Stress on Brittle Materials

When stresses are applied in more that one direction or when normal stress and shear stress are applied simultaneously, it is necessary to compute the principal stresses, σ_1 and σ_2, using Mohr's circle or the equations in Chapter 4. *Stress concentrations should be included in the applied stresses before preparing Mohr's circle for brittle materials.*

For safety, the *combination* of the two principal stresses must lie within the area shown in Figure 5–12 that graphically depicts the *modified Mohr theory*. The graph is a plot of the maximum principal stress, σ_1, on the horizontal axis (abscissa) and the minimum principal stress, σ_2, on the vertical axis (ordinate).

Note that the failure criteria depend on the quadrant in which the principal stresses lie. In the first quadrant, both principal stresses are tensile, and failure is predicted when either one exceeds the ultimate tensile strength, s_{ut}, of the material. Similarly, in the third quadrant, both principal stresses are compressive, and failure is predicted when either one exceeds the ultimate compressive strength, s_{uc}, of the material. The failure lines for the second and fourth quadrants are more complex and have been derived semi-empirically to correlate with test data. The

FIGURE 5–12
Modified Mohr diagram with example data and a load line plotted



ultimate tensile strength lines are extended from the first quadrant into the second and fourth quadrants to the point where each intersects the *shear diagonal*, drawn at 45 degrees through the origin. Then the failure line proceeds at an angle to the ultimate compressive strength.

For design, because of the many different shapes and dimensions of safe-stress zones in Figure 5–12, it is suggested that a rough plot be made of the pertinent part of the modified Mohr diagram from actual material strength data. Then the actual values of σ_1 and σ_2 can be plotted to ensure that they lie within the safe zone of the diagram.

A *load line* can be an aid in determining the design factor, N, using the modified Mohr diagram. The assumption is made that stresses increase proportionally as loads increase. Apply the following steps for an example stress state, A, for which $\sigma_{1A} = 15$ ksi and $\sigma_{2A} = -80$ ksi. The material is Grade 40 gray cast iron having $s_{ut} = 40$ ksi and $s_{uc} = 180$ ksi.

1. Draw the modified Mohr diagram as shown in Figure 5–12.
2. Plot point A at $(15, -80)$.
3. Draw the load line from the origin through point A until it intersects the failure line on the diagram at the point labeled A_f.
4. Determine the distances $OA = 81.4$ ksi and $OA_f = 112$ ksi by scaling the diagram.
5. Compute the design factor from $N = OA_f/OA = 112/81.4 = 1.38$.
6. Alternatively, the projections of points A and A_f on the σ_1 or σ_2 axes could be used because the value of N is a ratio and similar triangles are formed as shown in Figure 5–12.
7. In this example, the projections onto the σ_2 axis are: $OA' = -80$ ksi, $OA'_f = -110$ ksi. Then,

$$N = OA'_f/OA' = -110/-80.0 = 1.38.$$

Yield Strength Method for Uniaxial Static Normal Stresses on Ductile Materials

This is a simple application of the principle of yielding in which a component is carrying a direct tensile or compressive load in the manner similar to the conditions of the standard tensile or compressive test for the material. Failure is predicted when the actual applied stress exceeds the yield strength. Stress concentrations can normally be neglected for static stresses on ductile materials because the higher stresses near the stress concentrations are highly localized. When the local stress on a small part of the component reaches the yield strength of the material, it does in fact yield. In the process, the stress is redistributed to other areas and the component is still safe.

The following equations apply the yield strength principle to design.

For tensile stress: $\sigma < \sigma_d = s_{yt}/N$ **(5–11)**

For compressive stress: $\sigma < \sigma_d = s_{yc}/N$ **(5–12)**

For most wrought ductile metals, $s_{yt} = s_{yc}$.

Maximum Shear Stress Method for Biaxial Static Stress on Ductile Materials

The maximum shear stress method of failure prediction states that a ductile material begins to yield when the maximum shear stress in a load-carrying component exceeds that in a tensile-test specimen when yielding begins. A Mohr's circle analysis for the uniaxial tension test, discussed in Section 4–6, shows that the maximum shear stress is one-half of the applied tensile stress. At yield, then, $s_{sy} = s_y/2$. We use this approach in this book to estimate s_{sy}. Then, for design, use

$$\tau_{max} < \tau_d = s_{sy}/N = 0.5\, s_y/N \tag{5–13}$$

The maximum shear stress method of failure prediction has been shown by experimentation to be somewhat conservative for ductile materials subjected to a combination of normal and shear stresses. It is relatively easy to use and is often chosen by designers. For more precise analysis, the distortion energy method is preferred.

Distortion Energy Method for Static Biaxial or Triaxial Stress on Ductile Materials

The distortion energy method has been shown to be the best predictor of failure for ductile materials under static loads or completely reversed normal, shear, or combined stresses. It requires the definition of the new term, *von Mises stress*, indicated by the symbol, σ', that can be calculated for biaxial stresses, given the maximum and minimum principal stresses, σ_1 and σ_2, from

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \tag{5–14}$$

Failure is predicted when $\sigma' > s_y$. The biaxial stress approach requires that the applied stress in the third orthogonal direction, typically σ_z, is zero.

Credit is given to R. von Mises for the development of Equation 5–14 in 1913. Because of additional contributions by H. Hencky in 1925, the method is sometimes called the *von Mises-Hencky method*. Be aware that the results from many finite element analysis software packages include the von Mises stress. Another term used is the *octahedral shear stress*.

It is helpful to visualize the distortion energy failure prediction method by plotting a failure line on a graph with σ_1 on the horizontal axis and σ_2 on the vertical axis as shown in Figure 5–13. The failure line is an ellipse centered at the origin and passing through the yield strength on each axis, in both the tensile and compressive regions. It is necessary that the material has equal values for yield strength in tension and compression for direct use of this method. The numerical scales on the graph are normalized to the yield strength so the ellipse passes through $s_y/\sigma_1 = 1.0$ on the σ_1 axis and similarly on the other axes. *Combinations of principal stresses that lie within the distortion energy ellipse are predicted to be safe, while those outside would predict failure.*

For design, the design factor, N, can be applied to the yield strength. Then use

$$\sigma' < \sigma_d = s_y/N \tag{5–15}$$

For comparison, the failure prediction lines for the maximum shear stress method are shown also in Figure 5–13. With data showing that the distortion energy method is the best predictor, it can be seen that the maximum shear stress method is generally conservative and that it coincides with the distortion energy ellipse at six points. In other regions, it is as much as 16% low. Note the 45° diagonal line through the second and fourth quadrants, called the *shear diagonal*. It is the locus of points where $\sigma_1 = \sigma_2$ and its intersection with the failure ellipse is at the point $(-0.577, 0.577)$ in the second quadrant. This predicts failure when the shear stress is $0.577s_y$. The maximum shear stress method predicts failure at $0.50s_y$, thus quantifying the conservatism of the maximum shear stress method.

Also shown in Figure 5–13 are the failure prediction lines for the maximum principal stress method. It is coincident with the maximum shear stress lines in the first and third quadrants for which the two principal stresses have the same sign, either tensile (+) or

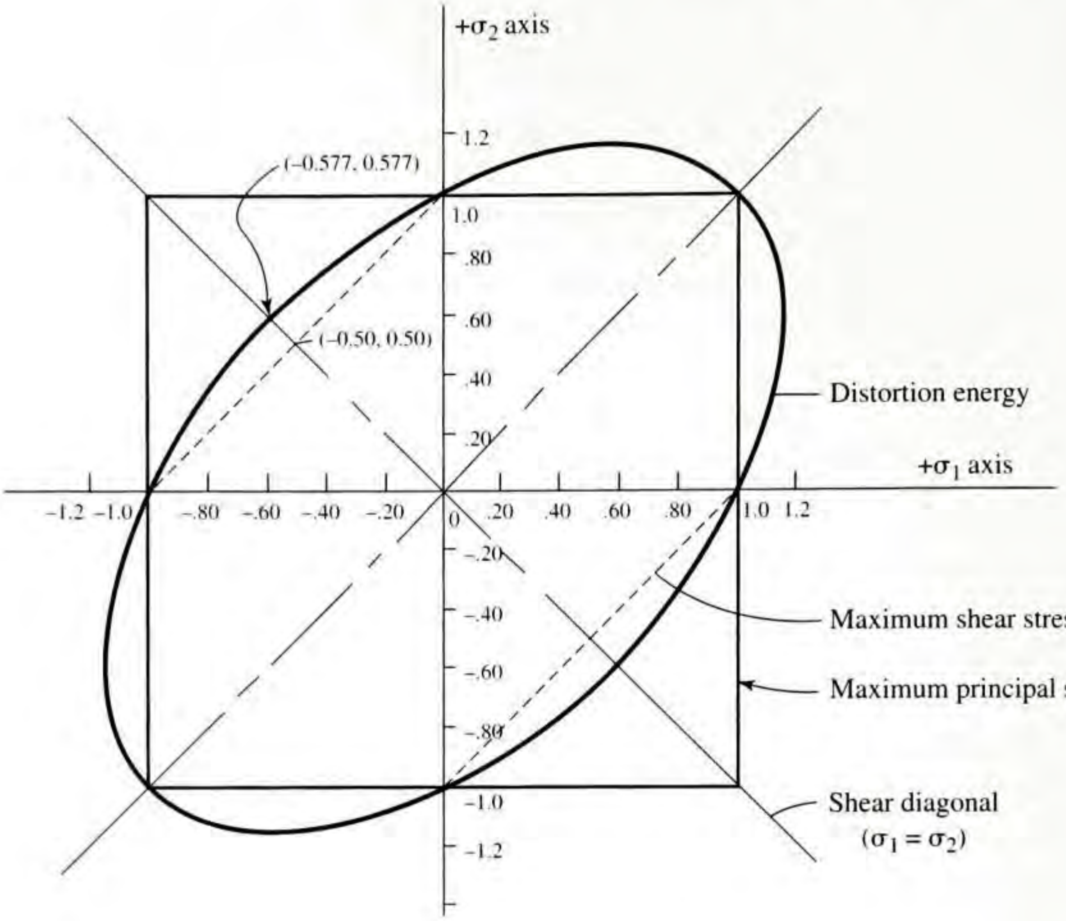


FIGURE 5–13 Distortion energy method compared with maximum shear stress and maximum principal stress methods

compressive (−). Therefore, it, too, is conservative in these regions. But note that it is dangerously nonconservative in the second and fourth quadrants.

Alternate Form for the von Mises Stress. Equation 5–14 requires that the two principal stresses be determined from Mohr's circle, equations 4–1 and 4–2, or from a finite element analysis. Often you will first determine the stresses in some convenient orthogonal directions, x and y, namely σ_x, σ_y, and τ_{xy}. The von Mises stress can then be calculated directly from

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \tag{5–16}$$

For uniaxial stress with shear, $\sigma_y = 0$, Equation (5–16) reduces to

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \tag{5–17}$$

Triaxial Distortion Energy Method. A more general expression of the von Mises (distortion energy) stress is required when principal stresses occur in all three directions, σ_1, σ_2, and σ_3. We normally order these stresses such that $\sigma_1 > \sigma_2 > \sigma_3$. Then,

$$\sigma' = \left(\sqrt{2}/2\right)\left[\sqrt{(\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2}\right] \tag{5–18}$$

Goodman Method for Fatigue Under Fluctuating Stress on Ductile Materials

Recall from Section 5–2 that the term *fluctuating stress* refers to the condition in which a load-carrying component is subjected to a nonzero mean stress with an alternating stress superimposed on the mean stress (see Figure 5–4). The Goodman method of failure pre-

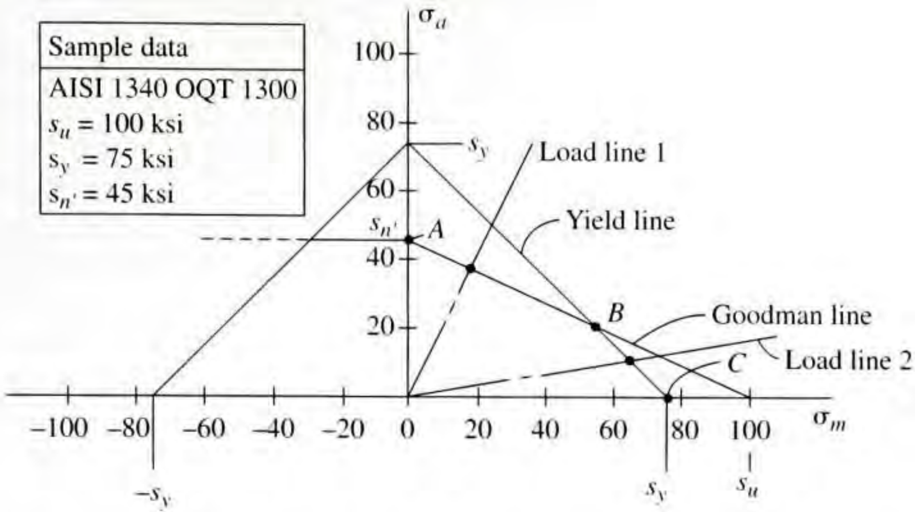


FIGURE 5–14 Modified Goodman diagram for fatigue of ductile materials

diction, sketched in Figure 5–14, has been shown to provide a good correlation with experimental data, falling just slightly below the scatter of data points.

The Goodman diagram plots the mean stresses on the horizontal axis and the alternating stresses on the vertical axis. Look first at the right part of the diagram representing fluctuating stresses with a tensile (+) mean stress. A straight line is drawn from the estimated actual endurance strength of the material, s_n', on the vertical axis to the ultimate tensile strength, s_u, on the horizontal axis. Combinations of mean stress, σ_m, and alternating stress, σ_a, above the line predict failure, while those below the line predict no failure from fatigue. The equation for the Goodman line is,

$$\frac{\sigma_a}{s_n'} + \frac{\sigma_m}{s_u} = 1 \tag{5–19}$$

Design Equation. We can modify Equation 5–`th the ultimate and endurance strength values, as shown in Figure 5–15, to depict a "safe stress" line. Furthermore, any stress concentration factor in the region of interest should be applied to the alternating component but not to the mean stress component, because experimental evidence shows that the presence of a stress concentration does not affect the contribution of the mean stress of fatigue failure. Making these adjustments to the equation for the Goodman line gives,

$$\frac{K_t\sigma_a}{s_n'} + \frac{\sigma_m}{s_u} = \frac{1}{N} \tag{5–20}$$

This is the design equation we will use in this book for fluctuating stresses.

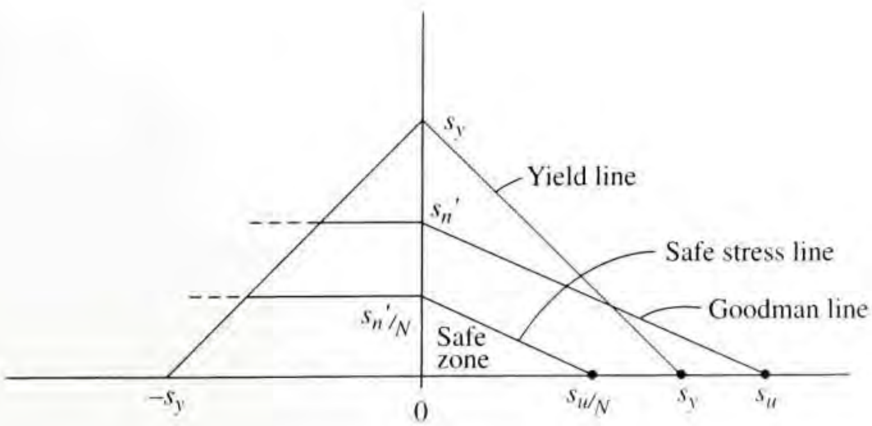


FIGURE 5–15 Modified Goodman diagram showing safe stress line

Checking for Early Cycle Yielding. The Goodman line presents a difficulty near the right end because it seems to allow a pure mean stress greater than the yield strength of the material. Furthermore, as some value of alternating stress is added to the mean stress, the actual maximum stress ranges above the mean and may cause yielding. From a pure fatigue consideration, this condition is acceptable, provided the application can tolerate some local yielding in areas of high maximum stress. Any yielding would occur within the early cycles of loading, perhaps from the first cycle and certainly at less than 1000 cycles. After yielding, the stresses would be redistributed and the component would continue to be safe.

However, most designers choose to *not* permit yielding anywhere. To accomplish this, the *yield line* is added to the Goodman diagram, drawn between the yield strength plotted on both axes. Now the line segments between points labeled A, B, and C define the failure line. Consider two load lines drawn from the origin and extended through intersections with all of the failure lines on the diagram. Load line 1 intersects the Goodman line first, indicating that fatigue failure governs. Load line 2 intersects the yield line first and failure would commence as yielding.

We recommend completing first the design based on fatigue using Equation 5–20 and then checking for yielding separately. The design equation for the yield line is,

$$\frac{K_t\sigma_a}{s_y} + \frac{K_t\sigma_m}{s_y} = \frac{1}{N} \tag{5–21}$$

Here we do apply the stress concentration factor to the mean stress to ensure that yielding does not occur. In many cases, the safe stress line for fatigue actually falls completely below the yield strength line, indicating that no yielding is expected. See Figure 5–15. However, there may be a lower effective design factor for yielding than on fatigue failure, and you will need to judge whether this is acceptable or not. Equation 5–21 can be solved for N based on yielding, giving,

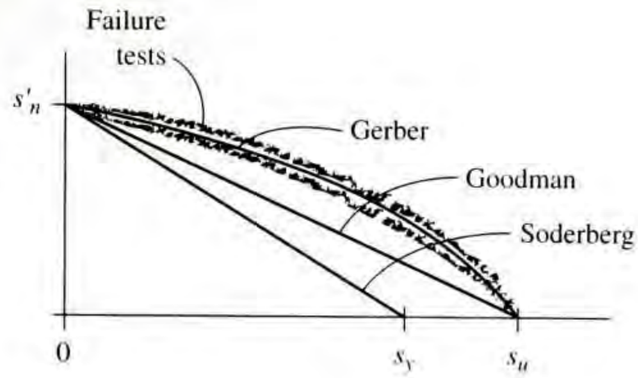
$$N = \frac{s_y}{K_t\,(\sigma_a + \sigma_m)} \tag{5–22}$$

Fluctuating Stresses with Compressive Mean Stress. The left part of the Goodman diagram represents fluctuating stresses with compressive (−) stresses. Experimental data show that the presence of compressive mean stress does not significantly degrade the fatigue life beyond that predicted by the alternating stress only. So the failure line extends horizontally to the left from the s_n' point on the alternating stress axis. Its limit is the yield line for compressive yielding.

Gerber Method for Fluctuating Stress on Ductile Materials

Those interested in a more precise predictor of fatigue failure propose the Gerber method, shown in Figure 5–16. The Goodman line is shown for comparison. The end points of each are the same, but the Gerber line is parabolic and follows generally among the experimentally determined failure points, whereas the Goodman line lies below them. (See References 11 to 13.) This means that some failure points will lie below the Gerber line, an undesirable result. For this reason, we will use the Goodman line for problem solutions in this book.

FIGURE 5–16
Comparison of Gerber, Goodman, and Soderberg methods for fluctuating stresses on ductile materials



The equation of the Gerber line is,

$$\frac{\sigma_a}{s'_n} + \left[\frac{\sigma_m}{s_u}\right]^2 = 1 \tag{5–23}$$

Soderberg Method for Fluctuating Stress on Ductile Materials

Another approach that has found significant use, and that was featured in earlier editions of this book, is called the *Soderberg method*. Figure 5–16 shows the Soderberg failure line in comparison with the Goodman and Gerber lines. The equation of the Soderberg line is,

$$\frac{K_t\sigma_a}{s'_n} + \frac{\sigma_m}{s_y} = 1 \tag{5–24}$$

Drawn between the endurance strength and the yield strength, the Soderberg line is the most conservative of the three. One advantage of the Soderberg line is that it protects directly against early cycle yielding, whereas the Goodman and Gerber methods require the secondary consideration of the yield line as discussed previously. However, the degree of conservatism is considered too great for competitive efficient design.

In summary, problem solving in this book will use the Goodman method for fluctuating stresses on ductile materials. It is only slightly conservative and its failure prediction line lies completely below the array of experimental failure data points.

5–9 DESIGN ANALYSIS METHODS

Here we summarize the recommended methods for design analysis based on the type of material (brittle or ductile), the nature of the loading (static or cyclical), and the type of stress (uniaxial or biaxial). The fact that 16 different cases are discussed is an indication of the large variety of approaches available. As you read about each case, refer to Figure 5–17 to follow the relationships among the factors to be considered.

For cases C, E, F, and I, which involve ductile materials under four different types of loading, both the maximum shear stress and distortion energy methods are included. Recall from the discussions in the previous section that the maximum shear stress method is the simpler to use but somewhat conservative. The distortion energy method is the most accurate predictor of failure, but it requires the additional step of computing the von Mises stress. Both methods will be illustrated in the example problems in this book; the distortion energy method is recommended.

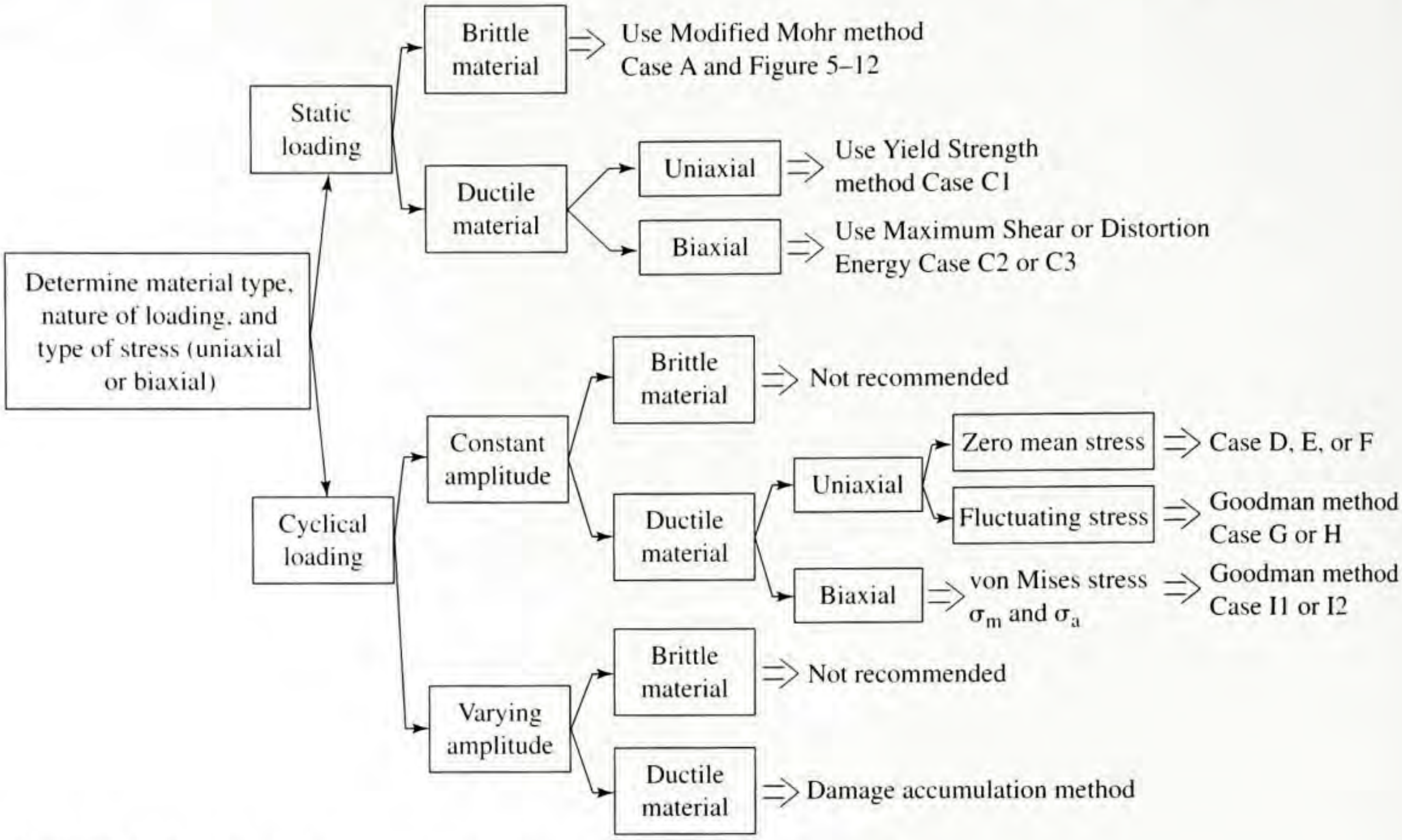


FIGURE 5–17 Logic diagram for visualizing methods of design analysis

Figure 5–17 includes a listing for the *damage accumulation method* for when ductile materials are subjected to cyclical loading with varying amplitude. This topic is discussed in Section 5–13.

The following symbols are used in the various cases.

s_u or s_{ut} = ultimate tensile strength
s_{uc} = ultimate compressive strength
s_y = yield strength or yield point
s_{sy} = yield strength in shear
s'_n = endurance strength of material under actual conditions
s'_{sn} = endurance strength in shear under actual conditions
σ = nominal applied stress, without K_t

Case A: Brittle Materials under Static Loads

When the actual applied stress, σ, is simple tension or compression in only one direction, use the maximum normal stress theory of failure. Because brittle materials do not yield, you should always apply stress concentration factors when computing the applied stress.

Case A1: Uniaxial Tensile Stress

$$K_t\sigma < \sigma_d = s_{ut}/N \tag{5–9}$$

Case A2: Uniaxial Compressive Stress

$$K_t\sigma < \sigma_d = s_{uc}/N \tag{5–10}$$

Case A3: Biaxial Stress. Use Mohr's circle to determine the principal stresses, σ_1 and σ_2. If both principal stresses are of the same sign, either tensile or compressive, use Case A1 or A2. If they are of different signs, use the modified Mohr method described in the preceding section and illustrated in Figure 5–12. Any stress concentration factors should be applied to the computed nominal stresses.

Case B: Brittle Materials under Fatigue Loads

No specific recommendation will be given for brittle materials under fatigue loads because it is usually not desirable to use a brittle material in such cases. When it is necessary to do so, testing should be done to ensure safety under actual conditions of service.

Case C: Ductile Materials under Static Loads

Three failure methods are listed. The yield strength method is only for uniaxial normal stresses. For shear or biaxial loads, the maximum shear stress method is simpler but somewhat conservative. The distortion energy method is the best failure predictor.

C1: Yield Strength Method for Uniaxial Static Normal Stresses

For tensile stress: $$\sigma < \sigma_d = s_{yt}/N \tag{5–11}$$

For compressive stress: $$\sigma < \sigma_d = s_{yc}/N \tag{5–12}$$

C2: Maximum Shear Stress Method. Used for shear stresses and combined stresses. Determine the maximum shear stress from Mohr's circle. Then the design equation is,

$$\tau_{max} < \tau_d = s_{sy}/N = 0.50\, s_y/N \tag{5–13}$$

C3: Distortion Energy Method. Used for shear stresses and combined stresses. Determine the maximum shear stress from Mohr's circle. Then compute the von Mises stress from,

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \tag{5–14}$$

The alternate equations (5–16), (5–17), or (5–18) from the preceding section can also be used. For design use,

$$\sigma' < \sigma_d = s_y/N \tag{5–15}$$

Stress concentrations are not needed for static loading if local yielding can be tolerated.

Case D: Reversed, Repeated Normal Stress

Figure 5–2 shows the general form of reversed, repeated normal stress. Note that the mean stress, σ_m, is zero and that the alternating stress, σ_a, is equal to the maximum stress, σ_{max}.

This case follows directly from the definition of the estimated actual endurance strength because the rotating beam testing method is used to acquire the strength data. Also, it is a special case of fluctuating stress, covered by Equation 5–20 in the preceding section. With a zero mean stress, the design equation becomes,

$$K_t\sigma_{\max} < \sigma_d = s_n'/N \tag{5–25}$$

Case E: Reversed, Repeated Shear Stress

Again the maximum shear stress theory or the distortion energy theory can be used. First compute the maximum repeated shear stress, $\tau_{\max}$, including any stress concentration factor. The discussion for Case D applies for shear stress as well.

Case E1: Maximum Shear Stress Theory

$$s_{sn}' = 0.5\, s_n' \text{ (estimate for endurance strength in shear)}$$
$$K_t\tau_{\max} < \tau_d = s_{sn}'/N = 0.5\, s_n'/N \tag{5–26}$$

Case E2: Distortion Energy Theory

$$s_{sn}' = 0.577\, s_n' \text{ (estimate for endurance strength in shear)}$$
$$K_t\tau_{\max} < \tau_d = s_{sn}'/N = 0.577\, s_n/N \tag{5–27}$$

Case F: Reversed Combined Stress

Use Mohr's circle to find the maximum shear stress and the two principal stresses by using the maximum values of the applied stresses.

Case F1: Maximum Shear Stress Theory. Use Equation 5–26.

Case F2: Distortion Energy Theory. Use Equation 5–27.

Case G: Fluctuating Normal Stresses: The Goodman Method

Use the Goodman method that was described in Section 5–8 and illustrated in Figure 5–15. A satisfactory design results if the combination of the mean stress and the alternating stress produces a point in the *safe zone* shown in Figure 5–15. Then you can use the Equation (5–20) to evaluate the design factor for fluctuating loads:

$$\frac{K_t\sigma_a}{s_n'} + \frac{\sigma_m}{s_u} = \frac{1}{N} \tag{5–20}$$

Case H: Fluctuating Shear Stresses

The preceding development of the Goodman method can also be done for fluctuating shear stresses instead of normal stresses. The design factor equation would then be

$$\frac{K_t\tau_a}{s_{sn}'} + \frac{\tau_m}{s_{su}} = \frac{1}{N} \tag{5–28}$$

In the absence of shear strength data, use the estimates, $s_{sn}' = 0.577\, s_n'$ and $s_{su} = 0.75\, s_u$.

Case I: Fluctuating Combined Stresses

The approach presented here is similar to the Goodman method described previously, but the effect of the combined stresses is first determined by using Mohr's circle.

Case I1. For the maximum shear stress theory, draw two Mohr's circles, one for the mean stresses and one for the alternating stresses. From the first circle, determine maximum mean shear stress, $(\tau_m)_{max}$. From the second circle, determine the maximum alternating shear stress, $(\tau_a)_{max}$. Then use these values in the design equation

$$\frac{K_t\,(\tau_a)_{max}}{s'_{sn}} + \frac{(\tau_m)_{max}}{s_{su}} = \frac{1}{N} \tag{5–29}$$

In the absence of shear strength data, use the estimates, $s'_{sn} = 0.577\, s'_n$ and $s_{su} = 0.75\, s_u$.

Case I2. For the distortion energy theory, draw two Mohr's circles, one for the mean stresses and one for the alternating stresses. From these circles, determine the maximum and minimum principal stresses. Then compute the von Mises stresses for both the mean and the alternating components from

$$\sigma'_m = \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}}$$
$$\sigma'_a = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a}}$$

The Goodman equation then becomes

$$\frac{K_t\sigma'_a}{s'_n} + \frac{\sigma'_m}{s_u} = \frac{1}{N} \tag{5–30}$$

5–10 GENERAL DESIGN PROCEDURE

The earlier parts of this chapter have provided guidance related to the many factors involved in design of machine elements that must be safe when carrying the applied loads. This section brings these factors together so that you can complete the design. The general design procedure described here is meant to give you a feel for the process. It is not practical to provide a completely general procedure. You will have to adapt it to the specific situations that you encounter.

The procedure is set up assuming that the following factors are known or can be specified or estimated:

- General design requirements: Objectives and limitations on size, shape, weight, desired precision, and so forth.
- Nature of the loads to be carried.
- Types of stresses produced by the loads.
- Type of material from which the element is to be made.
- General description of the manufacturing process to be used, particularly with regard to the surface finish that will be produced.
- Desired reliability.

General Design Procedure

1. Specify the objectives and limitations, if any, of the design, including desired life, size, shape, and appearance.
2. Determine the environment in which the element will be placed, considering such factors as corrosion potential and temperature.
3. Determine the nature and characteristics of the loads to be carried by the element, such as

   Static, dead, slowly applied loads.

   Dynamic, live, varying, repeated loads that may potentially cause fatigue failure.

   Shock or impact loads.
4. Determine the magnitudes for the loads and the operating conditions, such as

   Maximum expected load.

   Minimum expected load.

   Mean and alternating levels for fluctuating loads.

   Frequency of load application and repetition.

   Expected number of cycles of loading.
5. Analyze how loads are to be applied to determine the type of stresses produced, such as

   Direct normal stress, bending stress, direct shear stress, torsional shear stress, or some combination of stresses.
6. Propose the basic geometry for the element, paying particular attention to

   Its ability to carry the applied loads safely.

   Its ability to transmit loads to appropriate support points. Consider the *load paths*.

   The use of efficient shapes according to the nature of the loads and the types of stresses encountered. This applies to the general shape of the element and to each of its cross sections. Achieving efficiency involves optimizing the amount and the type of material involved. In Chapter 20, Section 20–2 gives some suggestions for efficient design of frames and members in bending and torsion.

   Providing appropriate attachments to supports and to other elements in the machine or structure.

   Providing for the positive location of other components that may be installed on the element being designed. This may call for shoulders, grooves, holes, retaining rings, keys and keyseats, pins, or other means of fastening or holding parts.
7. Propose the method of manufacturing the element with particular attention to the precision required for various features and the surface finish that is desired. Will it be cast, machined, ground, or polished, or produced by some other process? These design decisions have important impacts on the performance of the element, its ability to withstand fatigue loading, and the cost to produce it.
8. Specify the material from which the element is to be made, along with its condition. For metals the specific alloy should be specified, and the condition could include such processing factors as hot rolling, cold drawing, and a specific heat treatment. For nonmetals, it is often necessary to consult with vendors to specify the composition and the mechanical and physical properties of the desired material. Consult Chapter 2 and Section 20–2 in Chapter 20 for additional guidance.

9. Determine the expected properties of the selected material, for example

Ultimate tensile strength, s_u.

Ultimate compressive strength, s_{uc}, if appropriate.

Yield strength, s_y.

Ductility as represented by percent elongation.

Stiffness as represented by modulus of elasticity, E or G.

10. Specify an appropriate design factor, N, for the stress analysis using the guidelines discussed in Section 5–7.

11. Determine which stress analysis method outlined in Section 5–9 applies to the design being completed.

12. Compute the appropriate design stress for use in the stress analysis. If fatigue loading is involved, the actual expected endurance strength of the material should be computed as outlined in Section 5–4. This requires the consideration of the expected size of the section, the type of material to be used, the nature of the stress, and the desired reliability. Because the size of the section is typically unknown at the start of the design process, an estimate must be made to allow the inclusion of a reasonable size factor, C_s. You should check the estimate at the end of the design process to verify that reasonable values were assumed at this stage of the design.

13. Determine the nature of any stress concentrations that may exist in the design at places where geometry changes occur. Stress analysis should be considered at all such places because of the likelihood of localized high tensile stresses that may produce fatigue failure. If the geometry of the element in these areas is known, determine the appropriate stress concentration factor, K_t. If the geometry is not yet known, it is advisable to estimate the expected magnitude of K_t. The estimate must then be checked at the end of the design process.

14. Complete the required stress analyses at all points where the stress may be high and at changes of cross section to determine the minimum acceptable dimensions for critical areas.

15. Specify suitable, convenient dimensions for all features of the element. Many design decisions are required, such as

The use of preferred basic sizes as listed in Table A2–1.

The size of any part that will be installed on or attached to the element being analyzed. Examples of this are shown in Chapter 12 on shaft design where gears, chain sprockets, bearings, and other elements are to be installed on the shafts. But many machine elements have similar needs to accommodate mating elements.

Elements should not be significantly oversized without good reason in order to achieve an efficient overall design.

Sometimes the manufacturing process to be used has an effect on the dimensions. For example, a company may have a preferred set of cutting tools for use in producing the elements. Casting, rolling, or molding processes often have limitations on the dimensions of certain features such as the thickness of ribs, radii produced by machining or bending, variation in cross section within different parts of the element, and convenient handling of the element during manufacture.

Consideration should be given to the sizes and shapes that are commercially available in the desired material. This could result in significant cost reductions both in material and in processing.

Sizes should be compatible with standard company practices if practical.

16. After completing all necessary stress analyses and proposing the basic sizes for all features, check all assumptions made earlier in the design to ensure that the element is still safe and reasonably efficient. See Steps 7, 12, and 13.
17. Specify suitable tolerances for all dimensions, considering the performance of the element, its fit with mating elements, the capability of the manufacturing process, and cost. Chapter 13 may be consulted. The use of computer-based tolerance-analysis techniques may be appropriate.
18. Check to determine whether some part of the component may deflect excessively. If that is an issue, complete an analysis of the deflection of the element as designed to this point. Sometimes there are known limits for deflection based on the operation of the machine of which the element being designed is a part. In the absence of such limits, the following guidelines may be applied based on the degree of precision desired:

Deflection of a Beam Due to Bending

General machine part:	0.000 5 to 0.003 in/in of beam length
Moderate precision:	0.000 01 to 0.000 5 in/in
High precision:	0.000 001 to 0.000 01 in/in

Deflection (Rotation) Due to Torsion

General machine part:	0.001° to 0.01°/in of length
Moderate precision:	0.000 02° to 0.000 4°/in
High precision:	0.000 001° to 0.000 02°/in

See also Section 20–2 in Chapter 20 for additional suggestions for efficient design. The results of the deflection analysis may cause you to redesign the component. Typically, when high stiffness and precision are required, deflection, rather than strength, will govern the design.

19. Document the final design with drawings and specifications.
20. Maintain a careful record of the design analyses for future reference. Keep in mind that others may have to consult these records whether or not you are still involved in the project.

5–11 DESIGN EXAMPLES

Example design problems are shown here to give you a feel for the application of the process outlined in Section 5–10. It is not practical to illustrate all possible situations, and you must develop the ability to adapt the design procedure to the specific characteristics of each problem. Also note that there are many possible solutions to any given design problem. The selection of a final solution is the responsibility of you, the designer.

In most design situations, a great deal more information will be available than is given in the problem statements in this book. But, often, you will have to seek out that information. We will make certain assumptions in the examples that allow the design to proceed. In your job, you must ensure that such assumptions are appropriate. The design examples focus on only one or a few of the components of the given systems. In real situations, you must ensure that each design decision is compatible with the totality of the design.

Design Example 5–1 A large electrical transformer is to be suspended from a roof truss of a building. The total weight of the transformer is 32 000 lb. Design the means of support.

Solution Objective Design the means of supporting the transformer.

Given The total load is 32 000 lb. The transformer will be suspended below a roof truss inside a building. The load can be considered to be static. It is assumed that it will be protected from the weather and that temperatures are not expected to be severely cold or hot in the vicinity of the transformer.

Basic Design Decisions Two straight, cylindrical rods will be used to support the transformer, connecting the top of its casing to the bottom chord of the roof truss. The ends of the rod will be threaded to allow them to be secured by nuts or by threading them into tapped holes. This design example will be concerned only with the rods. It is assumed that appropriate attachment points are available to allow the two rods to share the load equally during service. However, it is possible that only one rod will carry the entire load at some point during installation. Therefore, each rod will be designed to carry the full 32 000 lb.

We will use steel for the rods, and because neither weight nor physical size is critical in this application, a plain, medium-carbon steel will be used. We specify AISI 1040 cold-drawn steel. From Appendix 3, we find that it has a yield strength of 71 ksi and moderately high ductility as represented by its 12% elongation. The rods should be protected from corrosion by appropriate coatings.

The objective of the design analysis that follows is to determine the size of the rod.

Analysis The rods are to be subjected to direct normal tensile stress. Assuming that the threads at the ends of the rods are cut or rolled into the nominal diameter of the rods, the critical place for stress analysis is in the threaded portion.

Use the direct tensile stress formula, Equation (3–1): $\sigma = F/A$. We will first compute the design stress and then compute the required cross-sectional area to maintain the stress in service below that value. Finally, a standard thread will be specified from the data in Chapter 18 on fasteners.

Case C1 from Section 5–9 applies for computing the design stress because the rod is made from a ductile steel and it carries a static load. The design stress is

$$\sigma_d = s_y/N$$

We specify a design factor of $N = 3$, because it is typical for general machine design and because there is some uncertainty about the actual installation procedures that may be used (see Section 5–7). Then

$$\sigma_d = s_y/N = (71\ 000\ \text{psi})/3 = 23\ 667\ \text{psi}$$

Results In the basic tensile stress equation, $\sigma = F/A$, we know F, and we will let $\sigma = \sigma_d$. Then the required cross-sectional area is

$$A = F/\sigma_d = (32\ 000\ \text{lb})/(23\ 667\ \text{lb/in}^2) = 1.35\ \text{in}^2$$

A standard size thread will now be specified from the data in Chapter 18 on fasteners. You should be familiar with such data from earlier courses. Table A2–2(b) lists the tensile stress area for American Standard threads. A $1\frac{1}{2}$–6 UNC thread ($1\frac{1}{2}$-in-diameter rod with 6 threads per in) has a tensile stress area of 1.405 in^2 which should be satisfactory for this application.

Comments The final design specifies a $1\frac{1}{2}$-in-diameter rod made from AISI 1040 cold-drawn steel with $1\frac{1}{2}$–6 UNC threads machined on each end to allow the attachment of the rods to the transformer and to the truss.

Design Example 5–2 A part of a conveyor system for a production operation is shown in Figure 5–18. Design the pin that connects the horizontal bar to the fixture. The empty fixture weighs 85 lb. A cast iron engine block weighing 225 lb is hung on the fixture to carry it from one process to another, where it is then removed. It is expected that the system will experience many thousands of cycles of loading and unloading of the engine blocks.

Solution Objective Design the pin for attaching the fixture to the conveyor system.

Given The general arrangement is shown in Figure 5–18. The fixture places a shearing load that is alternately 85 lb and 310 lb (85 + 225) on the pin many thousands of times in the expected life of the system.

Basic Design Decisions It is proposed to make the pin from AISI 1020 cold-drawn steel. Appendix 3 lists s_y = 51 ksi and s_u = 61 ksi. The steel is ductile with 15% elongation. This material is inexpensive, and it is not necessary to achieve a particularly small size for the pin.

The connection of the fixture to the bar is basically a clevis joint with two tabs at the top of the fixture, one on each side of the bar. There will be a close fit between the tabs and the bar to minimize bending action on the pin. Also, the pin will be a fairly close fit in the holes while still allowing rotation of the fixture relative to the bar.

Analysis Case H from Section 5–9 applies for completing the design analysis because fluctuating shearing stresses are experienced by the pin. Therefore, we will have to determine relationships for the mean and alternating stresses (τ_m and τ_a) in terms of the applied loads and the cross-sectional area of the bar. Note that the pin is in double shear, so two cross sections resist the applied shearing force. In general, $\tau = F/2A$.

Now we will use the basic forms of Equations (5–1) and (5–2) to compute the values for the mean and alternating forces on the pin:

$$F_m = (F_{ma} + F_{min})/2 = (310 + 85)/2 = 198 \text{ lb}$$
$$F_a = (F_{max} - F_{min})/2 = (310 - 85)/2 = 113 \text{ lb}$$

The stresses will be found from $\tau_m = F_m/2A$ and $\tau_a = F_a/2A$.

The material strength values needed in Equation (5–28) for Case H are

$$s_{su} = 0.75\, s_u = 0.75(51 \text{ ksi}) = 38.3 \text{ ksi} = 38\,300 \text{ psi}$$
$$s'_{sn} = 0.577\, s'_n$$

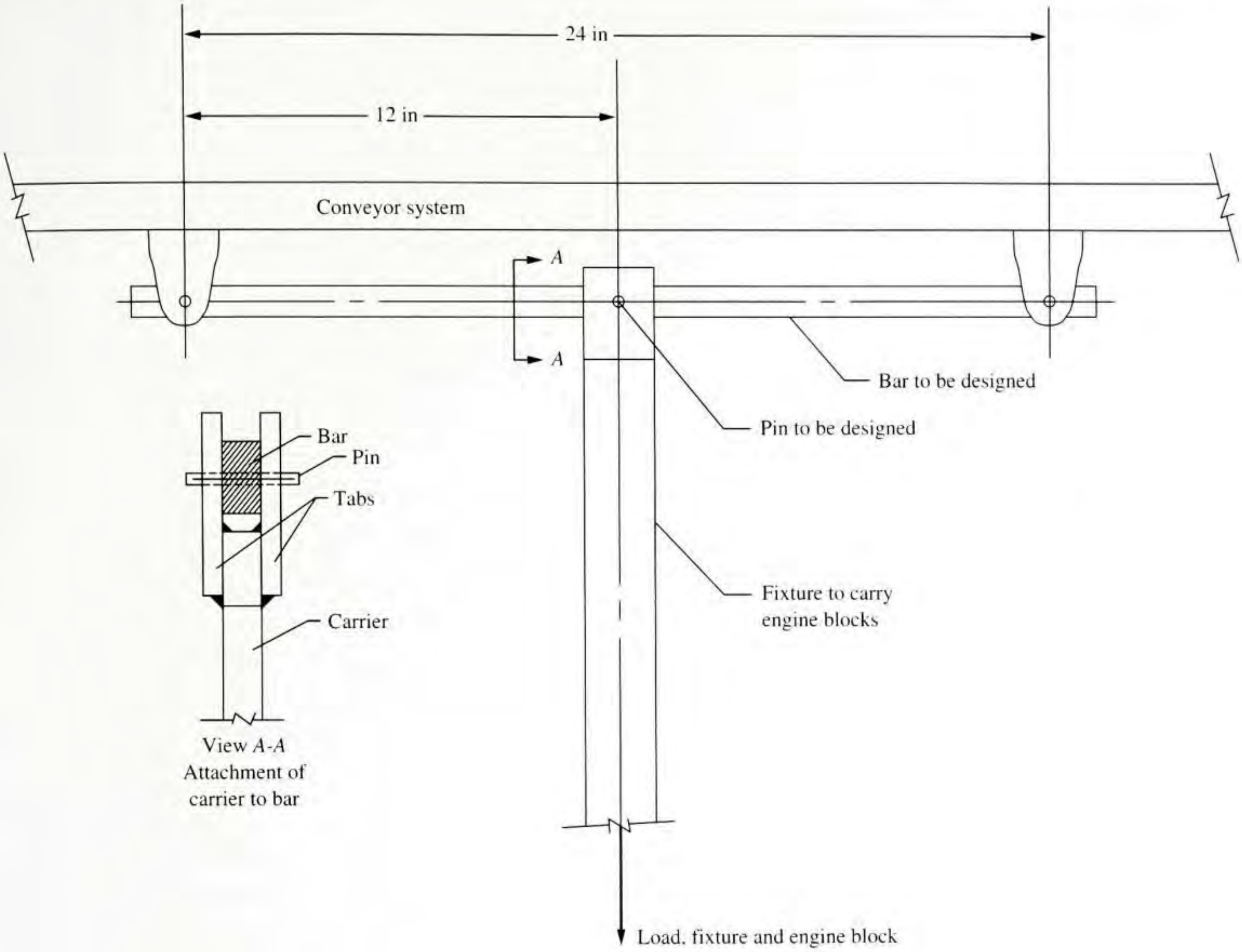


FIGURE 5–18 Conveyor system

We must find the value of s_n' using the method from Section 5–4. We find from Figure 5–8 that $s_n = 21$ ksi for the machined pin having a value of $s_u = 61$ ksi. It is expected that the pin will be fairly small, so we will use $C_s = 1.0$. The material is wrought steel rod, so $C_m = 1.0$. Let's use $C_{st} = 1.0$ to be conservative because there is little information about such factors for direct shearing stress. A high reliability is desired for this application, so let's use $C_R = 0.75$ to produce a reliability of 0.999 (see Table 5–1). Then

$$s_n' = C_R\,(s_n) = (0.75)(21\text{ ksi}) = 15.75\text{ ksi} = 15\,750\text{ psi}$$

Finally,

$$s_{sn}' = 0.577\,s_n' = 0.577\,(15\,750\text{ psi}) = 9088\text{ psi}$$

We can now apply Equation (5–28) from Case H:

$$\frac{1}{N} = \frac{\tau_m}{s_{su}} + \frac{K_t\tau_a}{s_{sn}'}$$

Because the pins will be of uniform diameter, $K_t = 1.0$.

Substituting $\tau_m = F_m/2A$ and $\tau_a = F_a/2A$ found earlier gives

$$\frac{1}{N} = \frac{F_m}{2As_{su}} + \frac{F_a}{2As'_{sn}}$$

Let's use $N = 4$ because mild shock can be expected.

Note that we now know all factors in this equation except the cross-sectional area of the pin, A. We can solve for the required area:

$$A = \frac{N}{2}\left[\frac{F_m}{s_{su}} + \frac{F_a}{s'_{sn}}\right]$$

Finally, we can compute the minimum allowable pin diameter, D, from $A = \pi D^2/4$ and $D = \sqrt{4A/\pi}$.

Results The required area is

$$A = \frac{4}{2}\left[\frac{198\text{ lb}}{38\,300\text{ lb/in}^2} + \frac{113\text{ lb}}{9\,088\text{ lb/in}^2}\right] = 0.0352\text{ in}^2$$

Now the required diameter is

$$D = \sqrt{4A/\pi} = \sqrt{4(0.0352\text{ in}^2/\pi} = 0.212\text{ in}$$

Final Design Decisions and Comments The computed value for the minimum required diameter for the pin, 0.212 in, is quite small. Other considerations such as bearing stress and wear at the surfaces that contact the tabs of the fixture and the bar indicate that a larger diameter would be preferred. Let's specify $D = 0.50$ in for the pin at this location. The pin will be of uniform diameter within the area of the bar and the tabs. It should extend beyond the tabs, and it could be secured with cotter pins or retaining rings.

This completes the design of the pin. But the next design example deals with the horizontal bar for this same system. There are pins at the conveyor hangers to support the bar. They would also have to be designed. However, note that each of these pins carries only half the load of the pin in the fixture connection. These pins would experience less relative motion as well, so wear should not be so severe. Thus, let's use pins with $D = 3/8$ in $= 0.375$ in at the ends of the horizontal bar.

Design Example 5–3 A part of a conveyor system for a production operation is shown in Figure 5–18. The complete system will include several hundred hanger assemblies like this one. Design the horizontal bar that extends between two adjacent conveyor hangers and that supports a fixture at its midpoint. The empty fixture weighs 85 lb. A cast iron engine block weighing 225 lb is hung on the fixture to carry it from one process to another, where it is then removed. It is expected that the bar will experience several thousand cycles of loading and unloading of the engine blocks. Design Example 5–2 considered this same system with the objective of specifying the diameter of the pins. The pin at the middle of the horizontal bar where the fixture is hung has been specified to have a diameter of 0.50 in.

Those at each end where the horizontal bar is connected to the conveyor hangers are each 0.375 in.

Solution Objective Design the horizontal bar for the conveyor system.

Given The general arrangement is shown in Figure 5–18. The bar is simply supported at points 24 in apart. A vertical load that is alternately 85 lb and 310 lb (85 + 225) is applied at the middle of the bar through the pin connecting the fixture to the bar. The load will cycle between these two values many thousands of times in the expected life of the bar. The pin at the middle of the bar has a diameter of 0.50 in, while the pins at each end are 0.375 in.

Basic Design Decisions It is proposed to make the bar from steel in the form of a rectangular bar with the long dimension of its cross section vertical. Cylindrical holes will be machined on the neutral axis of the bar at the support points and at its center to receive cylindrical pins that will attach the bar to the conveyor carriers and to the fixture. Figure 5–19 shows the basic design for the bar.

The thickness of the bar, t, should be fairly large to provide a good bearing surface for the pins and to ensure lateral stability of the bar when subjected to the bending stress. A relatively thin bar would tend to buckle along its top surface where the stress is compressive. As a design decision, we will use a thickness of $t = 0.50$ in. The design analysis will determine the required height of the bar, h, assuming that the primary mode of failure is stress due to bending. Other possible modes of failure are discussed in the comments at the end of this example.

An inexpensive steel is desirable because several hundred bars will be made. We specify AISI 1020 hot-rolled steel having a yield strength of $s_y = 30$ ksi and an ultimate strength of $s_u = 55$ ksi (Appendix 3).

Analysis Case G from Section 5–9 applies for completing the design analysis because fluctuating normal stress due to bending is experienced by the bar. Equation (5–20) will be used:

$$\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t\sigma_a}{s_n'}$$

In general, the bending stress in the bar will be computed from the flexure formula:

$$\sigma = M/S$$

where M = bending moment
S = section modulus of the cross section of the bar

Our approach will be to first determine the values for both the mean and the alternating bending moments experienced by the bar at its middle. Then the yield and endurance strength values for the steel will be found. Furthermore, as shown in Figure A15–3, the stress concentration factor for this case can be taken as $K_t = 1.0$ if the ratio of the hole diameter, d, to the height of the bar, h, is less than 0.50. We will make that assumption and check it later. Finally, Equation (5–20) includes the design factor, N. Based on the application conditions, let's use $N = 4$ as advised in item 4 in Section 5–7 because the actual use pattern for this conveyor system in a factory environment is somewhat uncertain and shock loading is likely.

Bending Moments Figure 5–19 shows the shearing force and bending moment diagrams for the bar when carrying just the fixture and then both the fixture and the engine block. The

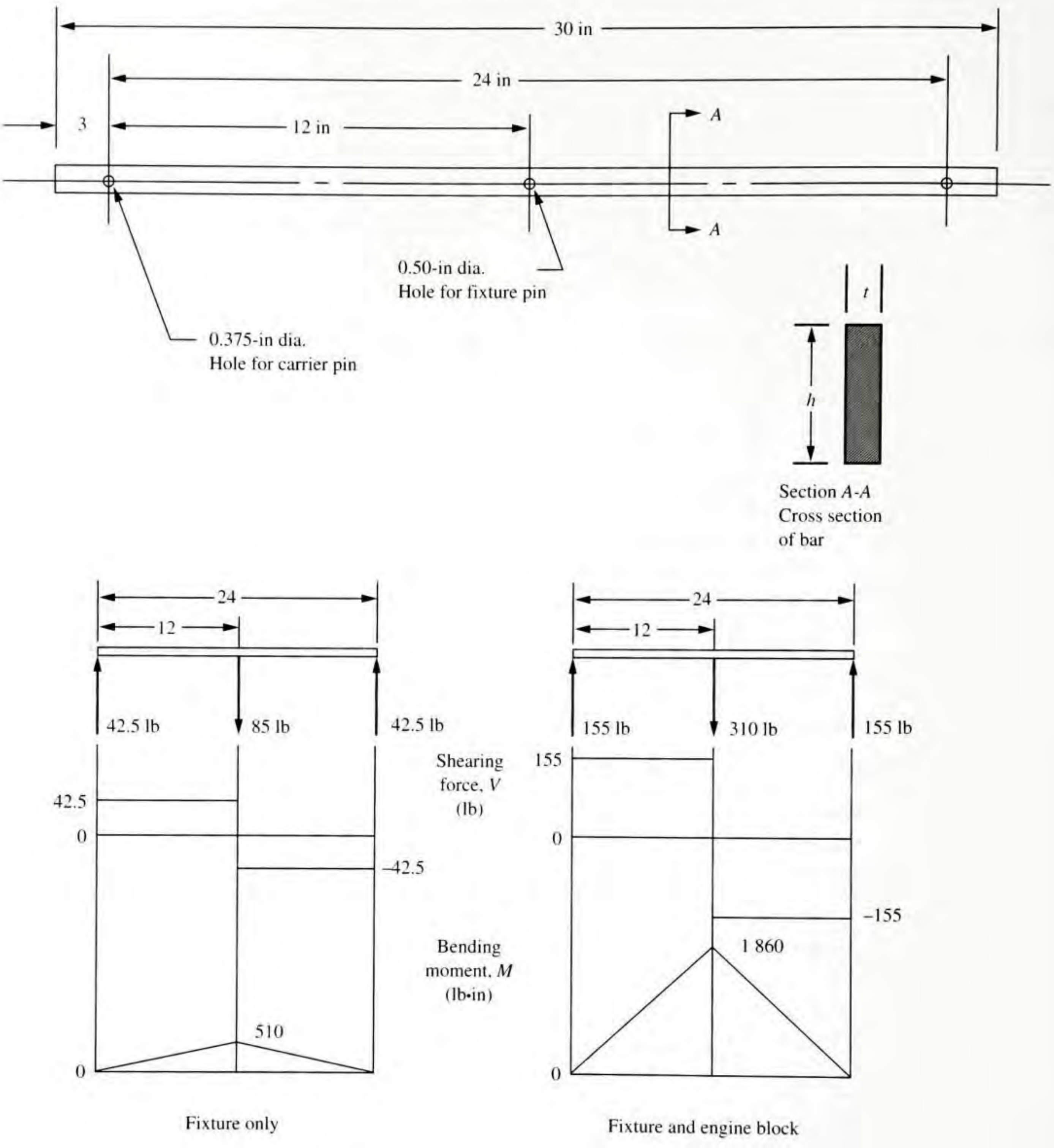


FIGURE 5–19 Basic design of the horizontal bar and the load, shearing force, and bending moment diagrams

maximum bending moment occurs at the middle of the bar where the load is applied. The values are $M_{max} = 1860$ lb·in with the engine block on the fixture and $M_{min} = 510$ lb·in for the fixture alone. Now the values for the mean and alternating bending moments are calculated using modified forms of Equations (5–1) and (5–2):

$$M_m = (M_{max} + M_{min})/2 = (1860 + 510)/2 = 1185 \text{ lb}\cdot\text{in}$$
$$M_a = (M_{max} - M_{min})/2 = (1860 - 510)/2 = 675 \text{ lb}\cdot\text{in}$$

The stresses will be found from $\sigma_m = M_m/S$ and $\sigma_a = M_a/S$.

Material Strength Values The material strength properties required are the ultimate strength s_u and the estimated actual endurance strength s_n'. We know that the ultimate strength s_u = 55 ksi. We now find s_n' using the method outlined in Section 5–4.

Size factor, Cs: From Section 5–4, Equation 5–8 defines an equivalent diameter, D_e, for the rectangular section as,

$$D_e = 0.808\sqrt{ht}$$

We have specified the thickness of the bar to be $t = 0.50$ in. The height is unknown at this time. As an estimate, let's assume $h \approx 2.0$ in. Then,

$$D_e = 0.808\sqrt{ht} = 0.808\sqrt{(2.0)(0.50)} = 0.808 \text{ in}$$

We can now use Figure 5–9 or the equations in Table 5–2 to find $C_s = 0.90$. This value should be checked later after a specific height dimension is proposed.

Material factor, C_m: Use $C_m = 1.0$ for the wrought, hot-rolled steel.

Stress-type factor, C_{st}: Use $C_{st} = 1.0$ for repeated bending stress.

Reliability factor, C_R: A high reliability is desired. Let's use $C_R = 0.75$ to achieve a reliability of 0.999 as indicated in Table 5–1.

The value of s_n = 20 ksi is found from Figure 5–8 for hot-rolled steel having an ultimate strength of 55 ksi.

Now, applying Equation (5–4) from Section 5–5, we have

$$s_n' = (C_m)(C_{st})(C_R)(C_s)\, s_n = (1.0)(1.0)(0.75)(0.90)(20 \text{ ksi}) = 13.5 \text{ ksi}$$

Solution for the Required Section Modulus At this point, we have specified all factors in Equation (5–20) except the section modulus of the cross section of the bar that is involved in each expression for stress as shown above. We will now solve the equation for the required value of S.

Recall that we showed earlier that $\sigma_m = M_m/S$ and $\sigma_a = M_a/S$. Then

$$\frac{1}{N} = \frac{\sigma_m}{s_u} + \frac{K_t\sigma_a}{s_n'} = \frac{M_m}{Ss_u} + \frac{K_t M_a}{Ss_n'} = \frac{1}{S}\left[\frac{M_m}{s_u} + \frac{K_t M_a}{s_n'}\right]$$

$$S = N\left[\frac{M_m}{s_u} + \frac{K_t M_a}{s_n'}\right] = 4\left[\frac{1185 \text{ lb}\cdot\text{in}}{55\,000 \text{ lb/in}^2} + \frac{1.0\,(675 \text{ lb}\cdot\text{in})}{13\,500 \text{ lb/in}^2}\right]$$

$$S = 0.286 \text{ in}^3$$

Results The required section modulus has been found to be $S = 0.286$ in^3. We observed earlier that $S = th^2/6$ for a solid rectangular cross section, and we decided to use this form to find an initial estimate for the required height of the section, h. We have specified $t = 0.50$ in. Then the estimated minimum acceptable value for the height h is

$$h = \sqrt{6S/t} = \sqrt{6(0.286 \text{ in}^3)/(0.50 \text{ in})} = 1.85 \text{ in}$$

The table of preferred basic sizes in the decimal-inch system (Table A2–1) recommends $h = 2.00$ in. We should first check the earlier assumption that the ratio $d/h < 0.50$ at the middle of the bar. The actual ratio is

$$d/h = (0.50 \text{ in})/(2.00 \text{ in}) = 0.25 \text{ (okay)}$$

This indicates that our earlier assumption that $K_t = 1.0$ is correct. Also, our assumed value of $C_s = 0.90$ is correct because the actual height, $h = 2.0$ in, is identical to our assumed value.

We will now compute the actual value for the section modulus of the cross section with the hole in it.

$$S = \frac{t(h^3 - d^3)}{6h} = \frac{(0.50 \text{ in})[(2.00 \text{ in})^3 - (0.50 \text{ in})^3]}{6(2.00 \text{ in})} = 0.328 \text{ in}^3$$

This value is larger than the minimum required value of 0.286 in^3. Therefore, the size of the cross section is satisfactory with regard to stress due to bending.

Final Design Decisions and Comments

In summary, the following are the design decisions for the horizontal bar of the conveyor hanger shown in Figure 5–19.

1. ***Material:*** AISI 1020 hot-rolled steel.
2. ***Size:*** Rectangular cross section. Thickness $t = 0.50$ in; height $h = 2.00$ in.
3. ***Overall design:*** Figure 5–19 shows the basic features of the bar.
4. ***Other considerations:*** Remaining to be specified are the tolerances on the dimensions for the bar and the finishing of its surfaces. The potential for corrosion should be considered and may call for paint, plating, or some other corrosion protection. The size of the cross section can likely be used with the as-received tolerances on thickness and height, but this is somewhat dependent on the design of the fixture that holds the engine block and the conveyor hangers. So the final tolerances will be left open pending later design decisions. The holes in the bar for the pins should be designed to produce a close sliding fit with the pins, and the details of specifying the tolerances on the hole diameters for such a fit are discussed in Chapter 13.
5. ***Other possible modes of failure:*** The analysis used in this problem assumed that failure would occur due to the bending stresses in the rectangular bar. The dimensions were specified to preclude this from happening. Other possible modes are discussed here:
   a. *Deflection of the bar as an indication of stiffness:* The type of conveyor system described in this problem should not be expected to have extreme rigidity because moderate deflection of members should not impair its operation. However, if the horizontal bar deflects so much that it appears to be rather flexible, it would be deemed unsuitable. This is a subjective judgment. We can use Case (a) in Table A14–2 to compute the deflection.

   $$y = FL^3/48EI$$

   In this design,

   $$F = 310 \text{ lb} = \text{maximum load on the bar}$$
   $$L = 24.0 \text{ in} = \text{distance between supports}$$
   $$E = 30 \times 10^6 \text{ psi} = \text{modulus of elasticity of steel}$$
   $$I = th^3/12 = \text{moment of inertia of the cross section}$$
   $$I = (0.50 \text{ in})(2.00 \text{ in})^3/12 = 0.333 \text{ in}^4$$

Then

$$y = \frac{(310\ \text{lb})(24.0\ \text{in})^3}{48(30 \times 10^6\ \text{lb/in}^2)(0.333\ \text{in}^4)} = 0.0089\ \text{in}$$

This value seems satisfactory. In Section 5–10, some guidelines were given for deflection of machine elements. One stated that bending deflections for general machine parts should be limited to the range of 0.000 5 to 0.003 in/in of beam length. For the bar in this design, the ratio of y/L can be compared to this range:

$$y/L = (0.0089\ \text{in})/(24.0\ \text{in}) = 0.0004\ \text{in/in of beam length}$$

Therefore, this deflection is well within the recommended range.

b. *Buckling of the bar:* When a beam with a tall, thin, rectangular cross section is subjected to bending, it would be possible for the shape to distort due to buckling before the bending stresses would cause failure of the material. This is called *elastic instability*, and a complete discussion is beyond the scope of this book. However, Reference 16 shows a method of computing the critical buckling load for this kind of loading. The pertinent geometrical feature is the ratio of the thickness t of the bar to its height h. It can be shown that the bar as designed will not buckle.

c. *Bearing stress on the inside surfaces of the holes in the beam:* Pins transfer loads between the bar and the mating elements in the conveyor system. It is possible that the bearing stress at the pin–hole interface could be large, leading to excessive deformation or wear. Reference 3 in Chapter 3 indicates that the allowable bearing stress for a steel pin in a steel hole is $0.90s_y$.

$$\sigma_{bd} = 0.90s_y = 0.90(30\ 000\ \text{psi}) = 27\ 000\ \text{psi}$$

The actual bearing stress at the center hole is found using the projected area, D_pt.

$$\sigma_b = F/D_pt = (310\ \text{lb})/(0.50\ \text{in})(0.50\ \text{in}) = 1240\ \text{psi}$$

Thus the pin and hole are very safe for bearing.

Design Example 5–4 A bracket is made by welding a rectangular bar to a circular rod as shown in Figure 5–20. Design the bar and the rod to carry a static load of 250 lb.

Solution Objective: The design process will be divided into two parts:

1. Design the rectangular bar for the bracket.
2. Design the circular rod for the bracket.

Rectangular Bar

Given: The bracket design is shown in Figure 5–20. The rectangular bar carries a load of 250 lb vertically downward at its end. Support is provided by the weld at its left end where the loads are transferred to the circular rod. The bar acts as a cantilever beam, 12 in long. The design task is to specify the material for the bar and the dimensions of its cross section.

FIGURE 5–20
Bracket design

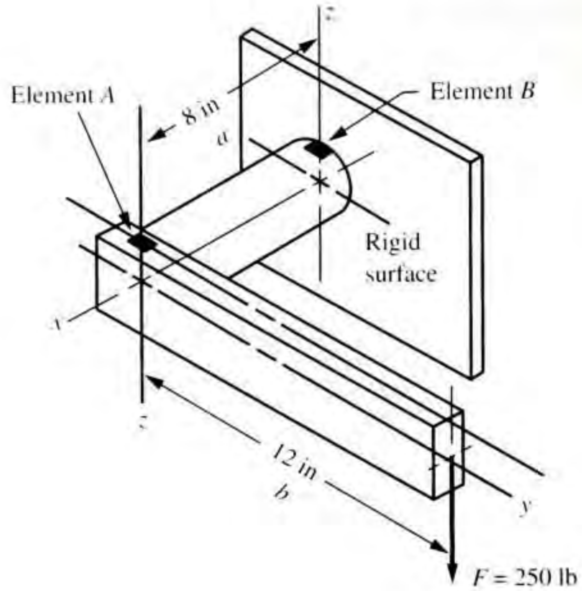
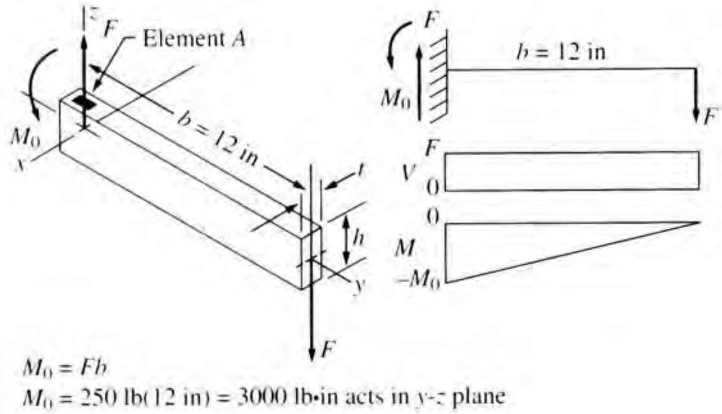


FIGURE 5–21 Free-body diagram of bar



Basic Design Decisions

We will use steel for both parts of the bracket because of its relatively high stiffness, the ease of welding, and the wide range of strengths available. Let's specify AISI 1340 annealed steel having $s_y = 63$ ksi and $s_u = 102$ ksi (Appendix 3). The steel is highly ductile, with a 26% elongation.

The objective of the design analysis that follows is to determine the size of the cross section of the rectangular bar. Assuming that the loading and processing conditions are well known, we will use a design factor of $N = 2$ because of the static load.

Analysis and Results

The free-body diagram of the cantilever bar is shown in Figure 5–21, along with the shearing force and bending moment diagrams. This should be a familiar case, leading to the judgment that the maximum tensile stress occurs at the top of the bar near to where it is supported by the circular rod. This point is labeled element A in Figure 5–21. The maximum bending moment there is $M = 3000$ lb · in. The stress at A is

$$\sigma_A = M/S$$

where S = section modulus of the cross section of the bar.

We will first compute the minimum allowable value for S and then determine the dimensions for the cross section.

Case C1 from Section 5–9 applies because of the static loading. We will first compute the design stress from

$$\sigma_d = s_y/N$$
$$\sigma_d = s_y/N = (63\,000 \text{ psi})/2 = 31\,500 \text{ psi}$$

Now we must ensure that the expected maximum stress $\sigma_A = M/S$ does not exceed the design stress. We can substitute $\sigma_A = \sigma_d$ and solve for S.

$$S = M/\sigma_d = (3000 \text{ lb}\cdot\text{in})/(31\,500 \text{ lb/in}^2) = 0.095 \text{ in}^3$$

The relationship for S is

$$S = th^2/6$$

As a design decision, let's specify the approximate proportion for the cross-sectional dimensions to be $h = 3t$. Then

$$S = th^2/6 = t(3t)^2/6 = 9t^3/6 = 1.5t^3$$

The required minimum thickness is then

$$t = \sqrt[3]{S/1.5} = \sqrt[3]{(0.095 \text{ in}^3)/1.5} = 0.399 \text{ in}$$

The nominal height of the cross section should be, approximately,

$$h = 3t = 3(0.399 \text{ in}) = 1.20 \text{ in}$$

Final Design Decisions and Comments

In the fractional-inch system, standard sizes are selected to be $t = 3/8$ in $= 0.375$ in and $h = 1\frac{1}{4}$ in $= 1.25$ in (see Table A2–1). Note that we chose a slightly smaller value for t but a slightly larger value for h. We must check to see that the resulting value for S is satisfactory.

$$S = th^2/6 = (0.375 \text{ in})(1.25 \text{ in})^2/6 = 0.0977 \text{ in}^3$$

This is larger than the required value of 0.095 in^3, so the design is satisfactory.

Circular Rod

Given

The bracket design is shown in Figure 5–20. The design task is to specify the material for the rod and the diameter of its cross section.

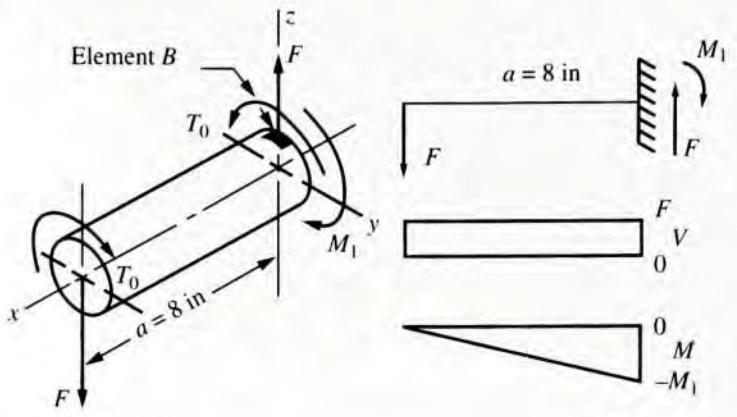
Basic Design Decisions

Let's specify AISI 1340 annealed steel, the same as that used for the rectangular bar. Its properties are $s_y = 63$ ksi and $s_u = 102$ ksi.

Analysis and Results

Figure 5–22 is the free-body diagram for the rod. The rod is loaded at its left end by the reactions at the end of the rectangular bar, namely, a downward force of 250 lb and a moment of

FIGURE 5–22 Free-body diagram of rod



$x = F = 250$ lb
$M_1 = Fa = 250$ lb(8 in) = 2000 lb•in (acts in y-z plane)
$T_0 = Fb = 250$ lb(12 in) = 3000 lb•in (see figure 5–21. T_0-M_0)

3000 lb · in. The figure shows that the moment acts as a torque on the circular rod, and the 250-lb force causes bending with a maximum bending moment of 2000 lb · in at the right end. Reactions are provided by the weld at its right end where the loads are transferred to the support. The rod then is subjected to a combined stress due to torsion and bending. Element B on the top of the rod is subjected to the maximum combined stress.

The manner of loading on the circular rod is identical to that analyzed earlier in Section 4–6 in Chapter 4. It was shown that when only bending and torsional shear occur, a procedure called the *equivalent torque method* can be used to complete the analysis. First we define the equivalent torque, T_e:

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2000)^2 + (3000)^2} = 3606 \text{ lb} \cdot \text{in}$$

Then the shear stress in the bar is

$$\tau = T_e/Z_p$$

where Z_p = polar section modulus

For a solid circular rod,

$$Z_p = \pi D^3/16$$

Our approach is to determine the design shear stress and T_e and then solve for Z_p. Case C2 using the maximum shear stress theory of failure can be applied. The design shear stress is

$$\tau_d = 0.50 s_y/N = (0.5)(63\,000 \text{ psi})/2 = 15\,750 \text{ psi}$$

We let $\tau = \tau_d$ and solve for Z_p:

$$Z_p = T_e/\tau_d = (3606 \text{ lb} \cdot \text{in})/(15\,750 \text{ lb/in}^2) = 0.229 \text{ in}^3$$

Now that we know Z_p, we can compute the required diameter from

$$D = \sqrt[3]{16Z_p/\pi} = \sqrt[3]{16(0.229 \text{ in}^3)/\pi} = 1.053 \text{ in}$$

This is the minimum acceptable diameter for the rod.

Final Design Decisions and Comments The circular rod is to be welded to the side of the rectangular bar, and we have specified the height of the bar to be $1\frac{1}{4}$ in. Let's specify the diameter of the circular rod to be machined to 1.10 in. This will allow welding all around its periphery.

5–12 STATISTICAL APPROACHES TO DESIGN

The design approaches presented in this chapter are somewhat deterministic in the sense that data are taken to be discrete values, and analyses use the data to determine specific results. The method of accounting for uncertainty with regard to the data themselves lies with the selection of an acceptable value for the design factor represented by the final design decision. Obviously this is a subjective judgment. Often decisions that are made to ensure the safety of a design cause many designs to be quite conservative.

Competitive pressures call for ever more efficient, less conservative designs. Throughout this book, recommendations are made to seek more reliable data for loads, material properties, and environmental factors, providing more confidence in the results of design analyses and allowing lower values for the design factor as discussed in Section 5–7. A more robust and reliable product results from testing samples of the actual material to be used in the product; performing extensive measurements of loads to be experienced; investing in more detailed performance testing, experimental stress analysis, and finite element analysis; exerting more careful control of manufacturing processes; and life testing of prototype products in realistic conditions where possible. All of these measures typically come with significant additional costs, and difficult decisions must be made about whether or not to implement them.

In combination with the approaches listed previously, a greater use of statistical methods (also called *stochastic methods*) is emerging to account for the inevitable variability of data by determining the mean values of critical parameters from several sets of data and quantifying the variability using the concepts of distributions and standard deviations. References 13 and 14 provide guidance in these methods. Section 2–4 provided a modest discussion of this approach to account for the variability of materials property data.

Industries such as automotive, aerospace, construction equipment, and machine tools devote considerable resources to acquiring useful data for operating conditions that will aid designers in producing more efficient designs.

Samples of Statistical Terminology and Tools

- Statistical methods analyze data to present useful information about the source of the data.
- Stochastic methods apply probability theories to characterize variability in the data.
- Data sets can be analyzed to determine the mean (average), the range of variation, and the standard deviation.
- Inferences can be made about the nature of the distribution of the data, such as normal or lognormal.

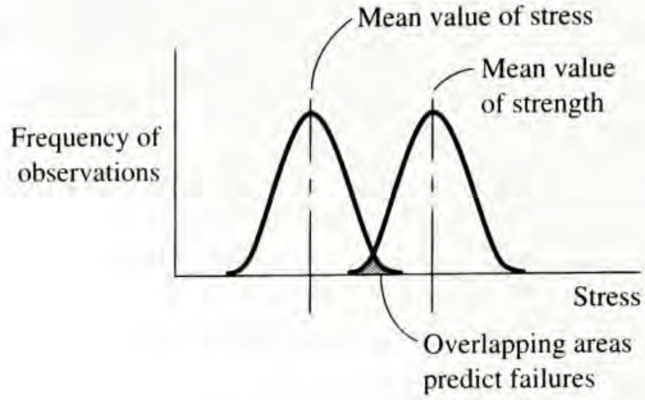


FIGURE 5–23
Illustration of statistical variation in failure potential

- Linear regression and other means of curve fitting can be employed to represent a set of data by mathematical functions.
- The distributions for applied loads and stresses can be compared with the distribution for the strength of the material to determine to what degree they overlap and the probability that a certain number of failures will occur. See Figure 5–23.
- The reliability of a component or a complete product can be quantified.
- The optimum assignment of tolerances can be made to reasonably ensure satisfactory performance of a product while allowing as broad a range of tolerances as practical.

5–13 FINITE LIFE AND DAMAGE ACCUMULATION METHOD

The fatigue design methods described thus far in this chapter have the goal to design a component for infinite life by using the estimated actual endurance strength as the basis for design and assuming that this value is the *endurance limit*. Applied repeated stress levels below this value will provide infinite life. Furthermore, the analyses were based on the assumption that the loading pattern was uniform over the life of the component. That is, the mean and alternating stresses do not vary over time.

There are, however, many common examples for which a finite life is adequate for the application and where the loading pattern does vary with time. Consider the following.

Finite Life Examples

First, let's discuss the concept of finite life. Refer to the endurance strength curves in Figure 5–7 in Section 5–3. Data are plotted on a stress vs. number of cycles to failure graph (σ *vs.* N) with both axes using logarithmic scales. For the materials that exhibit an endurance limit, it can be seen that the limit occurs for a life of approximately 10^6 cycles. How long would it take to accumulate 1 million cycles of load applications? Here are some examples.

Bicycle brake lever: Assume that the brake is applied every 5.0 minutes while riding 4.0 hours per day every day of each year. It would take more than 57 years to apply the brake 1 million times.

Lawn mower height-adjustment mechanism: Consider a lawn mower used by a commercial lawn maintenance company. Assume that the cutting height for the mower is adjusted to accommodate varying terrain three times per mowing and that the mower is used

40 times per week for all 12 months per year. It would take 160 years to accumulate 1 million cycles of load on the height adjustment mechanism.

Automotive lift in a service station: Assume that the service technician lifts four automobiles per hour, 10 hours per day, 6 days per week, each week of the year. It would take more than 80 years to accumulate 1 million cycles of load on the lift mechanism.

Each of these examples indicates that it may be appropriate to design the load-carrying members of the example systems for something less than infinite life. But many industrial examples do require design for infinite life. Here is an example.

Parts feeding device: On an automated assembly system, a feeding device inserts 120 parts per minute. If the system operates 16 hours per day, 6 days per week, each week of the year, it would take only 8.7 days to accumulate 1 million cycles of loading. It would see 35.9 million cycles in one year.

When it can be justified to design for a finite life less than the number of cycles corresponding to the endurance limit, you will need data similar to that in Figure 5–7 for the actual material to be used in the component. Testing the material yourself is preferred, but it would be a time-consuming and costly exercise to acquire sufficient data to make statistically valid σ-N curves. References 1, 3, 6, and 14 may provide suitable data, or an additional literature search may be required. Once reliable data are identified, use the endurance strength at the specified number of cycles as the starting point for computing the estimated actual endurance strength as described in Section 5–5. Then use that value in subsequent analyses as described in Section 5–9.

Varying Stress Amplitude Examples

Here we are looking for examples where the component experiences cyclical loading for a large number of cycles but for which the amplitude of the stress varies over time.

Bicycle brake lever: Let's reconsider the braking action on a bicycle. Sometimes you need to bring the bike to a stop very quickly from a high speed, requiring a rather high force on the brake lever. Other times you may apply a lighter force to simply slow down a bit to safely negotiate a curve.

Automotive suspension member: Suspension parts such as a strut, spring, shock absorber, control arm, or fastener pass loads from the wheel to the frame of a car. The magnitude of the load depends on vehicle speed, the condition of the road, and driver action. Roads may be smoothly paved, potholed, or rough surfaced gravel. The vehicle may even be driven off-road where violent peaks of stress will be encountered.

Machine tool drive system: Consider the lifetime of a milling machine. Its primary function is to cut metal, and it takes a certain amount of torque to drive the cutter depending on the machinability of the material, the depth of cut, and the feed rate. Surely the torque will vary significantly from job to job. During part of its operating time, there may be no cutting action at all as a new part is positioned or as the operation completes one cut and adjusts before making another. At times the cutter will encounter locally harder material, requiring higher torque for a short period of time.

Cranes, power shovels, bulldozers, and other construction equipment: Here there are obviously varying loads as the equipment is used for numerous tasks, such as hoisting large steel beams or small bracing members, digging through hard clay or soft sandy soil, rapidly grading a hillside or performing the final smoothing of a driveway, or encountering a tree stump or large rock.

How would you determine the loads that these devices would experience over time? One method involves building a prototype and instrumenting critical elements with strain gages, load cells, or accelerometers. Then the system would be "put through its paces" over a wide range of tasks while loads and stresses are recorded as a function of time. Similar vehicles could be monitored to determine the frequency that different kinds of loading would be encountered over its expected life. Combining such data would produce a record from which the total number of cycles of stress at any given level can be estimated. Statistical techniques such as spectrum analysis, Fast Fourier Transform analysis, and time compression analysis produce charts that summarize stress amplitude and frequency data that are useful for fatigue and vibration analysis. See Reference 14 for extended discussion of these techniques.

Damage Accumulation Method

The basic principle of damage accumulation is based on the assumption that any given level of stress applied for one cycle of loading contributes a certain amount of damage to a component. Refer again to Figure 5–7 and observe curve A for a particular heat-treated alloy steel. If this material were subjected to a repeated and reversed stress with a constant amplitude of 120 ksi, its predicted life is 5×10^4 cycles. If the specimen experiences 100 cycles at this stress level, it would exhibit damage amounting to the ratio of $100/(5\times10^4)$. A stress amplitude of 100 ksi corresponds to a life of approximately 1.8×10^5. A total of 2000 cycles at this stress level would produce damage of $2000/(1.8\times10^5)$. A stress below 82 ksi would not be expected to produce any damage because it is below the endurance limit of the material.

This kind of logic can be used to predict the total life of a component subjected to a sequence of loading levels. Let n_i represent the number of cycles of a specific stress level experienced by a component. Let N_i be the number of cycles to failure for this stress level as indicated by an σ-N curve such as those shown in Figure 5–7. Then the damage contribution from this loading is

$$D_i = n_i/N_i$$

When several stress levels are experienced for different numbers of cycles, the cumulative damage can be represented as,

$$D_c = \sum_{i=1}^{i=k} (n_i/N_i) \tag{5–31}$$

Failure is predicted when $D_c = 1.0$. This process is called the *Miner linear cumulative-damage rule* or simply *Miner's rule* in honor of his work in 1945. An example problem will now demonstrate the application of Miner's rule.

Example Problem 5–5 Determine the cumulative damage experienced by a ground circular rod, 1.50 in diameter, subjected to the combination of the cycles of loading and varying levels of reversed, repeated bending stress shown in Table 5–3. The bar is made from AISI 6150 OQT1100 alloy steel. The σ-N curve for the steel is shown in Figure 5–7, curve A for the standard, polished R. R. Moore type specimen.

TABLE 5–3 Loading pattern for Example Problem 5–5

Stress level (ksi)	Cycles n_i
80	4000
70	6000
65	10 000
60	25 000
55	15 000
45	1500

Solution Given

AISI 6150 OQT1100 alloy steel rod. $D = 1.50$ in. Ground surface.

Endurance strength data (σ-N) shown in Figure 5–7, curve A.

Loading is reversed, repeated bending. Load history shown in Table 5–3.

Analysis First adjust σ-N data for actual conditions using methods of Section 5–4. Use Miner's rule to estimate portion of life used by loading pattern.

Results

For AISI 6150 OQT1100, $s_u = 162$ ksi (Appendix A4–6)

From Figure 5–8, basic $s_n = 74$ ksi for ground surface

Material factor, $C_m = 1.00$ for wrought steel

Type-of-stress factor, $C_{st} = 1.0$ for reversed, rotating bending stress

Reliability factor, $C_R = 0.81$ (Table 5–1) for $R = 0.99$ (Design decision)

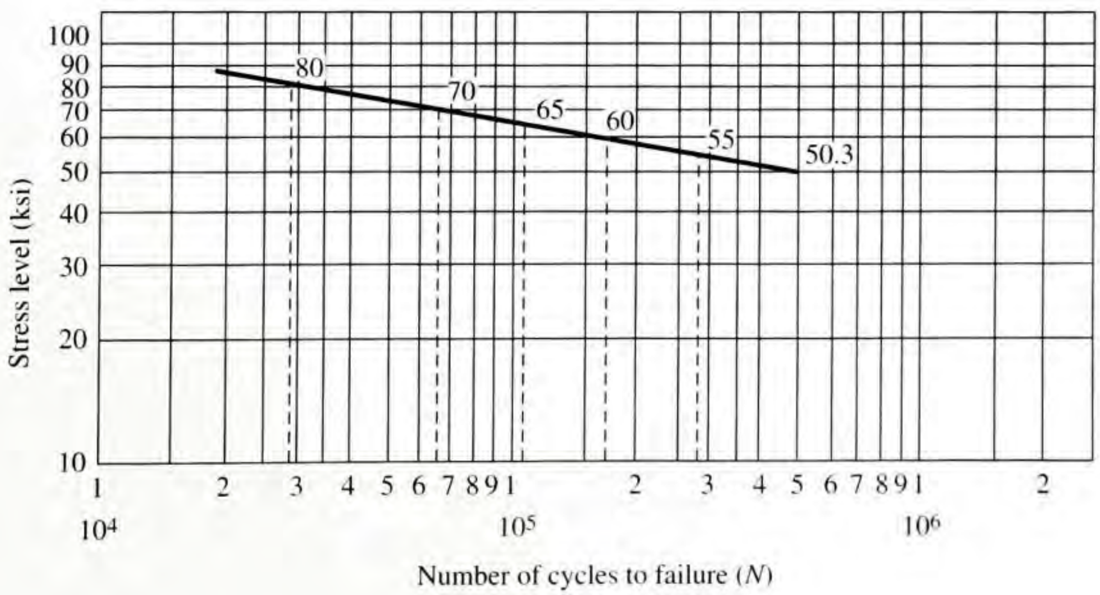
Size factor, $C_s = 0.84$ (Figure 5–9 and Table 5–2 for $D = 1.50$ in)

Estimated actual endurance strength, s_n' – Computed:

$$s_n' = s_n\, C_m\, C_{st}\, C_R\, C_s = (74\text{ ksi})(1.0)(1.0)(0.81)(0.84) = 50.3\text{ ksi}$$

This is the estimate for the endurance limit of the steel. In Figure 5–7, the endurance limit for the standard specimen is 82 ksi. The ratio of the actual to the standard data is, 50.3/82 = 0.61. We can now adjust the entire σ-N curve by this factor. The result is shown in Figure 5–24.

FIGURE 5–24 σ-N Curve for Example Problem 5–5



Now we can read the number of cycles of life, N_i, corresponding to each of the given loading levels from Table 5–3. The combined data for the number of applied load cycles, n_i, and the life cycles, N_i, are now used in Miner's rule, Equation 5–31, to determine the cumulative damage, D_c.

Stress level (ksi)	Cycles n_i	Life cycles N_i	n_i/N_i
80	4000	2.80×10^4	0.143
70	6000	6.60×10^4	0.0909
65	10 000	1.05×10^5	0.0952
60	25 000	1.70×10^5	0.147
55	15 000	2.85×10^5	0.0526
45	1500	∞	0.00
		Total:	0.529

Comment We can conclude from this number that approximately 53% of the life of the component has been accumulated by the given loading. For these data, the greatest damage occurs from the 60 ksi loading for 25 000 cycles. An almost equal amount of damage is caused by the 80 ksi loading for only 4000 cycles. Note that the cycles of loading at 45 ksi contributed nothing to the damage because they are below the endurance limit of the steel.

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PROBLEMS

Stress Ratio

For each of Problems 1–9, draw a sketch of the variation of stress versus time, and compute the maximum stress, minimum stress, mean stress, alternating stress, and stress ratio, R. For Problems 6–9, analyze the beam at the place where the maximum stress would occur at any time in the cycle.

1. A link in a mechanism is made from a round bar having a diameter of 10.0 mm. It is subjected to a tensile force that varies from 3500 to 500 N in a cyclical fashion as the mechanism runs.
2. A strut in a space frame has a rectangular cross section of 10.0 mm by 30.0 mm. It sees a load that varies from a tensile force of 20.0 kN to a compressive force of 8.0 kN.
3. A link in a packaging machine mechanism has a square cross section of 0.40 in on a side. It is subjected to a load that varies from a tensile force of 860 lb to a compressive force of 120 lb.
4. A circular rod with a diameter of 3/8 in supports part of a storage shelf in a warehouse. As products are loaded and unloaded, the rod is subjected to a tensile load that varies from 1800 to 150 lb.
5. A part of a latch for a car door is made from a circular rod having a diameter of 3.0 mm. With each actuation, it sees a tensile force that varies from 780 to 360 N.
6. A part of the structure for a factory automation system is a beam that spans 30.0 in as shown in Figure P5–6. Loads are applied at two points, each 8.0 in from a support. The left load $F_1 = 1800$ lb remains constantly applied, while the right load $F_2 = 1800$ lb is applied and removed frequently as the machine cycles.

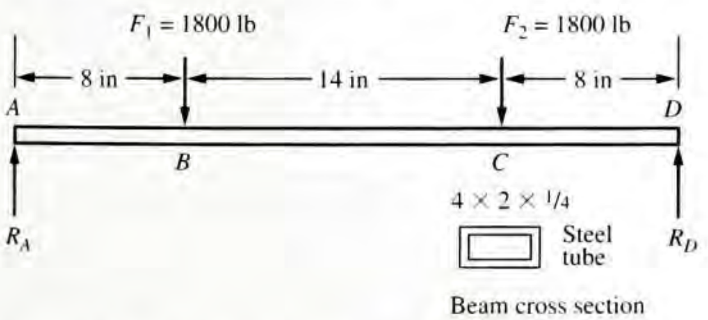


FIGURE P5–6 (Problems 6 and 23)

7. A cantilevered boom is part of an assembly machine and is made from an American Standard steel beam, S4 × 7.7. A tool with a weight of 500 lb moves continuously from the end of the 60-in beam to a point 10 in from the support.
8. A part of a bracket in the seat assembly of a bus is shown in Figure P5–8. The load varies from 1450 to 140 N as passengers enter and exit the bus.

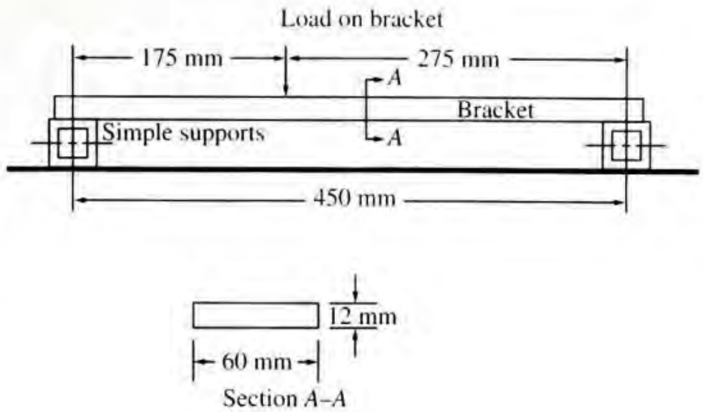


FIGURE P5–8 Seat bracket (Problems 8, 19, and 20)

9. A flat steel strip is used as a spring to maintain a force against part of a cabinet latch in a commercial printer as shown in Figure P5–9. When the cabinet door is open, the spring is deflected by the latch pin by an amount $y_1 = 0.25$ mm. The pin causes the deflection to increase to 0.40 mm when the door is closed.

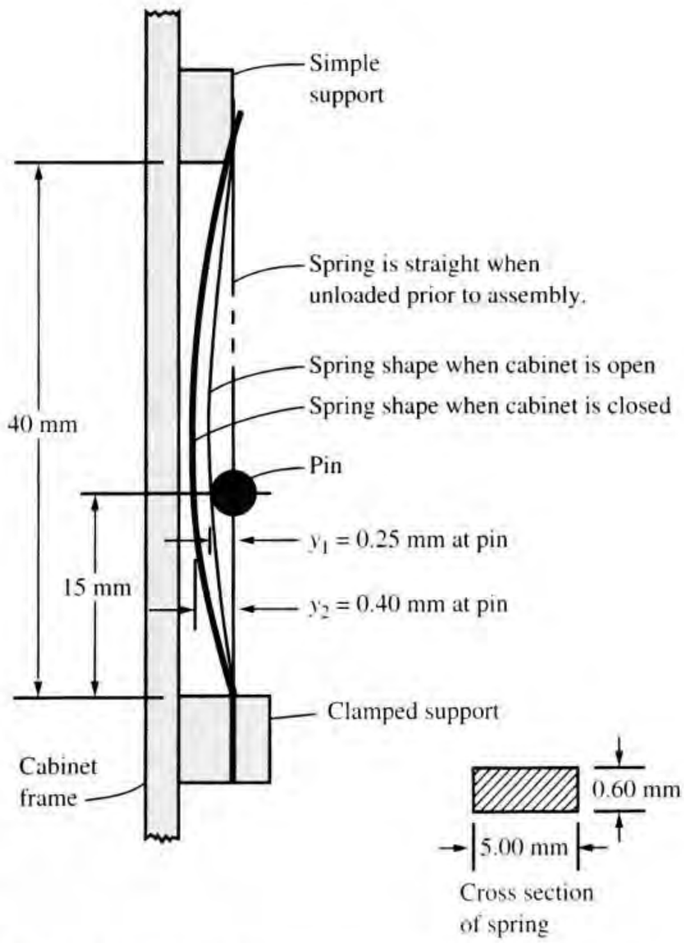


FIGURE P5–9 Cabinet latch spring (Problems 9 and 22)

Endurance Strength

For Problems 10–14, use the method outlined in Section 5–14 to determine the expected actual endurance strength for the material.

10. Compute the estimated actual endurance strength for a 0.75-in-diameter rod made from AISI 1040 cold-drawn steel. It is to be used in the as-rolled condition and subjected to repeated bending stress. A reliability of 99% is desired.

11. Compute the estimated actual endurance strength for AISI 5160 OQT 1300 steel rod with a diameter of 20.0 mm. It is to be machined and subjected to repeated bending stress. A reliability of 99% is desired.

12. Compute the estimated actual endurance strength for AISI 4130 WQT 1300 steel bar with a rectangular cross section of 20.0 mm by 60 mm. It is to be machined and subjected to repeated bending stress. A reliability of 99% is desired.

13. Compute the estimated actual endurance strength for AISI 301 stainless steel rod, 1/2 hard, with a diameter of 0.60 in. It is to be machined and subjected to repeated axial tensile stress. A reliability of 99.9% is desired.

14. Compute the estimated actual endurance strength for a machined rectangular steel bar (ASTM A242) 3.5 in high by 0.375 in thick, subjected to repeated bending stress. A reliability of 99% is desired.

Design and Analysis

15. A link in a mechanism is to be subjected to a tensile force that varies from 3500 to 500 N in a cyclical fashion as the mechanism runs. It has been decided to use AISI 1040 cold-drawn steel. Complete the design of the link, specifying a suitable cross-sectional shape and dimensions.

16. A circular rod is to support part of a storage shelf in a warehouse. As products are loaded and unloaded, the rod is subjected to a tensile load that varies from 1800 to 150 lb. Specify a suitable shape, material, and dimensions for the rod.

17. A strut in a space frame sees a load that varies from a tensile force of 20.0 kN to a compressive force of 8.0 kN. Specify a suitable shape, material, and dimensions for the strut.

18. A part of a latch for a car door is to be made from a circular rod. With each actuation, it sees a tensile force that varies from 780 to 360 N. Small size is important. Complete the design, and specify a suitable shape, material, and dimensions for the rod.

19. A part of a bracket in the seat assembly of a bus is shown in Figure P5–8. The load varies from 1450 to 140 N as passengers enter and exit the bus. The bracket is made from AISI 1020 hot-rolled steel. Determine the resulting design factor.

20. For the bus seat bracket described in Problem 19 and shown in Figure P5–8, propose an alternate design for the bracket, different from that shown in the figure, to achieve a lighter design with a design factor of approximately 4.0.

21. A cantilevered boom is part of an assembly machine. A tool with a weight of 500 lb moves continuously from the end of the 60-in beam to a point 10 in from the support. Specify a suitable design for the boom, giving the material, the cross-sectional shape, and the dimensions.

22. A flat steel strip is used as a spring to maintain a force against part of a cabinet latch in a commercial printer as shown in Figure P5–9. When the cabinet door is open, the spring is deflected by the latch pin by an amount $y_1 = 0.25$ mm. The pin causes the deflection to increase to 0.40 mm when the door is closed. Specify a suitable material for the spring if it is made to the dimensions shown in the figure.

23. A part of the structure for a factory automation system is a beam that spans 30.0 in as shown in Figure P5–6. Loads are applied at two points, each 8.0 in from a support. The left load $F_1 = 1800$ lb remains constantly applied, while the right load $F_2 = 1800$ lb is applied and removed frequently as the machine cycles. If the rectangular tube is made from ASTM A500 Grade B steel, is the proposed design satisfactory? Improve the design to achieve a lighter beam.

24. Figure P5–24 shows a hydraulic cylinder that pushes a heavy tool during the outward stroke, placing a compressive load of 400 lb in the piston rod. During the return stroke, the rod pulls on the tool with a force of 1500 lb. Compute the resulting design factor for the 0.60-in-diameter rod when subjected to this pattern of forces for many cycles. The material is AISI 4130 WQT 1300 steel. If the resulting design factor is much different from 4.0, determine the size of rod that would produce $N = 4.0$.

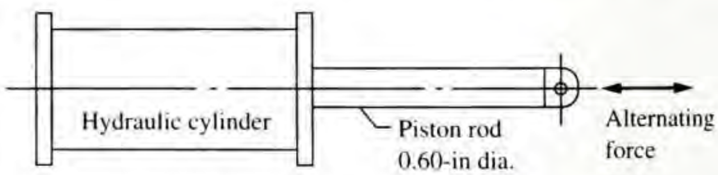


FIGURE P5–24 (Problem 24)

25. The cast iron cylinder shown in Figure P5–25 carries only an axial compressive load of 75 000 lb. (The torque $T = 0$.) Compute the design factor if it is made from gray cast iron, Grade 40, having a tensile ultimate strength of 40 ksi and a compressive ultimate strength of 140 ksi.

26. Repeat Problem 25, except using a tensile load with a magnitude of 12 000 lb.

27. Repeat Problem 25, except using a load that is a combination of an axial compressive load of 75 000 lb and a torsion of 20 000 lb · in.

28. The shaft shown in Figure P5–28 is supported by bearings at each end, which have bores of 20.0 mm. Design the shaft to carry the given load if it is steady and the shaft is stationary. Make the dimension a as large as possible while keeping the stress safe. Determine the required diameter in the middle portion. The maximum fillet per-

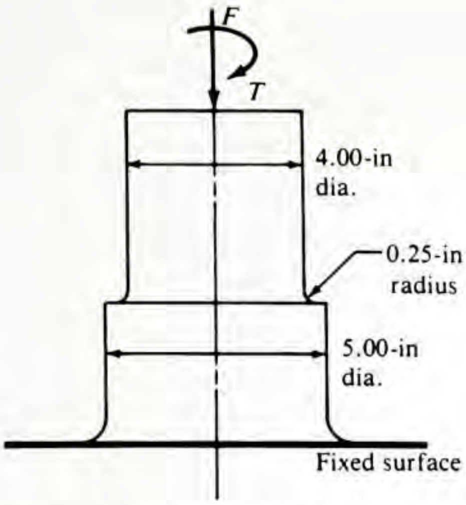


FIGURE P5–25 (Problems 25, 26, and 27)

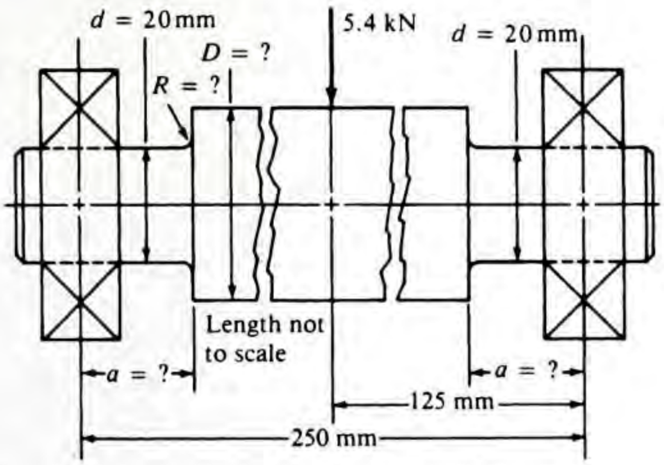


FIGURE P5–28 (Problems 28, 29, and 30)

missible is 2.0 mm. Use AISI 1137 cold-drawn steel. Use a design factor of 3.

29. Repeat Problem 28, except using a rotating shaft.

30. Repeat Problem 28, except using a shaft that is rotating and transmitting a torque of 150 N · m from the left bearing to the middle of the shaft. Also, there is a profile keyseat at the middle under the load.

31. Figure P5–31 shows a proposed design for a seat support. The vertical member is to be a standard pipe (see Table A16–6). Specify a suitable pipe to resist static loads simultaneously in the vertical and horizontal directions, as shown. The tube has properties similar to those of AISI 1020 hot-rolled steel. Use a design factor of 3.

32. A torsion bar is to have a solid circular cross section. It is to carry a fluctuating torque from 30 to 65 N · m. Use AISI 4140 OQT 1000 for the bar, and determine the required diameter for a design factor of 2. Attachments produce a stress concentration of 2.5 near the ends of the bar.

33. Determine the required size for a square bar to be made from AISI 1213 cold-drawn steel. It carries a constant axial tensile load of 1500 lb and a bending load that varies from zero to a maximum of 800 lb at the center of the 48-in length of the bar. Use a design factor of 3.

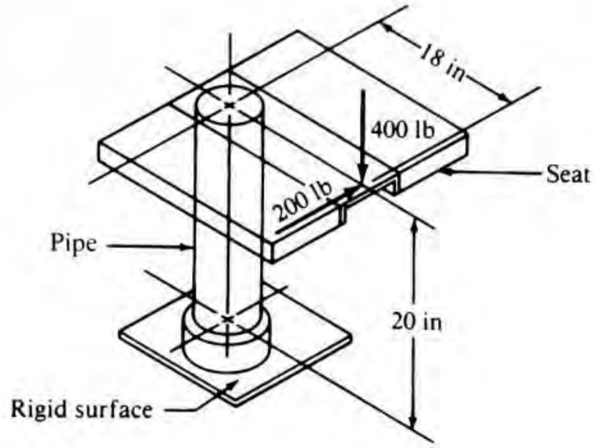


FIGURE P5–31 (Problem 31)

34. Repeat Problem 33, but add a constant torsional moment of 1200 lb · in to the other loads.

In some of the following problems, you are asked to compute the design factor resulting from the design proposed for the given loading. Unless stated otherwise, assume that the element being analyzed has a machined surface. If the design factor is significantly different from $N = 3$, redesign the component to achieve approximately $N = 3$. (See the figures in Chapter 3.)

35. A tensile member in a machine structure is subjected to a steady load of 4.50 kN. It has a length of 750 mm and is made from a steel tube, AISI 1040 hot-rolled, having an outside diameter of 18 mm and an inside diameter of 12 mm. Compute the resulting design factor.

36. A steady tensile load of 5.00 kN is applied to a square bar, 12 mm on a side and having a length of 1.65 m. Compute the stress in the bar and the resulting design factor if it is made from (a) AISI 1020 hot-rolled steel; (b) AISI 8650 OQT 1000 steel; (c) ductile iron A536-84 (60-40-18); (d) aluminum alloy 6061-T6; (e) titanium alloy Ti-6Al-4V, annealed; (f) rigid PVC plastic; and (g) phenolic plastic.

37. An aluminum rod, made from alloy 6061-T6, is made in the form of a hollow square tube, 2.25 in outside with a wall thickness of 0.125 in. Its length is 16.0 in. It carries an axial compressive force of 12 600 lb. Compute the resulting design factor. Assume that the tube does not buckle.

38. Compute the design factor in the middle portion only of the rod AC in Figure P3–8 if the steady vertical force on the boom is 2500 lb. The rod is rectangular, 1.50 in by 3.50 in, and is made from AISI 1144 cold-drawn steel.

39. Compute the forces in the two angled rods in Figure P3–9 for a steady applied force, $F = 1500$ lb, if the angle θ is 45°. Then design the middle portion of each rod to be circular and made from AISI 1040 hot-rolled steel. Specify a suitable diameter.

40. Repeat Problem 39 if the angle θ is 15°.

41. Figure 3–26 shows a portion of a circular bar that is subjected to a repeated and reversed force of 7500 N. If the bar is made from AISI 4140 OQT 1000, compute the resulting design factor.

42. Compute the torsional shear stress in a circular shaft having a diameter of 50 mm when subjected to a torque of 800 N·m. If the torque is completely reversed and repeated, compute the resulting design factor. The material is AISI 1040 WQT 1000.

43. If the torque in Problem 42 fluctuates from zero to the maximum of 800 N·m, compute the resulting design factor.

44. Compute the torsional shear stress in a circular shaft 0.40 inch in diameter that is due to a steady torque of 88.0 lb·in. Specify a suitable aluminum alloy for the rod.

45. Compute the required diameter for a solid circular shaft if it is transmitting a maximum of 110 hp at a speed of 560 rpm. The torque varies from zero to the maximum. There are no other significant loads on the shaft. Use AISI 4130 WQT 700.

46. Specify a suitable material for a hollow shaft with an outside diameter of 40 mm and an inside diameter of 30 mm when transmitting 28 kilowatts (kW) of steady power at a speed of 45 radians per second (rad/s).

47. Repeat Problem 46 if the power fluctuates from 15 to 28 kW.

48. Figure P5–48 shows part of a support bar for a heavy machine, suspended on springs to soften applied loads. The tensile load on the bar varies from 12 500 lb to a minimum of 7500 lb. Rapid cycling for many million cycles is expected. The bar is made from AISI 6150 OQT 1300 steel. Compute the design factor for the bar in the vicinity of the hole.

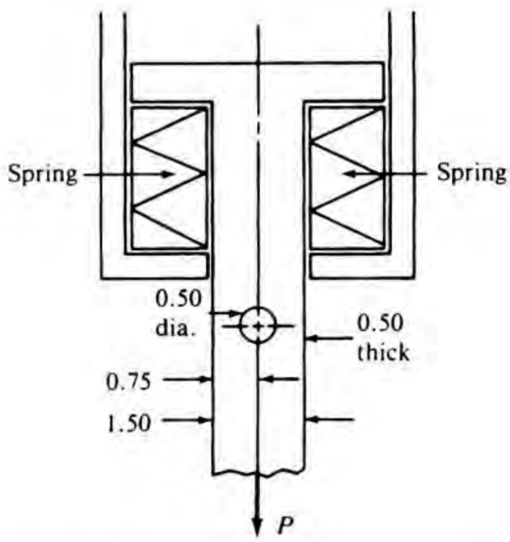


FIGURE P5–48 (Problem 48)

49. Figure P3–61 shows a valve stem from an engine subjected to an axial tensile load applied by the valve spring. The force varies from 0.80 to 1.25 kN. Compute the resulting design factor at the fillet under the shoulder. The valve is made from AISI 8650 OQT 1300 steel.

50. A conveyor fixture shown in Figure P3–62 carries three heavy assemblies (1200 lb each). The fixture is machined from AISI 1144 OQT 900 steel. Compute the resulting design factor in the fixture, considering stress concentrations at the fillets and assuming that the load acts axially. The load will vary from zero to the maximum as the conveyor is loaded and unloaded.

51. For the flat plate in tension in Figure P3–63, compute the minimum resulting design factor, assuming that the holes are sufficiently far apart that their effects do not interact. The plate is machined from stainless steel, UNS S17400 in condition H1150. The load is repeated and varies from 4000 to 6200 lb.

For Problems 52–56, select a suitable material for the member, considering stress concentrations, for the given loading to produce a design factor of $N = 3$.

52. Use Figure P3–64. The load is steady. The material is to be some grade of gray cast iron, ASTM A48.

53. Use Figure P3–65. The load varies from 20.0 to 30.3 kN. The material is to be titanium.

54. Use Figure P3–66. The torque varies from zero to 2200 lb·in. The material is to be steel.

55. Use Figure P3–67. The bending moment is steady. The material is to be ductile iron, ASTM A536.

56. Use Figure P3–68. The bending moment is completely reversed. The material is to be stainless steel.

57. Figure P5–57 shows part of an automatic screwdriver designed to drive several million screws. The maximum torque required to drive a screw is 100 lb·in. Compute the design factor for the proposed design if the part is made from AISI 8740 OQT 1000.

58. The beam in Figure P5–58 carries two steady loads, $P =$ 750 lb. Evaluate the design factor that would result if the beam were made from class 40 gray cast iron.

59. A tension link is subjected to a repeated, one-direction load of 3000 lb. Specify a suitable material if the link is to be steel and is to have a diameter of 0.50 in.

60. One member of an automatic transfer device in a factory must withstand a repeated tensile load of 800 lb and must not elongate more than 0.010 inch in its 25.0-in length. Specify a suitable steel material and the dimensions for the rod if it has a square cross section.

61. Figure P5–61 shows two designs for a beam to carry a repeated central load of 600 lb. Which design would have the highest design factor for a given material?

62. Refer to Figure P5–61. By reducing the 8.0-in dimension, redesign the beam in Part (b) of the figure so that it has a design factor equal to or higher than that for the design in Part (a).

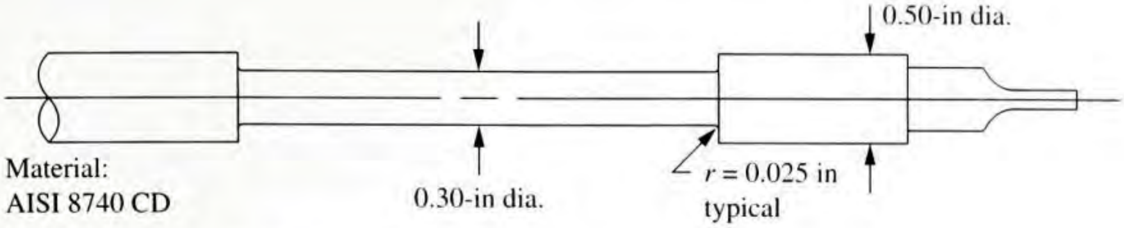


FIGURE P5–57 Screwdriver for Problem 57

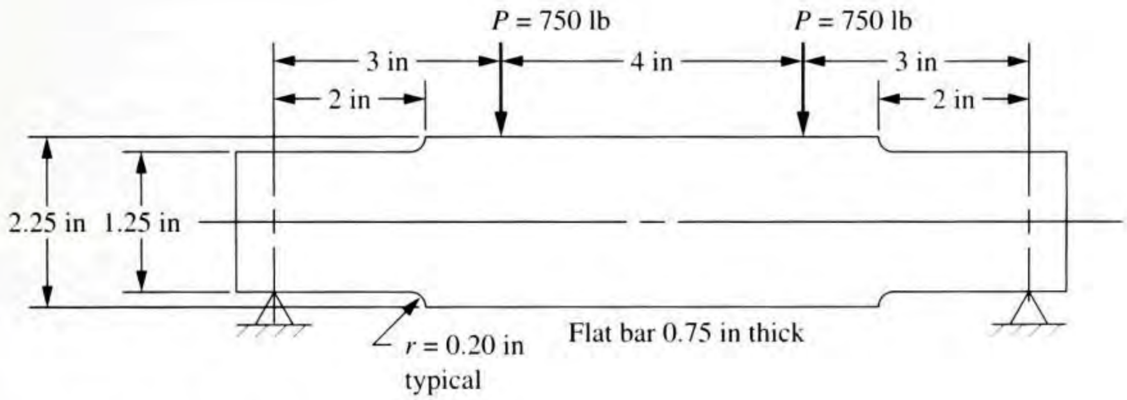


FIGURE P5–58 Beam for Problem 58

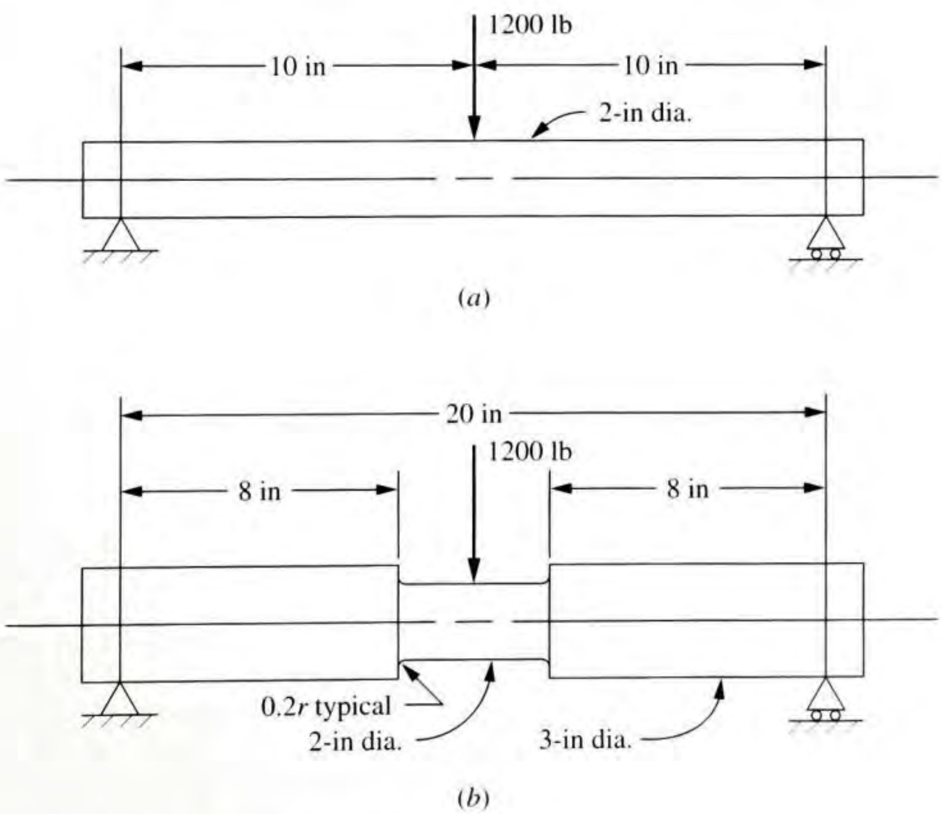


FIGURE P5–61 Beam for Problems 61, 62, and 63

63. Refer to Figure P5–61. Redesign the beam in Part (b) of the figure by first increasing the fillet radius to 0.40 in and then by reducing the 8.0-in dimension so that the new design has a design factor equal to or higher than that for the design in Part (a).

64. The part shown in Figure P5–64 is made from AISI 1040 HR steel. It is to be subjected to a repeated, one-direction force of 5000 lb applied through two 0.25-in-diameter pins in the holes at each end. Compute the resulting design factor.

65. For the part described in Problem 64, make at least three improvements in the design that will significantly reduce the stress without increasing the weight. The dimensions marked © are critical and cannot be changed. After the redesign, specify a suitable material to achieve a design factor of at least 3.

66. The link shown in Figure P5–66 is subjected to a tensile force that varies from 3.0 to 24.8 kN. Evaluate the design factor if the link is made from AISI 1040 CD steel.

67. The beam shown in Figure P5–67 carries a repeated, reversed load of 400 N applied at section C. Compute the resulting design factor if the beam is made from AISI 1340 OQT 1300.

68. For the beam described in Problem 67, change the steel's tempering temperature to achieve a design factor of at least 3.0.

69. The cantilever shown in Figure P5–69 carries a downward load that varies from 300 to 700 lb. Compute the resulting design factor if the bar is made from AISI 1050 HR steel.

70. For the cantilever described in Problem 69, increase the size of the fillet radius to improve the design factor to at least 3.0 if possible.

71. For the cantilever described in Problem 69, specify a suitable material to achieve a design factor of at least 3.0 without changing the geometry of the beam.

72. Figure P5–72 shows a rotating shaft carrying a steady downward load of 100 lb at C. Specify a suitable material.

73. The stepped rod shown in Figure P5–73 is subjected to a direct tensile force that varies from 8500 to 16 000 lb. If the rod is made from AISI 1340 OQT 700 steel, compute the resulting design factor.

74. For the rod described in Problem 73, complete a redesign that will achieve a design factor of at least 3.0. The two diameters cannot be changed.

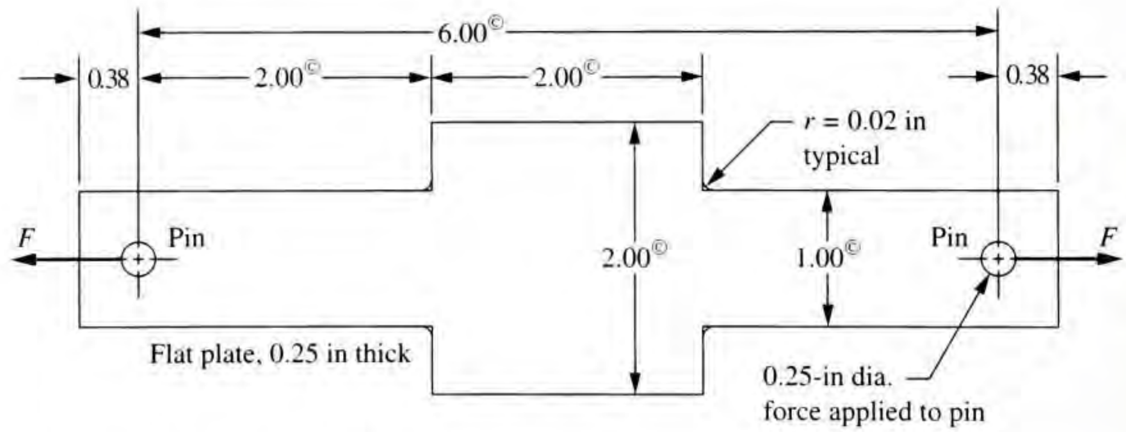


FIGURE P5–64 Beam for Problems 64 and 65

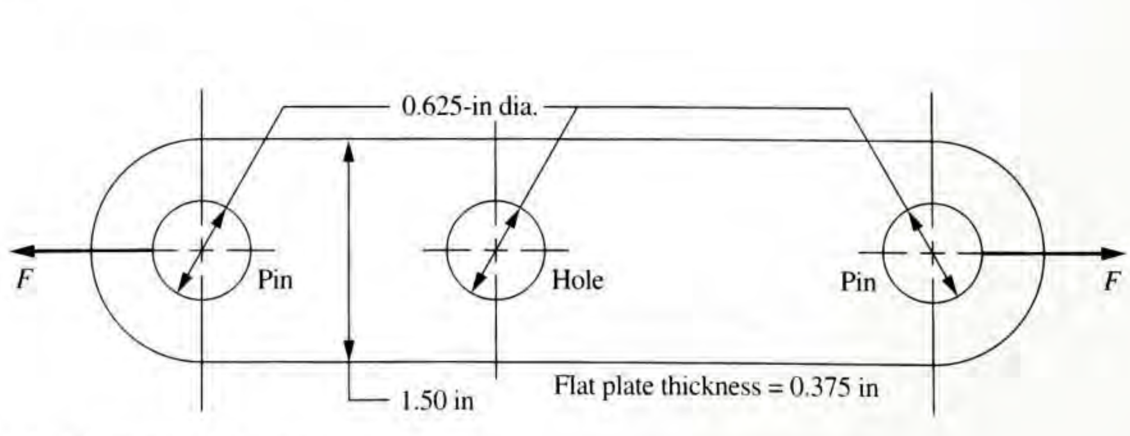


FIGURE P5–66 Link for Problem 66

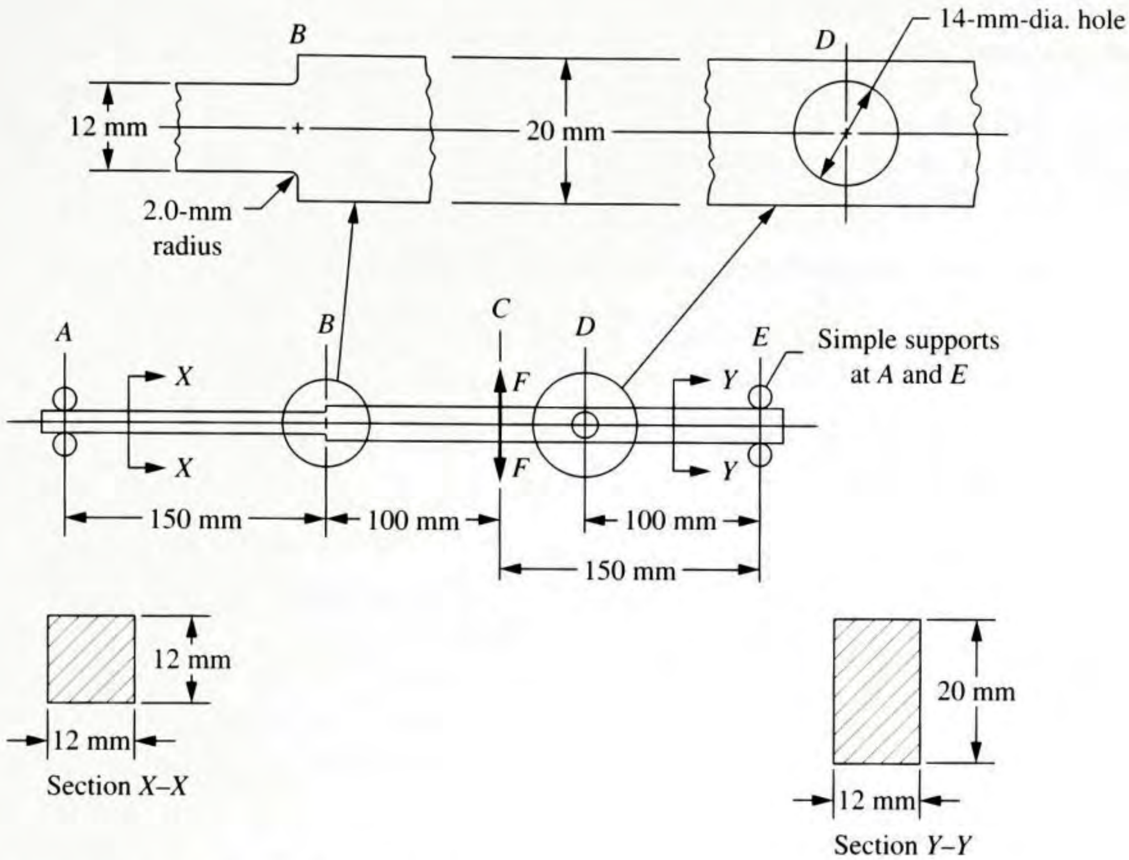


FIGURE P5–67 Beam for Problems 67 and 68

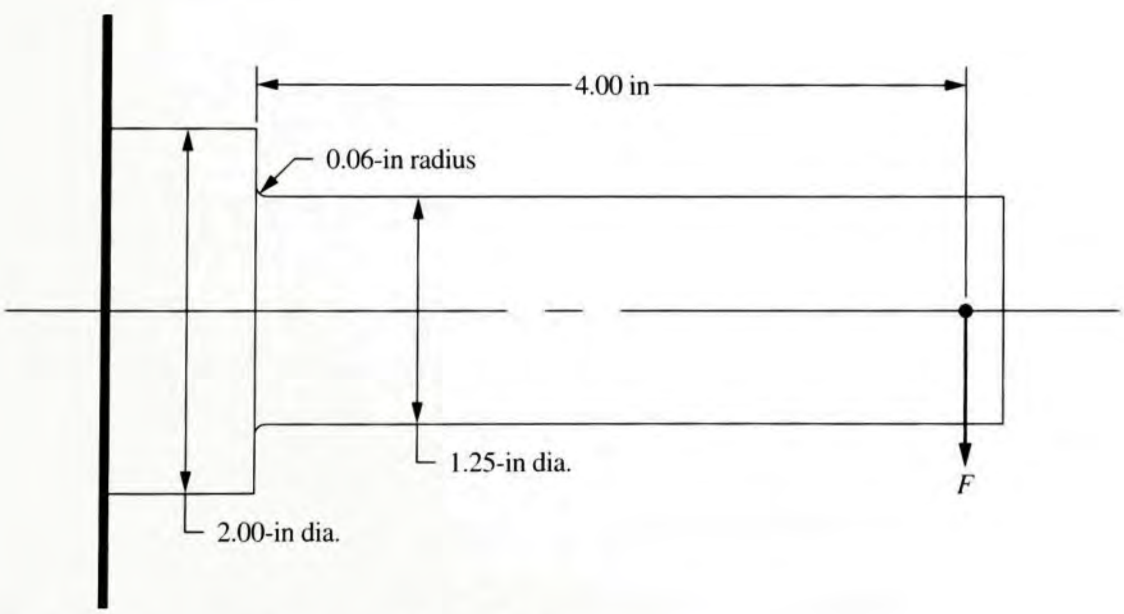


FIGURE P5–69 Cantilever for Problems 69, 70, and 71

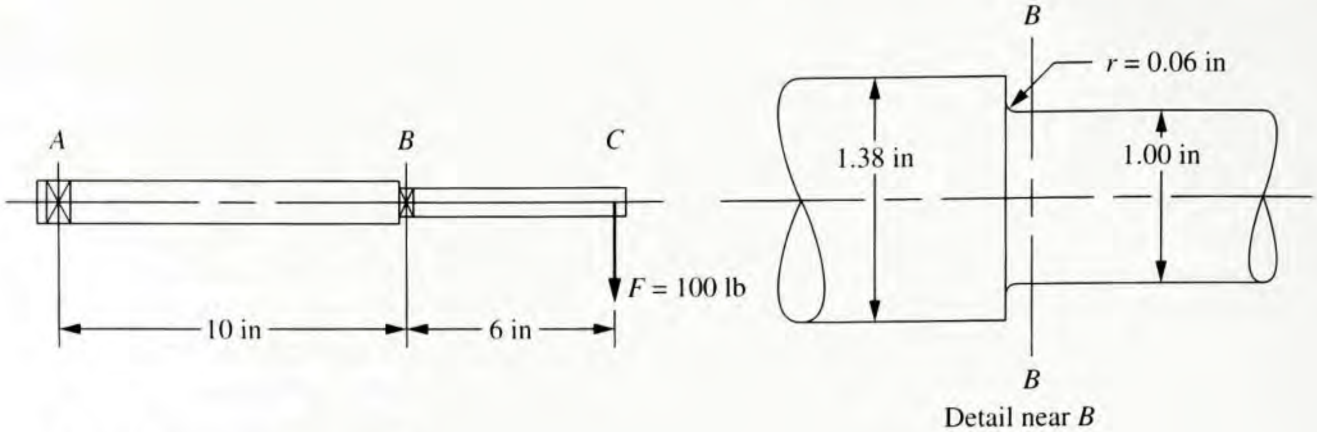


FIGURE P5–72 Shaft for Problem 72

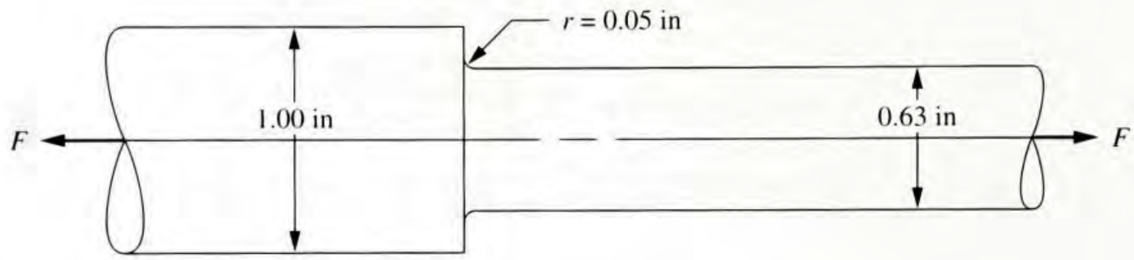


FIGURE P5–73 Rod for Problems 73 and 74

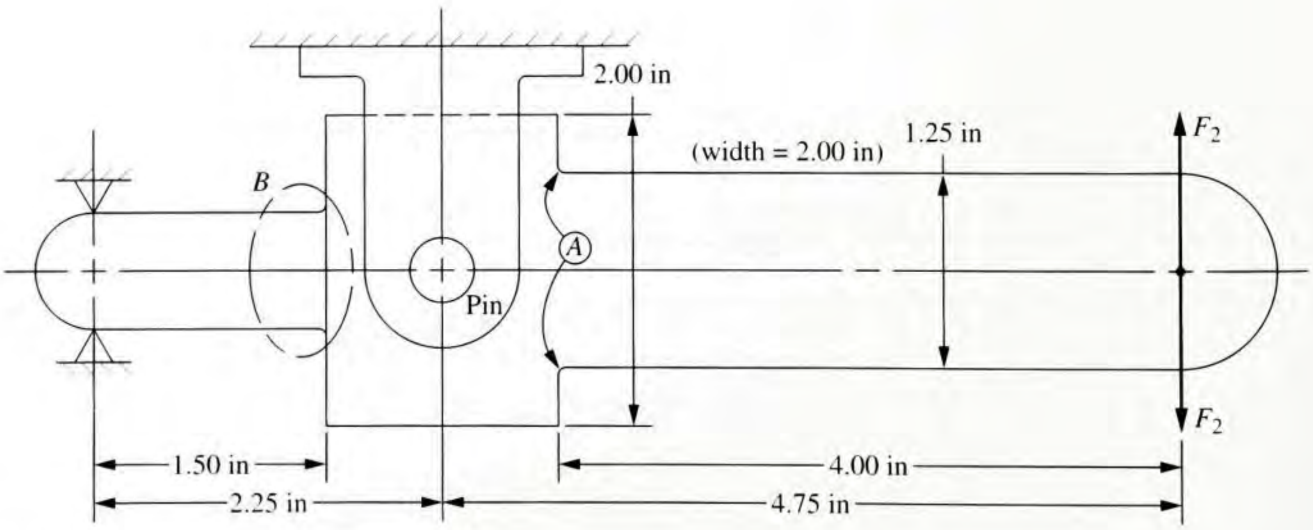


FIGURE P5–75 Beam for Problems 75 and 76

75. The beam shown in Figure P5–75 carries a repeated, reversed load of 800 lb alternately applied upward and downward. If the beam is made from AISI 1144 OQT 1100, specify the smallest acceptable fillet radius at *A* to ensure a design factor of 3.0.

76. For the beam described in Problem 75, design the section at *B* to achieve a minimum design factor of 3.0. Specify the shape, dimensions, and fillet radius where the smaller part joins the 2.00-by-2.00-in section.

Design Problems

For each of the following problems, complete the requested design to achieve a minimum design factor of 3.0. Specify the shape, the dimensions, and the material for the part to be designed. Work toward an efficient design that will have a low weight.

77. The link shown in Figure P5–77 carries a load of 3000 N that is applied and released many times. The link is machined from a square bar, 12.0 mm on a side, from AISI 1144 OQT 1100 steel. The ends must remain 12.0 mm square to facilitate the connection to mating parts. It is desired to reduce the size of the middle part of the link to reduce weight. Complete the design.

78. Complete the design of the beam shown in Figure P5–78 to carry a large hydraulic motor. The beam is attached to the two side rails of the frame of a truck. Because of the vertical accelerations experienced by the truck, the load on the beam varies from 1200 lb upward to 5000 lb downward. One-half of the load is applied to the beam by each foot of the motor.

79. A tensile member in a truss frame is subjected to a load that varies from 0 to 6500 lb as a traveling crane moves across the frame. Design the tensile member.

80. A hanger for a conveyor system extends outward from two supports as shown in Figure P5–80. The load at the right end varies from 600 to 3800 lb. Design the hanger.

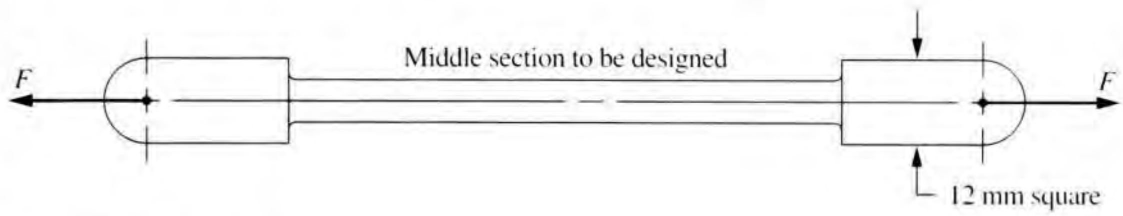


FIGURE P5–77 Link for Problem 77

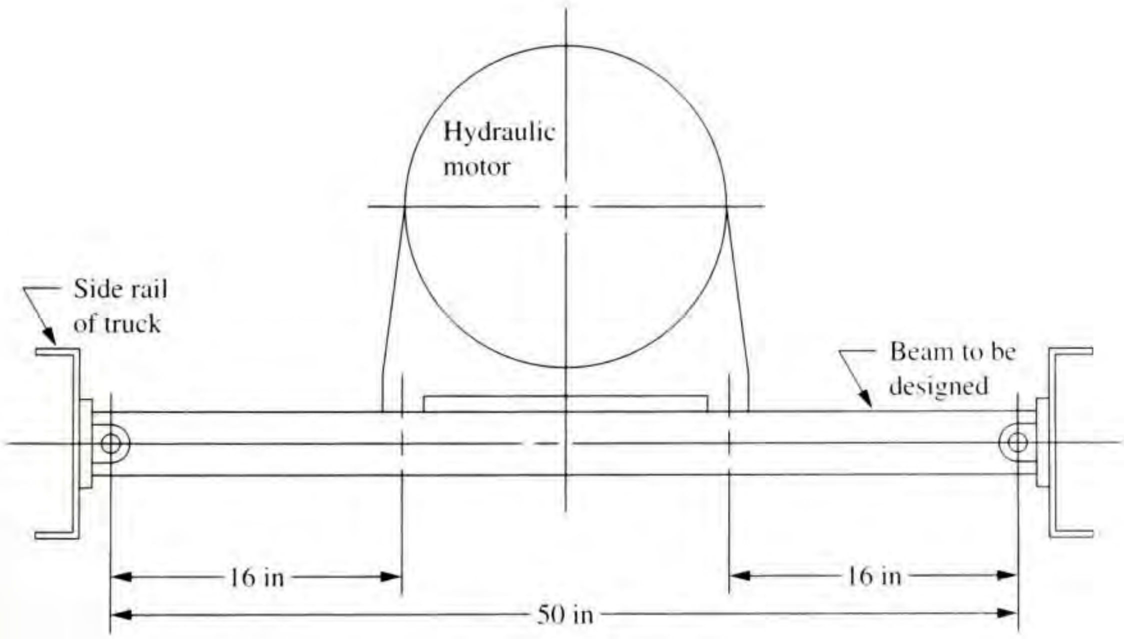


FIGURE P5–78 Beam for Problem 78

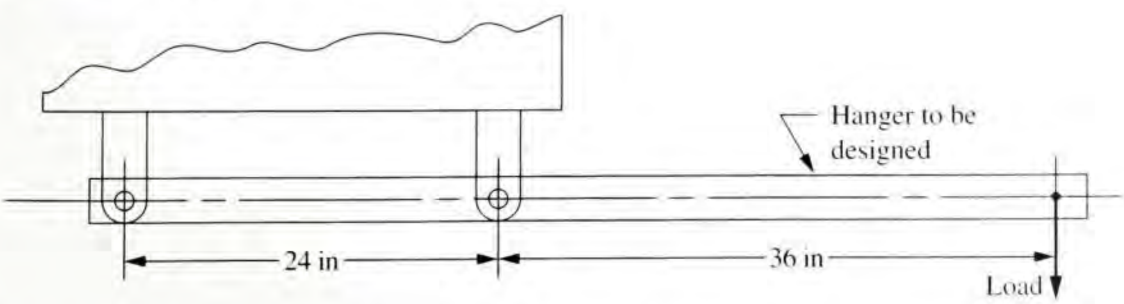


FIGURE P5–80 Hanger for the conveyor system for Problem 80

81. Figure P5–81 shows a yoke suspended beneath a crane beam by two rods. Design the yoke if the loads are applied and released many times.

82. For the system shown in Figure P5–81, design the two vertical rods if the loads are applied and released many times.

83. Design the connections between the rods and the yoke and the crane beam shown in Figure P5–81.

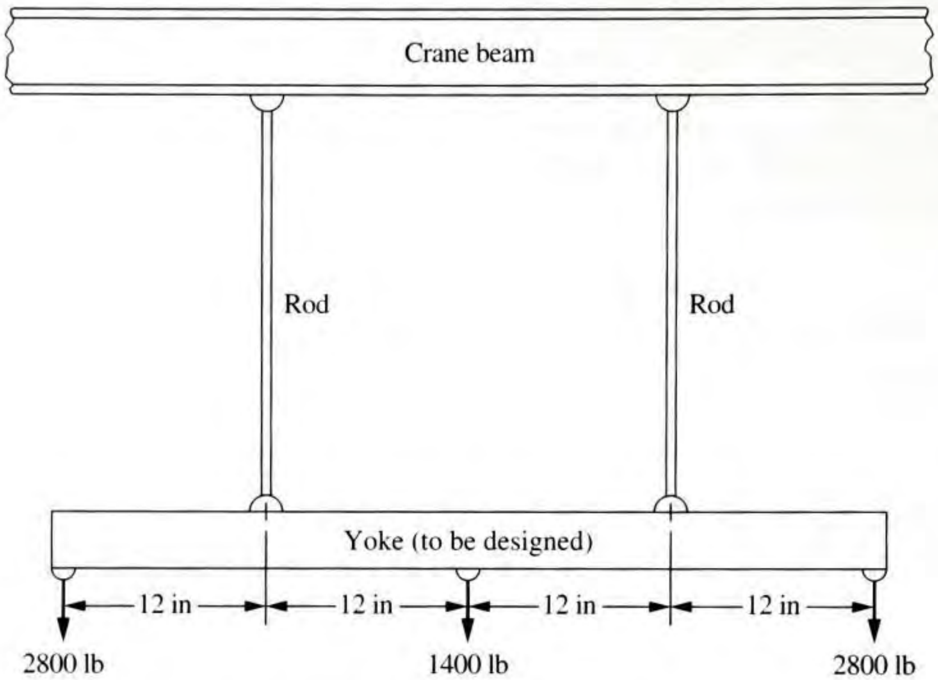


FIGURE P5–81 Yoke and rods for Problems 81, 82, and 83

6

Columns

The Big Picture

Columns

Discussion Map

- ☐ A column is a long, slender member that carries an axial compressive load and that fails due to buckling rather than due to failure of the material of the column.

Discover

Find at least 10 examples of columns. Describe them and how they are loaded, and discuss them with your colleagues.

Try to find at least one column that you can load conveniently by hand, and observe the buckling phenomenon.

Discuss with your colleagues the variables that seem to affect how a column fails and how much load it can carry before failing.

This chapter will help you acquire some of the analytical tools necessary to design and analyze columns.

A *column* is a structural member that carries an axial compressive load and that tends to fail by elastic instability, or buckling, rather than by crushing the material. *Elastic instability* is the condition of failure in which the shape of the column is insufficiently rigid to hold it straight under load. At the point of buckling, a radical deflection of the axis of the column occurs suddenly. Then, if the load is not reduced, the column will collapse. Obviously this kind of catastrophic failure must be avoided in structures and machine elements.

Columns are ideally straight and relatively long and slender. If a compression member is so short that it does not tend to buckle, failure analysis must use the methods presented in Chapter 5. This chapter presents several methods of analyzing and designing columns to ensure safety under a variety of loading conditions.

Take a few minutes to visualize examples of column buckling. Find any object that appears to be long and slender, for example, a meter stick, a plastic ruler, a long wooden dowel with a small diameter, a drinking straw, or a thin metal or plastic rod. Carefully apply a downward load on your column while resting the bottom on a desk or the floor. Try to make sure that it does not slide. Gradually increase the load, and observe the behavior of the column until it begins to bend noticeably in the middle. Then hold that level of load. Don't increase it much beyond that level, or the column will likely break!

Now release the load; the column should return to its original shape. The material should not have broken or yielded. But wouldn't you have considered the column to have failed at the point of buckling? Wouldn't it be important to keep the applied load well below the load that caused the buckling to be initiated?

Now look around you. Think of things you are familiar with, or take time to go out and find other examples of columns. Remember, look for relatively long, slender, load-carrying members subjected to compressive loads. Consider parts of furniture, buildings, cars, trucks, toys, play structures, industrial machinery, and construction machinery. Try to find at least 10 examples. Describe their appearance: the material from which they are made, the way they are supported, and the way they are loaded. Do this activity in the classroom or with colleagues; bring the descriptions to class next session for discussion.

Notice that you were asked to find *relatively long, slender*, load-carrying members. How will you know when a member is long and slender? At this point, you should just use your judgment. If the column is available and you are strong enough to load it to buckling, go ahead and try it. Later in this chapter, we will quantify what the terms *long* and *slender* mean.

If the columns you have seen buckle did not actually collapse, what property of the material is highly related to the phenomenon of buckling failure? Remember that the failure was described as *elastic instability*. Then it should seem that the *modulus of elasticity* of the material is a key property, and it is. Review the definition of this property from Chapter 1, and look up representative values in the tables of material properties in Appendices 3–13.

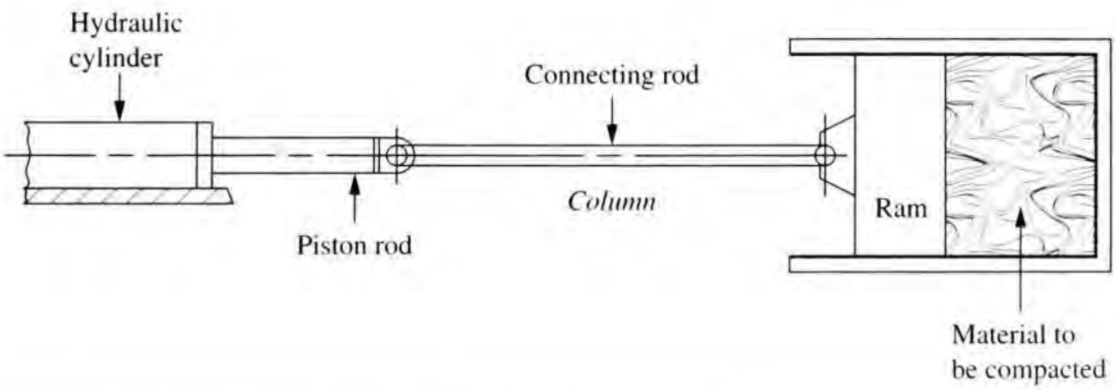
Also note that we specified that the columns are to be initially straight and that loads are to be applied axially. What if these conditions are not met? What if the column is a little crooked before loading? Do you think that it would carry as much compressive loading as one that was straight? Why or why not? What if the column is loaded *eccentrically*, that is, the load is directed off-center, away from the centroidal axis of the column? How will that affect the load-carrying ability? How does the manner of supporting the ends of the column affect its load-carrying ability? What standards exist that guide designers when dealing with columns?

These and other questions will be addressed in this chapter. Any time you are involved in a design in which a compressive load is applied, you should think about analyzing it as a column. The following **You Are the Designer** situation is a good example of such a machine design problem.

You Are the Designer

You are a member of a team that is designing a commercial compactor to reduce the volume of cardboard and paper waste so that the waste can be transported easily to a processing plant. Figure 6–1 is a sketch of the compaction ram that is driven by a hydraulic cylinder under several thousand pounds of force. The connecting rod between the hydraulic cylinder and the ram must be designed as a column because it is a relatively long, slender compression member. What shape should the cross section of the connecting rod be? From what material should it be made? How is it to be connected to the ram and to the hydraulic cylinder? What are the final dimensions of the rod to be? You, the designer, must specify all of these factors.

FIGURE 6–1 Waste paper compactor



6–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Recognize that any relatively long, slender compression member must be analyzed as a column to prevent buckling.
2. Specify efficient shapes for the cross section of columns.
3. Compute the *radius of gyration* of a column cross section.

4. Specify a suitable value for the *end-fixity factor*, K, and determine the *effective length* of a column.
5. Compute the *slenderness ratio* for columns.
6. Select the proper method of analysis or design for a column based on the manner of loading, the type of support, and the magnitude of the slenderness ratio.
7. Determine whether a column is *long* or *short* based on the value of the slenderness ratio in comparison with the *column constant*.
8. Use the *Euler formula* for the analysis and design of long columns.
9. Use the *J. B. Johnson formula* for the analysis and design of short columns.
10. Analyze crooked columns to determine the allowable load.
11. Analyze columns in which the load is applied with a modest amount of eccentricity to determine the maximum predicted stress and the maximum amount of deflection of the centerline of such columns under load.

6–2 PROPERTIES OF THE CROSS SECTION OF A COLUMN

The tendency for a column to buckle is dependent on the shape and the dimensions of its cross section, along with its length and the manner of attachment to adjacent members or supports. Cross-sectional properties that are important are as follows:

1. The cross sectional area, A.
2. The moment of inertia of the cross section, I, with respect to the axis about which the value of I is minimum.
3. The least value of the radius of gyration of the cross section, r.

The radius of gyration is computed from

Radius of Gyration

$$r = \sqrt{I/A} \tag{6–1}$$

A column tends to buckle about the axis for which the radius of gyration and the moment of inertia are minimum. Figure 6–2 shows a sketch of a column that has a rectangular cross section. The expected buckling axis is Y-Y because both I and r are much smaller for that axis than for the X-X axis. You can demonstrate this phenomenon by loading a common ruler or meter stick with an axial load of sufficient magnitude to cause buckling. See Appendix 1 for formulas for I and r for common shapes. See Appendix 16 for structural shapes.

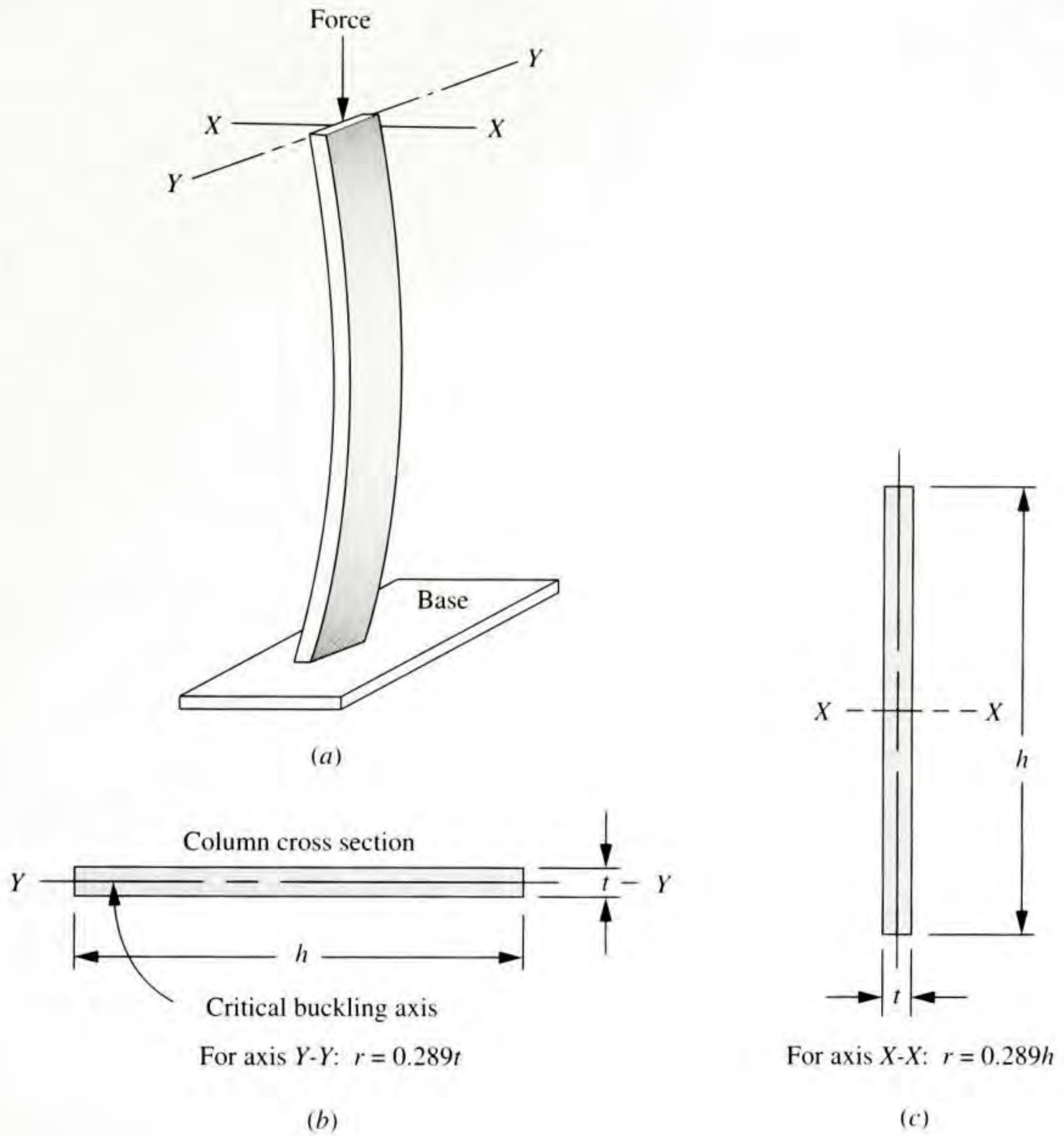
6–3 END FIXITY AND EFFECTIVE LENGTH

The term *end fixity* refers to the manner in which the ends of a column are supported. The most important variable is the amount of restraint offered at the ends of the column to the tendency for rotation. Three forms of end restraint are *pinned*, *fixed*, and *free*.

A *pinned-end* column is guided so that the end cannot sway from side to side, but it offers no resistance to rotation of the end. The best approximation of the pinned end would be a frictionless ball-and-socket joint. A cylindrical pin joint offers little resistance about one axis, but it may restrain the axis perpendicular to the pin axis.

A *fixed end* is one that is held against rotation at the support. An example is a cylindrical column inserted into a tight-fitting sleeve that itself is rigidly supported. The sleeve

FIGURE 6–2
Buckling of a thin, rectangular column. (a) General appearance of the buckled column. (b) Radius of gyration for *Y-Y* axis. (c) Radius of gyration for *X-X* axis



prohibits any tendency for the fixed end of the column to rotate. A column end securely welded to a rigid base plate is also a good approximation of a fixed-end column.

The *free end* can be illustrated by the example of a flagpole. The top end of a flagpole is unrestrained and unguided, the worst case for column loading.

The manner of support of both ends of the column affects the *effective length* of the column, defined as

Effective Length

$$L_e = KL \tag{6–2}$$

where L = actual length of the column between supports
K = constant dependent on the end fixity, as illustrated in Figure 6–3

The first values given for K are theoretical values based on the shape of the deflected column. The second values take into account the expected fixity of the column ends in real, practical structures. It is particularly difficult to achieve a true fixed-end column because of the lack of complete rigidity of the support or the means of attachment. Therefore, the higher value of K is recommended.

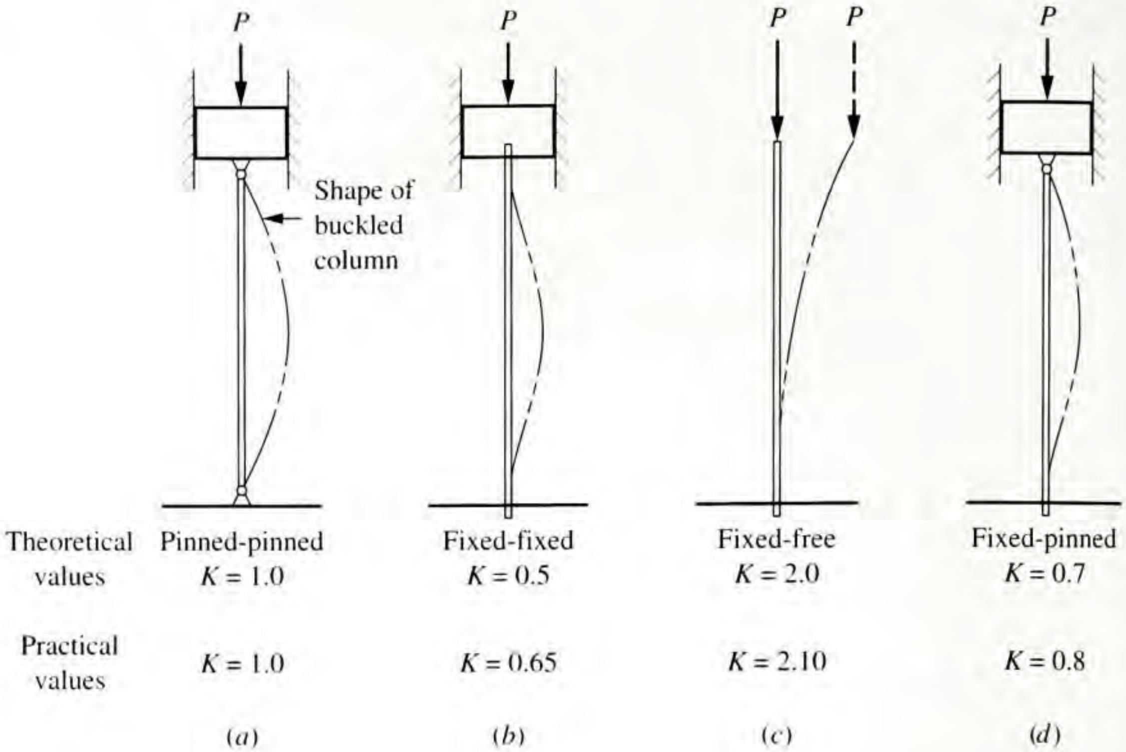


FIGURE 6–3 Values of K for effective length, $L_e = KL$, for different end connections

6–4 SLENDERNESS RATIO

Slenderness Ratio

The *slenderness ratio* is the ratio of the effective length of the column to its least radius of gyration. That is,

$$\text{Slenderness ratio} = L_e/r_{\min} = KL/r_{\min} \tag{6–3}$$

We will use the slenderness ratio to aid in the selection of the method of performing the analysis of straight, centrally loaded columns.

6–5 TRANSITION SLENDERNESS RATIO

In the following sections, two methods for analyzing straight, centrally loaded columns are presented: (1) the Euler formula for long, slender columns and (2) the J. B. Johnson formula for short columns.

The choice of which method to use depends on the value of the actual slenderness ratio for the column being analyzed in relation to the *transition slenderness ratio*, or *column constant*, C_c, defined as

Column Constant

$$C_c = \sqrt{\frac{2\pi^2 E}{s_y}} \tag{6–4}$$

where E = modulus of elasticity of the material of the column
s_y = yield strength of the material

The use of the column constant is illustrated in the following procedure for analyzing straight, centrally loaded columns.

Procedure for Analyzing Straight, Centrally Loaded Columns

1. For the given column, compute its actual slenderness ratio.
2. Compute the value of C_c.

3. Compare C_c with KL/r. Because C_c represents the value of the slenderness ratio that separates a long column from a short one, the result of the comparison indicates which type of analysis should be used.
4. If the actual KL/r is greater than C_c, the column is *long*. Use Euler's equation, as described in Section 6–6.
5. If KL/r is less than C_c, the column is *short*. Use the J. B. Johnson formula, described in Section 6–7.

Figure 6–4 is a logical flowchart for this procedure.

The value of the column constant, or transition slenderness ratio, is dependent on the material properties of modulus of elasticity and yield strength. For any given class of material, for example, steel, the modulus of elasticity is nearly constant. Thus, the value of C_c varies inversely as the square root of the yield strength. Figures 6–5 and 6–6 show the resulting values for steel and aluminum, respectively, for the range of yield strengths expected for each material. The figures show that the value of C_c decreases as the yield strength increases. The importance of this observation is discussed in the following section.

6–6 LONG COLUMN ANALYSIS: THE EULER FORMULA

Analysis of a long column employs the Euler formula (see Reference 3):

Euler Formula for Long Columns

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \tag{6–5}$$

DESIGN

The equation gives the critical load, P_{cr}, at which the column would begin to buckle.

An alternative form of the Euler formula is often desirable. Note that, from Equation (6–5),

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 EA}{(KL)^2/r^2} = \frac{\pi^2 EA\, r^2}{(KL)^2}$$

But, from the definition of the radius of gyration, r,

$$r = \sqrt{I/A}$$
$$r^2 = I/A$$

Then

Alternative Form of Euler Formula

$$P_{cr} = \frac{\pi^2 EA\, I}{(KL)^2 A} = \frac{\pi^2 EI}{(KL)^2} \tag{6–6}$$

This form of the Euler equation aids in a design problem in which the objective is to specify a size and a shape of a column cross section to carry a certain load. The moment of inertia for the required cross section can be easily determined from Equation (6–6).

Notice that the buckling load is dependent only on the geometry (length and cross section) of the column and the stiffness of the material represented by the modulus of elasticity. The strength of the material is not involved at all. For these reasons, it is often of no benefit to specify a high-strength material in a long column application. A lower-strength material having the same stiffness, E, would perform as well.

FIGURE 6–4
Analysis of a straight, centrally loaded column

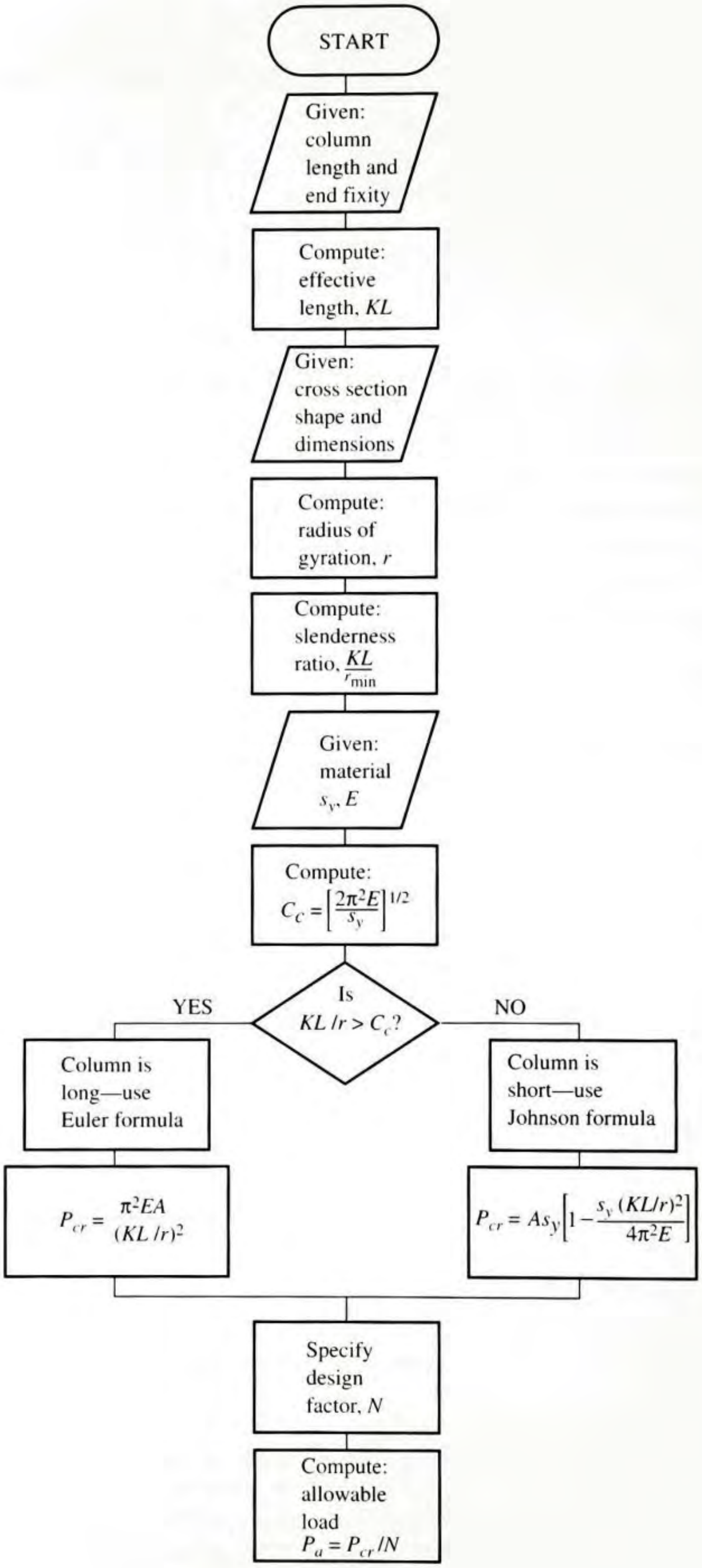


FIGURE 6–5
Transition slenderness ratio C_c vs. yield strength for steel

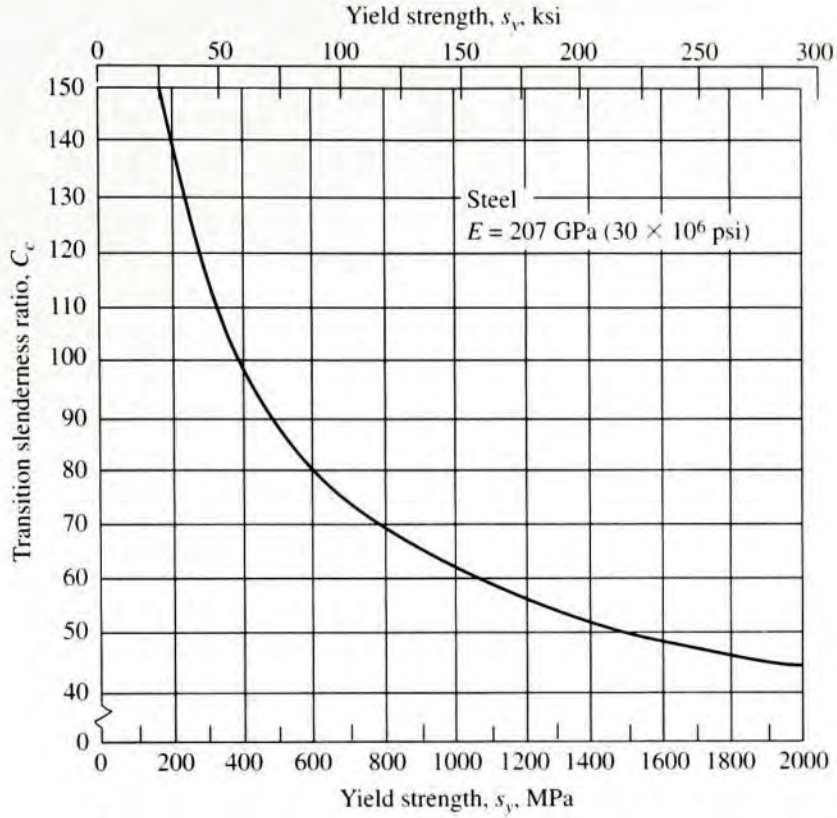
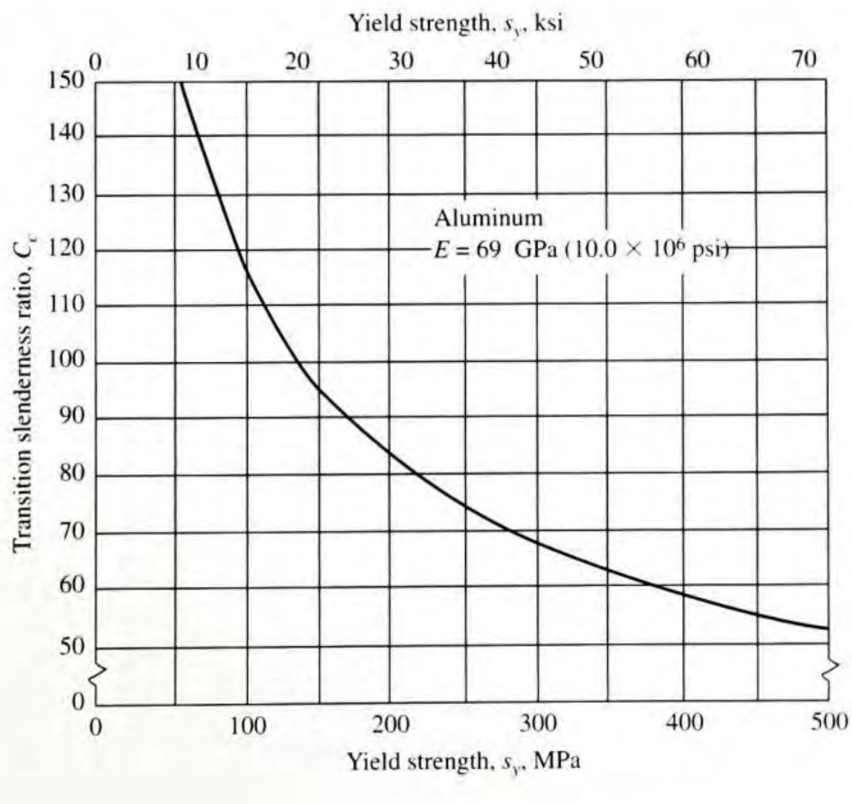


FIGURE 6–6
Transition slenderness ratio C_c vs. yield strength for aluminum



Design Factor and Allowable Load

Because failure is predicted to occur at a limiting load, rather than a stress, the concept of a design factor is applied differently than it is for most other load-carrying members. Rather than applying the design factor to the yield strength or the ultimate strength of the material, we apply it to the critical load, from Equation (6–5) or (6–6). For typical machine design applications, a design factor of 3 is used. For stationary columns with well-known loads and end fixity, a lower factor can be used, such as 2.0. A factor of 1.92 is used in some construction applications. Conversely, for very long columns, where there is some uncertainty about the loads or the end fixity, or where special dangers are presented, larger factors are advised. (See References 1 and 2.)

In summary, the objective of column analysis and design is to ensure that the load applied to a column is safe, well below the critical buckling load. The following definitions of terms must be understood:

$$P_{cr} = \text{critical buckling load}$$
$$P_a = \text{allowable load}$$
$$P = \text{actual applied load}$$
$$N = \text{design factor}$$

Then

Allowable Load

$$P_a = P_{cr}/N$$

The actual applied load, P, must be less than P_a.

Example Problem 6–1 A column has a solid circular cross section, 1.25 inches in diameter; it has a length of 4.50 ft and is pinned at both ends. If it is made from AISI 1020 cold-drawn steel, what would be a safe column loading?

Solution Objective: Specify a safe loading for the column.

Given: Solid circular cross section: diameter = d = 1.25 in; length = L = 4.50 ft.

Both ends of the column are pinned.

Material: AISI 1020 cold-drawn steel.

Analysis: Use the procedure in Figure 6–4.

Results: ***Step 1.*** For the pinned-end column, the end-fixity factor is $K = 1.0$. The effective length equals the actual length; $KL = 4.50\text{ ft} = 54.0\text{ in}$.

Step 2. From Appendix 1, for a solid round section,

$$r = D/4 = 1.25/4 = 0.3125\text{ in}$$

Step 3. Compute the slenderness ratio:

$$\frac{KL}{r} = \frac{1.0(54)}{0.3125} = 173$$

Step 4. Compute the column constant from Equation (6–4). For AISI 1020 cold-drawn steel, the yield strength is 51 000 psi, and the modulus of elasticity is 30×10^6 psi. Then

$$C_c = \sqrt{\frac{2\pi^2 E}{s_y}} = \sqrt{\frac{2\pi^2 (30 \times 10^6)}{51\,000}} = 108$$

Step 5. Because KL/r is greater than C_c, the column is long, and Euler's formula should be used. The area is

$$A = \frac{\pi D^2}{4} = \frac{\pi(1.25)^2}{4} = 1.23 \text{ in}^2$$

Then the critical load is

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2 (30 \times 10^6)(1.23)}{(173)^2} = 12\,200 \text{ lb}$$

At this load, the column should just begin to buckle. A safe load would be a reduced value, found by applying the design factor to the critical load. Let's use $N = 3$ to compute the *allowable load*, $P_a = P_{cr}/N$:

$$P_a = (12\,200)/3 = 4067 \text{ lb}$$

Comment The safe load on the column is 4067 lb.

6–7 SHORT COLUMN ANALYSIS: THE J. B. JOHNSON FORMULA

When the actual slenderness ratio for a column, KL/r, is less than the transition value, C_c, then the column is short, and the J. B. Johnson formula should be used. Use of the Euler formula in this range would predict a critical load greater than it really is.

The J. B. Johnson formula is written as follows:

J. B. Johnson Formula for Short Columns

$$P_{cr} = As_y\left[1 - \frac{s_y(KL/r)^2}{4\pi^2 E}\right] \tag{6–7}$$

Figure 6–7 shows a plot of the results of this equation as a function of the slenderness ratio, KL/r. Notice that it becomes tangent to the result of the Euler formula at the transition

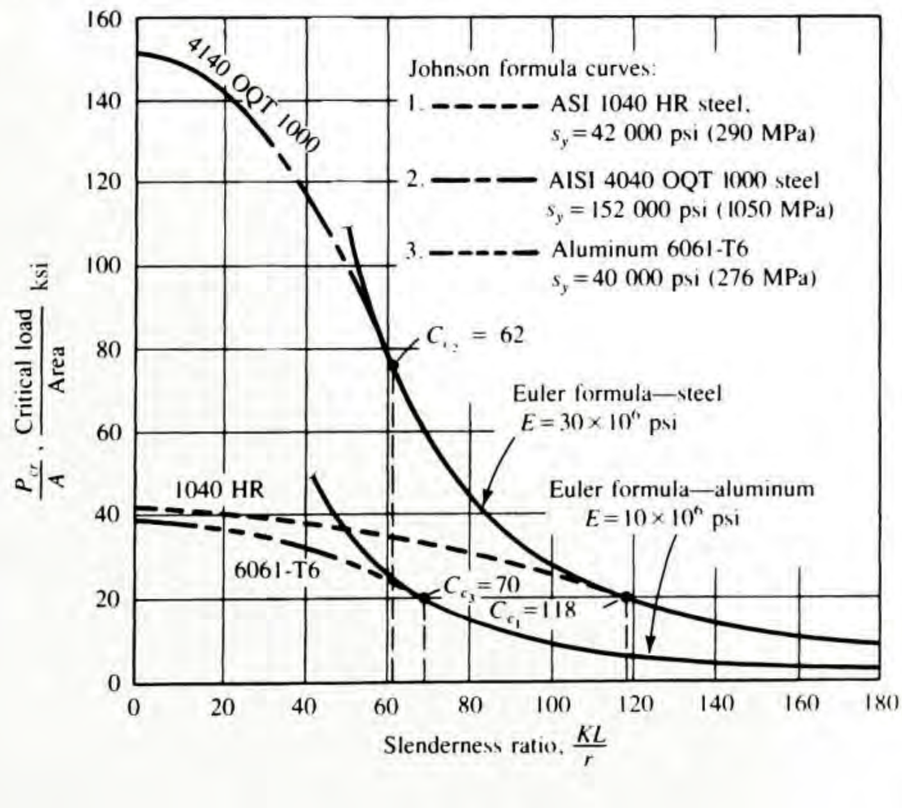


FIGURE 6–7 Johnson formula curves

slenderness ratio, the limit of its application. Also, at very low values for the slenderness ratio, the second term of the equation approaches zero, and the critical load approaches the yield load. Curves for three different materials are included in the figure to illustrate the effect of E and s_y on the critical load and the transition slenderness ratio.

The critical load for a short column is affected by the strength of the material in addition to its stiffness, E. As shown in the preceding section, strength is not a factor for a long column when the Euler formula is used.

Example Problem 6–2 Determine the critical load on a steel column having a rectangular cross section, 12 mm by 18 mm, and a length of 280 mm. It is proposed to use AISI 1040 hot-rolled steel. The lower end of the column is inserted into a close-fitting socket and is welded securely. The upper end is pinned (see Figure 6–8).

Solution Objective: Compute the critical load for the column.

Given: Solid rectangular cross section: $B = 12$ mm; $H = 18$ mm; $L = 280$ mm.

The bottom of column is fixed; the top is pinned (see Figure 6–8).

Material: AISI 1040 hot-rolled steel.

Analysis: Use the procedure in Figure 6–4.

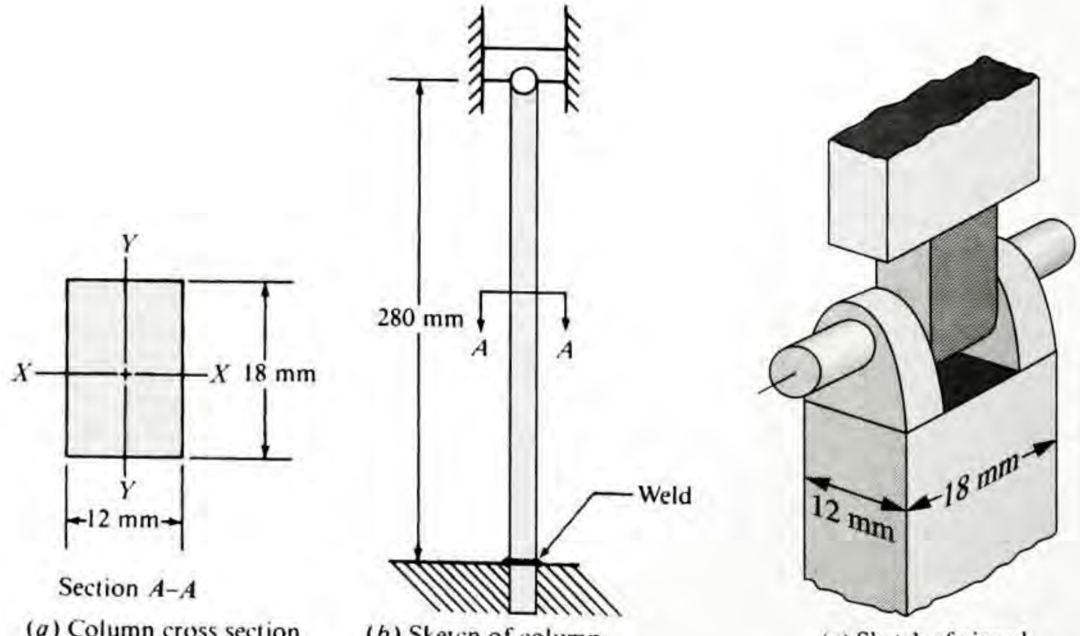
Results: ***Step 1.*** Compute the slenderness ratio. The radius of gyration must be computed about the axis that gives the least value. This is the Y-Y axis, for which

$$r = \frac{B}{\sqrt{12}} = \frac{12\text{ mm}}{\sqrt{12}} = 3.46\text{ mm}$$

The column has a fixed-pinned end fixity for which $K = 0.8$. Then

$$KL/r = [(0.8)(280)]/3.46 = 64.7$$

FIGURE 6–8 Steel column



(a) Column cross section (b) Sketch of column installation (c) Sketch of pinned connection

Step 2. Compute the transition slenderness ratio. For the AISI 1040 hot-rolled steel, $E = 207$ GPa and $s_y = 290$ MPa. Then, from Equation (6–4),

$$C_c = \sqrt{\frac{2\pi^2\,(207 \times 10^9\ \text{Pa})}{290 \times 10^6\ \text{Pa}}} = 119$$

Step 3. Then $KL/r < C_c$; thus the column is short. Use the J. B. Johnson formula to compute the critical load:

$$P_{cr} = As_y\left[1 - \frac{s_y(KL/r)^2}{4\pi^2 E}\right]$$

$$P_{cr} = (216\ \text{mm}^2)(290\ \text{N/mm}^2)\left[1 - \frac{(290 \times 10^6\ \text{Pa})(64.7)^2}{4\pi^2(207 \times 10^9\ \text{Pa})}\right] \tag{6–7}$$

$$P_{cr} = 53.3 \times 10^3\ \text{N} = 53.3\ \text{kN}$$

Comments This is the critical buckling load. We would have to apply a design factor to determine the allowable load. Specifying $N = 3$ results in $P_a = 17.8$ kN.

6–8 COLUMN ANALYSIS SPREADSHEET

Completing the process described in Figure 6–4 using a calculator, pencil, and paper is tedious. A spreadsheet automates the calculations after you have entered the pertinent data for the particular column to be analyzed. Figure 6–9 shows the output of a spreadsheet used to solve Example Problem 6–1. The layout of the spreadsheet could be done in many ways, and you are encouraged to develop your own style. The following comments describe the features of the given spreadsheet:

1. At the top of the sheet, instructions to the user are given for entering data and for units. This sheet is for U.S. Customary units only. A different sheet would be used if SI metric data were to be used. (See Figure 6–10, which gives the solution for Example Problem 6–2.)
2. On the left side of the sheet are listed the various data that must be provided by the user to run the calculations. On the right are listed the output values. Formulas for computing L_e, C_c, KL/r, and allowable load are written directly into the cell where the computed values show. The output data for the message "Column is: ***long***" and the critical buckling load are produced by *functions* set up within *macros* written in Visual Basic and placed on a separate sheet of the spreadsheet. Figure 6–11 shows the two macros used. The first (*LorS*) carries out the decision process to test whether the column is long or short as indicated by comparison of its slenderness ratio with the column constant. The second (*Pcr*) computes the critical buckling load using either the Euler formula or the J. B. Johnson formula, depending on the result of the *LorS* macro. These functions are called by statements in the cells where "long" and the computed value of the critical buckling load (12 197 lb) are located.
3. Having such a spreadsheet can enable you to analyze several design options quickly. For example, the given problem statement indicated that the ends were pinned, resulting in an end-fixity value of $K = 1$. What would happen if both ends were fixed? Simply changing the value of that one cell to $K = 0.65$ would cause the entire sheet to be recalculated, and the revised value of critical buckling load would be available almost instantly. The result is that $P_{cr} = 28\,868$ lb, an increase of 2.37 times the original value. With that kind of improvement, you, the designer, might be inclined to change the design to produce fixed ends.

COLUMN ANALYSIS PROGRAM		*Data from:* Example Problem 6–1
Refer to Figure 6–4 for analysis logic.		
Enter data for variables in *italics in shaded boxes.*		Use consistent U.S. Customary units.
Data to Be Entered:		**Computed Values:**
Length and End Fixity:		
Column length, L = 54 *in* *End fixity, K =* 1	→	Eq. length, $L_e = KL =$ 54.0 in
Material Properties:		
Yield strength, s_y = 51,000 *psi* *Modulus of elasticity, E =* 3.00E + 07 *psi*	→	Column constant, $C_c =$ 107.8
Cross Section Properties:		
[Note: Enter *r* or compute *r* = sqrt(*I*/*A*).] [Always enter Area.] [Enter zero for *I* or *r* if not used.]		
Area, A = 1.23 *in*^{2} *Moment of inertia, I =* 0 *in*^{4} ***Or*** *Radius of gyration, r =* 0.3125 *in*	→	Slender ratio, *KL*/*r* = 172.8
		Column is: ***long***
		Critical buckling load = *12,197 lb*
Design Factor:		
Design factor on load, N = 3	→	**Allowable load = *4,066 lb***

FIGURE 6–9 Spreadsheet for column analysis with data from Example Problem 6–1

COLUMN ANALYSIS PROGRAM		*Data from:* Example Problem 6–2
Refer to Figure 6–4 for analysis logic.		
Enter data for variables in *italics in shaded boxes.*		Use consistent SI metric units.
Data to Be Entered:		**Computed Values:**
Length and End Fixity:		
Column length, L = 280 mm *End fixity, K = 0.8*	→	Eq. length, L_e = KL = 224.0 mm
Material Properties:		
Yield strength, s_y = 290 MPa *Modulus of elasticity, E = 207 GPa*	→	Column constant, C_c = 118.7
Cross Section Properties:		
[Note: Enter *r* or compute *r* = sqrt(*I*/*A*).] [Always enter Area.] [Enter zero for *I* or *r* if not used.]		
Area, A = 216 mm^2 *Moment of inertia, I = 0 mm^4* ***Or*** *Radius of gyration, r = 3.5 mm*	→	Slender ratio, KL/r = 64.7
		Column is: ***short***
Design Factor:		**Critical buckling load = *53.32 kN***
Design factor on load, N = 3	→	**Allowable load = *17.77 kN***

FIGURE 6–10 Spreadsheet for column analysis with data from Example Problem 6–2

FIGURE 6–11 Macros used in the column analysis spreadsheet

```
' LorS Macro
' Determines if column is long or short.
Function LorS(SR, CC)
    If SR > CC Then
        LorS = "long"
    Else
        LorS = "short"
    End If
End Function

' Critical Load Macro
' Uses Euler formula for long columns
' Uses Johnson formula for short columns
Function Pcr(LorS, SR, E, A, Sy)
Const Pi = 3.1415926
    If LorS = "long" Then
        Pcr = Pi ^ 2 * E * A / SR ^ 2
        ' Euler Equation; Eq. (6-4)
    Else
        Pcr = A * Sy(1 - (Sy * SR ^ 2 / (4 * Pi ^ 2 * E)))
        ' Johnson Equation; Eq. (6-7)
    End If
End Function
```

6–9 EFFICIENT SHAPES FOR COLUMN CROSS SECTIONS

An *efficient shape* is one that provides good performance with a small amount of material. In the case of columns, the shape of the cross section and its dimensions determine the value of the radius of gyration, r. From the definition of the slenderness ratio, KL/r, we can see that as r gets larger, the slenderness ratio gets smaller. In the critical load equations, a smaller slenderness ratio results in a larger critical load, the most desirable situation. Therefore, it is desirable to maximize the radius of gyration to design an efficient column cross section.

Unless end fixity varies with respect to the axes of the cross section, the column will tend to buckle with respect to the axis with the *least* radius of gyration. So a column with equal values for the radius of gyration in any direction is desirable.

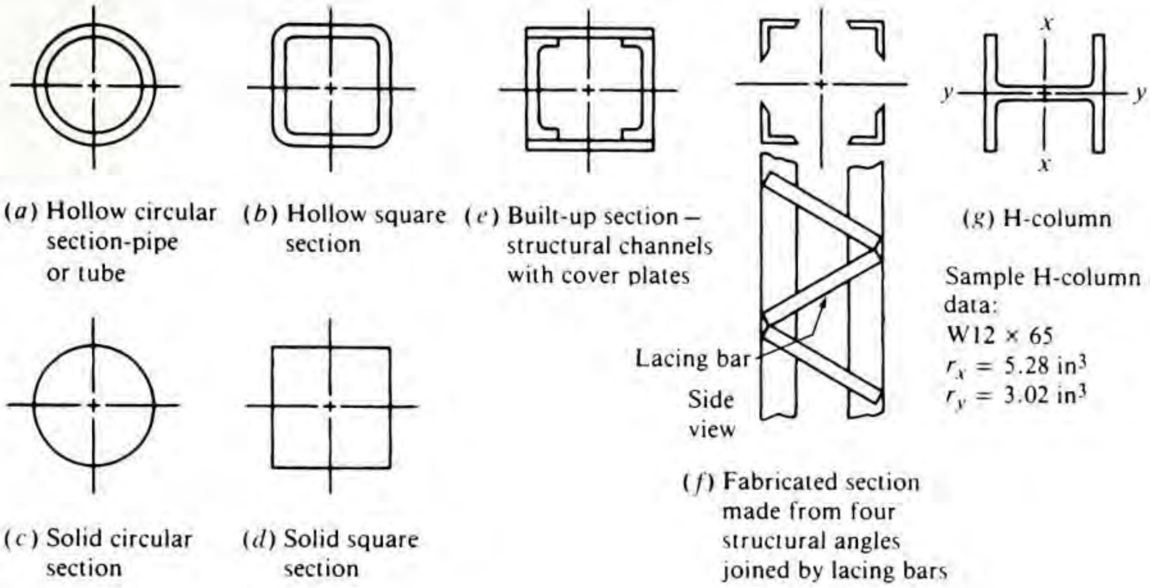
Review again the definition of the radius of gyration:

$$r = \sqrt{I/A}$$

This equation indicates that for a given area of material, we should try to maximize the moment of inertia to maximize the radius of gyration. A shape with a high moment of inertia has its area distributed far away from its centroidal axis.

Shapes that have the desirable characteristics described include circular hollow pipes and tubes, square hollow tubing, and fabricated column sections made from structural shapes placed at the outer boundaries of the section. Solid circular sections and solid square sections are also good, although not as efficient as the hollow sections. Figure 6–12(a–d) illustrates some of these shapes. The built-up section in (e) gives a rigid, boxlike section approximating the hollow square tube in larger sizes. In the case of the section in Figure 6–12(f), the angle sections at the corners provide the greatest contribution to the moment of inertia. The lacing bars merely hold the angles in position. The H-column in (g) has an equal depth and width and relatively heavy flanges and web. The moment of inertia with respect to the y-y axis is still smaller than for the x-x axis, but they are more nearly equal than

FIGURE 6–12
Column cross sections



for most other I-sections designed to be used as beams with bending in only one direction. Thus, this shape would be more desirable for columns.

6–10 THE DESIGN OF COLUMNS

DESIGN

In a design situation, the expected load on the column would be known, along with the length required by the application. The designer would then specify the following:

1. The manner of attaching the ends to the structure that affects the end fixity.
2. The general shape of the column cross section (for example, round, square, rectangular, and hollow tube).
3. The material for the column.
4. The design factor, considering the application.
5. The final dimensions for the column.

It may be desirable to propose and analyze several different designs to approach an optimum for the application. A computer program or spreadsheet facilitates the process.

It is assumed that items 1 through 4 are specified by the designer for any given trial. For some simple shapes, such as the solid round or square section, the final dimensions are computed from the appropriate formula: the Euler formula, Equation (6–5) or (6–6), or the J. B. Johnson formula, Equation (6–7). If an algebraic solution is not possible, iteration can be done.

In a design situation, the unknown cross-sectional dimensions make computing the radius of gyration and therefore the slenderness ratio, KL/r, impossible. Without the slenderness ratio, we cannot determine whether the column is long (Euler) or short (Johnson). Thus, the proper formula to use is not known.

We overcome this difficulty by making an assumption that the column is either long or short and proceeding with the corresponding formula. Then, after the dimensions are determined for the cross section, the actual value of KL/r will be computed and compared with C_c. This will show whether or not the correct formula has been used. If so, the computed answer is correct. If not, the alternate formula must be used and the computation repeated to determine new dimensions. Figure 6–13 shows a flowchart for the design logic described here.

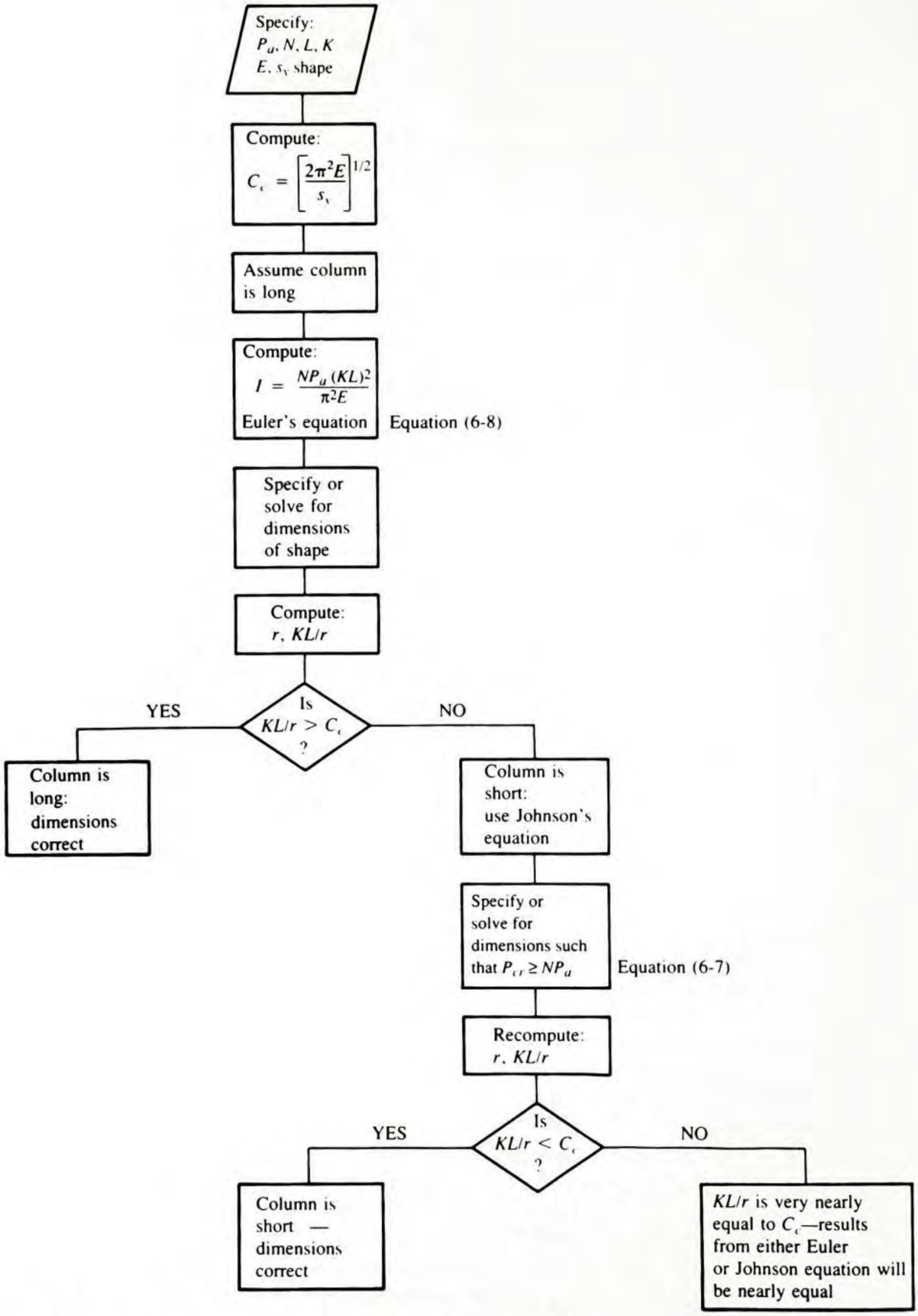


FIGURE 6–13 Design of a straight, centrally loaded column

Design: Assuming a Long Column

Euler's formula is used if the assumption is that the column is long. Equation (6–6) would be the most convenient form because it can be solved for the moment of inertia, I:

Euler's Formula Solved for Required Value of I

$$I = \frac{P_{cr}\,(KL)^2}{\pi^2\,E} = \frac{NP_a(KL)^2}{\pi^2\,E} \tag{6–8}$$

where $P_a =$ allowable load, usually set equal to the actual maximum expected load

Having the required value for I, we can determine the dimensions for the shape by additional computations or by scanning tables of data of the properties of commercially available sections.

The solid circular section is one for which it is possible to derive a final equation for the characteristic dimension, the diameter. The moment of inertia is

$$I = \frac{\pi D^4}{64}$$

Substituting this into Equation (6–8) gives

$$I = \frac{\pi D^4}{64} = \frac{NP_a\,(KL)^2}{\pi^2\,E}$$

Solving for D yields

Required Diameter for a Long, Solid Circular Column

$$D = \left[\frac{64NP_a\,(KL)^2}{\pi^3\,E}\right]^{1/4} \tag{6–9}$$

Design: Assuming a Short Column

The J. B. Johnson formula is used to analyze a short column. It is difficult to derive a convenient form for use in design. In the general case, then, trial and error is used.

For some special cases, including the solid circular section, it is possible to solve the Johnson formula for the characteristic dimension, the diameter:

$$P_{cr} = As_y\left[1 - \frac{s_y\,(KL/r)^2}{4\pi^2\,E}\right] \tag{6–7}$$

But

$$\begin{aligned} A &= \pi D^2/4 \\ r &= D/4 \text{ (from Appendix1)} \\ P_{cr} &= NP_a \end{aligned}$$

Then

$$NP_a = \frac{\pi D^2}{4}\,s_y\left[1 - \frac{s_y\,(KL)^2}{4\pi^2\,E(D/4)^2}\right]$$

$$\frac{4NP_a}{\pi s_y} = D^2\left[1 - \frac{s_y\,(KL)^2\,(16)}{4\pi^2\,ED^2}\right]$$

Solving for D gives

Required Diameter for a Short, Solid Circular Column

$$D = \left[\frac{4NP_a}{\pi s_y} + \frac{4s_y\,(KL)^2}{\pi^2\,E}\right]^{1/2} \tag{6–10}$$

Example Problem 6–3 Specify a suitable diameter of a solid, round cross section for a machine link if it is to carry 9800 lb of axial compressive load. The length will be 25 in, and the ends will be pinned. Use a design factor of 3. Use AISI 1020 hot-rolled steel.

Solution

Objective Specify a suitable diameter for the column.

Given

Solid circular cross section: $L = 25$ in; use $N = 3$.

Both ends are pinned.

Material: AISI 1020 hot-rolled steel.

Analysis Use the procedure in Figure 6–13. Assume first that the column is long.

Results From Equation (6–9),

$$D = \left[\frac{64NP_a(KL)^2}{\pi^3 E}\right]^{1/4} = \left[\frac{64(3)(9800)(25)^2}{\pi^3\,(30 \times 10^6)}\right]^{1/4}$$

$$D = 1.06 \text{ in}$$

The radius of gyration can now be found:

$$r = D/4 = 1.06/4 = 0.265 \text{ in}$$

The slenderness ratio is

$$KL/r = [(1.0)(25)]/0.265 = 94.3$$

For the AISI 1020 hot-rolled steel, $s_y = 30\,000$ psi. The graph in Figure 6–5 shows C_c to be approximately 138. Thus, the actual KL/r is less than the transition value, and the column must be redesigned as a short column, using Equation (6–10) derived from the Johnson formula:

$$D = \left[\frac{4NP_a}{\pi s_y} + \frac{4s_y\,(KL)^2}{\pi^2 E}\right]^{1/2}$$

$$D = \left[\frac{4(3)(9800)}{(\pi)(30\,000)} + \frac{4\,(30\,000)(25)^2}{\pi^2\,(30 \times 10^6)}\right]^{1/2} = 1.23 \text{ in} \qquad \textbf{(6–10)}$$

Checking the slenderness ratio again, we have

$$KL/r = [(1.0)(25)]/(1.23/4) = 81.3$$

Comments This is still less than the transition value, so our analysis is acceptable. A preferred size of $D = 1.25$ in could be specified.

An alternate method of using spreadsheets to design columns is to use an analysis approach similar to that shown in Figure 6–9 but to use it as a convenient "trial and error" tool. You could compute data by hand, or you could look them up in a table for A, I, and r for any desired cross-sectional shape and dimensions and insert the values into the spreadsheet. Then you could compare the computed allowable load with the required value and choose smaller or larger sections to bring the computed value close to the required value. Many iterations could be completed in a short amount of time. For shapes

Data to Be Entered		Computed Values
CIRCULAR COLUMN ANALYSIS		*Data from:* Example Problem 6–3
Refer to Figure 6–4 for analysis logic.		
Enter data for variables in *italics in shaded boxes.*		Use consistent U.S. Customary units.
Data to Be Entered:		**Computed Values:**
Length and End Fixity:		
Column length, L = *25 in* *End fixity, K =* *1*	→	Eq. length, $L_e = KL =$ 25.0 in
Material Properties:		
Yield strength, s_y *=* *30 000 psi* *Modulus of elasticity, E =* *3.00E + 07 psi*	→	Column constant, $C_c =$ 140.5
Cross Section Properties:		
[Note: *A* and *r* computed from] [dimensions for circular cross section] [in following section of this spreadsheet.]		
Area, A = *1.188 in²* *Radius of gyration, r =* *0.3075 in*	→	Slender ratio, $KL/r =$ 81.3
Properties for round column:		
Diameter for round column = *1.23 in* Area, *A* = 1.188 in² Radius of gyration, *r* = 0.3075 in		Column is: ***short*** **Critical buckling load = *29,679 lb***
Design Factor:		
Design factor on load, N = *3*	→	**Allowable load = *9,893 lb***

FIGURE 6–14 Spreadsheet for column analysis used as a tool to design a column with a round cross section

that allow computing r and A fairly simply, you could add a new section to the spreadsheet to calculate these values. An example is shown in Figure 6–14, where the differently shaded box shows the calculations for the properties of a round cross section. The data are from Example Problem 6–3, and the result shown was arrived at with only four iterations.

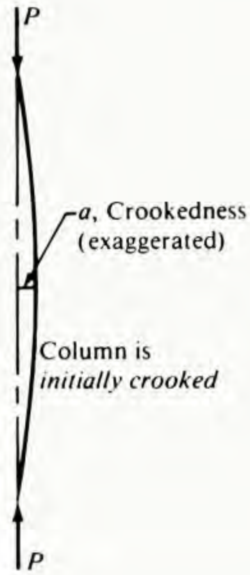


FIGURE 6–15
Illustration of crooked column

6–11 CROOKED COLUMNS

The Euler and Johnson formulas assume that the column is straight and that the load acts in line with the centroid of the cross section of the column. If the column is somewhat crooked, bending occurs in addition to the column action (see Figure 6–15).

The crooked column formula allows an initial crookedness, a, to be considered (see References 6, 7, and 8):

Crooked Column Formula

$$P_a^2 - \frac{1}{N}\left[s_y A + \left(1 + \frac{ac}{r^2}\right) P_{cr}\right] P_a + \frac{s_y A P_{cr}}{N^2} = 0 \qquad \textbf{(6–11)}$$

where c = distance from the neutral axis of the cross section about which bending occurs to its outer edge

P_{cr} is defined to be the critical load found from the *Euler formula*.

Although this formula may become increasingly inaccurate for shorter columns, it is not appropriate to switch to the Johnson formula as it is for straight columns.

The crooked column formula is a quadratic with respect to the allowable load P_a. Evaluating all constant terms in Equation (6–11) produces an equation of the form

$$P_a^2 + C_1 P_a + C_2 = 0$$

Then, from the solution for a quadratic equation,

$$P_a = 0.5\left[-C_1 - \sqrt{C_1^2 - 4C_2}\right]$$

The smaller of the two possible solutions is selected.

Example Problem 6–4 A column has both ends pinned and has a length of 32 in. It has a circular cross section with a diameter of 0.75 in and an initial crookedness of 0.125 in. The material is AISI 1040 hot-rolled steel. Compute the allowable load for a design factor of 3.

Solution Objective Specify the allowable load for the column.

Given Solid circular cross section: $D = 0.75$ in; $L = 32$ in; use $N = 3$.

Both are ends pinned. Initial crookedness $= a = 0.125$ in.

Material: AISI 1040 hot-rolled steel.

Analysis Use Equation (6–11). First evaluate C_1 and C_2. Then solve the quadratic equation for P_a.

Results

$$s_y = 42\,000 \text{ psi}$$

$$A = \pi D^2/4 = (\pi)(0.75)^2/4 = 0.442 \text{ in}^2$$

$$r = D/4 = 0.75/4 = 0.188 \text{ in}$$

$$c = D/2 = 0.75/2 = 0.375 \text{ in}$$

$$KL/r = [(1.0)(32)]/0.188 = 171$$

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} = \frac{\pi^2(30\,000\,000)(0.442)}{(171)^2} = 4476 \text{ lb}$$

$$C_1 = \frac{-1}{N}\left[s_y A + \left(1 + \frac{ac}{r^2}\right)P_{cr}\right] = -9649$$

$$C_2 = \frac{s_y A P_{cr}}{N^2} = 9.232 \times 10^6$$

The quadratic is therefore

$$P_a^2 - 9649\,P_a + 9.232 \times 10^6 = 0$$

Comment From this, $P_a = 1077$ lb is the allowable load.

Figure 6–16 shows the solution of Example Problem 6–4 using a spreadsheet. Whereas its appearance is similar to that of the earlier column analysis spreadsheets, the details follow the calculations needed to solve Equation (6–11). On the lower left, two special data values are needed: (1) the crookedness a and (2) the distance c from the neutral axis for buckling to the outer surface of the cross section. In the middle of the right part are listed some intermediate values used in Equation (6–11): C_1 and C_2 as defined in the solution to Example Problem 6–4. The result, the allowable load, P_a, is at the lower right of the spreadsheet. Above that, for comparison, the computed value of the critical buckling load is given for a straight column of the same design. Note that this solution procedure is most accurate for long columns. If the analysis indicates that the column is *short* rather than *long*, the designer should take note of how short it is by comparing the slenderness ratio, KL/r, with the column constant, C_c. If the column is quite short, the designer should not rely on the accuracy of the result from Equation (6–11).

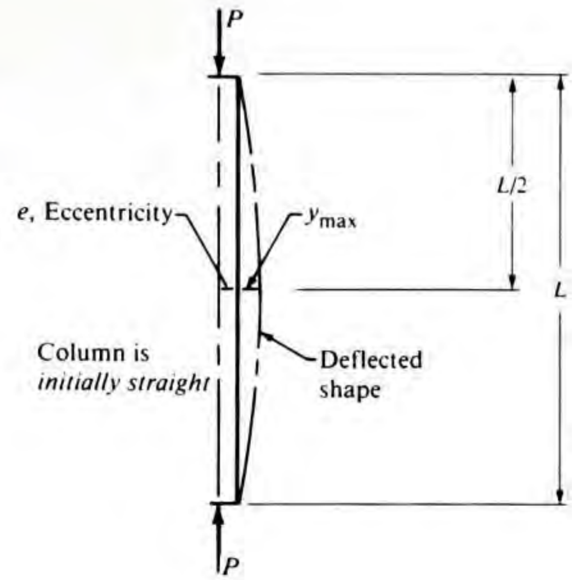
6–12 ECCENTRICALLY LOADED COLUMNS

An *eccentric load* is one that is applied away from the centroidal axis of the cross section of the column, as shown in Figure 6–17. Such a load exerts bending in addition to the column action that results in the deflected shape shown in the figure. The maximum stress in the deflected column occurs in the outermost fibers of the cross section at the midlength of

CROOKED COLUMN ANALYSIS		*Data from:* Example Problem 6–4
Solves Equation 6–11 for allowable load.		
Enter data for variables in *italics in shaded boxes.*		Use consistent U.S. Customary units.
Data to Be Entered:		**Computed Values:**
Length and End Fixity:		
Column length, L = *32 in* *End fixity, K =* *1*	→	Eq. length, $L_e = KL =$ 32.0 in
Material Properties:		
Yield strength, s_y *=* *42,000 psi* *Modulus of elasticity, E =* *3.00E+07 psi*	→	Column constant, $C_c =$ 18.7
Cross Section Properties:		Euler buckling load = 4,491 lb
[Note: Enter *r* or compute *r* = sqrt(*I*/*A*).] [Always enter Area.] [Enter zero for *I or r if not used.*]		C_1 in Eq. (6–11) = −9,678 C_2 in Eq. (6–11) = 9.259E+06
Area, A = *0.442 in*^{2} *Moment of inertia, I =* *0 in*^{4}		
Radius of gyration, r = *0.188 in*	→	Slender ratio, KL/r = 170.7
Values for Eq. (6–11):		
Initial crookedness = a = *0.125 in* *Neutral axis to outside = c =* *0.375 in*		Column is: ***long***
		Straight Column
		Critical buckling load = ***4,491 lb***
Design Factor:		***Crooked Column***
Design factor on load, N = *3*	→	**Allowable load =** ***1,076 lb***

FIGURE 6–16 Spreadsheet for analysis of crooked columns

FIGURE 6–17 Illustration of eccentrically loaded columns



DESIGN

the column where the maximum deflection, y_{max}, occurs. Let's denote the stress at this point as $\sigma_{L/2}$. Then, for any applied load, P,

Secant Formula for Eccentrically Loaded Columns

$$\sigma_{L/2} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{AE}}\right)\right] \tag{6–12}$$

(See References 4, 5, and 9.) Note that this stress is *not* directly proportional to the load. When evaluating the secant in this formula, note that its argument in the parentheses is in *radians*. Also, because most calculators do not have the secant function, recall that the secant is equal to 1/cosine.

For design purposes, we would like to specify a design factor, N, that can be applied to the *failure load* similar to that defined for straight, centrally loaded columns. However, in this case, failure is predicted when the maximum stress in the column exceeds the yield strength of the material. Let's now define a new term, P_y, to be the load applied to the eccentrically loaded column when the maximum stress is equal to the yield strength. Equation (6–12) then becomes

$$s_y = \frac{P_y}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P_y}{AE}}\right)\right]$$

Now, if we define the *allowable load* to be

$$P_a = P_y/N$$

or

$$P_y = NP_a$$

this equation becomes

Design Equation for Eccentrically Loaded Columns

$$\text{Required } s_y = \frac{NP_a}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{NP_a}{AE}}\right)\right] \tag{6–13}$$

This equation cannot be solved for either A or P_a. Therefore, an iterative solution is required, as will be demonstrated in Example Problem 6–6.

Another critical factor may be the amount of deflection of the axis of the column due to the eccentric load:

Maximum Deflection in an Eccentrically Loaded Column

$$y_{max} = e\left[\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{AE}}\right) - 1\right] \tag{6–14}$$

Note that the argument of the secant is the same as that used in Equation (6–12).

Example Problem 6–5 For the column of Example Problem 6–4, compute the maximum stress and deflection if a load of 1075 lb is applied with an eccentricity of 0.75 in. The column is initially straight.

Solution Objective Compute the stress and the deflection for the eccentrically loaded column.

Given Data from Example Problem 6–4, but eccentricity = e = 0.75 in.

Solid circular cross section: D = 0.75 in; L = 32 in; Initially straight

Both ends are pinned; KL = 32 in; r = 0.188 in; $c = D/2$ = 0.375 in.

Material: AISI 1040 hot-rolled steel; $E = 30 \times 10^6$ psi.

Analysis Use Equation (6–12) to compute maximum stress. Then use Equation (6–14) to compute maximum deflection.

Results All terms have been evaluated before. Then the maximum stress is found from Equation (6–12):

$$\sigma_{L/2} = \frac{1075}{0.422}\left[1 + \frac{(0.75)(0.375)}{(0.188)^2}\sec\left(\frac{32}{2(0.188)}\sqrt{\frac{1075}{(0.442)(30 \times 10^6)}}\right)\right]$$

$$\sigma_{L/2} = 29\,300 \text{ psi}$$

The maximum deflection is found from Equation (6–14):

$$y_{max} = 0.75\left[\sec\left(\frac{32}{2(0.188)}\sqrt{\frac{1075}{(0.442)(30 \times 10^6)}}\right) - 1\right] = 0.293 \text{ in}$$

Comments The maximum stress is 29 300 psi at the midlength of the column. The deflection there is 0.293 in from the original straight central axis of the column.

Example Problem 6–6 The stress in the column found in Example Problem 6–5 seems high for the AISI 1040 hot-rolled steel. Redesign the column to achieve a design factor of at least 3.

Solution Objective Redesign the eccentrically loaded column of Example Problem 6–5 to reduce the stress and achieve a design factor of at least 3.

Given Data from Example Problems 6–4 and 6–5.

Analysis Use a larger diameter. Use Equation (6–13) to compute the required strength. Then compare that with the strength of AISI 1040 hot-rolled steel. Iterate until the stress is satisfactory.

Results Appendix 3 gives the value for the yield strength of AISI 1040 HR to be 42 000 psi. If we choose to retain the same material, the cross-sectional dimensions of the column must be increased to decrease the stress. Equation (6–13) can be used to evaluate a design alternative.

The objective is to find suitable values for A, c, and r for the cross section such that $P_a = 1075$ lb; $N = 3$; $L_e = 32$ in; $e = 0.75$ in; and the value of the entire right side of the equation is less than 42 000 psi. The original design had a circular cross section with a diameter of 0.75 in. Let's try increasing the diameter to $D = 1.00$ in. Then

$$A = \pi D^2/4 = \pi(1.00\text{ in})^2/4 = 0.785\text{ in}^2$$
$$r = D/4 = (1.00\text{ in})/4 = 0.250\text{ in}$$
$$r^2 = (0.250\text{ in})^2 = 0.0625\text{ in}^2$$
$$c = D/2 = (1.00\text{ in})/2 = 0.50\text{ in}$$

Now let's call the right side of Equation (6–13) s'_y. Then

$$s'_y = \frac{3(1075)}{0.785}\left[1 + \frac{(0.75)(0.50)}{(0.0625)}\sec\left(\frac{32}{2(0.250)}\sqrt{\frac{(3)(1075)}{(0.785)(30\times 10^6)}}\right)\right]$$
$$s'_y = 37\,740\text{ psi} = \text{required value of } s_y$$

This is a satisfactory result because it is just slightly less than the value of s_y of 42 000 psi for the steel.

Now we can evaluate the expected maximum deflection with the new design using Equation (6–14):

$$y_{max} = 0.75\left[\sec\left(\frac{32}{2(0.250)}\sqrt{\frac{1075}{(0.785)(30\times 10^6)}}\right) - 1\right]$$
$$y_{max} = 0.076\text{ in}$$

Comments The diameter of 1.00 in is satisfactory. The maximum deflection for the column is 0.076 in.

Figure 6–18 shows the solution of the eccentric column problem of Example Problem 6–6 using a spreadsheet to evaluate Equations (6–13) and (6–14). It is a design aid that facilitates the iteration required to determine an acceptable geometry for a column to carry a specified load with a desired design factor. Note that the data are in U.S. Customary units. At the lower left of the spreadsheet, data required for Equations (6–13) and (6–14) are entered by the designer, along with the other data discussed for earlier column analysis spreadsheets. The **"FINAL RESULTS"** at the lower right show the computed value of the required yield strength of the material for the column and compare it with the given value entered by the designer near the upper left. The designer must ensure that the actual value is greater than the computed value. The last part of the right side of the spreadsheet gives the computed maximum deflection of the column that occurs at its midlength.

ECCENTRIC COLUMN ANALYSIS		*Data from:* Example Problem 6–6
Solves Equation (6–13) for design stress and Equation (6–14) for maximum deflection.		
Enter data for variables in *italics in shaded boxes.*		Use consistent U.S. Customary units.
Data to Be Entered:		**Computed Values:**
Length and End Fixity:		
Column length, L = *32 in* *End fixity, K =* *1*	→	Eq. length, $L_e = KL$ = 32.0 in
Material Properties:		
Yield strength, s_y = *42,000 psi* *Modulus of elasticity, E =* *3.00E + 07 psi*	→	Column constant, C_c = 118.7
Cross Section Properties:		Argument of sec = 0.749 for strength Value of secant = 1.3654 Argument of sec = 0.432 for deflection Value of secant = 1.1014
[Note: Enter *r* or compute *r* = sqrt(*I*/*A*).] [Always enter Area.] [Enter zero for *I or r if not used.*] *Area, A =* *0.785 in^2* *Moment of inertia, I =* *0 in^4* ***OR*** *Radius of gyration, r =* *0.250 in*	→	Slender ratio, KL/r = 128.0
Values for Eqs. (6–13) and (6–14): *Eccentricity, e =* *0.75 in* *Neutral axis to outside, c =* *0.5 in* *Allowable load, P_a =* *1,075 lb*		Column is: ***long*** **FINAL RESULTS**
Design Factor: *Design factor on load, N =* *3*		**Req'd yield strength =** ***37,764 psi*** ***Must be less than actual yield strength:*** **s_y = 42,000 psi**
		Max deflection, y_{max} = *0.076 in*

FIGURE 6–18 Spreadsheet for analysis of eccentric columns

REFERENCES

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PROBLEMS

1. A column has both ends pinned and has a length of 32 in. It is made of AISI 1040 HR steel and has a circular shape with a diameter of 0.75 in. Determine the critical load.

2. Repeat Problem 1 using a length of 15 in.

3. Repeat Problem 1 with the bar made of aluminum 6061-T4.

4. Repeat Problem 1 assuming both ends fixed.

5. Repeat Problem 1 using a square cross section, 0.65 in on a side, instead of the circular cross section.

6. Repeat Problem 1 with the bar made from high-impact acrylic plastic.

7. A rectangular steel bar has a cross section 0.50 by 1.00 in and is 8.5 in long. The bar has pinned ends and is made of AISI 4150 OQT 1000 steel. Compute the critical load.

8. A steel pipe has an outside diameter of 1.60 in, a wall thickness of 0.109 in, and a length of 6.25 ft. Compute the critical load for each of the end conditions shown in Figure 6–2. Use AISI 1020 HR steel.

9. Compute the required diameter of a circular bar to be used as a column carrying a load of 8500 lb with pinned ends. The length is 50 in. Use AISI 4140 OQT 1000 steel and a design factor of 3.0.

10. Repeat Problem 9 using AISI 1020 HR steel.

11. Repeat Problem 9 with aluminum 2014-T4.

12. In Section 6–10, equations were derived for the design of a solid circular column, either long or short. Perform the derivation for a solid square cross section.

13. Repeat the derivations called for in Problem 12 for a hollow circular tube for any ratio of inside to outside diameter. That is, let the ratio $R = ID/OD$, and solve for the required OD for a given load, material, design factor, and end fixity.

14. Determine the required dimensions of a column with a square cross section to carry an axial compressive load of 6500 lb if its length is 64 in and its ends are fixed. Use a design factor of 3.0. Use aluminum 6061-T6.

15. Repeat Problem 14 for a hollow aluminum tube (6061-T6) with the ratio of $ID/OD = 0.80$. Compare the weight of this column with that of Problem 14.

16. A toggle device is being used to compact scrap steel shavings, as illustrated in Figure P6–16. Design the two links of the toggle to be steel, AISI 5160 OQT 1000, with a circular cross section and pinned ends. The force P required to crush the shavings is 5000 lb. Use $N = 3.50$.

17. Repeat Problem 16, but propose a design that will be lighter than the solid circular cross section.

18. A sling, sketched in Figure P6–18, is to carry 18 000 lb. Design the spreader.

19. For the sling in Problem 18, design the spreader if the angle shown is changed from 30° to 15°.

20. A rod for a certain hydraulic cylinder behaves as a fixed-free column when used to actuate a compactor of industrial waste. Its maximum extended length will be 10.75 ft. If it is to be made of AISI 1144 OQT 1300 steel, determine the required diameter of the rod for a design factor of 2.5 for an axial load of 25 000 lb.

21. Design a column to carry 40 000 lb. One end is pinned, and the other is fixed. The length is 12.75 ft.

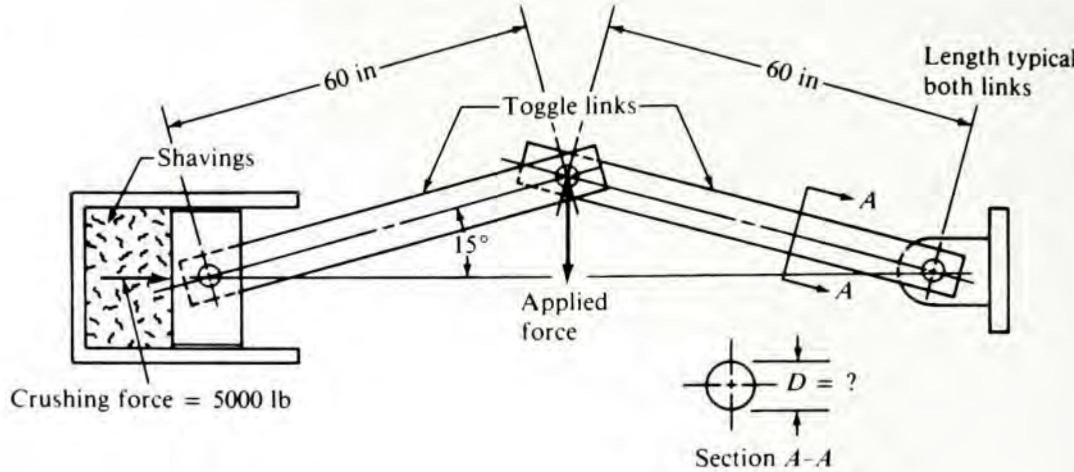
22. Repeat Problem 21 using a length of 4.25 ft.

23. Repeat Problem 1 if the column has an initial crookedness of 0.08 in. Determine the allowable load for a design factor of 3.

24. Repeat Problem 7 if the column has an initial crookedness of 0.04 in. Determine the allowable load for a design factor of 3.

25. Repeat Problem 8 if the column has an initial crookedness of 0.15 in. Determine the allowable load for a design factor of 3 and pinned ends only.

FIGURE P6–16 (Problems 16 and 17)



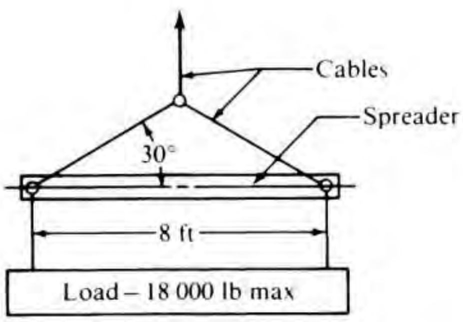


FIGURE P6–18 (Problems 18 and 19)

26. An aluminum (6063-T4) column is 42 in long and has a square cross section, 1.25 in on a side. If it carries a compressive load of 1250 lb, applied with an eccentricity of 0.60 in, compute the maximum stress in the column and the maximum deflection.

27. A steel (AISI 1020 hot-rolled) column is 3.2 m long and is made from a standard 3-in Schedule 40 steel pipe (see Table A16–6). If a compressive load of 30.5 kN is applied with an eccentricity of 150 mm, compute the maximum stress in the column and the maximum deflection.

28. A link in a mechanism is 14.75 in long and has a square cross section, 0.250 in on a side. It is made from annealed AISI 410 stainless steel. Use $E = 28 \times 10^6$ psi. If it carries a compressive load of 45 lb with an eccentricity of 0.30 in, compute the maximum stress and the maximum deflection.

29. A hollow square steel tube, 40 in long, is proposed for use as a prop to hold up the ram of a punch press during installation of new dies. The ram weighs 75 000 lb. The prop is made from 4 × 4 × 1/4 structural tubing. It is made from steel similar to structural steel, ASTM A500 Grade C. If the load applied by the ram could have an eccentricity of 0.50 in, would the prop be safe?

30. Determine the allowable load on a column 16.0 ft long made from a wide-flange beam shape, W5 × 19. The load will be centrally applied. The end conditions are somewhat between fixed and hinged, say, $K = 0.8$. Use a design factor of 3. Use ASTM A36 structural steel.

31. Determine the allowable load on a fixed-end column having a length of 66 in if it is made from a steel American Standard Beam, S4 × 7.7. The material is ASTM A36 structural steel. Use a design factor of 3.

32. Compute the maximum stress and deflection that can be expected in the steel machine member carrying an eccentric load as shown in Figure P6–32. The load P is 1000 lb. If a design factor of 3 is desired, specify a suitable steel.

33. Specify a suitable steel tube from Table A16–5 to support one side of a platform as shown in Figure P6–33. The ma-

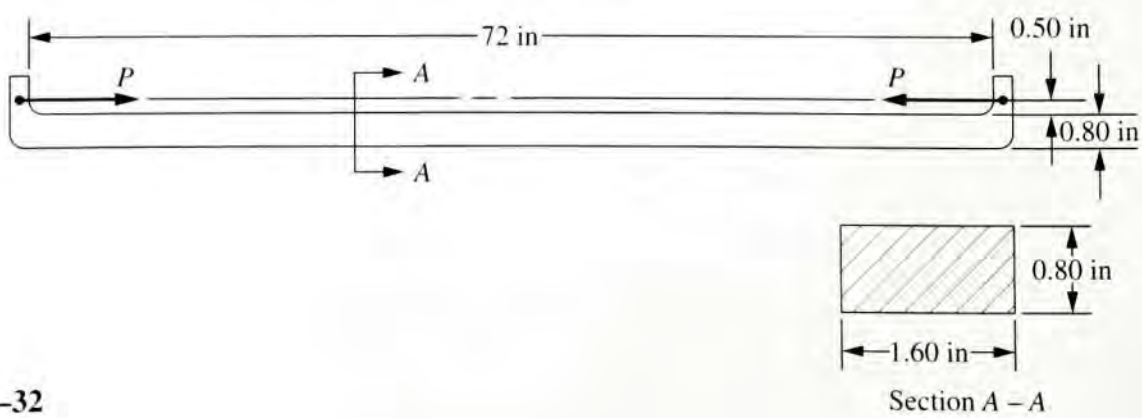


FIGURE P6–32

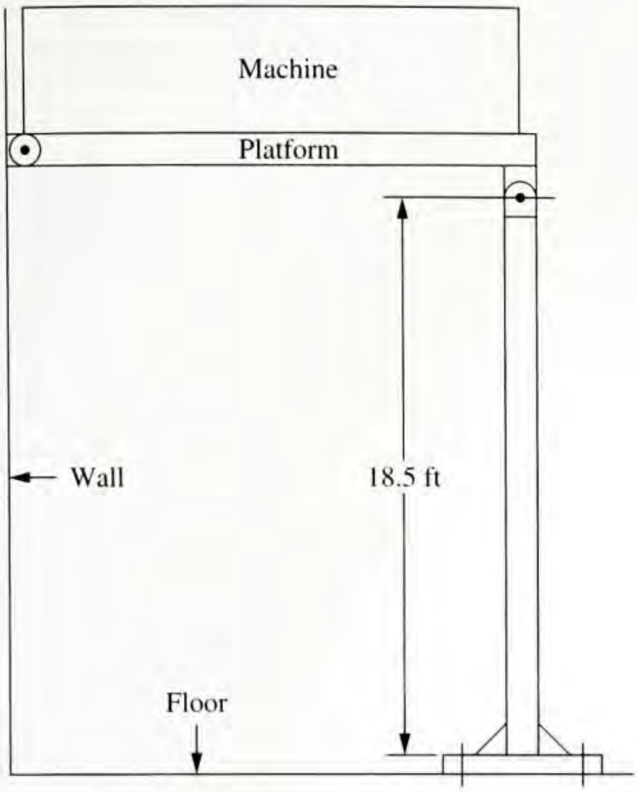


FIGURE P6–33

terial has a yield strength of 36 ksi. The total load on the platform is 55 000 lb, uniformly distributed.

34. Compute the allowable axial load on a steel channel, C5 × 9, made from ASTM A36 structural steel. The channel is 112 in long and can be considered to be pinned at its ends. Use a design factor of 3.

35. Repeat Problem 34 with the ends fixed rather than pinned.

36. Repeat Problem 34, except consider the load to be applied along the outside of the web of the channel instead of being axial.

37. Figure P6–37 shows a 4 × 4 × 1/2 steel column made from ASTM A500 Grade B structural steel. To accommodate a special mounting restriction, the load is applied eccentrically as shown. Determine the amount of load that the column can safely support. The column is supported laterally by the structure.

38. The device shown in Figure P6–38 is subjected to opposing forces F. Determine the maximum allowable load to achieve a design factor of 3. The device is made from aluminum 6061-T6.

39. A hydraulic cylinder is capable of exerting a force of 5200 N to move a heavy casting along a conveyor. The design of the pusher causes the load to be applied eccentrically to the piston rod as shown in Figure P6–39. Is the piston rod safe under this loading if it is made from AISI 416 stainless steel in the Q&T 1000 condition? Use E = 200 GPa.

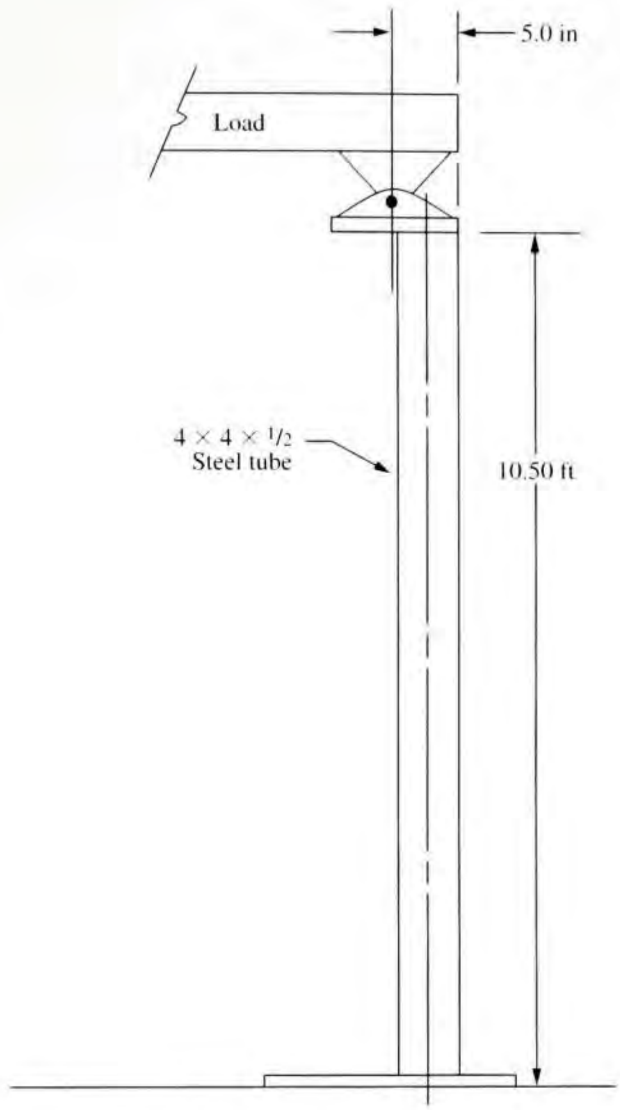


FIGURE P6–37

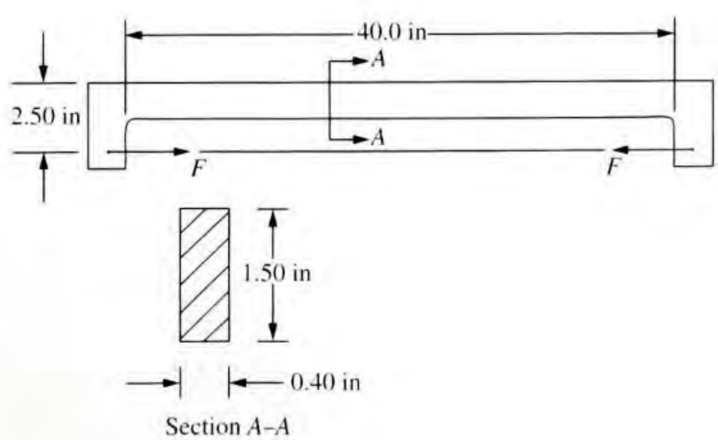


FIGURE P6–38

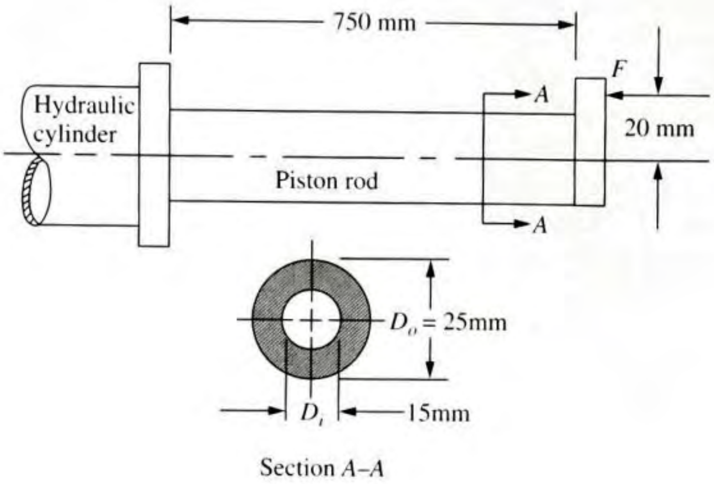


FIGURE P6–39

40. A standard 2-in schedule 40 steel pipe is proposed to be used to support the roof of a porch during renovation. Its length is 13.0 ft. The pipe is made from ASTM A501 structural steel.

(a) Determine the safe load on the pipe to achieve a design factor of 3 if the pipe is straight.

(b) Determine the safe load if the pipe has an initial crookedness of 1.25 in.

PART II

Design of a Mechanical Drive

OBJECTIVES AND CONTENT OF PART II

Part II of this book contains nine chapters (Chapters 7–15) that help you gain experience in approaching the design of an important complete device—a *mechanical drive*. The drive, sometimes called a *power transmission*, serves the following functions:

- It receives power from some kind of rotating source such as an electric motor, an internal combustion engine, a gas turbine engine, a hydraulic or pneumatic motor, a steam or water turbine, or even hand rotation provided by the operator.
- The drive typically causes some change in the speed of rotation of the shafts that make up the drive so that the output shaft operates more slowly or faster than the input shaft. Speed reducers are more prevalent than speed increasers.
- The active elements of the drive transmit the power from the input to the output shafts.
- When there is a speed reduction, there is a corresponding increase in the torque transmitted. Conversely, a speed increase causes a reduction in torque at the output compared with the input of the drive.

The chapters of Part II provide the detailed descriptions of the various machine elements that are typically used in power transmissions: *belt drives, chain drives, gears, shafts, bearings, keys, couplings, seals,* and *housings to hold all the elements together*. You will learn the important features of these elements and the methods of analyzing and designing them.

Of equal importance is the information provided on how the various elements interact with each other. You must be sensitive, for example, to how gears are mounted on shafts, how the shafts are supported by bearings, and how the bearings must be mounted securely in a housing that holds the system together. The final completed design must function as an integrated unit.

THE PROCESS OF DESIGNING A MECHANICAL DRIVE

In the design of a power transmission, you would typically know the following:

- ***The nature of the driven machine:*** It might be a machine tool in a factory that cuts metal parts for engines; an electric drill used by professional carpenters or home craft workers; the axle of a farm tractor; the propeller shaft of a turbojet for an airplane; the propeller shaft for a large ship; the wheels of a toy train; a mechanical timing mechanism; or any other of the numerous products that need a controlled-speed drive.

- ***The level of power to be transmitted:*** From the examples just listed, the power demanded may range from thousands of horsepower for a ship, hundreds of horsepower for a large farm tractor or airplane, or a few watts for a timer or a toy.
- ***The rotational speed of the drive motor or other prime mover:*** Typically the prime mover operates at a rather high speed of rotation. The shafts of standard electric motors rotate at about 1200, 1800, or 3600 revolutions per minute (rpm). Automotive engines operate from about 1000 to 6000 rpm. Universal motors in some hand tools (drills, saws, and routers) and household appliances (mixers, blenders, and vacuum cleaners) operate from 3500 to 20 000 rpm. Gas turbine engines for aircraft rotate many thousands of rpm.
- ***The desired output speed of the transmission:*** This is highly dependent on the application. Some gear motors for instruments rotate less than 1.0 rpm. Production machines in factories may run a few hundred rpm. Drives for assembly conveyors may run fewer than 100 rpm. Aircraft propellers may operate at several thousand rpm.

You, the designer, must then do the following:

- Choose the type of power transmission elements to be used: gears, belt drives, chain drives, or other types. In fact, some power transmission systems use two or more types in series to optimize the performance of each.
- Specify how the rotating elements are arranged and how the power transmission elements are mounted on shafts.
- Design the shafts to be safe under the expected torques and bending loads and properly locate the power transmission elements and the bearings. It is likely that the shafts will have several diameters and special features to accommodate keys, couplings, retaining rings, and other details. The dimensions of all features must be specified, along with the tolerances on the dimensions and surface finishes.
- Specify suitable bearings to support the shafts and determine how they will be mounted on the shafts and how they will be held in a housing.
- Specify keys to connect the shaft to the power transmission elements; couplings to connect the shaft from the driver to the input shaft of the transmission or to connect the output shaft to the driven machine; seals to effectively exclude contaminants from entering the transmission; and other accessories.
- Place all of the elements in a suitable housing that provides for the mounting of all elements and for their protection from the environment and their lubrication.

CHAPTERS THAT MAKE UP PART II

To guide you through this process of designing a mechanical drive, Part II includes the following chapters.

Chapter 7: Belt Drives and Chain Drives emphasizes recognizing the variety of commercially available belt and chain drives, the critical design parameters, and the methods used to specify reasonably optimum components of the drive systems.

Chapter 8: Kinematics of Gears describes and defines the important geometric features of gears. Methods of manufacturing gears are discussed, along with the importance of precision in the operation of the gears. The details of how a pair of gears operates are described, and the design and the operation of two or more gear pairs in a gear train are analyzed.

Chapter 9: Spur Gear Design illustrates how to compute forces exerted by one tooth of a gear on its mating teeth. Methods of computing the stresses in the gear teeth are presented, and design procedures for specifying gear-tooth geometry and material are given to produce a safe, long-life gear transmission system.

Chapter 10: Helical Gears, Bevel Gears, and Wormgearing contains approaches similar to those described for spur gears, with special attention to the unique geometry of these types of gears.

Chapter 11: Keys, Couplings, and Seals discusses how to design keys to be safe under the prevailing loads caused by the torque transmitted by them from the shaft to the gears or other elements. Couplings must be specified that accommodate the possible misalignment of connected shafts while transmitting the required torque at operating speeds. Seals must be specified for shafts that project through the sides of the housing and for bearings that must be kept free of contaminants. The maintenance of a reliable supply of clean lubricant for the active elements is essential.

Chapter 12: Shaft Design discusses the fact that in addition to being designed to safely transmit the required torque levels at given speeds, the shafts will probably have several diameters and special features to accommodate keys, couplings, retaining rings, and other details. The dimensions of all features must be specified, along with the tolerances on the dimensions and surface finishes. Completion of these tasks requires some of the skills developed in following chapters. So you will have to come back to this task later.

Chapter 13: Tolerances and Fits discusses the fit of elements that are assembled together and that may operate on one another; this fit is critical to the performance and life of the elements. In some cases, such as fitting the inner race of a ball or roller bearing onto a shaft, the bearing manufacturer specifies the allowable dimensional variation on the shaft so that it is compatible with the tolerances to which the bearing is produced. There is typically an interference fit between the bearing inner race and the shaft diameter where the bearing is to be mounted. But there is a close sliding fit between the outer race and the housing that holds the bearing in place. In general, it is important for you to take charge of specifying the tolerances for all dimensions to ensure proper operation while allowing economical manufacture.

Chapter 14: Rolling Contact Bearings focuses on commercially available rolling contact bearings such as ball bearings, roller bearings, tapered roller bearings, and others. You must be able to compute or specify the loads that the bearings will support, their speed of operation, and their expected life. From these data, standard bearings from manufacturers' catalogs will be specified. Then you must review the design process for the shafts as described for Chapter 12 to complete the specification of dimensions and tolerances. It is likely that iteration among the design processes for the power transmission elements, the shafts, and the bearings will be needed to achieve an optimum arrangement.

Chapter 15: Completion of the Design of a Power Transmission merges all of the preceding topics together. You will resolve the details of the design of each element and ensure the compatibility of mating elements. You will review all previous design decisions and assumptions and verify that the design meets specifications. After the individual elements have been analyzed and the iteration among them is complete, they must be packaged in a suitable housing to hold them securely, to protect them from contaminants, and to protect the people who may work around them. The housing must also be designed to be compatible with the driver and the driven machine. That often requires special fastening provisions and means of locating all connected devices relative to one another. Assembly and service must be considered. Then you will present a final set of specifications for the entire power transmission system and document your design with suitable drawings and a written report.

7

Belt Drives and Chain Drives

The Big Picture

Belt Drives and Chain Drives

Discussion Map

- Belts and chains are the major types of flexible power transmission elements. Belts operate on sheaves or pulleys, whereas chains operate on toothed wheels called *sprockets*.

Discover

Look around and identify at least one mechanical device having a belt drive and one having a chain drive system.

Describe each system, and make a sketch showing how it receives power from some source and how it transfers power to a driven machine.

Describe the differences between belt drives and chain drives.

In this chapter, you will learn how to select suitable components for belt drives and chain drives from commercially available designs.

Belts and chains represent the major types of flexible power transmission elements. Figure 7–1 shows a typical industrial application of these elements combined with a gear-type speed reducer. This application illustrates where belts, gear drives, and chains are each used to best advantage.

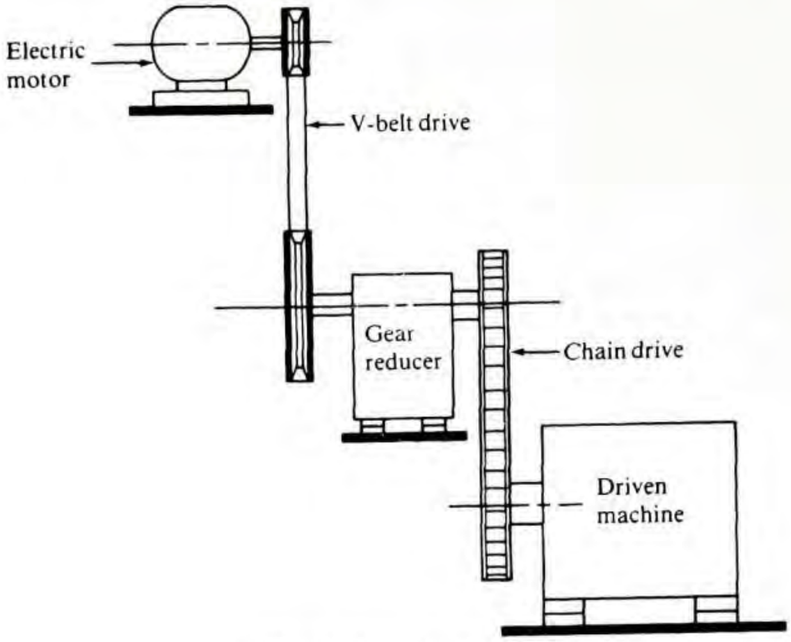
Rotary power is developed by the electric motor, but motors typically operate at too high a speed and deliver too low a torque to be appropriate for the final drive application. Remember, for a given power transmission, the torque is increased in proportion to the amount that rotational speed is reduced. So some speed reduction is often desirable. The high speed of the motor makes belt drives somewhat ideal for that first stage of reduction. A smaller drive pulley is attached to the motor shaft, while a larger diameter pulley is attached to a parallel shaft that operates at a correspondingly lower speed. Pulleys for belt drives are also called *sheaves*.

However, if very large ratios of speed reduction are required in the drive, gear reducers are desirable because they can typically accomplish large reductions in a rather small package. The output shaft of the gear-type speed reducer is generally at low speed and high torque. If both speed and torque are satisfactory for the application, it could be directly coupled to the driven machine.

However, because gear reducers are available only at discrete reduction ratios, the output must often be reduced more before meeting the requirements of the machine. At the low-speed, high-torque condition, chain drives become desirable. The high torque causes high tensile forces to be developed in the chain. The elements of the chain are typically metal, and they are sized to withstand the high forces. The links of chains are engaged in toothed wheels called *sprockets* to provide positive mechanical drive, desirable at the low-speed, high-torque conditions.

In general, belt drives are applied where the rotational speeds are relatively high, as on the first stage of speed reduction from an electric motor or engine. The linear speed of a belt is usually 2500 to 6500 ft/min, which results in relatively low tensile forces in the belt. At lower speeds, the tension in the belt becomes too large for typical belt cross sections, and slipping may occur between the sides of the belt and the sheave or pulley that carries it. At higher speeds, dynamic effects such as centrifugal forces, belt whip, and vibration reduce the effectiveness of the drive and its life. A speed of 4000 ft/min is generally ideal. Some belt designs employ high-strength, reinforcing strands and a cogged design that engages matching grooves in the pulleys to enhance their ability to transmit the high forces at low speeds. These designs compete with chain drives in many applications.

FIGURE 7–1
Combination drive employing V-belts, a gear reducer, and a chain drive [Source for Part (*b*): Browning Mfg. Division, Emerson Electric Co., Maysville, KY]



(*a*) Sketch of combination drive

(*b*) Photograph of an actual drive installation. Note that guards have been removed from the belt and chain drives to show detail.

Where have you seen belt drives? Consider mechanical devices around your home or office; vehicles; construction equipment; heating, air conditioning, and ventilation systems; and industrial machinery. Describe their general appearance. To what was the input pulley attached? Was it operating at a fairly high speed? What was the size of the next pulley? Did it cause the second shaft to rotate at a slower speed? How much slower? Were there more stages of reduction accomplished by belts or by some other reducer? Make a sketch of the layout of the drive system. Make measurements if you can get access to the equipment safely.

Where have you seen chain drives? One obvious place is likely to be the chain on a bicycle where the sprocket attached to the pedal-crank assembly is fairly large and that attached to the rear wheel is smaller. The drive sprocket and/or the driven sprocket assemblies may have several sizes to allow the rider to select many different speed ratios to permit optimum operation under different conditions of speed and hill-climbing demands. Where else have you seen chain drives? Again consider vehicles, construction equipment, and industrial machinery. Describe and sketch at least one chain drive system.

This chapter will help you learn to identify the typical design features of commercially available belt and chain drives. You will be able to specify suitable types and sizes to transmit a given level of power at a certain speed and to accomplish a specified speed ratio between the input and the output of the drive. Installation considerations are also described so that you can put your designs into successful systems.

You Are the Designer

A plant in Louisiana that produces sugar needs a drive system designed for a machine that chops long pieces of sugar cane into short lengths prior to processing. The machine's drive shaft is to rotate slowly at 30 rpm so that the cane is chopped smoothly and not beaten. The large machine requires a torque of 31 500 lb·in to drive the chopping blades.

Your company is asked to design the drive, and you are given the assignment. What kind of power source should be used? You might consider an electric motor, a gasoline engine, or a hydraulic motor. Most of these run at relatively high speeds, significantly higher than 30 rpm. Therefore, some type of speed reduction is needed. Perhaps you decide to use a drive similar to that shown in Figure 7–1.

Three stages of speed reduction are used. The input sheave of the belt drive rotates at the speed of the motor, while the larger driven sheave rotates at a slower speed and delivers the power to the input of the gear reducer. The larger part of the speed reduction is likely to be accomplished in the gear reducer, with the output shaft rotating slowly and providing a large torque. Remember, as the speed of rotation of a rotating shaft decreases, the torque delivered increases for a given power transmitted. But because there are only a limited number of reducer designs available, the output speed of the reducer will probably not be ideal for the cane chopper input shaft. The chain drive then provides the last stage of reduction.

As the designer, you must decide what type and size of belt drive to use and what the speed ratio between the driving and the driven sheave should be. How is the driving sheave attached to the motor shaft? How is the driven sheave attached to the input shaft of the gear reducer? Where should the motor be mounted in relation to the gear reducer, and what will be the resulting center distance between the two shafts? What speed reduction ratio will the gear reducer provide? What type of gear reducer should be used: helical gears, a worm and worm-gear drive, or bevel gears? How much additional speed reduction must the chain drive provide to deliver the proper speed to the cane-chopper shaft? What size and type of chain should be specified? What is the center distance between the output of the gear reducer and the input to the chopper? Then what length of chain is required? Finally, what motor power is required to drive the entire system at the stated conditions? The information in this chapter will help you answer questions about the design of power transmission systems incorporating belts and chains. Gear reducers are discussed in Chapter 8–10.

7–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Describe the basic features of a belt drive system.
2. Describe several types of belt drives.
3. Specify suitable types and sizes of belts and sheaves to transmit a given level of power at specified speeds for the input and output sheaves.
4. Specify the primary installation variables for belt drives, including center distance and belt length.
5. Describe the basic features of a chain drive system.
6. Describe several types of chain drives.

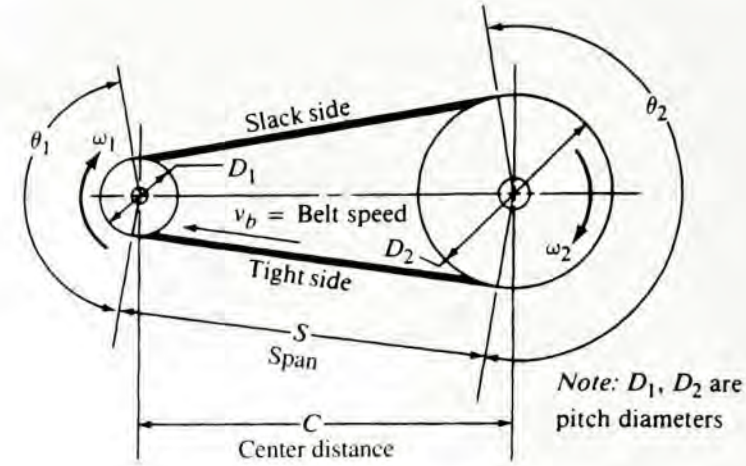


FIGURE 7–2 Basic belt drive geometry

7. Specify suitable types and sizes of chains and sprockets to transmit a given level of power at specified speeds for the input and output sprockets.
8. Specify the primary installation variables for chain drives, including center distance between the sheaves, chain length, and lubrication requirements.

7–2 TYPES OF BELT DRIVES

A belt is a flexible power transmission element that seats tightly on a set of pulleys or sheaves. Figure 7–2 shows the basic layout. When the belt is used for speed reduction, the typical case, the smaller sheave is mounted on the high-speed shaft, such as the shaft of an electric motor. The larger sheave is mounted on the driven machine. The belt is designed to ride around the two sheaves without slipping.

The belt is installed by placing it around the two sheaves while the center distance between them is reduced. Then the sheaves are moved apart, placing the belt in a rather high initial tension. When the belt is transmitting power, friction causes the belt to grip the driving sheave, increasing the tension in one side, called the "tight side," of the drive. The tensile force in the belt exerts a tangential force on the driven sheave, and thus a torque is applied to the driven shaft. The opposite side of the belt is still under tension, but at a smaller value. Thus, it is called the "slack side."

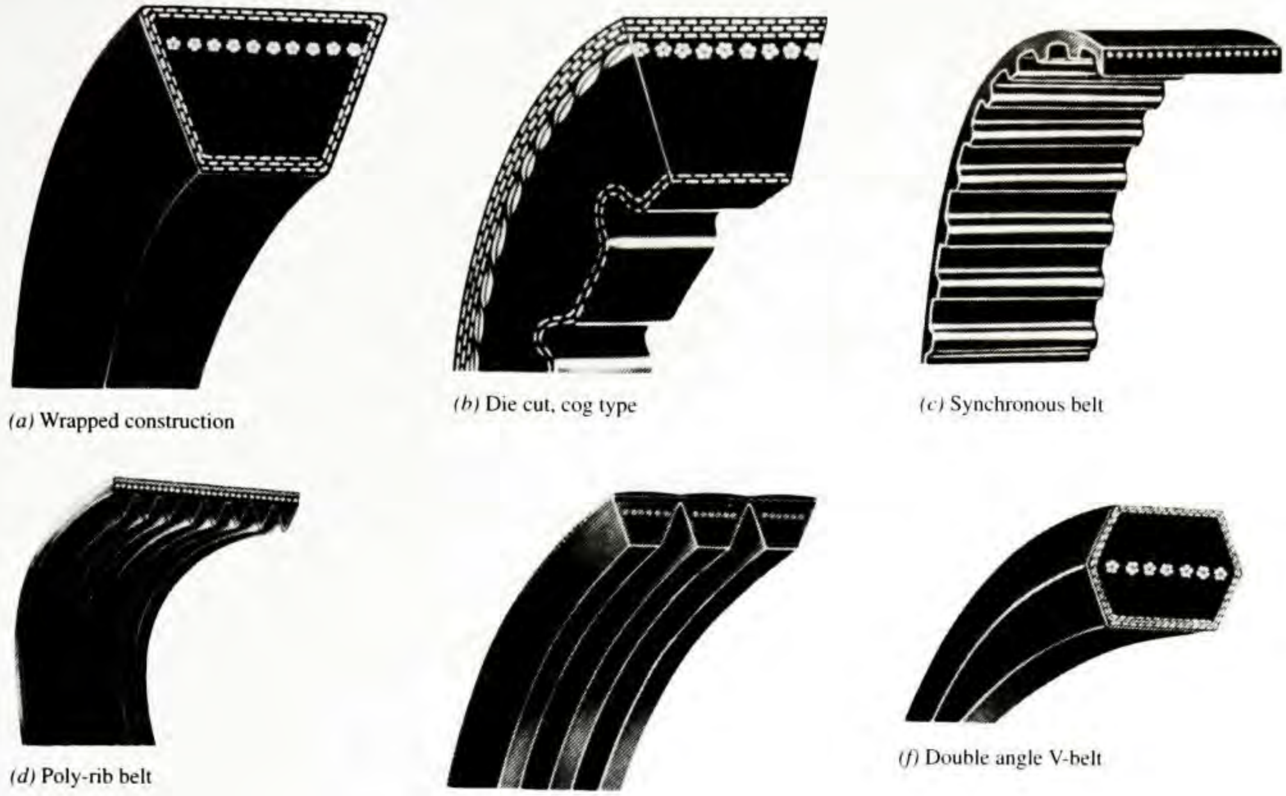
Many types of belts are available: flat belts, grooved or cogged belts, standard V-belts, double-angle V-belts, and others. See Figure 7–3 for examples. References 2–5 and 8–15 give more examples and technical data.

The *flat belt* is the simplest type, often made from leather or rubber-coated fabric. The sheave surface is also flat and smooth, and the driving force is therefore limited by the pure friction between the belt and the sheave. Some designers prefer flat belts for delicate machinery because the belt *will* slip if the torque tends to rise to a level high enough to damage the machine.

Synchronous belts, sometimes called *timing belts* [see Figure 7–3(c)], ride on sprockets having mating grooves into which the teeth on the belt seat. This is a positive drive, limited only by the tensile strength of the belt and the shear strength of the teeth.

Some cog belts, such as that shown in Figure 7–3(b), are applied to standard V-grooved sheaves. The cogs give the belt greater flexibility and higher efficiency compared with standard belts. They can operate on smaller sheave diameters.

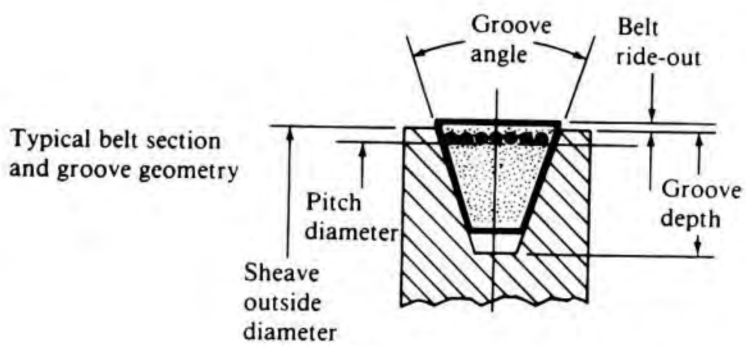
A widely used type of belt, particularly in industrial drives and vehicular applications, is the *V-belt drive*, shown in Figures 7–3(a) and 7–4. The V-shape causes the belt to wedge tightly into the groove, increasing friction and allowing high torques to be transmitted before slipping occurs. Most belts have high-strength cords positioned at the pitch diameter of the belt cross section to increase the tensile strength of the belt. The cords, made from natural fibers, synthetic strands, or steel, are embedded in a firm rubber compound to pro-



(a) Wrapped construction *(b)* Die cut, cog type *(c)* Synchronous belt *(d)* Poly-rib belt *(e)* Vee-band *(f)* Double angle V-belt

FIGURE 7–3 Examples of belt construction (Dayco Corp., Dayton, OH)

FIGURE 7–4 Cross section of V-belt and sheave groove



vide the flexibility needed to allow the belt to pass around the sheave. Often an outer fabric cover is added to give the belt good durability.

The selection of commercially available V-belt drives is discussed in the next section.

7–3 V-BELT DRIVES

The typical arrangement of the elements of a V-belt drive is shown in Figure 7–2. The important observations to be derived from this arrangement are summarized here:

1. The pulley, with a circumferential groove carrying the belt, is called a *sheave* (usually pronounced "shiv").
2. The size of a sheave is indicated by its pitch diameter, slightly smaller than the outside diameter of the sheave.
3. The speed ratio between the driving and the driven sheaves is inversely proportional to the ratio of the sheave pitch diameters. This follows from the observation

that there is no slipping (under normal loads). Thus, the linear speed of the pitch line of both sheaves is the same as and equal to the belt speed, v_b. Then

$$v_b = R_1\omega_1 = R_2\omega_2 \tag{7–1}$$

But $R_1 = D_1/2$ and $R_2 = D_2/2$. Then

$$v_b = \frac{D_1\omega_1}{2} = \frac{D_2\omega_2}{2} \tag{7–1A}$$

The angular velocity ratio is

$$\frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} \tag{7–2}$$

4. The relationships between pitch length, L, center distance, C, and the sheave diameters are

$$L = 2C + 1.57\,(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C} \tag{7–3}$$

$$C = \frac{B + \sqrt{B^2 - 32\,(D_2 - D_1)^2}}{16} \tag{7–4}$$

where $B = 4L - 6.28(D_2 + D_1)$

5. The angle of contact of the belt on each sheave is

$$\theta_1 = 180^\circ - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right] \tag{7–5}$$

$$\theta_2 = 180^\circ + 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right] \tag{7–6}$$

These angles are important because commercially available belts are rated with an assumed contact angle of 180°. This will occur only if the drive ratio is 1 (no speed change). The angle of contact on the smaller of the two sheaves will always be less than 180°, requiring a lower power rating.

6. The length of the span between the two sheaves, over which the belt is unsupported, is

$$S = \sqrt{C^2 - \left[\frac{D_2 - D_1}{2}\right]^2} \tag{7–7}$$

This is important for two reasons: You can check the proper belt tension by measuring the amount of force required to deflect the belt at the middle of the span by a given amount. Also, the tendency for the belt to vibrate or whip is dependent on this length.

7. The contributors to the stress in the belt are as follows:

(a) The tensile force in the belt, maximum on the tight side of the belt.

(b) The bending of the belt around the sheaves, maximum as the tight side of the belt bends around the smaller sheave.

(c) Centrifugal forces created as the belt moves around the sheaves.

The maximum total stress occurs where the belt enters the smaller sheave, and the bending stress is a major part. Thus, there are recommended minimum sheave diameters for standard belts. Using smaller sheaves drastically reduces belt life.

8. The design value of the ratio of the tight side tension to the slack side tension is 5.0 for V-belt drives. The actual value may range as high as 10.0.

Standard Belt Cross Sections

Commercially available belts are made to one of the standards shown in Figures 7–5 through 7–8. The alignment between the inch sizes and the metric sizes indicates that the paired sizes are actually the same cross section. A "soft conversion" was used to rename the familiar inch sizes with the number for the metric sizes giving the nominal top width in millimeters.

The nominal value of the included angle between the sides of the V-groove ranges from 30° to 42°. The angle on the belt may be slightly different to achieve a tight fit in the groove. Some belts are designed to "ride out" of the groove somewhat.

Many automotive applications use synchronous belt drives similar to that called a *timing belt* in Figure 7–3(c) or V-ribbed belts similar to that called a *poly-rib belt* in Figure

FIGURE 7–5 Heavy-duty industrial V-belts

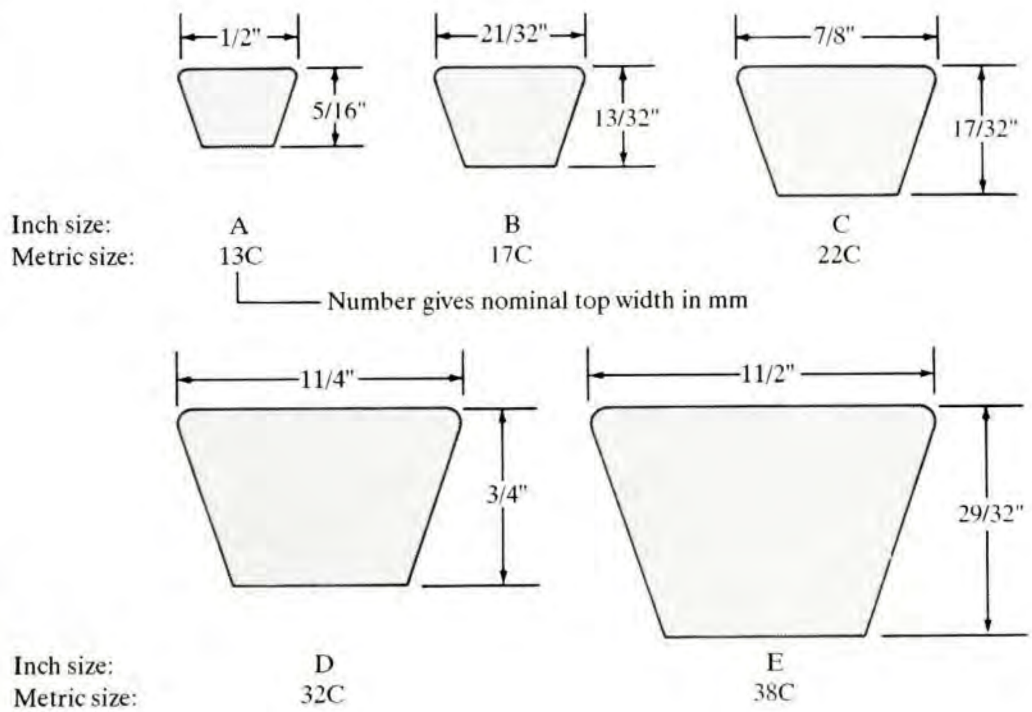
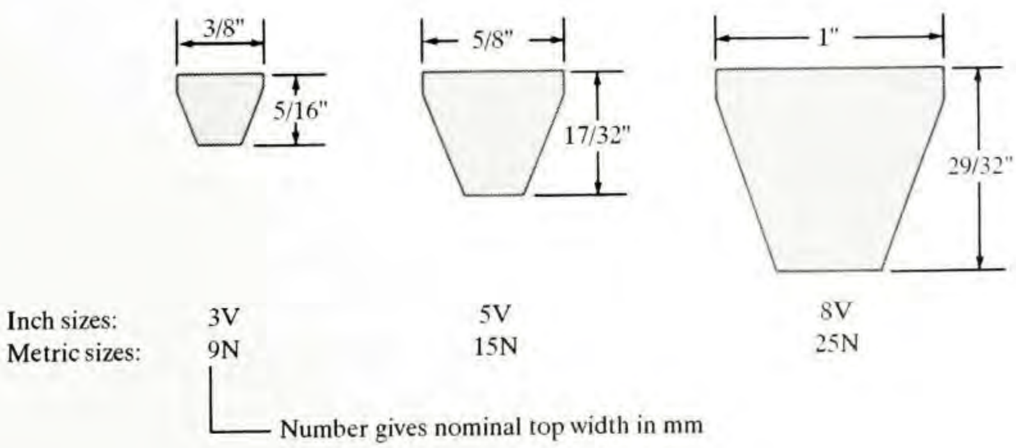


FIGURE 7–6 Industrial narrow-section V-belts



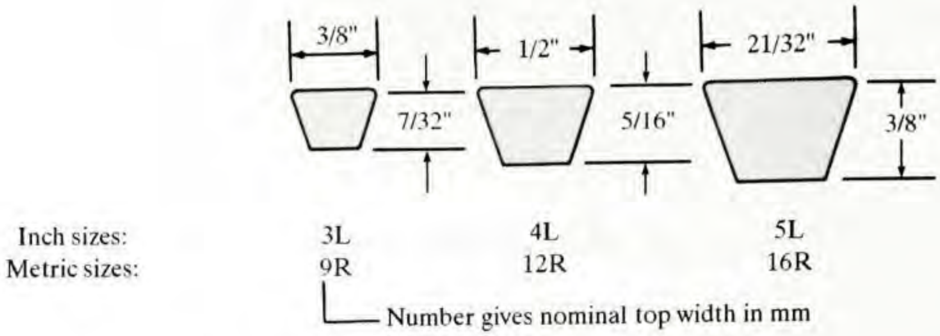


FIGURE 7–7 Light-duty, fractional horsepower (FHP) V-belts

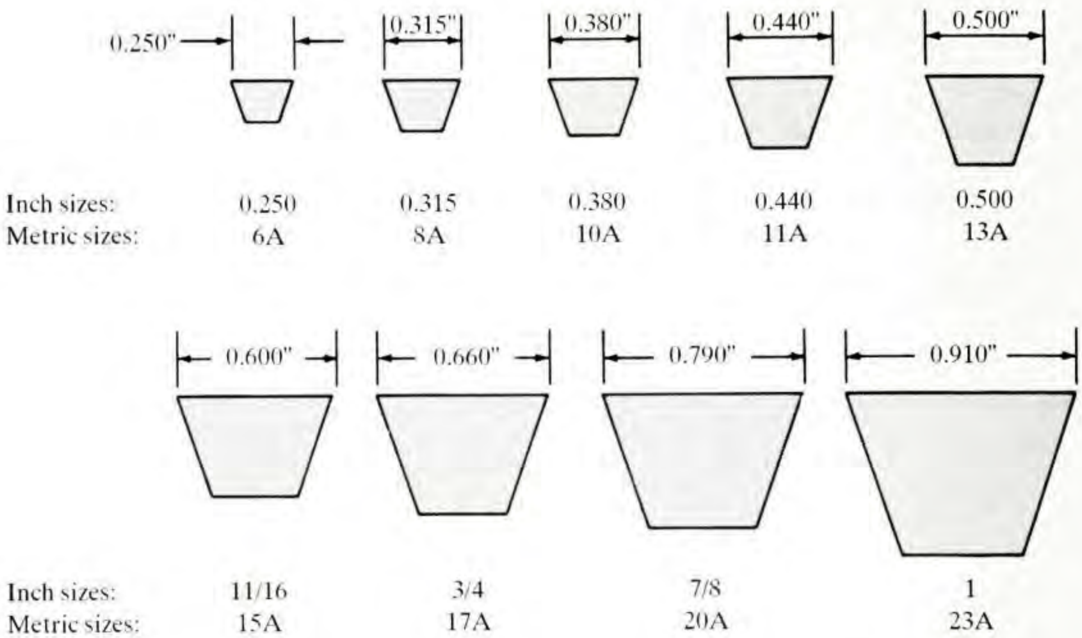


FIGURE 7–8 Automotive V-belts

7–3(d). The following standards of the Society of Automotive Engineers (SAE) give dimensions and performance standards for automotive belts.

SAE Standard J636: V-belts and pulleys

SAE Standard J637: Automotive V-belt drives

SAE Standard J1278: SI (metric) synchronous belts and pulleys

SAE Standard J1313: Automotive synchronous belt drives

SAE Standard J1459: V-ribbed belts and pulleys

7–4 V-BELT DRIVE DESIGN

MDESIGN

The factors involved in selection of a V-belt and the driving and driven sheaves and proper installation of the drive are summarized in this section. Abbreviated examples of the data available from suppliers are given for illustration. Catalogs contain extensive data, and step-by-step instructions are given for their use. The basic data required for drive selection are the following:

- The rated power of the driving motor or other prime mover
- The service factor based on the type of driver and driven load
- The center distance
- The power rating for one belt as a function of the size and speed of the smaller sheave
- The belt length
- The size of the driving and driven sheaves

- The correction factor for belt length
- The correction factor for the angle of wrap on the smaller sheave
- The number of belts
- The initial tension on the belt

Many design decisions depend on the application and on space limitations. A few guidelines are given here:

- Adjustment for the center distance must be provided in both directions from the nominal value. The center distance must be shortened at the time of installation to enable the belt to be placed in the grooves of the sheaves without force. Provision for increasing the center distance must be made to permit the initial tensioning of the drive and to take up for belt stretch. Manufacturers' catalogs give the data. One convenient way to accomplish the adjustment is the use of a take-up unit, as shown in Figure 14–10(b) and (c).
- If fixed centers are required, idler pulleys should be used. It is best to use a grooved idler on the inside of the belt, close to the large sheave. Adjustable tensioners are commercially available to carry the idler.
- The nominal range of center distances should be

$$D_2 < C < 3\,(D_2 + D_1) \tag{7–8}$$

- The angle of wrap on the smaller sheave should be greater than 120°.
- Most commercially available sheaves are cast iron, which should be limited to 6500-ft/min belt speed.
- Consider an alternative type of drive, such as a gear type or chain, if the belt speed is less than 1000 ft/min.
- Avoid elevated temperatures around belts.
- Ensure that the shafts carrying mating sheaves are parallel and that the sheaves are in alignment so that the belts track smoothly into the grooves.
- In multibelt installations, matched belts are required. Match numbers are printed on industrial belts, with 50 indicating a belt length very close to nominal. Longer belts carry match numbers above 50; shorter belts below 50.
- Belts must be installed with the initial tension recommended by the manufacturer. Tension should be checked after the first few hours of operation because seating and initial stretch occur.

Design Data

Catalogs typically give several dozen pages of design data for the various sizes of belts and sheave combinations to ease the job of drive design. The data typically are given in tabular form (see Reference 2). Graphical form is also used here so that you can get a feel for the variation in performance with design choices. Any design made from the data in this book should be checked against a particular manufacturer's ratings before use.

The data given here are for the narrow-section belts: 3V, 5V, and 8V. These three sizes cover a wide range of power transmission capacities. Figure 7–9 can be used to choose the basic size for the belt cross section. Note that the power axis is *design power*, the rated power of the prime mover times the service factor from Table 7–1.

Figures 7–10, 7–11, and 7–12 give the rated power per belt for the three cross sections as a function of the pitch diameter of the smaller sheave and its speed of rotation. The labeled vertical lines in each figure give the standard sheave pitch diameters available.

FIGURE 7–9
Selection chart for narrow-section industrial V-belts (Dayco Corp., Dayton, OH)

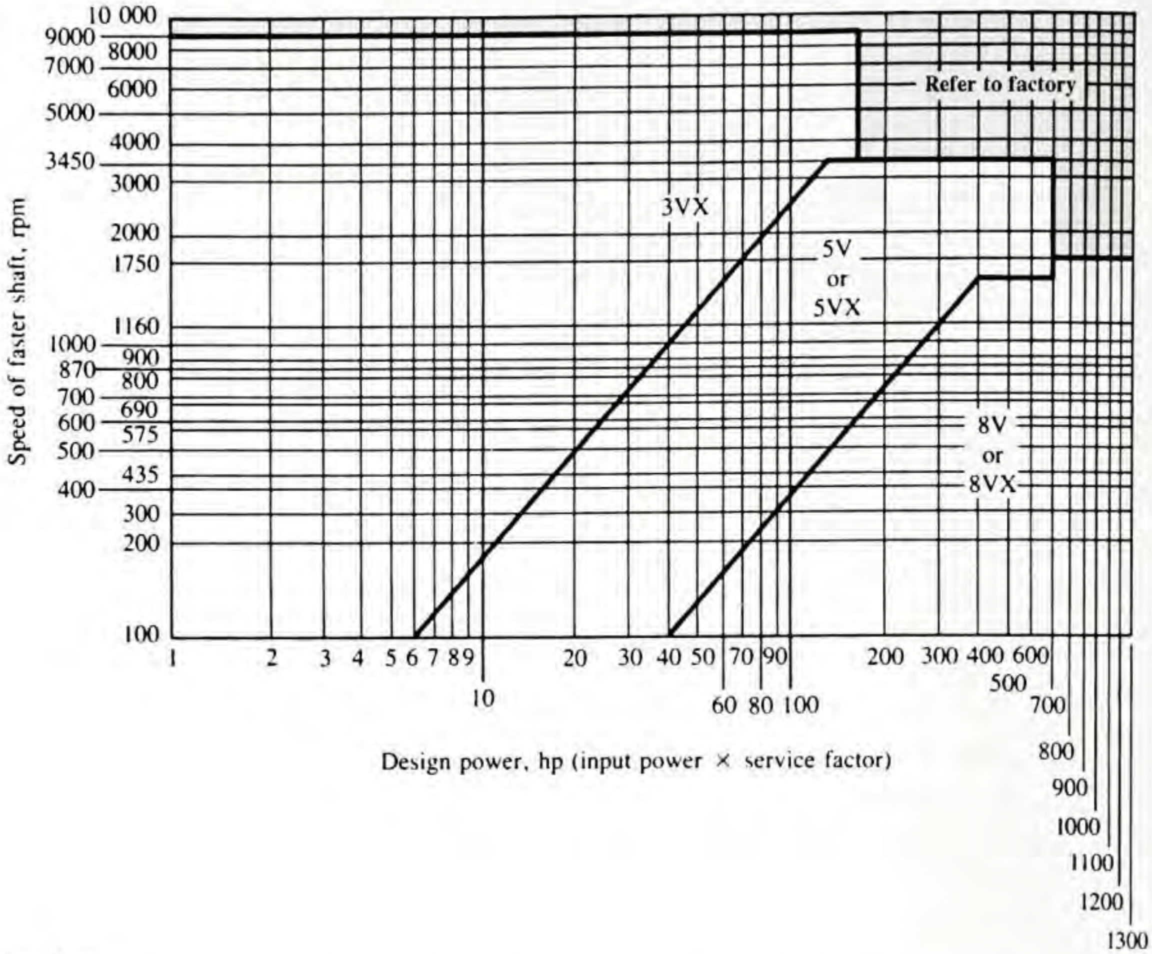


TABLE 7–1 V-belt service factors

Driven machine type	AC motors: Normal torque[a] DC motors: Shunt-wound Engines: Multiple-cylinder			AC motors: High torque[b] DC motors: Series-wound, compound-wound Engines: 4-cylinder or less		
	<6 h per day	6–15 h per day	>15 h per day	<6 h per day	6–15 h per day	>15 h per day
Agitators, blowers, fans, centrifugal pumps, light conveyors	1.0	1.1	1.2	1.1	1.2	1.3
Generators, machine tools, mixers, gravel conveyors	1.1	1.2	1.3	1.2	1.3	1.4
Bucket elevators, textile machines, hammer mills, heavy conveyors	1.2	1.3	1.4	1.4	1.5	1.6
Crushers, ball mills, hoists, rubber extruders	1.3	1.4	1.5	1.5	1.6	1.8
Any machine that can choke	2.0	2.0	2.0	2.0	2.0	2.0

Driver type (spans both driver groups).

[a]Synchronous, split-phase, three-phase with starting torque or breakdown torque less than 175% of full-load torque.
[b]Single-phase, three-phase with starting torque or breakdown torque greater than 175% of full-load torque.

The basic power rating for a speed ratio of 1.00 is given as the solid curve. A given belt can carry a greater power as the speed ratio increases, up to a ratio of approximately 3.38. Further increases have little effect and may also lead to trouble with the angle of wrap on the smaller sheave. Figure 7–13 is a plot of the data for power to be added to the basic rating as a function of speed ratio for the 5V belt size. The catalog data are given in a stepwise fashion.

FIGURE 7–10
Power rating: 3V belts

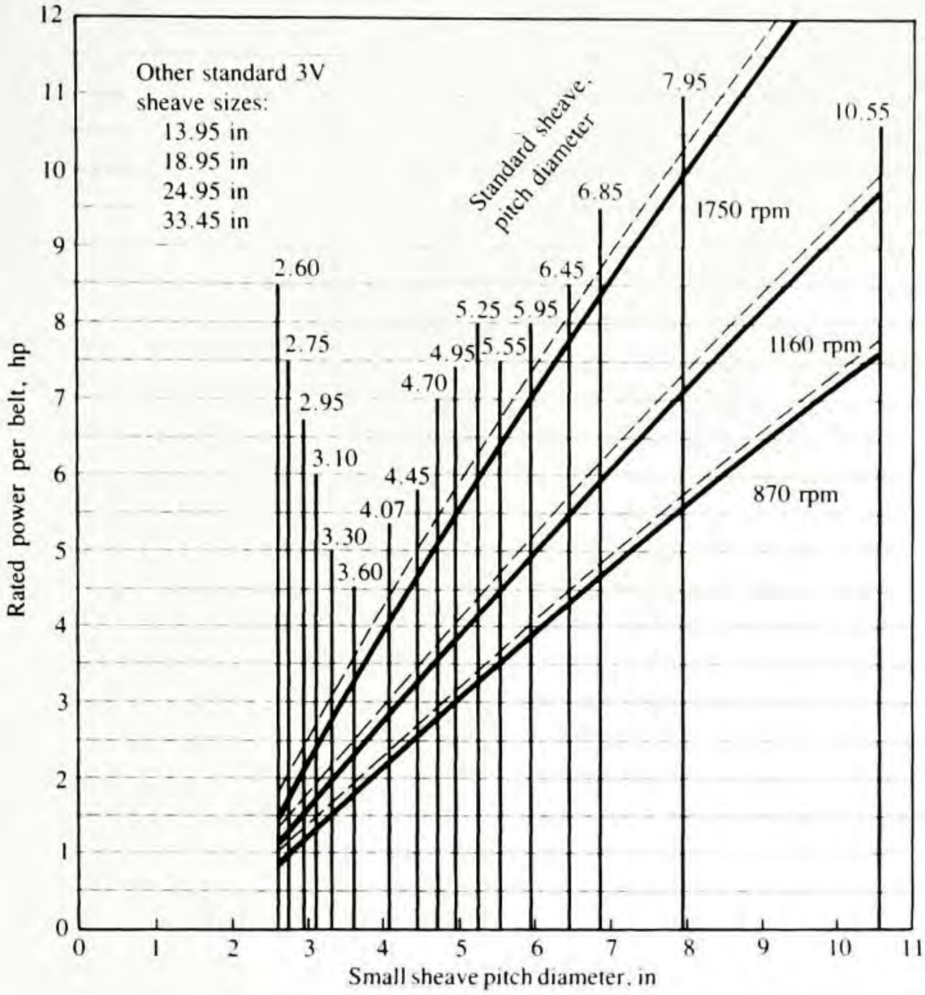


FIGURE 7–11
Power rating: 5V belts

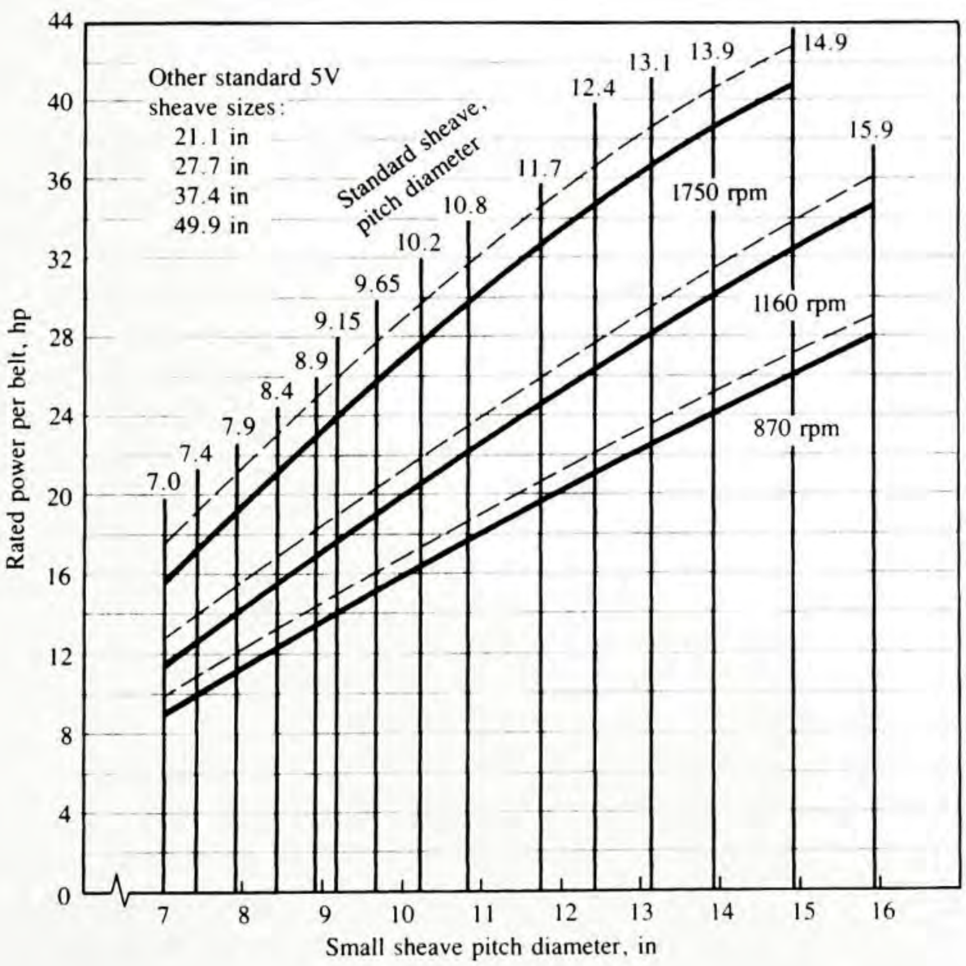


FIGURE 7–12
Power rating: 8V belts

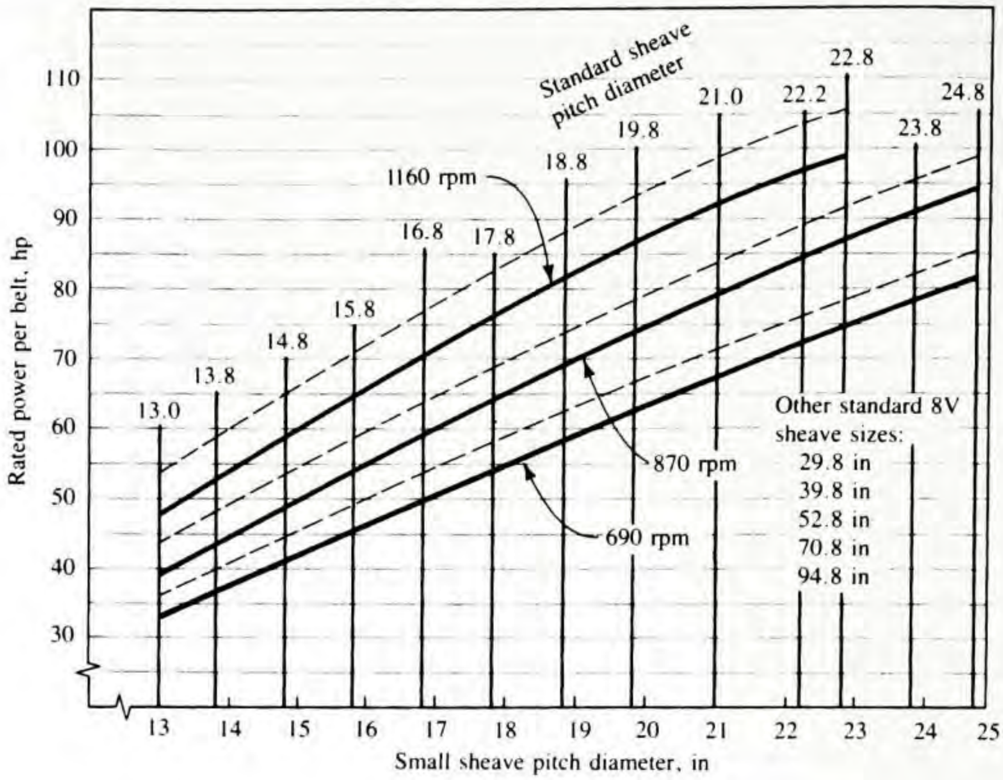
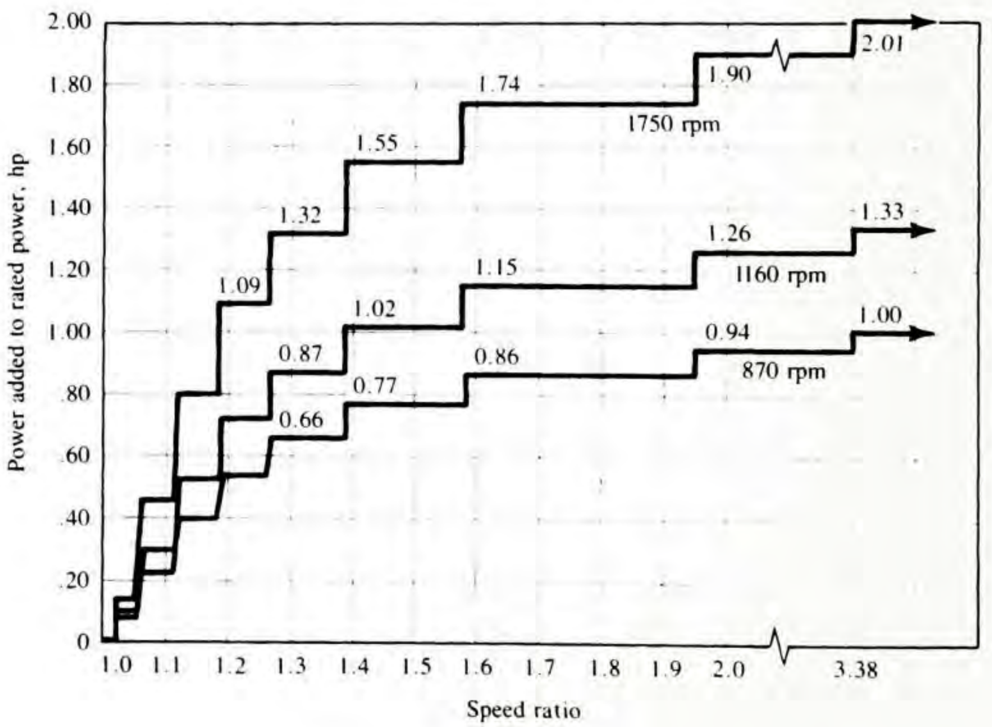


FIGURE 7–13
Power added versus speed ratio: 5V belts



The maximum power added, for ratios of above 3.38, was used to draw the dashed curves in Figures 7–10, 7–11, and 7–12. In most cases, a rough interpolation between the two curves is satisfactory.

Figure 7–14 gives the value of a correction factor, C_θ, as a function of the angle of wrap of the belt on the small sheave.

Figure 7–15 gives the value of the correction factor, C_L, for belt length. A longer belt is desirable because it reduces the frequency with which a given part of the belt encounters the stress peak as it enters the small sheave. Only certain standard belt lengths are available (Table 7–2).

Example Problem 7–1 illustrates the use of the design data.

FIGURE 7–14 Angle of wrap correction factor, C_θ

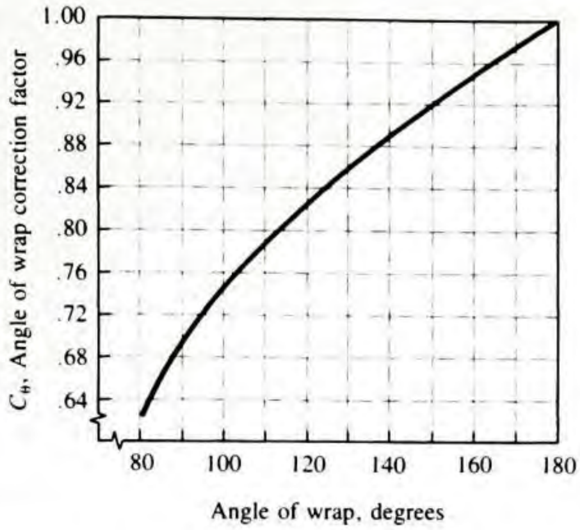


FIGURE 7–15 Belt length correction factor, C_L

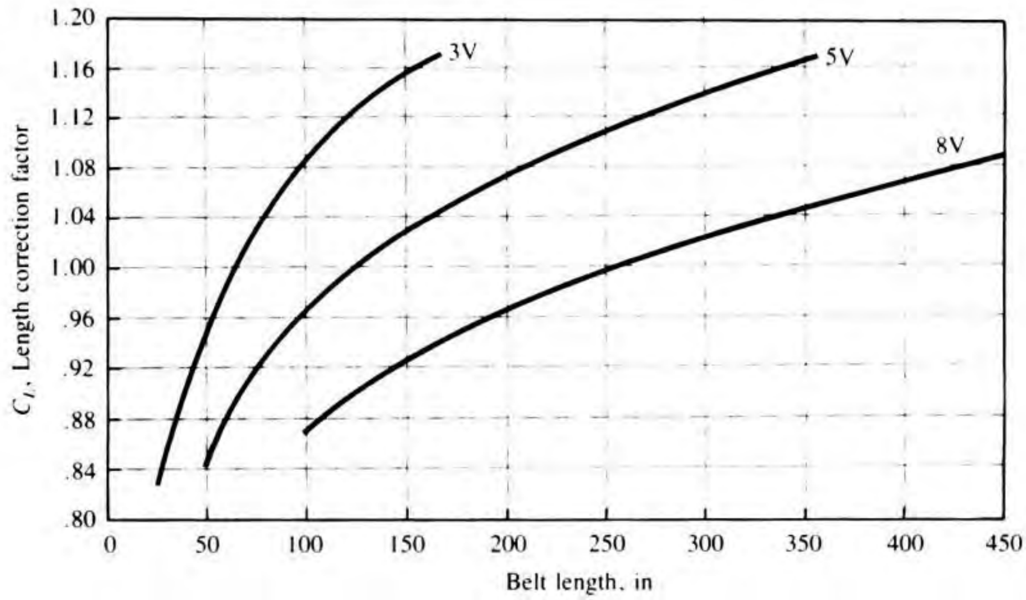


TABLE 7–2 Standard belt lengths for 3V, 5V, and 8V belts (in)

3V only	3V and 5V	3V, 5V, and 8V	5V and 8V	8V only
25	50	100	150	375
26.5	53	106	160	400
28	56	112	170	425
30	60	118	180	450
31.5	63	125	190	475
33.5	67	132	200	500
35.5	71	140	212	
37.5	75		224	
40	80		236	
42.5	85		250	
45	90		265	
47.5	95		280	
			300	
165			315	
			335	
			355	

Example Problem 7–1 Design a V-belt drive that has the input sheave on the shaft of an electric motor (normal torque) rated at 50.0 hp at 1160-rpm, full-load speed. The drive is to a bucket elevator in a potash plant that is to be used 12 hours (h) daily at approximately 675 rpm.

Solution Objective Design the V-belt drive.

Given

Power transmitted = 50 hp to bucket elevator

Speed of motor = 1160 rpm; output speed = 675 rpm

Analysis Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results

Step 1. Compute the design power. From Table 7–1, for a normal torque electric motor running 12 h daily driving a bucket elevator, the service factor is 1.30. Then the design power is 1.30(50.0 hp) = 65.0 hp.

Step 2. Select the belt section. From Figure 7–9, a 5V belt is recommended for 70.0 hp at 1160-rpm input speed.

Step 3. Compute the nominal speed ratio:

$$\text{Ratio} = 1160/675 = 1.72$$

Step 4. Compute the driving sheave size that would produce a belt speed of 4000 ft/min, as a guide to selecting a standard sheave:

$$\text{Belt speed} = v_b = \frac{\pi D_1 n_1}{12}\text{ft/min}$$

Then the required diameter to give $v_b = 4000$ ft/min is

$$D_1 = \frac{12\,v_b}{\pi n_1} = \frac{12(4000)}{\pi n_1} = \frac{15\,279}{n_1} = \frac{15\,279}{1160} = 13.17\text{ in}$$

Step 5. Select trial sizes for the input sheave, and compute the desired size of the output sheave. Select a standard size for the output sheave, and compute the actual ratio and output speed.

For this problem, the trials are given in Table 7–3 (diameters are in inches).

The two trials in **boldface** in Table 7–3 give only about 1% variation from the desired output speed of 675 rpm, and the speed of a bucket elevator is not critical. Because no space limitations were given, let's choose the larger size.

Step 6. Determine the rated power from Figure 7–10, 7–11, or 7–12.

For the 5V belt that we have selected, Figure 7–11 is appropriate. For a 12.4-in sheave at 1160 rpm, the basic rated power is 26.4 hp. Multiple belts will be required. The ratio is relatively high, indicating that some added power rating can be used. This value can be estimated from Figure 7–11 or taken directly from Figure 7–13 for the 5V belt. Power added is 1.15 hp. Then the actual rated power is 26.4 + 1.15 = 27.55 hp.

Step 7. Specify a trial center distance.

We can use Equation (7–8) to determine a nominal acceptable range for C:

$$D_2 < C < 3(D_2 + D_1)$$
$$21.1 < C < 3(21.1 + 12.4)$$
$$21.1 < C < 100.5\text{ in}$$

In the interest of conserving space, let's try $C = 24.0$ in.

TABLE 7–3 Trial sheave sizes for Example Problem 7–1

Standard driving sheave size, D_1	Approximate driven sheave size ($1.72D_1$)	Nearest standard sheave, D_2	Actual output speed (rpm)
13.10	22.5	21.1	720
12.4	**21.3**	**21.1**	**682**
11.7	20.1	21.1	643
10.8	18.6	21.1	594
10.2	17.5	15.9	744
9.65	16.6	15.9	704
9.15	**15.7**	**15.9**	**668**
8.9	15.3	14.9	693

Step 8. Compute the required belt length from Equation (7–3):

$$L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

$$L = 2(24.0) + 1.57(21.1 + 12.4) + \frac{(21.1 - 12.4)^2}{4(24.0)} = 101.4 \text{ in}$$

Step 9. Select a standard belt length from Table 7–2, and compute the resulting actual center distance from Equation (7–4).

In this problem, the nearest standard length is 100.0 in. Then, from Equation (7–4),

$$B = 4L - 6.28(D_2 + D_1) = 4(100) - 6.28\,(21.1 + 12.4) = 189.6$$

$$C = \frac{189.6 + \sqrt{(189.6)^2 - 32(21.1 - 12.4)^2}}{16} = 23.30 \text{ in}$$

Step 10. Compute the angle of wrap of the belt on the small sheave from Equation (7–5):

$$\theta_1 = 180^\circ - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right] = 180^\circ - 2\sin^{-1}\left[\frac{21.1 - 12.4}{2(23.30)}\right] = 158^\circ$$

Step 11. Determine the correction factors from Figures 7–14 and 7–15. For $\theta = 158^\circ$, $C_\theta = 0.94$. For $L = 100$ in, $C_L = 0.96$.

Step 12. Compute the corrected rated power per belt and the number of belts required to carry the design power:

$$\text{Corrected power} = C_\theta C_L P = (0.94)(0.96)(27.55 \text{ hp}) = 24.86 \text{ hp}$$

$$\text{Number of belts} = 65.0/24.86 = 2.61 \text{ belts (Use 3 belts.)}$$

Comments

Summary of Design

Input: Electric motor, 50.0 hp at 1160 rpm

Service factor: 1.4

Design power: 70.0 hp

Belt: 5V cross section, 100-in length, 3 belts

Sheaves: Driver, 12.4-in pitch diameter, 3 grooves, 5V. Driven, 21.1-in pitch diameter, 3 grooves, 5V

Actual output speed: 682 rpm

Center distance: 23.30 in

Belt Tension

The initial tension given to a belt is critical because it ensures that the belt will not slip under the design load. At rest, the two sides of the belt have the same tension. As power is being transmitted, the tension in the tight side increases while the tension in the slack side decreases. Without the initial tension, the slack side would go totally loose, and the belt would not seat in the groove; thus, it would slip. Manufacturers' catalogs give data for the proper belt-tensioning procedures.

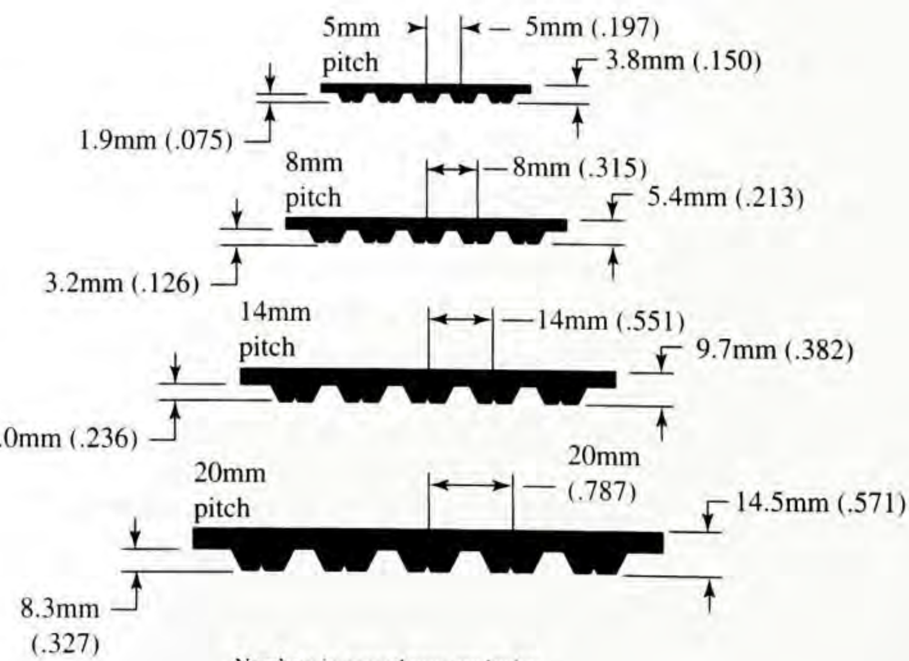
Synchronous Belt Drives

Synchronous belts are constructed with ribs or teeth across the underside of the belt, as shown in Figure 7–3(c). The teeth mate with corresponding grooves in the driving and driven pulleys, called *sprockets*, providing a positive drive without slippage. Therefore, there is a fixed relationship between the speed of the driver and the speed of the driven sprocket. For this reason synchronous belts are often called *timing belts*. In contrast, V-belts can creep or slip with respect to their mating sheaves, especially under heavy loads and varying power demand. Synchronous action is critical to the successful operation of such systems as printing, material handling, packaging, and assembly. Synchronous belt drives are increasingly being considered for applications in which gear drives or chain drives had been used previously.

Figure 7–16 shows a synchronous belt mating with the toothed driving sprocket. Typical driving and driven sprockets are shown in Figure 7–17. At least one of the two sprockets will have side flanges to ensure that the belt does not move axially. Figure 7–18 shows

FIGURE 7–16
Synchronous belt on driving sprocket (Copyright Rockwell Automation, used by permission)

FIGURE 7–17
Driving and driven sprockets for synchronous belt drive (Copyright Rockwell Automation, used by permission)



Numbers in parentheses are inches

FIGURE 7–18
Dimensions of standard synchronous belts

the four common tooth pitches and sizes for commercially available synchronous belts. The pitch is the distance from the center of one tooth to the center of the next adjacent tooth. Standard pitches are 5 mm, 8 mm, 14 mm, and 20 mm.

Figure 7–3(c) shows detail of the construction of the cross section of a synchronous belt. The tensile strength is provided predominantly by high-strength cords made from fiberglass or similar materials. The cords are encased in a flexible rubber backing material, and the teeth are formed integrally with the backing. Often a fabric covering is used on those parts of the belt that contact the sprockets to provide additional wear resistance and higher net shear strength for the teeth. Various widths of the belts are available for each given pitch to provide a wide range of power transmission capacity.

Commercially available sprockets typically employ split-taper bushings in their hubs with a precise bore that provides a clearance of only 0.001 to 0.002 in (0.025 to 0.050 mm) relative to the shaft diameter on which it is to be mounted. Smooth, balanced, concentric operation results.

The process of selecting appropriate components for a synchronous belt drive is similar to that already discussed for V-belt drives. Manufacturers provide selection guides similar to those shown in Figure 7–19 that give the relationship between design power and the rotational speed of the smaller sprocket. These are used to determine the basic belt pitch required. Also, numerous pages of performance data are given showing the power transmission capacity for many combinations of belt width, driving and driven sprocket size, and center distances between the axes of the sprockets for specific belt lengths. In general the selection process involves the following steps. Refer to data and design procedures for specific manufacturers as listed in Internet sites 2–5.

General Selection Procedure for Synchronous Belt Drives

1. Specify the speed of the driving sprocket (typically a motor or engine) and the desired speed of the driven sprocket.
2. Specify the rated power for the driving motor or engine.
3. Determine a service factor, using manufacturers' recommendations, considering the type of driver and the nature of the driven machine.
4. Calculate the design power by multiplying the driver rated power by the service factor.
5. Determine the required pitch of the belt from a specific manufacturer's data.
6. Calculate the speed ratio between the driver and the driven sprocket.
7. Select several candidate combinations of the number of teeth in the driver sprocket to that in the driven sprocket that will produce the desired ratio.
8. Using the desired range of acceptable center distances, determine a standard belt length that will produce a suitable value.
9. A belt-length correction factor may be required. Catalog data will show factors less than 1.0 for shorter center distances and greater than 1.0 for longer center distances. This reflects the frequency with which a given part of the belt encounters the high-stress area as it enters the smaller sprocket. Apply the factor to the rated power capacity for the belt.
10. Specify the final design details for the sprockets such as flanges, type and size of bushings in the hub, and the bore size to match the mating shafts.
11. Summarize the design, check compatibility with other components of the system, and prepare purchasing documents.

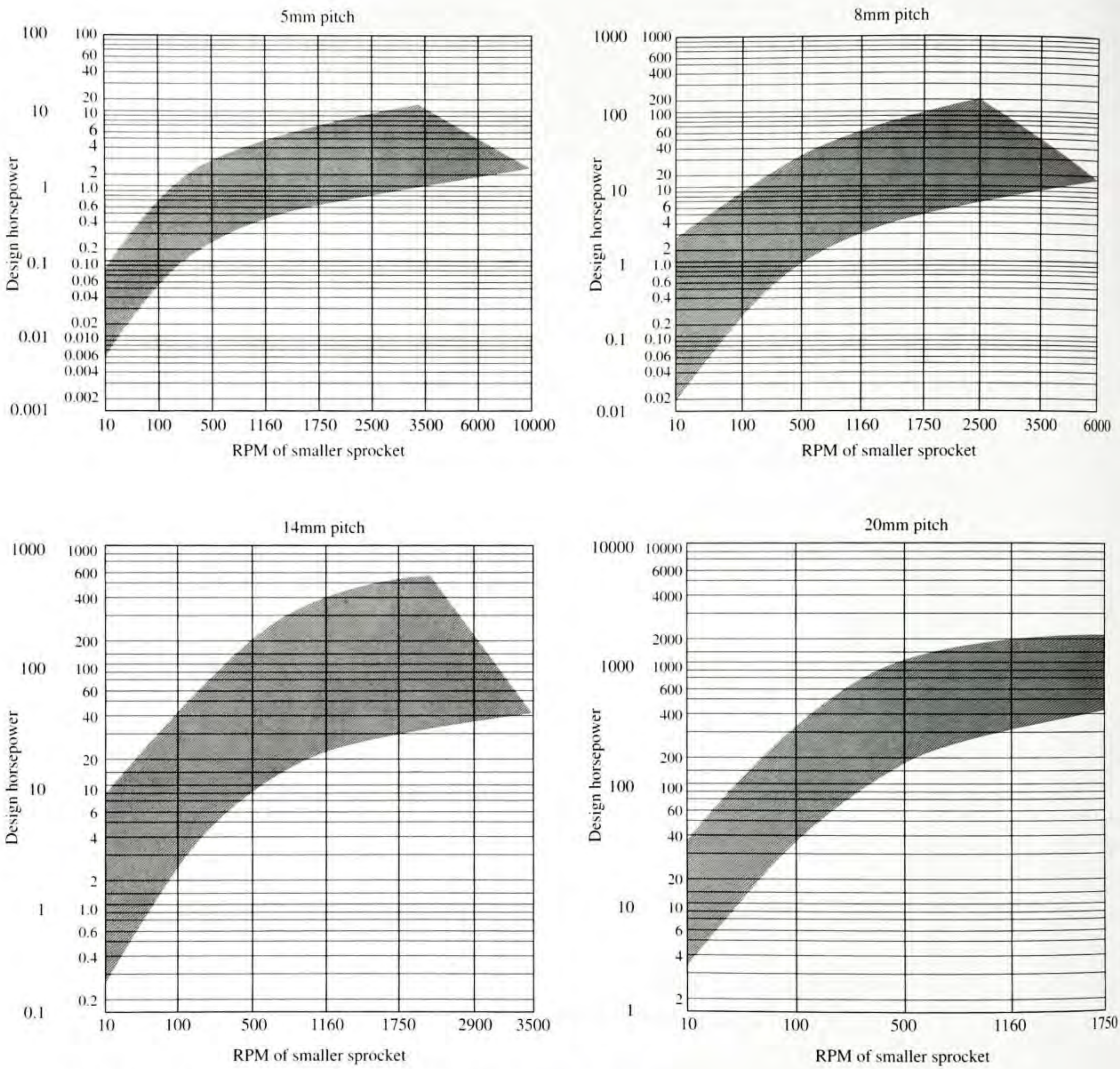


FIGURE 7–19 Belt pitch selection guide for synchronous belts

Installation of the sprockets and the belt requires a nominal amount of center distance allowance to enable the belt teeth to slide into the sprocket grooves without force. Subsequently, the center distance will normally have to be adjusted outward to provide a suitable amount of initial tension as defined by the manufacturer. The initial tension is typically less than that required for a V-belt drive. Idlers can be used to take up slack if fixed centers are required between the driver and driven sprockets. However, they may decrease the life of the belt. Consult the manufacturer.

In operation, the final tension in the tight side of the belt is much less than that developed by a V-belt and the slack side tension is virtually zero. The results are lower net forces in the belt, lower side loads on the shafts carrying the sprockets, and reduced bearing loads.

7–5 CHAIN DRIVES

A chain is a power transmission element made as a series of pin-connected links. The design provides for flexibility while enabling the chain to transmit large tensile forces. See References 1, 6, and 7 for more technical information and manufacturers' data.

When transmitting power between rotating shafts, the chain engages mating toothed wheels, called sprockets. Figure 7–20 shows a typical chain drive.

The most common type of chain is the *roller chain*, in which the roller on each pin provides exceptionally low friction between the chain and the sprockets. Other types include a variety of extended link designs used mostly in conveyor applications (see Figure 7–21).

Roller chain is classified by its *pitch*, the distance between corresponding parts of adjacent links. The pitch is usually illustrated as the distance between the centers of adjacent pins. Standard roller chain carries a size designation from 40 to 240, as listed in Table 7–4. The digits (other than the final zero) indicate the pitch of the chain in eighths of an inch, as in the table. For example, the no. 100 chain has a pitch of 10/8 or $1\frac{1}{4}$ in. A series of heavy-duty sizes, with the suffix *H* on the designation (60H–240H), has the same basic dimensions as the standard chain of the same number except for thicker side plates. In addition, there are the smaller and lighter sizes: 25, 35, and 41.

The average tensile strengths of the various chain sizes are also listed in Table 7–4. These data can be used for very low speed drives or for applications in which the function of the chain is to apply a tensile force or to support a load. It is recommended that only 10% of the average tensile strength be used in such applications. For power transmission, the rating of a given chain size as a function of the speed of rotation must be determined, as explained later in this chapter.

A wide variety of attachments are available to facilitate the application of roller chain to conveying or other material handling uses. Usually in the form of extended plates

FIGURE 7–20
Roller chain drive (Rexnord, Inc., Milwaukee, WI)

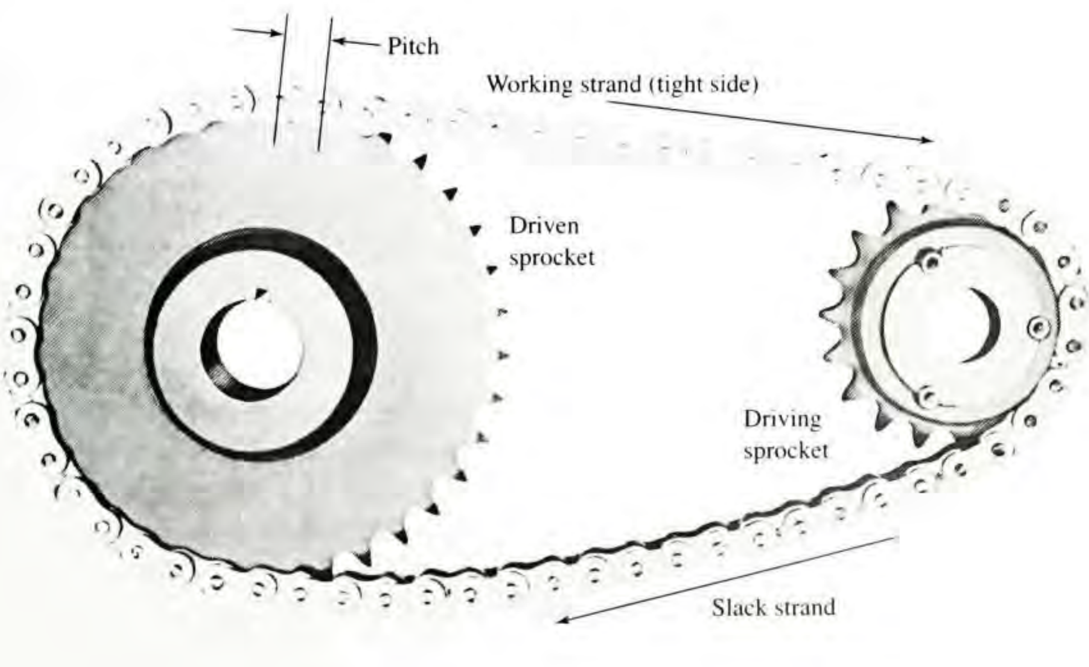


FIGURE 7–21 Some roller chain styles (Rexnord, Inc., Milwaukee, WI)

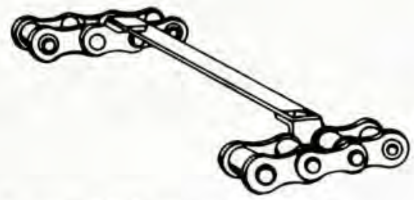
TABLE 7–4 Roller chain sizes

Chain number	Pitch (in)	Roller diameter	Roller width	Link plate thickness	Average tensile strength (lb)
25	1/4	None	–	0.030	925
35	3/8	None	–	0.050	2100
41	1/2	0.306	0.250	0.050	2000
40	1/2	0.312	0.312	0.060	3700
50	5/8	0.400	0.375	0.080	6100
60	3/4	0.469	0.500	0.094	8500
80	1	0.626	0.625	0.125	14 500
100	$1\frac{1}{4}$	0.750	0.750	0.156	24 000
120	$1\frac{1}{2}$	0.875	1.000	0.187	34 000
140	$1\frac{3}{4}$	1.000	1.000	0.219	46 000
160	2	1.125	1.250	0.250	58 000
180	$2\frac{1}{4}$	1.406	1.406	0.281	80 000
200	$2\frac{1}{2}$	1.562	1.500	0.312	95 000
240	3	1.875	1.875	0.375	130 000

or tabs with holes provided, the attachments make it easy to connect rods, buckets, parts pushers, part support devices, or conveyor slats to the chain. Figure 7–22 shows some attachment styles.

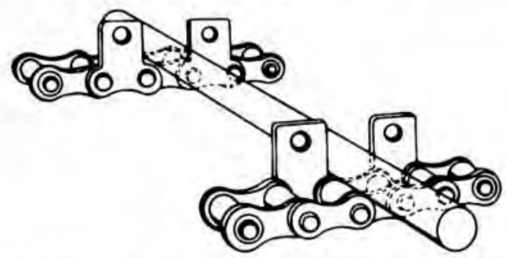
Figure 7–23 shows a variety of chain types used especially for conveying and similar applications. Such chain typically has a longer pitch than standard roller chain (usually twice the pitch), and the link plates are heavier. The larger sizes have cast link plates.

FIGURE 7–22 Chain attachments (Rexnord, Inc., Milwaukee, WI)



(*a*) Slats assembled to attachments to form a flat conveying surface

(*b*) *V* block assembled to attachments to convey round objects of varying diameters



(*c*) Attachments used as spacers to convey and position long objects

7–6 DESIGN OF CHAIN DRIVES

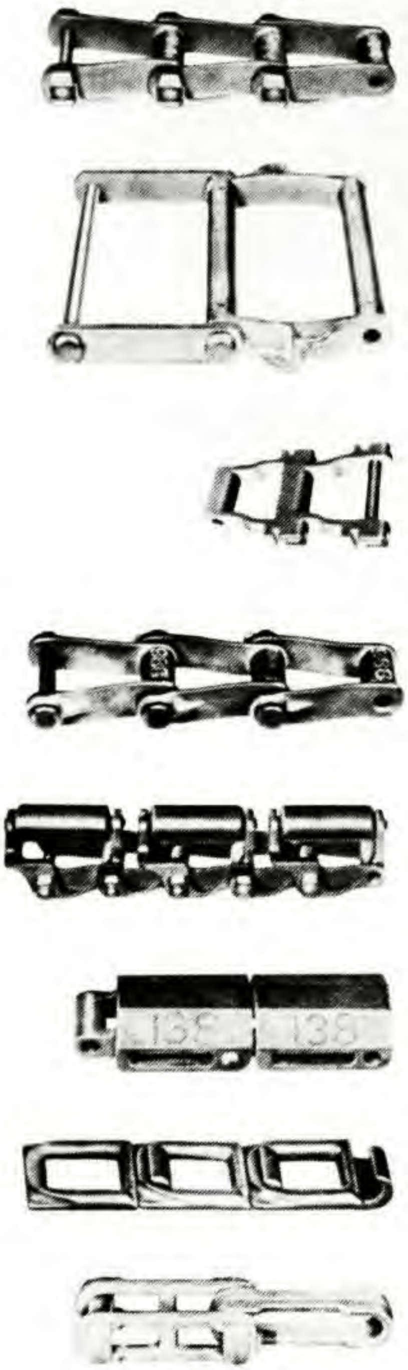
The rating of chain for its power transmission capacity considers three modes of failure: (1) fatigue of the link plates due to the repeated application of the tension in the tight side of the chain, (2) impact of the rollers as they engage the sprocket teeth, and (3) galling between the pins of each link and the bushings on the pins.

The ratings are based on empirical data with a smooth driver and a smooth load (service factor = 1.0) and with a rated life of approximately 15 000 h. The important variables are the pitch of the chain and the size and rotational speed of the smaller sprocket. Lubrication is critical to the satisfactory operation of a chain drive. Manufacturers recommend the type of lubrication method for given combinations of chain size, sprocket size, and speed. Details are discussed later.

Tables 7–5, 7–6, and 7–7 list the rated power for three sizes of standard chain: no. 40 (1/2 in), no. 60 (3/4 in), and no. 80 (1.00 in). These are typical of the types of data available for all chain sizes in manufacturers' catalogs. Notice these features of the data:

1. The ratings are based on the speed of the smaller sprocket and an expected life of approximately 15 000 hours.
2. For a given speed, the power capacity increases with the number of teeth on the sprocket. Of course, the larger the number of teeth, the larger the diameter of the sprocket. Note that the use of a chain with a small pitch on a large sprocket produces the quieter drive.
3. For a given sprocket size (a given number of teeth), the power capacity increases with increasing speed up to a point; then it decreases. Fatigue due to the tension in the chain governs at the low to moderate speeds; impact on the sprockets governs at the higher speeds. Each sprocket size has an absolute upper-limit speed due to the onset of galling between the pins and the bushings

FIGURE 7–23
Conveyor chains (Rexnord, Inc., Milwaukee, WI)



of the chain. This explains the abrupt drop in power capacity to zero at the limiting speed.

4. The ratings are for a single strand of chain. Although multiple strands do increase the power capacity, they do not provide a direct multiple of the single-strand capacity. Multiply the capacity in the tables by the following factors.

   Two strands: Factor = 1.7

   Three strands: Factor = 2.5

   Four strands: Factor = 3.3

5. The ratings are for a service factor of 1.0. Specify a service factor for a given application according to Table 7–8.

TABLE 7–5 Horsepower ratings—single strand roller chain no. 40

No. of teeth	0.500 inch pitch			Rotational speed of small sprocket, rev/min																	
	10	25	50	100	180	200	300	500	700	900	1000	1200	1400	1600	1800	2100	2500	3000	3500	4000	5000	6000	7000	8000	9000
11	0.06	0.14	0.27	0.52	0.91	1.00	1.48	2.42	3.34	4.25	4.70	5.60	6.49	5.57	4.66	3.70	2.85	2.17	1.72	1.41	1.01	0.77	0.61	0.50	0.00
12	0.06	0.15	0.29	0.56	0.99	1.09	1.61	2.64	3.64	4.64	5.13	6.11	7.09	6.34	5.31	4.22	3.25	2.47	1.96	1.60	1.15	0.87	0.69	0.57	0.00
13	0.07	0.16	0.31	0.61	1.07	1.19	1.75	2.86	3.95	5.02	5.56	6.62	7.68	7.15	5.99	4.76	3.66	2.79	2.21	1.81	1.29	0.98	0.78	0.00	
14	0.07	0.17	0.34	0.66	1.15	1.28	1.88	3.08	4.25	5.41	5.98	7.13	8.27	7.99	6.70	5.31	4.09	3.11	2.47	2.02	1.45	1.10	0.87	0.00	
15	0.08	0.19	0.36	0.70	1.24	1.37	2.02	3.30	4.55	5.80	6.41	7.64	8.86	8.86	7.43	5.89	4.54	3.45	2.74	2.24	1.60	1.22	0.97	0.00	
16	0.08	0.20	0.39	0.75	1.32	1.46	2.15	3.52	4.86	6.18	6.84	8.15	9.45	9.76	8.18	6.49	5.00	3.80	3.02	2.47	1.77	1.34	0.00		
17	0.09	0.21	0.41	0.80	1.40	1.55	2.29	3.74	5.16	6.57	7.27	8.66	10.04	10.69	8.96	7.11	5.48	4.17	3.31	2.71	1.94	1.47	0.00		
18	0.09	0.22	0.43	0.84	1.48	1.64	2.42	3.96	5.46	6.95	7.69	9.17	10.63	11.65	9.76	7.75	5.97	4.54	3.60	2.95	2.11	1.60	0.00		
19	0.10	0.24	0.46	0.89	1.57	1.73	2.56	4.18	5.77	7.34	8.12	9.66	11.22	12.64	10.59	8.40	6.47	4.92	3.91	3.20	2.29	0.09	0.00		
20	0.10	0.25	0.48	0.94	1.65	1.82	2.69	4.39	6.07	7.73	8.55	10.18	11.81	13.42	11.44	9.07	6.99	5.31	4.22	3.45	2.47	0.00			
21	0.11	0.26	0.51	0.98	1.73	1.91	2.83	4.61	6.37	8.11	8.98	10.69	12.40	14.10	12.30	9.76	7.52	5.72	4.54	3.71	2.65	0.00			
22	0.11	0.27	0.53	1.03	1.81	2.01	2.96	4.83	6.68	8.50	9.40	11.20	12.99	14.77	13.19	10.47	8.06	6.13	4.87	3.98	2.85	0.00			
23	0.12	0.28	0.56	1.08	1.90	2.10	3.10	5.05	6.98	8.89	9.83	11.71	13.58	15.44	14.10	11.19	8.62	6.55	5.20	4.26	3.05	0.00			
24	0.12	0.30	0.58	1.12	1.98	2.19	3.23	5.27	7.28	9.27	10.26	12.22	14.17	16.11	15.03	11.93	9.18	6.99	5.54	4.54	0.87	0.00			
25	0.13	0.31	0.60	1.17	2.06	2.28	3.36	5.49	7.59	9.66	10.69	12.73	14.76	16.78	15.98	12.68	9.76	7.43	5.89	4.82	0.00				
26	0.13	0.32	0.63	1.22	2.14	2.37	3.50	5.71	7.89	10.04	11.11	13.24	15.35	17.45	16.95	13.45	10.36	7.88	6.25	5.12	0.00				
28	0.14	0.35	0.67	1.31	2.31	2.55	3.77	6.15	8.50	10.82	11.97	14.26	16.53	18.79	18.94	15.03	11.57	8.80	6.99	5.72	0.00				
30	0.15	0.37	0.72	1.41	2.47	2.74	4.04	6.59	9.11	11.59	12.82	15.28	17.71	20.14	21.01	16.67	12.84	9.76	7.75	6.34	0.00				
32	0.16	0.40	0.77	1.50	2.64	2.92	4.31	7.03	9.71	12.38	13.68	16.30	18.89	21.48	23.14	18.37	14.14	10.76	8.54	1.41					
35	0.18	0.43	0.84	1.64	2.88	3.19	4.71	7.69	10.62	13.52	14.96	17.82	20.67	23.49	26.30	21.01	16.17	12.30	9.76	0.00					
40	0.21	0.50	0.96	1.87	3.30	3.65	5.38	8.79	12.14	15.45	17.10	20.37	23.62	26.85	30.06	25.67	19.76	15.03	0.00						
45	0.23	0.56	1.08	2.11	3.71	4.10	6.08	9.89	13.66	17.39	19.24	22.92	26.57	30.20	33.82	30.63	23.58	5.53	0.00						

Type A — Type B — Type C

Type A: Manual or drip lubrication
Type B: Bath or disc lubrication
Type C: Oil stream lubrication

Source: American Chain Association, Naples, FL

TABLE 7–6 Horsepower ratings—single strand roller chain no. 60

No. of teeth	0.750 inch pitch								Rotational speed of small sprocket, rev/min																
	10	25	50	100	120	200	300	400	500	600	800	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500	5000	5500	6000
11	0.19	0.46	0.89	1.72	2.05	3.35	4.95	6.52	8.08	9.63	12.69	15.58	11.85	9.41	7.70	6.45	5.51	3.94	3.00	2.38	1.95	1.63	1.39	1.21	0.00
12	0.21	0.50	0.97	1.88	2.24	3.66	5.40	7.12	8.82	10.51	13.85	17.15	13.51	10.72	8.77	7.35	6.28	4.49	3.42	2.71	2.22	1.86	1.59	1.38	0.00
13	0.22	0.54	1.05	2.04	2.43	3.96	5.85	7.71	9.55	11.38	15.00	18.58	15.23	12.08	9.89	8.29	7.08	5.06	3.85	3.06	2.50	2.10	1.79	0.00	
14	0.24	0.58	1.13	2.19	2.61	4.27	6.30	8.30	10.29	12.26	16.15	20.01	17.02	13.51	11.05	9.26	7.91	5.66	4.31	3.42	2.80	2.34	0.41	0.00	
15	0.26	0.62	1.21	2.35	2.80	4.57	6.75	8.90	11.02	13.13	17.31	21.44	18.87	14.98	12.26	10.27	8.77	6.28	4.77	3.79	3.10	2.60	0.00		
16	0.27	0.66	1.29	2.51	2.99	4.88	7.20	9.49	11.76	14.01	18.46	22.87	20.79	16.50	13.51	11.32	9.66	6.91	5.26	4.17	3.42	1.78	0.00		
17	0.29	0.70	1.37	2.66	3.17	5.18	7.65	10.08	12.49	14.88	19.62	24.30	22.77	18.07	14.79	12.40	10.58	7.57	5.76	4.57	3.74	0.00			
18	0.31	0.75	1.45	2.82	3.36	5.49	8.10	10.68	13.23	15.76	20.77	25.73	24.81	19.69	16.11	13.51	11.53	8.25	6.28	4.98	4.08	0.00			
19	0.33	0.79	1.53	2.98	3.55	5.79	8.55	11.27	13.96	16.63	21.92	27.16	26.91	21.35	17.48	14.65	12.50	8.95	6.81	5.40	0.20	0.00			
20	0.34	0.83	1.61	3.13	3.73	6.10	9.00	11.86	14.70	17.51	23.08	28.59	29.06	23.06	18.87	15.82	13.51	9.66	7.35	5.83	0.00				
21	0.36	0.87	1.69	3.29	3.92	6.40	9.45	12.46	15.43	18.38	24.23	30.02	31.26	24.81	20.31	17.02	14.53	10.40	7.91	6.28	0.00				
22	0.38	0.91	1.77	3.45	4.11	6.71	9.90	13.05	16.17	19.26	25.39	31.45	33.52	26.60	21.77	18.25	15.58	11.15	8.48	0.00					
23	0.40	0.95	1.85	3.61	4.29	7.01	10.35	13.64	16.90	20.13	26.54	32.88	35.84	28.44	23.28	19.51	16.66	11.92	9.07	0.00					
24	0.41	0.99	1.93	3.76	4.48	7.32	10.80	14.24	17.64	21.01	27.69	34.31	38.20	30.31	24.81	20.79	17.75	12.70	9.66	0.00					
25	0.43	1.04	2.01	3.92	4.67	7.62	11.25	14.83	18.37	21.89	28.85	35.74	40.61	32.23	26.38	22.11	18.87	13.51	10.27	0.00					
26	0.45	1.08	2.09	4.08	4.85	7.93	11.70	15.42	19.11	22.76	30.00	37.17	43.07	34.18	27.98	23.44	20.02	14.32	10.90	0.00					
28	0.48	1.16	2.26	4.39	5.23	8.54	12.60	16.61	20.58	24.51	32.31	40.03	47.68	38.20	31.26	26.20	22.37	16.01	0.00						
30	0.52	1.24	2.42	4.70	5.60	9.15	13.50	17.79	22.05	26.26	34.62	42.89	51.09	42.36	34.67	29.06	24.81	17.75	0.00						
32	0.55	1.33	2.58	5.02	5.98	9.76	14.40	18.98	23.52	28.01	36.92	45.75	54.50	46.67	38.20	32.01	27.33	19.56	0.00						
35	0.60	1.45	2.82	5.49	6.54	10.67	15.75	20.76	25.72	30.64	40.39	50.03	59.60	53.38	43.69	36.62	31.26	1.35	0.00						
40	0.69	1.66	3.22	6.27	7.47	12.20	18.00	23.73	29.39	35.02	46.16	57.18	68.12	65.22	53.38	44.74	38.20	0.00							
45	0.77	1.86	3.63	7.05	8.40	13.72	20.25	26.69	33.07	38.39	51.92	64.33	76.63	77.83	63.70	53.38	12.45	0.00							

Type A — Type B — Type C

Type A: Manual or drip lubrication
Type B: Bath of disc lubrication
Type C: Oil stream lubrication

Source: American Chain Association, Naples, FL

TABLE 7–7 Horsepower ratings—single strand roller chain no. 80

No. of teeth	10	25	50	75	88	100	200	300	400	500	600	700	800	900	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500
	1.000 inch pitch				Rotational speed of small sprocket, rev/min																				
11	0.44	1.06	2.07	3.05	3.56	4.03	7.83	11.56	15.23	18.87	22.48	26.07	27.41	22.97	19.61	14.92	11.84	9.69	8.12	6.83	4.96	3.77	3.00	2.45	0.00
12	0.48	1.16	2.26	3.33	3.88	4.39	8.54	12.61	16.82	20.59	24.53	28.44	31.23	26.17	22.35	17.00	13.49	11.04	9.25	7.90	5.65	4.30	3.41	2.79	0.00
13	0.52	1.26	2.45	3.61	4.21	4.76	9.26	13.66	18.00	22.31	26.57	30.81	35.02	29.51	25.20	19.17	15.21	12.45	10.43	8.91	6.37	4.85	3.85	3.15	
14	0.56	1.35	2.63	3.89	4.53	5.12	9.97	14.71	19.39	24.02	28.62	33.18	37.72	32.98	28.16	21.42	17.00	13.91	11.66	9.96	7.12	5.42	4.30	3.52	
15	0.60	1.45	2.82	4.16	4.86	5.49	10.68	15.76	20.77	25.74	30.66	35.55	40.41	36.58	31.23	23.76	18.85	15.43	12.93	11.04	7.90	6.01	4.77	0.00	
16	0.64	1.55	3.01	4.44	5.18	5.86	11.39	16.81	22.16	27.45	32.70	37.92	43.11	40.30	34.41	26.17	20.77	17.00	14.25	12.16	8.70	6.62	5.25	0.00	
17	0.68	1.64	3.20	4.72	5.50	6.22	12.10	17.86	23.54	29.17	34.75	40.29	45.80	44.13	37.68	28.66	22.75	18.62	15.60	13.32	9.53	7.25	0.00		
18	0.72	1.74	3.39	5.00	5.83	6.59	12.81	18.91	24.93	30.88	36.79	42.66	48.49	48.08	41.05	31.23	24.78	20.29	17.00	14.51	10.39	7.90	0.00		
19	0.76	1.84	3.57	5.28	6.15	6.95	13.53	19.96	26.31	32.60	38.84	45.03	51.19	52.15	44.52	33.87	26.88	22.00	18.44	15.74	11.26	0.36	0.00		
20	0.80	1.93	3.76	5.55	6.47	7.32	14.24	21.01	27.70	34.32	40.88	47.40	53.88	56.32	48.08	36.58	29.03	23.76	19.91	17.00	12.16	0.00			
21	0.84	2.03	3.95	5.83	6.80	7.69	14.95	22.07	29.08	36.03	42.92	49.77	56.58	60.59	51.73	39.36	31.23	25.56	21.42	18.29	13.09	0.00			
22	0.88	2.13	4.14	6.11	7.12	8.05	15.66	23.12	30.47	37.75	44.97	52.14	59.27	64.97	55.47	42.20	33.49	27.41	22.97	19.61	14.03				
23	0.92	2.22	4.33	6.39	7.45	8.42	16.37	24.17	31.85	39.46	47.01	54.51	61.97	69.38	59.30	45.11	35.80	29.30	24.55	20.97	15.00				
24	0.96	2.32	4.52	6.66	7.77	8.78	17.09	25.22	33.24	41.18	49.06	56.88	64.66	72.40	63.21	48.08	38.16	31.23	26.17	22.35	15.99				
25	1.00	2.42	4.70	6.94	8.09	9.15	17.80	26.27	34.62	42.89	51.10	59.25	67.35	75.42	67.20	51.12	40.57	33.20	27.83	23.76	8.16				
26	1.04	2.51	4.89	7.22	8.42	9.52	18.51	27.32	36.01	44.61	53.14	61.62	70.05	78.43	71.27	54.22	43.02	36.22	29.51	25.20	0.00				
28	1.12	2.71	5.27	7.77	9.06	10.25	19.93	29.42	38.78	48.04	57.23	66.36	75.44	84.47	79.65	60.59	48.08	39.36	32.98	28.16	0.00				
30	1.20	2.90	5.64	8.33	9.71	10.98	21.36	31.52	41.55	51.47	61.32	71.10	80.82	90.50	88.33	67.20	53.33	43.65	36.58	31.23					
32	1.28	3.09	6.02	8.89	10.36	11.71	22.78	33.62	44.32	54.91	65.41	75.84	86.21	96.53	97.31	74.03	58.75	48.08	40.30	5.65					
35	1.40	3.38	6.58	9.72	11.33	12.81	24.92	36.78	48.47	60.05	71.54	82.95	94.29	105.58	111.31	84.68	67.20	55.00	28.15	0.00					
40	1.61	3.87	7.53	11.11	12.95	14.64	28.48	42.03	55.40	68.63	81.76	94.80	107.77	120.67	133.51	103.46	82.10	40.16	0.00						
45	1.81	4.35	8.47	12.49	14.57	16.47	32.04	47.28	62.32	77.21	91.98	106.65	121.24	135.75	150.20	123.45	72.28	0.00							

Type A | Type B | Type C

Type A: Manual or drip lubrication
Type B: Bath of disc lubrication
Type C: Oil stream lubrication

Source: American Chain Association, Naples, FL

TABLE 7–8 Service factors for chain drives

	Type of driver		
Load type	Hydraulic drive	Electric motor or turbine	Internal combustion engine with mechanical drive
Smooth (agitators; fans; light, uniformly loaded conveyors)	1.0	1.0	1.2
Moderate shock (machine tools, cranes, heavy conveyors, food mixers and grinders)	1.2	1.3	1.4
Heavy shock (punch presses, hammer mills, reciprocating conveyors, rolling mill drive)	1.4	1.5	1.7

Design Guidelines for Chain Drives

The following are general recommendations for designing chain drives:

1. The minimum number of teeth in a sprocket should be 17 unless the drive is operating at a very low speed, under 100 rpm.
2. The maximum speed ratio should be 7.0, although higher ratios are feasible. Two or more stages of reduction can be used to achieve higher ratios.
3. The center distance between the sprocket axes should be approximately 30 to 50 pitches (30 to 50 times the pitch of the chain).
4. The larger sprocket should normally have no more than 120 teeth.
5. The preferred arrangement for a chain drive is with the centerline of the sprockets horizontal and with the tight side on top.
6. The chain length must be an integral multiple of the pitch, and an even number of pitches is recommended. The center distance should be made adjustable to accommodate the chain length and to take up for tolerances and wear. Excessive sag on the slack side should be avoided, especially on drives that are not horizontal. A convenient relation between center distance (C), chain length (L), number of teeth in the small sprocket (N_1), and number of teeth in the large sprocket (N_2), expressed in pitches, is

$$L = 2C + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C} \qquad \textbf{(7–9)}$$

The center distance for a given chain length, again in pitches, is

$$C = \frac{1}{4}\left[L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{N_2 + N_1}{2}\right]^2 - \frac{8\,(N_2 - N_1)^2}{4\pi^2}}\right] \qquad \textbf{(7–10)}$$

The computed center distance assumes no sag in either the tight or the slack side of the chain, and thus it is a *maximum*. Negative tolerances or adjustment must be provided. Adjustment for wear must also be provided.

7. The pitch diameter of a sprocket with N teeth for a chain with a pitch of p is

$$D = \frac{p}{\sin(180°/N)} \tag{7–11}$$

8. The minimum sprocket diameter and therefore the minimum number of teeth in a sprocket are often limited by the size of the shaft on which it is mounted. Check the sprocket catalog.

9. The arc of contact, θ_1, of the chain on the smaller sprocket should be greater than 120°.

$$\theta_1 = 180° - 2\sin^{-1}[(D_2 - D_1)/2C] \tag{7–12}$$

10. For reference, the arc of contact, θ_2, on the larger sprocket is,

$$\theta_2 = 180° + 2\sin^{-1}[(D_2 - D_1)/2C] \tag{7–13}$$

Lubrication

It is essential that adequate lubrication be provided for chain drives. There are numerous moving parts within the chain, along with the interaction between the chain and the sprocket teeth. The designer must define the lubricant properties and the method of lubrication.

Lubricant Properties. Petroleum-based lubricating oil similar to engine oil is recommended. Its viscosity must enable the oil to flow readily between chain surfaces that move relative to each other while providing adequate lubrication action. The oil should be kept clean and free of moisture. Table 7–9 gives the recommended lubricant for different ambient temperatures.

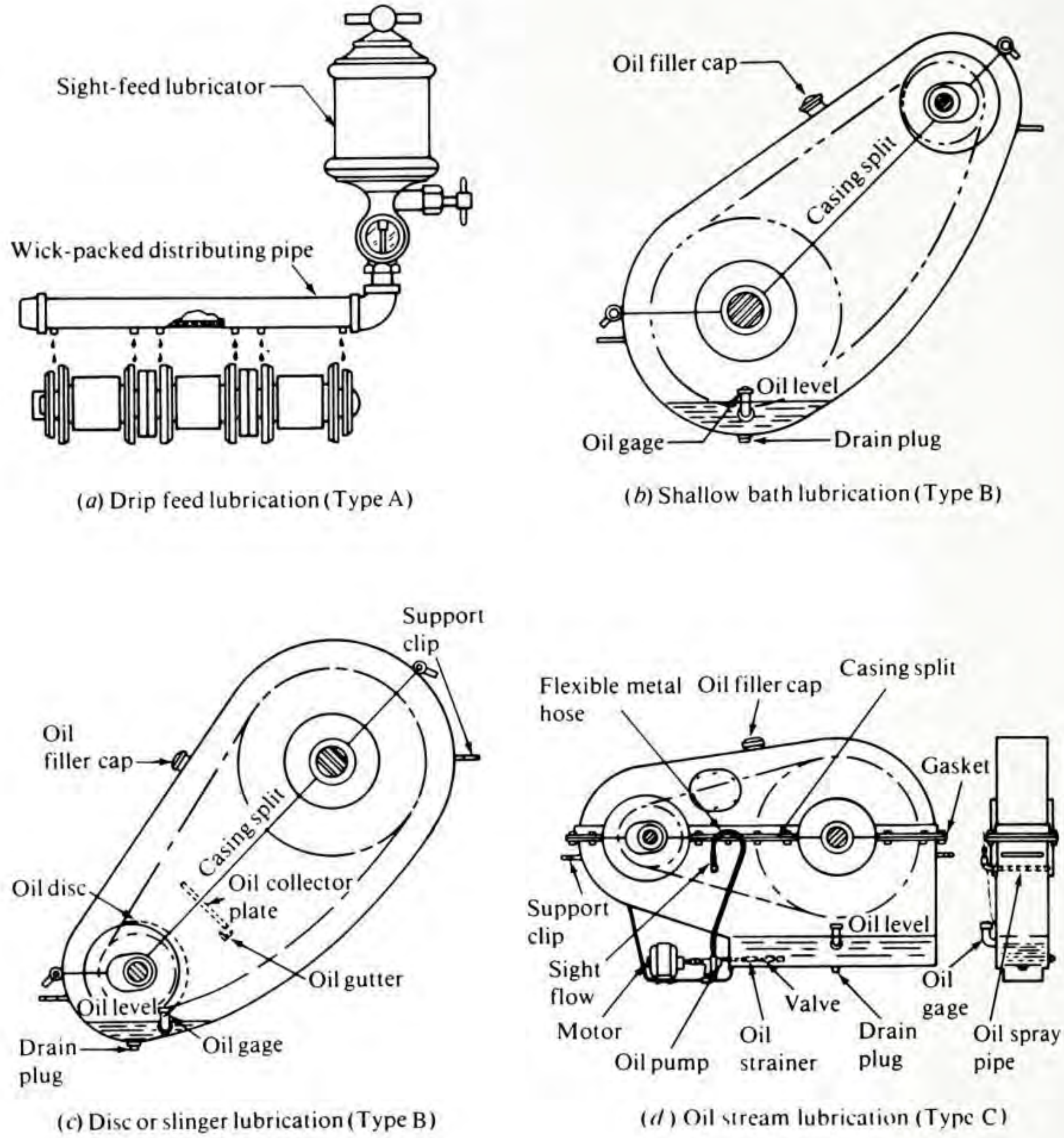
Method of Lubrication. The American Chain Association recommends three different types of lubrication depending on the speed of operation and the power being transmitted. See Tables 7–5 to 7–7 or manufacturer's catalogs for recommendations. Refer to the following descriptions of the methods and the illustrations in Figure 7–24.

Type A. *Manual or drip lubrication*: For manual lubrication, oil is applied copiously with a brush or a spout can, at least once every 8 h of operation. For drip feed lubrication, oil is fed directly onto the link plates of each chain strand.

TABLE 7–9 Recommended lubricant for chain drives

Ambient temperature		Recommended lubricant
°F	°C	
20 to 40	-7 to 5	SAE 20
40 to 100	5 to 38	SAE 30
100 to 120	38 to 49	SAE 40
120 to 140	49 to 60	SAE 50

FIGURE 7–24
Lubrication methods (American Chain Association, Naples, FL)



Type B. *Bath or disc lubrication*: The chain cover provides a sump of oil into which the chain dips continuously. Alternatively, a disc or a slinger can be attached to one of the shafts to lift oil to a trough above the lower strand of chain. The trough then delivers a stream of oil to the chain. The chain itself, then, does not need to dip into the oil.

Type C. *Oil stream lubrication*: An oil pump delivers a continuous stream of oil on the lower part of the chain.

Example Problem 7–2 Design a chain drive for a heavily loaded coal conveyor to be driven by a gasoline engine through a mechanical drive. The input speed will be 900 rpm, and the desired output speed is 230 to 240 rpm. The conveyor requires 15.0 hp.

Solution Objective Design the chain drive.

Given

Power transmitted = 15 hp to a coal conveyor

Speed of motor = 900 rpm; output speed range = 230 to 240 rpm

Analysis Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results ***Step 1.*** Specify a service factor and compute the design power. From Table 7–8, for moderate shock and a gasoline engine drive through a mechanical drive, $SF = 1.4$.

$$\text{Design power} = 1.4(15.0) = 21.0 \text{ hp}$$

Step 2. Compute the desired ratio. Using the middle of the required range of output speeds, we have

$$\text{Ratio} = (900 \text{ rpm})/(235 \text{ rpm}) = 3.83$$

Step 3. Refer to the tables for power capacity (Tables 7–5, 7–6, and 7–7), and select the chain pitch. For a single strand, the no. 60 chain with $p = 3/4$ in seems best. A 17-tooth sprocket is rated at 21.96 hp at 900 rpm by interpolation. At this speed, type B lubrication (oil bath) is required.

Step 4. Compute the required number of teeth on the large sprocket:

$$N_2 = N_1 \times \text{ratio} = 17(3.83) = 65.11$$

Let's use the integer: 65 teeth.

Step 5. Compute the actual expected output speed:

$$n_2 = n_1(N_1/N_2) = 900 \text{ rpm}(17/65) = 235.3 \text{ rpm (Okay!)}$$

Step 6. Compute the pitch diameters of the sprockets using Equation (7–11):

$$D_1 = \frac{p}{\sin(180°/N_1)} = \frac{0.75 \text{ in}}{\sin(180°/17)} = 4.082 \text{ in}$$

$$D_2 = \frac{p}{\sin(180°/N_2)} = \frac{0.75 \text{ in}}{\sin(180°/65)} = 15.524 \text{ in}$$

Step 7. Specify the nominal center distance. Let's use the middle of the recommended range, 40 pitches.

Step 8. Compute the required chain length in pitches from Equation (7–9):

$$L = 2C + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C}$$

$$L = 2(40) + \frac{65 + 17}{2} + \frac{(65 - 17)^2}{4\pi^2 (40)} = 122.5 \text{ pitches} \tag{7–9}$$

Step 9. Specify an integral number of pitches for the chain length, and compute the actual theoretical center distance. Let's use 122 pitches, an even number. Then, from Equation (7–10),

$$C = \frac{1}{4}\left[L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{N_2 + N_1}{2}\right]^2 - \frac{8\,(N_2 - N_1)^2}{4\pi^2}}\right]$$

$$C = \frac{1}{4}\left[122 - \frac{65 + 17}{2} + \sqrt{\left[122 - \frac{65 + 17}{2}\right]^2 - \frac{8(65 - 17)^2}{4\pi^2}}\right] \tag{7–10}$$

$C = 39.766$ pitches $= 39.766(0.75\text{ in}) = 29.825$ in

Step 10. Compute the angle of wrap of the chain for each sprocket using Equations (7–12) and (7–13). Note that the minimum angle of wrap should be 120 degrees.

For the small sprocket,

$$\theta_1 = 180^\circ - 2\sin^{-1}[(D_2 - D_1)/2C]$$
$$\theta_1 = 180^\circ - 2\sin^{-1}[(15.524 - 4.082)/(2(29.825))] = 158^\circ$$

Because this is greater than 120°, it is acceptable.

For the larger sprocket,

$$\theta_2 = 180^\circ + 2\sin^{-1}(D_2 - D_1)/2C]$$
$$\theta_2 = 180^\circ + 2\sin^{-1}[(15.524 - 4.082)/(2(29.825))] = 202^\circ$$

Comments

Summary of Design

Figure 7–25(a) shows a sketch of the design to scale.

Pitch: No. 60 chain, 3/4-in pitch

Length: 122 pitches = 122(0.75) = 91.50 in

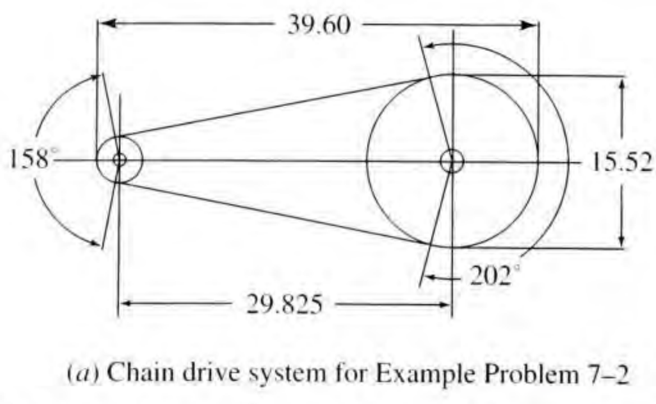
Center distance: $C = 29.825$ in (maximum)

Sprockets: Single-strand, no. 60, 3/4-in pitch

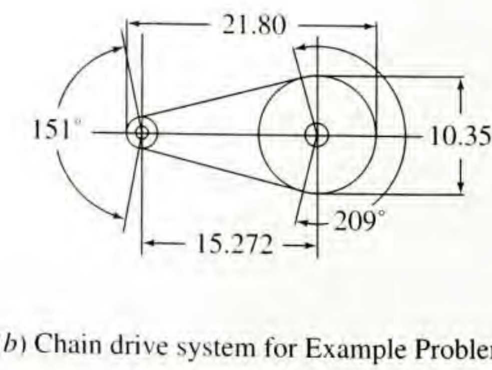
Small: 17 teeth, $D = 4.082$ in

Large: 65 teeth, $D = 15.524$ in

Type B lubrication is required. The large sprocket can dip into an oil bath.



(*a*) Chain drive system for Example Problem 7–2



(*b*) Chain drive system for Example Problem 7–3

FIGURE 7–25 Scale drawings of layouts for chain drives for Example Problems 7–2 and 7–3

Example Problem 7–3 Create an alternate design for the conditions of Example Problem 7–2 to produce a smaller drive.

Solution

Objective: Design a smaller chain drive for the application in Example Problem 7–2.

Given: Power transmitted = 15 hp to a conveyor
Speed of motor = 900 rpm; output speed range = 230 to 240 rpm

Analysis: Use a multistrand design to permit a smaller-pitch chain to be used to transmit the same design power (21.0 hp) at the same speed (900 rpm). Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results: Let's try a four-strand chain for which the power capacity factor is 3.3. Then the required power per strand is

$$P = 21.0/3.3 = 6.36\text{ hp}$$

From Table 7–5, we find that a no. 40 chain (1/2-in pitch) with a 17-tooth sprocket will be satisfactory. Type B lubrication, oil bath, can be used.

For the required large sprocket,

$$N_2 = N_1 \times \text{ratio} = 17(3.83) = 65.11$$

Let's use $N_2 = 65$ teeth.

The sprocket diameters are

$$D_1 = \frac{p}{\sin(180°/N_1)} = \frac{0.500\text{ in}}{\sin(180/17)} = 2.721\text{ in}$$

$$D_2 = \frac{p}{\sin(180°/N_2)} = \frac{0.500\text{ in}}{\sin(180°/65)} = 10.349\text{ in}$$

For the center distance, let's try the minimum recommended: $C = 30$ pitches.

$$30(0.50\text{ in}) = 15.0\text{ in}$$

The chain length is

$$L = 2(30) + \frac{65 + 17}{2} + \frac{(65 - 17)^2}{4\pi^2(30)} = 102.9\text{ pitches}$$

Specify the integer length, $L = 104$ pitches $= 104(0.50) = 52.0$ in. The actual maximum center distance is

$$C = \frac{1}{4}\left[104 - \frac{65 + 17}{2} + \sqrt{\left[104 - \frac{65 + 17}{2}\right]^2 - \frac{8(65 - 17)^2}{4\pi^2}}\right]$$

$$C = 30.54\text{ pitches} = 30.54(0.50) = 15.272\text{ in}$$

Compute the angle of wrap of the chain for each sprocket using Equations (7–12) and (7–13). Note that the minimum angle of wrap should be 120 degrees.

For the small sprocket,

$$\theta_1 = 180^\circ - 2\sin^{-1}[(D_2 - D_1)/2C]$$
$$\theta_1 = 180^\circ - 2\sin^{-1}[(10.349 - 2.721)/(2(15.272))] = 151.1^\circ$$

Because this is greater than 120°, it is acceptable.
For the larger sprocket,

$$\theta_1 = 180^\circ + 2\sin^{-1}[(D_2 - D_1)/2C]$$
$$\theta_2 = 180^\circ + 2\sin^{-1}[(10.349 - 2.721)/(2(15.272))] = 208.9^\circ$$

Comments

Summary

Figure 7–25(b) shows the new design to the same scale as the first design. The space reduction is significant.

Chain: No. 40, 1/2-in pitch, four-strand, 104 pitches, 52.0 in length

Sprockets: No. 40–4 (four strands), $\frac{1}{2}$-in pitch

Small: 17 teeth, $D_1 = 2.721$ in

Large: 65 teeth, $D_2 = 10.349$ in

Maximum center distance: 15.272 in

Type B lubrication (oil bath)

Spreadsheet for Chain Design

Figure 7–26 shows a spreadsheet that assists in the design of chain drives using the procedure developed in this section. The user enters data shown in italics in the gray-shaded cells. Refer to Tables 7–4 to 7–8 for required data. Results for Example Problem 7–3 are shown in the figure.

REFERENCES

1. American Chain Association. *Chains for Power Transmission and Material Handling*. New York: Marcel Dekker, 1982.
2. Dayco CPT. *Industrial V-Belt Drives Design Guide*. Dayton, OH: Carlisle Power Transmission Products.
3. Dayco Products. *Engineering Handbook for Automotive V-Belt Drives*. Rochester Hills, MI: Mark IV Automotive Co.
4. Emerson Power Transmission Company. *Power Transmission Equipment Catalog*. Maysville, KY: Browning Manufacturing Division.
5. The Gates Rubber Company. *V-Belt Drive Design Manual*. Denver, CO: The Gates Rubber Company.
6. Putnam Precision Molding. *Plastic Chain Products*. Putnam, CT.
7. Rexnord, Incorporated. *Catalog of Power Transmission and Conveying Components*. Milwaukee, WI: Rexnord.
8. Rockwell Automation/Dodge. *Power Transmission Products*. Greenville, SC. Rockwell Automation.
9. Rubber Manufacturers Association. Power Transmission Belt Publication IP-3-10. *V-Belt Drives with Twist and Non-Alignment Including Quarter Turn*. 3rd ed. Washington, DC: Rubber Manufacturers Association. 1999.
10. Society of Automotive Engineers. *SAE Standard J636—V-Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineers, 2001.
11. Society of Automotive Engineers. *SAE Standard J637—Automotive V-Belt Drives*. Warrendale, PA: Society of Automotive Engineers, 2001.

CHAIN DRIVE DESIGN		
Initial Input Data:	Example Problem 7–3—Multiple strands	
Application:	*Coal Conveyor*	
Drive/type:	*Engine-Mechanical drive*	
Driven machine:	*Heavily loaded conveyor*	
Power input:	*15* hp	
Service factor:	*1.4*	Table 7–8
Input speed:	*900* rpm	
Desired output speed:	*235* rpm	
Computed Data:		
Design power:	21 hp	
Speed ratio:	3.83	
Design Decisions—Chain Type and Teeth Numbers:		
Number of strands:	*4*	1 2 3 4
Strand factor:	*3.3*	1.0 1.7 2.5 3.3
Required power per strand:	6.36 hp	
Chain number:	*40*	Tables 7–5, 7–6, or 7–7
Chain pitch:	*0.5* in	
Number of teeth-Driver sprocket:	*17*	
Computed no. of teeth-Driven sprocket:	65.11	
Enter: Chosen number of teeth:	*65*	
Computed Data:		
Actual output speed:	235.4 rpm	
Pitch diameter-Driver sprocket:	2.721 in	
Pitch diameter-Driven sprocket:	10.349 in	
Center Distance, Chain Length and Angle of Wrap:		
Enter: Nominal center distance:	*30* pitches	30 to 50 pitches recommended
Computed nominal chain length:	102.9 pitches	
Enter: Specified no. of pitches:	*104* pitches	Even number recommended
Actual chain length:	52.00 in	
Computed actual center distance:	30.545 pitches	
Actual center distance:	15.272 in	
Angle of wrap–Driver sprocket:	151.1 degrees	Should be greater than 120 degrees
Angle of wrap–Driven sprocket:	208.9 degrees	

FIGURE 7–26 Spread sheet for chain design

12. Society of Automotive Engineers. *SAE Standard J1278—SI (Metric) Synchronous Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineers, 1993.

13. Society of Automotive Engineers. *SAE Standard J1313—Automotive Synchronous Belt Drives*. Warrendale, PA: Society of Automotive Engineers, 1993.

14. Society of Automotive Engineers. *SAE Standard J1459—V-Ribbed Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineering, 2001.

15. T. B. Wood's Sons Company. *V-Belt Drive Manual*. Chambersburg, PA: T. B. Wood's Sons Company.

INTERNET SITES RELATED TO BELT DRIVES AND CHAIN DRIVES

1. **American Chain Association.** *www.americanchainassn.org* A national trade organization for companies providing products for the chain drive industry. Publishes standards and design aids for designing, applying, and maintaining chain drives and engineering chain conveyor systems.
2. **Dayco Belt Drives.** *www.dayco.com* and *www.markivauto.com* Manufacturer of Dayco industrial belt drive systems under Carlisle Power Transmission Products, Inc., and Dayco automotive belt drive systems under the MarkIV Automotive Company.
3. **Dodge Power Transmission.** *www.dodge-pt.com* Manufacturer of numerous power transmission components, including V-belt and synchronous belt drive systems. Part of Rockwell Automation, Inc., which includes Reliance Electric motors and drives and Allen-Bradley controls.
4. **Emerson Power Transmission.** *www.emerson-ept.com* Manufacturer of numerous power transmission components including V-belt drives, synchronous belt drives, and roller chain drives through their Browning and Morse divisions.
5. **Gates Rubber Company.** *www.gates.com* Rubber products for the automotive and industrial markets including V-belt drives and synchronous belt drives.
6. **Power Transmission** *www.powertransmission.com* A comprehensive website for companies providing products for the power transmission industry, many of which supply belt and chain drive systems.
7. **Putnam Precision Molding, Inc.** *www.putnamprecisionmolding.com* Producer of plastic injection molded mechanical drive components, including plastic chain, sprockets, and synchronous belt pulleys.
8. **Rexnord Corporation.** *www.rexnord.com* Manufacturer of power transmission and conveying components, including roller chain drives and engineered chain drive systems.
9. **Rubber Manufacturers Association.** *www.rma.org* National trade association for the finished rubber products industry. Provides many standards and technical publications for the application of rubber products, including V-belt drives.
10. **SAE International.** *www.sae.org* The Society of Automotive Engineers, the engineering society for advancing mobility on land or sea, in air or space. Offers standards on V-belts, synchronous belts, pulleys, and drives for automotive applications.
11. **T. B. Wood's Sons Company** *www.tbwoods.com* Manufacturer of many mechanical drives products, including V-belt drives, synchronous belt drives, and adjustable speed drives.

PROBLEMS

V-Belt Drives

1. Specify the standard 3V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 5.25 in and 13.95 in with a center distance of no more than 24.0 in.
2. For the standard belt specified in Problem 1, compute the actual center distance that would result.
3. For the standard belt specified in Problem 1, compute the angle of wrap on both of the sheaves.
4. Specify the standard 5V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 8.4 in and 27.7 in with a center distance of no more than 60.0 in.
5. For the standard belt specified in Problem 4, compute the actual center distance that would result.
6. For the standard belt specified in Problem 4, compute the angle of wrap on both of the sheaves.
7. Specify the standard 8V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 13.8 in and 94.8 in with a center distance of no more than 144 in.
8. For the standard belt specified in Problem 7, compute the actual center distance that would result.
9. For the standard belt specified in Problem 7, compute the angle of wrap on both of the sheaves.
10. If the small sheave of Problem 1 is rotating at 1750 rpm, compute the linear speed of the belt.
11. If the small sheave of Problem 4 is rotating at 1160 rpm, compute the linear speed of the belt.
12. If the small sheave of Problem 7 is rotating at 870 rpm, compute the linear speed of the belt.
13. For the belt drive from Problems 1 and 10, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.
14. For the belt drive from Problems 4 and 11, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.
15. For the belt drive from Problems 7 and 12, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.

TABLE 7–10

Problem number	Driver type	Driven machine	Service (h/day)	Input speed (rpm)	Input power (hp)	Nominal output speed (rpm)
18.	AC motor (HT)	Hammer mill	8	870	25	310
19.	AC motor (NT)	Fan	22	1750	5	725
20.	6-cylinder engine	Heavy conveyor	16	1500	40	550
21.	DC motor (compound)	Milling machine	16	1250	20	695
22.	AC motor (HT)	Rock crusher	8	870	100	625

Note: NT indicates a normal-torque electric motor. *HT* indicates a high-torque electric motor.

16. Describe a standard 15N belt cross section. To what size belt (inches) would it be closest?

17. Describe a standard 17A belt cross section. To what size belt (inches) would it be closest?

For Problems 18–22 (Table 7–10), design a V-belt drive. Specify the belt size, the sheave sizes, the number of belts, the actual output speed, and the center distance.

Roller Chain

23. Describe a standard roller chain, no. 140.

24. Describe a standard roller chain, no. 60.

25. Specify a suitable standard chain to exert a static pulling force of 1250 lb.

26. Roller chain is used in a hydraulic forklift truck to elevate the forks. If two strands support the load equally, which size would you specify for a design load of 5000 lb?

27. List three typical failure modes of roller chain.

28. Determine the power rating of a no. 60 chain, single-strand, operating on a 20-tooth sprocket at 750 rpm. Describe the preferred method of lubrication. The chain connects a hydraulic drive with a meat grinder.

29. For the data of Problem 28, what would be the rating for three strands?

30. Determine the power rating of a no. 40 chain, single-strand, operating on a 12-tooth sprocket at 860 rpm. Describe the preferred method of lubrication. The small sprocket is applied to the shaft of an electric motor. The output is to a coal conveyor.

31. For the data of Problem 30, what would be the rating for four strands?

32. Determine the power rating of a no. 80 chain, single-strand, operating on a 32-tooth sprocket at 1160 rpm. Describe the preferred method of lubrication. The input is an internal combustion engine, and the output is to a fluid agitator.

33. For the data of Problem 32, what would be the rating for two strands?

34. Specify the required length of no. 60 chain to mount on sprockets having 15 and 50 teeth with a center distance of no more than 36 in.

35. For the chain specified in Problem 34, compute the actual center distance.

36. Specify the required length of no. 40 chain to mount on sprockets having 11 and 45 teeth with a center distance of no more than 24 in.

37. For the chain specified in Problem 36, compute the actual center distance.

For Problems 38–42 (Table 7–11), design a roller chain drive. Specify the chain size, the sizes and number of teeth in the sprockets, the number of chain pitches, and the center distance.

TABLE 7–11

Problem number	Driver type	Driven machine	Input speed (rpm)	Input power (hp)	Nominal output speed (rpm)
38.	AC motor	Hammer mill	310	25	160
39.	AC motor	Agitator	750	5	325
40.	6-cylinder engine	Heavy conveyor	500	40	250
41.	Steam turbine	Centrifugal pump	2200	20	775
42.	Hydraulic drive	Rock crusher	625	100	225

8

Kinematics of Gears

The Big Picture

Kinematics of Gears

Discussion Map

- ☐ Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another.
- ☐ Most gear drives cause a change in the speed of the output gear relative to the input gear.
- ☐ Some of the most common types of gears are *spur gears, helical gears, bevel gears, and worm/wormgear sets.*

Discover

Identify at least two machines or devices that employ gears. Describe the operation of the machines or devices and the appearance of the gears.

This chapter will help you learn about the features of different kinds of gears, the kinematics of a pair of gears operating together, and the operation of gear trains having more than two gears.

Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another. The teeth of a driving gear mesh accurately in the spaces between teeth on the driven gear as shown in Figure 8–1. The driving teeth push on the driven teeth, exerting a force perpendicular to the radius of the gear. Thus, a torque is transmitted, and because the gear is rotating, power is also transmitted.

Speed Reduction Ratio. Often gears are employed to produce a change in the speed of rotation of the driven gear relative to the driving gear. In Figure 8–1, if the smaller top gear, called a *pinion*, is driving the larger lower gear, simply called the *gear*, the larger gear will rotate more slowly. The amount of speed reduction is dependent on the ratio of the number of teeth in the pinion to the number of teeth in the gear according to this relationship:

$$n_P/n_G = N_G/N_P \tag{8–1}$$

The basis for this equation will be shown later in this chapter. But to show an example of its application here, consider that the pinion in Figure 8–1 is rotating at 1800 rpm. You can count the number of teeth in the pinion to be 11 and the number of teeth in the gear to be 18. Then we can compute the rotational speed of the gear by solving Equation (8–1) for n_G:

$$n_G = n_P(N_P/N_G) = (1800\text{ rpm})(11/18) = 1100\text{ rpm}$$

When there is a reduction in the speed of rotation of the gear, there is a simultaneous proportional *increase* in the torque transmitted to the shaft carrying the gear. More will be said about this later, also.

Kinds of Gears. Several kinds of gears having different tooth geometries are in common use. To acquaint you with the general appearance of some, their basic descriptions are given here. Later we will describe their geometry more completely.

Figure 8–2 shows a photograph of many kinds of gears. Labels indicate the major types of gears that are discussed in this chapter: *spur gears, helical gears, bevel gears*, and *worm/wormgear sets.* Obviously, the shafts that would carry the gears are not included in this photograph.

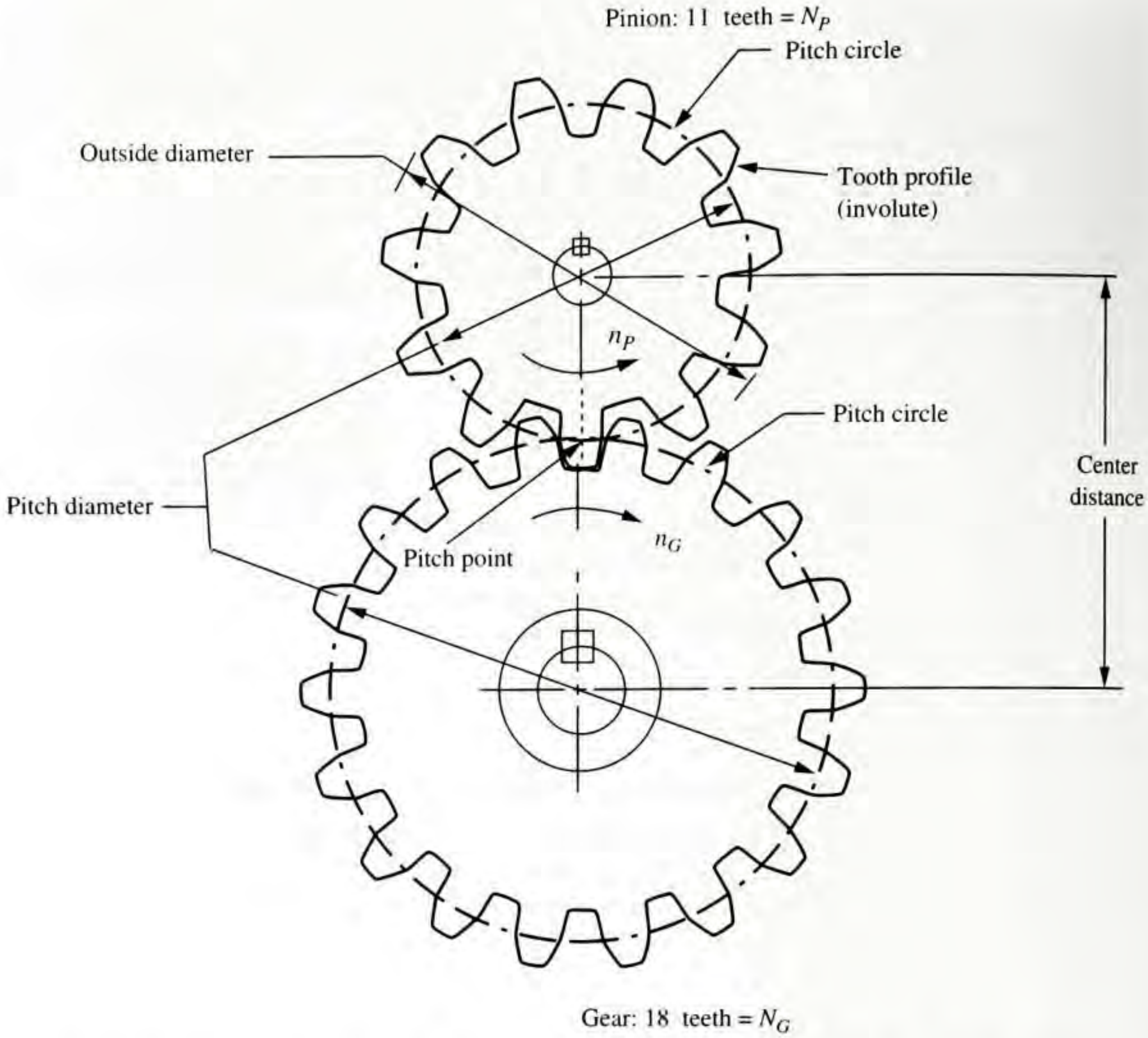


FIGURE 8–1 Pair of spur gears. The pinion drives the gear.

Spur gears have teeth that are straight and arranged parallel to the axis of the shaft that carries the gear. The curved shape of the faces of the spur gear teeth have a special geometry called an *involute curve,* described later in this chapter. This shape makes it possible for two gears to operate together with smooth, positive transmission of power. Figure 8–1 also shows the side view of spur gear teeth, and the involute curve shape is evident there. The shafts carrying the gears are parallel.

The teeth of *helical gears* are arranged so that they lie at an angle with respect to the axis of the shaft. The angle, called the *helix angle,* can be virtually any angle. Typical helix angles range from approximately 10° to 30°, but angles up to 45° are practical. The helical teeth operate more smoothly than equivalent spur gear teeth, and stresses are lower. Therefore, a smaller helical gear can be designed for a given power-transmitting capacity as compared with spur gears. One disadvantage of helical gears is that an axial force, called a *thrust force,* is generated in addition to the driving force that acts tangent to the basic cylinder on which the teeth are arranged. The designer must consider the thrust force when selecting bearings that will hold the shaft during operation. Shafts carrying helical gears are typically arranged parallel to each other. However, a special design, called *crossed helical* gears, has 45° helix angles, and their shafts operate 90° to each other.

Bevel gears have teeth that are arranged as elements on the surface of a cone. The teeth of straight bevel gears appear to be similar to spur gear teeth, but they are tapered, being wider at the outside and narrower at the top of the cone. Bevel gears typically operate on shafts that are 90° to each other. Indeed, this is often the reason for specifying bevel gears in a drive system. Specially designed bevel gears can operate on shafts that are at some angle other than 90°. When bevel gears are made with teeth that form a helix angle similar to

FIGURE 8–2 A variety of gear types (Boston Gear, Quincy, MA)

that in helical gears, they are called *spiral bevel gears.* They operate more smoothly than straight bevel gears and can be made smaller for a given power transmission capacity. When both bevel gears in a pair have the same number of teeth, they are called *miter gears* and are used only to change the axes of the shafts to 90 degrees. No speed change occurs.

A *rack* is a straight gear that moves linearly instead of rotating. When a circular gear is mated with a rack, as shown toward the right side of Figure 8–2, the combination is called *a rack and pinion drive*. You may have heard that term applied to the steering mechanism of a car or to a part of other machinery. See Section 8–6 for more discussion about a rack.

A *worm and its mating wormgear* operate on shafts that are at 90° to each other. They typically accomplish a rather large speed reduction ratio compared with other types of gears. The worm is the driver, and the wormgear is the driven gear. The teeth on the worm appear similar to screw threads, and, indeed, they are often called *threads* rather than *teeth.* The teeth of the wormgear can be straight like spur gear teeth, or they can be helical. Often the shape of the tip of the wormgear teeth is enlarged to partially wrap around the threads of the worm to improve the power transmission capacity of the set. One disadvantage of the worm/wormgear drive is that it has a somewhat lower mechanical efficiency than most other kinds of gears because there is extensive rubbing contact between the surfaces of the worm threads and the sides of the wormgear teeth.

Where Have You Observed Gears? Think of examples where you have seen gears in actual equipment. Describe the operation of the equipment, particularly the power transmission system. Sometimes, of course, the gears and the shafts are enclosed in a housing, making it difficult for you to observe the actual gears. Perhaps you can find a manual for the equipment that shows the drive system. Or look elsewhere in this chapter and in Chapters 9 and 10 for some photographs of commercially available gear reducers. ***(Note:** If the equipment you are observing is operating, be very careful not to come in contact with any moving parts!)* Try to answer these questions:

- What was the source of the power? An electric motor, a gasoline engine, a steam turbine, a hydraulic motor? Or were the gears operated by hand?
- How were the gears arranged together, and how were they attached to the driving source and the driven machine?
- Was there a speed change? Can you determine how much of a change?
- Were there more than two gears in the drive system?
- What types of gears were used? (You should refer to Figure 8–2.)
- What materials were the gears made from?
- How were the gears attached to the shafts that supported them?
- Were the shafts for mating gears aligned parallel to each other, or were they perpendicular to one another?
- How were the shafts themselves supported?
- Was the gear transmission system enclosed in a housing? If so, describe it.

This chapter will help you learn the basic geometries and kinematics of gears and pairs of gears operating together. You will also learn how to analyze gear trains having more than two gears so that you can describe the motion of each gear. Then you will learn how to devise a gear train to produce a given speed reduction ratio. In later chapters, you will learn how to analyze gears for their power transmission capacity and to design gear trains to transmit a given amount of power at a specified ratio of the speed of the input shaft to the speed of the output shaft.

You Are the Designer

A gear-type speed reducer was described in Chapter 1, and a sketch of the layout of the gears within the reducer was shown in Figure 1–1. You are advised to review that discussion now because it will help you understand how the present chapter on *gear geometry* and *kinematics* fits into the design of the complete speed reducer.

Assume that you are responsible for the design of a speed reducer that will take the power from the shaft of an electric motor rotating at 1750 rpm and deliver it to a machine that is to operate at approximately 292 rpm. You have decided to use gears to transmit the power, and you are proposing a double-reduction speed reducer like that shown in Figure 8–3. This chapter will give you the information you need to define the general nature of the gears, including their arrangement and their relative sizes.

The input shaft (shaft 1) is coupled to the motor shaft. The first gear of the gear train is mounted on this shaft and rotates at the same speed as the motor, 1750 rpm. Gear 1 drives the mating gear 2, which is larger, causing the speed of rotation of shaft 2 to be slower than that of shaft 1. But the speed is not yet down to 292 rpm as desired.

The next step is to mount a third gear (gear 3) on shaft 2 and mate it with gear 4 mounted on the output shaft, shaft 3. With proper sizing of all four gears, you should be able to produce an output speed equal or quite close to the desired speed. This process requires knowledge of the concept of *velocity ratio* and the techniques of designing gear trains as presented in this chapter.

But you will also need to specify the appearance of the gears and the geometry of the several features that make up each gear. Whereas the final specification also requires the information from following chapters, you will learn how to recognize common forms of gears and to compute the dimensions of key features. This will be important when completing the design for strength and wear resistance in later chapters.

Let's say that you have chosen to use spur gears in your design. What design decisions must you make to complete the specification of all four gears? The following list gives some of the important parameters for each gear:

- The number of teeth
- The form of the teeth
- The size of the teeth as indicated by the *pitch*
- The width of the face of the teeth
- The style and dimensions of the gear blank into which the gear teeth are to be machined
- The design of the hub for the gear that facilitates its mounting to the shaft
- The degree of precision of the gear teeth and the corresponding method of manufacture that can produce that precision
- The means of attaching the gear to its shaft
- The means of locating the gear axially on the shaft

To make reliable decisions about these parameters, you must understand the special geometry of spur gears as presented first in this chapter. However, there are other forms of gears that you could choose. Later sections give the special geometry of helical gears, bevel gears, and worm/wormgear sets. The methods of analyzing the forces on these various kinds of gears are described in later chapters, including the stress analysis of the gear teeth and recommendations on material selection to ensure safe operation with long life.

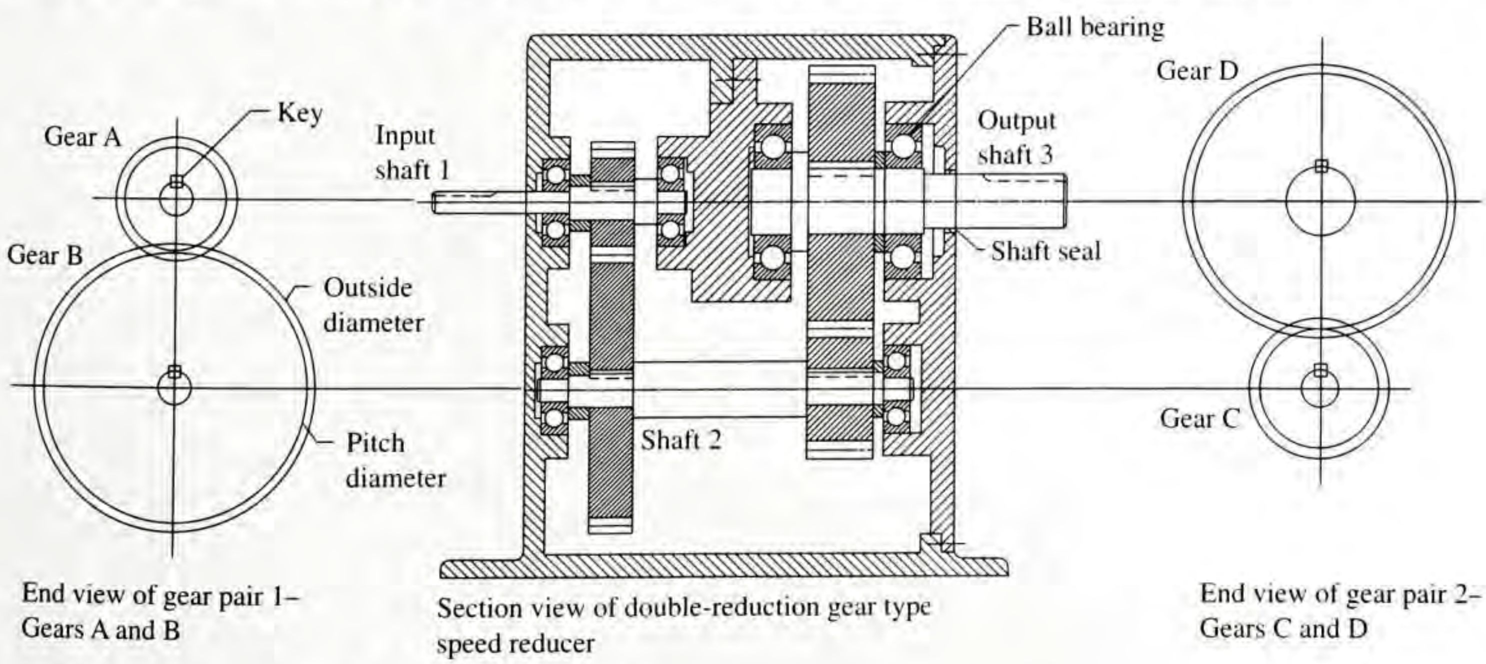


FIGURE 8–3 Conceptual design for a speed reducer

8–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Recognize and describe the main features of *spur gears, helical gears, bevel gears,* and *worm/wormgear sets.*
2. Describe the important operating characteristics of these various types of gears with regard to the similarities and differences among them and their general advantages and disadvantages.
3. Describe the *involute-tooth form* and discuss its relationship to the *law of gearing.*
4. Describe the basic functions of the American Gear Manufacturers Association (AGMA) and identify pertinent standards developed and published by this organization.
5. Define *velocity ratio* as it pertains to two gears operating together.
6. Specify appropriate numbers of teeth for a mating pair of gears to produce a given velocity ratio.
7. Define *train value* as it pertains to the overall speed ratio between the input and output shafts of a gear-type speed reducer (or speed increaser) that uses more than two gears.

8–2 SPUR GEAR STYLES

Figure 8–4 shows several different styles of commercially available spur gears. When gears are large, the spoked design in Part (a) is often used to save weight. The gear teeth are machined into a relatively thin rim that is held by a set of spokes connecting to the hub. The bore of the hub is typically designed to be a close sliding fit with the shaft that carries the

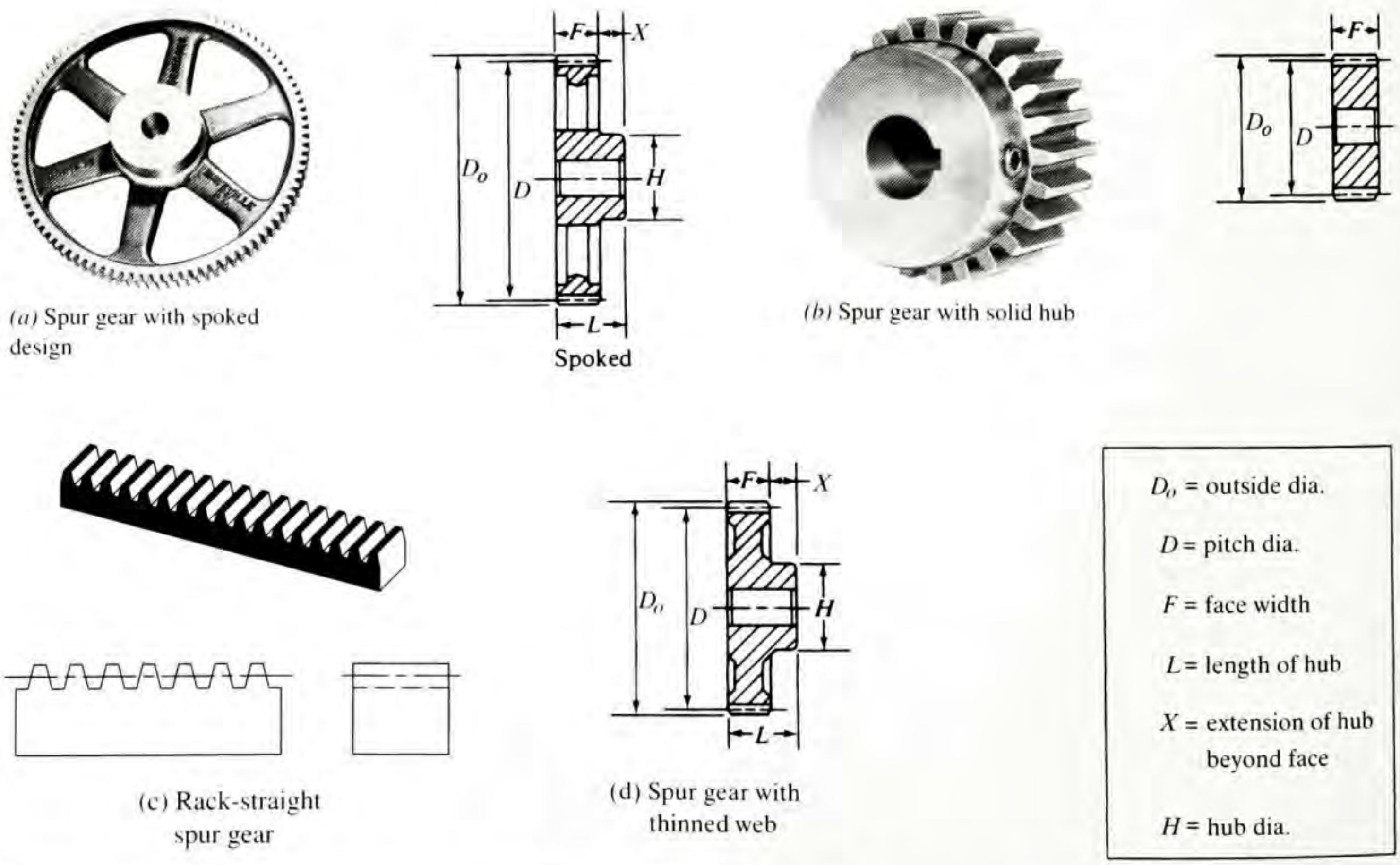


FIGURE 8–4 Spur gears (Emerson Power Transmission Corporation, Browning Division, Maysville, KY)

gear. A keyway is usually machined into the bore to allow a key to be inserted for positive transmission of torque. The illustration does not include a keyway because this gear is sold as a stock item, and the ultimate user finishes the bore to match a given piece of equipment.

The solid hub design in Figure 8–4(b) is typical of smaller spur gears. Here the finished bore with a keyway is visible. The set screw over the keyway allows the locking of the key in place after assembly.

When spur gear teeth are machined into a straight, flat bar, the assembly is called a *rack*, as shown in Figure 8–4(c). The rack is essentially a spur gear with an infinite radius. In this form, the teeth become straight-sided, rather than the curved, involute form typical of smaller gears.

Gears with diameters between the small solid form [Part (b)] and the larger spoked form [Part (a)] are often produced with a thinned web as shown in Part (d), again to save weight.

You as a designer may create special designs for gears that you implement into a mechanical device or system. One useful approach is to machine the gear teeth of small pinions directly into the surface of the shaft that carries the gear. This is very often done for the input shaft of gear reducers.

8–3 SPUR GEAR GEOMETRY INVOLUTE-TOOTH FORM

The most widely used spur gear tooth form is the full-depth involute form. Its characteristic shape is shown in Figure 8–5.

The involute is one of a class of geometric curves called *conjugate curves*. When two such gear teeth are in mesh and rotating, there is a *constant angular velocity* ratio between them: From the moment of initial contact to the moment of disengagement, the speed of the driving gear is in a constant proportion to the speed of the driven gear. The resulting action of the two gears is very smooth. If this were not the case, there would be some speeding up and slowing down during the engagement, with the resulting accelerations causing vibration, noise, and dangerous torsional oscillations in the system.

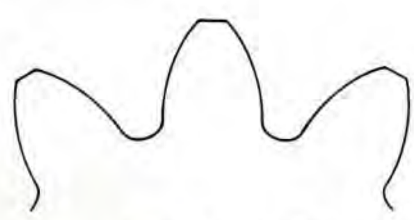
You can easily visualize an involute curve by taking a cylinder and wrapping a string around its circumference. Tie a pencil to the end of the string. Then start with the pencil tight against the cylinder, and hold the string taut. Move the pencil away from the cylinder while keeping the string taut. The curve that you will draw is an involute. Figure 8–6 is a sketch of the process.

The circle represented by the cylinder is called the *base circle*. Notice that at any position on the curve, the string represents a line tangent to the base circle and, at the same time, perpendicular to the involute. Drawing another base circle along the same centerline in such a position that the resulting involute is tangent to the first one, as shown in Figure 8–7, demonstrates that at the point of contact, the two lines tangent to the base circles are coincident and will stay in the same position as the base circles rotate. This is what happens when two gear teeth are in mesh.

It is a fundamental principle of *kinematics*, the study of motion, that if the line drawn perpendicular to the surfaces of two rotating bodies at their point of contact always crosses the centerline between the two bodies at the same place, the angular velocity ratio of the two bodies will be constant. This is a statement of the *law of gearing*. As demonstrated here, the gear teeth made in the involute-tooth form obey the law.

Of course, only the part of the gear tooth that actually comes into contact with the mating tooth needs to be in the involute form.

FIGURE 8–5
Involute-tooth form



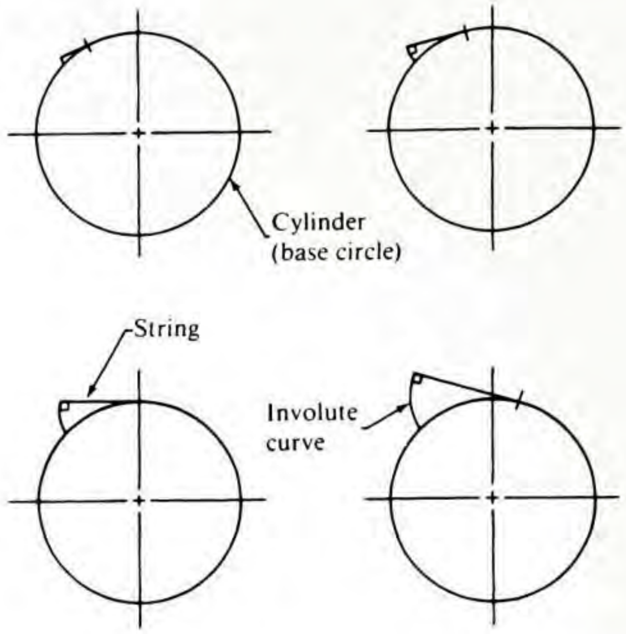


FIGURE 8–6 Graphical generation of an involute curve

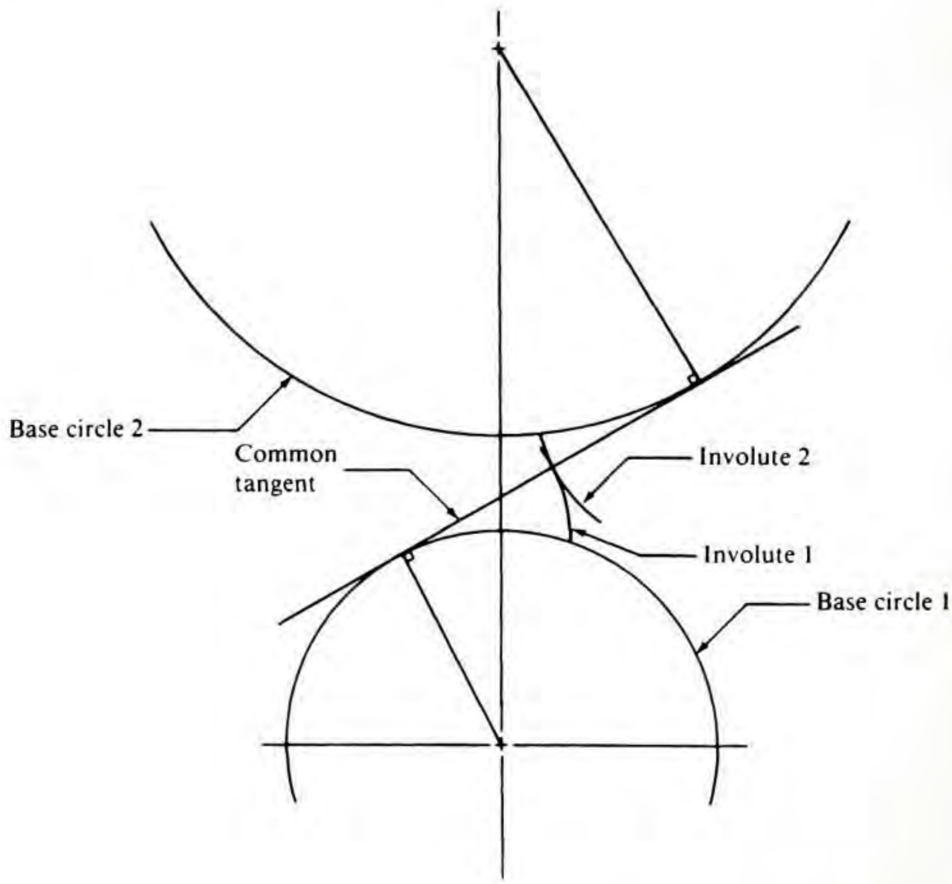


FIGURE 8–7 Mating involutes

8–4 SPUR GEAR NOMENCLATURE AND GEAR-TOOTH FEATURES

MDESIGN

This section describes several features of individual spur gear teeth and complete gears. Terms and symbols used conform to American Gear Manufacturers Association (AGMA) standards. (See Reference 1 for a more complete set of definitions.) Figure 8–8 shows drawings of spur gear teeth, with the symbols for the various features indicated. These features are described next.

Pitch Diameter

Figure 8–9 shows teeth from two gears in mesh to demonstrate the relative positions of the teeth at several stages of engagement. One of the most important observations that can be

FIGURE 8–8 Spur gear teeth features

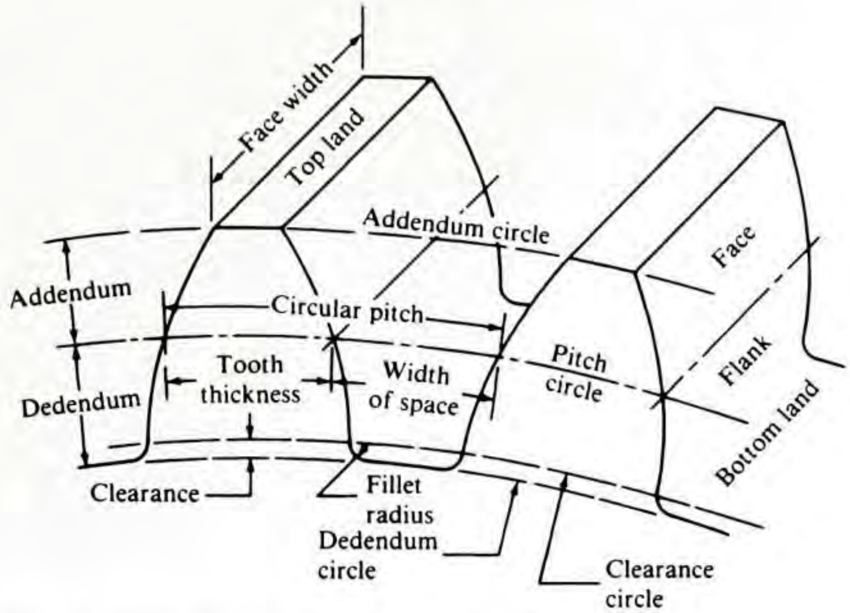
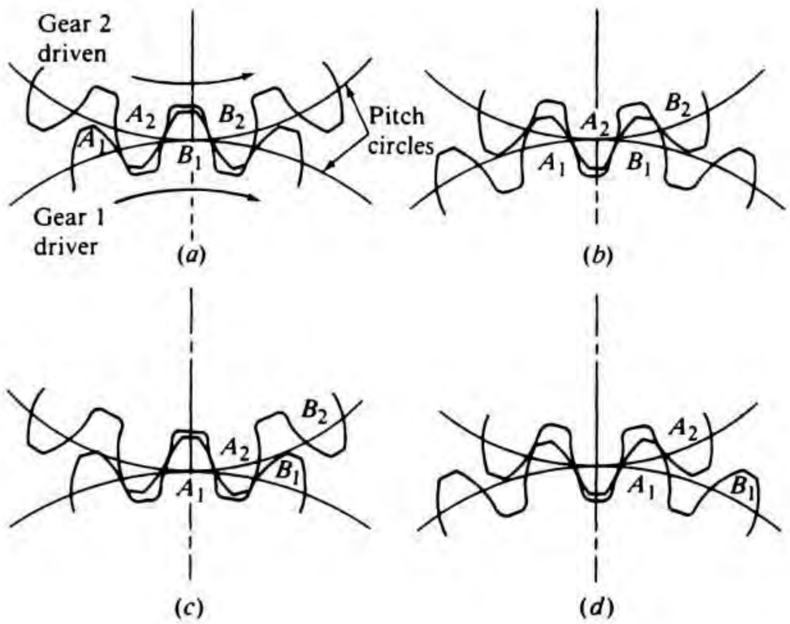


FIGURE 8–9 Cycle of engagement of gear teeth



made from Figure 8–9 is that throughout the engagement cycle there are two circles, one from each gear, that remain tangent. These are called the *pitch circles.* The diameter of the pitch circle of a gear is its *pitch diameter;* the point of tangency is the *pitch point.*

When two gears mesh, the smaller gear is called the *pinion,* and the larger is the *gear.* We will use the symbol D_P to indicate the pitch diameter of the pinion, and the symbol D_G for the pitch diameter of the gear. When referring to the number of teeth, we will use N_P for the pinion and N_G for the gear.

Notice that the pitch diameter lies somewhere within the height of the gear tooth, and thus it is not possible to measure its diameter directly. It must be calculated from other known features of the gear; this calculation depends on understanding the concept of *pitch,* discussed in the following section.

Pitch

The spacing between adjacent teeth and the size of the teeth are controlled by the pitch of the teeth. Three types of pitch designation systems are in common use for gears: (1) circular pitch, (2) diametral pitch, and (3) the metric module.

Circular Pitch, *p*.

The distance from a point on a tooth of a gear at the pitch circle to a corresponding point on the next adjacent tooth, measured along the pitch circle, is the circular pitch (see Figure 8–8).

Note that it is an arc length, usually in inches. To compute the value of the circular pitch, take the circumference of the pitch circle and divide it into a number of equal parts corresponding to the number of teeth in the gear. Using N for the number of teeth, we have

Circular Pitch

$$p = \pi D/N \tag{8–2}$$

Notice that the tooth size increases as the value of the circular pitch increases because there is a larger pitch circle for the same number of teeth. Also note that the basic sizes of the mating gear teeth must be the same for them to mesh properly. This observation leads to a very important rule:

The pitch of two gears in mesh must be identical.

This must be true whether the pitch is indicated as the circular pitch, the diametral pitch, or the metric module. Then Equation (8–2) can be written in terms of either the pinion or the gear diameter:

Circular Pitch

$$p = \pi D_G/N_G = \pi D_P/N_P \tag{8–3}$$

Circular pitch is infrequently used now. It is sometimes an advantage to use this system when large gears are to be made by casting. To facilitate the layout of the pattern for the casting, lay off the chord of the arc length of the circular pitch. Also, some machines and product lines of machines have traditionally used circular pitch gears and continue to do so. Table 8–1 lists the recommended standard circular pitches for large gear teeth.

Diametral Pitch, P_d. The most common pitch system used today in the United States is the *diametral pitch* system, the number of teeth per inch of pitch diameter. Its basic definition is

Diametral Pitch

$$P_d = N_G/D_G = N_P/D_P \tag{8–4}$$

As such, it has units of in $^{-1}$. However, the units are rarely reported, and gears are referred to as 8-pitch or 20-pitch, for example. One of the advantages of the diametral pitch system is that there is a set list of standard pitches, and most of the pitches have integer values. Table 8–2 lists the recommended standard pitches, with those of 20 and above called *fine pitch* and those below 20 called *coarse pitch*.

TABLE 8–1 Standard circular pitches (in)

10.0	7.5	5.0
9.5	7.0	4.5
9.0	6.5	4.0
8.5	6.0	3.5
8.0	5.5	

TABLE 8–2 Standard diametral pitches (teeth/in)

Coarse pitch ($P_d < 20$)				Fine pitch ($P_d \geq 20$)	
1	2	5	12	20	72
1.25	2.5	6	14	24	80
1.5	3	8	16	32	96
1.75	4	10	18	48	120
				64	

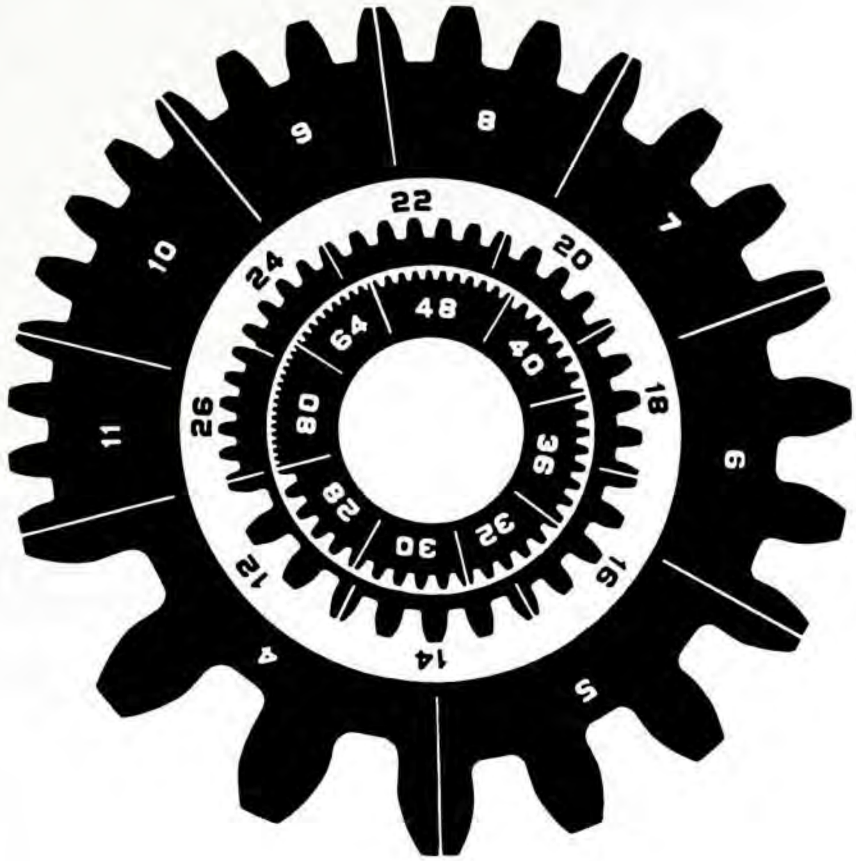


FIGURE 8–10 Gear-tooth size as a function of diametral pitch (Barber-Colman Company, Loves Park, IL)

Other intermediate values are available, but most manufacturers produce gears from this list of pitches. In any case, it is advisable to check availability before finally specifying a pitch. In problem solutions in this book, it is expected that one of the pitches listed in Table 8–2 will be used if possible.

As stated before, the pitch of the gear teeth determines their size, and two mating gears must have the same pitch. Figure 8–10 shows the profiles of some of the standard diametral pitch gear teeth, drawn actual size. That is, you can lay a given gear down on the page and compare its size with the drawing to obtain a good estimate of the pitch of the teeth. Notice that as the numerical value of the diametral pitch increases, the physical size of the tooth decreases, and vice versa.

Sometimes it is necessary to convert from diametral pitch to circular pitch, or vice versa. Their definitions provide a simple means of doing this. Solving for the pitch diameter in both Equations (8–2) and (8–4) gives

$$D = Np/\pi$$
$$D = N/P_d$$

Equating these two gives

Relation between Circular and Diametral Pitches

$$N/P_d = Np/\pi \quad \text{or} \quad P_d p = \pi \tag{8–5}$$

From this equation, the equivalent circular pitch for a gear having a diametral pitch of 1 is $p = \pi/1 = 3.1416$. Referring to Tables 8–1 and 8–2, notice that the circular pitches listed are for the larger gear teeth, being preferred when the diametral pitch is less than 1. Diametral pitch is preferred for sizes equivalent to 1 pitch or smaller.

Metric Module System. In the SI, a common unit of length is the *millimeter.* The pitch of gears in the metric system is based on this unit and is designated the *module, m.* To find the module of a gear, divide the pitch diameter of the gear in millimeters by the number of teeth. That is,

Metric Module

$$m = D_G/N_G = D_p/N_p \tag{8–6}$$

There is rarely a need to convert from the module system to the diametral pitch system. However, it is important to have a feel for the physical size of gear teeth. Because at this time people are more familiar with the standard diametral pitches, as shown in Figure 8–10, we will develop the relationship between m and P_d. From their definitions, Equations (8–4) and (8–6), we can say

$$m = 1/P_d$$

But recall that diametral pitch uses the inch unit, and module uses the millimeter. Therefore, the conversion factor of 25.4 mm per inch must be applied.

$$m = \frac{1}{P_d \text{in}^{-1}} \cdot \frac{25.4 \text{ mm}}{\text{in}}$$

This reduces to

Relation between Module and Diametral Pitch

$$m = 25.4/P_d \tag{8–7}$$

For example, if a gear has a diametral pitch of 10, the equivalent module is

$$m = 25.4/10 = 2.54 \text{ mm}$$

This is not a standard value for module, but it is close to the standard value of 2.5. So it can be concluded that a 10-pitch gear is of similar size to a gear with module 2.5. Table 8–3 gives selected standard modules with their equivalent diametral pitches.

Gear-Tooth Features

In design and inspection of gear teeth, several special features must be known. Figure 8–8, presented earlier, and Figure 8–11, which shows segments of two gears in mesh, identify these features. They are defined in the list that follows. Table 8–4 gives the relationships needed to compute their values. See References 1, 2, 4, 7, and 8 for the relevant AGMA standards. References 9, 10, and 12 provide much additional data. Note that many of the computations involve the diametral pitch, again illustrating that the physical size of a gear tooth is determined by its diametral pitch. The definitions are for external gears. Internal gears are discussed in Section 8–6.

- ***Addendum (a):*** The radial distance from the pitch circle to the outside of a tooth.
- ***Dedendum (b):*** The radial distance from the pitch circle to the bottom of the tooth space.
- ***Clearance (c):*** The radial distance from the top of a tooth to the bottom of the tooth space of the mating gear when the tooth is fully engaged. Note that

Clearance

$$c = b - a \tag{8–8}$$

TABLE 8–3 Standard modules

Module (mm)	Equivalent P_d	Closest standard P_d (teeth/in)
0.3	84.667	80
0.4	63.500	64
0.5	50.800	48
0.8	31.750	32
1	25.400	24
1.25	20.320	20
1.5	16.933	16
2	12.700	12
2.5	10.160	10
3	8.466	8
4	6.350	6
5	5.080	5
6	4.233	4
8	3.175	3
10	2.540	2.5
12	2.117	2
16	1.587	1.5
20	1.270	1.25
25	1.016	1

FIGURE 8–11 Gear pair features

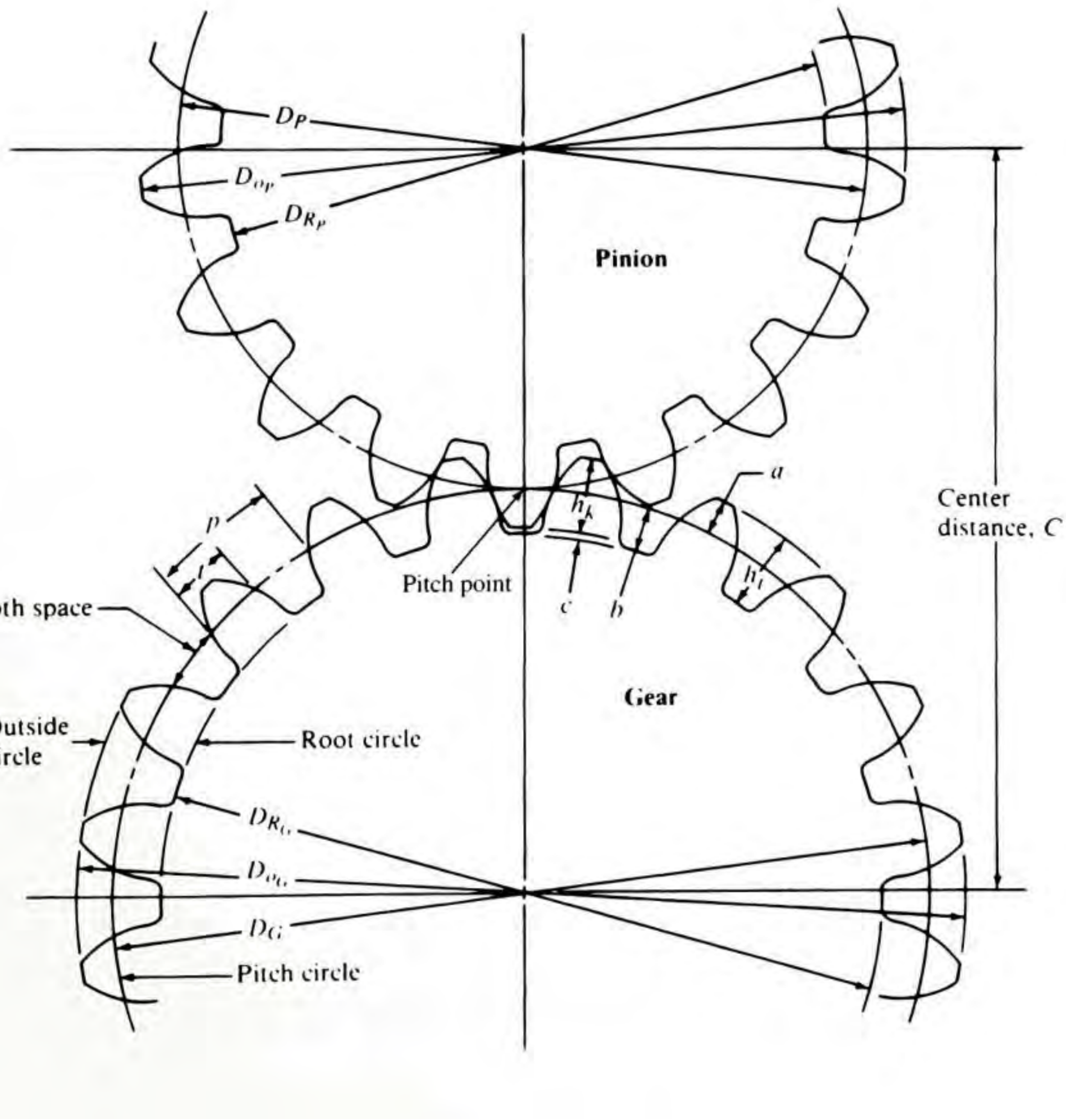


TABLE 8–4 Formulas for gear-tooth features for 20° pressure angle

		Full-depth involute system		
Feature	Symbol	Coarse pitch ($P_d < 20$)	Fine pitch ($P_d \geq 20$)	Metric module system
Addendum	a	$1/P_d$	$1/P_d$	$1.00m$
Dedendum	b	$1.25/P_d$	$1.200/P_d + 0.002$	$1.25m$
Clearance	c	$0.25/P_d$	$0.200/P_d + 0.002$	$0.25m$

- ***Outside diameter (D_o):*** The diameter of the circle that encloses the outside of the gear teeth. Note that

Outside Diameter Basic Definition

$$D_o = D + 2a \tag{8–9}$$

Also note that both the pitch diameter, D, and the addendum, a, are defined in terms of the diametral pitch, P_d. Making these substitutions produces a very useful form of the equation for outside diameter:

Outside Diameter in Terms of P_d and N

$$D_o = \frac{N}{P_d} + 2\frac{1}{P_d} = \frac{N+2}{P_d} \tag{8–10}$$

In the metric module system, a similar equation can be derived:

Outside Diameter in Metric Module system

$$D_o = mN + 2m = m(N + 2) \tag{8–11}$$

- ***Root diameter (D_R):*** The diameter of the circle that contains the bottom of the tooth space; this circle is called the *root circle.* Note that

Root Diameter

$$D_R = D - 2b \tag{8–12}$$

- ***Whole depth (h_t):*** The radial distance from the top of a tooth to the bottom of the tooth space. Note that

Whole Depth

$$h_t = a + b \tag{8–13}$$

- ***Working depth (h_k):*** The radial distance that a gear tooth projects into the tooth space of the mating gear. Note that

Working Depth

$$h_k = a + a = 2a \tag{8–14}$$

and

Whole Depth

$$h_t = h_k + c \tag{8–15}$$

- ***Tooth thickness (t):*** The arc length, measured on the pitch circle from one side of a tooth to the other side. This is sometimes called the *circular thickness* and has the theoretical value of one-half of the circular pitch. That is,

Tooth Thickness

$$t = p/2 = \pi/2P_d \tag{8–16}$$

- ***Tooth space:*** The arc length, measured on the pitch circle, from the right side of one tooth to the left side of the next tooth. Theoretically, the tooth space equals the tooth thickness. But for practical reasons, the tooth space is made larger (see "Backlash").

- ***Backlash:*** If the tooth thickness were made identical in value to the tooth space, as it theoretically is, the tooth geometry would have to be absolutely precise for the gears to operate, and there would be no space available for lubrication of the tooth surfaces. To alleviate these problems, practical gears are made with the tooth space slightly larger than the tooth thickness, the difference being called the *backlash.* To provide backlash, the cutter generating the gear teeth can be fed more deeply into the gear blank than the theoretical value on either or both of the mating gears. Alternatively, backlash can be created by adjusting the center distance to a larger value than the theoretical value.

  The magnitude of backlash depends on the desired precision of the gear pair and on the size and the pitch of the gears. It is actually a design decision, balancing cost of production with desired performance. The American Gear Manufacturers Association (AGMA) provides recommendations for backlash in their standards. (See Reference 2.) Table 8–5 lists recommended ranges for several values of pitch.
- ***Face width (F):*** The width of the tooth measured parallel to the axis of the gear.
- ***Fillet:*** The arc joining the involute-tooth profile to the root of the tooth space.
- ***Face:*** The surface of a gear tooth from the pitch circle to the outside circle of the gear.
- ***Flank:*** The surface of a gear tooth from the pitch circle to the root of the tooth space, including the fillet.

TABLE 8–5 Recommended minimum backlash for coarse pitch gears

A. Diametral pitch system (backlash in inches)

	Center distance, C (in)				
P_d	2	4	8	16	32
18	0.005	0.006			
12	0.006	0.007	0.009		
8	0.007	0.008	0.010	0.014	
5		0.010	0.012	0.016	
3		0.014	0.016	0.020	0.028
2			0.021	0.025	0.033
1.25				0.034	0.042

B. Metric module system (backlash in millimeters)

	Center distance, C (mm)				
Module, m	50	100	200	400	800
1.5	0.13	0.16			
2	0.14	0.17	0.22		
3	0.18	0.20	0.25	0.35	
5		0.26	0.31	0.41	
8		0.35	0.40	0.50	0.70
12			0.52	0.62	0.82
18				0.80	1.00

Source: Extracted from AGMA 2002-B88 Standard, *Tooth Thickness Specification and Measurement*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

- ***Center distance (C):*** The distance from the center of the pinion to the center of the gear; the sum of the pitch radii of two gears in mesh. That is, because radius = diameter/2,

Center Distance

$$C = D_G/2 + D_P/2 = (D_G + D_P)/2 \tag{8–17}$$

Also note that both pitch diameters can be expressed in terms of the diametral pitch:

Center Distance in terms of N_G, N_P, and P_d

$$C = \frac{1}{2}\left[\frac{N_G}{P_d} + \frac{N_P}{P_d}\right] = \frac{(N_G + N_P)}{2\,P_d} \tag{8–18}$$

It is recommended that Equation (8–18) be used for center distance because all of the terms are usually integers, giving greater accuracy in the computation. In the metric module system, a similar equation can be derived:

$$C = (D_G + D_P)/2 = (mN_G + mN_P)/2 = [(N_G + N_P)m]/2 \tag{8–19}$$

Pressure Angle

> ***The* pressure angle *is the angle between the tangent to the pitch circles and the line drawn normal (perpendicular) to the surface of the gear tooth (see Figure 8–12).***

The normal line is sometimes referred to as the *line of action.* When two gear teeth are in mesh and are transmitting power, the force transferred from the driver to the driven gear tooth acts in a direction along the line of action. Also, the actual shape of the gear tooth depends on the pressure angle, as illustrated in Figure 8–13. The teeth in this figure were drawn according to the proportions for a 20-tooth, 5-pitch gear having a pitch diameter of 4.000 in.

All three teeth have the same tooth thickness because, as stated in Equation (8–16), the thickness at the pitch line depends only on the pitch. The difference between the three teeth shown is due to the different pressure angles because the pressure angle determines the size of the base circle. Remember that the base circle is the circle from which the involute is generated. The line of action is always tangent to the base circle. Therefore, the size of the base circle can be found from

Base Circle Diameter

$$D_b = D\cos\phi \tag{8–20}$$

FIGURE 8–12
Pressure angle

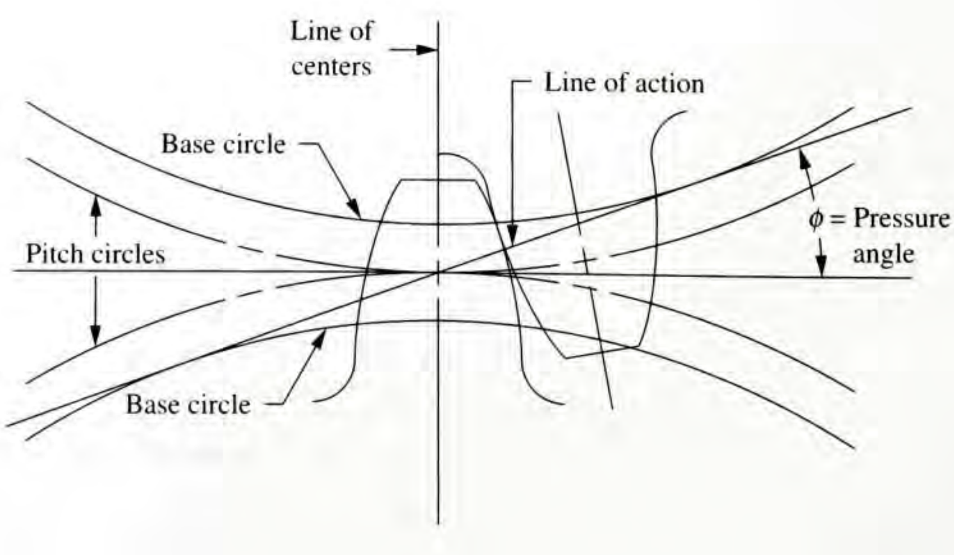
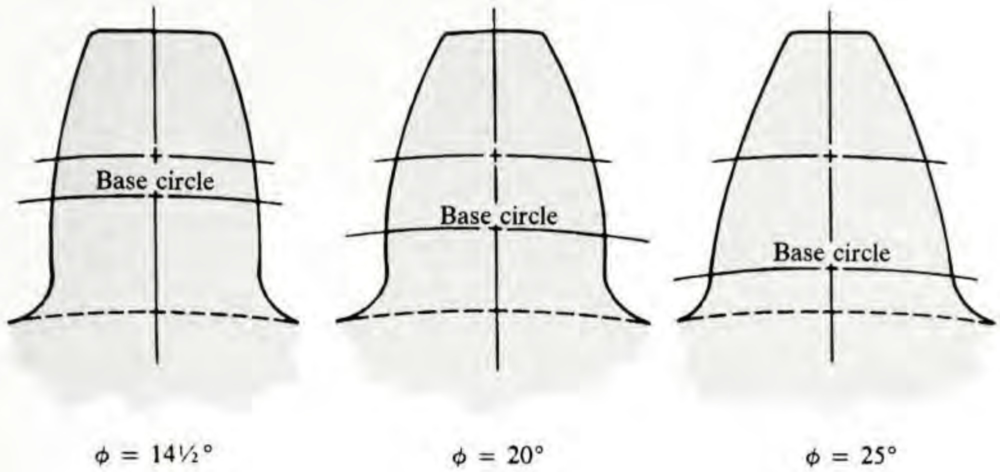


FIGURE 8–13 Full-depth, involute-tooth form for varying pressure angles



Standard values of the pressure angle are established by gear manufacturers, and the pressure angles of two gears in mesh must be the same. Current standard pressure angles are $14\frac{1}{2}°$, 20°, and 25° as illustrated in Figure 8–13. Actually, the $14\frac{1}{2}°$, tooth form is considered to be obsolete. Although it is still available, it should be avoided for new designs. The 20° tooth form is the most readily available at this time. The advantages and disadvantages of the different values of pressure angle relate to the strength of the teeth, the occurrence of interference, and the magnitude of forces exerted on the shaft. Interference is discussed in Section 8–5. The other points are discussed in a later chapter.

Contact Ratio

When two gears mesh, it is essential for smooth operation that a second tooth begin to make contact before a given tooth disengages. The term *contact ratio* is used to indicate the average number of teeth in contact during the transmission of power. A recommended minimum contact ratio is 1.2 and typical spur gear combinations often have values of 1.5 or higher.

The contact ratio is defined as the ratio of the length of the line-of–action to the base pitch for the gear. The line-of-action is the straight-line path of a tooth from where it encounters the outside diameter of the mating gear to the point where it leaves engagement. The base pitch is the diameter of the base circle divided by the number of teeth in the gear. A convenient formula for computing the contact ratio, m_f is,

$$m_f = \frac{\sqrt{R_{oP}^2 - R_{bP}^2} + \sqrt{R_{oG}^2 - R_{bG}^2} - C \sin \phi}{p \cos \phi}$$

where,

ϕ = Pressure angle

R_{oP} = Outside radius of the pinion $= D_{oP}/2 = (N_P + 2)/(2P_d)$

R_{bP} = Radius of the base circle for the pinion $= D_{bP}/2 = (D_P/2) \cos \phi = (N_P/2P_d) \cos \phi$

R_{oG} = Outside radius of the gear $= D_{oG}/2 = (N_G + 2)/(2P_d)$

R_{bG} = Radius of the base circle for the gear $= D_{bG}/2 = (D_G/2) \cos \phi = (N_G/2P_d) \cos \phi$

C = Center distance $= (N_P + N_G)/(2P_d)$

p = Circular pitch $= (\pi D_P/N_P) = \pi/P_d$

For example, consider a pair of gears with the following data:

$$N_P = 18, N_G = 64, P_d = 8, \phi = 20°$$

Then,

$$R_{oP} = (N_P + 2)/(2P_d) = (18 + 2)/[2(8)] = 1.250 \text{ in}$$
$$R_{bP} = (N_P/2P_d)\cos\phi = 18/[2(8)]\cos 20^\circ = 1.05715 \text{ in}$$
$$R_{oG} = (N_G + 2)/(2P_d) = (64 + 2)/[2(8)] = 4.125 \text{ in}$$
$$R_{bG} = (N_G/2P_d)\cos\phi = 64/[2(8)]\cos 20^\circ = 3.75877 \text{ in}$$
$$C = (N_P + N_G)/(2P_d) = (18 + 64)/[2(8)] = 5.125 \text{ in}$$
$$p = \pi/P_d = \pi/8 = 0.392699 \text{ in}$$

Finally, the contact ratio is,

$$m_f = \frac{\sqrt{(1.250)^2 - (1.05715)^2} + \sqrt{(4.125)^2 - (3.75877)^2} - (5.125)\sin 20^\circ}{(0.392699)\cos 20^\circ}$$
$$m_f = 1.66$$

This value is comfortably above the recommended minimum value of 1.20.

Example Problem 8–1 For the pair of gears shown in Figure 8–1, compute all of the features of the gear teeth described in this section. The gears conform to the standard AGMA form and have a diametral pitch of 12 and a 20° pressure angle.

Solution Given $P_d = 12$; $N_P = 11$; $N_G = 18$; $\phi = 20^\circ$.

Analysis We use Equations (8–2) through (8–20) and Table 8–4 to compute the features. Note that gears are precision mechanical components. Dimensions are typically produced to at least the nearest thousandth of an inch (0.001 in). Often, for more accurate gears, controlling to the nearest 0.0001 in is important. Also, in the inspection of gear features using metrology techniques, it is important to know the standard dimension to a high degree of precision.

The results for this problem are presented to a minimum of three decimal places or, for small dimensions, to four decimal places. A similar level of precision is expected in practice problems using this book.

Results *Pitch Diameters*
For the pinion,

$$D_P = N_P/P_d = 11/12 = 0.9167 \text{ in}$$

For the gear,

$$D_G = N_G/P_d = 18/12 = 1.500 \text{ in}$$

Circular Pitch
Three different approaches could be used. First, using Equation (8–5) is preferred.

$$p = \pi/P_d = \pi/12 = 0.2618 \text{ in}$$

Now, we can also use Equation (8–5): Note that either the pinion or the gear data may be used. For the pinion,

$$p = \pi D_P/N_P = \pi(0.9167\text{ in})/11 = 0.2618\text{ in}$$

For the gear,

$$p = \pi D_G/N_G = \pi(1.500\text{ in})/18 = 0.2618\text{ in}$$

Addendum
From Table 8–4,

$$a = 1/P_d = 1/12 = 0.833\text{ in}$$

Dedendum
From Table 8–4, note that the 12-pitch gear is considered to be coarse. Thus,

$$b = 1.25/P_d = 1.25/12 = 0.1042\text{ in}$$

Clearance
From Table 8–4,

$$c = 0.25/P_d = 0.25/12 = 0.0208\text{ in}$$

Outside Diameters
Use of Equation (8–10) is preferred for accuracy. For the pinion,

$$D_{oP} = (N_P + 2)/P_d = (11 + 2)/12 = 1.0833\text{ in}$$

For the gear,

$$D_{oG} = (N_G + 2)/P_d = (18 + 2)/12 = 1.6667\text{ in}$$

Root Diameters
We use Equation (8–12). First, for the pinion,

$$D_{RP} = D_P - 2b = 0.9167\text{ in} - 2(0.1042\text{ in}) = 0.7083\text{ in}$$

For the gear,

$$D_{RG} = D_G - 2b = 1.500\text{ in} - 2(0.1042\text{ in}) = 1.2917\text{ in}$$

Whole Depth
Using Equation (8–13), we have

$$h_t = a + b = 0.0833\text{ in} + 0.104\text{ in} = 0.1875\text{ in}$$

Working Depth
Using Equation (8–14), we have

$$h_k = 2a = 2(0.0833\text{ in}) = 0.1667\text{ in}$$

Tooth Thickness
Using Equation (8–16), we have

$$t = \pi/2P_d = \pi/2(12) = 0.1309 \text{ in}$$

Center Distance
Use of Equation (8–18) is preferred:

$$C = (N_G + N_P)/(2P_d) = (18 + 11)/[2(12)] = 1.2083 \text{ in}$$

Base Circle Diameter
Using Equation (8–20), we have

$$D_{bP} = D_P \cos \phi = (0.9167 \text{ in}) \cos (20°) = 0.8614 \text{ in}$$
$$D_{bG} = D_G \cos \phi = (1.500 \text{ in}) \cos (20°) = 1.4095 \text{ in}$$

8–5 INTERFERENCE BETWEEN MATING SPUR GEAR TEETH

For certain combinations of numbers of teeth in a gear pair, there is interference between the tip of the teeth on the pinion and the fillet or root of the teeth on the gear. Obviously this cannot be tolerated because the gears simply will not mesh. The probability that interference will occur is greatest when a small pinion drives a large gear, with the worst case being a small pinion driving a rack. A *rack* is a gear with a straight pitch line; it can be thought of as a gear with an infinite pitch diameter [see Figure 8–4(c)].

It is the designer's responsibility to ensure that interference does not occur in a given application. The surest way to do this is to control the minimum number of teeth in the pinion to the limiting values shown on the left side of Table 8–6. With this number of teeth or a greater number, there will be no interference with a rack or with any other gear. A designer who desires to use fewer than the listed number of teeth can use a graphical layout to test the combination of pinion and gear for interference. Texts on kinematics provide the necessary procedure. The right side of Table 8–6 indicates the maximum number of gear teeth that you can use for a given number of pinion teeth to avoid interference. (See References 9 and 11.)

Using the information in Table 8–6, we can draw the following conclusions:

1. If a designer wants to be sure that there will not be interference between any two gears when using the $14\frac{1}{2}°$, full-depth, involute system, the pinion of the gear pair must have no fewer than 32 teeth.

TABLE 8–6 Number of pinion teeth to ensure no interference

For a pinion meshing with a rack		For a 20°, full-depth pinion meshing with a gear	
Tooth form	Minimum number of teeth	Number of pinion teeth	Maximum number of gear teeth
$14\frac{1}{2}°$, involute, full-depth	32	17	1309
20°, involute, full-depth	18	16	101
25°, involute, full-depth	12	15	45
		14	26
		13	16

2. For the 20°, full-depth, involute system, using no fewer than 18 teeth will ensure that no interference occurs.
3. For the 25°, full-depth, involute system, using no fewer than 12 teeth will ensure that no interference occurs.
4. If a designer desires to use fewer than 18 teeth in a pinion having 20°, full-depth teeth, there is an upper limit to the number of teeth that can be used on the mating gear without interference. For 17 teeth in the pinion, any number of teeth on the gear can be used up to 1309, a very high number. Most gear drive systems use no more than about 200 teeth in any gear. But a 17-tooth pinion *would* have interference with a *rack* which is effectively a gear with an infinite number of teeth or an infinite pitch diameter. Similarly, the following requirements apply for 20° full-depth teeth:

   A 16-tooth pinion requires a gear having 101 or fewer teeth, producing a maximum velocity ratio of $N_G/N_P = 101/16 = 6.31$.

   A 15-tooth pinion requires a gear having 45 or fewer teeth, producing a maximum velocity ratio of $45/15 = 3.00$.

   A 14-tooth pinion requires a gear having 26 or fewer teeth, producing a maximum velocity ratio of $26/14 = 1.85$.

   A 13-tooth pinion requires a gear having 16 or fewer teeth, producing a maximum velocity ratio of $16/13 = 1.23$.

As noted earlier, the $14\frac{1}{2}°$ system is considered to be obsolete. The data in Table 8–6 indicate one of the main disadvantages with that system: its potential for causing interference.

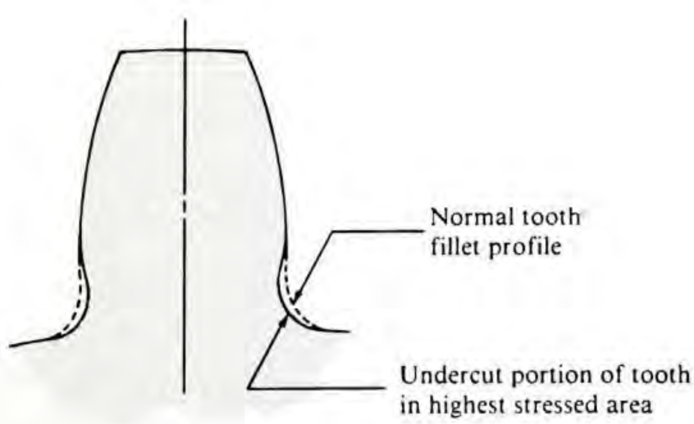
Overcoming Interference

If a proposed design encounters interference, there are ways to make it work. But caution should be exercised because the tooth form or the alignment of the mating gears is changed, causing the stress and wear analysis to be inaccurate. With this in mind, the designer can provide for undercutting, modification of the addendum on the pinion or the gear, or modification of the center distance:

> ***Undercutting is the process of cutting away the material at the fillet or root of the gear teeth, thus relieving the interference.***

Figure 8–14 shows the result of undercutting. It should be obvious that this process weakens the tooth; this point is discussed further in the section on stresses in gear teeth.

FIGURE 8–14
Undercutting of a gear tooth



To alleviate the problem of interference, increase the addendum of the pinion while decreasing the addendum of the gear. The center distance can remain the same as its theoretical value for the number of teeth in the pair. But the resulting gears are, of course, nonstandard. (See Reference 10.) It is possible to make the pinion of a gear pair larger than standard while keeping the gear standard if the center distance for the pair is enlarged. (See Reference 9.)

8–6 VELOCITY RATIO AND GEAR TRAINS

A* gear train *is one or more pairs of gears operating together to transmit power.

Normally there is a speed change from one gear to the next due to the different sizes of the gears in mesh. The fundamental building block of the total speed change ratio in a gear train is the *velocity ratio* between two gears in a single pair.

Velocity Ratio

The* velocity ratio *(VR) is defined as the ratio of the rotational speed of the input gear to that of the output gear for a single pair of gears.

To develop the equation for computing the velocity ratio, it is helpful to view the action of two gears in mesh, as shown in Figure 8–15. The action is equivalent to the action of two smooth wheels rolling on each other without slipping, with the diameters of the two wheels equal to the pitch diameters of the two gears. Remember that when two gears are in mesh, their pitch circles are tangent, obviously, the gear teeth prohibit any slipping.

As shown in Figure 8–15, without slipping there is no relative motion between the two pitch circles at the pitch point, and therefore the linear velocity of a point on either pitch circle is the same. We will use the symbol v_t for this velocity. The linear velocity of a point that is in rotation at a distance R from its center of rotation and rotating with an angular velocity, ω, is found from

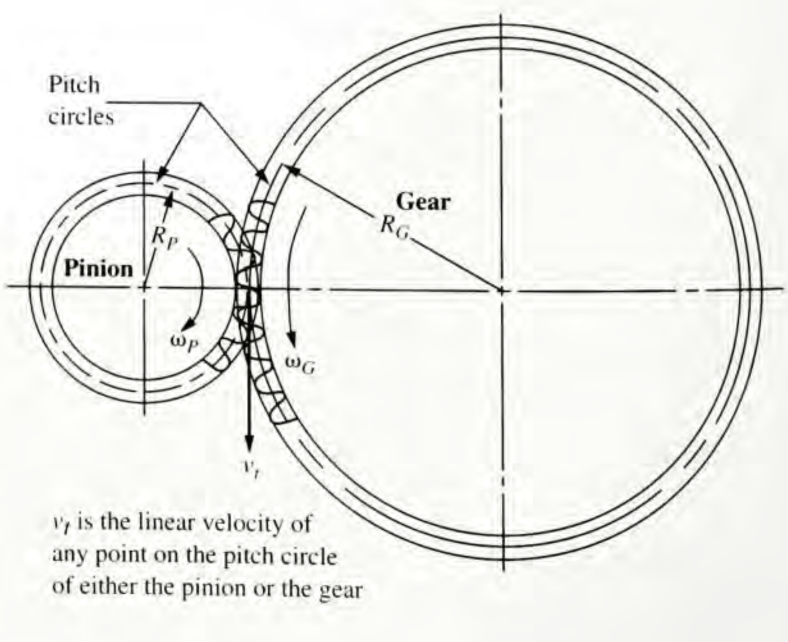
Pitch Line Speed of a Gear

$$v_t = R\omega \tag{8–21}$$

Using the subscript P for the pinion and G for the gear for two gears in mesh, we have

$$v_t = R_P\omega_P \quad \text{and} \quad v_t = R_G\omega_G$$

FIGURE 8–15 Two gears in mesh



This set of equations says that the pitch line speeds of the pinion and the gear are the same. Equating these two and solving for ω_P/ω_G gives our definition for the velocity ratio, VR:

$$VR = \omega_P/\omega_G = R_G/R_P$$

In general, it is convenient to express the velocity ratio in terms of the pitch diameters, the rotational speeds, or the numbers of teeth of the two gears in mesh. Remember that

$$R_G = D_G/2$$
$$R_P = D_P/2$$
$$D_G = N_G/P_d$$
$$D_P = N_P/P_d$$
$$n_P = \text{rotational speed of the pinion} \quad \text{(in rpm)}$$
$$n_G = \text{rotational speed of the gear} \quad \text{(in rpm)}$$

The velocity ratio can then be defined in any of the following ways:

Velocity Ratio for Gear Pair

$$VR = \frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = \frac{R_G}{R_P} = \frac{D_G}{D_P} = \frac{N_G}{N_P} = \frac{\text{speed}_P}{\text{speed}_G} = \frac{\text{size}_G}{\text{size}_P} \quad \textbf{(8–22)}$$

Most gear drives are *speed reducers;* that is, their output speed is lower than their input speed. This results in a velocity ratio greater than 1. If a *speed increaser* is desired, then VR is less than 1. Note that not all books and articles use the same definition for velocity ratio. Some define it as the ratio of the output speed to the input speed, the inverse of our definition. It is thought that the use of VR greater than 1 for the reducer—that is, the majority of the time—is more convenient.

Train Value

When more than two gears are in mesh, the term* train value *(TV) refers to the ratio of the input speed (for the first gear in the train) to the output speed (for the last gear in the train). By definition the train value is the product of the values of VR for each* gear pair *in the train. In this definition, a gear pair is any set of two gears with a driver and a follower (driven) gear.

Again, TV will be greater than 1 for a reducer and less than 1 for an increaser. For example, consider the gear train sketched in Figure 8–16. The input is through the shaft carrying gear A. Gear A drives gear B. Gear C is on the same shaft with gear B and rotates at the same speed. Gear C drives gear D, which is connected to the output shaft. Then gears A and B constitute the first gear pair, and gears C and D constitute the second pair. The velocity ratios are

$$VR_1 = n_A/n_B$$
$$VR_2 = n_C/n_D$$

The train value is

$$TV = (VR_1)(VR_2) = \frac{n_A}{n_B}\frac{n_C}{n_D}$$

But, because they are on the same shaft, $n_B = n_C$, and the preceding equation reduces to

$$TV = n_A/n_D$$

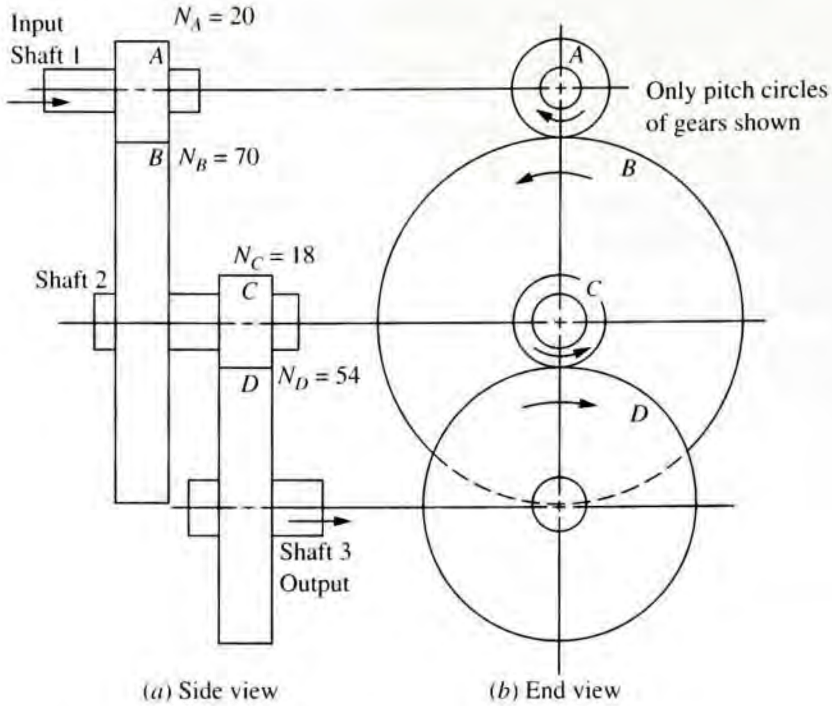


FIGURE 8–16
Double-reduction gear train

This is the input speed divided by the output speed, the basic definition of the train value. This process can be expanded to any number of stages of reduction in a gear train.

Remember that any of the forms for velocity ratio shown in Equation (8–22) can be used for computing the train value. In design, it is often most convenient to express the velocity ratio in terms of the numbers of teeth in each gear because they must be integers. Then, once the diametral pitch or module is defined, the values of the diameters or radii can be determined.

The train value of the double-reduction gear train in Figure 8–16 can be expressed in terms of the numbers of teeth in the four gears as follows:

$$VR_1 = N_B/N_A$$

Note that this is the number of teeth in the *driven gear B* divided by the number of teeth in the *driving gear A*. This is the typical format for velocity ratio. Then VR_2 can be found similarly:

$$VR_2 = N_D/N_C$$

Thus, the train value is

$$TV = (VR_1)(VR_2) = (N_B/N_A)(N_D/N_C)$$

This is usually shown in the form

Train Value

$$TV = \frac{N_B}{N_A}\frac{N_D}{N_C} = \frac{\text{product of number of teeth in the driven gears}}{\text{product of number of teeth in the driving gears}} \tag{8–23}$$

This is the form for train value that we will use most often.

The direction of rotation can be determined by observation, noting that there is a direction reversal for each pair of external gears.

We will use the term* positive train value *to refer to one in which the input and output gears rotate in the same direction. Conversely, if they rotate in the opposite direction, the train value will be negative.

Example Problem 8–2 For the gear train shown in Figure 8–16, if the input shaft rotates at 1750 rpm clockwise, compute the speed of the output shaft and its direction of rotation.

Solution We can find the output speed if we can determine the train value:

$$TV = n_A/n_D = \text{input speed/output speed}$$

Then

$$n_D = n_A/TV$$

But

$$TV = (VR_1)(VR_2) = \frac{N_B}{N_A}\frac{N_D}{N_C} = \frac{70}{20}\frac{54}{18} = \frac{3.5}{1}\frac{3.0}{1} = \frac{10.5}{1} = 10.5$$

Now

$$n_D = n_A/TV = (1750 \text{ rpm})/10.5 = 166.7 \text{ rpm}$$

Gear A rotates clockwise; gear B rotates counterclockwise.
Gear C rotates counterclockwise; gear D rotates clockwise.

Thus, the train in Figure 8–16 is a positive train.

Example Problem 8–3 Determine the train value for the train shown in Figure 8–17. If the shaft carrying gear A rotates at 1750 rpm clockwise, compute the speed and the direction of the shaft carrying gear E.

Solution Look first at the direction of rotation. Remember that a gear pair is defined as any two gears in mesh (a driver and a follower). There are actually three gear pairs:

Gear A drives gear B: A rotates clockwise; B, counterclockwise.

Gear C drives gear D: C rotates counterclockwise; D, clockwise.

Gear D drives gear E: D rotates clockwise; E, counterclockwise.

Because gears A and E rotate in opposite directions, the train value is negative. Now

$$TV = -(VR_1)(VR_2)(VR_3)$$

In terms of the number of teeth,

$$TV = -\frac{N_B}{N_A}\frac{N_D}{N_C}\frac{N_E}{N_D}$$

Note that the number of teeth in gear D appears in both the numerator and the denominator and thus can be canceled. The train value then becomes

$$TV = -\frac{N_B}{N_A}\cdot\frac{N_E}{N_C} = -\frac{70}{20}\cdot\frac{50}{18} = -\frac{3.5}{1}\frac{3.0}{1} = -10.5$$

Gear D is called an *idler.* As demonstrated here, it has no effect on the magnitude of the train value, but it does cause a direction reversal. The output speed is then found from

$$TV = n_A/n_E$$
$$n_E = n_A/TV = (1750 \text{ rpm})/(-10.5) = -166.7 \text{ rpm} \quad \text{(counterclockwise)}$$

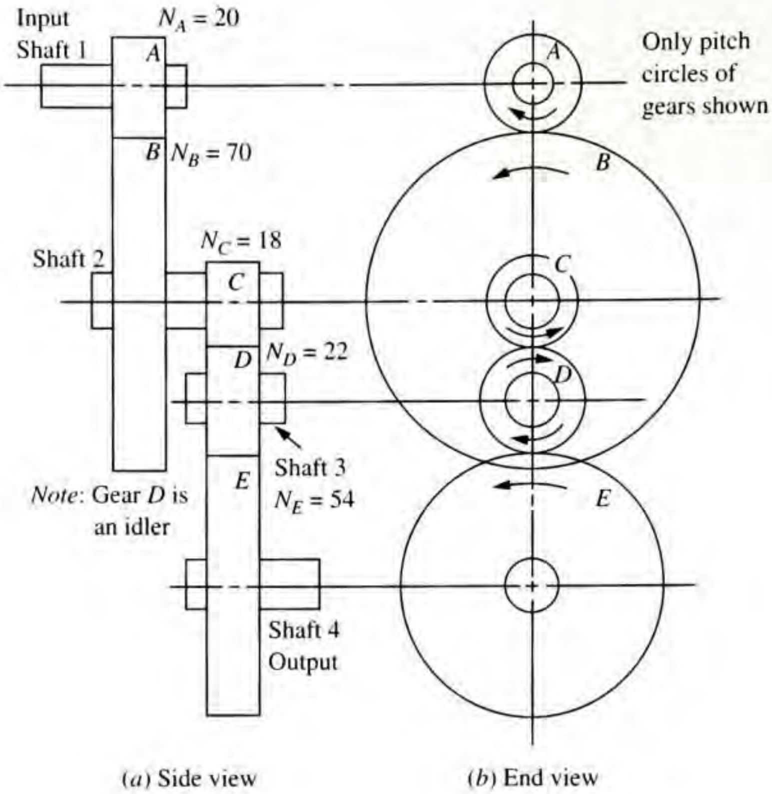


FIGURE 8–17 Double-reduction gear train with an idler. Gear D is an idler.

Idler Gear

Example Problem 8–2 introduced the concept of an *idler gear,* defined as follows:

> ***Any gear in a gear train that performs as both a driving gear and a driven gear is called an* idler gear *or simply an* idler.**

The main features of an idler are as follows:

1. An idler does not affect the train value of a gear train because, since it is both a driver and a driven gear, its number of teeth appears in both the numerator and the denominator of the train value equation, Equation (8–23). Thus, any pitch diameter size and any number of teeth may be used for the idler.
2. Placing an idler in a gear train causes a direction reversal of the output gear.
3. An idler gear may be used to fill a space between two gears in a gear train when the desired distance between their centers is greater than the center distance for the two gears alone.

Internal Gear

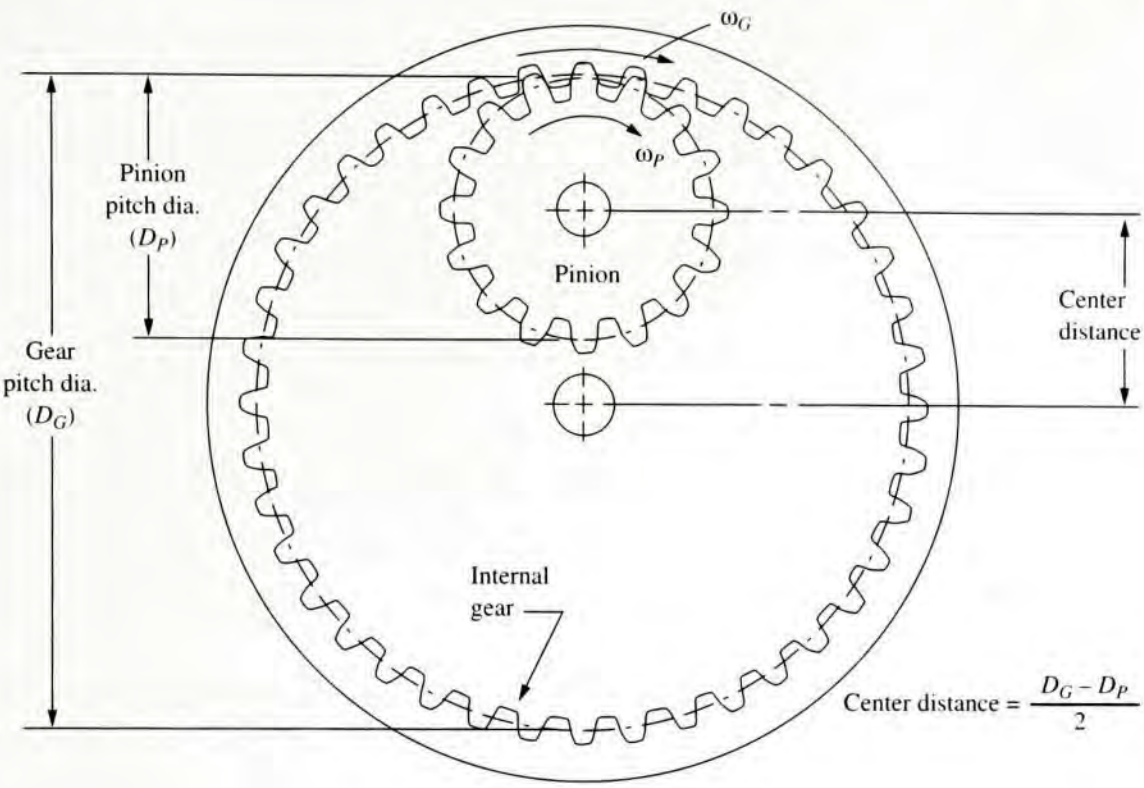
> ***An* internal gear *is one for which the teeth are machined on the inside of a ring instead of on the outside of a gear blank.***

An internal gear mating with a standard, external pinion is illustrated at the lower left in Figure 8–2, along with a variety of other kinds of gears.

Figure 8–18 is a sketch of an external pinion driving an internal gear. Note the following:

1. The gear rotates in the *same direction* as the pinion. This is different from the case when an external pinion drives an external gear.

FIGURE 8–18
Internal gear driven by an external pinion



2. The center distance is

Center Distance Internal Gear

$$C = D_G/2 - D_P/2 = (D_G - D_P)/2 = (N_G/P_d - N_P/P_d)/2 = (N_G - N_P)/(2P_d) \quad \textbf{(8–24)}$$

The last form is preferred because its factors are all integers for typical gear trains.

3. The descriptions of most other features of internal gears are the same as those for external gears presented earlier. Exceptions for an internal gear are as follows:

The addendum, a, is the radial distance from the pitch circle to the inside of a tooth.

The inside diameter, D_i, is

$$D_i = D - 2a$$

The root diameter, D_R, is

$$D_R = D + 2b$$

where b = dedendum

Internal gears are used when it is desired to have the same direction of rotation for the input and the output. Also note that less space is taken for an internal gear mating with an external pinion compared with two external gears in mesh.

Velocity of a Rack

Figure 8–19 shows the basic configuration of a *rack-and-pinion* drive. The function of such a drive is to produce a linear motion of the rack from the rotational motion of the driving pinion. The opposite is also true: If the driver produces the linear motion of the rack, it produces a rotational motion of the pinion.

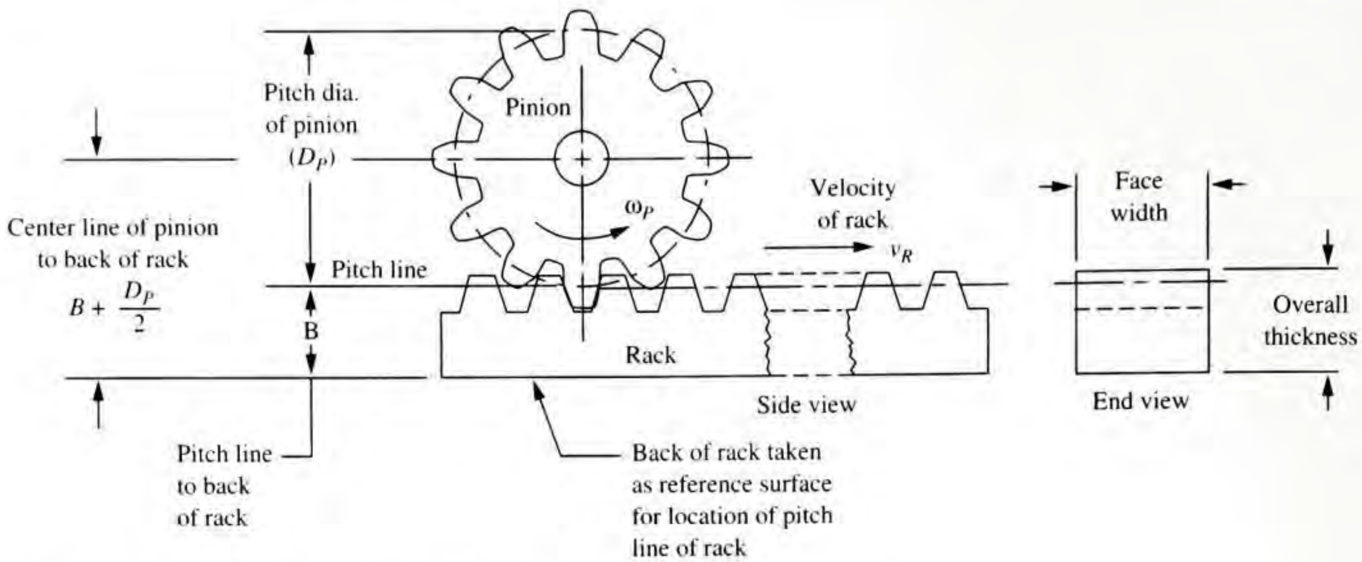


FIGURE 8–19 Rack driven by a pinion

The linear velocity of the rack, v_R, must be the same as the pitch line velocity of the pinion, v_t, as defined by Equation (8–21), repeated here. Recall that w_P is the angular velocity of the pinion:

$$v_R = v_t = R_P\omega_P = (D_P/2)\omega_P$$

You must carefully consider units when using this equation. The angular velocity ω_P should be expressed in rad/s. Then the units for linear velocity will be in/s if the pitch diameter is in inches. If the pitch diameter is in mm, as in the metric module system, then the units for velocity will be mm/s. These units could be converted to m/s if that is more convenient.

The concept of center distance does not apply directly for a rack-and-pinion set because the center of the rack is at infinity. But it is critical that the pitch circle of the pinion be tangent to the pitch line of the rack as shown in Figure 8–19. The rack will be machined so that there is a specified dimension between the pitch line and a reference surface, typically the back of the rack. This is dimension B in Figure 8–19. Then the location of the center of the pinion can be computed using the relationships shown in the figure.

Example Problem 8–4 Determine the linear velocity of the rack in Figure 8–19 if the driving pinion rotates at 125 rpm. The pinion has 24 teeth and a diametral pitch of 6.

Solution We will use Equation (8–21). First the pitch diameter of the pinion is computed using Equation (8–4):

$$D_P = N_P/P_d = 24/6 = 4.000 \text{ in}$$

Now the rotational speed is converted to rad/s:

$$\omega_P = (125 \text{ rev/min}(2\pi \text{ rad/rev})(1 \text{ min/60 s}) = 13.09 \text{ rad/s}$$

Then the pitch line speed of the pinion and the linear velocity of the rack are both equal to

$$v_R = v_t = (D_P/2)\omega_P = (4.000 \text{ in}/2)(13.09 \text{ rad/s}) = 26.2 \text{ in/s}$$

8–7 HELICAL GEAR GEOMETRY

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Helical and spur gears are distinguished by the orientation of their teeth. On spur gears, the teeth are straight and are aligned with the axis of the gear. On helical gears, the teeth are inclined at an angle with the axis, that angle being called the *helix angle*. If the gear were very wide, it would appear that the teeth wind around the gear blank in a continuous, helical path. However, practical considerations limit the width of the gears so that the teeth normally appear to be merely inclined with respect to the axis. Figure 8–20 shows two examples of commercially available helical gears.

The forms of helical gear teeth are very similar to those discussed for spur gears. The basic task is to account for the effect of the helix angle.

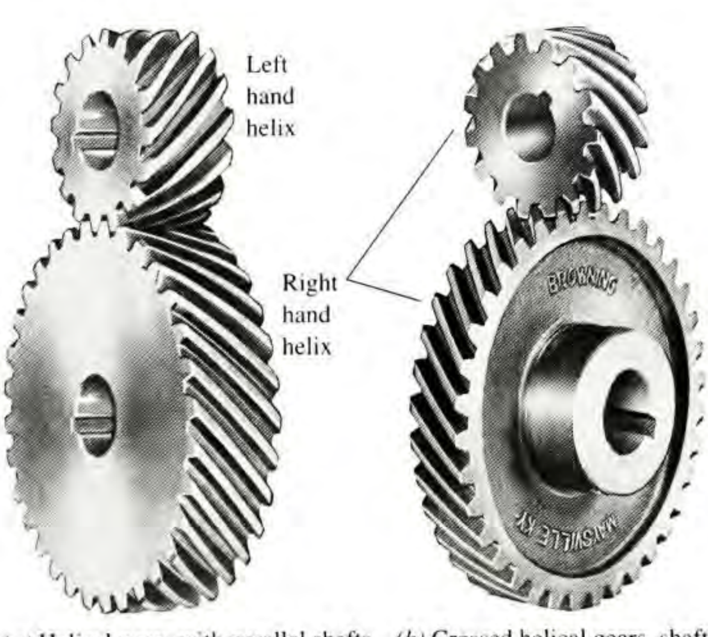
Helix Angle

The helix for a given gear can be either *left-hand* or *right-hand*. The teeth of a right-hand helical gear would appear to lean to the right when the gear is lying on a flat surface. Conversely, the teeth of a left-hand helical gear would lean to the left. In normal installation, helical gears would be mounted on parallel shafts as shown in Figure 8–20(a). To achieve this arrangement, it is required that one gear be of the right-hand design and that the other be left-hand with an equal helix angle. If both gears in mesh are of the same hand, as shown in Figure 8–20(b), the shafts will be at 90 to each other. Such gears are called *crossed helical gears*.

The parallel shaft arrangement for helical gears is preferred because it results in a much higher power-transmitting capacity for a given size of gear than the crossed helical arrangement. In this book, we will assume that the parallel shaft arrangement is being used unless otherwise stated.

Figure 8–21(a) shows the pertinent geometry of helical gear teeth. To simplify the drawing, only the pitch surface of the gear is shown. The pitch surface is the cylinder that passes through the gear teeth at the pitch line. Thus, the diameter of the cylinder is equal to the pitch diameter of the gear. The lines drawn on the pitch surface represent elements of each tooth where the surface would cut into the face of the tooth. These elements are inclined with respect to a line parallel to the axis of the cylinder, and the angle of inclination is the *helix angle*, ψ (the Greek letter *psi*).

FIGURE 8–20
Helical gears. These gears have a 45° helix angle. (Emerson Power Transmission Corporation, Browning Division, Maysville, KY)



(*a*) Helical gears with parallel shafts (*b*) Crossed helical gears, shafts at right angle

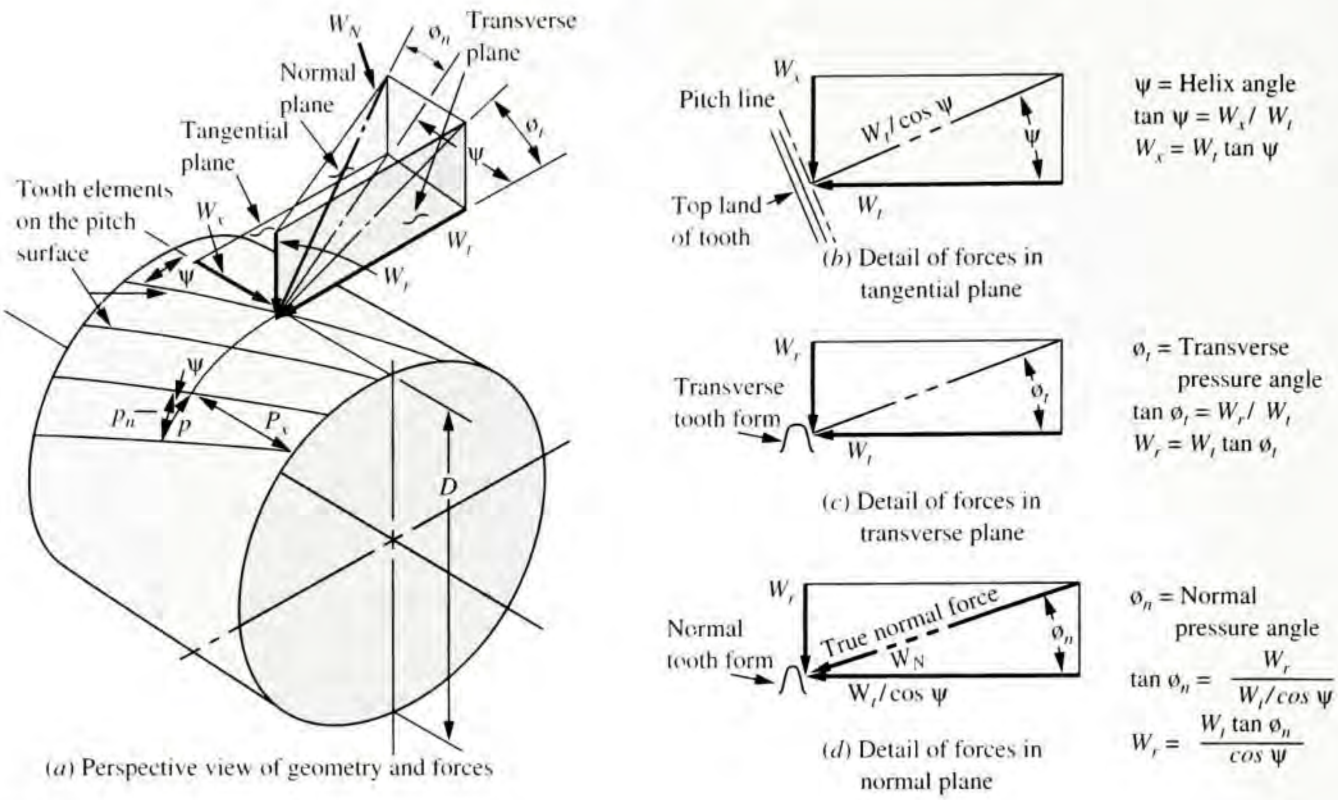


FIGURE 8–21 Helical gear geometry and forces

The main advantage of helical gears over spur gears is smoother engagement because a given tooth assumes its load gradually instead of suddenly. Contact starts at one end of a tooth near the tip and progresses across the face in a path downward across the pitch line to the lower flank of the tooth, where it leaves engagement. Simultaneously, other teeth are coming into engagement before a given tooth leaves engagement, with the result that a larger average number of teeth are engaged and are sharing the applied loads compared with a spur gear. The lower average load per tooth allows a greater power transmission capacity for a given size of gear, or a smaller gear can be designed to carry the same power.

The main disadvantage of helical gears is that an *axial thrust load* is produced as a natural result of the inclined arrangement of the teeth. The bearings that hold the shaft carrying the helical gear must be capable of reacting against the thrust load.

The helix angle is specified for each given gear design. A balance should be sought to take advantage of the smoother engagement of the gear teeth when the helix angle is high while maintaining a reasonable value of the axial thrust load that increases with increasing helix angle. A typical range of values of helix angles is from 15° to 45°.

Pressure Angles, Primary Planes, and Forces for Helical Gears

To completely describe the geometry of helical gear teeth, we need to define two different pressure angles in addition to the helix angle. The two pressure angles are related to the three primary planes that are illustrated in Figure 8–21: (1) the *tangential plane,* (2) the *transverse plane,* and (3) the *normal plane.* Note that these planes contain the three orthogonal components of the true total normal force that is exerted by a tooth of one gear on a tooth of the mating gear. It may help you to understand the geometry of the teeth, and the importance of that geometry, to see how it affects the forces.

We refer first to the *true normal force* as W_N. It acts normal (perpendicular) to the curved surface of the tooth. In reality, we typically do not use the normal force itself in analyzing the performance of the gear. Instead we use its three orthogonal components:

- The *tangential force* (also called the *transmitted force*), W_t, acts tangential to the pitch surface of the gear and perpendicular to the axis of the shaft carrying the gear. This is the force that actually drives the gear. Stress analysis and pitting resistance are both related to the magnitude of the tangential force. It is similar to W_t used in spur gear design and analysis.
- The *radial force*, W_r, acts toward the center of the gear along a radius and tends to separate the two gears in mesh. It is similar to W_r used in spur gear design and analysis.
- The *axial force*, W_x, acts in the tangential plane parallel to the axis of the shaft carrying the gear. Another name for the axial force is the *thrust*. It tends to push the gear along the shaft. The thrust must be reacted by one of the bearings that carry the shaft, so this force is generally undesirable. Spur gears generate no such force because the teeth are straight and are parallel to the axis of the gear.

The plane containing the tangential force, W_t, and the axial force, W_x, is the *tangential plane* [see Figure 8–21(b)]. It is tangential to the pitch surface of the gear and acts through the pitch point at the middle of the face of the tooth being analyzed.

The plane containing the tangential force, W_t, and the radial force, W_r, is the *transverse plane* [see Figure 8–21(c)]. It is perpendicular to the axis of the gear and acts through the pitch point at the middle of the face of the tooth being analyzed. The *transverse pressure angle*, ϕ_t, is defined in this plane as shown in the figure.

The plane containing the true normal force, W_N, and the radial force, W_r, is the *normal plane* [see Figure 8–21(d)]. The angle between the normal plane and the transverse plane is the helix angle, ψ. Within the normal plane we can see that the angle between the tangential plane and the true normal force, W_N, is the *normal pressure angle*, ϕ_n.

In design of a helical gear, there are three angles of interest: (1) the helix angle, ψ; (2) the *normal pressure angle*, ϕ_n; and (3) the *transverse pressure angle*, ϕ_t. Designers must specify the helix angle and one of the two pressure angles. The other pressure angle can be computed from the following relationship:

$$\tan \phi_n = \tan \phi_t \cos \psi \tag{8–25}$$

For example, one manufacturer's catalog offers standard helical gears with a normal pressure angle of $14\frac{1}{2}°$ and a 45° helix angle. Then the transverse pressure angle is found from

$$\begin{aligned}
\tan \phi_n &= \tan \phi_t \cos \psi \\
\tan \phi_t &= \tan \phi_n / \cos \psi = \tan(14.5°)/\cos(45°) = 0.3657 \\
\phi_t &= \tan^{-1}(0.3657) = 20.09°
\end{aligned}$$

Pitches for Helical Gears

To obtain a clear picture of the geometry of helical gears, you must understand the following five different pitches.

Circular Pitch, *p*. *Circular pitch* is the distance from a point on one tooth to the corresponding point on the next adjacent tooth, measured at the pitch line in the transverse plane. This is the same definition used for spur gears. Then

Circular Pitch

$$p = \pi D/N \tag{8–26}$$

Normal Circular Pitch, p_n. *Normal circular pitch* is the distance between corresponding points on adjacent teeth measured on the pitch surface in the normal direction. Pitches p and p_n are related by the following equation:

Normal Circular Pitch

$$p_n = p \cos \psi \tag{8–27}$$

Diametral Pitch, P_d. *Diametral pitch* is the ratio of the number of teeth in the gear to the pitch diameter. This is the same definition as the one for spur gears; it applies in considerations of the form of the teeth in the diametral or transverse plane. Thus, this pitch is sometimes called the *transverse diametral pitch:*

Diametral Pitch

$$P_d = N/D \tag{8–28}$$

Normal Diametral Pitch, P_{nd}. Normal diametral pitch is the equivalent diametral pitch in the plane normal to the teeth:

Normal Diametral Piltch

$$P_{nd} = P_d/\cos \psi \tag{8–29}$$

It is helpful to remember these relationships:

$$P_d p = \pi \tag{8–30}$$

$$P_{nd} P_n = \pi \tag{8–31}$$

Axial Pitch, P_x. *Axial pitch* is the distance between corresponding points on adjacent teeth, measured on the pitch surface in the axial direction:

Axial Pitch

$$P_x = p/\tan \psi = \pi/(P_d \tan \psi) \tag{8–32}$$

It is necessary to have at least two axial pitches in the face width to have the benefit of full helical action and its smooth transfer of the load from tooth to tooth.

The use of equations (8–25) through (8–29) and Equation (8–32) is now illustrated in the following example problem.

Example Problem 8–5 A helical gear has a transverse diametral pitch of 12, a transverse pressure angle of $14\frac{1}{2}°$, 28 teeth, a face width of 1.25 in, and a helix angle of 30°. Compute circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and the normal pressure angle. Compute the number of axial pitches in the face width.

Solution *Circular Pitch*

Use Equation (8–30):

$$p = \pi/P_d = \pi/12 = 0.262 \text{ in}$$

Normal Circular Pitch
Use Equation (8–27):

$$p_n = p\cos\psi = (0.262)\cos(30) = 0.227 \text{ in}$$

Normal Diametral Pitch
Use Equation (8–29):

$$P_{nd} = P_d/\cos\psi = 12/\cos(30) = 13.856$$

Axial Pitch
Use Equation (8–32):

$$P_x = p/\tan\psi = 0.262/\tan(30) = 0.453 \text{ in}$$

Pitch Diameter
Use Equation (8–28):

$$D = N/P_d = 28/12 = 2.333 \text{ in}$$

Normal Pressure Angle
Use Equation (8–25):

$$\phi_n = \tan^{-1}(\tan\phi_t\cos\psi)$$
$$\phi_n = \tan^{-1}[\tan(14\tfrac{1}{2})\cos(30)] = 12.62^\circ$$

Number of Axial Pitches in the Face Width

$$F/P_x = 1.25/0.453 = 2.76 \text{ pitches}$$

Since this is greater than 2.0, there will be full helical action.

8–8 BEVEL GEAR GEOMETRY

Bevel gears are used to transfer motion between nonparallel shafts, usually at 90° to one another. The four primary styles of bevel gears are straight bevel, spiral bevel, zero spiral bevel, and hypoid. Figure 8–22 shows the general appearance of these four types of bevel gear sets. The surface on which bevel gear teeth are machined is inherently a part of a cone. The differences occur in the specific shape of the teeth and in the orientation of the pinion relative to the gear. (See References 3, 5, 13, and 14.)

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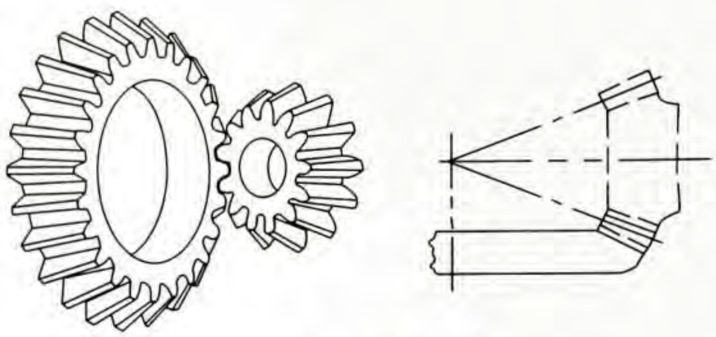
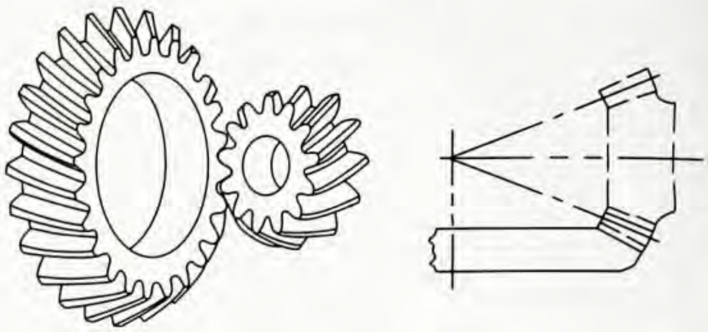
Straight Bevel Gears

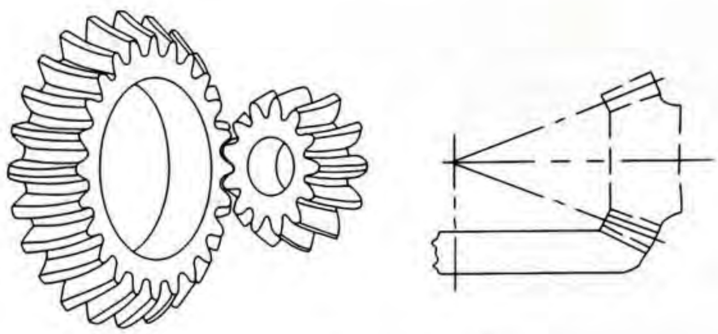
The teeth of a straight bevel gear are straight and lie along an element of the conical surface. Lines along the face of the teeth through the pitch circle meet at the apex of the pitch cone. As shown in Figure 8–22(f), the centerlines of both the pinion and the gear also meet at this apex. In the standard configuration, the teeth are tapered toward the center of the cone.



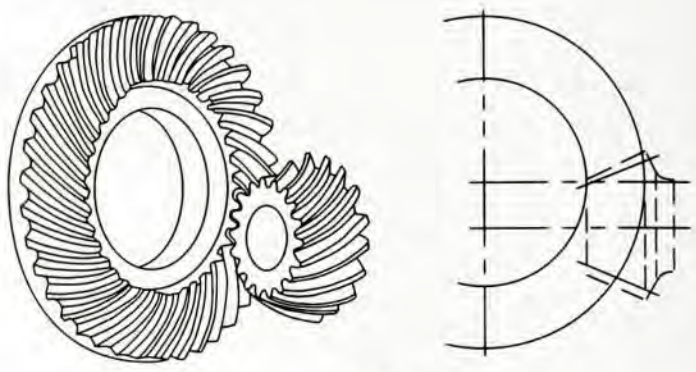
(*a*) Straight bevel



(*b*) Spiral bevel

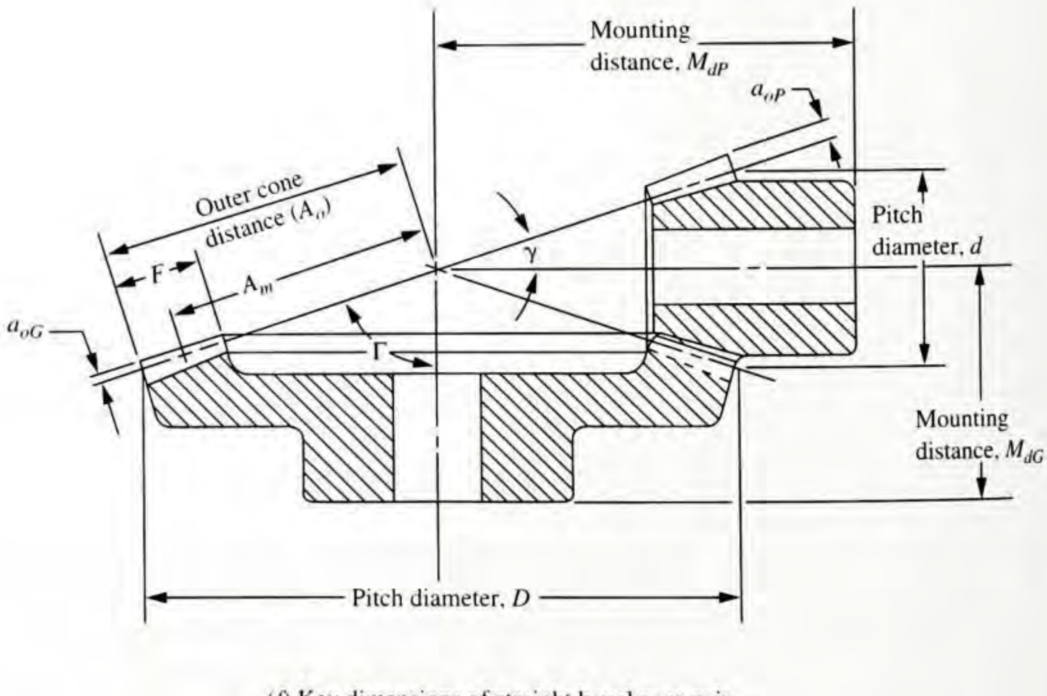


(*c*) Zero sprial bevel



(*d*) Hypoid

(*e*) Photograph of straight bevel gear pair



(*f*) Key dimensions of straight bevel gear pair

FIGURE 8–22 Types of bevel gears [Parts (*a*) through (*d*) extracted from ANSI/AGMA 2005-C96, *Design Manual for Bevel Gears,* with the permission of the publisher, the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314. Photograph in (*e*) courtesy of Emerson Power Transmission Corporation, Browning Division, Maysville, KY

Key dimensions are specified either at the outer end of the teeth or at the mean, mid-face position. The relationships that control some of these dimensions are listed in Table 8–7 for the case when the shafts are at the 90° angle. The pitch cone angles for the pinion and the gear are determined by the ratio of the number of teeth, as shown in the table. Note that their sum is 90°. Also, for a pair of bevel gears having a ratio of unity, each has a pitch cone angle of 45°. Such gears, called *miter gears*, are used simply to change the direction of the shafts in a machine drive without affecting the speed of rotation.

You should understand that many more features need to be specified before the gears can be produced. Furthermore, many successful, commercially available gears are made in some nonstandard form. For example, the addendum of the pinion is often made longer than that of the gear. Some manufacturers modify the slope of the root of the teeth to produce a uniform depth, rather than using the standard, tapered form. Reference 5 gives many more data.

TABLE 8–7 Geometrical features of straight bevel gears

Given Diametral pitch = $P_d = N_P/d = N_G/D$

where N_P = number of teeth in pinion
N_G = number of teeth in gear

Dimension	Formula
Gear ratio	$m_G = N_G/N_P$
Pitch diameters:	
Pinion	$d = N_P/P_d$
Gear	$D = N_G/P_d$
Pitch cone angles:	
Pinion	$\gamma = \tan^{-1}(N_P/N_G)$ (lowercase Greek *gamma*)
Gear	$\Gamma = \tan^{-1}(N_G/N_P)$ (uppercase Greek *gamma*)
Outer cone distance	$A_o = 0.5D/\sin(\Gamma)$
Face width must be specified.	$F =$
Nominal face width	$F_{nom} = 0.30A_o$
Maximum face width	$F_{max} = A_o/3$ or $F_{max} = 10/P_d$ (whichever is less)
Mean cone distance	$A_m = A_o - 0.5F$
	(*Note:* A_m is defined for the gear, also called A_{mG}.)
Mean circular pitch	$p_m = (\pi/P_d)(A_m/A_o)$
Mean working depth	$h = (2.00/P_d)(A_m/A_o)$
Clearance	$c = 0.125h$
Mean whole depth	$h_m = h + c$
Mean addendum factor	$c_1 = 0.210 + 0.290/(m_G)^2$
Gear mean addendum	$a_G = c_1 h$
Pinion mean addendum	$a_P = h - a_G$
Gear mean dedendum	$b_G = h_m - a_G$
Pinion mean dedendum	$b_P = h_m - a_P$
Gear dedendum angle	$\delta_G = \tan^{-1}(b_G/A_{mG})$
Pinion dedendum angle	$\delta_P = \tan^{-1}(b_P/A_{mG})$
Gear outer addendum	$a_{oG} = a_G + 0.5F\tan\delta_P$
Pinion outer addendum	$a_{oP} = a_P + 0.5F\tan\delta_G$
Gear outside diameter	$D_o = D + 2a_{oG}\cos\Gamma$
Pinion outside diameter	$d_o = d + 2a_{oP}\cos\gamma$

Source: Extracted from ANSI/AGMA 2005-C96, *Design Manual for Bevel Gears*, with the permission of the publisher, the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

The pressure angle, ϕ, is typically 20°, but 22.5° and 25° are often used to avoid interference. The minimum number of teeth for straight bevel gears is typically 12. More is said about the design of straight bevel gears in Chapter 10.

The mounting of bevel gears is critical if satisfactory performance is to be achieved. Most commercial gears have a defined mounting distance similar to that shown in Figure 8–22(f). It is the distance from some reference surface, typically the back of the hub of the gear, to the apex of the pitch cone. Because the pitch cones of the mating gears have coincident apexes, the mounting distance also locates the axis of the mating gear. If the gear is mounted at a distance smaller than the recommended mounting distance, the teeth will likely bind. If it is mounted at a greater distance, there will be excessive backlash, causing noisy and rough operation.

Spiral Bevel Gears

The teeth of a spiral bevel gear are curved and sloped with respect to the surface of the pitch cone. Spiral angles, ψ, of 20° to 45° are used, with 35° being typical. Contact starts at one end of the teeth and moves along the tooth to its end. For a given tooth form and number of teeth, more teeth are in contact for spiral bevel gears than for straight bevel gears. The gradual transfer of loads and the greater average number of teeth in contact make spiral bevel gears smoother and allow smaller designs than for typical straight bevel gears. Recall that similar advantages were described for a helical gear relative to a spur gear.

The pressure angle, ϕ, is typically 20° for spiral bevel gears, and the minimum number of teeth is typically 12 to avoid interference. But nonstandard spiral gears allow as few as five teeth in the pinion of high-ratio sets if the tips of the teeth are trimmed to avoid interference. The rather high average number of teeth in contact (high contact ratio) for spiral gears makes this approach acceptable and can result in a very compact design. Reference 5 gives the relationships for computing the geometric features of spiral bevel gears that are extensions of those given in Table 8–7.

Zero Spiral Bevel Gears

The teeth of a zero spiral bevel gear are curved somewhat as in a spiral bevel gear, but the spiral angle is zero. These gears can be used in the same mountings as straight bevel gears, but they operate more smoothly. They are sometimes called ZEROL® bevel gears

Hypoid Gears

The major difference between hypoid gears and the others just described is that the centerline of the pinion for a set of hypoid gears is offset either above or below the centerline of the gear. The teeth are designed specially for each combination of offset distance and spiral angle of the teeth. A major advantage is the more compact design that results, particularly when applied to vehicle drive trains and machine tools. (See References 5, 13, and 14 for more data.)

The hypoid gear geometry is the most general form, and the others are special cases. The hypoid gear has an offset axis for the pinion, and its curved teeth are cut at a spiral angle. Then the spiral bevel gear is a hypoid gear with a zero offset distance. A ZEROL® bevel gear is a hypoid gear with a zero offset and a zero spiral angle. A straight bevel gear is a hypoid gear with a zero offset, a zero spiral angle, and straight teeth.

Example Problem 8–6 Compute the values for the geometrical features listed in Table 8–7 for a pair of straight bevel gears having a diametral pitch of 8, a 20° pressure angle, 16 teeth in the pinion, and 48 teeth in the gear. The shafts are at 90°.

Solution Given $P_d = 8$; $N_p = 16$; $N_G = 48$.

Computed Values *Gear Ratio*

$$m_G = N_G/N_P = 48/16 = 3.000$$

Pitch Diameter
For the pinion,

$$d = N_P/P_d = 16/8 = 2.000 \text{ in}$$

For the gear,

$$D = N_G/P_d = 48/8 = 6.000 \text{ in}$$

Pitch Cone Angles
For the pinion,

$$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(16/48) = 18.43^\circ$$

For the gear,

$$\Gamma = \tan^{-1}(N_G/N_P) = \tan^{-1}(48/16) = 71.57^\circ$$

Outer Cone Distance

$$A_o = 0.5\ D/\sin(\Gamma) = 0.5(6.00 \text{ in})/\sin(71.57^\circ) = 3.162 \text{ in}$$

Face Width
The face width must be specified:

$$F = 1.000 \text{ in}$$

Based on the following guidelines:
Nominal face width:

$$F_{\text{nom}} = 0.30A_o = 0.30(3.162 \text{ in}) = 0.949 \text{ in}$$

Maximum face width:

$$F_{\text{max}} = A_o/3 = (3.162 \text{ in})/3 = 1.054 \text{ in}$$

or

$$F_{max} = 10/P_d = 10/8 = 1.25 \text{ in}$$

Mean Cone Distance

$$A_m = A_{mG} = A_o - 0.5\ F = 3.162 \text{ in} - 0.5(1.00 \text{ in}) = 2.662 \text{ in}$$

Ratio $A_m/A_o = 2.662/3.162 = 0.842$ (This ratio occurs in several following calculations.)

Mean Circular Pitch

$$p_m = (\pi/P_d)(A_m/A_o) = (\pi/8)(0.842) = 0.331 \text{ in}$$

Mean Working Depth

$$h = (2.00/P_d)(A_m/A_o) = (2.00/8)(0.842) = 0.210 \text{ in}$$

Clearance

$$c = 0.125h = 0.125(0.210 \text{ in}) = 0.026 \text{ in}$$

Mean Whole Depth

$$h_m = h + c = 0.210 \text{ in} + 0.026 \text{ in} = 0.236 \text{ in}$$

Mean Addendum Factor

$$c_1 = 0.210 + 0.290/(m_G)^2 = 0.210 + 0.290/(3.00)^2 = 0.242$$

Gear Mean Addendum

$$a_G = c_1 h = (0.242)(0.210 \text{ in}) = 0.051 \text{ in}$$

Pinion Mean Addendum

$$a_p = h - a_G = 0.210 \text{ in} - 0.051 \text{ in} = 0.159 \text{ in}$$

Gear Mean Dedendum

$$b_G = h_m - a_G = 0.236 \text{ in} - 0.051 \text{ in} = 0.185 \text{ in}$$

Pinion Mean Dedendum

$$b_P = h_m - a_P = 0.236 \text{ in} - 0.159 \text{ in} = 0.077 \text{ in}$$

Gear Dedendum Angle

$$\delta_G = \tan^{-1}(b_G/A_{mG}) = \tan^{-1}(0.185/2.662) = 3.98°$$

Pinion Dedendum Angle

$$\delta_P = \tan^{-1}(b_P/A_{mG}) = \tan^{-1}(0.077/2.662) = 1.66°$$

Gear Outer Addendum

$$a_o G = a_G + 0.5\, F \tan \delta_P$$
$$a_{oG} = (0.051 \text{ in}) + (0.5)(1.00 \text{ in}) \tan(1.657°) = 0.0655 \text{ in}$$

Pinion Outer Addendum

$$a_o P = a_P + 0.5\, F \tan \delta_G$$
$$a_o P = (0.159 \text{ in}) + (0.5)(1.00 \text{ in}) \tan(3.975°) = 0.1937 \text{ in}$$

Gear Outside Diameter

$$D_o = D + 2a_{oG} \cos \Gamma$$
$$D_o = 6.000 \text{ in} + 2(0.0655 \text{ in}) \cos(71.57°) = 6.041 \text{ in}$$

Pinion Outside Diameter

$$d_o = d + 2a_{oP} \cos \gamma$$
$$d_o = 2.000 \text{ in} + 2(0.1937 \text{ in}) \cos(18.43°) = 2.368 \text{ in}$$

8–9 TYPES OF WORMGEARING

Wormgearing is used to transmit motion and power between nonintersecting shafts, usually at 90° to each other. The drive consists of a worm on the high-speed shaft which has the general appearance of a power screw thread: a cylindrical, helical thread. The worm drives a wormgear, which has an appearance similar to that of a helical gear. Figures 8–23 shows a typical worm and wormgear set. Sometimes the wormgear is referred to as a *worm wheel* or simply a *wheel* or *gear*. (See Reference 6.) Worms and wormgears can be provided with either right hand or left hand threads on the worm and correspondingly designed teeth on the wormgear affecting the rotational direction of the wormgear.

Several variations of the geometry of wormgear drives are available. The most common one, shown in Figures 8–23 and 8–24, employs a cylindrical worm mating with a wormgear having teeth that are throated, wrapping partially around the worm. This is called a *single-enveloping type* of wormgear drive. The contact between the threads of the worm and wormgear teeth is along a line, and the power transmission capacity is quite good. Many manufacturers offer this type of wormgear set as a stocked item. Installation of the worm is relatively easy because axial alignment is not very critical. However, the wormgear must be carefully aligned radially in order to achieve the benefit of the enveloping action. Figure 8–25 shows a cutaway of a commercial worm-gear reducer.

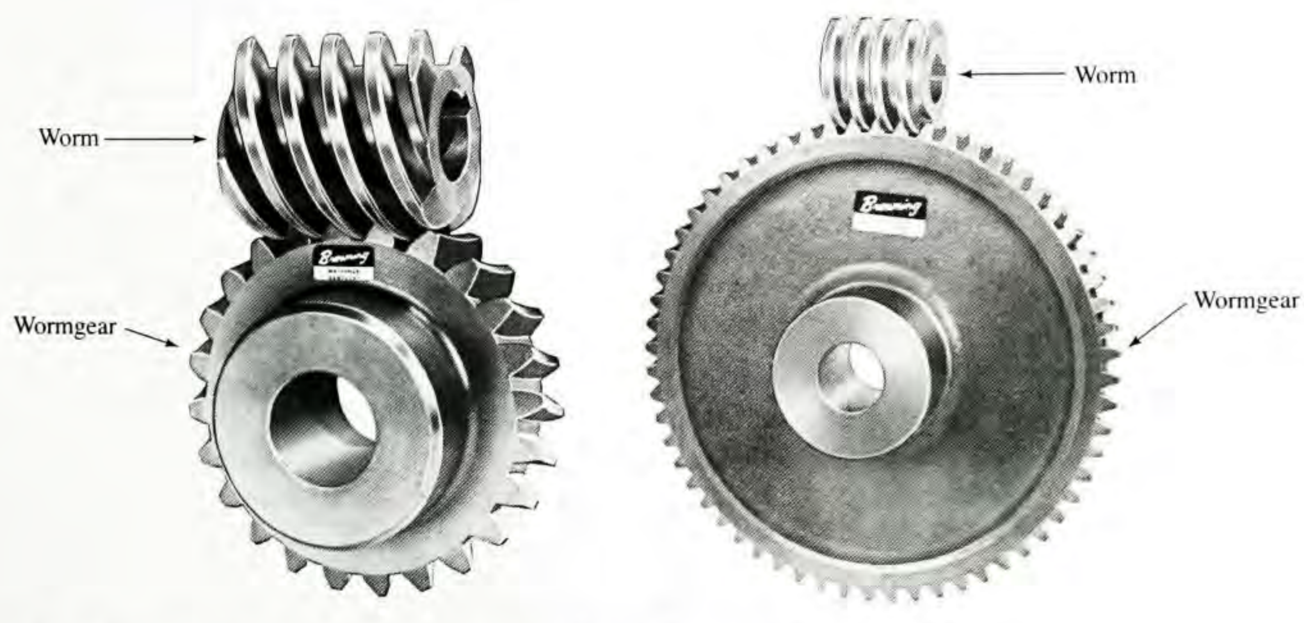


FIGURE 8–23 Worms and wormgears (Emerson Power Transmission Corporation, Browning Division, Maysville, KY)

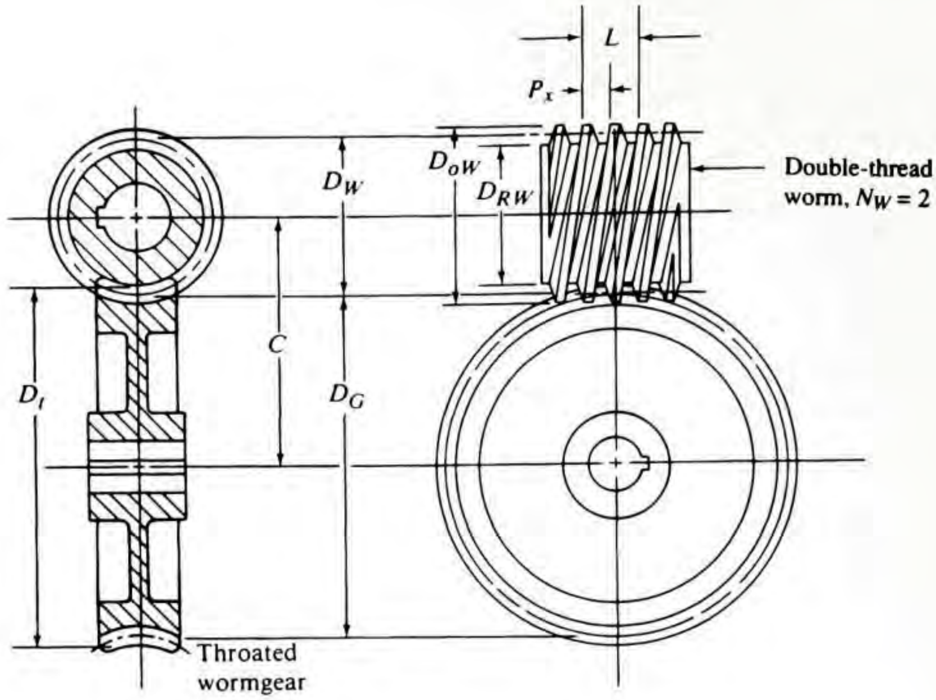


FIGURE 8–24
Single-enveloping wormgear set

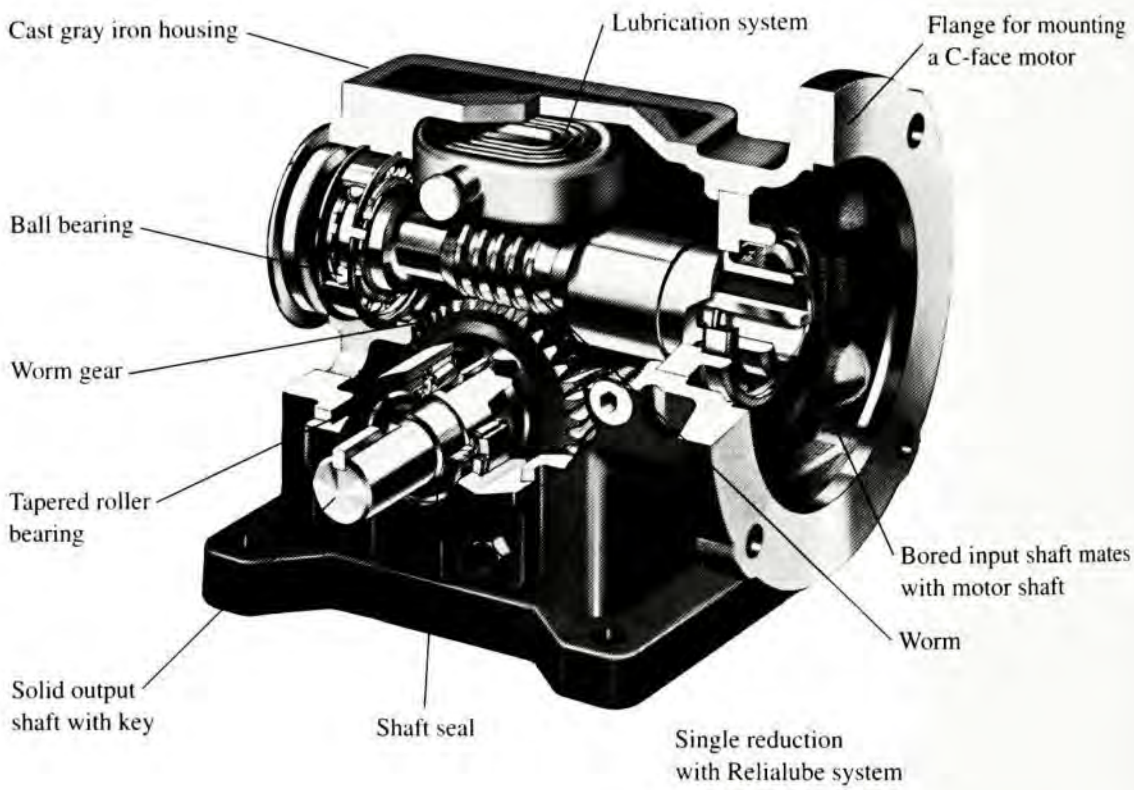


FIGURE 8–25
Wormgear reducer (Rockwell Automation/Dodge Greenville, SC)

A simpler form of wormgear drive allows a special cylindrical worm to be used with a standard spur gear or helical gear. Neither the worm nor the gear must be aligned with great accuracy, and the center distance is not critical. However, the contact between the worm threads and the wormgear teeth is theoretically a point, drastically reducing the power transmission capacity of the set. Thus, this type is used mostly for nonprecision positioning applications at low speeds and low power levels.

A third type of wormgear set is the *double-enveloping type* in which the worm is made in an hourglass shape and mates with an enveloping type of wormgear. This results in area contact rather than line or point contact and allows a much smaller system to transmit a given power at a given reduction ratio. However, the worm is more difficult to manufacture, and the alignment of both the worm and the wormgear is very critical.

8–10 GEOMETRY OF WORMS AND WORMGEARS

MDESIGN

Pitches, p and P_d

A basic requirement of the worm and wormgear set is that the *axial pitch* of the worm must be equal to the *circular pitch* of the wormgear in order for them to mesh. Figure 8–24 shows the basic geometric features of a single-enveloping worm and wormgear set. *Axial pitch*, P_x, is defined as the distance from a point on the worm thread to the corresponding point on the next adjacent thread, measured axially on the pitch cylinder. As before, the circular pitch is defined for the wormgear as the distance from a point on a tooth on the pitch circle of the gear to the corresponding point on the next adjacent tooth, measured along the pitch circle. Thus, the circular pitch is an arc distance that can be calculated from

Circular Pitch

$$p = \pi D_G / N_G \qquad \textbf{(8–33)}$$

where D_G = pitch diameter of the gear
N_G = number of teeth in the gear

Some wormgears are made according to the circular pitch convention. But, as noted with spur gears, commercially available wormgear sets are usually made to a diametral pitch convention with the following pitches readily available: 48, 32, 24, 16, 12, 10, 8, 6, 5, 4, and 3. The diametral pitch is defined for the gear as

Diametral Pitch

$$P_d = N_G / D_G \qquad \textbf{(8–34)}$$

The conversion from diametral pitch to circular pitch can be made from the following equation:

$$P_d p = \pi \qquad \textbf{(8–35)}$$

Number of Worm Threads, N_W

Worms can have a single thread, as in a typical screw, or multiple threads, usually 2 or 4, but sometimes 3, 5, 6, 8, or more. It is common to refer to the number of threads as N_W and then to treat that number as if it were the number of teeth in the worm. The number of threads in the worm is frequently referred to as the number of *starts;* this is convenient because if you look at the end of a worm, you can count the number of threads that start at the end and wind down the cylindrical worm.

Lead, L

The *lead* of a worm is the axial distance that a point on the worm would move as the worm is rotated one revolution. Lead is related to the axial pitch by

Lead

$$L = N_W P_x \qquad \textbf{(8–36)}$$

Lead Angle, λ

The *lead angle* is the angle between the tangent to the worm thread and the line perpendicular to the axis of the worm. To visualize the method of calculating the lead angle, refer to Figure 8–26, which shows a simple triangle that would be formed if one thread of the worm

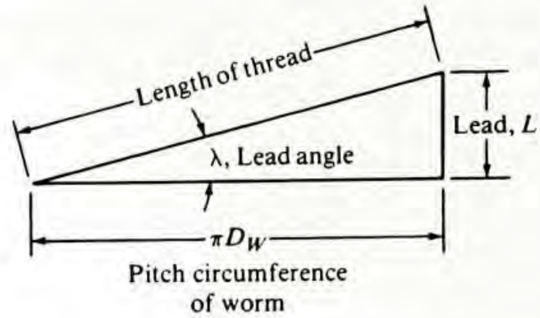


FIGURE 8–26 Lead angle

were unwrapped from the pitch cylinder and laid flat on the paper. The length of the hypotenuse is the length of the thread itself. The vertical side is the lead, L. The horizontal side is the circumference of the pitch cylinder, πD_W, where D_W is the pitch diameter of the worm. Then

Lead Angle

$$\tan \lambda = L/\pi D_W \tag{8–37}$$

Pitch Line Speed, v_t

As before, the pitch line speed is the linear velocity of a point on the pitch line for the worm or the wormgear. For the worm having a pitch diameter D_W in, rotating at n_W rpm,

Pitch Line Speed for worm

$$v_{tW} = \frac{\pi D_W n_W}{12} \text{ ft/min}$$

For the wormgear having a pitch diameter D_G in, rotating at n_G rpm,

Pitch Line Speed for Gear

$$V_{tG} = \frac{\pi D_G n_G}{12} \text{ ft/min}$$

Note that these two values for pitch line speed are *not* equal.

Velocity Ratio, *VR*

It is most convenient to calculate the velocity ratio of a worm and wormgear set from the ratio of the input rotational speed to the output rotational speed:

Velocity Ratio for Worm/Wormgear Set

$$VR = \frac{\text{speed of worm}}{\text{speed of gear}} = \frac{n_W}{n_G} = \frac{N_G}{N_W} \tag{8–38}$$

Example Problem 8–7 A wormgear has 52 teeth and a diametral pitch of 6. It mates with a triple-threaded worm that rotates at 1750 rpm. The pitch diameter of the worm is 2.000 in. Compute the circular pitch, the axial pitch, the lead, the lead angle, the pitch diameter of the wormgear, the center distance, the velocity ratio, and the rotational speed of the wormgear.

Solution *Circular Pitch*

$$p = \pi/P_d = \pi/6 = 0.5236 \text{ in}$$

Axial Pitch

$$P_x = p = 0.5236 \text{ in}$$

Lead

$$L = N_W P_x = (3)(0.5236) = 1.5708 \text{ in}$$

Lead Angle

$$\lambda = \tan^{-1}(L/\pi D_W) = \tan^{-1}(1.5708/\pi 2.000)$$
$$\lambda = 14.04^\circ$$

Pitch Diameter

$$D_G = N_G/P_d = 52/6 = 8.667 \text{ in}$$

Center Distance

$$C = (D_W + D_G)/2 = (2.000 + 8.667)/2 = 5.333 \text{ in}$$

Velocity Ratio

$$VR = N_G/N_W = 52/3 = 17.333$$

Gear rpm

$$n_G = n_W/VR = 1750/17.333 = 101 \text{ rpm}$$

Pressure Angle

Most commercially available wormgears are made with pressure angles of $14\frac{1}{2}^\circ$, 20°, 25°, or 30°. The low pressure angles are used with worms having a low lead angle and/or a low diametral pitch. For example, a $14\frac{1}{2}^\circ$ pressure angle may be used for lead angles up to about 17°. For higher lead angles and with higher diametral pitches (smaller teeth), the 20° or 25° pressure angle is used to eliminate interference without excessive undercutting. The 20° pressure angle is the preferred value for lead angles up to 30°. From 30° to 45° of lead angle, the 25° pressure angle is recommended. Either the normal pressure angle, ϕ_n, or the transverse pressure angle, ϕ_t, may be specified. These are related by

Pressure Angle

$$\tan\phi_n = \tan\phi_t \cos\lambda \qquad \textbf{(8–39)}$$

Self-Locking Wormgear Sets

Self-locking is the condition in which the worm drives the wormgear, but if torque is applied to the gear shaft, the worm does not turn. It is locked! The locking action is produced by the friction force between the worm threads and the wormgear teeth, and this is highly dependent on the lead angle. It is recommended that a lead angle no higher than about 5.0°

be used in order to ensure that self-locking will occur. This lead angle usually requires the use of a single-threaded worm. Note that the triple-threaded worm in Example Problem 8–7 has a lead angle of 14.04°. It is *not* likely to be self-locking.

8–11 TYPICAL GEOMETRY OF WORMGEAR SETS

Considerable latitude is permissible in the design of wormgear sets because the worm and wormgear combination is designed as a unit. However, there are some guidelines.

General Guidelines for Worm and Wormgear Dimensions

Typical Tooth Dimensions

Table 8–8 shows typical values used for the dimensions of worm threads and gear teeth.

Worm Diameter

The diameter of the worm affects the lead angle, which in turn affects the efficiency of the set. For this reason, small diameters are desirable. But for practical reasons and proper proportion with respect to the wormgear, it is recommended that the worm diameter be approximately $C^{0.875}/2.2$, where C is the center distance between the worm and the wormgear. Variation of about 30% is allowed. (See Reference 6.) Thus, the worm diameter should fall in the range

$$1.6 < \frac{C^{0.875}}{D_W} < 3.0 \qquad \textbf{(8–40)}$$

But some commercially available wormgear sets fall outside this range, especially in the smaller sizes. Also, those worms designed to have a through-hole bored in them for installation on a shaft are typically larger than you would find from Equation (8–40). Proper proportion and efficient use of material should be the guide. The worm shaft must also be checked for deflection under operating loads. For worms machined integral with the shaft, the root of the worm threads determines the minimum shaft diameter. For worms having bored holes, sometimes called *shell worms,* care must be exercised to leave sufficient material between the thread root and the keyway in the bore. Figure 8–27 shows the recommended thickness above the keyway to be a minimum of one-half the whole depth of the threads.

TABLE 8–8 Typical tooth dimensions for worms and wormgears

Dimension	Formula
Addendum	$a = 0.3183P_x = 1/P_d$
Whole depth	$h_t = 0.6866P_x = 2.157/P_d$
Working depth	$h_k = 2a = 0.6366P_a = 2/P_d$
Dedendum	$b = h_t - a = 0.3683P_x = 1.157/P_d$
Root diameter of worm	$D_{rW} = D_W - 2b$
Outside diameter of worm	$D_{oW} = D_W + 2a = D_W + h_k$
Root diameter of gear	$D_{rG} = D_G - 2b$
Throat diameter of gear	$D_t = D_G + 2a$

Source: Standard AGMA *Design Manual Cylindrical Wormgearing*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

FIGURE 8–27 Shell worm

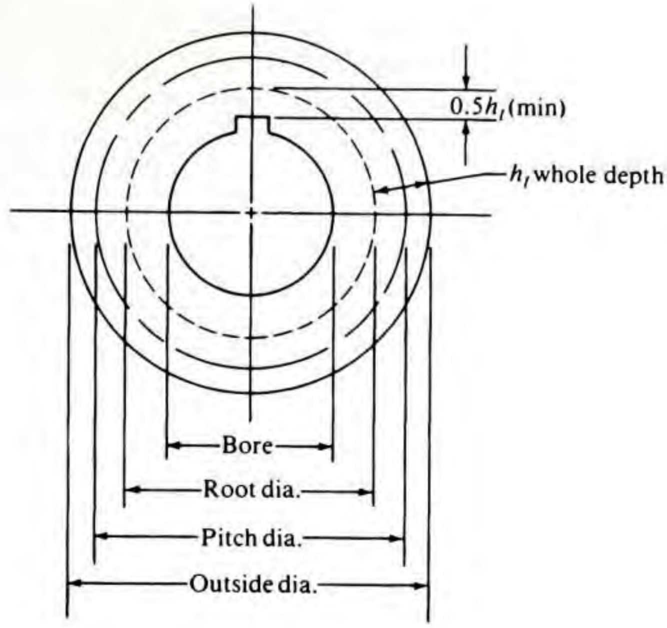
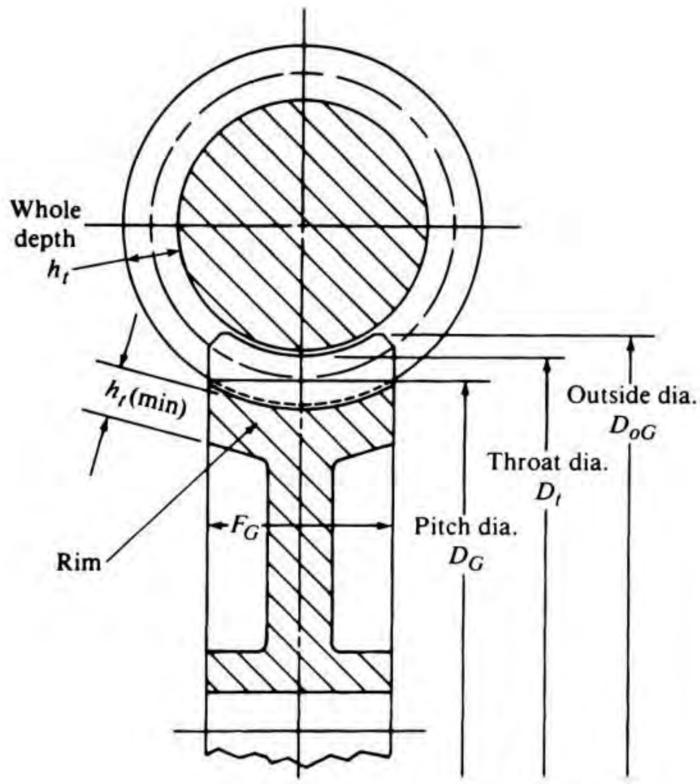


FIGURE 8–28 Wormgear details



Wormgear Dimensions

We are concerned here with the single-enveloping type of wormgear, as shown in Figure 8–28. Its addendum, dedendum, and depth dimensions are assumed to be the same as those listed in Table 8–8, measured at the throat of the wormgear teeth. The throat is in line with the vertical centerline of the worm. The recommended face width for the wormgear is

Face Width of Wormgear

$$F_G = (D_{oW}^2 - D_W^2)^{1/2} \tag{8–41}$$

This corresponds to the length of the line tangent to the pitch circle of the worm and limited by the outside diameter of the worm. Any face width beyond this value would not be effective in resisting stress or wear, but a convenient value slightly greater than the minimum should be used. The outer edges of the wormgear teeth should be chamfered approximately as shown in Figure 8–28.

Another recommendation, which is convenient for initial design, is that the face width of the gear should be approximately 2.0 times the circular pitch. Because we are working in the diametral pitch system, we will use

$$F_G = 2p = 2\pi/P_d \tag{8–42}$$

However, since this is only approximate and 2π is approximately 6, we will use

$$F_G = 6/P_d \tag{8–43}$$

If the gear web is thinned, a rim thickness at least equal to the whole depth of the teeth should be left.

Face Length of the Worm

For maximum load sharing, the worm face length should extend to at least the point where the outside diameter of the worm intersects the throat diameter of the wormgear. This length is

Face Length of Worm

$$F_W = 2[D_o/2)^2 - (D_G/2 - a)^2]^{1/2} \tag{8–44}$$

Example Problem 8–8 A worm and wormgear set is to be designed to produce a velocity ratio of 40. It has been proposed that the diametral pitch of the wormgear be 8, based on the torque that must be transmitted. (This will be discussed in Chapter 10.) Using the relationships presented in this section, specify the following:

Worm diameter, D_W

Number of threads in the worm, N_W

Number of teeth in the gear, N_G

Actual center distance, C

Face width of the gear, F_G

Face length of the worm, F_W

Minimum thickness of the rim of the gear

Solution Many design decisions need to be made, and multiple solutions could satisfy the requirements. Presented here is one solution, along with comparisons with the various guidelines discussed in this section. This type of analysis precedes the stress analysis and the determination of the power-transmitting capacity of the worm and wormgear drive which is discussed in Chapter 10.

Trial Design: Let's specify a double-threaded worm: $N_W = 2$. Then there must be 80 teeth in the wormgear to achieve a velocity ratio of 40. That is,

$$VR = N_G/N_W = 80/2 = 40$$

With the known diametral pitch, $P_d = 8$, the pitch diameter of the wormgear is

$$D_G = N_G/P_d = 80/8 = 10.000 \text{ in}$$

An initial estimate for the magnitude of the center distance is approximately $C = 6.50$ in. We know that it will be greater than 5.00 in, the radius of the wormgear. Using Equation (8–40), the recommended minimum size of the worm is

$$D_W = C^{0.875}/3.0 = 1.71 \text{ in}$$

Similarly, the maximum diameter should be

$$D_W = C^{0.875}/1.6 = 3.21 \text{ in}$$

A small worm diameter is desirable. Let's specify $D_W = 2.25$ in. The actual center distance is

$$C = (D_W + D_G)/2 = 6.125 \text{ in}$$

Worm Outside Diameter

$$D_{oW} = D_W + 2a = 2.25 + 2(1/P_d) = 2.25 + 2(1/8) = 2.50 \text{ in}$$

Whole Depth

$$h_t = 2.157/P_d = 2.157/8 = 0.270 \text{ in}$$

Face Width for Gear
Let's use Equation (8–41):

$$F_G = (D_{oW}^2 - D_W^2)^{1/2} = (2.50^2 - 2.25^2) = 1.090 \text{ in}$$

Let's specify $F_G = 1.25$ in.

Addendum

$$a = 1/P_d = 1/8 = 0.125 \text{ in}$$

Throat Diameter of Wormgear

$$D_t = D_G + 2a = 10.000 + 2(0.125) = 10.250 \text{ in}$$

Recommended Minimum Face Length of Worm

$$F_W = 2[(D_t/2)^2 - (D_G/2 - a)^2]^{1/2} = 3.16 \text{ in}$$

Let's specify $F_W = 3.25$ in.

Minimum Thickness of the Rim of the Gear
The rim thickness should be greater than the whole depth:

$$h_t = 0.270 \text{ in}$$

8–12 TRAIN VALUE FOR COMPLEX GEAR TRAINS

The concept of train value was introduced in Section 8–6 and was applied to gear trains having all spur gears and a modest reduction ratio. Trains of two to five gears having a single speed reduction or a double reduction were included. This section expands on the concept of train value to include a wider variety of gear types, higher reduction ratios, and the opportunity to use several different arrangements of gears.

The basic definition of *train value* as shown in Equation (8–23) will continue to be used. Its general form is repeated here:

$$TV = \frac{\text{product of number of teeth in the driven gears}}{\text{product of number of teeth in the driving gears}} = \frac{\text{input speed}}{\text{output speed}}$$

This is equivalent to saying that the train value is the product of the velocity ratios of the individual gear pairs in the train.

When spur, helical, and bevel gears are used, the specific geometry of the gears and their teeth does not affect the train value. Also, when a worm/wormgear set is a part of the train, its data can be entered into the equation if we recall that the number of *threads* in the worm can be considered equivalent to the number of *teeth* in the driver of that set.

Sketches of the gear trains are valuable to illustrate the arrangement of gears and to enable you to track the flow of power through the train, that is, how the motion is transferred from the input shaft through the entire train to the output shaft. The sketches can be somewhat schematic, but they should show the relative positions of the gears and the shafts that carry the gears. Although sizes need not be drawn to scale, it is helpful to suggest the nominal size of the gears to aid in judging whether a given pair will produce a speed reduction or a speed increase. Showing the pitch diameters of the gears will suffice. Recall that the motion of one gear on its mating gear is kinematically similar to the rolling of smooth cylinders on one another without slipping where the diameters for the cylinders are equal to the pitch diameters of the gears. But be sure to understand that any two gears in mesh must have compatible tooth forms. Only the kinematics of the train are being considered here. Later, when the actual gears, shafts, bearings, and the housing are designed, many other geometric features and operating parameters must be considered.

The sketching technique shown in Figure 8–29 will be used in this section and in the Problems section of this chapter. Gears are given letter designations, and the shafts carrying the gears are numbered. The data for the train shown in the figure should be interpreted as follows:

- Gears A, B, C, and D are external type, either spur or helical. Their shafts (shafts 1, 2, and 3) are parallel and are indicated only by their centerlines. Shaft 1 is the input to the train. It would typically be connected directly to a drive such as an electric motor.
- Gear A drives gear B with a speed reduction because gear A is smaller than gear B. A change in direction of rotation occurs. Gear C is on the same shaft as gear B and rotates at the same speed and in the same direction. Gear C drives gear D with a speed reduction and a direction reversal.
- Gears E and F are bevel gears, straight, or spiral, or some other tooth form. Their shafts (3 and 4) are perpendicular.
- Gear G is a worm, and it is driving wormgear H. Their shafts (4 and 5) are perpendicular, with shaft 5 directed out of the page. Note that we are seeing the side view of the pitch cylinder of the worm rotating about a vertical axis (shaft 4). We see the end view of wormgear H.
- The small pinion I is also mounted on shaft 5 and rotates at the same speed and in the same direction as wormgear H.
- Pinion I drives the internal gear J mounted on shaft 6 which is the output gear of the train. Note that shaft 6 rotates in the same direction as shaft 5.

Data for the gears in the train may be provided in a number of ways. If only speed of rotation is to be considered, the number of teeth in each gear will normally be given. If physical size of the train, center distances, bearing placement, housing design, and detailed layout are to be considered, then other geometrical features such as diametral pitch, face width, gear blank style, hub style, and gear diameters will have to be specified. Most of these features are discussed in later chapters. We will give only the numbers of teeth in most cases in this chapter. But notice that if pitch diameter and diametral pitch of a pinion and gear are given, their numbers of teeth can be determined from

$$P_d = N_P/D_P = N_G/D_G$$
$$N_P = P_d D_P$$
$$N_G = P_d D_G$$

Also, we can use the definition of diametral pitch to show the relationship between the ratio of the numbers of teeth in a pair of gears and the ratio of their diameters:

$$P_d = N_P/D_P = N_G/D_G$$
$$N_G/N_P = D_G/D_P$$

This shows that the diameter ratio could replace the teeth ratio in the train value equation.

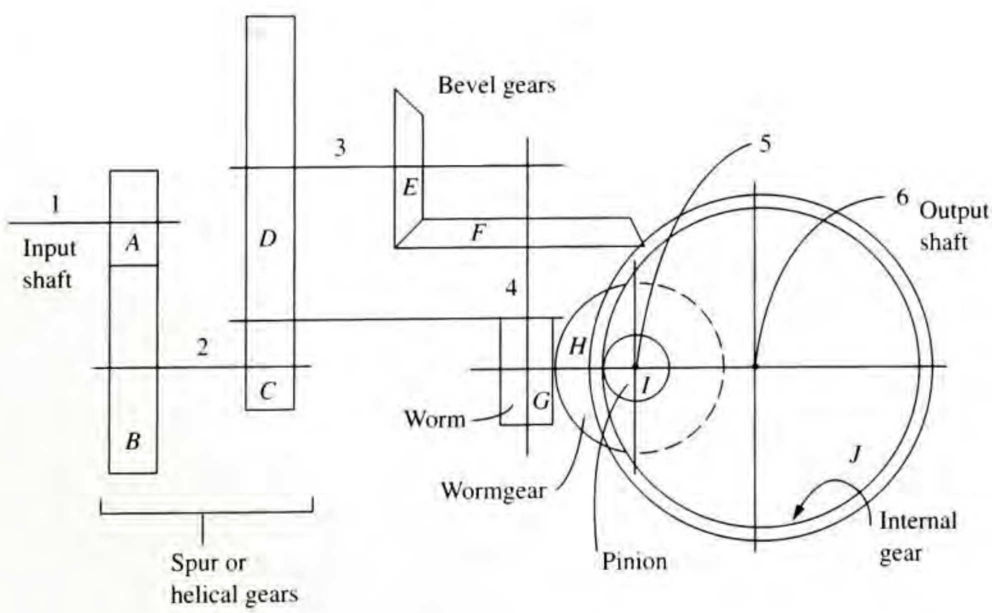
Example Problem 8–9 Refer to Figure 8–29. Shaft 1 is a motor shaft and rotates at 1160 rpm. Compute the rotational speed of the output shaft, shaft 6. Data for the gears are as follows:

$N_A = 18$	$N_B = 34$	$N_C = 20$	$N_D = 62$
$N_E = 30$	$N_F = 60$	$N_G = 2$ (worm threads)	$N_H = 40$
		$N_I = 16$	$N_J = 88$

Solution We will compute the train value first:

$$TV = \frac{(N_B)(N_D)(N_F)(N_H)(N_J)}{(N_A)(N_C)(N_E)(N_G)(N_I)} = \frac{(34)(62)(60)(40)(88)}{(18)(20)(30)(2)(16)} = 1288.2$$

FIGURE 8–29 Gear train for Example Problem 8–9



But $TV = n_1/n_6$. Then

$$n_6 = n_1/TV = (1160 \text{ rpm})/1288.2 = 0.900 \text{ rpm}$$

Shaft 6 rotates 0.900 rpm.

8–13 DEVISING GEAR TRAINS

Now we will show several methods for devising gear trains to produce a desired train value. The result will typically be the specification of the number of teeth in each gear and the general arrangement of the gears relative to each other. The determination of the types of gears will generally not be considered except for how they may affect the direction of rotation or the general alignment of the shafts. Additional details can be specified after completion of the study of the design procedures in later chapters.

A few general principles that were discussed earlier in this chapter are reviewed first.

General Principles for Devising Gear Trains

1. The velocity ratio for any pair of gears can be computed in a variety of ways as indicated in Equation (8–22).
2. The number of teeth in any gear must be an integer.
3. Gears in mesh must have the same tooth form and the same pitch.
4. When external gears mesh, there is a direction reversal of their shafts.
5. When an external pinion meshes with an internal gear, their shafts rotate in the same direction.
6. An idler is a gear that performs as both a driver and a driven gear in the same train. Its size and number of teeth have no effect on the magnitude of the train value, but the direction of rotation is changed.
7. Spur and helical gears operate on parallel shafts.
8. Bevel gears and a worm/wormgear set operate on shafts perpendicular to each other.
9. The number of teeth in the pinion of a gear pair should not be such that it causes interference with the teeth of its mating gear. Refer to Table 8–6.
10. In general, the number of teeth in the gear should not be larger than about 150. This is somewhat arbitrary, but it is typically more desirable to use a double-reduction gear train rather than a very large, single-reduction gear pair.

Hunting Tooth

Some designers recommend that integer velocity ratios be avoided, if possible, because the same two teeth would come into contact frequently and produce uneven wear patterns on the teeth. For example when using a velocity ratio of exactly 2.0, a given tooth on the pinion would contact the same two teeth on the gear with every two revolutions. In Chapter 9 you will learn that the pinion teeth are often made harder than the gear because the pinion experiences higher stresses. As the gears rotate, the pinion teeth tend to smooth any inherent roughness of the gear teeth, a process sometimes called *wearing in*. Each tooth on the pinion has a slightly different geometry causing unique wear patterns on the few teeth with which it mates.

A more uniform wear pattern will result if the velocity ratio in not an integer. Adding or subtracting one tooth from the number of teeth in the gear produces the result that each pinion tooth would contact a different gear tooth with each revolution and the wear pattern would be more uniform. The added or subtracted tooth is called the *hunting tooth*. Obviously the velocity ratio for the gear pair will be slightly different, but that is often not a concern unless precise timing between the driver and driven gears is required. Consider the following example.

An initial design for a gear pair calls for the pinion to be mounted to the shaft of an electric motor having a nominal speed of 1750 rpm. The pinion has 18 teeth and the gear has 36 teeth, resulting in a velocity ratio of 36/18 or 2.000. The output speed would then be,

$$\text{Initial design:} \qquad n_2 = n_1\,(N_P/N_G) = 1750\text{ rpm }(18/36) = 875\text{ rpm}$$

Now consider adding or subtracting one tooth from the gear. The output speeds would be,

$$\text{Modified design:} \qquad n_2 = n_1\,(N_P/N_G) = 1750\text{ rpm }(18/35) = 900\text{ rpm}$$

$$\text{Modified design:} \qquad n_2 = n_1\,(N_P/N_G) = 1750\text{ rpm }(18/37) = 851\text{ rpm}$$

The output speeds for the modified designs are less than 3.0 percent different from the original design. You would have to decide if that is acceptable in a given design project. However, be aware that the motor speed is typically not exactly 1750 rpm. As discussed in Chapter 21, 1750 rpm is the *full load speed* of a four-pole alternating current electric motor. When operating at a torque less than the full load torque the speed would be greater than 1750 rpm. Conversely, a greater torque would result in a slower speed. When precise speeds are required, a variable speed drive that can be adjusting according to actual loads is recommended.

Several design procedures will now be demonstrated through example problems. It is not practical to outline a completely general procedure because of the many variables in any given design situation. You are advised to study the examples for their general approach, which you can adapt to future problems as needed.

Design of a Single Pair of Gears to Produce a Desired Velocity Ratio

Example Problem 8–10 Devise a gear train to reduce the speed of rotation of a drive from an electric motor shaft operating at 3450 rpm to approximately 650 rpm.

Solution First we will compute the nominal train value:

$$TV = (\text{input speed})/(\text{output speed}) = 3450/650 = 5.308$$

If a single pair of gears is used then the train value is equal to the velocity ratio for that pair. That is, $TV = VR = N_G/N_P$.

Let's decide that spur gears having 20°, full-depth, involute teeth are to be used. Then we can refer to Table 8–6 and determine that no fewer than 16 teeth should be used for the pinion in order to avoid interference. We can specify the number of teeth in the pinion and use the velocity ratio to compute the number of teeth in the gear:

$$N_G = (VR)(N_P) = (5.308)(N_P)$$

Some examples are given in Table 8–9.

Conclusion and Comments The combination of $N_P = 26$ and $N_G = 138$ gives the most ideal result for the output speed. But all of the trial values give output speeds reasonably close to the desired value. Only two

TABLE 8–9

N_P	Computed $N_G = (5.308)(N_P)$	Nearest integer N_G	Actual VR: $VR = N_G/N_P$	Actual output speed (rpm): $n_G = n_P/VR = n_P(N_P/N_G)$
16	84.92	85	85/16 = 5.31	649.4
17	90.23	90	90/17 = 5.29	651.7
18	95.54	96	96/18 = 5.33	646.9
19	100.85	101	101/19 = 5.32	649.0
20	106.15	106	106/20 = 5.30	650.9
21	111.46	111	111/21 = 5.29	652.7
22	116.77	117	117/22 = 5.32	648.7
23	122.08	122	122/23 = 5.30	650.4
24	127.38	127	127/24 = 5.29	652.0
25	132.69	133	133/25 = 5.32	648.5
26	138.00	138	138/26 = 5.308	650.0 Exact
27	143.31	143	143/27 = 5.30	651.4
28	148.61	149	149/28 = 5.32	648.3
29	153.92	154	**Too large**	

are more than 2.0 rpm off the desired value. It remains a design decision as to how close the output speed must be to the stated value of 650 rpm. Note that the input speed is given as 3450 rpm, the full load speed of an electric motor. But how accurate is that? The actual speed of the input will vary depending on the load on the motor. Therefore, it is not likely that the ratio must be precise.

Equal Reduction Ratios for Compound Gear Trains

Example Problem 8–11 Devise a gear train for a machine tool drive. The input is a shaft that rotates at exactly 1800 rpm. The output speed must be within the range of 31.5 and 32.5 rpm. Use 20°, full-depth, involute teeth and no more than 150 teeth in any gear.

Solution *Permissible Train Values*

First let's compute the nominal train value that will produce an output speed of 32.0 rpm at the middle of the allowable range:

$$TV_{\text{nom}} = (\text{input speed})/(\text{nominal output speed}) = 1800/32 = 56.25$$

Similarly, we can compute the minimum and maximum allowable ratio:

$$TV_{\min} = (\text{input speed})/(\text{maximum output speed}) = 1800/32.5 = 55.38$$
$$TV_{\max} = (\text{input speed})/(\text{minimum output speed}) = 1800/31.5 = 57.14$$

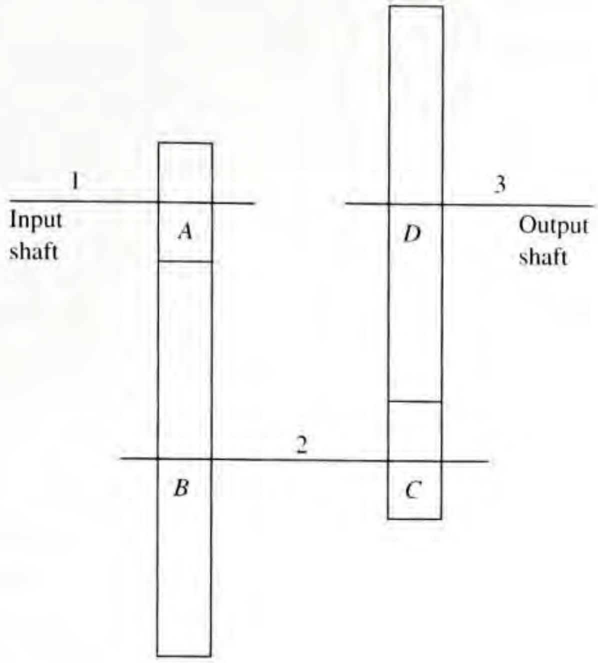
Possible Ratio for Single Pair

The maximum ratio that any one pair of gears can produce occurs when the gear has 150 teeth and the pinion has 17 teeth (see Table 8–6). Then

$$VR_{\max} = N_G/N_P = 150/17 = 8.82$$

This is too low.

FIGURE 8–30
General layout of proposed gear train



Possible Train Value for Double-Reduction Train
If a double-reduction train is proposed, the train value will be

$$TV = (VR_1)(VR_2)$$

But the maximum value for either VR is 8.82. Then the maximum train value for the double-reduction train is

$$TV_{max} = (8.82)(8.82) = (8.82)^2 = 77.9$$

It can be concluded that a double-reduction train should be practical.

Optional Designs
The general layout of the proposed train is shown in Figure 8–30. Its train value is

$$TV = (VR_1)(VR_2) = (N_B/N_A)(N_D/N_C)$$

We need to specify the number of teeth in each of the four gears to achieve a train value within the range just computed. Our approach is to specify two ratios, VR_1 and VR_2, such that their product is within the desired range. One possibility is to let the two ratios be equal. To produce the middle train value, each velocity ratio must be the square root of the target ratio, 56.25. That is,

$$VR_1 = VR_2 = \sqrt{56.25} = 7.50$$

As shown in Table 8–10, we will use a process similar to that used in the previous example problem to select possible numbers of teeth.

Any of the possible designs shown in Table 8–10 would produce acceptable results. For example, we could specify

$$N_A = 18 \qquad N_B = 135 \qquad N_C = 18 \qquad N_D = 135$$

This combination would produce an output speed of exactly 32.0 rpm when the input speed was exactly 1800 rpm.

TABLE 8–10

N_P	Computed $N_G = (7.5)(N_P)$	Nearest integer N_G	Actual VR: $VR = N_G/N_P$	Actual output speed (rpm): $n_G = n_P(VR)^2$
17	127.5	128	128/17 = 7.529	31.75
17	127.5	127	127/17 = 7.470	32.25
18	135	135	135/18 = 7.500	32.00
19	142.5	143	143/19 = 7.526	31.78
19	142.5	142	142/19 = 7.474	32.23
20	150	150	150/20 = 7.500	32.00

Factoring Approach for Compound Gear Trains

Example Problem 8–12 Devise a gear train for a recorder for a precision measuring instrument. The input is a shaft that rotates at exactly 3600 rpm. The output speed must be within the range of 11.0 and 11.5 rpm. Use 20°, full-depth, involute teeth; no fewer than 18 teeth; and no more than 150 teeth in any gear.

Solution *Nominal Target TV*

$$TV_{nom} = 3600/11.25 = 320$$

Maximum TV

$$TV_{max} = 3600/11.0 = 327.3$$

Minimum TV

$$TV_{min} = 3600/11.5 = 313.0$$

Maximum Single VR

$$VR_{max} = 150/18 = 8.33$$

Maximum TV for Double Reduction

$$TV_{max} = (8.333)^2 = 69.4 \text{ (too low)}$$

Maximum TV for Triple Reduction

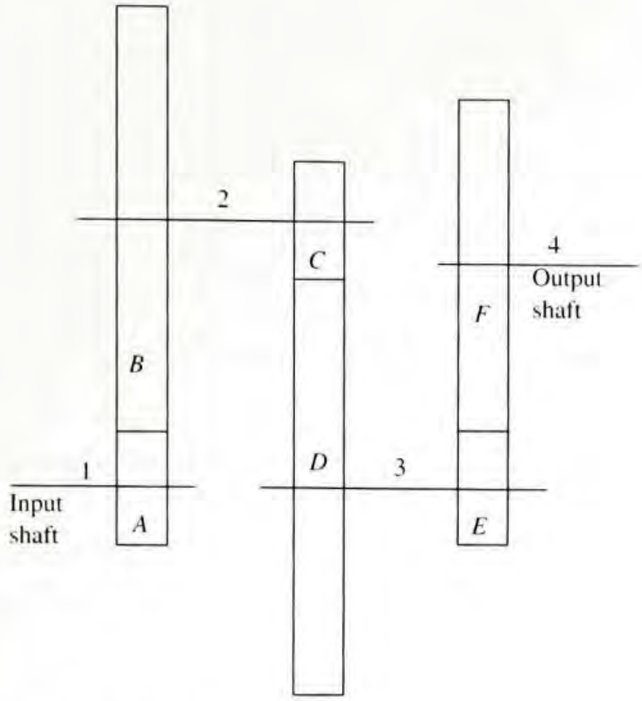
$$TV_{max} = (8.333)^3 = 578 \text{ (okay)}$$

Design a triple-reduction gear train such as that shown in Figure 8–31. The train value is the product of the three individual velocity ratios:

$$TV = (VR_1)(VR_2)(VR_3)$$

If we can find three factors of 320 that are within the range of the possible ratio for a single pair of gears, they can be specified for each velocity ratio.

FIGURE 8–31
Triple-reduction gear train



Factors of 320

One method is to divide by the smallest prime numbers that will divide evenly into the given number, typically 2, 3, 5, or 7. For example,

$$320/2 = 160$$
$$160/2 = 80$$
$$80/2 = 40$$
$$40/2 = 20$$
$$20/2 = 10$$
$$10/2 = 5$$

Then the prime factors of 320 are 2, 2, 2, 2, 2, 2, and 5. We desire a set of three factors, which we can find by combining each set of three "2" factors into their product. That is,

$$(2)(2)(2) = 8$$

Then the three factors of 320 are

$$(8)(8)(5) = 320$$

Now let the number of teeth in the pinion of each pair be 18. The number of teeth in the gears will then be $(8)(18) = 144$ or $(5)(18) = 90$. Finally, we can specify

$$N_A = 18 \qquad N_C = 18 \qquad N_E = 18$$
$$N_B = 144 \qquad N_D = 144 \qquad N_F = 90$$

Residual Ratio

Example Problem 8–13 Devise a gear train for a conveyor drive. The drive motor rotates at 1150 rpm, and it is desired that the output speed for the shaft that drives the conveyor be in the range of 24 to 28 rpm. Use a double-reduction gear train. Power transmission analysis indicates that it would be desirable for the reduction ratio for the first pair of gears to be somewhat greater than that for the second pair.

Solution The start of this problem is similar to that for Example Problem 8–12.

Permissible Train Values
First let's compute the nominal train value that will produce an output speed of 26.0 rpm at the middle of the allowable range:

$$TV_{\text{nom}} = (\text{input speed})/(\text{nominal output speed}) = 1150/26 = 44.23$$

Now we can compute the minimum and maximum allowable speed ratio:

$$TV_{\min} = (\text{input speed})/(\text{maximum output speed}) = 1150/24 = 47.92$$
$$TV_{\max} = (\text{input speed})/(\text{minimum output speed}) = 1150/28 = 41.07$$

Possible Ratio for Single Pair
The maximum ratio that any one pair of gears can produce occurs when the gear has 150 teeth and the pinion has 17 teeth (see Table 8–6). Then

$$VR_{\max} = N_G/N_P = 150/17 = 8.82 \text{ (too low)}$$

Possible Train Value for Double-Reduction Train

$$TV = (VR_1)(VR_2)$$

But the maximum value for either VR is 8.82. Then the maximum train value is

$$TV_{\max} = (8.82)(8.82) = (8.82)^2 = 77.9$$

A double-reduction train is practical.

Optional Designs
The general layout of the proposed train is shown in Figure 8–30. Its train value is

$$TV = (VR_1)(VR_2) = (N_B/N_A)(N_D/N_C)$$

We need to specify the number of teeth in each of the four gears to achieve a train value within the range just computed. Our approach is to specify two ratios, VR_1 and VR_2, such that their product is within the desired range. If the two ratios were equal as before, each would be the square root of the target ratio, 44.23. That is,

$$VR_1 = VR_2 = \sqrt{44.23} = 6.65$$

But we want the first ratio to be somewhat larger than the second. Let's specify

$$VR_1 = 8.0 = (N_B/N_A)$$

If we let pinion A have 17 teeth, the number of teeth in gear B must be

$$N_B = (N_A)(8) = (17)(8) = 136$$

Then the second ratio should be approximately

$$(VR_2) = TV(VR_1) = 44.23/8.0 = 5.53$$

This is the *residual ratio* left after the first ratio has been specified. Now if we specify 17 teeth for pinion C, gear D must be

$$VR_2 = 5.53 = N_D/N_C = N_D/17$$
$$N_D = (5.53)(17) = 94.01$$

Rounding this off to 94 is likely to produce an acceptable result. Finally,

$$N_A = 17 \qquad N_B = 136 \qquad N_C = 17 \qquad N_D = 94$$

We should check the final design:

$$TV = (136/17)(94/17) = 44.235 = n_A/n_D$$

The actual output speed is

$$n_D = n_A/TV = (1150 \text{ rpm})/44.235 = 26.0 \text{ rpm}$$

This is right in the middle of the desired range.

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INTERNET SITES RELATED TO KINEMATICS OF GEARS

1. **American Gear Manufacturers Association (AGMA).** *www.agma.org* Develops and publishes voluntary, consensus standards for gears and gear drives. Some standards are jointly published with the American National Standards Institute (ANSI).
2. **Boston Gear Company.** *www.bostongear.com* A manufacturer of gears and complete gear drives. Part of the Colfax Power Transmission Group. Data provided for spur, helical, bevel, and worm gearing.
3. **Emerson Power Transmission Corporation.** *www.emerson-ept.com* The Browning Division produces spur, helical, bevel, and worm gearing and complete gear drives.
4. **Gear Industry Home Page.** *www.geartechnology.com* Information source for many companies that manufacture or use gears or gearing systems. Includes gear machinery, gear cutting tools, gear materials, gear drives, open gearing, tooling

& supplies, software, training and education. Publishes *Gear Technology Magazine*, *The Journal of Gear Manufacturing*.

5. **Power Transmission Home Page.** *www.powertransmission.com* Clearinghouse on the Internet for buyers, users, and sellers of power transmission-related products and services. Included are gears, gear drives, and gearmotors.

6. **Rockwell Automation/Dodge.** *www.dodge-pt.com* Manufacturer of many power transmission components, including complete gear-type speed reducers, bearings, and components such as belt drives, chain drives, clutches, brakes, and couplings.

PROBLEMS

Gear Geometry

1. A gear has 44 teeth of the 20°, full-depth, involute form and a diametral pitch of 12. Compute the following:

(a) Pitch diameter

(b) Circular pitch

(c) Equivalent module

(d) Nearest standard module

(e) Addendum

(f) Dedendum

(g) Clearance

(h) Whole depth

(i) Working depth

(j) Tooth thickness

(k) Outside diameter

Repeat Problem 1 for the following gears:

2. $N = 34$; $P_d = 24$

3. $N = 45$; $P_d = 2$

4. $N = 18$; $P_d = 8$

5. $N = 22$; $P_d = 1.75$

6. $N = 20$; $P_d = 64$

7. $N = 180$; $P_d = 80$

8. $N = 28$; $P_d = 18$

9. $N = 28$; $P_d = 20$

For Problems 10–17, repeat Problem 1 for the following gears in the metric module system. Replace Part (c) with equivalent P_d and Part (d) with nearest standard P_d.

10. $N = 34$; $m = 3$

11. $N = 45$; $m = 1.25$

12. $N = 18$; $m = 12$

13. $N = 22$; $m = 20$

14. $N = 20$; $m = 1$

15. $N = 180$; $m = 0.4$

16. $N = 28$; $m = 1.5$

17. $N = 28$; $m = 0.8$

18. Define *backlash*, and discuss the methods used to produce it.

19. For the gears of Problems 1 and 12, recommend the amount of backlash.

Velocity Ratio

20. An 8-pitch pinion with 18 teeth mates with a gear having 64 teeth. The pinion rotates at 2450 rpm. Compute the following:

(a) Center distance

(b) Velocity ratio

(c) Speed of gear

(d) Pitch line speed

Repeat Problem 20 for the following data:

21. $P_d = 4$; $N_P = 20$; $N_G = 92$; $n_P = 225$ rpm

22. $P_d = 20$; $N_P = 30$; $N_G = 68$; $n_P = 850$ rpm

23. $P_d = 64$; $N_P = 40$; $N_G = 250$; $n_P = 3450$ rpm

24. $P_d = 12$; $N_P = 24$; $N_G = 88$; $n_P = 1750$ rpm

25. $m = 2$; $N_P = 22$; $N_G = 68$; $n_P = 1750$ rpm

26. $m = 0.8$; $N_P = 18$; $N_G = 48$; $n_P = 1150$ rpm

27. $m = 4$; $N_P = 36$; $N_G = 45$; $n_P = 150$ rpm

28. $m = 12$; $N_P = 15$; $N_G = 36$; $n_P = 480$ rpm

For Problems 29–32, all gears are made in standard 20°, full-depth, involute form. Tell what is wrong with the following statements:

29. An 8-pitch pinion having 24 teeth mates with a 10-pitch gear having 88 teeth. The pinion rotates at 1750 rpm, and the gear at approximately 477 rpm. The center distance is 5.900 in.

30. A 6-pitch pinion having 18 teeth mates with a 6-pitch gear having 82 teeth. The pinion rotates at 1750 rpm, and the gear at approximately 384 rpm. The center distance is 8.3 in.

31. A 20-pitch pinion having 12 teeth mates with a 20-pitch gear having 62 teeth. The pinion rotates at 825 rpm, and the gear at approximately 160 rpm. The center distance is 1.850 in.

32. A 16-pitch pinion having 24 teeth mates with a 16-pitch gear having 45 teeth. The outside diameter of the pinion is 1.625 in. The outside diameter of the gear is 2.938 in. The center distance is 2.281 in.

Housing Dimensions

33. The gear pair described in Problem 20 is to be installed in a rectangular housing. Specify the dimensions X and Y as sketched in Figure P8–33 that would provide a minimum clearance of 0.10 in.
34. Repeat Problem 33 for the data of Problem 23.
35. Repeat Problem 33 for the data of Problem 26, but make the clearance 2.0 mm.
36. Repeat Problem 33 for the data of Problem 27, but make the clearance 2.0 mm.

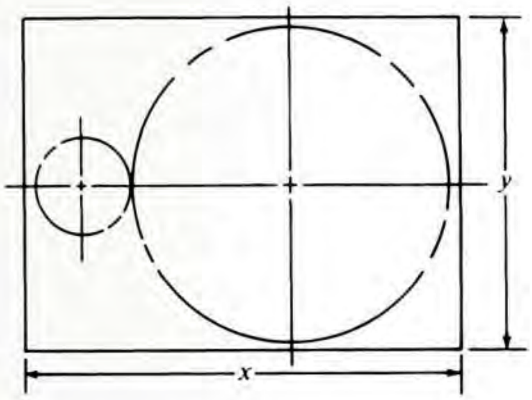


FIGURE P8–33 (Problems 33, 34, 35, and 36)

Analysis of Simple Gear Trains

For the gear trains sketched in the given figures, compute the output speed and the direction of rotation of the output shaft if the input shaft rotates at 1750 rpm clockwise.

37. Use Figure P8–37.

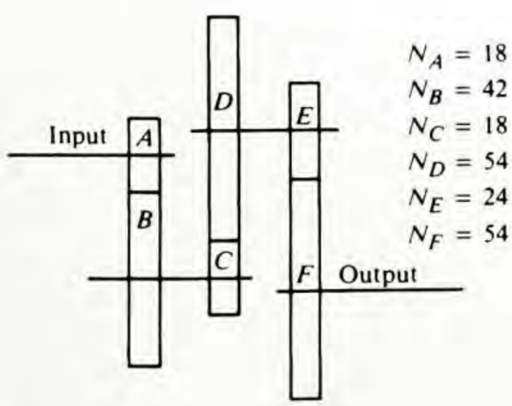


FIGURE P8–37

38. Use Figure P8–38.

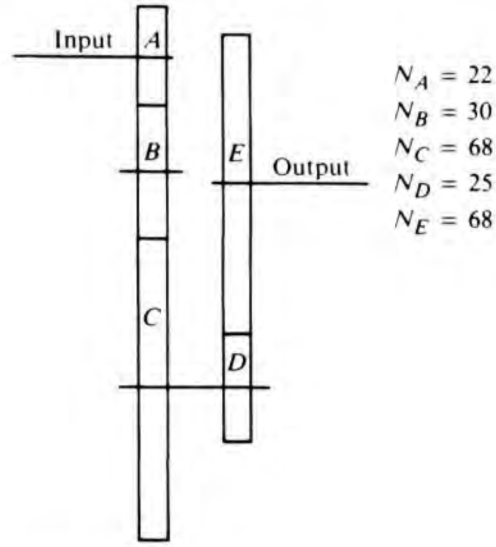


FIGURE P8–38

39. Use Figure P8–39.

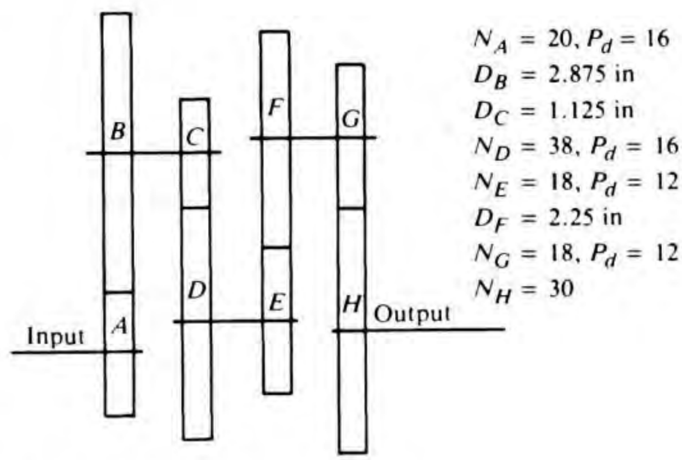


FIGURE P8–39

40. Use Figure P8–40.

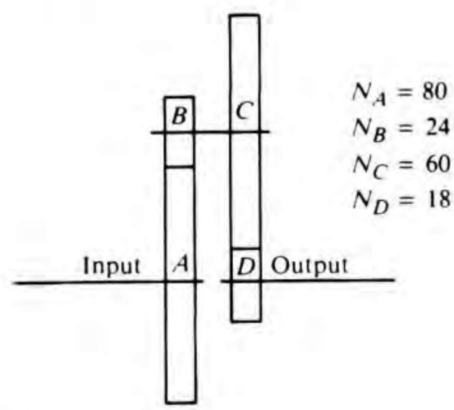


FIGURE P8–40

Helical Gearing

41. A helical gear has a transverse diametral pitch of 8, a transverse pressure angle of $14\frac{1}{2}°$, 45 teeth, a face width of 2.00 in, and a helix angle of 30°. Compute the circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and normal pressure angle. Then compute the number of axial pitches in the face width.

42. A helical gear has a normal diametral pitch of 12, a normal pressure angle of 20°, 48 teeth, a face width of 1.50 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, transverse diametral pitch, axial pitch, pitch diameter, and transverse pressure angle. Then compute the number of axial pitches in the face width.

43. A helical gear has a transverse diametral pitch of 6, a transverse pressure angle of $14\frac{1}{2}°$, 36 teeth, a face width of 1.00 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and normal pressure angle. Then compute the number of axial pitches in the face width.

44. A helical gear has a normal diametral pitch of 24, a normal pressure angle of $14\frac{1}{2}°$, 72 teeth, a face width of 0.25 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, transverse diametral pitch, axial pitch, pitch diameter, and transverse pressure angle. Then compute the number of axial pitches in the face width.

Bevel Gears

45. A straight bevel gear pair has the following data: $N_P = 15$; $N_G = 45$; $P_d = 6$; 20° pressure angle. Compute all of the geometric features from Table 8–7.

46. Draw the gear pair of Problem 45 to scale. The following additional dimensions are given (refer to Figure 8–22). Mounting distance (M_{dP}) for the pinion = 5.250 in; M_{dG} for the gear = 3.000 in; face width = 1.250 in. Supply any other needed dimensions.

47. A straight bevel gear pair has the following data: $N_P = 25$; $N_G = 50$; $P_d = 10$; 20° pressure angle. Compute all of the geometric features from Table 8–7.

48. Draw the gear pair of Problem 47 to scale. The following additional dimensions are given (refer to Figure 8–22). Mounting distance (M_{dP}) for the pinion = 3.375 in; M_{dG} for the gear = 2.625 in; face width = 0.700 in. Supply any other needed dimensions.

49. A straight bevel gear pair has the following data: $N_P = 18$; $N_G = 72$; $P_d = 12$; 20° pressure angle. Compute all of the geometric features from Table 8–7.

50. A straight bevel gear pair has the following data: $N_P = 16$; $N_G = 64$; $P_d = 32$; 20° pressure angle. Compute all of the geometric features from Table 8–7.

51. A straight bevel gear pair has the following data: $N_P = 12$; $N_G = 36$; $P_d = 48$; 20° pressure angle. Compute all of the geometric features from Table 8–7.

Wormgearing

52. A wormgear set has a single-thread worm with a pitch diameter of 1.250 in, a diametral pitch of 10, and a normal pressure angle of 14.5°. If the worm meshes with a wormgear having 40 teeth and a face width of 0.625 in, compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, worm root diameter, gear pitch diameter, center distance, and velocity ratio.

53. Three designs are being considered for a wormgear set to produce a velocity ratio of 20 when the wormgear rotates at 90 rpm. All three have a diametral pitch of 12, a worm pitch diameter of 1.000 in, a gear face width of 0.500 in, and a normal pressure angle of 14.5°. One has a single-thread worm and 20 wormgear teeth; the second has a double-thread worm and 40 wormgear teeth; the third has a four-thread worm and 80 wormgear teeth. For each design, compute the lead, axial pitch, circular pitch, lead angle, gear pitch diameter, and center distance.

54. A wormgear set has a double-threaded worm with a normal pressure angle of 20°, a pitch diameter of 0.625 in, and a diametral pitch of 16. Its mating wormgear has 100 teeth and a face width of 0.3125 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.

55. A wormgear set has a four-threaded worm with a normal pressure angle of $14\frac{1}{2}°$, a pitch diameter of 2.000 in, and a diametral pitch of 6. Its mating wormgear has 72 teeth and a face width of 1.000 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.

56. A wormgear set has a single-threaded worm with a normal pressure angle of $14\frac{1}{2}°$, a pitch diameter of 4.000 in, and a diametral pitch of 3. Its mating wormgear has 54 teeth and a face width of 2.000 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.

57. A wormgear set has a four-threaded worm with a normal pressure angle of 25°, a pitch diameter of 0.333 in, and a diametral pitch of 48. Its mating wormgear has 80 teeth and a face width of 0.156 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.

Analysis of Complex Gear Trains

58. The input shaft for the gear train shown in Figure P8–58 rotates at 3450 rpm. Compute the rotational speed of the output shaft.

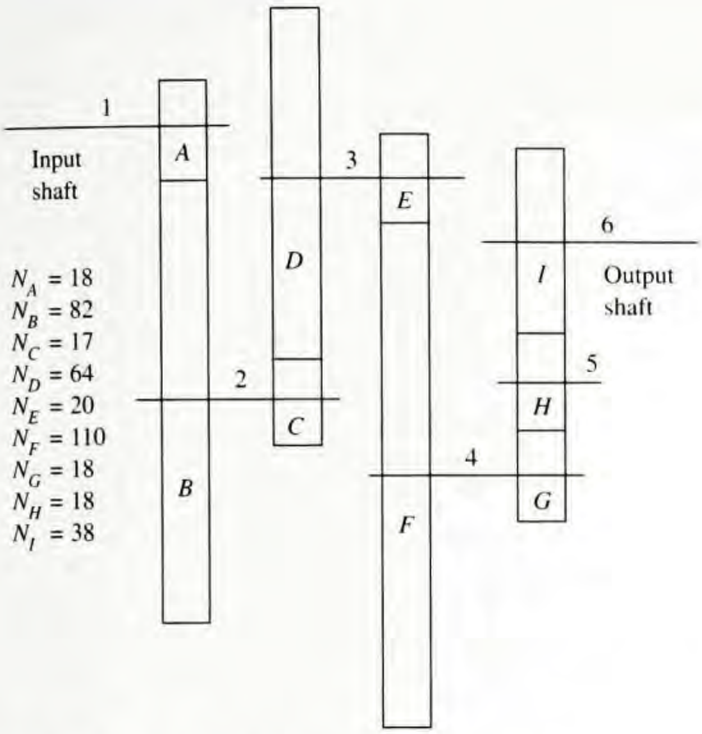


FIGURE P8–58 Gear train for Problem 58

59. The input shaft for the gear train shown in Figure P8–59 rotates at 12 200 rpm. Compute the rotational speed of the output shaft.

60. The input shaft for the gear train shown in Figure P8–60 rotates at 6840 rpm. Compute the rotational speed of the output shaft.

61. The input shaft for the gear train shown in Figure P8–61 rotates at 2875 rpm. Compute the rotational speed of the output shaft.

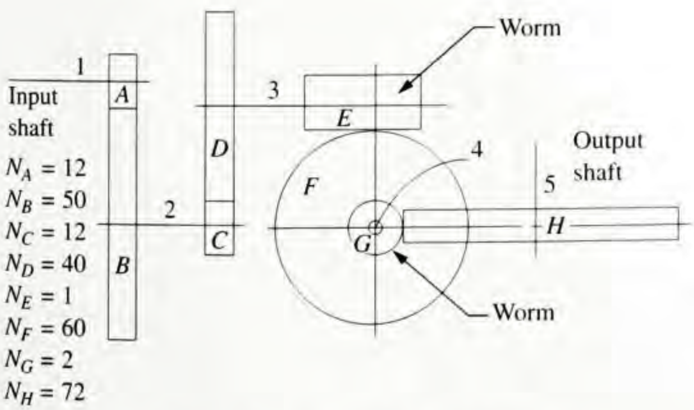


FIGURE P8–59 Gear train for Problem 59

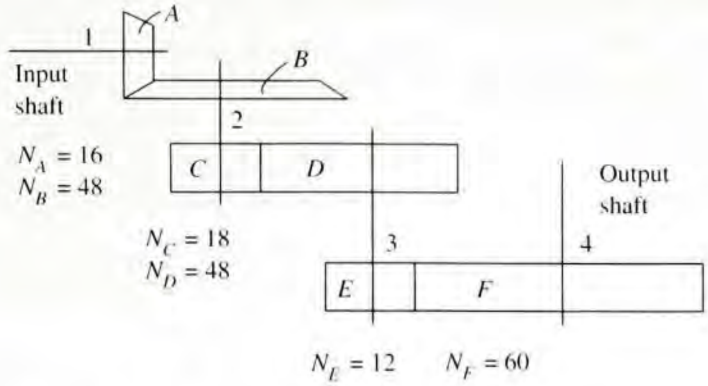


FIGURE P8–60 Gear train for Problem 60

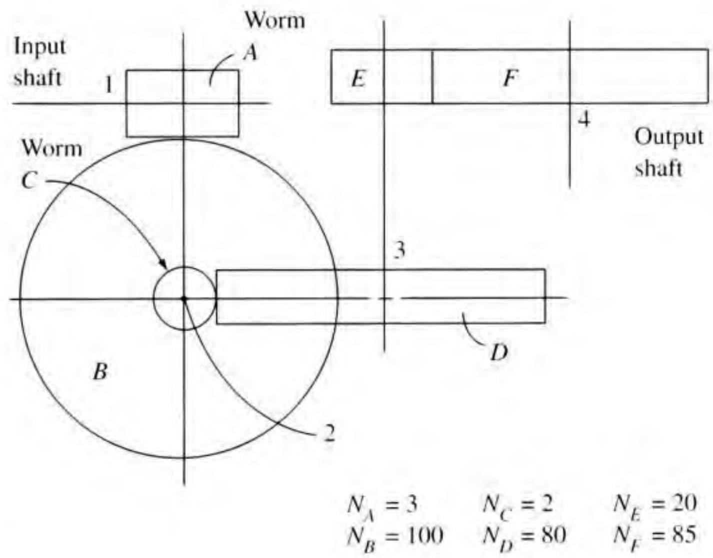


FIGURE P8–61 Gear train for Problem 61

Kinematic Design of a Single Gear Pair

62. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of π as closely as possible. Use no fewer than 16 teeth nor more than 24 teeth in the pinion.

63. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of $\sqrt{3}$ as closely as possible. Use no fewer than 16 teeth nor more than 24 teeth in the pinion.

64. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of $\sqrt{38}$ as closely as possible. Use no fewer than 18 teeth nor more than 24 teeth in the pinion.

65. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of 7.42 as closely as possible. Use no fewer than 18 teeth nor more than 24 teeth in the pinion.

Kinematic Design of Gear Trains

For Problems 66–75, devise a gear train using all external gears on parallel shafts. Use 20 full-depth involute teeth and no more than 150 teeth in any gear. Ensure that there is no interference. Sketch the layout for your design.

Problem no.	Input speed (rpm)	Output speed range (rpm)
66.	1800	2.0 Exactly
67.	1800	21.0 to 22.0
68.	3360	12.0 Exactly
69.	4200	13.0 to 13.5
70.	5500	221 to 225
71.	5500	13.0 to 14.0
72.	1750	146 to 150
73.	850	40.0 to 44.0
74.	3000	548 to 552 Use two pairs
75.	3600	3.0 to 5.0

For Problems 76–80, devise a gear train using any type of gears. Try for the minimum number of gears while avoiding interference and having no more that 150 teeth in any gear. Sketch your design.

Problem no.	Input speed (rpm)	Output speed (rpm)
76.	3600	3.0 to 5.0
77.	1800	8.0 Exactly
78.	3360	12.0 Exactly
79.	4200	13.0 to 13.5
80.	5500	13.0 to 14.0

9

Spur Gear Design

The Big Picture

Spur Gear Design

Discussion Map

- ☐ A spur gear has involute teeth that are straight and parallel to the axis of the shaft that carries the gear.

Discover

Describe the action of the teeth of the driving gear on those of the driven gear. What kinds of stresses are produced?

How do the geometry of the gear teeth, the materials from which they are made, and the operating conditions affect the stresses and the life of the drive system?

This chapter will help you acquire the skills to perform the necessary analyses and to design safe spur gear drive systems that demonstrate long life.

A *spur gear* is one of the most fundamental types of gears. Its teeth are straight and parallel to the axis of the shaft that carries the gear. The teeth have the involute form described in Chapter 8. So, in general, the action of one tooth on a mating tooth is like that of two convex, curved members in contact: As the driving gear rotates, its teeth exert a force on the mating gear that is tangential to the pitch circles of the two gears. Because this force acts at a distance equal to the pitch radius of the gear, a torque is developed in the shaft that carries the gear. When the two gears rotate, they transmit power that is proportional to the torque. Indeed, that is the primary purpose of the spur gear drive system.

Consider the action described in the preceding paragraph:

- How does that action relate to the design of the gear teeth? Look back at Figure 8–1 in Chapter 8 as you consider this question and those that follow.
- As the force is exerted by the driving tooth on the driven tooth, what kinds of stresses are produced in the teeth? Consider both the point of contact of one tooth on the other and the whole tooth. Where are stresses a maximum?
- How could the teeth fail under the influence of these stresses?
- What material properties are critical to allow the gears to carry such loads safely and with a reasonable life span?
- What important geometric features affect the level of stress produced in the teeth?
- How does the precision of the tooth geometry affect its operation?
- How does the nature of the application affect the gears? What if the machine that the gears drive is a rock crusher that takes large boulders and reduces them to gravel made up of small stones? How would that loading compare with that of a gear system that drives a fan providing ventilation air to a building?
- What is the influence of the driving machine? Would the design be different if an electric motor were the driver or if a gasoline engine were used?
- The gears are typically mounted on shafts that deliver power from the driver to the input gear of a gear train and that take power from the output gear and transmit it to the driven machine. Describe various ways that the gears can be attached to the shafts and located with respect to each other. How can the shafts be supported?

This chapter contains the kinds of information that you can use to answer such questions and to complete the analysis and design of spur gear power transmission systems.

Later chapters cover similar topics for helical gears, bevel gears, and wormgearing, along with the design and specification of keys, couplings, seals, shafts, and bearings—all of which are needed to design a complete mechanical drive.

You Are the Designer

You have already made the design decision that a spur gear type of speed reducer is to be used for a particular application. How do you complete the design of the gears themselves?

This is a continuation of a design scenario that was started in Chapter 1 of this book when the original goals were stated and when an overview of the entire book was given. The introduction to Part II continued this theme by indicating that the arrangement of the chapters is aligned with the design process that you could use to complete the design of the speed reducer.

Then in Chapter 8, you, as the designer, dealt with the kinematics of a gear reducer that would take power from the shaft of an electric motor rotating at 1750 rpm and deliver it to a machine that was to operate at approximately 292 rpm. There you limited your interest to the decisions that affected motion and the basic geometry of the gears. It was decided that you would use a double-reduction gear train to reduce the speed of rotation of the drive system in two stages using two pairs of gears in series. You also learned how to specify the layout of the gear train, along with key design decisions such as the numbers of teeth in all of the gears and the relationships among the diametral pitch, the number of teeth in the gears, the pitch diameters, and the distance between the centers of the shafts that carry those gears. For a chosen diametral pitch, you learned how to compute the dimensions of key features of the gear teeth such as the addendum, dedendum, and tooth width.

But the design is not complete until you have specified the material from which the gears are to be made and until you have verified that the gears will withstand the forces exerted on the gears as they transmit power and the corresponding torque. The teeth must not break, and they must have a sufficiently long life to meet the needs of the customer who uses the reducer.

To complete the design, you need more data: How much power is to be transmitted? To what kind of machine is the power from the output of the reducer being delivered? How does that affect the design of the gears? What is the anticipated duty cycle for the reducer in terms of the number of hours per day, days per week, and years of life expected? What options do you have for materials that are suitable for gears? Which material will you specify, and what will be its heat treatment?

You are the designer. The information in this chapter will help you complete the design.

9–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to demonstrate the competencies listed below. They are presented in the order that they are covered in this chapter. The primary objectives are numbers 6, 7, and 8, which involve (a) the calculation of the bending strength and the ability of the gear teeth to resist pitting and (b) the design of gears to be safe with regard to both strength and pitting resistance. The competencies are as follows:

1. Compute the forces exerted on gear teeth as they rotate and transmit power.
2. Describe various methods for manufacturing gears and the levels of precision and quality to which they can be produced.
3. Specify a suitable level of quality for gears according to the use to which they are to be put.
4. Describe suitable metallic materials from which to make the gears, in order to provide adequate performance for both strength and pitting resistance.
5. Use the standards of the American Gear Manufacturers Association (AGMA) as the basis for completing the design of the gears.
6. Use appropriate stress analyses to determine the relationships among the applied forces, the geometry of the gear teeth, the precision of the gear teeth, and other factors specific to a given application, in order to make final decisions about those variables.

7. Perform the analysis of the tendency for the contact stresses exerted on the surfaces of the teeth to cause pitting of the teeth, in order to determine an adequate hardness of the gear material that will provide an acceptable level of pitting resistance for the reducer.
8. Complete the design of the gears, taking into consideration both the stress analysis and the analysis of pitting resistance. The result will be a complete specification of the gear geometry, the material for the gear, and the heat treatment of the material.

9–2 CONCEPTS FROM PREVIOUS CHAPTERS

As you study this chapter, it is assumed that you are familiar with the geometry of gear features and the kinematics of one gear driving another as presented in Chapter 8 and illustrated in Figures 8–1, 8–8, 8–11, 8–12, 8–13, and 8–15. (See also References 4 and 23.) Key relationships that you should be able to use include the following:

$$\text{Pitch line speed} = v_t = R\omega = (D/2)\omega$$

where R = radius of the pitch circle
D = pitch diameter
ω = angular velocity of the gear

Because the pitch line speed is the same for both the pinion and the gear, values for R, D, and can be for either. In the computation of stresses in gear teeth, it is usual to express the pitch line speed in the units of ft/min, while the size of the gear is given as its pitch diameter expressed in inches. Speed of rotation is typically given as n rpm—that is, n rev/min. Let's compute the unit-specific equation that gives pitch line speed in ft/min:

Pitch Line Speed

$$v_t = (D/2)\omega = \frac{D\text{ in}}{2} \cdot \frac{n\text{ rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{\text{rev}} \cdot \frac{1\text{ ft}}{12\text{ in}} = (\pi D n/12)\text{ ft/min} \qquad \textbf{(9–1)}$$

The velocity ratio can be expressed in many ways. For the particular case of a pinion driving a larger gear,

Velocity Ratio

$$\text{Velocity ratio} = VR = \frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = \frac{R_G}{R_P} = \frac{D_G}{D_P} = \frac{N_G}{N_P} \qquad \textbf{(9–2)}$$

A related ratio, m_G, called the *gear ratio*, is often used in analysis of the performance of gears. It is always defined as the ratio of the number of teeth in the larger gear to the number of teeth in the pinion, regardless of which is the driver. Thus, m_G is always greater than or equal to 1.0. When the pinion is the driver, as it is for a speed reducer, m_G is equal to VR. That is,

Gear Ratio

$$\text{Gear ratio} = m_G = N_G/N_P \geq 1.0 \qquad \textbf{(9–3)}$$

The diametral pitch, P_d, characterizes the physical size of the teeth of a gear. It is related to the pitch diameter and the number of teeth as follows:

Diametral Pitch

$$P_d = N_G/D_G = N_P/D_P \qquad \textbf{(9–4)}$$

The pressure angle, ϕ, is an important feature that characterizes the form of the involute curve that makes up the active face of the teeth of standard gears. See Figure 8–13. Also notice in Figure 8–12 that the angle between a normal to the involute curve and the tangent to the pitch circle of a gear is equal to the pressure angle.

9–3 FORCES, TORQUE, AND POWER IN GEARING

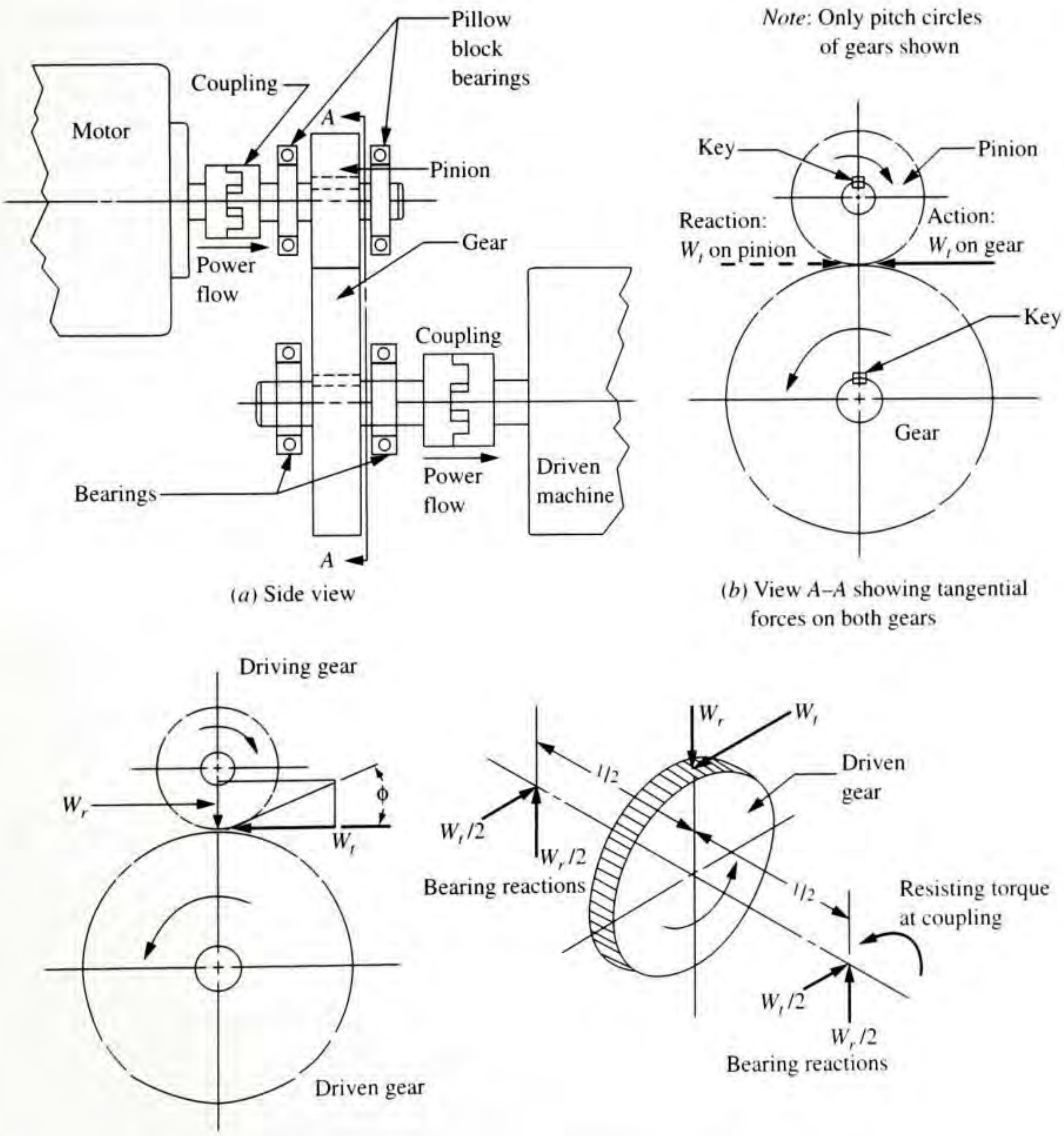
Torque

MDESIGN

To understand the method of computing stresses in gear teeth, consider the way power is transmitted by a gear system. For the simple single-reduction gear pair shown in Figure 9–1, power is received from the motor by the input shaft rotating at motor speed. Thus, torque in the shaft can be computed from the following equation:

$$\text{Torque} = \text{power/rotational speed} = P/n \tag{9–5}$$

The input shaft transmits the power from the coupling to the point where the pinion is mounted. The power is transmitted from the shaft to the pinion through the key. The teeth of the pinion drive the teeth of the gear and thus transmit the power to the gear. But again, power transmission actually involves the application of a torque during rotation at a given speed. The torque is the product of the force acting tangent to the pitch circle of the pinion times the pitch radius of the pinion. We will use the symbol W_t to indicate the *tangential force*. As described,



(*a*) Side view

(*b*) View *A–A* showing tangential forces on both gears

(*c*) View A–A showing tangential and radial forces on the driven gear

(*d*) Forces on a shaft carrying a spur gear

FIGURE 9–1 Power flow through a gear pair

W_t is the force exerted *by the pinion teeth on the gear teeth*. But if the gears are rotating at a constant speed and are transmitting a uniform level of power, the system is in equilibrium. Therefore, there must be an equal and opposite tangential force exerted by the gear teeth back on the pinion teeth. This is an application of the principle of action and reaction.

To complete the description of the power flow, the tangential force on the gear teeth produces a torque on the gear equal to the product of W_t times the pitch radius of the gear. Because W_t is the same on the pinion and the gear, but the pitch radius of the gear is larger than that of the pinion, the torque on the gear (the output torque) is greater than the input torque. However, note that the power transmitted is the same or slightly less because of mechanical inefficiencies. The power then flows from the gear through the key to the output shaft and finally to the driven machine.

From this description of power flow, we can see that gears transmit power by exerting a force by the driving teeth on the driven teeth while the reaction force acts back on the teeth of the driving gear. Figure 9–2 shows a single gear tooth with the tangential force W_t acting on it. But this is not the total force on the tooth. Because of the involute form of the tooth, the total force transferred from one tooth to the mating tooth acts normal to the involute profile. This action is shown as W_n. The tangential force W_t is actually the horizontal component of the total force. To complete the picture, note that there is a vertical component of the total force acting radially on the gear tooth, indicated by W_r.

We will start the computation of forces with the transmitted force, W_t, because its value is based on the given data for power and speed. It is convenient to develop unit-specific equations for W_t because standard practice typically calls for the following units for key quantities pertinent to the analysis of gear sets:

Forces in pounds (lb)

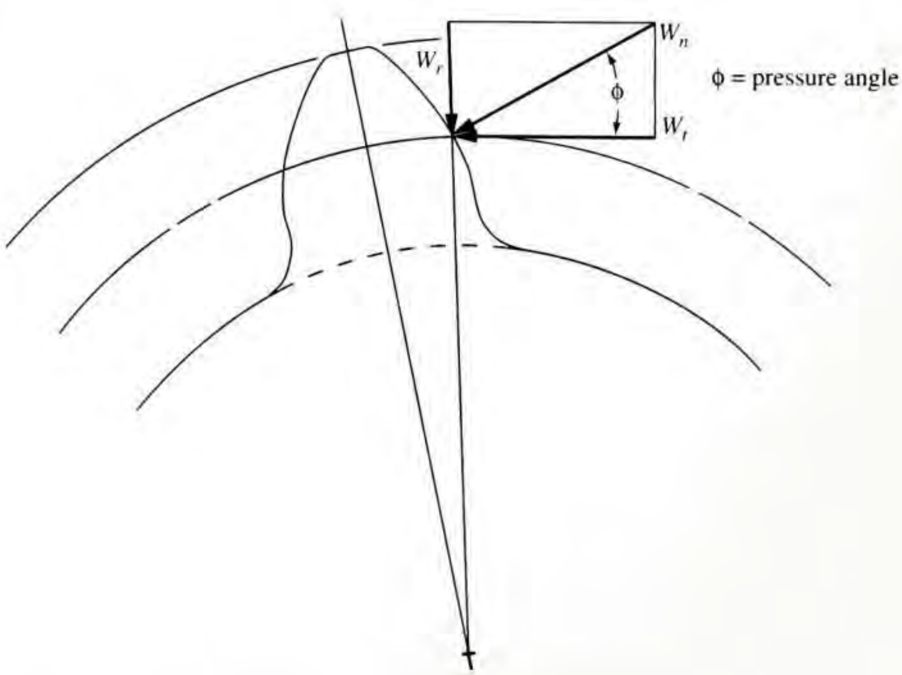
Power in horsepower (hp) (Note that 1.0 hp = 550 lb · ft/s.)

Rotational speed in rpm, that is, rev/min

Pitch line speed in ft/min

Torque in lb · in

FIGURE 9–2 Forces on gear teeth



The torque exerted on a gear is the product of the transmitted load, W_t, and the pitch radius of the gear. The torque is also equal to the power transmitted divided by the rotational speed. Then

$$T = W_t(R) = W_t(D/2) = P/n$$

Then we can solve for the force, and the units can be adjusted as follows:

$$W_t = \frac{2P}{Dn} = \frac{2P(\text{hp})}{D(\text{in})\cdot \text{n (rev/min)}} \cdot \frac{550\ \text{lb}\cdot\text{ft/s}}{(\text{hp})} \cdot \frac{1.0\ \text{rev}}{2\pi\ \text{rad}} \cdot \frac{60\ \text{s/min}}{} \cdot \frac{12\ \text{in}}{\text{ft}}$$

Tangential Force

$$W_t = (126\,000)(P)/(n\,D)\ \text{lb} \tag{9–6}$$

Data for either the pinion or the gear can be used in this equation. Other relationships are now developed because they are needed in other parts of the process of analyzing the gears or the shafts that carry them.

Power is also the product of transmitted force, W_t, and the pitch line velocity:

$$P = W_t \cdot v_t$$

Then, solving for the force and adjusting units, we have

Tangential Force

$$W_t = \frac{P}{v_t} = \frac{P\ (\text{hp})}{v_t\ (\text{ft/min})} \cdot \frac{550\ \text{lb}\cdot\text{ft/s}}{1.0\ \text{hp}} \cdot \frac{60\text{s/min}}{} = 33\,000\ (P)/(v_t)\ \text{lb} \tag{9–7}$$

We may also need to compute torque in lb · in:

$$T = \frac{P}{\omega} = \frac{P\ (\text{hp})}{n\ (\text{rev/min})} \cdot \frac{550\ \text{lb}\cdot\text{ft/s}}{1.0\ \text{hp}} \cdot \frac{1.0\ \text{rev}}{2\pi\ \text{rad}} \cdot \frac{60\ \text{s/min}}{} \cdot \frac{12\ \text{in}}{\text{ft}}$$

Torque

$$T = 63\,000\ (P)/n\ \text{lb}\cdot\text{in} \tag{9–8}$$

These values can be computed for either the pinion or the gear by appropriate substitutions. Remember that the pitch line speed is the same for the pinion and the gear and that the transmitted loads on the pinion and the gear are the same, except that they act in opposite directions.

The normal force, W_n, and the radial force, W_r, can be computed from the known W_t by using the right triangle relations evident in Figure 9–2:

Radial Force

$$W_r = W_t \tan\phi \tag{9–9}$$

Normal Force

$$W_n = W_t/\cos\phi \tag{9–10}$$

where ϕ = pressure angle of the tooth form

In addition to causing the stresses in the gear teeth, these forces act on the shaft. In order to maintain equilibrium, the bearings that support the shaft must provide the reactions. Figure 9–1(d) shows the free-body diagram of the output shaft of the reducer.

Power Flow and Efficiency

The discussion thus far has focused on power, torque, and forces for a single pair of gears. For compound gearing having two or more pairs of gears, the flow of power and the overall efficiency become increasingly important.

Power losses in gear drives made from spur, helical, and bevel gears depend on the action of each tooth on its mating tooth, a combination of rolling and sliding. For accurate, well-lubricated gears, the power loss ranges from 0.5% to 2.0% and is typically taken to be approximately 1.0% (See Reference 26). *Because this is quite small, it is customary to neglect it in sizing individual gear pairs; we do that in this book.*

Compound gear drives employ several pairs of gears in series to produce large speed reduction ratios. With 1.0% power loss in each pair, the accumulated power loss for the system can become significant, and it can affect the size of motor to drive the system or the ultimate power and torque available for use at the output. Furthermore, the power loss is transferred to the environment or into the gear lubricant and, for large power transmissions, the management of the heat generated is critical to the overall performance of the unit. The viscosity and load-carrying ability of lubricants is degraded with increasing temperature.

Tracking power flow in a simple or compund gear train is simple, the power is transferred from one gear pair to the next with only a small power loss at each mesh. More complex designs may split the power flow at some point to two or more paths. This is typical of planetary gear trains. In such cases, you should consider the basic relationship among power, torque, and rotational speed shown in Equation (9–5), $P = T \times n$. We can present this in another form. Let the rotational speed, n, that is typically taken to be in the units of rpm, be the more general term *angular velocity*, ω, in the units of rad/s. Now express the torque in terms of the transmitted forces, W_t, and the pitch radius of the gear, R. That is, $T = W_tR$. Equation (9–5) then becomes,

$$P = T \times n = W_t R \omega$$

But $R\,\omega$ is the pitch line velocity for the gears, v_t. Then,

$$P = W_t\, R\, \omega = W_t\, v_t$$

Knowing how the power splits enables the determination of the transmitted load at each mesh.

9–4 GEAR MANUFACTURE

The discussion of gear manufacture will begin with the method of producing the gear blank. Small gears are frequently made from wrought plate or bar, with the hub, web, spokes, and rim machined to final or near-final dimensions before the gear teeth are produced. The face width and the outside diameter of the gear teeth are also produced at this stage. Other gear blanks may be forged, sand cast, or die cast to achieve the basic form prior to machining. A few gears in which only moderate precision is required may be die cast with the teeth in virtually final form.

Large gears are frequently fabricated from components. The rim and the portion into which the teeth are machined may be rolled into a ring shape from a flat bar and then welded. The web or spokes and the hub are then welded inside the ring. Very large gears may be made in segments with the final assembly of the segments by welding or by mechanical fasteners.

The popular methods of machining the gear teeth are form milling, shaping, and hobbing. (See References 23 and 25.)

FIGURE 9–3 A variety of gear cutting tools (Gleason Cutting Tools Corporation, Loves Park, IL)

(a) Form milling cutter

(b) Spur gear shaper cutter

(c) Hob for small pitch gears having large teeth

(d) Hob for high pitch gears having small teeth

In *form milling* [Figure 9–3(a)], a milling cutter that has the shape of the tooth space is used, and each space is cut completely before the gear blank is indexed to the position of the next adjacent space. This method is used mostly for large gears, and great care is required to achieve accurate results.

Shaping [Figures 9–3(b) and 9–4] is a process in which the cutter reciprocates, usually on a vertical spindle. The shaping cutter rotates as it reciprocates and is fed into the gear blank. Thus, the involute-tooth form is generated gradually. This process is frequently used for internal gears.

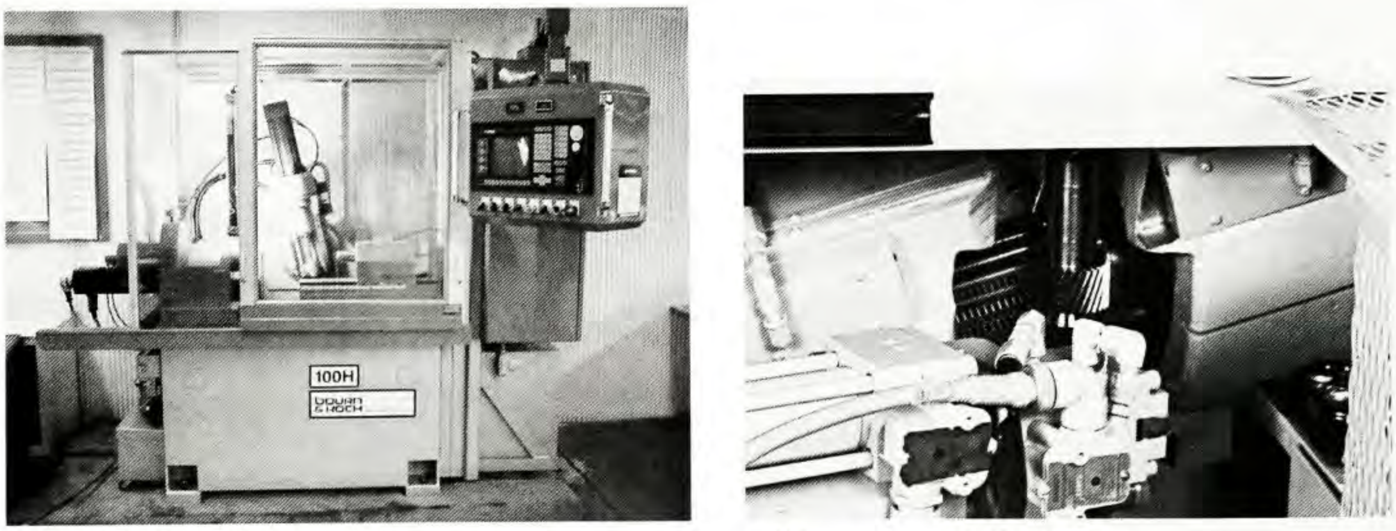
Hobbing [Figures 9–3(c) and (d) and 9–5] is a process similar to milling except that the workpiece (the gear blank) and the cutter (the hob) rotate in a coordinated fashion. Here also, the tooth form is generated gradually as the hob is fed into the blank.

The gear teeth are finished to greater precision after form milling, shaping, or hobbing by the processes of grinding, shaving, and honing. Being products of secondary processes, they are expensive and should be used only where the operation requires high accuracy in the tooth form and spacing. Figure 9–6 shows a gear grinding machine.

(a) Shaping small external gear

(b) Shaping large internal gear

FIGURE 9–4 Gear shaping operations (Bourn & Koch, Inc., Rockford, IL)



(a) Gear hobbing machine

(b) Close-up view of hobbing process

FIGURE 9–5 CNC 4-axis fear hobber close-up of gear hobbing process (Bourn & Koch, Inc., Rockford, IL)

9–5 GEAR QUALITY

Quality in gearing is the precision of specific features of a single gear or the composite error of a gear rotating in mesh with a precise master gear. The factors typically measured to determine quality are:

Index variation: The difference between the actual location of a point on the face of a gear tooth at the pitch circle and the corresponding point of a reference tooth, measured on the pitch circle. The variation causes inaccuracy in the action of mating gear teeth.

(a) Gear grinding machine (b) Close-up view of grinding process

FIGURE 9–6 CNC gear grinder and close-up of gear grinding setup (Bourn & Koch, Inc., Rockford, IL)

Tooth alignment: The deviation of the actual line on the gear tooth surface at the pitch circle from the theoretical line. Measurements are made across the face from one end to the other. For a spur gear the theoretical line is straight. For a helical gear it is a part of a helix. Measurement of tooth alignment is sometimes called the *helix* measurement. It is important because excessive misalignment causes nonuniform loading on the gear teeth.

Tooth profile: The measurement of the actual profile of the surface of a gear tooth from the point of the start of the active profile to the tip of the tooth. The theoretical profile is a true involute curve. Variations of the actual profile from the theoretical profile cause variations in the instantaneous velocity ratio between the two gears in mesh, affecting the smoothness of the motion.

Root radius: The radius of the fillet at the base of the tooth. Variations from the theoretical value can affect the meshing of mating gears, creating possible interference, and the stress concentration factors related to bending stress in the tooth.

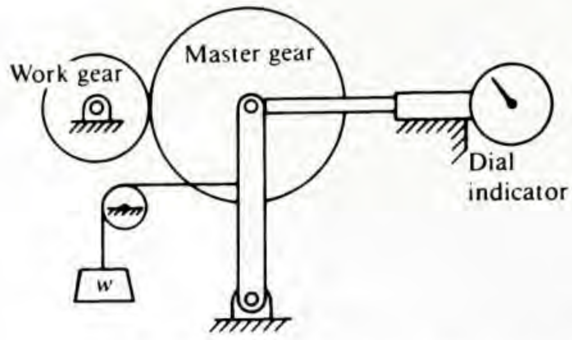
Runout: A measure of the eccentricity and out-of-roundness of a gear. Excessive runout causes the contact point on mating gear teeth to move radially during each revolution.

Total composite variation: A measure of the variation in the center distance between a precise master gear and the test gear for a full revolution. The shaft of one gear is fixed and the shaft of the mating gear is permitted to move while the teeth are kept in tight mesh. Figure 9–7(a) shows a sketch of one arrangement.

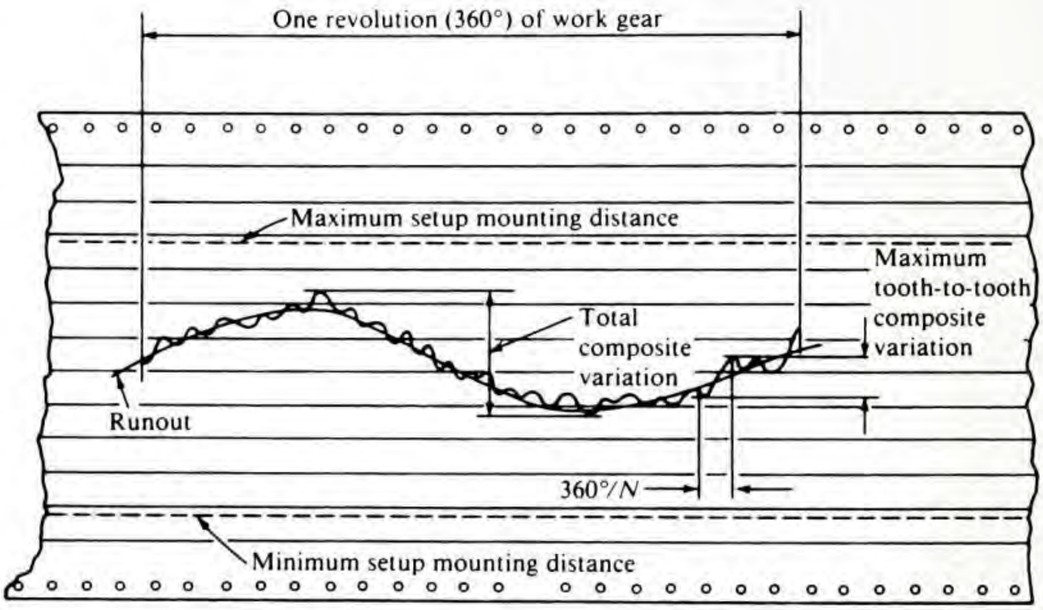
Standards for Gear Quality

The allowable amounts of variations of the actual tooth form from the theoretical form, or the composite variation, are specified by the AGMA as a *quality number.* Detailed charts giving the tolerances for many features are included in AGMA Standard 2000-A88 *Gear Classification and Inspection Handbook, Tolerances and Measuring Methods for Unassembled Spur and Helical Gears.* The quality numbers range from 5 to 15 with increasing precision.

FIGURE 9–7
Recording of errors in gear geometry (Extracted from AGMA Standard 2000-A88, *Gear Classification and Inspection Handbook, Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



(*a*) Schematic diagram of a typical gear rolling fixture



(*b*) Chart of gear-tooth errors of a typical gear when run with a specified gear in a rolling fixture

The actual tolerances are a function of the quality number, diametral pitch of the gear teeth, and the number of teeth of the gear. Table 9–1 shows representative data for the total composite tolerance for several quality numbers.

The International Organization for Standardization (ISO) defines a different set of quality numbers in its Standard 1328-1-1995, *Cylindrical gears–ISO system of accuracy–Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth* and Standard 1328-2-1997 *Cylindrical gears–ISO system of accuracy–Part 2: Definitions and allowable values of deviations relevant to radical composite deviations and runout information.* These standards differ greatly from the AGMA Standard 2000-A88. One major difference is that the quality numbering system is reversed. Whereas in the AGMA Standard 2008, higher numbers indicate greater precision, in the ISO standard lower numbers indicate greater precision.

The AGMA released two new standards just prior to the time this book was prepared. AGMA 2015-1-A01 *Accuracy Classification System–Tangential Measurements for Cylindrical Gears* applies a system in which lower grade numbers indicate lower tolerance values, similar but not identical to the ISO method. Comparisons are shown in this standard among the new AGMA 2015, the former AGMA 2008, and ISO 1328. AGMA 915-1-A02 *Inspection Practices–Part 1: Cylindrical Gears–Tangential Measurements,* deals with im-

TABLE 9–1 Selected values for total composite tolerance

AGMA quality number	Diametral pitch, P_d	Number of gear teeth 20	40	60	100	200
Q5	2	0.0260	0.0290	0.0320	0.0350	0.0410
	8	0.0120	0.0130	0.0140	0.0150	0.0170
	20	0.0074	0.0080	0.0085	0.0092	0.0100
	32	0.0060	0.0064	0.0068	0.0073	0.0080
Q8	2	0.0094	0.0110	0.0120	0.0130	0.0150
	8	0.0043	0.0047	0.0050	0.0055	0.0062
	20	0.0027	0.0029	0.0031	0.0034	0.0037
	32	0.0022	0.0023	0.0025	0.0027	0.0029
Q10	2	0.0048	0.0054	0.0059	0.0066	0.0076
	8	0.0022	0.0024	0.0026	0.0028	0.0032
	20	0.0014	0.0015	0.0016	0.0017	0.0019
	32	0.0011	0.0012	0.0013	0.0014	0.0015
Q12	2	0.0025	0.0028	0.0030	0.0034	0.0039
	8	0.0011	0.0012	0.0013	0.0014	0.0016
	20	0.000 71	0.000 77	0.000 81	0.000 87	0.000 97
	32	0.000 57	0.000 60	0.000 64	0.000 69	0.000 76
Q14	2	0.0013	0.0014	0.0015	0.0017	0.0020
	8	0.000 57	0.000 62	0.000 67	0.000 73	0.000 82
	20	0.000 36	0.000 39	0.000 41	0.000 45	0.000 50
	32	0.000 29	0.000 31	0.000 33	0.000 35	0.000 39

Source: Extracted from AGMA Standard 2000-A88, *Gear Classification and Inspection Handbook, Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

plementing the new grades. While the quality grades are not precisely the same, a rough set of equivalents follows.

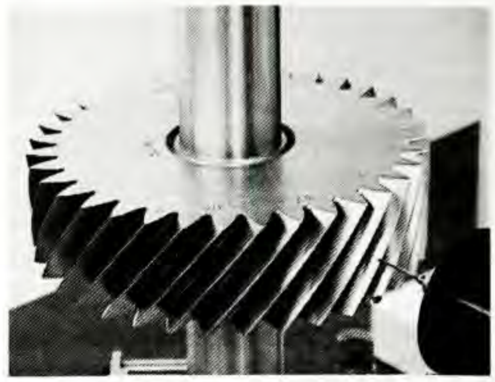
AGMA 2008	*AGMA 2015*	*ISO 1328*	*AGMA 2008*	*AGMA 2015*	*ISO 1328*
Q5	—	12	Q11	A6	6
Q6	A11	11	Q12	A5	5
Q7	A10	10	Q13	A4	4
Q8	A9	9	Q14	A3	3
Q9	A8	8	Q15	A2	2
Q10	A7	7		(Most precise)	

Note that the sum of the quality number from AGMA 2008 and the corresponding accuracy classification number from AGMA 2015 or ISO 1328 is always 17.

Some European manufacturers employ standards of the German DIN (Deutsche Industrie Normen) system whose quality numbers are similar to those of the ISO, although the detailed specifications of tolerances and measurement methods are not identical.

In this book, because of the recent introduction of AGMA 2015, we use the quality numbers from AGMA 2000-A88 unless stated otherwise. Also, AGMA 2008 is integrated into the gear design methodology that follows in terms of the dynamic factor, K_v.

(a) Overall view



(b) Closeup view of probe and test gear

FIGURE 9–8 Analytical measurement system for gear quality (Process Equipment Company, Tipp City, Ohio)

Methods of Gear Measurement

Two different approaches to determining gear quality are in use, functional measurement and analytical measurement.

Functional measurement typically uses a system such as that sketched in Figure 9–7(a) to measure total composite error. The variation in center distance is recorded for a complete revolution as shown in Figure 9–7(b). The total composite variation is the maximum spread between the highest and lowest points on the chart. In addition, the maximum spread on the chart for any two adjacent teeth is determined as a measure of the tooth-to-tooth composite variation. The runout can also be determined from the total excursion of the mean line through the plot as shown. These data allow the determination of the AGMA quality number based primarily on the total composite variation and are often considered adequate for general-purpose gears in industrial machinery.

Analytical measurement measures individual errors of *index*, *alignment (helix)*, *involute profile*, and other features. The equipment is a specially designed coordinate measurement system (CMM) with a highly accurate probe that scans the various important surfaces of the test gear and produces electronic and printed records of the variations. Figure 9–8 shows one commercially available model of an analytical measurement system. Part (a) is an overall view while part (b) shows the probe engaging the teeth of the test gear. Figure 9–9 shows two different types of output charts from an analytical measurement system. The *index variation* chart (a) shows the amount of index variation for each tooth relative to a specified datum tooth. The *profile variation* chart (b) shows a plot of the deviation of the actual tooth profile from the true involute. The tooth alignment chart, measuring the helix accuracy, is similar. Tabulated data are also given along with the corresponding quality number related to each measurement.

Comparisons are automatically made with the theoretical tooth forms and with tolerance values to report the resulting quality number in the AGMA, ISO, DIN, or a user-

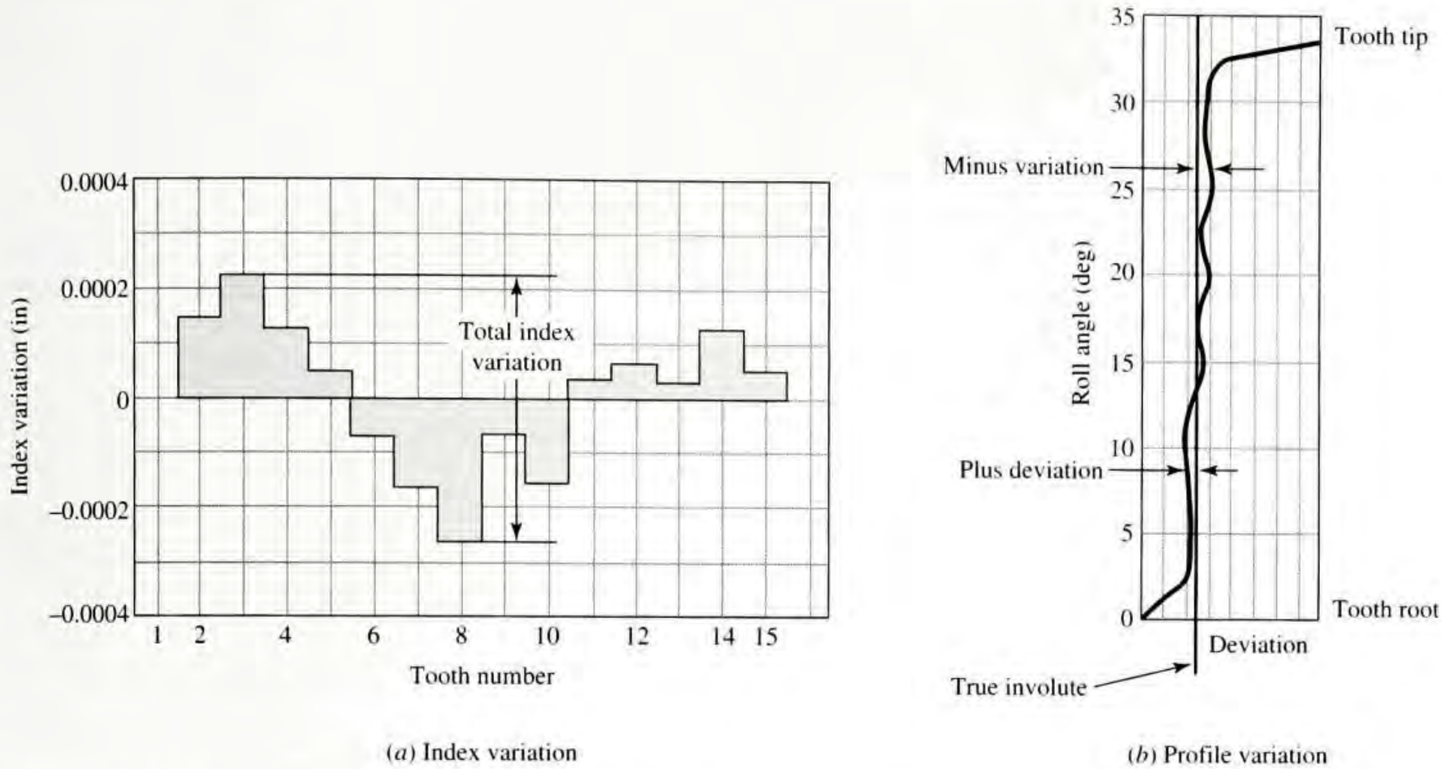


FIGURE 9–9 Typical output charts from an analytical measurement system (Process Equipment Company, Tipp City, Ohio)

defined standard. Besides giving the quality numbers, the detailed data from analytical measurement systems are useful to the manufacturing staff in making adjustments to cutters or equipment settings to improve the accuracy of the total process.

Using the general capabilities of the analytical measurement system, dimensions of features other than those of the gear teeth may also be determined while the gear is in its fixture. For example, when a gear is machined onto a shaft, key shaft diameters and geometric features may be checked for dimensions, perpendicularity, parallelism, and concentricity. Gear segments, composite gears having two or more gears on the same shaft, splines, cam surfaces, and other special features can be inspected along with the gear teeth.

Recommended Quality Numbers

The data in Table 9–1 should impress you with the precision that is normally exercised in the manufacture and installation of gears. The design of the entire gear system, including the shafts, bearings, and housing, must be consistent with this precision. Of course, the system should not be made more precise than necessary because of cost. For this reason, manufacturers have recommended quality numbers that will give satisfactory performance at a reasonable cost for a variety of applications. Table 9–2 lists several of these recommendations.

Also shown in Table 9–2 are recommendations for quality numbers for machine tool drives. Because this is such a wide range of specific applications, the recommended quality numbers are related to the *pitch line speed*, defined as the linear velocity of a point on the pitch circle of the gear. Use Equation (9–1). We recommend using these values for any high-accuracy machinery.

TABLE 9–2 Recommended AGMA quality numbers

Application	Quality number	Application	Quality number
Cement mixer drum drive	3–5	Small power drill	7–9
Cement kiln	5–6	Clothes washing machine	8–10
Steel mill drives	5–6	Printing press	9–11
Grain harvester	5–7	Computing mechanism	10–11
Cranes	5–7	Automotive transmission	10–11
Punch press	5–7	Radar antenna drive	10–12
Mining conveyor	5–7	Marine propulsion drive	10–12
Paper-box-making machine	6–8	Aircraft engine drive	10–13
Gas meter mechanism	7–9	Gyroscope	12–14

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0–800	6–8	0–4
800–2000	8–10	4–11
2000–4000	10–12	11–22
Over 4000	12–14	Over 22

9–6 ALLOWABLE STRESS NUMBERS

Later in this chapter, design procedures are presented in which two forms of gear-tooth failure are considered.

A gear tooth acts like a cantilever beam in resisting the force exerted on it by the mating tooth. The point of highest tensile bending stress is at the root of the tooth where the involute curve blends with the fillet. The AGMA has developed a set of *allowable bending stress numbers*, called s_{at}, which are compared to computed bending stress levels in the tooth to rate the acceptability of a design.

A second, independent form of failure is the pitting of the surface of the teeth, usually near the pitch line, where high contact stresses occur. The transfer of force from the driving to the driven tooth theoretically occurs across a line contact because of the action of two convex curves on each other. Repeated application of these high contact stresses can cause a type of fatigue failure of the surface, resulting in local fractures and an actual loss of material. This is called *pitting*. The AGMA has developed a set of *allowable contact stress numbers*, called s_{ac}, which are compared to computed contact stress levels in the tooth to rate the acceptability of a design. (See References 10–12.)

Representative data for s_{at} and s_{ac} are given in the following section for general information and use in problems in this book. More extensive data are given in the AGMA standards listed at the end of the chapter. (See References 6, 8, and 9.)

Many of the data given in this book for the design of spur and helical gears are taken from the AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314. That document should be consulted for details beyond the discussion in this book. Data in the U.S. Customary System only are given here. A separate document, AGMA Standard 2101-C95, has been published as a Metric Edition of AGMA 2001-C95. A brief summary of the differences between the terminology in these two standards is given later in this chapter.

9–7 METALLIC GEAR MATERIALS

DESIGN

Gears can be made from a wide variety of materials to achieve properties appropriate to the application. From a mechanical design standpoint, strength and pitting resistance are the most important properties. But, in general, the designer should consider the producibility of the gear, taking into account all of the manufacturing processes involved, from the preparation of the gear blank, through the forming of the gear teeth, to the final assembly of the gear into a machine. Other considerations are weight, appearance, corrosion resistance, noise, and, of course, cost. This section discusses several types of metals used for gears. Plastics are covered in a later section.

Steel Gear Materials

Through-Hardened Steels. Gears for machine tool drives and many kinds of medium- to heavy-duty speed reducers and transmissions are typically made from medium-carbon steels. Among a wide range of carbon and alloy steels used are

AISI 1020	AISI 1040	AISI 1050	AISI 3140
AISI 4140	AISI 4340	AISI 4620	AISI 5120
AISI 6150	AISI 8620	AISI 8650	AISI 9310

(See Reference 17.) AGMA Standard 2001-C95 gives data for the allowable bending stress number, s_{at}, and the allowable contact stress number, s_{ac}, for steels in the through-hardened condition. Figures 9–10 and 9–11 are graphs relating the stress numbers to the Brinell hardness number for the teeth. Notice that only knowledge of the hardness is required because of the direct relationship between hardness and the tensile strength of steels. See Appendix 19

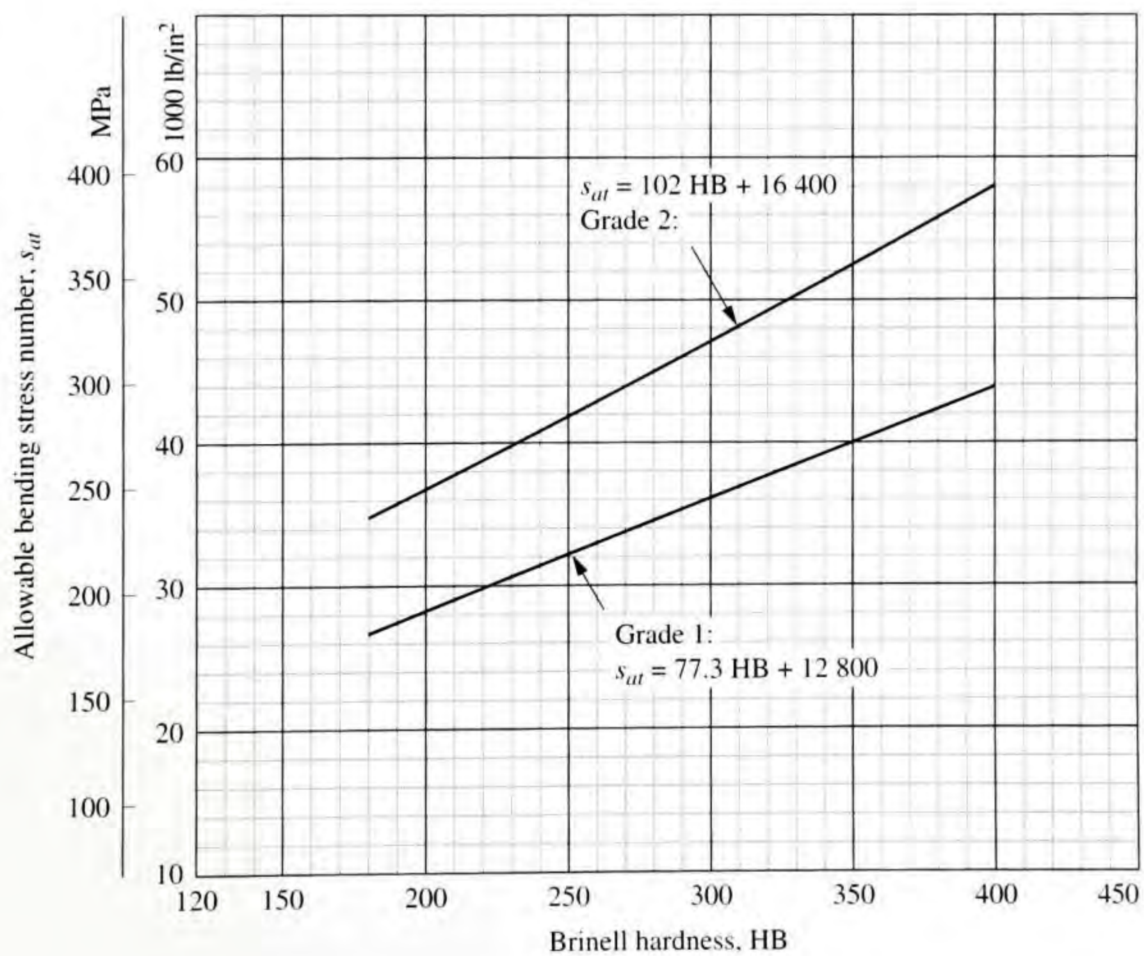


FIGURE 9–10 Allowable bending stress number for through-hardened steel gears, s_{at} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

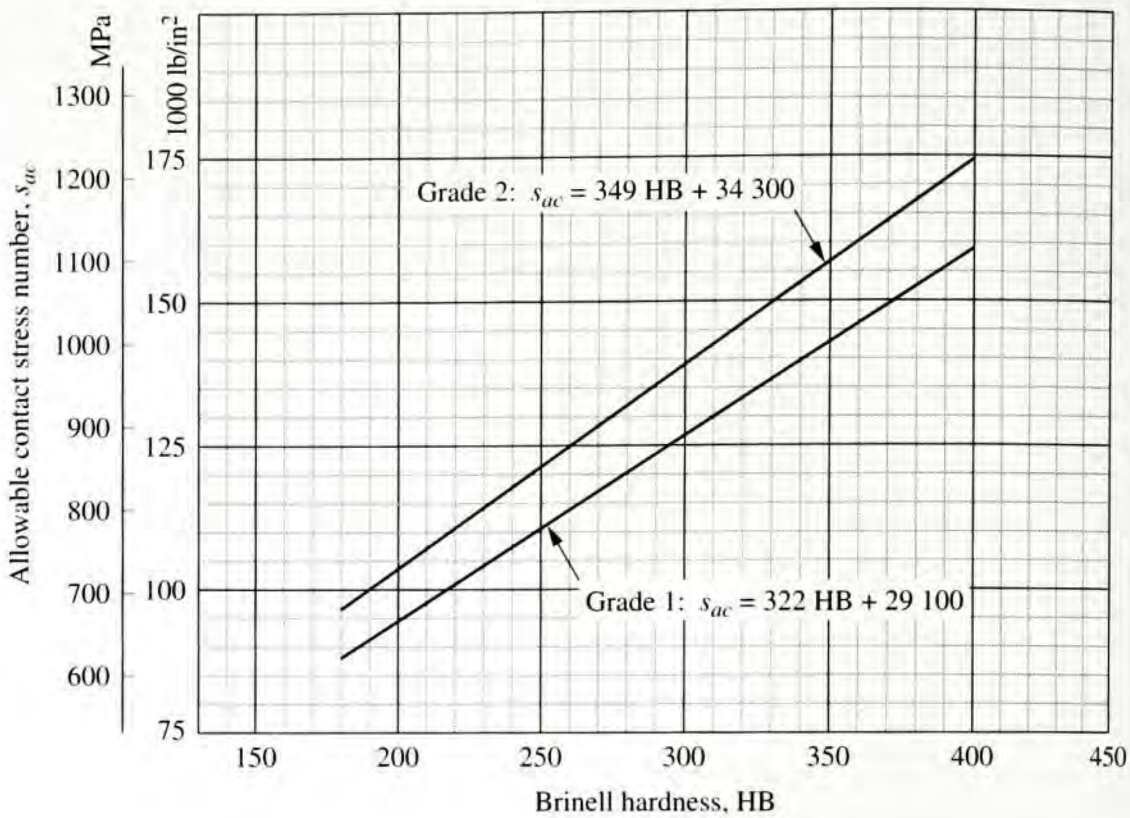


FIGURE 9–11 Allowable contact stress number for through-hardened steel gears, s_{ac} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

for data that correlate the Brinell hardness number, HB, with the tensile strength of steel in ksi. The range of hardnesses covered by the AGMA data is from 180 to 400 HB, corresponding to a tensile strength of approximately 87 to 200 ksi. It is not recommended to use through-hardening above 400 HB because of inconsistent performance of the gears in service. Typically, case hardening is used when there is a desire to achieve a surface hardness above 400 HB. This is described later in this section.

The hardness measurement for the allowable bending stress number is to be taken at the root of the teeth because that is where the highest bending stress occurs. The allowable contact stress number is related to the surface hardness on the face of the gear teeth where the mating teeth experience high contact stresses.

When selecting a material for gears, the designer must specify one that can be hardened to the desired hardness. Review Chapter 2 for discussions about heat treatment techniques. Consult Appendices 3 and 4 for representative data. For the higher hardnesses, say, above 250 HB, a medium-carbon-alloy steel with good hardenability is desirable. Examples are AISI 3140, 4140, 4340, 6150, and 8650. Ductility is also rather important because of the numerous cycles of stress experienced by gear teeth and the likelihood of occasional overloads, impact, or shock loading. A percent elongation value of 12% or higher is desired.

The curves in Figures 9–10 and 9–11 include two grades of steel: Grade 1 and Grade 2. *Grade 1 is considered to be the basic standard and will be used for problem solutions in this book.* Grade 2 requires a higher degree of control of the microstructure, alloy composition, greater cleanliness, prior heat treatment, nondestructive testing performed, core hardness values, and other factors. See AGMA Standard 2001-C95 (Reference 6) for details.

Case-Hardened Steels. Flame hardening, induction hardening, carburizing, and nitriding are processes used to produce a high hardness in the surface layer of gear teeth. See Figure 2–13 and the related discussion in Section 2–6. These processes provide surface

TABLE 9–3 Allowable stress numbers for case-hardened steel gear materials

Hardness at surface	Allowable bending stress number, s_{at} (ksi) Grade 1	Grade 2	Grade 3	Allowable contact stress number, s_{ac} (ksi) Grade 1	Grade 2	Grade 3
Flame- or induction-hardened:						
50 HRC	45	55		170	190	
54 HRC	45	55		175	195	
Carburized and case-hardened:						
55–64 HRC	55			180		
58–64 HRC	55	65	75	180	225	275
Nitrided, through-hardened steels:						
83.5 HR15N	See Figure 9–14.			150	163	175
84.5 HR15N	See Figure 9–14.			155	168	180
Nitrided, nitralloy 135M:[a]						
87.5 HR15N	See Figure 9–15.					
90.0 HR15N	See Figure 9–15.			170	183	195
Nitrided, nitralloy N:[a]						
87.5 HR15N	See Figure 9–15.					
90.0 HR15N	See Figure 9–15.			172	188	205
Nitrided, 2.5% chrome (no aluminum):						
87.5 HR15N	See Figure 9–15.			155	172	189
90.0 HR15N	See Figure 9–15.			176	196	216

Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

[a]Nitralloy is a proprietary family of steels containing approximately 1.0% aluminum which enhances the formation of hard nitrides.

hardness values from 50 to 64 HRC (Rockwell C) and correspondingly high values of s_{at} and s_{ac}, as shown in Table 9–3. Special discussions are given below for each of the types of case-hardening processes.

In addition to Grade 1 and Grade 2 as described earlier, case-hardened steel gears can be produced to Grade 3 which requires an even higher standard of control of the metallurgy and processing of the material. See AGMA Standard 2001-C95 (References 6 and 20) for details.

Flame- and Induction-Hardened Gear Teeth. Recall that these processes involve the local heating of the surface of the gear teeth by high-temperature gas flames or electrical induction coils. By controlling the time and energy input, the manufacturer can control the depth of heating and the depth of the resulting case. It is essential that the heating occur around the entire tooth, producing the hard case on the face of the teeth *and in the fillet and root areas*, in order to use the stress values listed in Table 9–3. This may require a special design for the flame shape or the induction heater.

The specifications for flame- or induction-hardened steel gear teeth call for a resulting hardness of HRC 50 to 54. Because these processes rely on the inherent hardenability of the steels, you must specify a material that can be hardened to these levels. Normally, medium-carbon-alloy steels (approximately 0.40% to 0.60% carbon) are specified. Appendices 3 and 4 list some suitable materials.

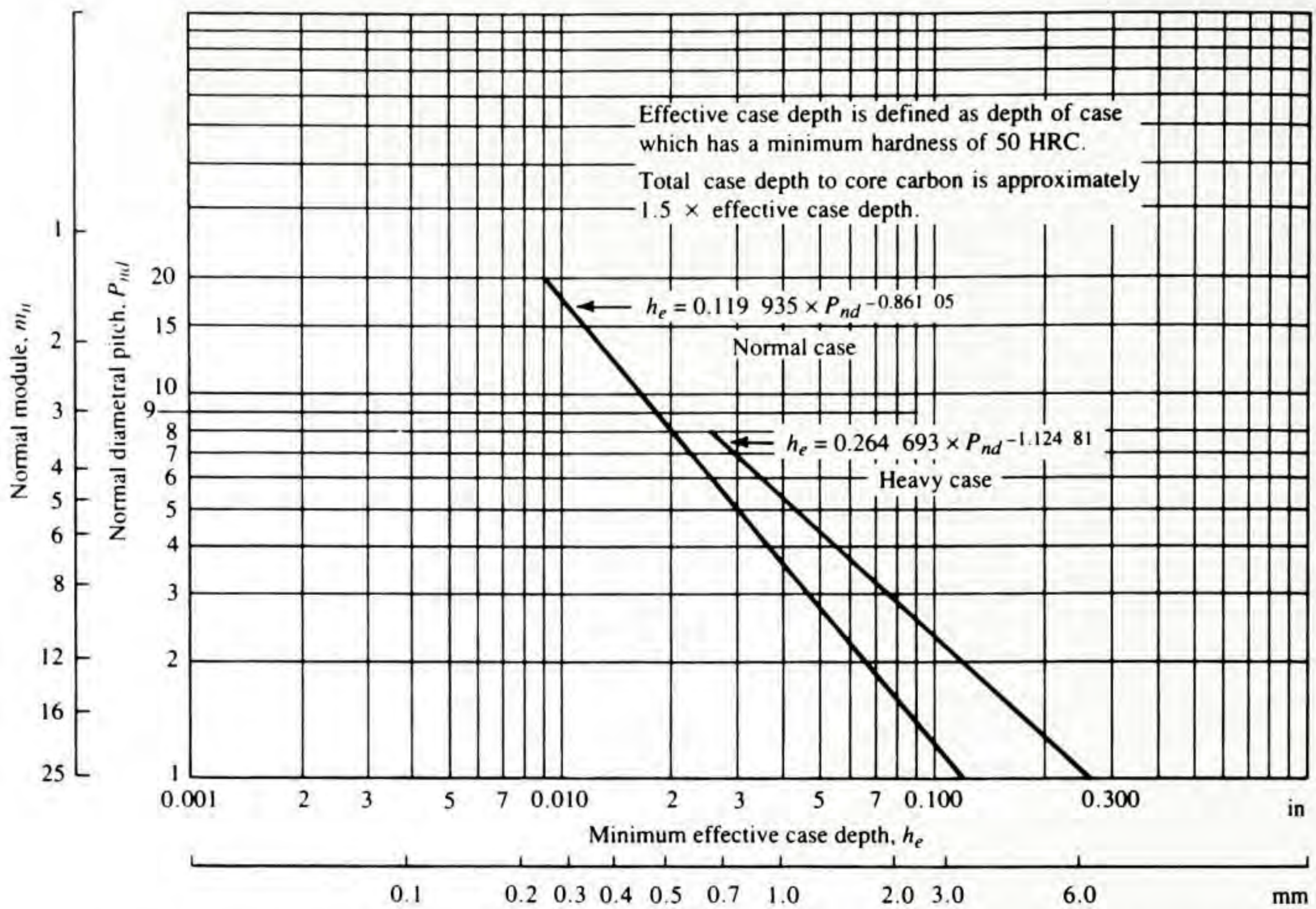
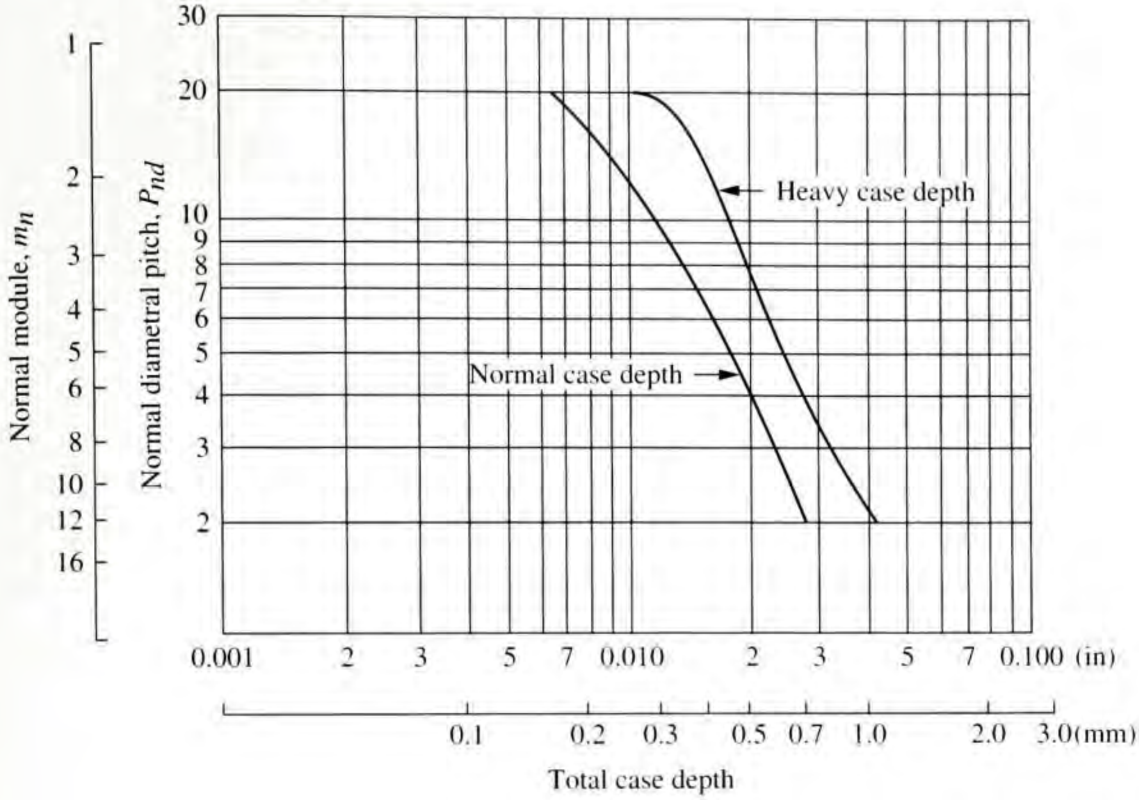


FIGURE 9–12 Effective case depth for carburized gears, h_e (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

Carburizing. Carburizing produces surface hardnesses in the range of 55 to 64 HRC. It results in some of the highest strengths in common use for gears. Special carburizing steels are listed in Appendix 5. Figure 9–12 shows the AGMA recommendation for the thickness of the case for carburized gear teeth. The effective case depth is defined as the depth from the surface to the point where the hardness has reached 50 HRC.

Nitriding. Nitriding produces a very hard *but very thin* case. It is specified for applications in which loads are smooth and well known. Nitriding should be avoided when overloading or shock can be experienced, because the case is not sufficiently strong or well supported to resist such loads. Because of the thin case, the Rockwell 15N scale is used to specify hardness.

Figure 9–13 shows the AGMA recommendation for the case depth of nitrided gears, defined as the depth below the surface at which the hardness has dropped to 110% of that at the core of the teeth. The values for the allowable bending stress number, s_{at}, are dependent on the conditions of the material in the core of the teeth because of the thin case for nitrided gears. Figure 9–14 gives the values for the general group of alloy steels used for gears that are through-hardened and then nitrided. Examples are AISI 4140 and AISI 4340 and similar alloys. As with other through-hardened materials, the primary variable is



Equations for Minimum Total Case Depth for Nitrided, Case-Hardened Gears

Normal case depth (in inches):
$h_{c\,\min} = 0.0432896 - 0.00968115*P_d + 0.00120185*P_d^2 - 6.79721 \times 10^{-5}*P_d^3 + 1.37117 \times 10^{-6}*P_d^4$

Heavy case depth (in inches):
$h_{c\,\min} = 0.0660090 - 0.0162224P_d + 0.00209361P_d^2 - 1.17755 \times 10^{-4}P_d^3 + 2.33160 \times 10^{-6}P_d^4$

Note: $P_d = P_{nd}$ for helical gear teeth

FIGURE 9–13 Recommended case depth for nitrided gears, h_c (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

the Brinell hardness number HB. Also, special alloys have been developed for use in gears with the nitriding process. Data are shown in Table 9–3 and Figure 9–15 for *nitralloy* and an alloy called *2.5% chrome*.

Iron and Bronze Gear Materials

Cast Irons. Two types of iron used for gears are *gray cast iron* and *ductile* (sometimes called *nodular*) iron. Table 9–4 gives the common ASTM grades used, with their corresponding allowable bending stress numbers and contact stress numbers. Remember that gray cast iron is brittle, so care should be exercised when shock loading is possible. Also, the higher-strength forms of the other irons have low ductility. Austempered ductile iron (ADI) is being used in some important automotive applications. However, standardized allowable stress numbers have not yet been specified.

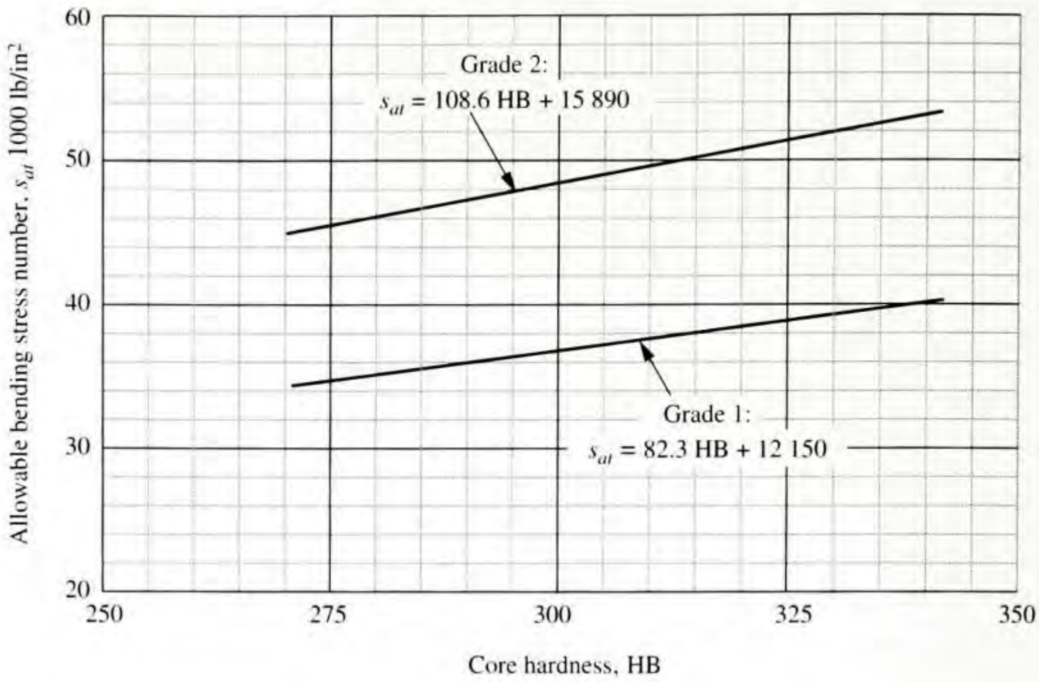


FIGURE 9–14 Allowable bending stress numbers for nitrided, through-hardened steel gears (that is, AISI 4140, AISI 4340), s_{at} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association,1500 King Street, Suite 201, Alexandria, VA 22314)

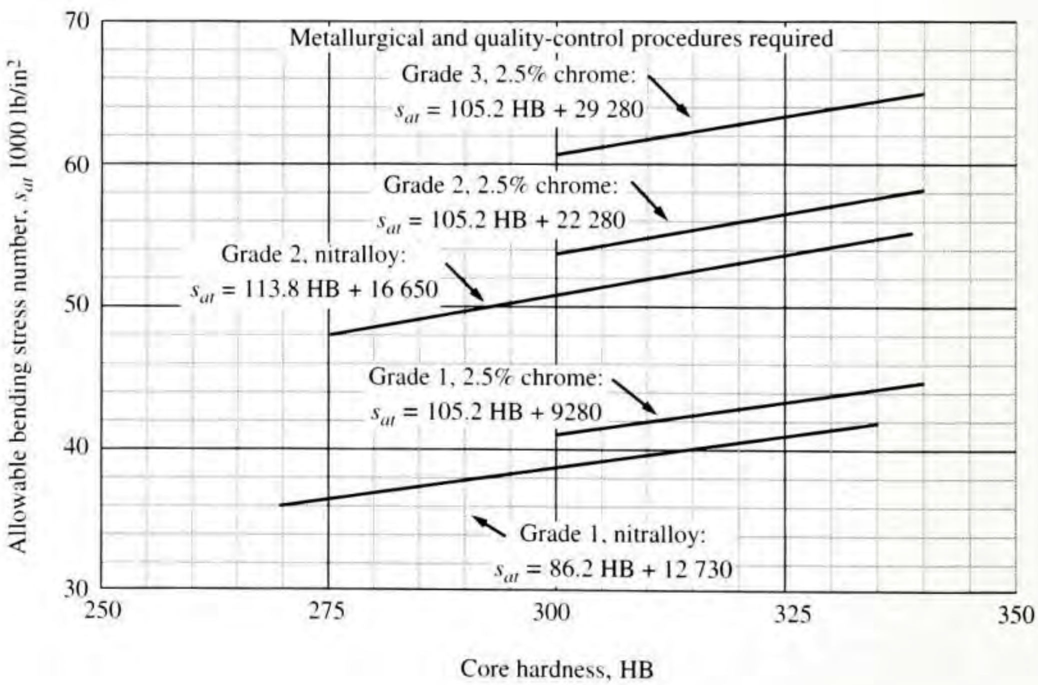


FIGURE 9–15 Allowable bending stress numbers for nitriding steel gears, s_{at} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201. Alexandira, VA 22314

Bronzes. Four families of bronzes are typically used for gears: (1) phosphor or tin bronze, (2) manganese bronze, (3) aluminum bronze, and (4) silicon bronze. Yellow brass is also used. Most bronzes are cast, but some are available in wrought form. Corrosion resistance, good wear properties, and low friction coefficients are some reasons for choosing bronzes for gears. Table 9–4 shows allowable stress numbers for one bronze alloy in two common forms.

TABLE 9–4 Allowable stress numbers for iron and bronze gears

Material designation	Minimum hardness at surface (HB)	Allowable bending stress number (ksi)	Allowable bending stress number (MPa)	Allowable contact stress number (ksi)	Allowable contact stress number (MPa)
Gray cast iron, ASTM A48, as cast					
Class 20		5	35	50	345
Class 30	174	8.5	59	65	448
Class 40	201	13	90	75	517
Ductile (nodular) iron, ASTM A536					
60-40-18 annealed	140	22	152	77	530
80-55-06 quenched and tempered	179	22	152	77	530
100-70-03 quenched and tempered	229	27	186	92	634
120-90-02 quenched and tempered	269	31	214	103	710
Bronze, sand-cast, $s_{u\,min}$ = 40 ksi (275 MPa)		5.7	39	30	207
Bronze, heat-treated, $s_{u\,min}$ = 90 ksi (620 MPa)		23.6	163	65	448

Source: Extracted from AGMA Standard 2001-C95 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

9–8 STRESSES IN GEAR TEETH

The stress analysis of gear teeth is facilitated by consideration of the orthogonal force components, W_t and W_r, as shown in Figure 9–2.

The tangential force, W_t, produces a bending moment on the gear tooth similar to that on a cantilever beam. The resulting bending stress is maximum at the base of the tooth in the fillet that joins the involute profile to the bottom of the tooth space. Taking the detailed geometry of the tooth into account, Wilfred Lewis developed the equation for the stress at the base of the involute profile, which is now called the *Lewis equation*:

Lewis Equation for Bending Stress In Gear Teeth

$$\sigma_t = \frac{W_t P_d}{FY} \qquad \textbf{(9–12)}$$

where W_t = tangential force
P_d = diametral pitch of the tooth
F = face width of the tooth
Y = *Lewis form factor*, which depends on the tooth form, the pressure angle, the diametral pitch, the number of teeth in the gear, and the place where W_t acts

While the theoretical basis for the stress analysis of gear teeth is presented, the Lewis equation must be modified for practical design and analysis. One important limitation is that it does not take into account the stress concentration that exists in the fillet of the tooth. Figure 9–16 is a photograph of a photoelastic stress analysis of a model of a gear tooth. It indicates a stress concentration in the fillet at the root of the tooth as well as high contact stresses at the mating surface (the contact stress is discussed in the following section). Comparing the actual stress at the root with that predicted by the Lewis equation enables

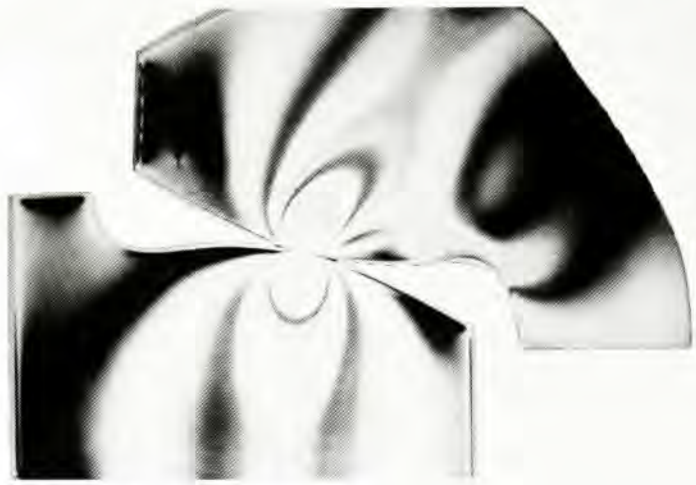


FIGURE 9–16
Photoelastic study of gear teeth under load (Measurements Group, Inc., Raleigh, NC)

us to determine the stress concentration factor, K_t, for the fillet area. Placing this into equation (9–12) gives

$$\sigma_t = \frac{W_t P_d K_t}{FY} \tag{9–13}$$

The value of the stress concentration factor is dependent on the form of the tooth, the shape and size of the fillet at the root of the tooth, and the point of application of the force on the tooth. Note that the value of the Lewis form factor, Y, also depends on the tooth geometry. Therefore, the two factors are combined into one term, the *geometry factor, J*, where $J = Y/K_t$. The value of J also, of course, varies with the location of the point of application of the force on the tooth because Y and K_t vary.

Figure 9–17 shows graphs giving the values for the geometry factor for 20° and 25°, full-depth, involute teeth. The safest value to use is the one for the load applied at the tip of the tooth. However, this value is overly conservative because there is some load sharing by another tooth at the time that the load is initially applied at the tip of a tooth. The critical load on a given tooth occurs when the load is at the highest point of single-tooth contact, when the tooth carries the entire load. The upper curves in Figure 9–17 give the values for J for this condition.

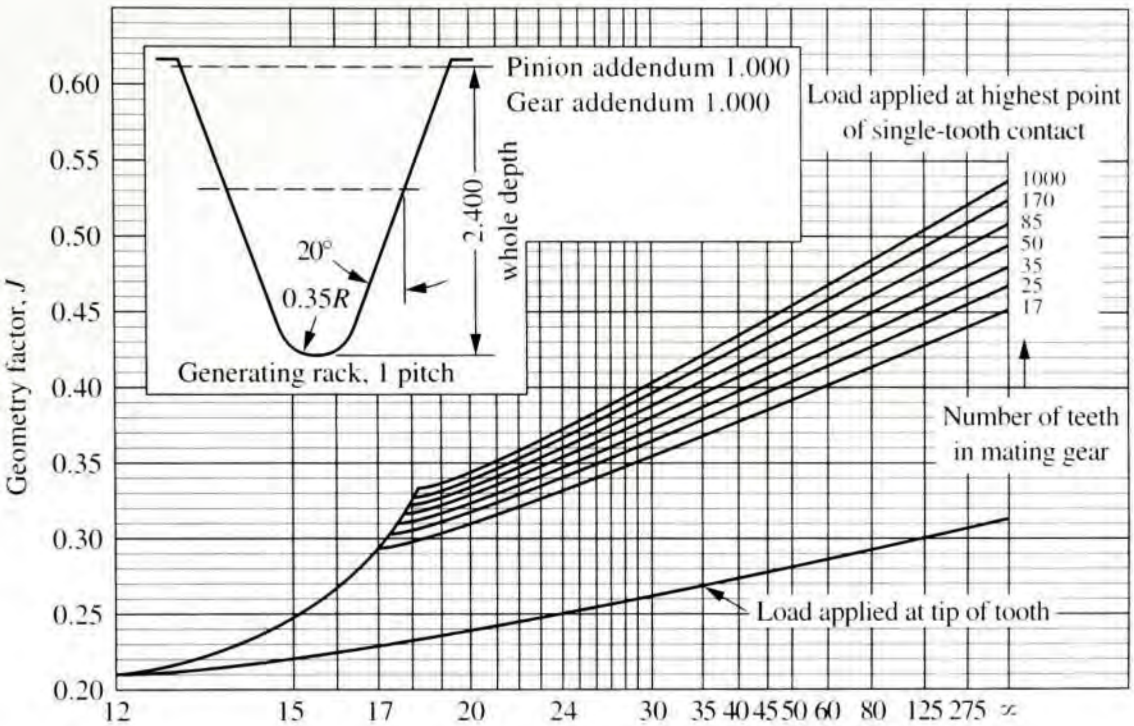
Using the geometry factor, J, in the stress equation gives

$$\sigma_t = \frac{W_t P_d}{FJ} \tag{9–14}$$

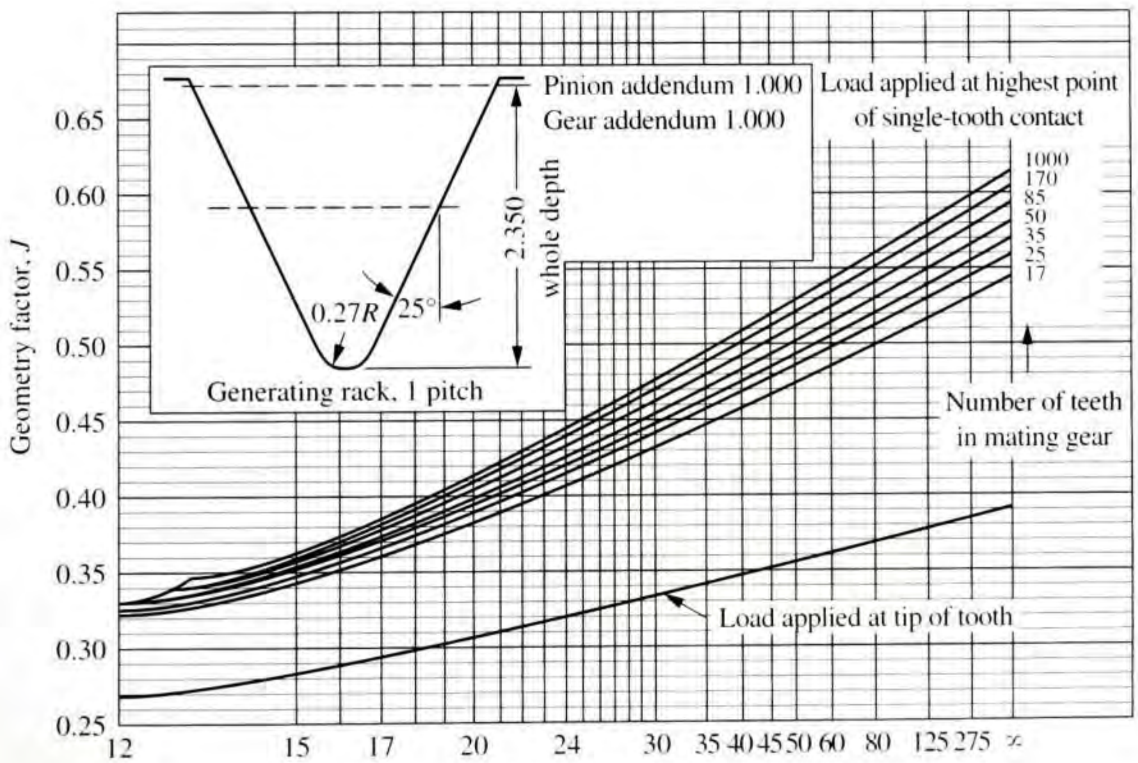
The graphs in Figure 9–17 are taken from the former AGMA Standard 218.01 which has been superseded by the two new standards: AGMA 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, 1995, and AGMA 908-B89 (R1995), *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, 1995. Standard 908-B89 includes an analytical method for calculating the geometry factor, J. But the values for J are unchanged from those in the former standard. Rather than graphs, the new standard reports values for J for a variety of tooth forms in tables. The graphs from the former standard are shown in Figure 9–17 so that you can visualize the variation of J with the number of teeth in the pinion and the gear.

Note also that J factors for only two tooth forms are included in Figure 9–17 and that the values are valid only for those forms. Designers must ensure that J factors for

FIGURE 9–17
Geometry factor, J (Extract from AGMA 218.01 Standard, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



(*a*) 20° spur gear: standard addendum



(*b*) 25° spur gear: standard addendum

the tooth form actually used, including the form of the fillet, are included in the stress analysis.

Equation (9–14) can be called the *modified Lewis equation*. Other modifications to the equation are recommended by the AGMA in Standard 2001-C95 for practical design to account for the variety of conditions that can be encountered in service.

The approach used by the AGMA is to apply a series of additional modifying factors to the bending stress from the modified Lewis equation to compute a value called the *bending stress number*, s_t. These factors represent the degree to which the actual loading case differs from the theoretical basis of the Lewis equation. The result is a better estimate of the real level of bending stress that is produced in the teeth of the gear and the pinion.

Then, separately, the allowable bending stress number, s_{at}, is modified by a series of factors that affect that value when the environment is different from the nominal situation assumed when the values for s_{at} are set. The result here is a better estimate of the real level of the bending strength of the material from which the gear or the pinion is made.

The design is completed in a manner that ensures that the bending stress number is less than the modified allowable bending stress number. This process should be completed for both the pinion and the gear of a given pair because materials may be different; the geometry factor, J, is different; and other operating conditions may be different. This is demonstrated in example problems later in this chapter.

Often the major decision to be made is the specification of suitable materials from which to make the pinion and the gear. In such cases, the required basic allowable bending stress number, s_{at}, will be computed. When steel is used, the required hardness of the material is found from the data described in Section 9–7. Finally, the material and its heat treatment are specified to ensure that it will have at least the required hardness.

We proceed now with the discussion of the bending stress number, s_t.

Bending Stress Number, s_t

The design analysis method used here is based primarily on AGMA Standard 2001-C95. However, because values for some of the factors are not included in the standard, data from other sources are added. These data illustrate the kinds of conditions that affect the final design. The designer ultimately has the responsibility for making appropriate design decisions.

The following equation will be used in this book:

Bending Stress Number, s_t

$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v \tag{9–15}$$

where K_o = overload factor for bending strength
K_s = size factor for bending strength
K_m = load distribution factor for bending strength
K_B = rim thickness factor
K_v = dynamic factor for bending strength

Methods for specifying values for these factors are discussed below.

Overload Factor, K_o

Overload factors consider the probability that load variations, vibrations, shock, speed changes, and other application-specific conditions may result in peak loads greater than W_t being applied to the gear teeth during operation. A careful analysis of actual conditions should be made, and the AGMA Standard 2001-C95 gives no specific values for K_o. Reference 15 gives some recommended values, and many industries have established suitable values based on experience.

For problem solutions in this book, we will use the values shown in Table 9–5. The primary considerations are the nature of *both* the driving power source and the driven machine. An overload factor of 1.00 would be applied for a perfectly smooth electric motor driving a perfectly smooth generator through a gear-type speed reducer. Any rougher conditions call for a value of K_o greater than 1.00. For power sources we will use the following:

Uniform: Electric motor or constant-speed gas turbine

Light shock: Water turbine, variable-speed drive

Moderate shock: Multicylinder engine

Examples of the roughness of driven machines include the following:

Uniform: Continuous-duty generator

Light shock: Fans and low-speed centrifugal pumps, liquid agitators, variable-duty generators, uniformly loaded conveyors, rotary positive displacement pumps

Moderate shock: High-speed centrifugal pumps, reciprocating pumps and compressors, heavy-duty conveyors, machine tool drives, concrete mixers, textile machinery, meat grinders, saws

Heavy shock: Rock crushers, punch press drives, pulverizers, processing mills, tumbling barrels, wood chippers, vibrating screens, railroad car dumpers

Size Factor, K_s

The AGMA indicates that the size factor can be taken to be 1.00 for most gears. But for gears with large-size teeth or large face widths, a value greater than 1.00 is recommended. Reference 15 recommends a value of 1.00 for diametral pitches of 5 or greater or for a metric module of 5 or smaller. For larger teeth, the values shown in Table 9–6 can be used.

Load-Distribution Factor, K_m

The determination of the load-distribution factor is based on many variables in the design of the gears themselves as well as in the shafts, bearings, housings, and the structure in which the gear drive is installed. Therefore, it is one of the most difficult factors to specify. Much analytical and experimental work is continuing on the determination of values for K_m.

TABLE 9–5 Suggested overload factors, K_o

	Driven Machine			
Power source	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

TABLE 9–6 Suggested size factors, K_s

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

If the intensity of loading on all parts of all teeth in contact at any given time were uniform, the value of K_m would be 1.00. However, this is seldom the case. Any of the following factors can cause misalignment of the teeth on the pinion relative to those on the gear:

1. Inaccurate gear teeth
2. Misalignment of the axes of shafts carrying gears
3. Elastic deformations of the gears, shafts, bearings, housings, and support structures
4. Clearances between the shafts and the gears, the shafts and the bearings, or the bearings and the housing
5. Thermal distortions during operation
6. Crowning or end relief of gear teeth

AGMA Standard 2001-C95 presents extensive discussions of methods of determining values for K_m. One is empirical and considers gears up to 40 in (1000 mm) wide. The other method is analytical and considers the stiffness and mass of individual gears and gear teeth and the total mismatch between mating teeth. We will not provide so much detail. However, rough guidelines are given below.

The designer can minimize the load-distribution factor by specifying the following:

1. Accurate teeth (a high quality number)
2. Narrow face widths
3. Gears centered between bearings (straddle mounting)
4. Short shaft spans between bearings
5. Large shaft diameters (high stiffness)
6. Rigid, stiff housings
7. High precision and small clearances on all drive components

You are advised to study the details of AGMA Standard 2001-C95 which covers a wide range of physical sizes for gear systems. But the gear designs discussed in this book are of moderate size, typical of power transmissions in light industrial and vehicular applications. A more limited set of data are reported here to illustrate the concepts that must be considered in gear design.

We will use the following equation for computing the value of the load-distribution factor:

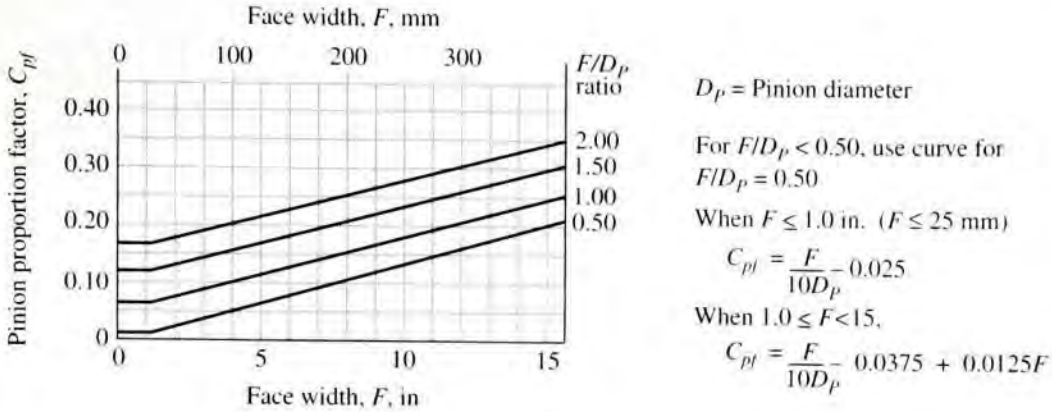
$$K_m = 1.0 + C_{pf} + C_{ma} \tag{9–16}$$

where C_{pf} = pinion proportion factor (see Figure 9–18)
C_{ma} = mesh alignment factor (see Figure 9–19)

In this book, we are limiting designs to those with face widths of 15 in or less. Wider face widths call for additional factors. Also, some commercially successful designs employ modifications to the basic tooth form to achieve a more uniform meshing of the teeth. Such methods are not discussed in this book.

Figure 9–18 shows that the pinion proportion factor is dependent on the actual face width of the pinion and on the ratio of the face width to the pinion pitch diameter. Figure 9–19 relates the mesh alignment factor to expected accuracy of different methods of applying gears. *Open gearing* refers to drive systems in which the shafts are supported in bearings that are mounted on structural elements of the machine with the expectation that relatively large misalignments will result. In *commercial-quality enclosed gear units*, the bearings are mounted in a specially designed housing that provides more rigidity than for

FIGURE 9–18 Pinion proportion factor, C_{pf} (Extracted from AGMA 2001-C95 *Standard, Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



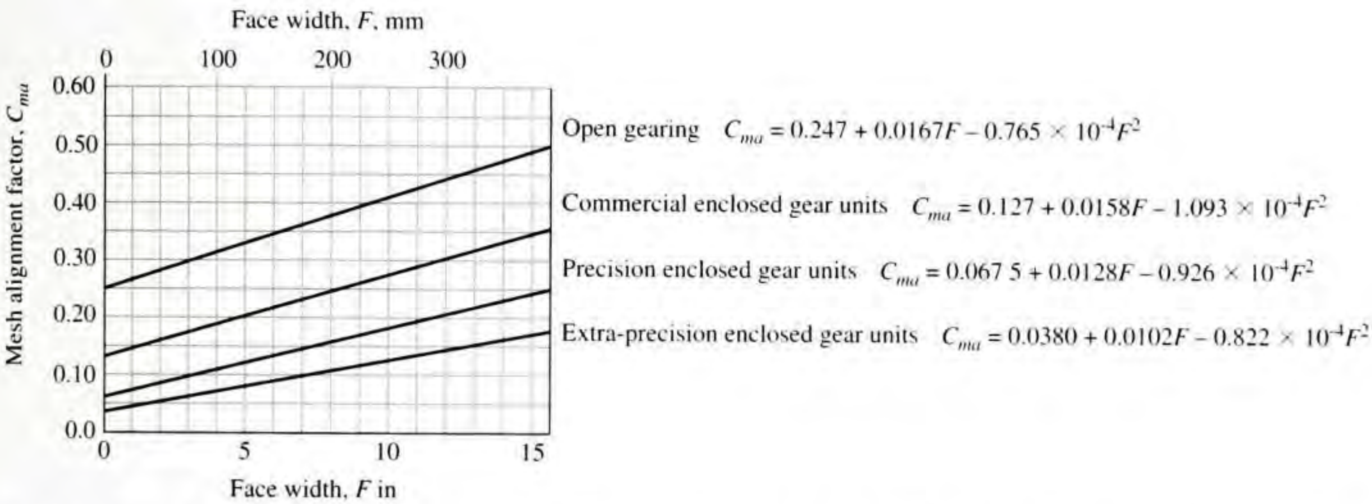


FIGURE 9–19 Mesh alignment factor, C_{ma} (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

open gearing, but for which the tolerances on individual dimensions are fairly loose. The *precision enclosed gear units* are made to tighter tolerances. *Extra-precision enclosed gear* units are made to exacting precision and are often adjusted at assembly to achieve excellent alignment of the gear teeth. Experience with similar units in the field will help you gain better understanding among the different types of designs.

Rim Thickness Factor, K_B

The basic analysis used to develop the Lewis equation assumes that the gear tooth behaves as a cantilever attached to a perfectly rigid support structure at its base. If the rim of the gear is too thin, it can deform and cause the point of maximum stress to shift from the area of the gear-tooth fillet to a point within the rim.

Figure 9–20 can be used to estimate the influence of rim thickness. The key geometry parameter is called the *backup ratio*, m_B, where

$$m_B = t_R/h_t$$

t_R = rim thickness

h_t = whole depth of the gear tooth

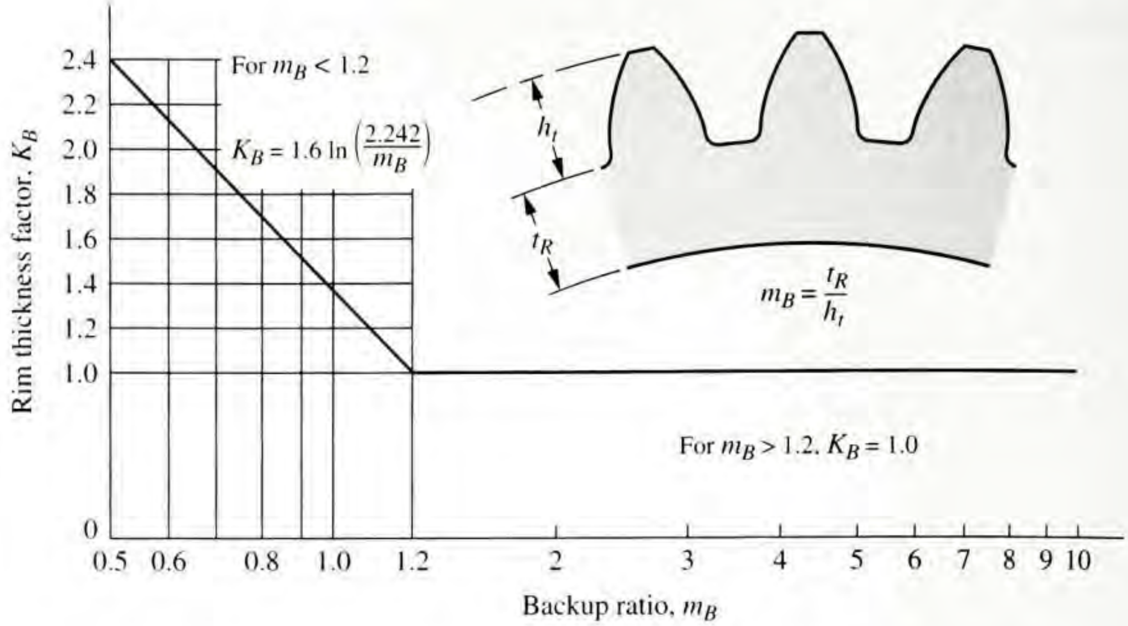


FIGURE 9–20 Rim thickness factor, K_B (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

For $m_B > 1.2$, the rim is sufficiently strong and stiff to support the tooth, and $K_B = 1.0$. The K_B factor can also be used in the vicinity of a keyseat where a small thickness of metal occurs between the top of the keyseat and the bottom of the tooth space.

Dynamic Factor, K_v

The dynamic factor accounts for the fact that the load is assumed by a tooth with some degree of impact and that the actual load subjected to the tooth is higher than the transmitted load alone. The value of K_v depends on the accuracy of the tooth profile, the elastic properties of the tooth, and the speed with which the teeth come into contact.

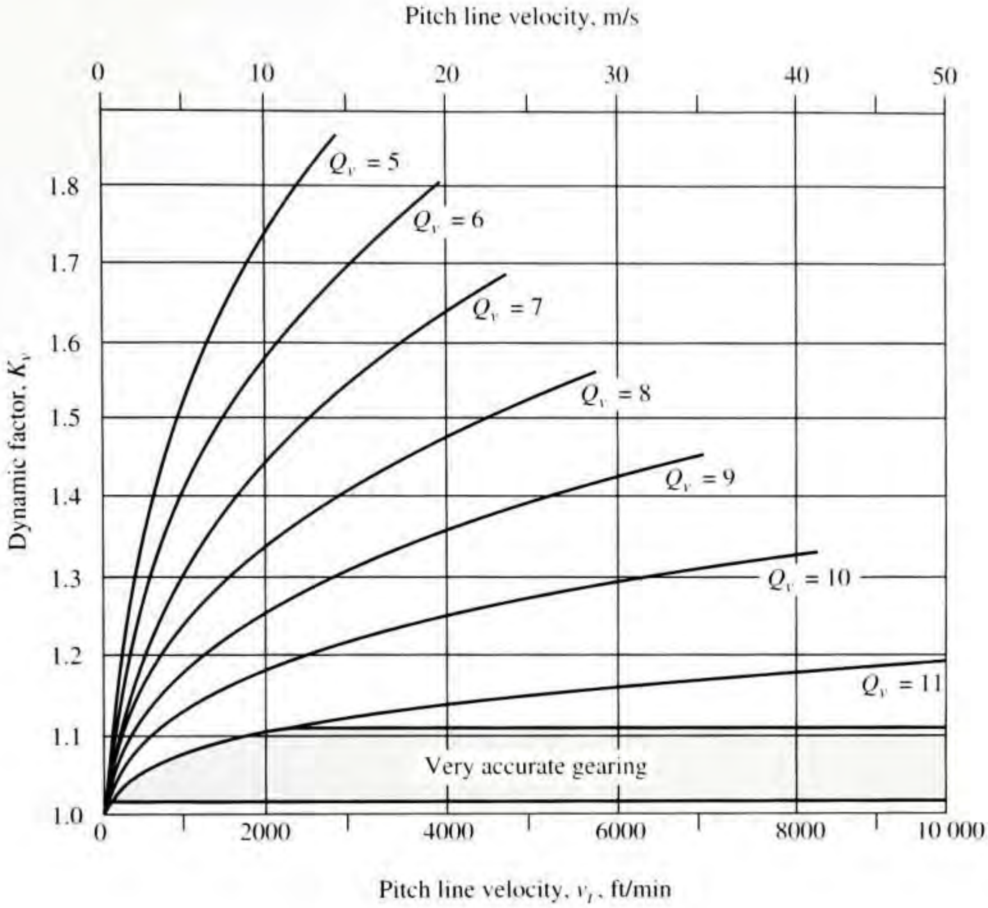
Figure 9–21 shows a graph of the AGMA-recommended values for K_v, where the Q_v numbers are the AGMA-quality numbers referred to earlier in Section 9–5. Gears in typical machine design would fall into the classes represented by curves 5, 6, or 7, which are for gears made by hobbing or shaping with average to good tooling. If the teeth are finish-ground or shaved to improve the accuracy of the tooth profile and spacing, curve 8, 9, 10, or 11 should be used. Under special conditions where teeth of high precision are used in applications where there is little chance of developing external dynamic loads, the shaded area can be used. If the teeth are cut by form milling, factors lower than those found from curve 5 should be used. Note that the quality 5 gears should not be used at pitch line speeds above 2500 ft/min. Note that the dynamic factors are approximate. For severe applications, especially those operating above 4000 ft/min, approaches taking into account the material properties, the mass and inertia of the gears, and the actual error in the tooth form should be used to predict the dynamic load. (See References 12, 15, and 18.)

Example Problem 9–1 Compute the bending stress numbers for the pinion and the gear of the pair of gears in Figure 9–1. The pinion rotates at 1750 rpm, driven directly by an electric motor. The driven machine is an industrial saw requiring 25 hp. The gear unit is enclosed and is made to commercial standards. Gears are straddle-mounted between bearings. The following gear data apply:

$$N_P = 20 \quad N_G = 70 \quad P_d = 8 \quad F = 1.50 \text{ in} \quad Q_v = 6$$

The gear teeth are 20°, full-depth, involute teeth, and the gear blanks are solid.

FIGURE 9–21
Dynamic factor, K_V (Extracted from AGMA 2001-C95 Standard *Fundamental, Rating Factors and Calculation Methods for Involute Spur and Helical Gear, Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



$v_{t\,max} = [A + (Q_v - 3)]^2$ (U.S. units)

$v_{t\,max} = \dfrac{[A + (Q_v - 3)]^2}{200}$ (SI units)

where $v_{t\,max}$ = end-point of K_v curves (ft/min or m/s)

Curves 5–11

$K_v = \left(\dfrac{A + \sqrt{v_t}}{A}\right)^B$ (U.S. units)

$K_v = \left(\dfrac{A + \sqrt{200 v_t}}{A}\right)^B$ (SI units)

where $A = 50 + 56(1.0 - B)$

$B = \dfrac{(12 - Q_v)^{0.667}}{4}$

Q_v = transmission accuracy grade number

Solution We will use Equation (9–15) to compute the expected stress:

$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v$$

We can first use the principles from Section 9–3 to compute the transmitted load on the gear teeth:

$$D_P = N_P/P_d = 20/8 = 2.500 \text{ in}$$
$$v_t = \pi D_P n_P/12 = \pi(2.5)(1750)/12 = 1145 \text{ ft/min}$$
$$W_t = 33\,000(P)/v_t = (33\,000)(25)/(1145) = 720 \text{ lb}$$

From Figure 9–17, we find that $J_P = 0.335$ and $J_G = 0.420$.

The overload factor is found from Table 9–5. For a smooth, uniform electric motor driving an industrial saw generating moderate shock, $K_o = 1.50$ is a reasonable value.

The size factor $K_s = 1.00$ because the gear teeth with $P_d = 8$ are relatively small. See Table 9–6.

The load distribution factor, K_m, can be found from Equation (9–16) for commercial enclosed gear drives. For this design, $F = 1.50$ in, and

$$\begin{aligned} F/D_P &= 1.50/2.50 = 0.60 \\ C_{pf} &= 0.04 \text{ (Figure 9–18)} \\ C_{ma} &= 0.15 \text{ (Figure 9–19)} \\ K_m &= 1.0 + C_{pf} + C_{ma} = 1.0 + 0.04 + 0.15 = 1.19 \end{aligned}$$

The rim thickness factor, K_B, can be taken as 1.00 because the gears are to be made from solid blanks.

The dynamic factor can be read from Figure 9–21. For $v_t = 1145$ ft/min and $Q_v = 6$, $K_v = 1.45$.

The stress can now be computed from Equation (9–15). We will compute the stress in the pinion first:

$$s_{tP} = \frac{(720)(8)}{(1.50)(0.335)}(1.50)(1.0)(1.19)(1.0)(1.45) = 29\,700 \text{ psi}$$

Notice that all factors in the stress equation are the same for the gear except the value of the geometry factor, J. Then the stress in the gear can be computed from

$$s_{tG} = \sigma_{tP}(J_P/J_G) = (29\,700)(0.335/0.420) = 23\,700 \text{ psi}$$

The stress in the pinion teeth will always be higher than the stress in the gear teeth because the value of J increases as the number of teeth increases.

9–9 SELECTION OF GEAR MATERIAL BASED ON BENDING STRESS

For safe operation, it is the designer's responsibility to specify a material that has an allowable bending stress greater than the computed stress due to bending from Equation (9–15). Recall that in Section 9–6, allowable stress numbers, s_{at}, were given for a variety of commonly used gear materials. Then it is necessary that

$$s_t < s_{at}$$

These data are valid for the following conditions:

Temperature less than 250°F

10^7 cycles of tooth loading

Reliability of 99%: Less than one failure in 100

Safety factor of 1.00

In this book we will assume that the operating temperature of the gears is less than 250°F. For higher temperatures, testing is recommended to determine the degree of reduction in the strength of the gear material.

Adjusted Allowable Bending Stress Numbers, s'_{at}

Data have been generated for different levels of expected life and reliability as discussed below. Designers may also choose to apply a factor of safety to the allowable bending stress number to account for uncertainties in the design analysis, material characteristics, or manufacturing tolerances, or to provide an extra measure of safety in critical applications. These factors are applied to the value of s_{at} to produce an *adjusted allowable bending stress number* which we will refer to as s'_{at}:

$$s'_{at} = s_{at}\, Y_N/(SF \cdot K_R) \tag{9–17}$$

Stress Cycle Factor, Y_N

Figure 9–22 allows the determination of the life adjustment factor, Y_N, if the teeth of the gear being analyzed are expected to experience a number of cycles of loading much different from 10^7. Note that the general type of material is a factor in this chart for the lower number of cycles. For the higher number of cycles, a range is indicated by a shaded area. General design practice would use the upper line of this range. Critical applications where pitting and tooth wear must be minimal may use the lower part of the range.

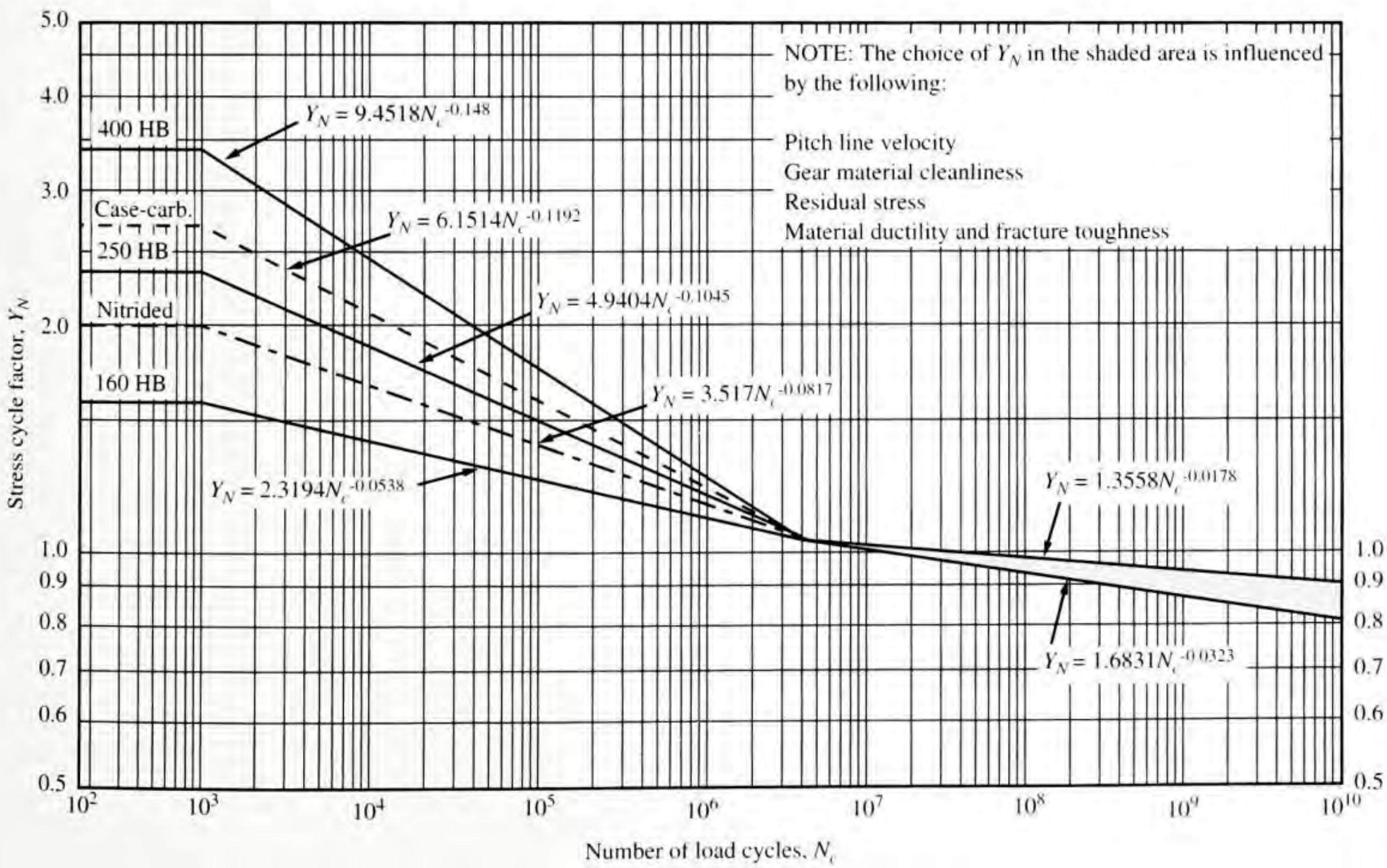


FIGURE 9–22 Bending strength stress cycle factor, Y_N (Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

Calculation of the expected number of cycles of loading can be done using

$$N_c = (60)(L)(n)(q) \tag{9–18}$$

where N_c = expected number of cycles of loading
L = design life in hours
n = rotational speed of the gear in rpm
q = number of load applications per revolution

Design life is, indeed, a design decision based on the application. As a guideline, we will use a set of data created for use in bearing design and reported here as Table 9–7. Unless stated otherwise, we will use a design life of $L = 20\,000$ h as listed for general industrial machines. The normal number of load applications per revolution for any given tooth is typically, of course, one. But consider the case of an idler gear that serves as both a driven and a driving gear in a gear train. It receives two cycles of load per revolution: one as it receives power from and one as it delivers power to its mating gears. Also, in certain types of gear trains, one gear may deliver power to two or more gears mating with it. Gears in a planetary gear train often have this characteristic.

As an example of the application of Equation (9–18), consider that the pinion in Example Problem 9–1 is designed to have a life of 20 000 h. Then

$$N_c = (60)(L)(n)(q) = (60)(20\,000)(1750)(1) = 2.1 \times 10^9 \text{ cycles}$$

Because this is higher than 10^7, an adjustment in the allowable bending stress number must be made.

Reliability Factor, K_R

Table 9–8 gives data that adjust for the design reliability desired. These data are based on statistical analyses of failure data.

Factor of Safety, *SF*

The factor of safety may be used to account for the following:

TABLE 9–7 Recommended design life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000

Source: Eugene A. Avallone and Theodore Baumeister III, eds. *Marks' Standard Handbook for Mechanical Engineers.* 9th ed. New York: McGraw-Hill, 1986.

TABLE 9–8 Reliability factor, K_R

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

- Uncertainties in the design analysis
- Uncertainties in material characteristics
- Uncertainties in manufacturing tolerances

It may also be used to provide an extra measure of safety in critical applications.

No general guidelines are published, and designers must evaluate the conditions of each application. Note, however, that many of the factors often considered to be a part of a factor of safety in general design practice have already been included in the calculations for s_t and s_{at}. Therefore, a modest value for factor of safety should suffice, say, between 1.00 and 1.50.

Procedure for Selecting Gear Materials for Bending Stress

The logic of the material selection process can be summarized as follows: The bending stress number from Equation (9–15) must be less than the adjusted allowable bending stress number from Equation (9–17). That is,

$$s_t < s'_{at}$$

Let's equate the expressions for these two values:

$$\frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v = s_t < s_{at} \frac{Y_N}{SF \cdot K_R} \tag{9–19}$$

To use this relationship for material selection, it is convenient to solve for s_{at}:

$$\frac{K_R\,(SF)}{Y_N} s_t < s_{at} \tag{9–20}$$

We will use this equation for selecting gear materials based on bending stress. The list below summarizes the terms included in Equations (9–19) and (9–20) for your reference. You should review the more complete discussion of each before doing practice designs:

W_t = tangential force on gear teeth = $(63\,000)P/n$

P_d = diametral pitch of the gear

F = face width of the gear

J = geometry factor for bending stress (see Figure 9–17)

K_o = overload factor (see Table 9–5)

K_s = size factor (see Table 9–6)

K_m = mesh alignment factor = $1.0 + C_{pf} + C_{ma}$ (see Figures 9–18 and 9–19)

K_B = rim thickness factor (see Figure 9–20)

K_v = velocity factor (see Figure 9–21)

K_R = reliability factor (see Table 9–8)

SF = factor of safety (design decision)

Y_N = bending strength stress cycle number (see Figure 9–22)

Completing the calculation of the value of the left side of Equation (9–20) gives the required value for the allowable bending stress number, s_{at}. You should then go to the data in Section 9–7, "Gear Materials," to select a suitable material. Consider first whether the material should be steel, cast iron, or bronze. Then consult the related tables of data.

For steel materials, the following review should aid your selection:

1. Start by checking Figure 9–10 to see whether a through-hardened steel will give the needed s_{at}. If so, determine the required hardness. Then specify a steel material and its heat treatment by referring to Appendices 3 and 4.
2. If a higher s_{at} is needed, see Table 9–3 and Figures 9–14 and 9–15 for properties of case-hardened steels.
3. Appendix 5 will aid in the selection of carburized steels.
4. If flame or induction hardening is planned, specify a material with a good hardenability, such as AISI 4140 or 4340 or similar medium-carbon-alloy steels. See Appendix 4.
5. Refer to Figure 9–12 or 9–13 for recommended case depths for surface-hardened steels.

For cast iron or bronze, refer to Table 9–4.

Example Problem 9–2 Specify suitable materials for the pinion and the gear from Example Problem 9–1. Design for a reliability of fewer than one failure in 10 000. The application is an industrial saw that will be fully utilized on a normal, one-shift, five-day-per-week operation.

Solution The results of Example Problem 9–1 include the expected bending stress number for both the pinion and the gear as follows:

$$s_{tP} = 29\,700 \text{ psi} \quad s_{tG} = 23\,700 \text{ psi}$$

We should consider the number of stress cycles, the reliability, and the safety factor to complete the calculation indicated in Equation (9–20).

Stress Cycle Factor, Y_N: From the problem statement, $n_P = 1750$ rpm, $N_P = 20$ teeth, and $N_G = 70$ teeth. Let's use these data to determine the expected number of cycles of stress that the pinion and gear teeth will experience. The application conforms to common industry practice, calling for a design life of approximately 20 000 h as suggested in Table 9–7. The number of stress cycles for the pinion is

$$N_{cP} = (60)(L)(n_P)(q) = (60)(20\,000)(1750)(1) = 2.10 \times 10^9 \text{ cycles}$$

The gear rotates more slowly because of the speed reduction. Then

$$n_G = n_P(N_P/N_G) = (1750 \text{ rpm})(20/70) = 500 \text{ rpm}$$

We can now compute the number of stress cycles for each gear tooth:

$$N_{cG} = (60)(L)(n_G)(q) = (60)(20\,000)(500)(1) = 6.00 \times 10^8 \text{ cycles}$$

Because both values are above the nominal value of 10^7 cycles, a value of Y_N must be determined from Figure 9–22 for both the pinion and the gear:

$$Y_{NP} = 0.92 \quad Y_{NG} = 0.96$$

Reliability Factor, K_R: For the design goal of fewer than one failure in 10 000, Table 9–8 recommends $K_R = 1.50$.

Factor of Safety: This is a design decision. Reviewing the discussion of factors throughout Example Problem 9–1 and this problem, we see that virtually all factors typically considered in adjusting stress on the teeth and strength of the material have been taken into account. Furthermore, when we select a material, it will likely have strength and hardness values somewhat above the minimum acceptable values. Therefore, as a design decision, let's use $SF = 1.00$.

Adjusted Value of s_{at}: We can now complete Equation (9–20) and use it for material selection. For the pinion,

$$\frac{K_R\,(SF)}{Y_{NP}}\,s_t = \frac{(1.50)(1.00)}{0.92}\,(29\,700\text{ psi}) = 48\,450\text{ psi} < s_{at}$$

For the gear,

$$\frac{K_R\,(SF)}{Y_{NG}}\,s_t = \frac{(1.50)(1.00)}{0.96}\,(23\,700\text{ psi}) = 37\,050\text{ psi} < s_{at}$$

Now, referring to Figure 9–10, and deciding to use Grade 1 steel, we find that the required allowable bending stress number for the pinion is greater than permitted for a through-hardened steel. But Table 9–3 indicates that a carburized, case-hardened steel with a case hardness of 55 to 64 HRC would be satisfactory, having a value of $s_{at} = 55$ ksi = 55 000 psi. Referring to Appendix 5, we see that virtually any of the listed carburized materials could be used. Let's specify AISI 4320 SOQT 300, having a core tensile strength of 218 ksi, 13% elongation, and a case hardness of 62 HRC.

For the gear, Figure 9–10 indicates that a through-hardened steel with a hardness of 320 HB would be satisfactory. From Appendix 3, let's specify AISI 4340 OQT 1000 having a hardness of 363 HB, a tensile strength of 171 ksi, and 16% elongation.

Comments These materials should provide satisfactory service, considering bending strength. The following section considers the other major failure mode: pitting resistance. It is possible, perhaps likely, that the requirements to meet that condition will be more severe than for bending.

9–10 PITTING RESISTANCE OF GEAR TEETH

MDESIGN

In addition to being safe from bending, gear teeth must also be capable of operating for the desired life without significant pitting of the tooth form. *Pitting* is the phenomenon in which small particles are removed from the surface of the tooth faces because of the high contact stresses, causing fatigue. Refer again to Figure 9–16 showing the high, localized contact stresses. Prolonged operation after pitting begins causes the teeth to roughen, and eventually the form is deteriorated. Rapid failure follows. Note that both the driving and driven teeth are subjected to these high contact stresses.

The action at the contact point on gear teeth is that of two externally curved surfaces. If the gear materials were infinitely rigid, the contact would be a simple line. Actually, because of the elasticity of the materials, the tooth shape deforms slightly, resulting in the transmitted force acting on a small rectangular area. The resulting stress is called a *contact stress* or *Hertz stress*. Reference 24 gives the following form of the equation for the Hertz stress,

Hertz Contact Stress on Gear Teeth

$$\sigma_c = \sqrt{\frac{W_c}{F}\frac{1}{\pi\{[(1-\nu_1^2)/E_1] + [(1-\nu_2^2)/E_2]\}}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} \qquad \textbf{(9–21)}$$

where the subscripts 1 and 2 refer to the materials of the two bodies in contact. The tensile modulus of elasticity is E and the Poisson's ratio is ν. W_c is the contact force exerted between the two bodies, and F is the length of the contacting surfaces. The radii of curvature of the two surfaces are called r_1 and r_2.

When Equation 9–21 is applied to gears, F is the face width of the gear teeth and W_c is the normal force delivered by the driving tooth on the driven tooth, found from Equation (9–10) to be,

$$W_N = W_t/\cos\phi$$

The second term is Equation 9–21 (including the square root) can be computed if the elastic properties of the materials for the pinion and gear are known. It is given the name *elastic coefficient*, C_P. That is,

Elastic Coefficient

$$C_P = \sqrt{\frac{1}{\pi\{[(1-\nu_P^2)/E_P] + [(1-\nu_G^2)/E_G]\}}} \qquad \textbf{(9–22)}$$

Table 9–9 gives values for the most common combinations of materials for pinions and gears.

TABLE 9–9 Elastic coefficient, C_p

		Gear material and modulus of elasticity, E_G, lb/in² (MPa)					
Pinion material	Modulus of elasticity, E_P, lb/in² (MPa)	Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6	2300	2180	2160	2100	1950	1900
	(2×10^5)	(191)	(181)	(179)	(174)	(162)	(158)
Mall. iron	25×10^6	2180	2090	2070	2020	1900	1850
	(1.7×10^5)	(181)	(174)	(172)	(168)	(158)	(154)
Nod. iron	24×10^6	2160	2070	2050	2000	1880	1830
	(1.7×10^5)	(179)	(172)	(170)	(166)	(156)	(152)
Cast iron	22×10^6	2100	2020	2000	1960	1850	1800
	(1.5×10^5)	(174)	(168)	(166)	(163)	(154)	(149)
Al. bronze	17.5×10^6	1950	1900	1880	1850	1750	1700
	(1.2×10^5)	(162)	(158)	(156)	(154)	(145)	(141)
Tin bronze	16×10^6	1900	1850	1830	1800	1700	1650
	(1.1×10^5)	(158)	(154)	(152)	(149)	(141)	(137)

Source: Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.
Note: Poisson's ratio = 0.30; units for C_p are $(\text{lb/in}^2)^{0.5}$ or $(\text{MPa})^{0.5}$.

The terms r_1 and r_2 are the radii of curvature of the involute tooth forms of the two mating teeth. These radii change continuously during the meshing cycle as the contact point moves from the top of the tooth through the pitch circle, and onto the lower flank of the tooth before leaving engagement. We can write the following equations for the radius of curvature when contact is at the pitch point,

$$r_1 = (D_P/2)\sin\phi \quad \text{and} \quad r_2 = (D_G/2)\sin\phi \tag{9–23}$$

However, the AGMA calls for the computation of the contact stress to be made at the lowest point of single tooth contact (LPSTC) because above that point, the load is being shared with other teeth. Computation of the radii of curvature for the LPSTC is somewhat complex. A geometry factor for pitting, I, is defined by the AGMA to include the radii of curvature terms and the cos φ term in Equation (9–21) because they all involve the specific geometry of the tooth. The variables required to compute I are the pressure angle ϕ, the gear ratio $m_G = N_G/N_P$, and the number of teeth in the pinion N_p. Another factor is the pinion diameter that is not included in I. The contact stress equation then becomes,

$$\sigma_c = C_p\sqrt{\frac{W_t}{FD_PI}} \tag{9–24}$$

Values for the elastic coefficient, I, for a few common cases are graphed in Figure 9–23 and should be used for problem solving in this book. Appendix 19 provides an approach to computing the value for I for spur gears as given in Reference 3.

As with the equation for bending stress in gear teeth, several factors are added to the equation for contact stress as shown below. The resulting quantity is called the *contact stress number*, s_c:

Contact Stress Number

$$s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{FD_PI}} \tag{9–25}$$

This is the form of the contact stress equation that we will use in problem solutions.

The values for the overload factor, K_o; the size factor, K_s; the load-distribution factor, K_m; and the dynamic factor, K_v, can be taken to be the same as the corresponding values for the bending stress analysis in the preceding sections.

Example Problem 9–3 Compute the contact stress number for the gear pair described in Example Problem 9–1.

Solution Data from Example Problem 9–1 are summarized as follows:

$N_P = 20$	$N_G = 70,$	$F = 1.50$ in	$W_t = 720$ lb	$D_P = 2.500$ in
$K_o = 1.50$	$K_s = 1.00$	$K_m = 1.19$	$K_v = 1.45$	

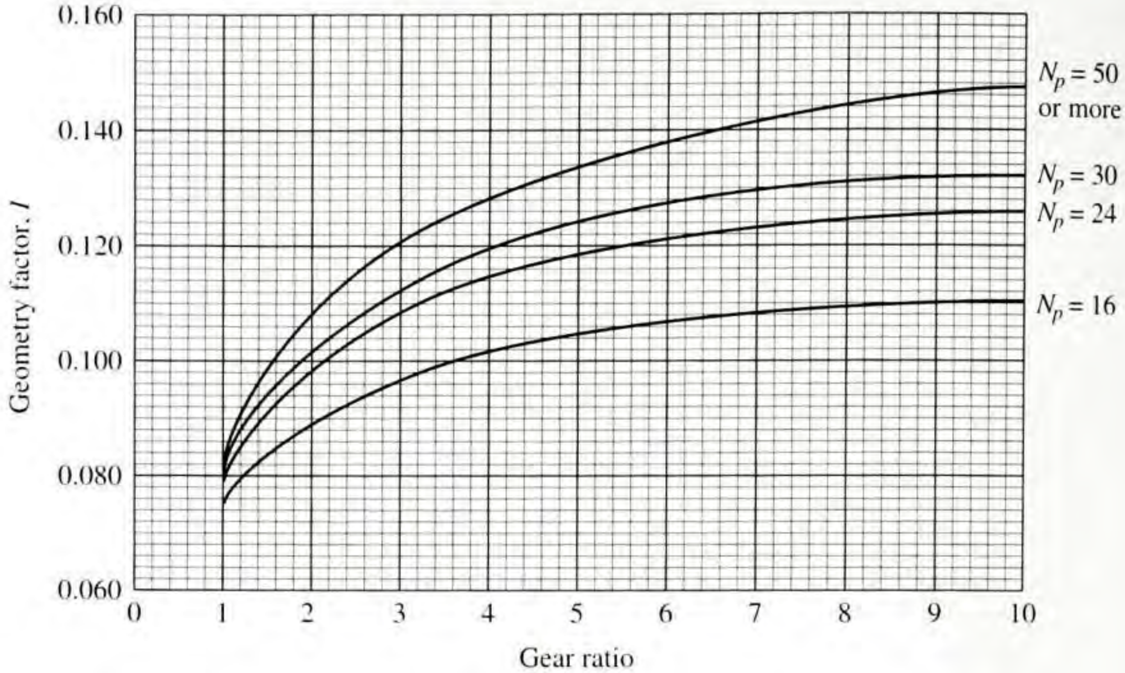
The gear teeth are 20°, full-depth, involute teeth. We also need the geometry factor for pitting resistance, I. From Figure 9–23(a), at a gear ratio of $m_G = N_G/N_P = 70/20 = 3.50$ and for $N_P = 20$, we read $I = 0.108$, approximately.

The design analysis for bending strength indicated that two steel gears should be used. Then, from Table 9–9, we find that $C_p = 2300$. Then the contact stress number is

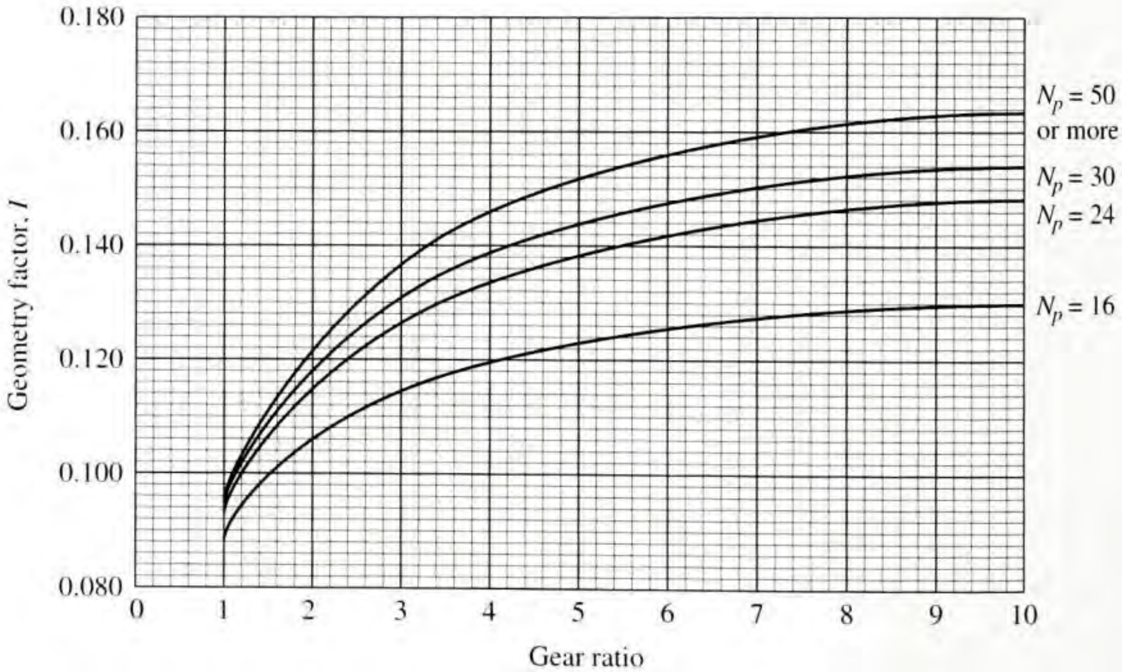
$$s_c = C_p\sqrt{\frac{W_tK_oK_sK_mK_v}{FD_PI}} = 2300\sqrt{\frac{(720)(1.50)(1.0)(1.19)(1.45)}{(1.50)(2.50)(0.108)}}$$

$$s_c = 156\,000 \text{ psi}$$

FIGURE 9–23 External spur pinion geometry factor, I, for standard center distances. All curves area for the lowest point of single-tooth contact on the pinion. (Extracted from AGMA Standard 218.01, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)



(a) 20° pressure angle, full-depth teeth (standard addendum = $1/P_d$)



(b) 25° pressure angle, full-depth teeth (standard addendum = $1/P_d$)

9–11 SELECTION OF GEAR MATERIAL BASED ON CONTACT STRESS

Because the pitting resulting from the contact stress is a different failure phenomenon from tooth failure due to bending, an independent specification for suitable materials for the pinion and the gear must now be made. In general, the designer must specify a material having an allowable contact stress number, s_{ac}, greater than the computed contact stress number, s_c. That is,

$$s_c < s_{ac}$$

In Section 9–6, values for s_{ac} were given for several materials that are valid for 10^7 cycles of loading at a reliability of 99% if the material temperature is under 250°F. For different life expectancy and reliability, other factors are added:

$$s_c < s_{ac}\frac{Z_N C_H}{(SF)K_R} \tag{9–26}$$

Designs in this book are limited to applications where the operating temperature is less than 250°F and so no temperature factor is applied. Data for the reduction in hardness and strength as a function of temperature should be sought if higher temperatures are experienced.

The reliability factor, K_R, is the same as that for bending stress; it is given in Table 9–8. The other factors in Equation (9–26) are discussed below.

Pitting Resistance Stress Cycle Factor, Z_N

The term Z_N is the *pitting resistance stress cycle factor* and accounts for an expected number of contacts different from 10^7 as was assumed when the data were produced for the allowable contact stress number. Figure 9–24 shows values for Z_N where the solid-line curve is for most steels and the dashed-line curve is for nitrided steels. The number of cycles of contact is computed from Equation (9–18) and is the same as that used for bending. For the higher number of cycles, a range is indicated by the shaded area. General design practice would use the upper line of this range. Critical applications where pitting and tooth wear must be minimal may use the lower part of the range.

Factor of Safety, *SF*

The factor of safety is based on the same conditions as described for bending, and often the same value would be used for both bending and pitting resistance. Review that discussion

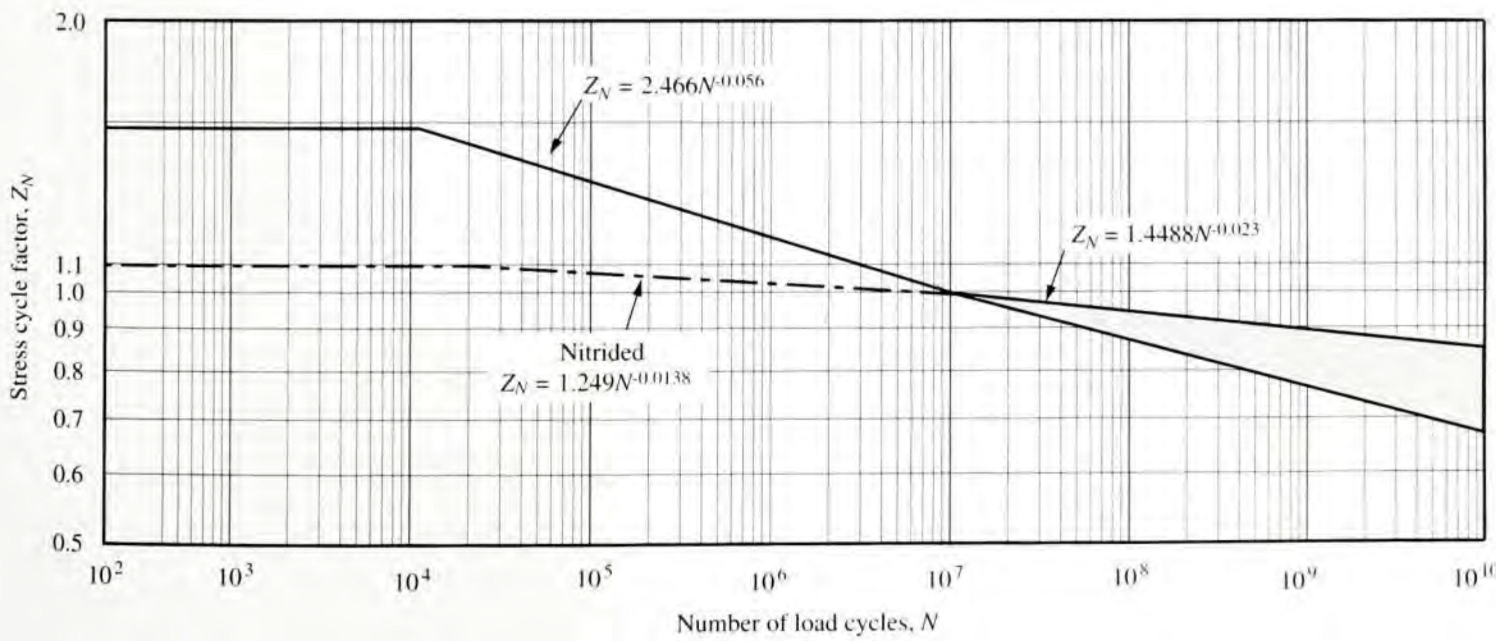


FIGURE 9–24 Pitting resistance stress cycle factor, Z_N (Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

in Section 9–9. However, if there are different levels of uncertainty, a different value should be chosen. No general guidelines are published. Because many factors are already considered in the pitting resistance calculations, a modest value for factor of safety should suffice, say, between 1.00 and 1.50.

Hardness Ratio Factor, C_H

Good gear design practice calls for making the pinion teeth harder than the gear teeth so that the gear teeth are smoothed and work-hardened during operation. This increases the gear capacity with regard to pitting resistance and is accounted for by the factor C_H. Figure 9–25 shows data for C_H for through-hardened gears that depend on the ratio of the hardness of the pinion and the hardness of the gear, expressed as the Brinell hardness number, and on the gear ratio where $m_G = N_G/N_P$. Use the given curves for hardness ratios between 1.2 and 1.7. For hardness ratios under 1.2, use $C_H = 1.00$. For hardness ratios over 1.7, use the value of C_H for 1.7, as no substantial additional improvement is gained.

Figure 9–26 shows data for C_H when pinions are surface-hardened to 48 HRC or higher and the gear is through-hardened up to 400 HB. The parameters are the Brinell hardness number for the gear and the surface finish of the pinion teeth, expressed as f_p and measured as the average roughness, R_a. Smoother teeth give a higher value of hardness factor and generally increase the pitting resistance of the gear teeth.

Note that C_H is applied for the calculations for only the gear, not the pinion.

When designing gears, the specification of the materials for the pinion and the gear is the final step. Therefore, the hardness for the two gears is unknown and a specific value for C_H cannot be determined. It is recommended that an initial value of $C_H = 1.00$ be used. Then, after specifying the materials, a final value for C_H can be determined and used in Equation 9–27 to determine the final value of S_{ac}.

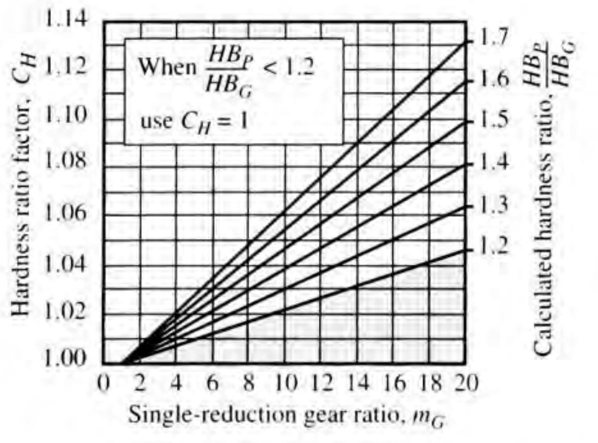


FIGURE 9–25 Hardness ratio facator, C_H (through-hardenced gears) (Extract from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

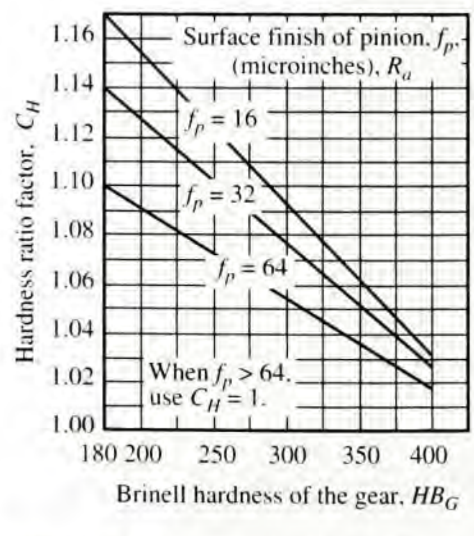


FIGURE 9–26 Hardness ratio factor, C_H (surface-hardened pinions) (Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

Procedure for Selecting Gear Materials for Pitting Resistance

We can refer to the value on the right side of Equation (9–26) as the *modified allowable contact stress number*, as it accounts for nonstandard conditions under which the gears operate that are different from those assumed when the data for s_{ac} were determined as reported in Section 9–6.

Transposing the modifying factors from Equation (9–26) to the left side of the equation gives

Required Allowable Contact Stress Number

$$\frac{K_R\,(SF)}{Z_N C_H} s_c < s_{ac} \tag{9–27}$$

This is the equation we will use to determine the required properties of most metallic materials used for gears. We can summarize the procedure as follows.

Procedure for Determining the Required Properties of Most Metallic Materials

1. Solve for the contact stress number, s_c, from Equation (9–25) using the same factors as those used for bending stress number.
2. Use the value for K_R from the bending stress analysis, or evaluate it from Table 9–8.
3. Use Figure 9–22 to find Z_N.
4. Assume an initial value for the hardness ratio factor $C_H = 1.00$.
5. Specify a safety factor, typically between 1.00 and 1.50, considering the degree of uncertainty in material property data, the gear precision, the severity of the application, or the danger presented by the application.
6. Compute s_{ac} from Equation (9–27).
7. Refer to data in Section 9–7, "Metallic Gear Materials," to select a suitable material. You should consider first whether the material should be steel, cast iron, or bronze. Then consult the related tables of data.

For cast iron or bronze, refer to Table 9–4. For steel materials, the following review should aid your selection:

1. Start by checking Figure 9–11 to see whether a through-hardened steel will give the needed s_{ac}. If so, determine the required hardness. Then specify a steel material and its heat treatment by referring to Appendices 3 and 4.
2. If a higher s_{ac} is needed, see Table 9–3 for properties of case-hardened steels.
3. Appendix 5 will aid in the selection of carburized steels.
4. If flame or induction hardening is planned, specify a material with a good hardenability, such as AISI 4140 or 4340 or similar medium-carbon-alloy steels. See Appendix 4.
5. Refer to Figure 9–12 or 9–13 for recommended case depths for surface-hardened steels.
6. If the specified pinion material has a significantly harder surface than the gear, refer to Figures 9–25 or 9–26 to determine a value for the hardness ratio factor C_H. Use Equation 9–27 to recompute the required s_{ac} and adjust the material selection if the data warrant a change.

Example Problem 9–4 Specify suitable materials for the pinion and the gear from Example Problem 9–3 based on contact stress. Application conditions are described in Example Problems 9–1 and 9–2.

Solution In Example Problem 9–3, we found that the expected contact stress number is $s_c = 156\,000$ psi. This should be modified as indicated in Equation (9–27).

In Example Problem 9–2, we determined that a carburized, case-hardened steel would be used for the pinion, and a through-hardened steel should be used for the gear. We must complete the material selection for pitting resistance independently from the bending stress analysis. However, we could not specify a material with lower properties than those specified in Example Problem 9–2 because then the bending strength would be inadequate.

In Example Problem 9–2, we used $K_R = 1.50$ for the desired reliability of fewer than one failure in 10 000. We decided to use $S_F = 1.00$ because we anticipated no unusual factors in the application that have not already been taken into account by other factors.

We can find Z_N from Figure 9–24 using 2.10×10^9 cycles of loading on the pinion and 6.00×10^8 cycles for the gear as computed in Example Problem 9–2. We find, then, that

$$Z_{NP} = 0.88 \quad Z_{NG} = 0.91$$

Let's complete the analysis for the pinion first. The hardness ratio factor does not apply to the pinion. Then Equation (9–27) gives

$$\frac{K_R(SF)}{Z_N} s_c = \frac{(1.50)(1.00)}{(0.88)}(156\,000 \text{ psi}) = 265\,900 \text{ psi} < s_{ac}$$

This value is quite high. Referring to Table 9–3, note that the only suitable listed material is a Grade 3 carburized and case-hardened steel having an allowable contact stress number of 275 ksi. Let's complete the calculations for the gear material and then discuss the results.

Assume an initial value for the hardness ratio factor $C_H = 1.00$. Then

Equation (9–27) gives

$$\frac{K_R(SF)}{Z_N C_H} s_c = \frac{(1.50)(1.00)}{(0.91)(1.00)}(156\,000 \text{ psi}) = 257\,100 \text{ psi} < s_{ac}$$

This value is also quite high, requiring the same Grade 3 carburized and case-hardened steel to provide adequate pitting resistance.

Comments and Design Decisions Specifying the Grade 3 carburized and case-hardened steel would be expected to provide adequate strength and pitting resistance for this pair of gears. However, the design is marginal, and it would be expensive because of the special requirements of cleanliness for the material and other guarantees related to material composition and microstructure. Most designs are executed using Grade 1 steel. It is recommended that the gears be redesigned to produce a lower bending stress and contact stress. In general, that can be achieved by using larger teeth (a smaller value for diametral pitch, P_d), a larger diameter for each gear, and a larger face width. Greater precision in the manufacture of the gears, producing a higher quality number, Q_v, would lower the dynamic factor and therefore reduce the bending stress number and the contact stress number. See section 9–15.

The next section outlines a design methodology for gears and provides guidelines for iterating on a design to produce alternatives from which we can select an optimum design for given conditions. In Section 9–14, a spreadsheet is developed that is useful for producing multiple iterations.

In Section 9–15, we use the same design requirements for the gear pair from Example Problems 9–1 through 9–4 as an example problem, making the necessary adjustments to our design decisions to ensure a satisfactory, economical gear design.

Then, in Chapter 15, this same design is used as the basis for a comprehensive discussion of the completion of the design of a power transmission. Considered there are the final selection of the gear design parameters, the design of the shafts for both the pinion and the gear (Chapter 12), the selection of two rolling contact bearings for each shaft (Chapter 14), and the enclosure of the transmission components in a suitable housing. Consideration is given for the inclusion of belt or chain drives for the input or output shafts of the gear reducer to provide more flexibility of its use. Chapter 15, then, is the culmination of virtually all of the design procedures presented in Part II of the book from Chapter 7 through Chapter 14.

9–12 DESIGN OF SPUR GEARS

MDESIGN

In designs involving gear drives, normally the required speeds of rotation of the pinion and the gear and the amount of power that the drive must transmit are known. These factors are determined from the application. Also, the environment and operating conditions to which the drive will be subjected must be understood. It is particularly important to know the type of driving device and the driven machine, in order to judge the proper value for the overload factor.

The designer must decide the type of gears to use; the arrangement of the gears on their shafts; the materials of the gears, including their heat treatment; and the geometry of the gears: numbers of teeth, diametral pitch, pitch diameters, tooth form, face width, and quality numbers.

This section presents a design procedure that accounts for the bending fatigue strength of the gear teeth and the pitting resistance, called *surface durability*. This procedure makes extensive use of the design equations presented in the preceding sections of the chapter and of the tables of material properties in Appendices 3 through 5, 8, and 12.

You should understand that there is no one best solution to a gear design problem; several good designs are possible. Your judgment and creativity and the specific requirements of the application will greatly affect the final design selected. The purpose here is to provide a means of approaching the problem to create a reasonable design.

Design Objectives

Some overall objectives of a design are listed below. The resulting drive should

- Be compact and small
- Operate smoothly and quietly
- Have long life
- Be low in cost
- Be easy to manufacture
- Be compatible with the other elements in the machine, such as bearings, shafts, the housing, the driver, and the driven machine

The major objective of the design procedure is to define a safe, long-lasting gear drive. General steps and guidelines are outlined here to produce a reasonable initial design. However, because of the numerous variables involved, several iterations are typically made to work toward an optimum design. Details of the procedure are presented in Example Problem 9–5.

Procedure for Designing a Safe and Long-Lasting Gear Drive

1. From the design requirements, identify the input speed of the pinion, n_P, the desired output speed of the gear, n_G, and the power to be transmitted, P.
2. Choose the type of material for the gears, such as steel, cast iron, or bronze.
3. Considering the type of driver and the driven machine, specify the overload factor, K_o, using Table 9–5. The primary concern is the expected level of shock or impact loading.
4. Specify a trial value for the diametral pitch. When steel gears are used, Figure 9–27 provides initial guidance. The graph of design power transmitted versus the pinion rotational speed was derived for selected pitches and pinion diameters. Design power, $P_{des} = K_o P$. Steel that is through hardened to HB 300 is used. Because of the numerous variables involved, the value of P_d read from the figure is only an initial target value. Subsequent iterations may require considering a different value.
5. Specify the face width within the following recommended range for general machine drive gears:

Nominal Face Width

$$8/P_d < F < 16/P_d$$

$$\text{Nominal value of } F = 12/P_d \qquad \textbf{(9–28)}$$

   The upper limit given tends to minimize alignment problems and ensure reasonably uniform loading across the face. When the face width is less than the lower limit, it is probable that a more compact design can be achieved with a different pitch. Also, the face width normally is less than twice the pitch diameter of the pinion.

6. Compute or specify the transmitted load, pitch line speed, quality number, geometry factor, and other factors required in the equations for bending stress and contact stress.
7. Compute the bending stress and the contact stress on the pinion and gear teeth. Judge whether the stresses are reasonable (nether too low nor too high) in terms of being able to specify a suitable material. If not, select a new pitch or revise the number of teeth, pitch diameter, or face width. Typically the contact stress on the pinion is the limiting value for gears designed for a long life.
8. Iterate the design process to seek more optimum designs. It is not unusual to make several trials before settling on a particular design. Using computer aids such as the spreadsheets described in Section 9–12 can make successive trials quickly.

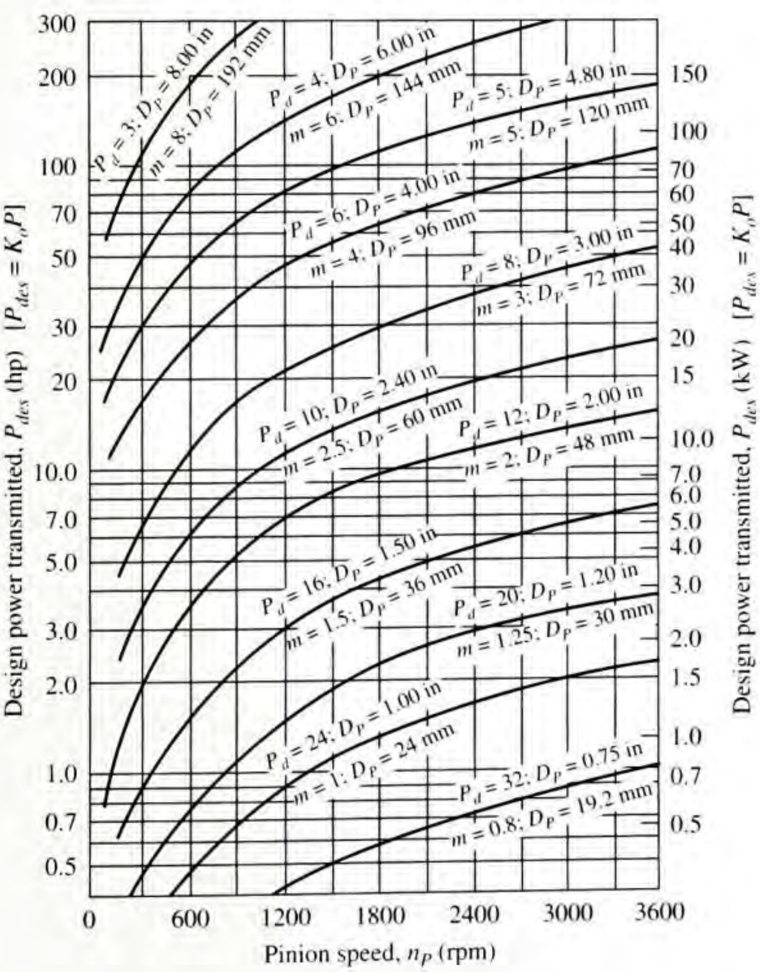
Guidelines for Adjustments in Successive Iterations

The following relationships should help you determine what changes in your design assumptions you should make after the first set of calculations to achieve a more optimum design:

- Decreasing the numerical value of the diametral pitch results in larger teeth and generally lower stresses. Also, the lower value of the pitch usually means a larger face width, which decreases stress and increases surface durability.
- Increasing the diameter of the pinion decreases the transmitted load, generally lowers the stresses, and improves the surface durability.

- Increasing the face width lowers the stress and improves the surface durability, but to a generally lesser extent than either the pitch or the pitch diameter changes discussed previously.
- Gears with more and smaller teeth tend to run more smoothly and quietly than gears with fewer and larger teeth.
- Standard values of diametral pitch should be used for ease of manufacture and lower cost (see Table 8–2).
- Using high-alloy steels with high surface hardness results in the most compact system, but the cost in higher.
- Using very accurate gears (with ground or shaved teeth) results in a higher quality number, lower dynamic loads, and consequently lower stresses and improved surface durability, but the cost is higher.
- The number of teeth in the pinion should generally be as small as possible to make the system compact. But the possibility of interference is greater with fewer teeth. Check Table 8–6 to ensure that no interference will occur. (See Reference 22.)

FIGURE 9–27
Design power transmitted vs. pinion speed for spur gears with different pitches and diameters



For all curves: 20° full depth teeth:
$N_P = 24$; $N_G = 96$; $m_G = 4.00$; $F = 12/P_d$; $Q_v = 6$
Steel gears, HB 300; $s_{at} = 36000$ psi; $s_{ac} = 126000$ psi

Example Problem 9–5 Design a pair of spur gears to be used as a part of the drive for a chipper to prepare pulpwood for use in a paper mill. Intermittent use is expected. An electric motor transmits 3.0 horsepower to the pinion at 1750 rpm and the gear must rotate between 460 and 465 rpm. A compact design is desired.

Solution and General Design Procedure

Step 1. Considering the transmitted power, P, the pinion speed, n_P, and the application, refer to Figure 9–27 to determine a trial value for the diametral pitch, P_d. The overload factor, K_o, can be determined from Table 9–5, considering both the power source and the driven machine.

For this problem, $P = 3.0$ hp and $n_P = 1750$ rpm, $K_o = 1.75$ (uniform driver, heavy shock driven machine). Then $P_{des} = (1.75)\,(3.0 \text{ hp}) = 5.25$ hp. Try $P_d = 12$ for the initial design.

Step 2. Specify the number of teeth in the pinion. For small size, use 17 to 20 teeth as a start.

For this problem, let's specify $N_P = 18$.

Step 3. Compute the nominal velocity ratio from $VR = n_P/n_G$.

For this problem, use $n_G = 462.5$ rpm at the middle of the acceptable range.

$$VR = n_P/n_G = 1750/462.5 = 3.78$$

Step 4. Compute the approximate number of teeth in the gear from $N_G = N_P(VR)$.

For this problem, $N_G = N_P(VR) = 18(3.78) = 68.04$. Specify $N_G = 68$.

Step 5. Compute the actual velocity ratio from $VR = N_G/N_P$

For this problem, $VR = N_G/N_P = 68/18 = 3.778$.

Step 6. Compute the actual output speed from $n_G = n_P\,(N_P/N_G)$.

For this problem, $n_G = n_P\,(N_P/N_G) = (1750 \text{ rpm})(18/68) = 463.2$ rpm. OK.

Step 7. Compute the pitch diameters, center distance, pitch line speed, and transmitted load and judge the general acceptability of the results.

For this problem, the pitch diameters are:

$$D_P = N_P/P_d = 18/12 = 1.500 \text{ in}$$

$$D_G = N_G/P_d = 68/12 = 5.667 \text{ in}$$

Center distance:

$$C = (N_P + N_G)/(2P_d) = (18 + 68)/(24) = 3.583 \text{ in}$$

Pitch line speed: $v_t = \pi D_P n_P/12 = [\pi(1.500)(1.750)]/12 = 687$ ft/min

Transmitted load: $W_t = 33\,000(P)/v_t = 33\,000(3.0)/687 = 144$ lb

These values seem to be acceptable.

Step 8. Specify the face width of the pinion and the gear using Equation (9–28) as a guide.

For this problem: Lower limit = $8/P_d = 8/12 = 0.667$ in.

Upper limit = $16/P_d = 16/12 = 1.333$ in

Nominal value = $12/P_d = 12/12 = 1.00$ in. Use this value.

Step 9. Specify the type of material for the gears and determine C_p from Table 9–9.

For this problem, specify two steel gears. $C_p = 2300$.

Step 10. Specify the quality number, Q_v, using Table 9–2 as a guide. Determine the dynamic factor from Figure 9–21.

For this problem, specify $Q_v = 6$ for a wood chipper. $K_v = 1.35$.

Step 11. Specify the tooth form, the bending geometry factors for the pinion and the gear from Figure 9–17 and the pitting geometry factor from Figure 9–23.

For this problem, specify 20° full depth teeth. $J_P = 0.325$, $J_G = 0.410$, $I = 0.104$.

Step 12. Determine the load distribution factor, K_m, from Equation (9–16) and Figures 9–18 and 9–19. The precision class of the gear system design must be specified. Values may be computed from equations in the figures or read from the graphs.

For this problem: $F = 1.00$ in, $D_P = 1.500$. $F/D_P = 0.667$. Then $C_{pf} = 0.042$.

Specify open gearing for the wood chipper, mounted to the frame. $C_{ma} = 0.264$.

Compute: $K_m = 1.0 + C_{pf} + C_{ma} + 0.042 + 0.264 = 1.31$

Step 13. Specify the size factor, K_s from Table 9–6.

For this problem, $K_s = 1.00$ for $P_d = 12$

Step 14. Specify the rim thickness factor, K_B, from Figure 9–20.

For this problem, specify a solid gear blank. $K_B = 1.00$.

Step 15. Specify a service factor, SF, typically from 1.00 to 1.50, based on uncertainty of data.

For this problem, there is no unusual uncertainty. Let $SF = 1.00$.

Step 16. Specify a hardness ratio factor, C_H, for the gear, if any. Use $C_H = 1.00$ for the early trials until materials have been specified. Then adjust C_H if significant differences exist in the hardness of the pinion and the gear.

Step 17. Specify a reliability factor using Table 9–8 as a guideline.

For this problem, specify a reliability of 0.99. $K_R = 1.00$.

Step 18. Specify a design life. Compute the number of loading cycles for the pinion and the gear. Determine the stress cycle factors for bending (Y_N) and pitting (Z_N) for the pinion and the gear.

For this problem, intermittent use is expected. Specify the design life to be 3000 hours, similar to agricultural machinery. The numbers of loading cycles are:

$$N_{cP} = (60)(3000\text{ hr})(1750\text{ rpm})(1) = 3.15 \times 10^8\text{ cycles}$$
$$N_{cG} = (60)(3000\text{ hr})(462.5\text{ rpm})(1) = 8.33 \times 10^7\text{ cycles}$$

Then, from Figure 9–22, $Y_{NP} = 0.96$, $Y_{NG} = 0.98$. From Figure 9–24, $Z_{NP} = 0.92$, $Z_{NG} = 0.95$.

Step 19. Compute the expected bending stresses in the pinion and the gear using Equation (9–15).

$$s_{tP} = \frac{W_t P_d}{FJ_P} K_o K_s K_m K_B K_v = \frac{(144)(12)}{(1.00)(0.325)}(1.75)(1.0)(1.31)(1.0)(1.35) = 16\,400 \text{ psi}$$

$$s_{tG} = s_{tP}(J_P/J_G) = (16\,400)(0.325/0.410) = 13\,000 \text{ psi}$$

Step 20. Adjust the bending stresses using Equation 9–20.

For this problem, for the pinion:

$$S_{atP} > S_{tP}\frac{K_R(SF)}{Y_{NP}} = (16\,400)\frac{(1.00)(1.00)}{0.96} = 17\,100 \text{ psi}$$

For the gear:

$$S_{atG} > S_{tG}\frac{K_R(SF)}{Y_{NG}} = (13\,000)\frac{(1.00)(1.00)}{0.98} = 13\,300 \text{ psi}$$

Step 21. Compute the expected contact stress in the pinion and the gear from Equation (9–25). Note that this value will be the same for both the pinion and the gear.

$$s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{FD_pI}} = 2300\sqrt{\frac{(144)(1.75)(1.0)(1.31)(1.35)}{(1.00)(1.50)(0.104)}} = 122\,900 \text{ psi}$$

Step 22. Adjust the contact stresses for the pinion and the gear using Equation (9–27).

$$S_{acP} > S_{cP}\frac{K_R(SF)}{Z_{NP}} = (122\,900)\frac{(1.00)(1.00)}{(0.92)} = 133\,500 \text{ psi}$$

For the gear:

$$S_{acG} > S_{cG}\frac{K_R(SF)}{Z_{NG}C_H} = (122\,900)\frac{(1.00)(1.00)}{(0.95)(1.00)} = 129\,300 \text{ psi}$$

Step 23. Specify materials for the pinion and the gear that will have suitable through hardening or case hardening to provide allowable bending and contact stresses greater than those required from Steps 20 and 22. Typically the contact stress is the controlling factor. Refer to Figures 9–10 and 9–11 and Tables 9–3 and 9–4 for data on required hardness. Refer to Appendices 3 to 5 for properties of steel to specify a particular alloy and heat treatment.

For this problem contact stress is the controlling factor. Figure 9–11 shows that through hardening of steel with a hardness of HB 320 is required for both the pinion and the gear. From Figure A4–4, we can specify AISI 4140 OQT 1000 steel that has a hardness of HB 341, giving a value of $s_{ac} = 140\,000$ psi. Ductility is adequate as indicated by the 18% elongation value. Other materials could be specified.

9–13 GEAR DESIGN FOR THE METRIC MODULE SYSTEM

Section 8-4, "Gear Nomenclature and Gear-Tooth Features," described the metric module system of gearing and its relation to the diametral pitch system. As the design process was being developed in Sections 9–6 through 9–11, the data for stress analysis and surface durability analysis were taken from charts using U.S. Customary units (in, lb, hp, ft/min, and ksi). Data for the metric module system were also available in the charts in units of millimeters (mm), newtons (N), kilowatts (kW), meters per second (m/s), and megapascals (MPa). But to use the SI data, we must modify some of the formulas.

The following example problem uses SI units. The procedure will be virtually the same as that used to design with U.S. Customary units. Those formulas that are converted to SI units are identified.

Example Problem 9–6 A gear pair is to be designed to transmit 15.0 kilowatts (kW) of power to a large meat grinder in a commercial meat processing plant. The pinion is attached to the shaft of an electric motor rotating at 575 rpm. The gear must operate at 270 to 280 rpm. The gear unit will be enclosed and of commercial quality. Commercially hobbed (quality number 5), 20°, full-depth, involute gears are to be used in the metric module system. The maximum center distance is to be 200 mm. Specify the design of the gears. Use $K_R = C_H = SF = Z_N = 1.00$.

Solution The nominal velocity ratio is

$$VR = 575/275 = 2.09$$

Specify an overload factor of $K_o = 1.50$ from Table 9–5 for a uniform power source and moderate shock for the meat grinder. Then computer design power,

$$P_{des} = K_o P = (1.50)(15\text{ kW}) = 22.5\text{ kW}$$

From Figure 9–27, $m = 4$ is a reasonable trial module. Then

$$N_P = 18 \quad \text{(design decision)}$$
$$D_P = N_P m = (18)(4) = 72\text{ mm}$$
$$N_G = N_p(VR) = (18)(2.09) = 37.6 \quad \text{(Use 38.)}$$
$$D_G = N_G m = (38)(4) = 152\text{ mm}$$
$$\text{Final output speed} = n_G = n_P(N_P/N_G)$$
$$n_G = 575\text{ rpm} \times (18/38) = 272\text{ rpm} \quad \text{(okay)}$$
$$\text{Center distance} = C = (N_P + N_G)m/2 \quad \text{[Equation (8–18)]}$$
$$C = (18 + 38)(4)/2 = 112\text{ mm} \quad \text{(okay)}$$

In SI units, the pitch line speed in meters per second (m/s) is

$$v_t = \pi D_P n_P/(60\,000)$$

where D_P is in mm and n_P is in revolutions per minute (rpm). Then

$$v_t = [(\pi)(72)(575)]/(60\,000) = 2.17\text{ m/s}$$

In SI units, the transmitted load, W_t, is in newtons (N). If the power, P, is in kW, and v_t is in m/s,

$$W_t = 1000(P)/v_t = (1000)(15)/(2.17) = 6920\text{ N}$$

In the U.S. Customary Unit System, it was recommended that the face width be approximately $F = 12/P_d$ in. The equivalent SI value is $F = 12(m)$ mm. For this problem, $F = 12(4) = 48$ mm. Let's use $F = 50$ mm.

Other factors are found as before.

$$K_s = K_B = 1.00$$
$$K_v = 1.34 \quad \text{(Figure 9–21)}$$
$$K_m = 1.21 \quad \text{(Figures 9–19 and 9–19)} \quad (F/D_P = 50/72 = 0.69)$$
$$J_P = 0.315 \quad J_G = 0.380 \quad [\text{Figure 9–17(a)}]$$

Then the stress in the pinion is found from Equation (9–15), modified by letting $P_d = 1/m$:

$$S_{tP} = \frac{W_t\, K_o\, K_s\, K_B\, K_m\, K_v}{Fm\, J_P} = \frac{(6920)(1.50)(1)(1.21)(1.34)}{(50)(4)(0.315)} = 269 \text{ MPa}$$

This is a reasonable stress level. The required hardness of grade 1 material is HB 360, as found in Figure 9–10. Proceed with design for pitting resistance.

For two steel gears,

$$K_s = 1.0$$
$$C_p = 191 \quad \text{(Table 9–10)}$$
$$I = 0.092 \quad \text{(Figure 9–23)}$$
$$K_v = 1.34$$
$$K_o = 1.50$$
$$K_m = 1.21$$

The contact stress [Equation (9–25)] gives

$$s_c = C_P\sqrt{\frac{W_t\, K_o\, K_s\, K_m\, K_v}{FD_P I}} = 191\sqrt{\frac{(6920)(1.50)(1.0)(1.21)(1.34)}{(50)(72)(0.092)}} = 1367 \text{ MPa}$$

Converting to ksi gives

$$s_c = 1367 \text{ MPa} \times 1 \text{ ksi}/6.895 \text{ MPa} = 198 \text{ ksi}$$

From Table 9–3, the required surface hardness is HRC 58-64, case-carburized, Grade 2.

Material selection from Appendix 5, for carburized steels, is as follows:

AISI 4320 SOQT 300; $s_u = 1500$ MPa; 13% elongation, Grade 2

Case-harden by carburizing to HRC 58 minimum

Case depth: 0.6 mm minimum (Figure 9–12)

Comment: Redesigning the gears to permit using Grade 1 material is recommended.

9–14 COMPUTER-AIDED SPUR GEAR DESIGN AND ANALYSIS

This section presents one approach to assisting the gear designer with the many calculations and judgments that must be made to produce an acceptable design. The spreadsheet shown in Figure 9–28 facilitates the completion of a prospective design for a pair of gears in a few minutes by an experienced designer. You must have studied all of the material in Chapters 8 and 9 in order to understand the data needed in the spreadsheet and to use it effectively.

The recommended use of the spreadsheet is to create a series of design iterations that allow you to progress toward an optimum design in a short amount of time. It follows the process outlined in Section 9–12 up to the point of computing the required allowable bending stress number and the allowable contact stress number for both the pinion and the gear. The designer must use those data to specify suitable materials for the gears and their heat treatments.

Given below is a discussion of the essential features of the spreadsheet. In general, it first calls for the input of basic performance data, allowing a proposed geometry to be specified. The final result is the completion of the stress analyses for bending and pitting resistance for both the pinion and the gear. Equations (9–19) and (9–20) are combined for the bending analysis. The analysis of pitting resistance uses Equations (9–25), and (9–27). The designer must provide data for the several factors in those equations taken from appropriate figures and charts or based on design decisions. Virtually all computations are performed by the spreadsheet, allowing the designer to exercise judgment based on the intermediate results.

The format used for the spreadsheet helps the designer follow the process. After defining the problem at the top of the sheet, the first column at the left calls for several pieces of input data. Any value in italics within a gray-shaded area must be entered by the designer. White areas offer the results of calculations and provide guidance. The upper part of the second column also guides the designer in determining values for the several factors needed to complete the stress analyses for bending and pitting resistance. The area at the lower right of the spreadsheet gives the primary output data on which the design decisions for materials and heat treatments are based.

The data in Figure 9–28 are taken from Example Problem 9–5 which was completed in the traditional manner in Section 9–12.

Discussion of the Use of the Spur Gear Design Spreadsheet

1. ***Describing the application:*** In the heading of the sheet, the designer is asked to describe the application for identification purposes and to focus on the basic uses for the gears. Of particular interest is the nature of the prime mover and the driven machine.
2. ***Initial input data:*** It is assumed that designers begin with a knowledge of the power transmission requirement, the rotational speed of the pinion of the gear pair, and the desired output speed. Use Figure 9–27 to determine a trial value of the diametral pitch based on the design power to be transmitted and the rotational speed of the pinion. The number of teeth in the pinion is a critical design decision because the size of the system depends on this value. Ensure against interference.
3. ***Number of gear teeth:*** The spreadsheet computes the approximate number of gear teeth to produce the desired output speed from $N_G = N_P(n_G/n_P)$. But, of course, the number of teeth in any gear must be an integer, and the actual value of N_G is entered by the designer.

DESIGN SPUR OF GEARS

Initial Input Data:

Input power:	$P =$	*3*	*hp*
Input speed:	$n_P =$	*1,750.*	*rpm*
Diametral pitch:	$P_d =$	*12*	
Number of pinion teeth:	$N_P =$	*18*	
Desired output speed:	$n_G =$	*462.5*	*rpm*
Computed number of gear teeth:		68.1	
Enter: Chosen no. of gear teeth:	$N_G =$	*68*	

Computed Data:

Actual output speed:	$n_G =$	463.2	rpm
Gear ratio:	$m_G =$	3.78	
Pitch diameter, pinion:	$D_P =$	1.500	in
Pitch diameter, gear:	$D_G =$	5.667	in
Center distance:	$C =$	3.583	in
Pitch line speed:	$v_t =$	687	ft/min
Transmitted load:	$W_t =$	144	lb

Secondary Input Data:

	Min.	Nom.	Max.
Face width guidelines (in):	0.667	1.000	1.333

Enter: Face width:	$F =$ *1.000 in*	
Ratio: Face width/pinion diameter:	$F/D_P = 0.67$	
Recommended range of ratio:	$F/D_P < 2.00$	
Enter: Elastic coefficient:	$C_p =$ *2300*	Table 9–9
Enter: Quality number:	$Q_v = 6$	Table 9–2
Enter: Bending geometry factors:		
Pinion:	$J_P =$ *0.325*	Fig. 9–17
Gear:	$J_G =$ *0.410*	Fig. 9–17
Enter: Pitting geometry factor:	$I =$ *0.104*	Fig. 9–23

FIGURE 9–28 Spreadsheet solution for Example Problem 9–5

Application:	*Wood chipper driven by an electric motor* *Example Problem 9–5*			
Factors in Design Analysis:				
Alignment factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P = 0.67$	
Pinion proportion factor, C_{pf} =	0.042	0.042	[$0.50 < F/D_P < 2.00$]	
Enter: C_{pf} =	*0.042*		Fig. 9–18	
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh alignment factor, C_{ma} =	0.264	0.143	0.080	0.048
Enter: C_{ma} =	*0.264*		Fig. 9–19	
Alignment factor: K_m =	1.31		[Computed]	
Overload factor: K_o =	*1.75*		Table 9–5	
Size factor: K_s =	*1.00*		Table 9–6: Use 1.00 if $P_d \geq 5$.	
Pinion rim thickness factor: K_{BP} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Gear rim thickness factor: K_{BG} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Dynamic factor: K_v =	1.35		[Computed: See Fig. 9–21.]	
Service factor: SF =	*1.00*		Use 1.00 if no unusual conditions.	
Hardness ratio factor: C_H =	*1.00*		Fig. 9–25 or 9–26; gear only	
Reliability factor: K_R =	*1.00*		Table 9–8: Use 1.00 for $R = 0.99$.	
Enter: Design life: =	*3000 hours*		See Table 9–7.	
Pinion—Number of load cycles: N_P =	$3.2E+08$		Guidelines: Y_N, Z_N	
Gear—Number of load cycles: N_G =	$8.3E+07$		10^7 cycles	$>10^7$	$<10^7$
Bending stress cycle factor: Y_{NP} =	*0.96*		1.00	0.96	Fig. 9–22
Bending stress cycle factor: Y_{NG} =	*0.98*		1.00	0.98	Fig. 9–22
Pitting stress cycle factor: Z_{NP} =	*0.92*		1.00	0.92	Fig. 9–24
Pitting stress cycle factor: Z_{NG} =	*0.95*		1.00	0.95	Fig. 9–24
Stress Analysis: Bending				
Pinion: Required s_{at} =	17,102 psi	See Fig. 9–10 or		
Gear: Required s_{at} =	13,280 psi	Table 9–3 or 9–4.		
Stress Analysis: Pitting				
Pinion: Required s_{ac} =	133,471 psi	See Fig. 9–11 or		
Gear: Required s_{ac} =	129,256 psi	Table 9–3 or 9–4.		
Specify materials, alloy and heat treatment, for most severe requirement.				
One possible material specification:				
Pinion: Requires HB > 320; AISI 4140 OQT 1000, HB = 341, S_{ac} = 140,000 psi				
Gear: Requires HB > 320; AISI 4140 OQT 1000, HB = 341, S_{ac} = 140,000 psi				

FIGURE 9–28 *Continued*

4. ***Computed data:*** The seven values reported in the middle of the first column are all determined from the input data, and they allow the designer to evaluate the suitability of the geometry of the proposed design at this point. Changes to the input data can be made at this time if any value is out of the desired range in the judgment of the designer.

5. ***Secondary input data:*** When a suitable geometry for the gears is obtained, the designer enters the data called for at the lower part of the first column of the spreadsheet. The locations of data in pertinent tables and figures are listed.

6. ***Factors in design analysis:*** The stress analysis requires many factors to account for the unique situation of the design being pursued. Again, guidance is offered, but the designer must enter the values of the required factors. Many of the factors can have a value of 1.00 for normal conditions or to produce a conservative result.

7. ***Alignment factor:*** The alignment factor depends on two other factors: the pinion proportion factor and the mesh alignment factor as shown in Figures 9–18 and 9–19. The suggested values in the white areas are computed from the equations given in the figures. Note the listed value of F/D_P. If $F/D_P < 0.50$, use $F/D_P = 0.50$ to find C_{pf}. The designer must decide on the type of gearing to be used (open or closed) and the degree of precision to be designed into the system. The final result is computed from the input data.

8. ***Overload, size, and rim thickness factors:*** Consult Tables 9–5 and 9–6 along with Figure 9–20. Note that the rim thickness factor can be different for the pinion and the gear. Sometimes the smaller pinion is made from a solid blank while the larger gear can use a rim-and-spoke design.

9. ***Dynamic factor:*** The spreadsheet uses the equations included in Figure 9–21 to compute the dynamic factor using the quality number and pitch line speed found from data in the first column.

10. ***Service factor:*** This is a design decision as discussed in Section 9–12. Often a value of 1.00 is used if no unusual conditions are expected that are not already accounted for in other factors. Larger service factors allow for a higher degree of safety or to account for uncertainties.

11. ***Hardness ratio factor:*** This value depends on the ratio of the hardness of the pinion to the hardness of the gear teeth as shown in Figures 9–25 and 9–26. At the start of the design, these data are not known, and it is suggested that a value of $C_H = 1.00$ be used initially. Then, after one or more iterations are completed with tentative specifications for the pinion and gear materials, the value can be adjusted to refine the design. The factor C_H is applied only in the pitting resistance stress analysis for the gear, and it may allow the use of a less expensive or more ductile material with a lower hardness.

12. ***Reliability factor:*** The designer must select a value from Table 9–8 according to the desired level of reliability.

13. ***Stress cycle factors:*** Here the designer must specify the design life in hours of operation for the gear pair being designed. Table 9–7 provides suggestions according to the use of the system. The number of cycles of stress is then computed for both the pinion and the gear, assuming the normal case of one cycle of one-direction stress per revolution. If the gears operate in a reversing mode, as idlers, or in planetary gear trains, this calculation must be adjusted to account for the multiple cycles of stress experienced in each revolution. Guidelines recommend factors of 1.00 for 10^7 cycles

for which the allowable stress numbers are computed. For a larger number of cycles, equations given in Figures 9–22 and 9–24 are used to compute the recommended factors. Because a variety of data are given for the case of fewer than 10^7 cycles, the designer is referred to the figures to determine the factors. In any case, the user of the spreadsheet must enter the selected values.

14. ***Stress analyses for bending and pitting resistance:*** Finally, the required allowable bending stress number and the required allowable contact stress number are computed using Equations (9–20) and (9–27), adjusted for the special values of factors for the pinion and the gear.
15. ***Specification of the materials and their heat treatment:*** The final step is left to the designer to use the computed values from the stress analyses and to specify materials that will provide an adequate strength and surface hardness of the gear teeth. Pertinent data are listed in Figures 9–10 and 9–11 and Tables 9–3 and 9–4. The Appendices tables for material properties may also be consulted once the required hardnesses of the materials are determined.

9–15 USE OF THE SPUR GEAR DESIGN SPREADSHEET

The spreadsheet developed in Section 9–14 is a useful tool that aids the designer in the process of completing a design for a pair of gears to be safe with regard to bending stresses in the teeth of the gears and for pitting resistance. The use of the spreadsheet was demonstrated for the data in Example Problem 9–5 as shown in Figure 9–28.

A more important use for the spreadsheet is to propose and analyze several design alternatives and to work toward a goal of optimizing the design with regard to size, cost, or other parameters important to a particular design objective.

In this section, we use the data for Example Problems 9–1 through 9–4 to work toward an improved solution. The basic requirements were to produce a satisfactory design to transmit 25 hp with a pinion speed of 1750 rpm and a gear speed of 500 rpm. The initial trial started in Example Problem 9–1 and carried through the other problems called for gears with a diametral pitch of 8 and 20 teeth in the pinion and 70 teeth in the gear. A quality number of 6 was chosen. Although those sound like reasonable choices, it was shown that the resulting stresses were significantly higher than desirable, particularly the required allowable contact stress number. It was shown in Example Problem 9–4 that a Grade 3 steel, case-hardened by carburizing, was required. This is a very expensive design because of the extreme controls on the material composition and cleanliness and because of the time-consuming heat treatment process.

Designers for typical machine and vehicle drives would plan to use Grade 1 steels and standard quenching and tempering heat treatments. Where small size is critical or where cost is not a major concern, case hardening by carburizing, induction or flame hardening, or nitriding can be used. Therefore, it is usually desirable to produce several design alternatives that can be analyzed for cost and manufacturability. Then the final selection can be made with assurance that a reasonably optimum design has been identified.

The spreadsheet in Figure 9–29 shows the combined results of Example Problems 9–1 through 9–4. The resulting stresses are slightly different because of small differences in the factors computed by the spreadsheet formulas and those read from charts. This summary can lead you to alternatives that meet the design goal of using only Grade 1 steels and achieving a cost-effective design.

Successive Iterations. We now continue the design process by making carefully selected changes in design decisions using the *Guidelines for Adjustments in Successive Iterations* from Section 9–12, just before Example Problem 9–5. The goal in this case is to reduce the required contact stress number to permit the use of lower cost Grade 1 steels.

Figures 9–30, 9–31, and 9–32 show three additional trial designs for the system described in Example Problem 9–1. Each of these designs reaches the goal of producing a practical design that allows the use of Grade 1 steels. You should study these designs to ascertain the differences among the design decisions and the resulting required allowable stress numbers for both bending and pitting resistance. Figure 9–33 summarizes the major results from all of the trials.

DESIGN SPUR OF GEARS				
Initial Input Data:				
Input power:	$P =$	*25*	*hp*	
Input speed:	$n_P =$	*1750*	*rpm*	
Diametral pitch:	$P_d =$	*8*		
Number of pinion teeth:	$N_P =$	*20*		
Desired output speed:	$n_G =$	*500*	*rpm*	
Computed number of gear teeth:		70.0		
Enter: Chosen no. of gear teeth:	$N_G =$	*70*		
Computed Data:				
Actual output speed:	$n_G =$	500.0	rpm	
Gear ratio:	$m_G =$	3.50		
Pitch diameter, pinion:	$D_P =$	2.500	in	
Pitch diameter, gear:	$D_G =$	8.750	in	
Center distance:	$C =$	5.625	in	
Pitch line speed:	$v_t =$	1145	ft/min	
Transmitted load:	$W_t =$	720	lb	
Secondary Input Data:				
	Min.	Nom.	Max.	
Face width guidelines (in):	1.000	1.500	2.000	
Enter: Face width:	$F =$	*1.500*	*in*	
Ratio: Face width/pinion diameter:	$F/D_P =$	0.60		
Recommended range of ratio:	$F/D_P <$	2.00		
Enter: Elastic coefficient:	$C_p =$	*2300*		Table 9–9
Enter: Quality number:	$Q_v =$	6		Table 9–2
Enter: Bending geometry factors:				
Pinion:	$J_P =$	*0.335*		Fig. 9–17
Gear:	$J_G =$	*0.420*		Fig. 9–17
Enter: Pitting geometry factor:	$I =$	*0.108*		Fig. 9–23

FIGURE 9–29 Spreadsheet solution for Example Problem 9–1 through 9–4

Application:	*Industrial saw driven by an electric motor* *Example Problems 9–1 through 9–4*			
Factors in Design Analysis:				
Alignment factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If *F* < *1.0*	If *F* > *1.0*	$F/D_P = 0.60$	
Pinion proportion factor, C_{pf} =	0.035	0.041	[0.50 < F/D_P < 2.00]	
Enter: C_{pf} =	*0.041*		Fig. 9–18	
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh alignment factor, C_{ma} =	0.272	0.150	0.086	0.053
Enter: C_{ma} =	*0.15*		Fig. 9–19	
Alignment factor: K_m =	1.19		[Computed]	
Overload factor: K_o =	*1.50*		Table 9–5	
Size factor: K_s =	*1.00*		Table 9–6: Use 1.00 if $P_d \geq 5$.	
Pinion rim thickness factor: K_{BP} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Gear rim thickness factor: K_{BG} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Dynamic factor: K_v =	1.45		[Computed: See Fig. 9–21.]	
Service factor: *SF* =	*1.00*		Use 1.00 if no unusual conditions.	
Hardness ratio factor: C_H =	*1.00*		Fig. 9–25 or 9–26; gear only	
Reliability factor: K_R =	*1.50*		Table 9–8: Use 1.00 for R = 0.99.	
Enter: Design life: =	*20,000 hours*		See Table 9–7.	

		Guidelines: Y_N, Z_N		
Pinion—Number of load cycles: N_P =	2.1*E*+09			
Gear—Number of load cycles: N_G =	6.0*E*+08	10^7 cycles	$>10^7$	$<10^7$
Bending stress cycle factor: Y_{NP} =	*0.93*	1.00	0.93	Fig. 9–22
Bending stress cycle factor: Y_{NG} =	*0.95*	1.00	0.95	Fig. 9–22
Pitting stress cycle factor: Z_{NP} =	*0.88*	1.00	0.88	Fig. 9–24
Pitting stress cycle factor: Z_{NG} =	*0.91*	1.00	0.91	Fig. 9–24

Stress Analysis: Bending

Pinion: Required s_{at} =	47,871 psi	See Fig. 9–10 or
Gear: Required s_{at} =	37,379 psi	Table 9–3 or 9–4.

Stress Analysis: Pitting

Pinion: Required s_{ac} =	265,989 psi	See Fig. 9–11 or
Gear: Required s_{ac} =	257,170 psi	Table 9–3 or 9–4.

Specify materials, alloy and heat treatment, for most severe requirement.

One possible material specification:

Pinion: Requires grade 3 carburized, case hardened steel

Gear: Requires grade 3 carburized, case hardened steel

FIGURE 9–29 *Continued*

Note that in all of the trials, the contact stress is critical. That is, the required hardness or case-hardening treatment for the gear teeth is most severe with regard to achieving adequate pitting resistance. The bending stress numbers are quite moderate. This situation is typical for steel gears, and designers often work first toward a satisfactory contact stress and then check to ensure that the bending stress is acceptable.

The trials shown in Figure 9–33 are arranged in order of increasing pinion diameter and increasing center distance, with the consequent decreasing of the bending and contact

DESIGN SPUR OF GEARS

Initial Input Data:

Input power:	$P =$	*25*	*hp*
Input speed:	$n_P =$	*1750*	*rpm*
Diametral pitch:	$P_d =$	*6*	
Number of pinion teeth:	$N_P =$	*18*	
Desired output speed:	$n_G =$	*500*	*rpm*
Computed number of gear teeth:		63.0	
Enter: Chosen no. of gear teeth:	$N_G =$	*63*	

Computed Data:

Actual output speed:	$n_G =$	500.0	rpm
Gear ratio:	$m_G =$	3.50	
Pitch diameter, pinion:	$D_P =$	3.000	in
Pitch diameter, gear:	$D_G =$	10.500	in
Center distance:	$C =$	6.750	in
Pitch line speed:	$v_t =$	1374	ft/min
Transmitted load:	$W_t =$	600	lb

Secondary Input Data:

	Min.	Nom.	Max.
Face width guidelines (in):	1.333	2.000	2.667

Enter: Face width:	$F =$ *2.750 in*		
Ratio: Face width/pinion diameter:	$F/D_P =$	0.92	
Recommended range of ratio:	$F/D_P <$	2.00	
Enter: Elastic coefficient:	$C_p =$	*2300*	Table 9–9
Enter: Quality number:	$Q_v =$	8	Table 9–2
Enter: Bending geometry factors:			
Pinion:	$J_P =$	*0.325*	Fig. 9–17
Gear:	$J_G =$	*0.410*	Fig. 9–17
Enter: Pitting geometry factor:	$I =$	*0.105*	Fig. 9–23

FIGURE 9–30 Redesign for data of Example Problem 9–1, Trial 1

Application:	*Industrial saw driven by an electric motor* *Redesign for data of Example Problem 9–1, Trial 1*

Factors in Design Analysis:

Alignment factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P = 0.92$	
Pinion proportion factor, C_{pf} =	0.067	0.089	[$0.50 < F/D_P < 0.92$]	
Enter: C_{pf} =	*0.089*		Fig. 9–18	
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh alignment factor, C_{ma} =	0.292	0.170	0.102	0.065
Enter: C_{ma} =	*0.17*		Fig. 9–19	
Alignment factor: K_m =	1.26		[Computed]	
Overload factor: K_o =	*1.50*		Table 9–5	
Size factor: K_s =	*1.00*		Table 9–6: Use 1.00 if $P_d \geq 5$.	
Pinion rim thickness factor: K_{BP} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Gear rim thickness factor: K_{BG} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Dynamic factor: K_v =	1.30		[Computed: See Fig. 9–21.]	
Service factor: SF =	*1.00*		Use 1.00 if no unusual conditions.	
Hardness ratio factor: C_H =	*1.00*		Fig. 9–25 or 9–26; gear only	
Reliability factor: K_R =	*1.50*		Table 9–8: Use 1.00 for R = 0.99.	
Enter: Design life: =	*20 000 hours*		See Table 9–7.	

		Guidelines: Y_N, Z_N		
Pinion—Number of load cycles: N_P =	2.1E+09			
Gear—Number of load cycles: N_G =	6.0E+08	10^7 cycles	$>10^7$	$<10^7$
Bending stress cycle factor: Y_{NP} =	*0.93*	1.00	0.93	Fig. 9–22
Bending stress cycle factor: Y_{NG} =	*0.95*	1.00	0.95	Fig. 9–22
Pitting stress cycle factor: Z_{NP} =	*0.88*	1.00	0.88	Fig. 9–24
Pitting stress cycle factor: Z_{NG} =	*0.91*	1.00	0.91	Fig. 9–24

Stress Analysis: Bending

Pinion: Required s_{at} =	16,009 psi	See Fig. 9–10 or
Gear: Required s_{at} =	12,423 psi	Table 9–3 or 9–4.

Stress Analysis: Pitting

Pinion: Required s_{ac} =	161,968 psi	See Fig. 9–11 or
Gear: Required s_{ac} =	156,629 psi	Table 9–3 or 9–4.

Specify materials, alloy and heat treatment, for most severe requirement.

One possible material specification:

Pinion: AISI 4140 Induction Hardened to 50 HRC, Grade 1

Gear: AISI 4140 OQT 800, HB 429, Grade 1

FIGURE 9–30 *Continued*

stresses. These are the most critical factors. Diametral pitch, face width, and quality numbers are also varied, but their effects are secondary.

The designs represented by Figures 9–30, 9–31, and 9–32 could all use the same material, provided that it could be hardened to the required levels. For example, specifying AISI 4340 Grade 1 steel or some similar medium-carbon-alloy steel would ensure that it could be flame or induction-hardened to a minimum of 50 HRC or through-hardened by quenching and tempering to the required 350 to 400 HB hardness on the Brinell scale.

DESIGN SPUR OF GEARS

Initial Input Data:

Input power:	$P =$	*25*	*hp*
Input speed:	$n_P =$	*1750*	*rpm*
Diametral pitch:	$P_d =$	*8*	
Number of pinion teeth:	$N_P =$	*28*	
Desired output speed:	$n_G =$	*500*	*rpm*
Computed number of gear teeth:		98.0	
Enter: Chosen no. of gear teeth:	$N_G =$	*98*	

Computed Data:

Actual output speed:	$n_G =$	500.0	rpm
Gear ratio:	$m_G =$	3.50	
Pitch diameter, pinion:	$D_P =$	3.500	in
Pitch diameter, gear:	$D_G =$	12.250	in
Center distance:	$C =$	7.875	in
Pitch line speed:	$v_t =$	1604	ft/min
Transmitted load:	$W_t =$	514	lb

Secondary Input Data:

	Min.	Nom.	Max.
Face width guidelines (in):	1.000	1.500	2.000

Enter: Face width:	$F =$ *2.000 in*		
Ratio: Face width/pinion diameter:	$F/D_P = 0.57$		
Recommended range of ratio:	$F/D_P < 2.00$		
Enter: Elastic coefficient:	$C_p =$ *2300*	Table 9–9	
Enter: Quality number:	$Q_v =$ 8	Table 9–2	
Enter: Bending geometry factors:			
Pinion:	$J_P =$ *0.380*	Fig. 9–17	
Gear:	$J_G =$ *0.440*	Fig. 9–17	
Enter: Pitting geometry factor:	$I =$ *0.115*	Fig. 9–23	

FIGURE 9–31 Redesign for data for Example Problem 9–1, Trial 2

Application:	*Industrial saw driven by an electric motor* *Redesign for data of Example Problem 9–1, Trial 2*			
Factors in Design Analysis:				
Alignment factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P = 0.57$	
Pinion proportion factor, C_{pf} =	0.032	0.045	[$0.50 < F/D_P < 2.00$]	
Enter: C_{pf} =	*0.045*	Fig. 9–18		
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh alignment factor, C_{ma} =	0.280	0.158	0.093	0.058
Enter: C_{ma} =	*0.158*	Fig. 9–19		
Alignment factor: K_m =	1.20	[Computed]		
Overload factor: K_o =	*1.50*	Table 9–5		
Size factor: K_s =	*1.00*	Table 9–6: Use 1.00 if $P_d \geq 5$.		
Pinion rim thickness factor: K_{BP} =	*1.00*	Fig. 9–20: Use 1.00 if solid blank.		
Gear rim thickness factor: K_{BG} =	*1.00*	Fig. 9–20: Use 1.00 if solid blank.		
Dynamic factor: K_v =	1.30	[Computed: See Fig. 9–21.]		
Service factor: SF =	*1.00*	Use 1.00 if no unusual conditions.		
Hardness ratio factor: C_H =	*1.00*	Fig. 9–25 or 9–26; gear only		
Reliability factor: K_R =	*1.50*	Table 9–8: Use 1.00 for R = 0.99.		
Enter: Design life: =	*20,000 hours*	See Table 9–7.		
Pinion—Number of load cycles: N_P =	2.1*E*+09	Guidelines: Y_N, Z_N		
Gear—Number of load cycles: N_G =	6.0*E*+08	10^7 cycles	$>10^7$	$<10^7$
Bending stress cycle factor: Y_{NP} =	*0.93*	1.00	0.93	Fig. 9–22
Bending stress cycle factor: Y_{NG} =	*0.95*	1.00	0.95	Fig. 9–22
Pitting stress cycle factor: Z_{NP} =	*0.88*	1.00	0.88	Fig. 9–24
Pitting stress cycle factor: Z_{NG} =	*0.91*	1.00	0.91	Fig. 9–24
Stress Analysis: Bending				
Pinion: Required s_{at} =	20,915 psi	See Fig. 9–10 or Table 9–3 or 9–4.		
Gear: Required s_{at} =	17,682 psi			
Stress Analysis: Pitting				
Pinion: Required s_{ac} =	153,363 psi	See Fig. 9–11 or Table 9–3 or 9–4.		
Gear: Required s_{ac} =	148,307 psi			
Specify materials, alloy and heat treatment, for most severe requirement.				
One possible material specification:				
Pinion: AISI 4140 OQT 800, HB 429, Grade 1				
Gear: AISI 4140 OQT 900, HB 388, Grade 1				

FIGURE 9–31 *Continued*

DESIGN SPUR OF GEARS

Initial Input Data:

Input power:	$P =$	*25*	*hp*
Input speed:	$n_P =$	*1,750*	*rpm*
Diametral pitch:	$P_d =$	*6*	
Number of pinion teeth:	$N_P =$	*24*	
Desired output speed:	$n_G =$	*500*	*rpm*
Computed number of gear teeth:		84.0	
Enter: Chosen no. of gear teeth:	$N_G =$	*84*	

Computed Data:

Actual output speed:	$n_G =$	500.0	rpm
Gear ratio:	$m_G =$	3.50	
Pitch diameter, pinion:	$D_P =$	4.000	in
Pitch diameter, gear:	$D_G =$	14.000	in
Center distance:	$C =$	9.000	in
Pitch line speed:	$v_t =$	1833	ft/min
Transmitted load:	$W_t =$	450	lb

Secondary Input Data:

	Min.	Nom.	Max.
Face width guidelines (in):	1.333	2.000	2.667

Enter: Face width:	$F =$	*2.000 in*	
Ratio: Face width/pinion diameter:	$F/D_P =$	0.50	
Recommended range of ratio:	$F/D_P <$	2.00	
Enter: Elastic coefficient:	$C_p =$	*2300*	Table 9–9
Enter: Quality number:	$Q_v =$	6	Table 9–2
Enter: Bending geometry factors:			
Pinion:	$J_P =$	*0.360*	Fig. 9–17
Gear:	$J_G =$	*0.430*	Fig. 9–17
Enter: Pitting geometry factor:	$I =$	*0.112*	Fig. 9–23

FIGURE 9–32 Redesign for data for Example Problem 9–1, Trial 3

Application:	*Industrial saw driven by an electric motor*			
	Redesign for data of Example Problem 9–1, Trial 3			
Factors in Design Analysis:				
Alignment factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P = 0.50$	
Pinion proportion factor, C_{pf} =	0.025	0.038	$[0.50 < F/D_P < 2.00]$	
Enter: C_{pf} =	*0.038*		Fig. 9–18	
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh alignment factor, C_{ma} =	0.280	0.158	0.093	0.058
Enter: C_{ma} =	*0.158*		Fig. 9–19	
Alignment factor: K_m =	1.20		[Computed]	
Overload factor: K_o =	*1.50*		Table 9–5	
Size factor: K_s =	*1.00*		Table 9–6: Use 1.00 if $P_d \geq 5$.	
Pinion rim thickness factor: K_{BP} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Gear rim thickness factor: K_{BG} =	*1.00*		Fig. 9–20: Use 1.00 if solid blank.	
Dynamic factor: K_v =	1.56		[Computed: See Fig. 9–21.]	
Service factor: SF =	*1.00*		Use 1.00 if no unusual conditions.	
Hardness ratio factor: C_H =	*1.00*		Fig. 9–25 or 9–26; gear only	
Reliability factor: K_R =	*1.50*		Table 9–8: Use 1.00 for R = 0.99.	
Enter: Design life: =	*20,000 hours*		See Table 9–7.	

		Guidelines: Y_N, Z_N		
Pinion—Number of load cycles: N_P =	2.1E+09	10^7 cycles	$>10^7$	$<10^7$
Gear—Number of load cycles: N_G =	6.0E+08			
Bending stress cycle factor: Y_{NP} =	*0.93*	1.00	0.93	Fig. 9–22
Bending stress cycle factor: Y_{NG} =	*0.95*	1.00	0.95	Fig. 9–22
Pitting stress cycle factor: Z_{NP} =	*0.88*	1.00	0.88	Fig. 9–24
Pitting stress cycle factor: Z_{NG} =	*0.91*	1.00	0.91	Fig. 9–24

Stress Analysis: Bending

Pinion: Required s_{at} =	16,961 psi	See Fig. 9–10 or
Gear: Required s_{at} =	13,901 psi	Table 9–3 or 9–4.

Stress Analysis: Pitting

Pinion: Required s_{ac} =	147,128 psi	See Fig. 9–11 or
Gear: Required s_{ac} =	142,277 psi	Table 9–3 or 9–4.

Specify materials, alloy and heat treatment, for most severe requirement.

One possible material specification:

Pinion: AISI 4140 OQT 800, HB 429, Grade 1

Gear: AISI 4140 OQT 900, HB 388, Grade 1

FIGURE 9–32 *Continued*

COMPARISON OF DESIGN ALTERNATIVES FOR DATA OF EXAMPLE PROBLEMS 9–1 THROUGH 9–4

P = 25 *hp* Pinion speed = 1750 rpm Gear speed = 500 rpm

	Original Solution and Alternative Trials				
	Problems 9–1 through 9–4	**Alternative Trial 1**	**Alternative Trial 2**	**Alternative Trial 3**	
Geometry, Quality, and Transmitted Load:					**Comments:**
N_P	20	18	28	24	All designs produce same gear ratio.
N_G	70	63	98	84	
P_d	8	6	8	6	Note change in diametral pitch.
D_P (in)	2.500	3.000	3.500	4.000	Increasing pinion diameter.
D_G (in)	8.750	10.500	12.250	14.000	Increasing gear diameter.
C (in)	5.625	6.750	7.875	9.000	Increasing center distance.
W_t (lb)	720	600	514	450	Decreasing transmitted load.
F (in)	1.50	2.75	2.00	2.00	Face width varies.
Q_v	6	8	8	6	Note change in quality number.
K_v	1.45	1.30	1.33	1.56	Note change in dynamic factor.
Stresses:					
s_{atP} (psi)	47,900	16,000	20,900	17,000	Moderate bending stress.
s_{atG} (psi)	37,400	12,450	17,700	13,900	Moderate bending stress.
s_{acP} (psi)	266,000	162,000	153,400	147,150	Contact stress critical.
s_{acG} (psi)	257,100	156,700	148,300	142,300	Contact stress critical.
Materials:					
Pinion	Grade 3 steel, carburized, case-hardened	Grade 1 steel, induction-hardened, 50 HRC	Grade 1 steel, through-hardened, 400 HB	Grade 1 steel, through-hardened, 400 HB	Design goal is to use only Grade 1 steels.
Gear	Grade 3 steel, carburized, case-hardened	Grade 1 steel, through-hardened, 400 HB	Grade 1 steel, through-hardened, 370 HB	Grade 1 steel, through-hardened, 350 HB	

FIGURE 9–33 Comparison of design alternative trials for data of Example Problem 9–1

9–16 POWER-TRANSMITTING CAPACITY

It is sometimes desirable to compute the amount of power that a gear pair can safely transmit after it has been completely defined. The *power-transmitting capacity* is the capacity when the tangential load causes the expected stress to equal the allowable stress number with all of the modifying factors considered. The capacity should be computed for both bending and pitting resistance and for both the pinion and the gear.

When similar materials are used for both the pinion and the gear, it is likely that the pinion will be critical for bending stress. But the most critical condition is usually pitting resistance. The following relationships can be used to compute the power-transmitting capacity. In this analysis, it is assumed that the operating temperature of the gears and their lubricants is 250°F and that gears are produced with the appropriate surface finish.

Bending

We start with Equation (9–19) in which the computed bending stress number is compared with the modified allowable bending stress number for the gear:

$$\frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v = s_t < s_{at}\frac{Y_N}{(SF)K_R}$$

But solving for W_t gives

$$W_t = \frac{s_{at} Y_N FJ}{(SF)K_R K_o K_s K_m K_B K_v P_d} \tag{9–29}$$

It was shown in Equation (9–6) that

$$W_t = (126\,000)(P)/(n_P D_P)$$

Then substituting into Equation (9–29) gives

$$\frac{(126\,000)(P)}{n_P D_P} = \frac{s_{at} Y_N FJ}{(SF)K_R K_o K_s K_m K_B K_v P_d}$$

Solving for P, we have

$$P = \frac{s_{at}\, Y_N\, FJ\, n_P D_P}{(126\,000)(P_d)(SF)\, K_R\, K_o\, K_s\, K_m\, K_B\, K_v} \tag{9–30}$$

This equation should be solved for both the pinion and the gear. Most variables will be the same except for s_{at}, Y_N, J, and possibly K_B.

Pitting Resistance

Here we start with Equations (9–25), (9–26), and (9–27) in which the computed contact stress number is compared with the modified allowable contact stress number for the gear. Equation (9–26) can be expressed in the form

$$s_c = C_P\sqrt{\frac{W_t K_o K_s K_m K_v}{D_P FI}} = \frac{s_{ac} Z_N C_H}{(SF)K_R}$$

Squaring both sides of this equation and solving for W_t gives

$$\frac{W_t K_o K_s K_m K_v}{D_P FI} = \left[\frac{s_{ac} Z_N C_H}{(SF)K_R C_P}\right]^2$$

$$W_t = \frac{D_P FI}{K_o K_s K_m K_v}\left[\frac{s_{ac} Z_N C_H}{(SF)K_R C_P}\right]^2$$

Now substituting this into Equation (9–6) and solving for the power P gives

$$P = \frac{W_t D_P n_P}{126\,000} = \frac{D_P n_P D_P F I}{126\,000\, K_o K_s K_m K_v}\left[\frac{s_{ac} Z_N C_H}{(SF) K_R C_p}\right]^2$$

$$P = \frac{n_P F I}{126\,000\, K_o K_s K_m K_v}\left[\frac{s_{ac}\, D_P Z_N C_H}{(SF)\, K_R C_P}\right]^2 \qquad \textbf{(9–31)}$$

Equations (9–30) and (9–31) should be used to compute the power-transmitting capacity for a pair of gears of known design with particular materials.

9–17 PRACTICAL CONSIDERATIONS FOR GEARS AND INTERFACES WITH OTHER ELEMENTS

It is important to consider the design of the entire gear system when designing the gears because they must work in harmony with the other elements in the system. This section will briefly discuss some of these practical considerations and will show commercially available speed reducers.

Our discussion so far has been concerned primarily with the gear teeth, including the tooth form, pitch, face width, material selection, and heat treatment. Also to be considered is the type of gear blank. Figures 8–2 and 8–4 show several styles of blanks. Smaller gears and lightly loaded gears are typically made in the plain style. Gears with pitch diameters of approximately 5.0 in through 8.0 in are frequently made with thinned webs between the rim and the hub for lightening, with some having holes bored in the webs for additional lightening. Larger gears, typically with pitch diameters greater than 8.0 in, are made from cast blanks with spokes between the rim and the hub.

In many precision special machines and gear systems produced in large quantities, the gears are machined integral with the shaft carrying the gears. This, of course, eliminates some of the problems associated with mounting and location of the gears, but it may complicate the machining operations.

In general machine design, gears are usually mounted on separate shafts, with the torque transmitted from the shaft to the gear through a key. This setup provides a positive means of transmitting the torque while permitting easy assembly and disassembly. The axial location of the gear must be provided by another means, such as a shoulder on the shaft, a retaining ring, or a spacer (see Chapters 11 and 12).

Other considerations include the forces exerted on the shaft and the bearings that are due to the action of the gears. These subjects are discussed in Section 9–3. The housing design must provide adequate support for the bearings and protection of the interior components. Normally, it must also provide a means of lubricating the gears.

See References 12–15 and 18–19 for additional practical considerations.

Lubrication

The action of spur gear teeth is a combination of rolling and sliding. Because of the relative motion, and because of the high local forces exerted at the gear faces, adequate lubrication is critical to smoothness of operation and gear life. A continuous supply of oil at the pitch line is desirable for most gears unless they are lightly loaded or operate only intermittently.

In splash-type lubrication, one of the gears in a pair dips into an oil supply sump and carries the oil to the pitch line. At higher speeds, the oil may be thrown onto the inside surfaces of the case; then it flows down, in a controlled fashion, onto the pitch line. Simultaneously, the oil can be directed to the bearings that support the shafts. One difficulty with the splash type of lubrication is that the oil is churned; at high gear speeds, excessive heat can be generated, and foaming can occur.

A positive oil circulation system is used for high-speed and high-capacity systems. A separate pump draws the oil from the sump and delivers it at a controlled rate to the meshing teeth.

The primary functions of gear lubricants are to reduce friction at the mesh and to keep operating temperatures at acceptable levels. It is essential that a continuous film of lubricant be maintained between the mating tooth surfaces of highly loaded gears and that there be a sufficient flow rate and total quantity of oil to maintain cool temperatures. Heat is generated by the meshing gear teeth, by the bearings, and by the churning of the oil. This heat must be dissipated from the oil to the case or to some other external heat-exchange device in order to keep the oil itself below 160°F (approximately 70°C). Above this temperature, the lubricating ability of the oil, as indicated by its viscosity, is severely decreased. Also, chemical changes can be produced in the oil, decreasing its lubricity. Because of the wide variety of lubricants available and the many different conditions under which they must operate, it is recommended that suppliers of lubricants be consulted for proper selection. (See also Reference 10.)

The AGMA, in Reference 10, defines several types of lubricants for use in gear drives.

- ***Rust and oxidation inhibited gear oils*** (called R & O) are petroleum based with chemical additives.
- ***Compounded gear lubricants*** (Comp) blend 3% to 10% of fatty oils with petroleum oils.
- ***Extreme pressure lubricants*** (EP) include chemical additives that inhibit scuffing of gear tooth faces.
- ***Synthetic gear lubricants*** (S) are special chemical formulations applied mostly in severe operating conditions.

R & O lubricants are supplied in 14 viscosity grades (0 to 13) where the lower numbers refer to the lower viscosities. Similar numbers are used for the other types with modified grade designations carrying suffixes *Comp*, *EP*, or *S*. The recommended lubricant grade depends on the ambient temperature around the drive and the pitch line velocity of the lowest speed pair of gears in a reducer. See the Table 9–10. Wormgear drives call for higher viscosity grades.

Commercially Available Gear-Type Speed Reducers

By studying the design of commercially available gear-type speed reducers, you should get a better feel for design details and the relationships among the component parts: the gears, the shafts, the bearings, the housing, the means of providing lubrication, and the coupling to the driving and driven machines.

Figure 9–34 shows a double-reduction spur gear speed reducer with an electric motor rigidly attached. Such a unit is often called a *gear motor*. Figure 9–35 is similar, except that one of the stages of reduction uses helical gears (discussed in the next chapter). The cross-sectional drawing shown with Figure 9–36 gives a clear picture of the several components of a reducer.

The planetary reducer in Figure 9–37 has quite a different design to accommodate the placement of the sun, planet, and ring gears. Figure 9–38 shows the eight-speed transmission from a large farm tractor and illustrates the high degree of complexity that may be involved in the design of transmissions.

TABLE 9–10 Recommended lubricant grade for spur, helical, herringbone, and bevel gear drives.

Pitch line velocity	Ambient temperature –40°F to 14°F –40°C to –10°C	 14°F to 50°F –10°C to 10°C	 50°F to 95°F 10°C to 35°C	 95°F to 131°F 35°C to 55°C
	Lubricant grade			
Less than 1000 ft/min Less than 5m/s	3S	4	6	8
1000 to 3000 ft/min 5 to 15 m/s	3S	3	5	7
3000 to 5000 ft/min 15 to 25 m/s	2S	2	4	6
Over 5000 ft/min Over 25 m/s	0S	0	2	3

Extracted from AGMA Standard 9005-D94, *Industrial Gear Lubrication*, with permission of the publisher, American Gear Manufactures Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

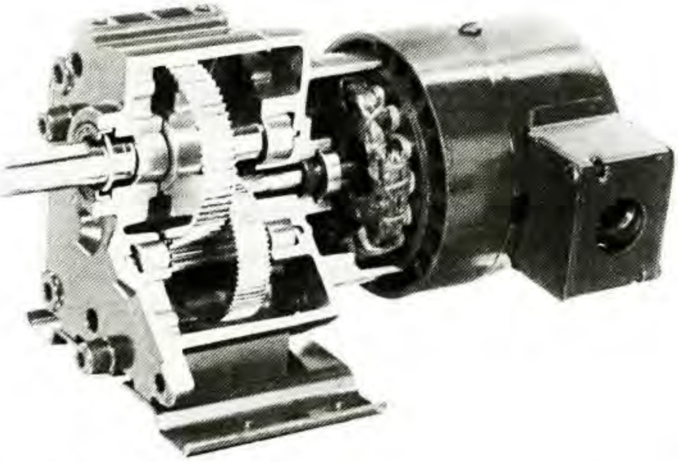


FIGURE 9–34 Double-reduction spur gear reducer (Bison Gear & Engineering Corporation, Downers Grove, IL)

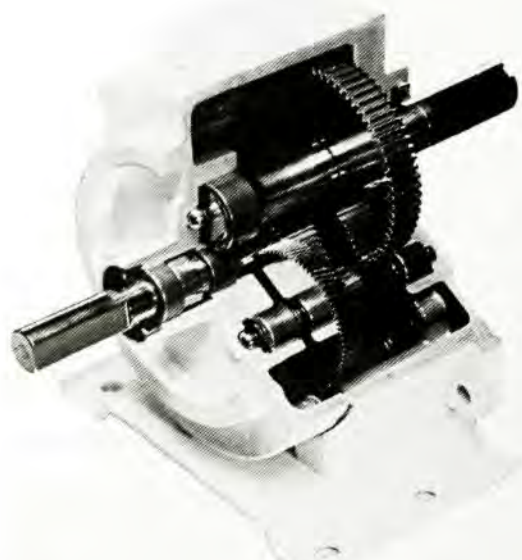
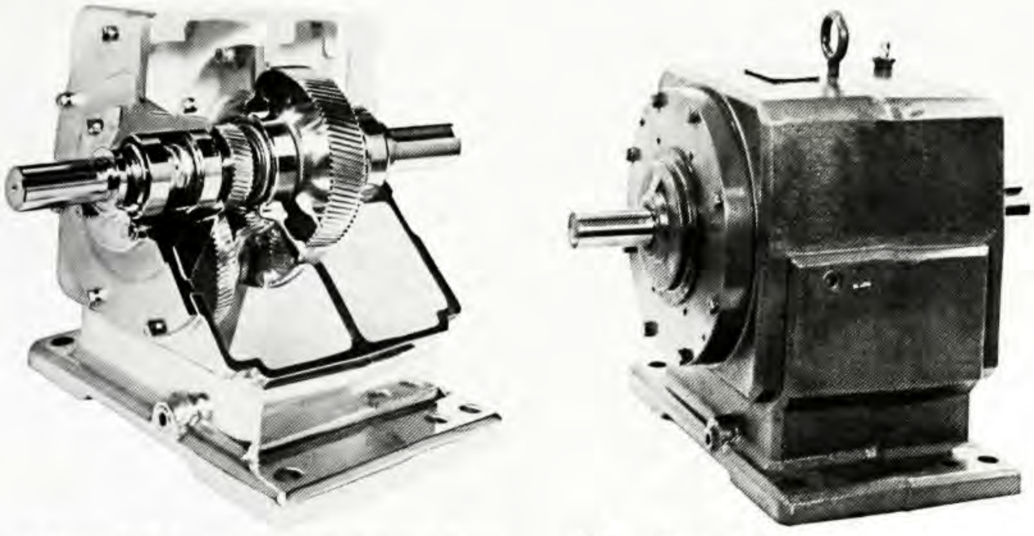
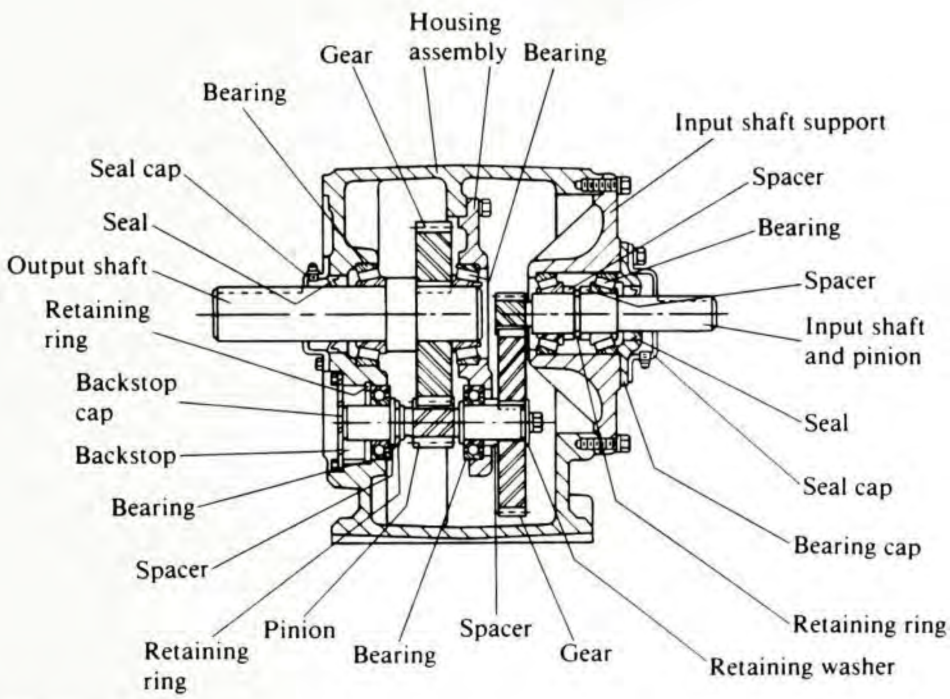


FIGURE 9–35 Double-reduction gear reducer. First stage, helical gears; second stage, spur gears, (Bison Gear & Engineering Corporation, Downers Grove, IL)

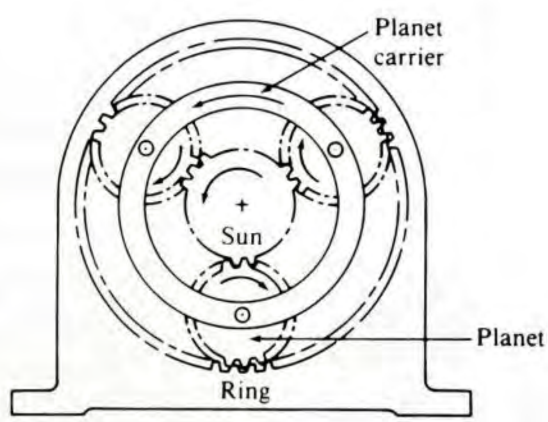
FIGURE 9–36
Concentric helical gear reducer (Peerless-Winsmith Subsidiary HBD Industries, Springville, NY)



(a) Cutaway of a concentric helical gear reducer *(b)* Complete reducer



(c) Parts index



(b) Schematic arrangement of planetary gearing

FIGURE 9–37 Planetary gear reducer (Rexnord, Milwaukee, WI)

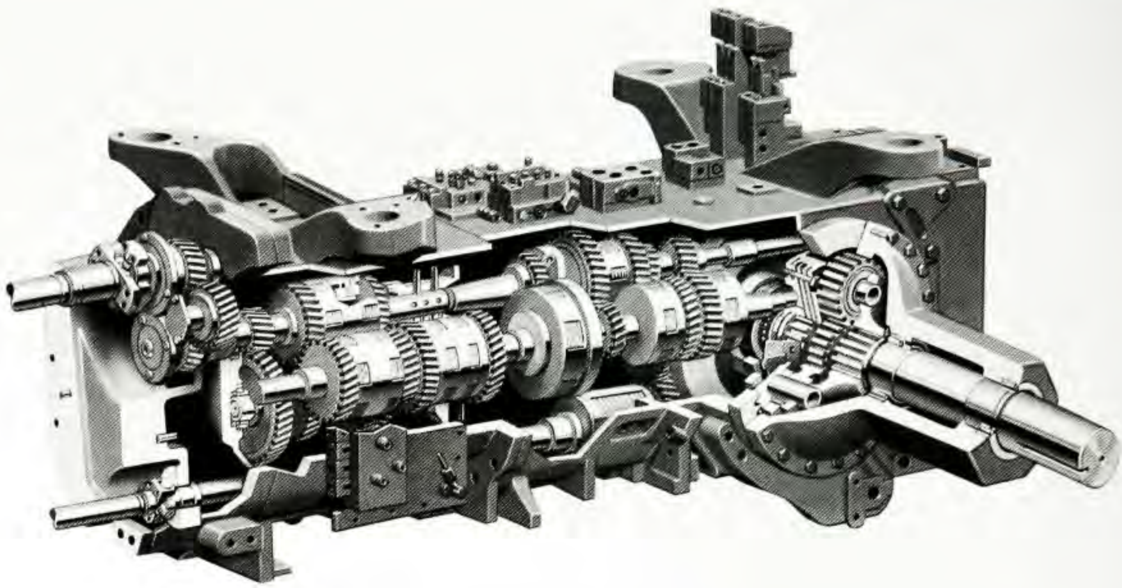


FIGURE 9–38
Eight-speed tractor transmission (Case IH, Racine, WI)

9–18 PLASTICS GEARING

Plastics are satisfying an important and growing part of the applications for gearing. Some of the numerous advantages of plastics in gearing systems compared with steels and other metals are:

- Lighter weight
- Lower inertia
- Possibility of running with little or no external lubrication
- Quieter operation
- Low sliding friction, which results in efficient gear meshing
- Chemical resistance and ability to operate in corrosive environments
- Ability to operate well under conditions of moderate vibration, shock, and impact
- Relatively low cost when made in large quantities
- Ability to combine several features into one part
- Accommodation of larger tolerances because of resiliency
- Material properties that can be tailored to meet the needs of the application
- Less wear among some plastics compared to metals in certain applications

The advantages must be weighed against disadvantages such as:

- Relatively lower strength of plastics as compared with metals
- Lower modulus of elasticity
- Higher coefficients of thermal expansion
- Difficulty operating at high temperatures
- Initial high cost for design, development, and mold manufacture
- Dimensional change with moisture absorption that varies with conditions
- Wide range of possible material formulations, which makes design more difficult

Some plastic gears are cut using hobbing or shaping processes similar to those used to cut metallic gears. However, most plastic gears are produced with the injection molding process because of its ability to make large quantities rapidly with low unit cost. Mold design is critical because it must accommodate the shrinking that occurs as the molten plastic solidifies. The typical successful approach accounts for predicted shrinkage by making the die larger than the required finished gear size. However, the allowance is not uniform throughout the gear, and significant amounts of data are required about the material molding properties and the molding process itself to produce plastic gears with high dimensional accuracy. Computer-assisted mold design software that simulates the flow of molten plastic through the mold cavities and the curing process is often used. The gear mold or the gear cutting tools are designed to produce dimensionally accurate gear teeth with tooth thickness controlled to produce a proper amount of backlash during operation. The electrical discharge machining process (EDM) is typically used to produce accurate gear tooth forms in molds made from high-hardness, wear-resistant steels to ensure that large production runs can be made without replacing tooling.

Plastic Materials for Gears

The great variety of plastics available makes material selection difficult, and it is recommended that gear system designers consult with material suppliers, mold designers, and manufacturing staff during the design process. While simulation can aid in reaching a suitable design, it is recommended that testing be done in realistic conditions before committing the design to production. Some of the more popular types of materials used for gears are

Nylon	Acetal	ABS (acrylonitrile-butadiene-styrene)
Polycarbonate	Polyurethane	Polyester thermoplastic
Polyimide	Phenolic	Polyphenylene sulfide
Polysufones	Phenylene oxides	Styrene-acrylonitrile (SAN)

Designers must seek a balance of material characteristics appropriate to the application, considering, for example:

- Strength in flexure under fatigue conditions
- High modulus of elasticity for stiffness
- Impact strength and toughness
- Wear and abrasion resistance
- Dimensional stability under expected temperatures
- Dimensional stability due to moisture absorption from liquids and humidity
- Frictional performance and need for lubrication, if any
- Operation in vibration environments
- Chemical resistance and compatibility with the operating environment
- Sensitivity to ultraviolet radiation
- Creep resistance if operated under load for long periods of time
- Flame retarding ability
- Cost
- Ease of processing and molding
- Assembly and disassembly considerations
- Compatibility with mating parts
- Environmental impact during processing, use, recycling, and disposal

The basic plastic materials listed previously are typically modified with fillers and additives to produce optimum as-molded properties. Some of these are:

> ***Fillers for strength reinforcement, toughness, moldability, long-term stability, thermal conductivity, and dimensional stability:*** Long glass fibers, chopped glass fibers, milled glass, woven glass fibers, carbon fibers, glass beads, aluminum flake, mineral, cellulose, rubber modifiers, wood flour, cotton, fabric, mica, talc, and calcium carbonate.
>
> ***Fillers to improve lubricity and overall frictional performance:*** PTFE (polytetrafluoroethylene), silicone, carbon fibers, graphite powders, and molybdenum disulfide (MoS_2).

Refer also to Section 2–17 in Chapter 2 for additional discussion about plastic materials, their properties, and special considerations for selecting plastics.

Design Strength for Plastic Gear Materials

Data are provided here for typical plastic materials used for gears. They can be applied to problem solving in this book. However, verification of properties for materials to be actually used in a commercial application, with due regard for the operating conditions, should be acquired from the material supplier. The effects of temperature on strength, modulus, toughness, chemical stability, and dimensional precision are particularly important. Manufacturing processes must be controlled to ensure that final properties are consistent with prescribed values.

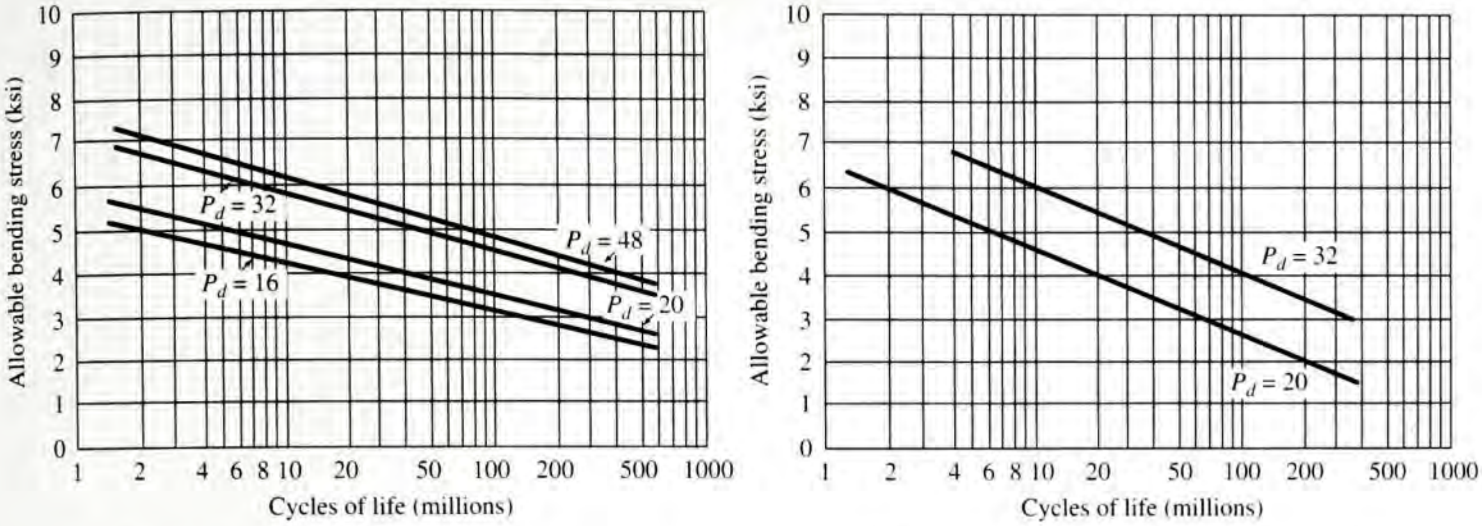
Table 9–11 lists some selected data for allowable tooth bending stress in plastic gears. Much additional data for other materials can be found in References 2, 15, and 16. Note the significant increase in allowable strength provided by the glass reinforcement. The combination of glass fibers and the basic plastic matrix performs like a composite material with the amount of filler typically ranging from 20% to 50%.

Material suppliers may be able to provide fatigue data for plastics in charts such as those shown in Figure 9–39, showing allowable bending stress versus number of cycles to failure for DuPont Zytel© nylon resin and Delrin© acetal resin. These data are for molded gears operating at room temperature with diametral pitches shown, pitch line velocity below 4000 ft/min, and continuous lubrication. Reductions should be applied for cut gears, higher temperatures, different pitches, and different lubrication conditions. See Reference 16.

TABLE 9–11 Approximate allowable tooth bending stress in plastic gears

	Approximate allowable bending stress, ksi (MPa)	
Material	Unfilled	Glass-filled
ABS	3000 (21)	6000 (41)
Acetal	5000 (34)	7000 (48)
Nylon	6000 (41)	12 000 (83)
Polycarbonate	6000 (41)	9000 (62)
Polyester	3500 (24)	8000 (55)
Polyurethane	2500 (17)	

Source: Plastics Gearing. Manchester, CT: ABA/PGT Publishing, 1994.



(*a*) Fatigue life data for DuPont© Zytel nylon resin

(*b*) Fatigue life data for DuPont© Delrin acetal resin

FIGURE 9–39 Fatigue life data for two types of plastic materials used for gears (DuPont Polymers, Wilmington, DE)

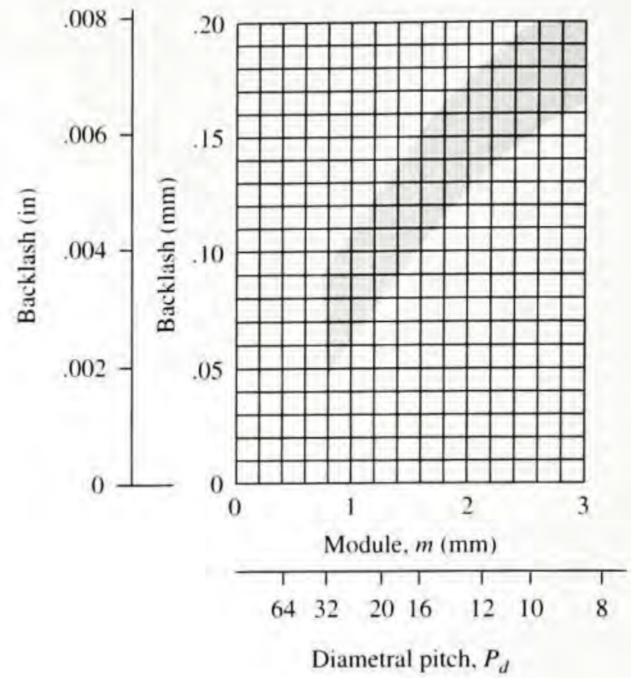
Tooth Geometry

In general, standard tooth geometry for plastic gears conforms to the configurations described in Section 8–4 in Chapter 8. Standard diametral pitches from Table 8–2 and standard metric modules from Table 8–3 should be used unless there are major advantages to using other values. Suppliers' ability to provide nonstandard pitches should be investigated. Pressure angles of 14½°, 20°, and 25° are used, with 20° usually preferred. Standard formulas for addendum, dedendum, and clearance for full-depth involute teeth are listed in Table 8–4. Gear quality values are set similarly to those for metallic gears as discussed in Section 9–5. The typical AGMA quality number produced by injection molding is in range of 6 to 10.

Designers sometimes use special tooth forms to tailor the strength of plastic gear teeth to the demands of particular applications. The 20° stub tooth system provides a shorter, broader tooth than the standard 20° full-depth tooth system, decreasing tooth-bending stress. The Plastics Gearing Technology unit of the ABA-PGT company has developed another system that is finding favor with some designers. See References 1 and 2.

Many designers of plastic gears prefer to use a longer addendum on the pinion and a shorter addendum on the mating gear to produce more favorable operation because of the grater flexibility of plastics as compared with metals. Tooth thickness is typically thinned on either or both of the pinion and gear to provide acceptable backlash and to ensure that mating gears do not bind. Binding may result from deflection of the teeth under load or from expansions due to increased temperature or moisture absorption from exposure to water or high humidity. Enlarging the center distance is another method employed to adjust for backlash. Designers must specify these feature sizes on drawings and in specifications. Consult AGMA Standard 1106-A97 *Tooth Proportions for Plastic Gears* for details. Reference 2 provides useful tables of formulas and data for adjustments to tooth form and center distance. Reference 16 recommends the range of backlash values shown in Figure 9–40.

FIGURE 9–40 Recommended backlash for plastic gears



Shrinkage

During manufacture of plastic gears using injection molding, enlarging the effective diametral pitch and the pitch diameter of the gear teeth cut into the mold accommodates shrinkage. The pressure is also adjusted. The nominal corrections are computed as follows:

$$P_{dc} = \frac{P_d}{(1 + S)} \tag{9–32}$$

$$\cos \phi_1 = \frac{\cos \phi}{(1 + S)} \tag{9–33}$$

$$D_c = N/P_{dc} \tag{9–34}$$

where S = Shrinkage of material
P_d = Standard diametral pitch for the gear
P_{dc} = Modified diametral pitch of the teeth in the mold
ϕ = Standard pressure angle for the gear
ϕ_1 = Modified pressure angle of the teeth in the mold
N = Number of teeth
D_c = Modified pitch diameter of teeth in the mold

After molding, the teeth should very nearly conform to standard geometry. Additional adjustments are sometimes made, relieving the tips of the teeth for smoother engagement and increasing the tooth width at the base near the point of highest bending stress.

Stress Analysis

Bending stress analysis for plastic gears relies on the basic Lewis formula introduced in Section 9–8, Equation (9–12). The modifying factors called for by the AGMA standards for

TABLE 9–12 Lewis tooth form factor, Y, for load near the pitch point

	Tooth Form		
Number Teeth	14½° Full Depth	20° Full Depth	20° Stub
14	–	–	0.540
15	–	–	0.566
16	–	–	0.578
17	–	0.512	0.587
18	–	0.521	0.603
19	–	0.534	0.616
20	–	0.544	0.628
22	–	0.559	0.648
24	0.509	0.572	0.664
26	0.522	0.588	0.678
28	0.535	0.597	0.688
30	0.540	0.606	0.698
34	0.553	0.628	0.714
38	0.566	0.651	0.729
43	0.575	0.672	0.739
50	0.588	0.694	0.758
60	0.604	0.713	0.774
75	0.613	0.735	0.792
100	0.622	0.757	0.808
150	0.635	0.779	0.830
300	0.650	0.801	0.855
Rack	0.660	0.823	0.881

Source: DuPont Polymers, Wilmington, DE

steel gears are not specified for plastic gears at this time. We can account for uncertainty or shock loading by inserting a safety factor. The overload factor from Table 9–5 can be used a guide. Testing of the proposed design in realistic conditions should be completed. The bending stress equation then becomes

$$\sigma_t = \frac{W_t P_d (SF)}{FY} \tag{9–35}$$

Values for the Lewis form factor, Y, shown in Table 9–12, describe the geometry of the involute gear teeth acting as a cantilever beam with the load applied near the pitch point. Thus Equation (9–35) gives the bending stress at the root of the tooth. Most plastic gear designs call for a generous fillet radius between the start of the active involute profile on the flank of the tooth and the root, resulting in little if any stress concentration.

Wear Considerations

Wear of tooth surfaces in plastic gear teeth is a function of the contact stress between mating teeth as it is with metal teeth. Equation (9–21) can be used compute the contact stress. However, published data are lacking for allowable contact stress values.

In reality, lubrication and the *combination of materials in mating gears* play major roles in the wear life of the pair. Presented here are some general guidelines from References 2 and 16. Communication with material suppliers and testing of proposed designs is recommended.

- Continuously lubricated gearing promotes the longest life.
- With continuous lubrication and light loads, fatigue resistance, not wear, typically determines life.
- Unlubricated gears tend to fail by wear, not fatigue, provided proper design bending stresses are used.
- When continuous lubrication is not practical, initially lubricating the gearing can aid in the run-in process and add life compared with gears that are never lubricated.
- When continuous lubrication is not practical, the combination of a nylon pinion and an acetal gear exhibits low friction and wear.
- Excellent wear performance for relatively high loads and pitch line speeds can be obtained by using a lubricated pair of a hardened steel pinion (HRC > 50) mating with a plastic gear made from nylon, acetal, or polyurethane.
- Wear accelerates when operating temperatures rise. Cooling to promote heat dissipation can increase life.

Gear Shapes and Assembly

References 2 and 16 include many recommendations for the geometric design of gears considering strength, inertia, and molding conditions. Many smaller gears are simply made with uniform thickness equal to the face width of the gear teeth. Larger gears often have a rim to support the teeth, a thinned web for lightening and material savings, and a hub to facilitate mounting on a shaft. Figure 9–41 shows recommended proportions. Symmetrical cross sections are preferred, along with balanced section thicknesses to promote good flow of material and to minimize distortion during molding.

Fastening gears to shafts requires careful design. Keys placed in shaft key seats and keyways in the hub of the gear provide reliable transmission of torque. For light torques, setscrews can be used, but slippage and damage of the shaft surface are possible. The bore of the gear hub can be lightly press fit onto the shaft with care to ensure that a sufficient torque can be transmitted while not overstressing the plastic hub. Knurling the shaft before pressing the gear on increases the torque capability. Some designers prefer to use metal hubs to facilitate the use of keys. Plastic is then molded onto the hub to form the rim and gear teeth.

Design Procedure

Design of plastic gearing should consider a variety of possibilities, and it is likely to be an iterative process. The following procedure outlines the steps for a given trial using U.S. Customary units.

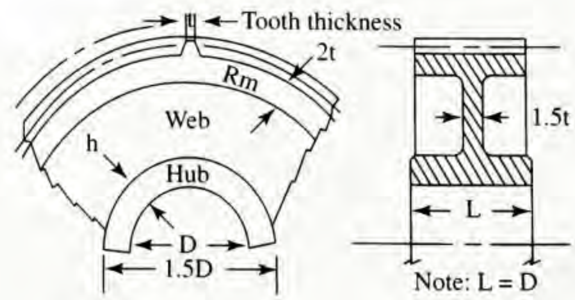


FIGURE 9–41
Suggested plastic gear proportions (DuPont Polymers, Wilmington, DE)

Procedure for Designing Plastic Gears

1. Determine the required horsepower, P, to be transmitted and the speed of rotation, n_P, of the pinion in rpm.
2. Specify the number of teeth, N, and select a trial diametral pitch diameter for the pinion.
3. Compute the pinion diameter from $D_P = N_P/P_d$.
4. Compute the transmitted load, W_t (in lb) from Equation (9–6), repeated here.

$$W_t = (126\,000)(P)/(n_P D_P)$$

5. Specify the tooth form and determine the Lewis form factor Y from Table 9–12.
6. Specify a safety factor, SF. Refer to Table 9–5 for guidance.
7. Specify the material to be used and determine the allowable stress from Table 9–10 or Figure 9–39.
8. Solve Equation (9–35) for the face width, F, and compute its value from,

$$F = \frac{W_t P_d (SF)}{s_{at} Y} \qquad \textbf{(9–36)}$$

9. Judge the suitability of the computed face width as it relates to the application. Consider its mounting on a shaft, space available in the diametral and axial directions, and whether the general proportions are acceptable for injection molding. See References 2 and 16. No general recommendations are published for the face width of plastic gears and often they are narrower than similar metallic gears.
10. Repeat steps 2 to 9 until a satisfactory design for the pinion is achieved. Specify convenient dimensions for the final value of the face width and other features of the pinion.
11. Considering the desired velocity ratio between the pinion and the gear, compute the required number of teeth in the gear and repeat steps 3 to 9 using the same diametral pitch as the pinion. Using the same face width as for the pinion, the stress in the gear teeth will always be lower than in the pinion because the form factor Y will increase and all other factors will be the same. When the same material is to be used for the gear, it will always be safe. Alternatively, you could compute the bending stress directly from Equation 9–35 and specify a different material for the gear that has a suitable allowable bending stress.

Example Problem 9–6 Design a pair of plastic gears for a paper shredder to transmit 0.25 horsepower at a pinion speed of 1160 rpm. The pinion will be mounted on the shaft of an electric motor that has a diameter of 0.625 in with a keyway for a 3/16 × 3/16 in key. The gear is to rotate approximately 300 rpm.

Given Data

$P = 0.25$ hp, $n_P = 1160$ rpm,

Shaft diameter $= D_s = 0.625$ in, Keyway for a 3/16 × 3/16 in key.

Approximate gear speed $= n_G = 300$ rpm

Solution Use the design procedure outlined in this section.

Step 1: Consider the given data.

Step 2: Specify $N_P = 18$ and $P_d = 16$

Step 3: $D_P = N_P/P_d = 18/16 = 1.125$ in. This seems reasonable for mounting on the 0.625 in motor shaft.

Step 4: Compute the transmitted load,

$$W_t = (126\,000)(P)/(n_P D_P) = (126\,000)(0.25)/[(1160)(1.125)] = 24.1 \text{ lb}$$

Step 5: Specify 20° full depth teeth. Then $Y = 0.521$ for 18 teeth from Table 9–12.

Step 6: Specify a safety factor, SF. The shredder will likely experience light shock; the preference is to operate the gears without lubrication. Specify $SF = 1.50$ from Table 9–5.

Step 7: Specify unfilled nylon. From Table 9–11, $s_{at} = 6000$ psi.

Step 8: Compute the required face width using Equation (9–36).

$$F = \frac{W_t P_d (SF)}{s_{at} Y} = \frac{(24.1)(16)(1.50)}{(6000)(0.521)} = 0.185 \text{ in}$$

Step 9: The dimensions seem reasonable.

Step10: Appendix 2 lists a preferred size for the face width of 0.200 in.

Comment In summary, the proposed pinion has the following features:

$P_d = 16$, $N_P = 18$ teeth, $D_P = 1.125$ in, $F = 0.200$ in, Bore $= 0.625$ in,

Keyway for a 3/16 × 3/16 in key. Unfilled nylon material.

Step 11: Gear design: Specify $F = 0.200$ in, $P_d = 16$. Compute the number of teeth in the gear.

$N_G = N_P (n_p/n_G) = 18(1160/300) = 69.6$ teeth

Specify $N_G = 70$ teeth

Pitch diameter of gear $= D_G = N_G/P_d = 70/16 = 4.375$ in

From Table 9–12, $Y_G = 0.728$ by interpolation.
Stress in gear teeth using Equation 9–35:

$$\sigma_t = \frac{W_t P_d (SF)}{FY} = \frac{(24.1)(16)(1.50)}{(0.200)(0.728)} = 3973 \text{ psi}$$

Comment This stress level is safe for nylon. The gear could also be made from acetal to achieve better wear performance.

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INTERNET SITES RELATED TO SPUR GEAR DESIGN

1. **ABA-PGT, Inc.** *www.abapgt.com* The ABA division produces molds for making plastic gears using injection molding; the PGT division is dedicated to plastic gearing technology.
2. **American Gear Manfuactures Association (AGMA)** *www.agma.org* Develops and publishes voluntary, consensus standards for gears and gear drives.
3. **Bison Gear, Inc.** *www.bisongear.com* Manufacturer of fractional horsepower gear reducers and gear motors.
4. **Boston Gear, Company.** *www.bostongear.com* Manufacturer of gears and complete gear drives. Part of the Colfax Power Transmission Group. Data provided for spur, helical, and worm gearing.
5. **DuPont Polymers** *www.plastics.dupont.com* Information and data on plastics and their properties. Searchable database by type of plastic or application.
6. **Emerson Power Transmission Corporation.** *www.emerson-ept.com* The Browning Division produces spur, helical, bevel, and worm gearing and complete gear drives.
7. **Gear Industry Home Page.** *www.geartechnology.com* Information source for many companies that manufacture or use gears or gearing systems. Includes gear machinery, gear cutting tools, gear materials, gear drives, open gearing, tooling & supplies, software, training and education. Publishes

Gear Technology Magazine: The Journal of Gear Manufacturing.

8. **Power Transmission Home Page.** *www.powertransmission.com* Clearinghouse on the Internet for buyers, users and sellers of power transmission-related products and services. Included are gears, gear drives, and gear motors.
9. **Rockwell Automation/Dodge.** *www.dodge-pt.com* Manufacturer of many power transmission components, including complete gear-type speed reducers, bearings, and components such as belt drives, chain drives, clutches, brakes, and couplings.
10. **Stock Drive Products–Sterling Instruments.** *www.sdp-si.com* Manufacturer and distributor of commercial and precision mechanical components, including gear reducers. Site includes an extensive handbook of design and information on metallic and plastic gears.
11. **Peerless-Winsmith, Inc.** *www.winsmith.com* Manufacturer of a wide variety of gear reducers and power transmission products, including worm gearing, planetary gearing, and combined helical/worm gearing. Subsidiary of HBD Industries, Inc.
12. **Drivetrain Technology Center.** *www.arl.psu.edu/areas/drivetrain/drivetrain.html* Research center for gear-type drivetrain technolgy. Part of the Applied Research Laboratory of Penn State University.
13. **Gleason Corporation.** *www.gleason.com* Manufacturer of many types of gear cutting machines for hobbing, milling, and grinding. The Gleason Cutting Tools Corporation manufactures a wide variety of milling cutters, hobs, shaper cutters, shaving cutters, and grinding wheels for gear production equipment.
14. **Bourn & Koch, Inc.** *www.bourn-koch.com* Manufacturer of hobbing, grinding, and other types of machines to produce gears, including the Barber-Colman line. Also provides remanufacturing services for a wide variety of existing macahine tools.
15. **Star-SU, Inc.** *www.star-su.com* Manufacturer of a wide variety of cutting tools for the gear industry, including hobs, shaping cutters, shaving cutters, bevel gear cutting tools, and grinding tools.

PROBLEMS

Forces on Spur Gear Teeth

1. A pair of spur gears with 20°, full-depth, involute teeth transmits 7.5 hp. The pinion is mounted on the shaft of an electric motor operating at 1750 rpm. The pinion has 20 teeth and a diametral pitch of 12. The gear has 72 teeth. Compute the following:
   - **a.** The rotational speed of the gear
   - **b.** The velocity ratio and the gear ratio for the gear pair
   - **c.** The pitch diameter of the pinion and the gear
   - **d.** The center distance between the shafts carrying the pinion and the gear
   - **e.** The pitch line speed for both the pinion and the gear
   - **f.** The torque on the pinion shaft and on the gear shaft
   - **g.** The tangential force acting on the teeth of each gear
   - **h.** The radial force acting on the teeth of each gear
   - **i.** The normal force acting on the teeth of each gear
2. A pair of spur gears with 20°, full-depth, involute teeth transmits 50 hp. The pinion is mounted on the shaft of an electric motor operating at 1150 rpm. The pinion has 18 teeth and a diametral pitch of 5. The gear has 68 teeth. Compute the following:
   - **a.** The rotational speed of the gear
   - **b.** The velocity ratio and the gear ratio for the gear pair
   - **c.** The pitch diameter of the pinion and the gear
   - **d.** The center distance between the shafts carrying the pinion and the gear
   - **e.** The pitch line speed for both the pinion and the gear
   - **f.** The torque on the pinion shaft and on the gear shaft
   - **g.** The tangential force acting on the teeth of each gear
   - **h.** The radial force acting on the teeth of each gear
   - **i.** The normal force acting on the teeth of each gear
3. A pair of spur gears with 20°, full-depth, involute teeth transmits 0.75 hp. The pinion is mounted on the shaft of an electric motor operating at 3450 rpm. The pinion has 24 teeth and diametral pitch of 24. The gear has 110 teeth. Compute the following:
   - **a.** The rotational speed of the gear
   - **b.** The velocity ratio and the gear ratio for the gear pair
   - **c.** The pitch diameter of the pinion and the gear
   - **d.** The center distance between the shafts carrying the pinion and the gear
   - **e.** The pitch line speed for both the pinion and the gear
   - **f.** The torque on the pinion shaft and on the gear shaft
   - **g.** The tangential force acting on the teeth of each gear
   - **h.** The radial force acting on the teeth of each gear
   - **i.** The normal force acting on the teeth of each gear

4. For the data of Problem 1, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.
5. For the data of Problem 2, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.
6. For the data of Problem 3, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.

Gear Manufacture and Quality

7. List three methods for producing gear teeth, and describe each method. Include a description of the cutter for each method along with its motion relative to the gear blank.
8. Specify a suitable quality number for the gears in the drive for a grain harvester. List the total composite tolerance for a pinion in the drive having a diametral pitch of 8 and 40 teeth and for its mating gear having 100 teeth.
9. Specify a suitable quality number for the gears in the drive for a high-speed printing press. List the total composite tolerance for a pinion in the drive having a diametral pitch of 20 and 40 teeth and for its mating gear having 100 teeth.
10. Specify a suitable quality number for the gears in the drive for an automotive transmission. List the total composite tolerance for a pinion in the drive having a diametral pitch of 8 and 40 teeth and its mating gear having 100 teeth.
11. Specify a suitable quality number for the gears in the drive for a gyroscope used in the guidance system for a spacecraft. List the total composite tolerance for a pinion in the drive having a diametral pitch of 32 and 40 teeth and its mating gear having 100 teeth.
12. Compare the values of total composite tolerance for the gears of Problems 8 and 10.
13. Compare the values of total composite tolerance for the gears of Problems 8, 9, and 11.
14. Specify a suitable quality number for the gears of Problem 1 if the drive is part of a precision machine tool.
15. Specify a suitable quality number for the gears of Problem 2 if the drive is part of a precision machine tool.
16. Specify a suitable quality number for the gears of Problem 3 if the drive is part of a precision machine tool.

Gear Materials

17. Identify the two major types of stresses that are created in gear teeth as they transmit power. Describe how the stresses are produced and where the maximum values of such stresses are expected to occur.
18. Describe the nature of the data contained in AGMA standards that relate to the ability of a given gear tooth to withstand the major types of stresses that it sees in operation.
19. Describe the general nature of steels that are typically used for gears, and list at least five examples of suitable alloys.
20. Describe the range of hardness that can typically be produced by through-hardening techniques and used successfully in steel gears.
21. Describe the general nature of the differences among steels produced as Grade 1, Grade 2, and Grade 3.
22. Suggest at least three applications in which Grade 2 or Grade 3 steel might be appropriate.
23. Describe three methods of producing gear teeth with strengths greater than can be achieved with through-hardening.
24. What AGMA standard should be consulted for data on the allowable stresses for steels used for gears?
25. In the AGMA standard identified in Problem 24, for what other materials besides steels are strength data provided?
26. Determine the allowable bending stress number and the allowable contact stress number for the following materials:
    a. Through-hardened, Grade 1 steel with a hardness of 200 HB
    b. Through-hardened, Grade 1 steel with a hardness of 300 HB
    c. Through-hardened, Grade 1 steel with a hardness of 400 HB
    d. Through-hardened, Grade 1 steel with a hardness of 450 HB
    e. Through-hardened, Grade 2 steel with a hardness of 200 HB
    f. Through-hardened, Grade 2 steel with a hardness of 300 HB
    g. Through-hardened, Grade 2 steel with a hardness of 400 HB
27. If the design of a steel gear indicates that an allowable bending stress number of 36 000 psi is needed, specify a suitable hardness level for Grade 1 steel. What hardness level would be required for Grade 2 steel?
28. What level of hardness can be expected for gear teeth that are case-hardened by carburizing?
29. Name three typical steels that are used in carburizing.
30. What is the level of hardness that can be expected for gear teeth that are case-hardened by flame or induction hardening?
31. Name three typical steels that are used for flame or induction hardening. What is an important property of such steels?
32. What level of hardness can be expected for gear teeth that are nitrided?
33. Determine the allowable bending stress number and the allowable contact stress number for the following materials:

a. Flame-hardened AISI 4140 steel, Grade 1, with a surface hardness of 50 HRC

b. Flame-hardened AISI 4140 steel, Grade 1, with a surface hardness of 54 HRC

c. Carburized and case-hardened AISI 4620 Grade 1 steel, DOQT 300

d. Carburized and case-hardened AISI 4620 Grade 2 steel, DOQT 300

e. Carburized and case-hardened AISI 1118 Grade 1 steel, SWQT 350

f. Nitrided, through-hardened, Grade 1 steel with a surface hardness of 84.5 HRN and a core hardness of 325 HB

g. Nitrided, through-hardened, Grade 2 steel with a surface hardness of 84.5 HRN and a core hardness of 325 HB

h. Nitrided, 2.5% chrome, Grade 3 steel with a surface hardness of 90.0 HRN and a core hardness of 325

i. Gray cast iron, class 20

j. Gray cast iron, class 40

k. Ductile iron, 100-70-03

l. Sand-cast bronze with a minimum tensile strength of 40 ksi (275 MPa)

m. Heat-treated bronze with a minimum tensile strength of 90 ksi (620 MPa)

n. Glass-filled nylon

o. Glass-filled polycarbonate

34. What depth should be specified for the case for a carburized gear tooth having a diametral pitch of 6?

35. What depth should be specified for the case for a carburized gear tooth having a metric module of 6?

Bending Stresses in Gear Teeth

For Problems 36–41, compute the bending stress number, s_t, using Equation (9–15). Assume that the gear blank is solid unless otherwise stated. *(Note that the data in these problems are used in later problems through Problem 59. You are advised to keep solutions to earlier problems accessible so that you can use data and results in later problems. The four problems that are keyed to the same set of data require the analysis of bending stress and contact stress and the corresponding specification of suitable materials based on those stresses. Later design problems, 60–70, use the complete analysis within each problem.)*

36. A pair of gears with 20°, full-depth, involute teeth transmits 10.0 hp while the pinion rotates at 1750 rpm. The diametral pitch is 12, and the quality number is 6. The pinion has 18 teeth, and the gear has 85 teeth. The face width is 1.25 in. The input power is from an electric motor, and the drive is for an industrial conveyor. The drive is a commercial enclosed gear unit.

37. A pair of gears with 20°, full-depth, involute teeth transmits 40 hp while the pinion rotates at 1150 rpm. The diametral pitch is 6, and the quality number is 6. The pinion has 20 teeth, and the gear has 48 teeth. The face width is 2.25 in. The input power is from an electric motor, and the drive is for a cement kiln. The drive is a commercial enclosed gear unit.

38. A pair of gears with 20°, full-depth, involute teeth transmits 0.50 hp while the pinion rotates at 3450 rpm. The diametral pitch is 32, and the quality number is 10. The pinion has 24 teeth, and the gear has 120 teeth. The face width is 0.50 in. The input power is from an electric motor, and the drive is for a small machine tool. The drive is a precision enclosed gear unit.

39. A pair of gears with 25° full-depth, involute teeth transmits 15.0 hp while the pinion rotates at 6500 rpm. The diametral pitch is 10, and the quality number is 12. The pinion has 30 teeth, and the gear has 88 teeth. The face width is 1.50 in. The input power is from a universal electric motor, and the drive is for an actuator on an aircraft. The drive is an extra-precision, enclosed gear unit.

40. A pair of gears with 25°, full-depth, involute teeth transmits 125 hp while the pinion rotates at 2500 rpm. The diametral pitch is 4, and the quality number is 8. The pinion has 32 teeth, and the gear has 76 teeth. The face width is 1.50 in. The input power is from a gasoline engine, and the drive is for a portable industrial water pump. The drive is a commercial enclosed gear unit.

41. A pair of gears with 25°, full-depth, involute teeth transmits 2.50 hp while the pinion rotates at 680 rpm. The diametral pitch is 10, and the quality number is 6. The pinion has 24 teeth, and the gear has 62 teeth. The face width is 1.25 in. The input power is from a vane-type fluid motor, and the drive is for a small lawn and garden tractor. The drive is a commercial enclosed gear unit.

Required Allowable Bending Stress Number

For Problems 42–47, compute the required allowable bending stress number, s_{at}, using Equation (9–20). Assume that no unusual conditions exist unless stated otherwise. That is, use a service factor, SF, of 1.00. Then specify a suitable steel and its heat treatment for both the pinion and the gear based on bending stress.

42. Use the data and results from Problem 36. Design for a reliability of 0.99 and a design life of 20 000 h.

43. Use the data and results from Problem 37. Design for a reliability of 0.99 and a design life of 8000 h.

44. Use the data and results from Problem 38. Design for a reliability of 0.9999 and a design life of 12 000 h. Consider that the machine tool is a critical part of a production system calling for a service factor of 1.25 to avoid unexpected down time.

45. Use the data and results from Problem 39. Design for a reliability of 0.9999 and a design life of 4000 h.

46. Use the data and results from Problem 40. Design for a reliability of 0.99 and a design life of 8000 h.

47. Use the data and results from Problem 41. Design for a reliability of 0.90 and a design life of 2000 h. The uncertainty of the actual use of the tractor calls for a service factor of 1.25. Consider using cast iron or bronze if the conditions permit.

Pitting Resistance

For Problems 48–53, compute the expected contact stress number, s_c, using Equation (9–25). Assume that both gears are to be steel unless stated otherwise.

48. Use the data and results from Problems 36 and 42.
49. Use the data and results from Problems 37 and 43.
50. Use the data and results from Problems 38 and 44.
51. Use the data and results from Problems 39 and 45.
52. Use the data and results from Problems 40 and 46.
53. Use the data and results from Problems 41 and 47.

Required Allowable Contact Stress Number

For Problems 54–59, compute the required allowable contact stress number, s_{ac}, using Equation (9–27). Use a service factor, *SF*, of 1.00 unless stated otherwise. Then specify suitable material for the pinion and the gear based on pitting resistance. Use steel unless an earlier decision has been made to use another material. Then evaluate whether the earlier decision is still valid. If not, specify a different material according to the most severe requirement. If no suitable material can be found, consider redesigning the original gears to enable reasonable materials to be used.

54. Use the data and results from Problems 36, 42, and 48.
55. Use the data and results from Problems 37, 43, and 49.
56. Use the data and results from Problems 38, 44, and 50.
57. Use the data and results from Problems 39, 45, and 51.
58. Use the data and results from Problems 40, 46, and 52.
59. Use the data and results from Problems 41, 47, and 53.

Design Problems

Problems 60–70, describe design situations. For each, design a pair of spur gears, specifying (at least) the diametral pitch, the number of teeth in each gear, the pitch diameters of each gear, the center distance, the face width, and the material from which the gears are to be made. Design for recommended life with regard to both strength and pitting resistance. Work toward designs that are compact. Use standard values of diametral pitch, and avoid designs for which interference could occur. See Example Problem 9–5. Assume that the input to the gear pair is from an electric motor unless otherwise stated.

If the data are given in SI units, complete the design in the metric module system with dimensions in millimeters, forces in newtons, and stresses in megapascals. See Example Problem 9–6.

60. A pair of spur gears is to be designed to transmit 5.0 hp while the pinion rotates at 1200 rpm. The gear must rotate between 385 and 390 rpm. The gear drives a reciprocating compressor.

61. A gear pair is to be a part of the drive for a milling machine requiring 20.0 hp with the pinion speed at 550 rpm and the gear speed to be between 180 and 190 rpm.

62. A drive for a punch press requires 50.0 hp with the pinion speed of 900 rpm and the gear speed of 225 to 230 rpm.

63. A single-cylinder gasoline engine has the pinion of a gear pair on its output shaft. The gear is attached to the shaft of a small cement mixer. The mixer requires 2.5 hp while rotating at approximately 75 rpm. The engine is governed to run at approximately 900 rpm.

64. A four-cylinder industrial engine runs at 2200 rpm and delivers 75 hp to the input gear of a drive for a large wood chipper used to prepare pulpwood chips for paper making. The output gear must run between 4500 and 4600 rpm.

65. A small commercial tractor is being designed for chores such as lawn mowing and snow removal. The wheel drive system is to be through a gear pair in which the pinion runs at 600 rpm while the gear, mounted on the hub of the wheel, runs at 170 to 180 rpm. The wheel is 300 mm in diameter. The gasoline engine delivers 3.0 kW of power to the gear pair.

66. A water turbine transmits 75 kW of power to a pair of gears at 4500 rpm. The output of the gear pair must drive an electric power generator at 3600 rpm. The center distance for the gear pair must not exceed 150 mm.

67. A drive system for a large commercial band saw is to be designed to transmit 12.0 hp. The saw will be used to cut steel tubing for automotive exhaust pipes. The pinion rotates at 3450 rpm, while the gear must rotate between 725 and 735 rpm. It has been specified that the gears are to be made from AISI 4340 steel, oil quenched and tempered. Case hardening is *not* to be used.

68. Repeat Problem 67, but consider a case-hardened carburized steel from Appendix 5. Try to achieve the smallest practical design. Compare the result with the design from Problem 67.

69. A gear drive for a special-purpose, dedicated machine tool is being designed to mill a surface on a rough steel casting. The drive must transmit 20 hp with a pinion speed of 650 rpm and an output speed between 110 and 115 rpm. The mill is to be used continuously, two shifts per day, six days per week, for at least five years. Design

the drive to be as small as practical to permit its being mounted close to the milling head.

70. A cable drum for a crane is to rotate between 160 and 166 rpm. Design a gear drive for 25 hp in which the input pinion rotates at 925 rpm and the output rotates with the drum. The crane is expected to operate with a 50% duty cycle for 120 hours per week for at least 10 years. The pinion and the gear of the drive must fit within the 24-in inside diameter of the drum, with the gear mounted on the drum shaft.

Power-Transmitting Capacity

71. Determine the power-transmitting capacity for a pair of spur gears having 20°, full-depth teeth, a diametral pitch of 10, a face width of 1.25 in, 25 teeth in the pinion, 60 teeth in the gear, and an AGMA quality class of 8. The pinion is made from AISI 4140 OQT 1000, and the gear is made from AISI 4140 OQT 1100. The pinion will rotate at 1725 rpm on the shaft of an electric motor. The gear will drive a centrifugal pump.

72. Determine the power-transmitting capacity for a pair of spur gears having 20°, full-depth teeth, a diametral pitch of 6, 35 teeth in the pinion, 100 teeth in the gear, a face width of 2.00 in, and an AGMA quality class of 6. A gasoline engine drives the pinion at 1500 rpm. The gear drives a conveyor for crushed rock in a quarry. The pinion is made from AISI 1040 WQT 800. The gear is made from gray cast iron, ASTM A48-83, class 30. Design for 15 000 hr life.

73. It was found that the gear pair described in Problem 72 wore out when driven by a 25-hp engine. Propose a redesign that would be expected to give indefinite life under the conditions described. Design for 15 000 hr life.

Design of Double-Reduction Drives

74. Design a double-reduction gear train that will transmit 10.0 hp from an electric motor running at 1750 rpm to an assembly conveyor whose drive shaft must rotate between 146 and 150 rpm. Note that this will require the design of two pairs of gears. Sketch the arrangement of the train, and compute the actual output speed.

75. A commercial food waste grinder in which the final shaft rotates at between 40 and 44 rpm is to be designed. The input is from an electric motor running at 850 rpm and delivering 0.50 hp. Design a double-reduction spur gear train for the grinder.

76. A small, powered hand drill is driven by an electric motor running at 3000 rpm. The drill speed is to be approximately 550 rpm. Design the speed reduction for the drill. The power transmitted is 0.25 hp.

77. The output from the drill described in Problem 76 provides the drive for a small bench-scale band saw similar to the one in Figure 1–1. The saw blade is to move with a linear velocity of 375 ft/min. The saw blade rides on 9.0-in-diameter wheels. Design a spur gear reduction to drive the band saw. Consider using plastic gears.

78. Design a rack-and-pinion drive to lift a heavy access panel on a furnace. A fluid power motor rotating 1500 rpm will provide 5.0 hp at the input to the drive. The linear speed of the rack is to be at least 2.0 ft/s. The rack moves 6.0 ft each way during the opening and closing of the furnace doors. More than one stage of reduction may be used, but attempt to design with the fewest number of gears. The drive is expected to operate at least six times per hour for three shifts per day, seven days per week, for at least 15 years.

79. Design the gear drive for the wheels of an industrial lift truck. Its top speed is to be 20 mph. It has been decided that the wheels will have a diameter of 12.0 in. A DC motor supplies 20 hp at a speed of 3000 rpm. The design life is 16 hours per day, six days per week, for 20 years.

Plastics Gearing

80. Design a pair of plastic gears to drive a small band saw. The input is from a 0.50 hp electric motor rotating at 860 rpm, and the pinion will be mounted on its 0.75-inch diameter shaft with a keyway for a 3/16 × 3/16 in key. The gear is to rotate between 265 and 267 rpm.

81. Design a pair of plastic gears to drive a paper feed roll for an office printer. The pinion rotates at 88 rpm and the gear must rotate between 20 and 22 rpm. The power required is 0.06 hp. Work toward the smallest practical size.

82. Design a pair of plastic gears to drive the wheels of a small remote control car. The gear is mounted on the axle of the wheel and must rotate between 120 and 122 rpm. The pinion rotates at 430 rpm. The power required is 0.025 hp. Work toward the smallest practical size using unfilled nylon.

83. Design a pair of plastic gears to drive a commercial food-chopping machine. The input is from a 0.65 hp electric motor rotating at 1560 rpm, and the pinion will be mounted on its 0.875-inch diameter shaft with a keyway for a 1/4 × 1/4 in key. The gear is to rotate between 468 and 470 rpm.

10

Helical Gears, Bevel Gears, and Wormgearing

The Big Picture

Helical Gears, Bevel Gears, and Wormgearing

Discussion Map

- ☐ The geometry of helical gears, bevel gears, and wormgearing was described in Chapter 8.
- ☐ The principles of stress analysis of gears were discussed in Chapter 9 for spur gears. Much of that information is applicable to the types of gears discussed in this chapter.

Discover

Review Chapters 8 and 9 now.

Recall some of the discussion at the beginning of Chapter 8 about uses for gears that you see in your world. Review that information now, and focus your discussion on helical gears, bevel gears, and wormgearing.

In this chapter, you acquire the skills to perform the necessary analyses to design safe gear drives that use helical gears, bevel gears, and wormgearing and that demonstrate long life.

Much was said in Chapters 8 and 9 about the kinematics of gears and about the stress analysis and design of spur gears. That information is important to the objectives of this chapter, in which we extend the application of those concepts to the analysis and design of helical gears, bevel gears, and wormgearing.

The basic geometry of helical gears was described in Section 8–7. The force system on helical gear teeth was also described, and that system is important to your understanding of the stresses and modes of potential failure for helical gears that we discuss in this chapter.

In Chapter 9, you learned how to analyze spur gears for bending strength and resistance to pitting of the surface of the teeth. This chapter modifies that same approach for application to the special geometry of helical gears. In fact, AGMA Standard 2001-C95, which we often referred to in Chapter 9 for spur gears, is the same reference that we use for helical gears. So you will need to refer to Chapter 9 from time to time.

Similarly, the geometry of bevel gears was described in Section 8–8, and wormgearing was described in Sections 8–9 and 8–10. This chapter includes information about stresses in bevel gears and wormgearing.

Figure 10–1 shows an example of a large, commercially available, double-reduction, parallel-shaft reducer employing helical gears. Notice that the shafts are supported in tapered roller bearings that have the ability to withstand the thrust loads created by the helical gears. Chapter 14 elaborates on the selection of such bearings.

Figure 10–2 shows another form of helical gear reducer in which the drive motor is mounted above the reducer, and the shaft of the driven machine is inserted directly through the hollow output shaft of the reducer. This allows the reducer to be supported by the frame of the driven machine.

Refer to Figure 8–22 for drawings and photographs of bevel gear systems. The three-dimensional force system that acts between the teeth of bevel gears requires great care in the installation and alignment of the gears and requires the use of bearings that can withstand forces in all directions. The stress analysis is adapted from the approach described in Chapter 9, modified for the geometry of the bevel gear teeth.

Refer to Figure 8–25 for a photograph of a commercially available wormgear reducer. Notice the use of tapered roller bearings here also to withstand the thrust forces created by

FIGURE 10–1
Parallel shaft reducer (Emerson Power Transmission Corporation, Drive and Component Division, Ithaca, NY)

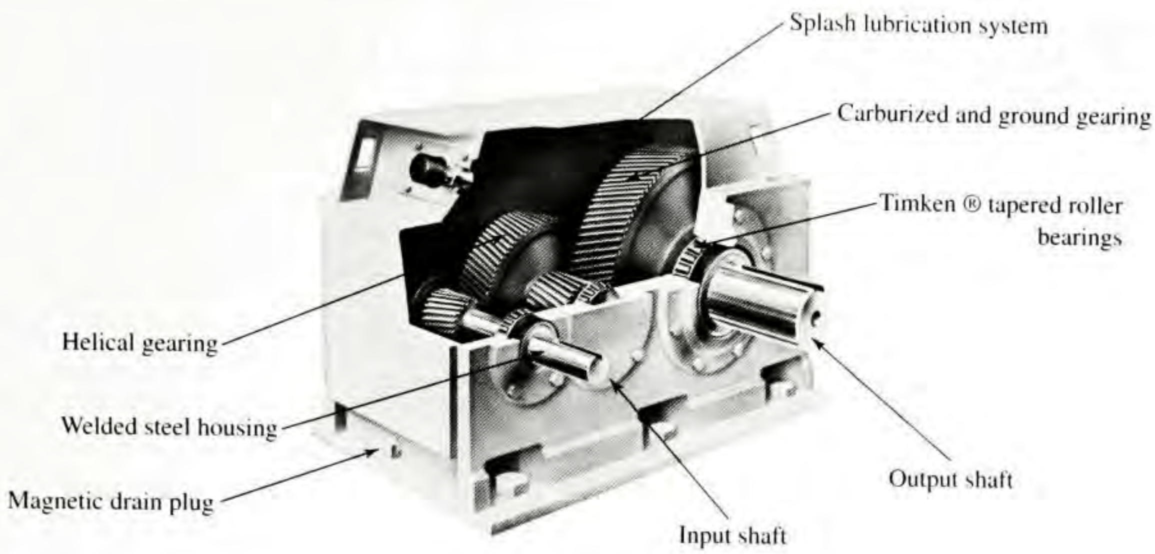
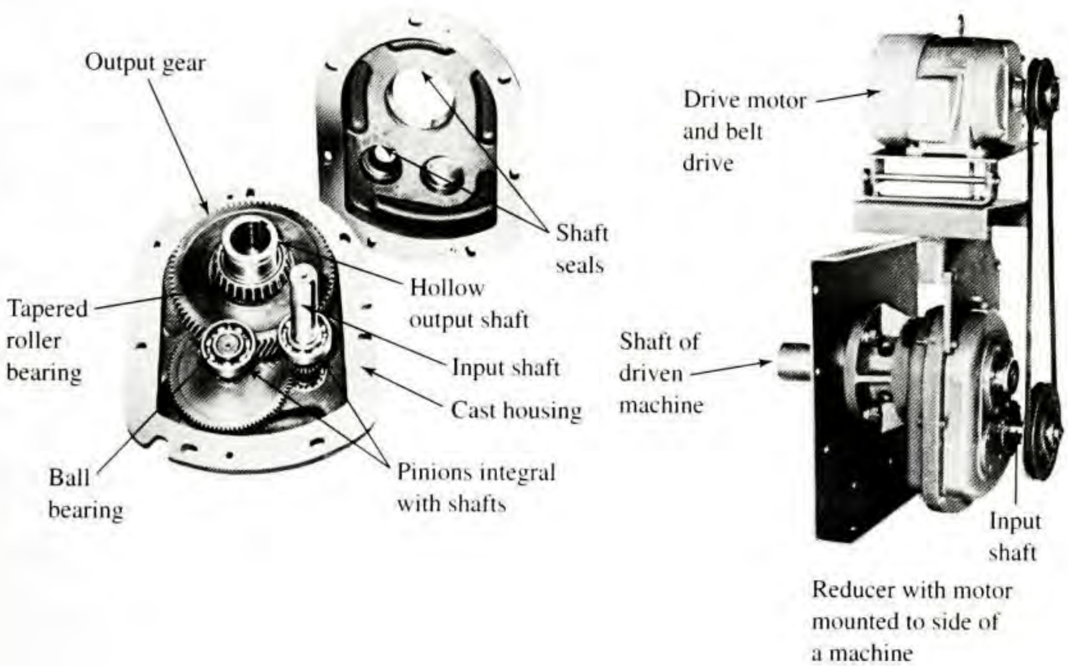


FIGURE 10–2
Helical shaft mount reducer (Emerson Power Transmission Corporation, Drive and Component Division, Ithaca, NY)



wormgearing. The lubrication of the gears is highlighted in the photograph. Lubrication is important because sliding action between the worm threads and the wormgear teeth inherently occurs, causing frictional heat to be generated. Consistent lubrication of the gear mesh is critical to the performance, efficiency, and life of the system.

This chapter presents more information about these factors in wormgearing system design. The design for strength and pitting resistance is presented for each type of gear, with an analysis of the geometry and the forces exerted on the gears.

You Are the Designer

The gear drives that were designed in Chapter 9 all assumed that spur gears would be used to accomplish the speed reduction or speed increase between the input and the output of the drive. But many other types of gears could have been used. Assume that you are the designer of the drive for the wood chipper described in Example Problem 9–5. How would the design be different if helical gears were used instead of spur gears? What forces would be created and transferred to the shafts carrying the gears and to the bearings carrying the shafts? Would you be able to use smaller gears? How is the geometry of helical gears different from that of spur gears?

Rather than having the input and output shafts parallel as they were in designs up to this time, how can we design drives that deliver power to an output shaft at right angles to the input shaft? What special analysis techniques are applied to bevel gears and wormgearing?

The information in this chapter will help you answer these and other questions.

10–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Describe the geometry of helical gears and compute the dimensions of key features.
2. Compute the forces exerted by one helical gear on its mating gear.
3. Compute the stress due to bending in helical gear teeth and specify suitable materials to withstand such stresses.
4. Design helical gears for surface durability.
5. Describe the geometry of bevel gears and compute the dimensions of key features.
6. Analyze the forces exerted by one bevel gear on another and show how those forces are transferred to the shafts carrying the gears.
7. Design and analyze bevel gear teeth for strength and surface durability.
8. Describe the geometry of worms and wormgears.
9. Compute the forces created by a wormgear drive system and analyze their effect on the shafts carrying the worm and the wormgear.
10. Compute the efficiency of wormgear drives.
11. Design and analyze wormgear drives to be safe for bending strength and wear. References 3, 4, 7, 8, and 17–22 are recommended for general guidelines for design and application of helical gears, bevel gears, and wormgearing.

10–2 FORCES ON HELICAL GEAR TEETH

Figure 10–3 shows a photograph of two helical gears in mesh and designed to be mounted on parallel shafts. This is the basic configuration that we analyze in this chapter. Refer to Figure 10–4 for a representation of the force system that acts between the teeth of two helical gears in mesh. In Chapter 8, using the same figure, we defined the following forces:

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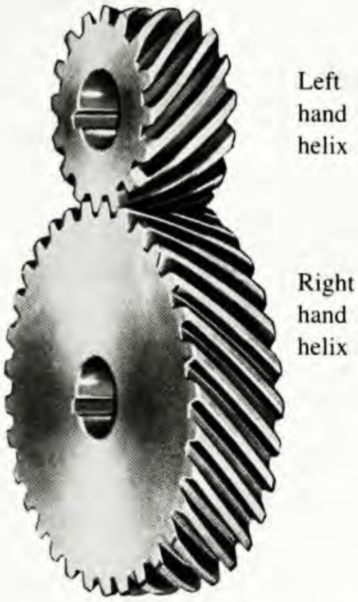
- W_N is the *true normal force* that acts perpendicular to the face of the tooth in the plane normal to the surface of the tooth. The normal plane is shown in Part (d) of Figure 10–4. We seldom need to use the value of W_N because its three orthogonal components, defined next, are used in the analyses performed for helical gears. The values for the orthogonal components depend on the following three angles that help define the geometry of the helical gear teeth:

  Normal pressure angle: ϕ_n
  Transverse pressure angle: ϕ_t
  Helix angle: ψ

FIGURE 10–3
Helical gears. These gears have a 45° helix angle. (Emerson Power Transmission Corporation, Drive and Component Division, Ithaca, NY)



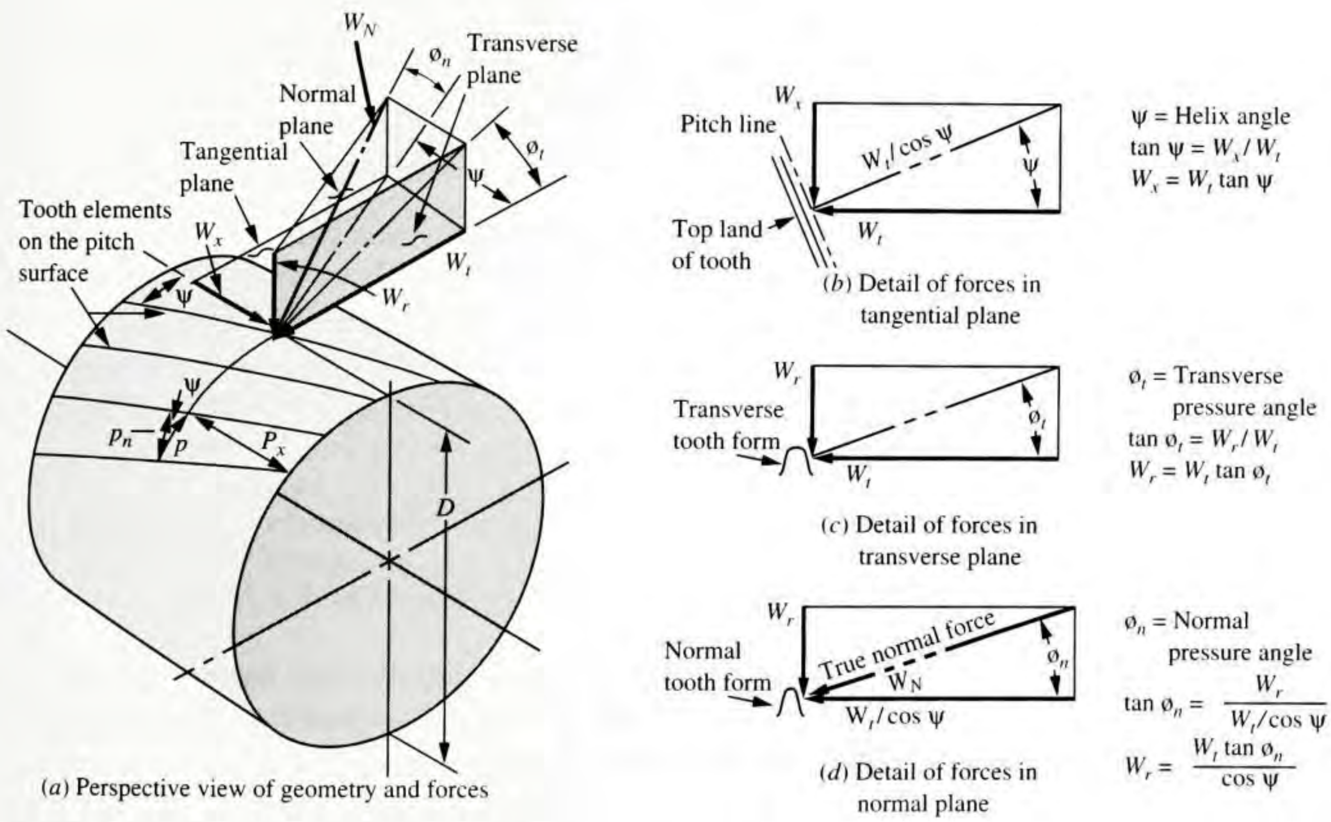


FIGURE 10–4 Helical gear geometry and forces

For helical gears, the helix angle and one of the other two are specified. The third angle can be computed from

$$\tan\phi_n = \tan\phi_t\cos\psi \tag{10–1}$$

- W_t is the *tangential force* that acts in the transverse plane and tangent to the pitch circle of the helical gear and that causes the torque to be transmitted from the driver to the driven gear. Therefore, this force is often called the *transmitted force*.

It is functionally similar to W_t used in the analysis of spur gears in Chapter 8. We can compute its value from the same equations, as follows:

If the torque being transmitted (T) and the size of the gear (D) are known,

$$W_t = T/(D/2) \tag{10–2}$$

If the power being transmitted (P) and the rotational speed (n) are known,

$$T = (P/n) \tag{10–3}$$

For the unit-specific situation where power is expressed in horsepower and the rotational speed is in rpm, the torque in lb·in is

$$T = 63\,000(P)/n \tag{10–4}$$

Then the tangential force can also be expressed as

$$W_t = 63\,000(P)/[(n)(D/2)] = 126\,000(P)/[(n)(D)] \tag{10–5}$$

If the pitch line speed, v_t (ft/min) of the gear is known, along with the power being transmitted (hp), the tangential load is

$$W_t = 33\,000(P)/v_t \tag{10–6}$$

The value of the tangential load is the most fundamental of the three orthogonal components of the true normal force. The calculation of the bending stress number and the pitting resistance of the gear teeth depends on W_t.

- W_r is the *radial force* that acts toward the center of the gear perpendicular to the pitch circle and to the tangential force. It tends to push the two gears apart. As can be seen in Figure 10–4(c),

$$W_r = W_t \tan \phi_t \tag{10–7}$$

where ϕ_t = transverse pressure angle for the helical teeth

- W_x is the *axial force* that acts parallel to the axis of the gear and causes a thrust load that must be resisted by the bearings carrying the shaft. With the tangential force known, the axial force is computed from

$$W_x = W_t \tan \psi \tag{10–8}$$

Example Problem 10–1 A helical gear has a normal diametral pitch of 8, a normal pressure angle of 20°, 32 teeth, a face width of 3.00 in, and a helix angle of 15°. Compute the diametral pitch, the transverse pressure angle, and the pitch diameter. If the gear is rotating at 650 rpm while transmitting 7.50 hp, compute the pitch line speed, the tangential force, the axial force, and the radial force.

Solution *Diametral Pitch [Equation (8–28)]*

$$P_d = P_{nd} \cos \psi = 8 \cos (15) = 7.727$$

Transverse Pressure Angle [Equation (10–1)]

$$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi)$$
$$\phi_t = \tan^{-1}[\tan(20)/\cos(15)] = 20.65^\circ$$

Pitch Diameter [Equation (8–27)]

$$D = N/P_d = 32/7.727 = 4.141 \text{ in}$$

Pitch Line Speed, v_t [Equation (9–1)]

$$v_t = \pi Dn/12 = \pi(4.141)(650)/12 = 704.7 \text{ ft/min}$$

Tangential Force, W_t [Equation (10–6)]

$$W_t = 33\,000(P)/v_t = 33\,000(7.5)/704.7 = 351 \text{ lb}$$

Axial Force, W_x [Equation (10–8)]

$$W_x = W_t \tan \psi = 351 \tan(15) = 94 \text{ lb}$$

Radial Force, W_r [Equation (10–7)]

$$W_r = W_t \tan \phi_t = 351 \tan(20.65) = 132 \text{ lb}$$

10–3 STRESSES IN HELICAL GEAR TEETH

We will use the same basic equation for computing the bending stress number for helical gear teeth as we did for spur gear teeth in Chapter 9, given in Equation (9–15) and repeated here:

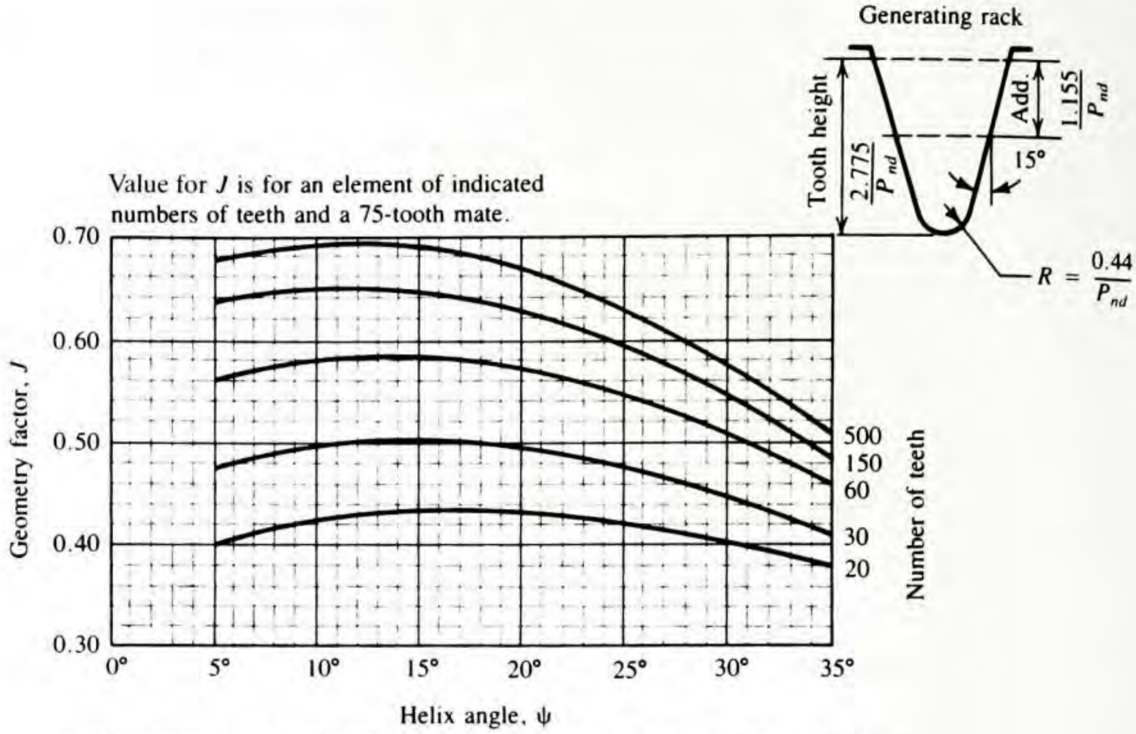
$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v$$

Figures 10–5, 10–6, and 10–7 show the values for the geometry factor, J, for helical gear teeth with 15°, 20°, and 22° normal pressure angles, respectively.[1] The K factors are the same as those used for spur gears. See References 9 and 18 and the following locations for values:

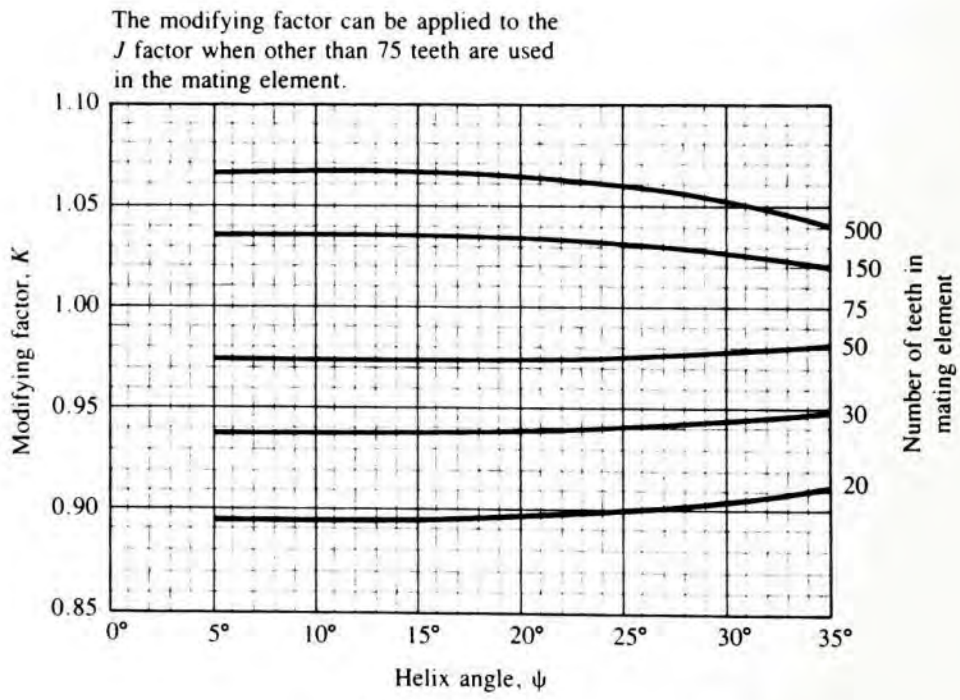
K_o = overload factor (Table 9–5)
K_s = size factor (Table 9–6)
K_m = load-distribution factor [Figures 9–18 and 9–19 and Equation (9–16)]
K_B = rim thickness factor (Figure 9–20)
K_v = dynamic factor (Figure 9–21)

[1] Figures 10–5, 10–6, and 10–7:
Graphs for the geometry factor, J, for helical gears have been taken from AGMA Standard 218.01-1982, *Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314. This standard has been superseded by two standards: (1) Standard 908-B89 (R1995), *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, 1989; (2) Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, 1995. The method of calculating the value for J has not been changed. However, the new standards do not contain the graphs. Users are cautioned to ensure that geometry factors for a given design conform to the specific cutter geometry used to manufacture the gears. Standards 908-B89 (R 1995) and 2001-C95 should be consulted for the details of computing the values for J and for rating the performance of the gear teeth.

FIGURE 10–5
Geometry factor (J) for 15° normal pressure angle



(a) Geometry factor (J) for 15° normal pressure angle and indicated addendum



(b) J factor multipliers

For design, a material must be specified that has an allowable bending stress number, s_{at}, greater than the computed bending stress number, s_t. Design values of s_{at} can be found:

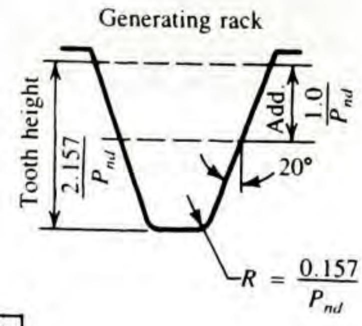
Figure 9–10: Steel, through-hardened, Grades 1 and 2

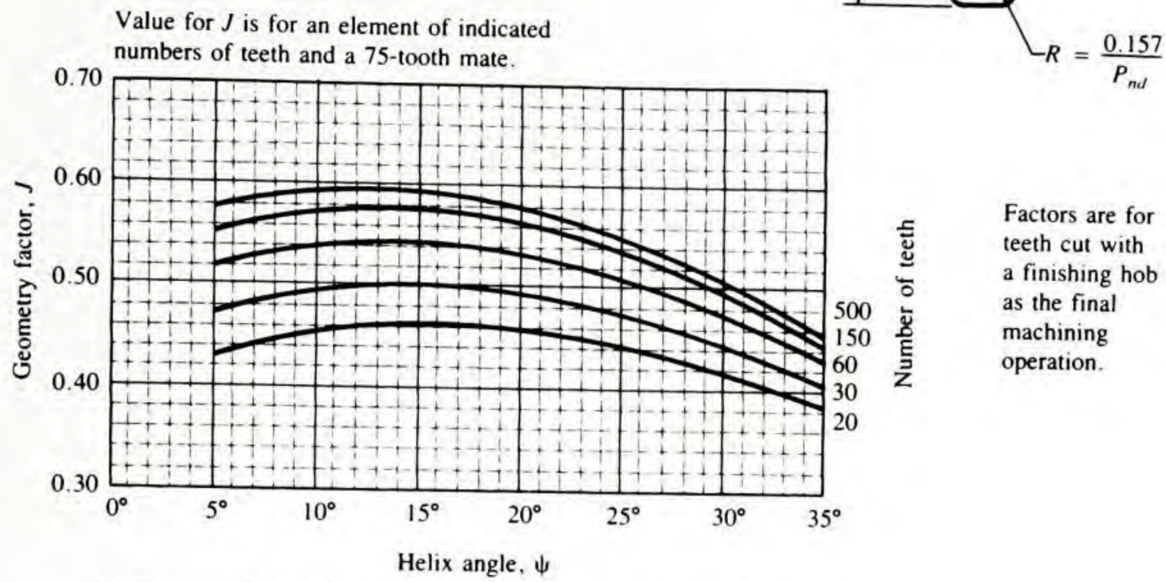
Table 9–3: Case-hardened steels

Figures 9–14 and 9–15: Nitrided gears

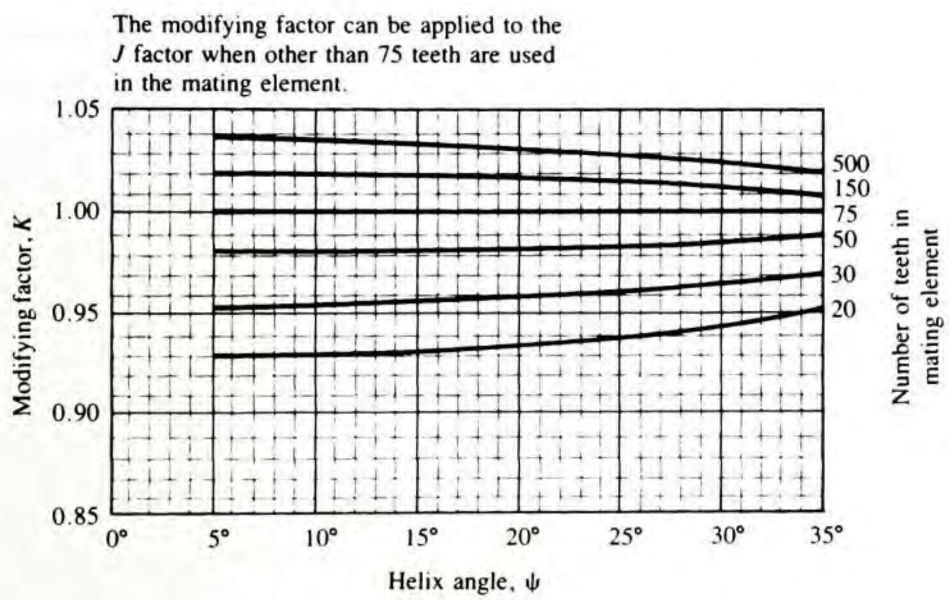
Table 9–4: Cast iron and bronze

FIGURE 10–6
Geometry factor (J) for 20° normal pressure angle





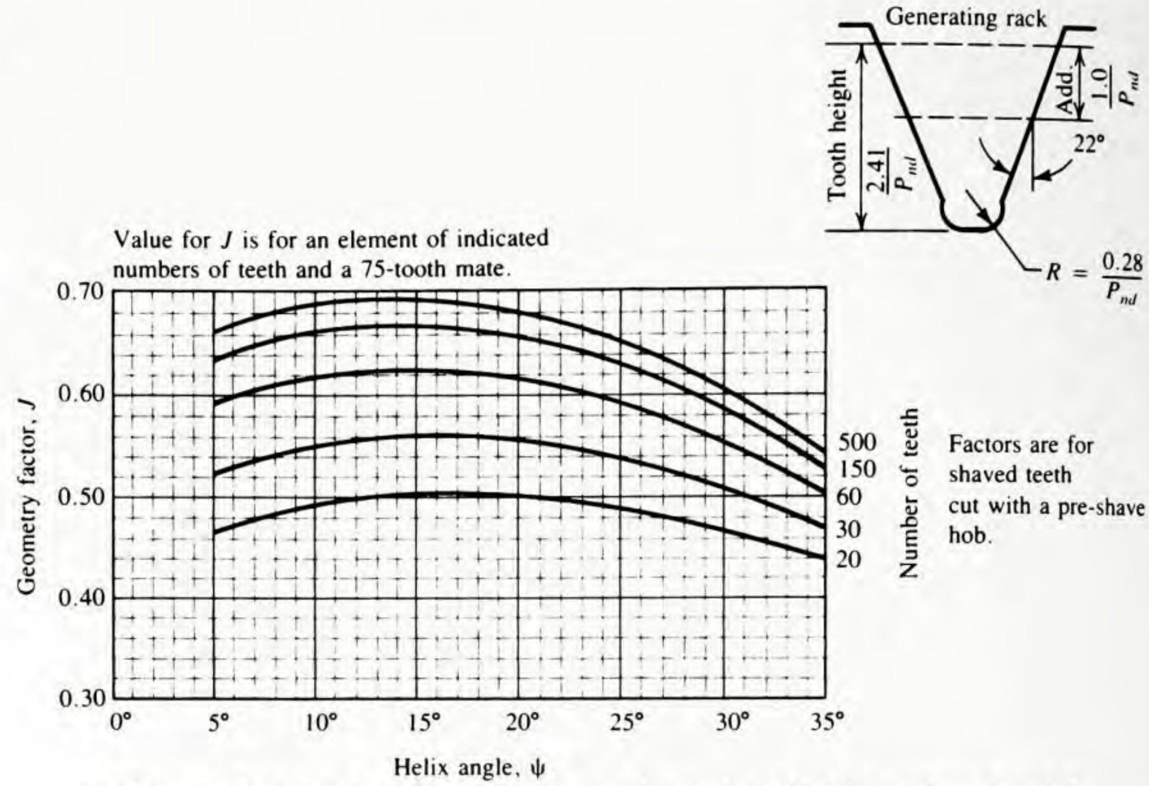
(a) Geometry factor (J) for 20° normal pressure angle, standard addendum, and finishing hob



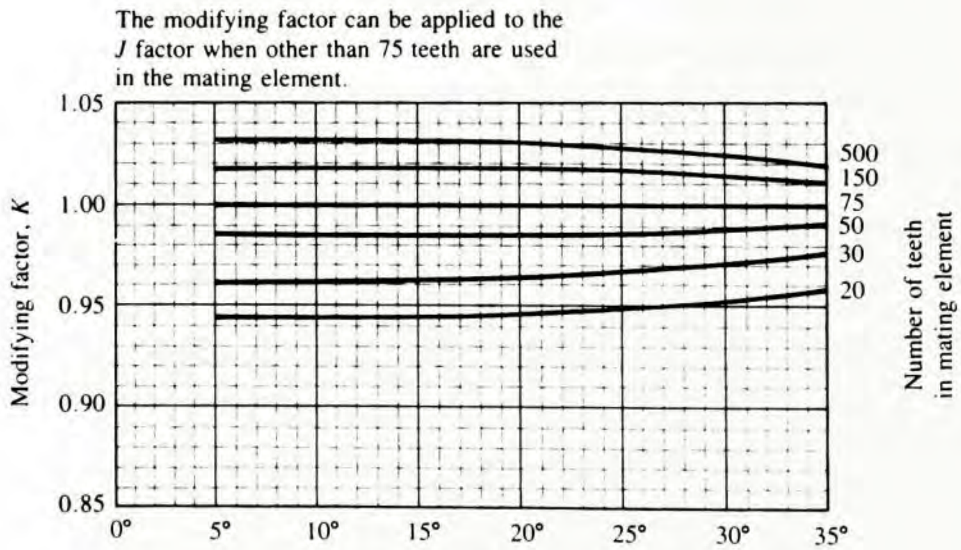
(b) J factor multipliers

(See also References 11, 16, and 21.) The data for steel, iron, and bronze apply to a design life of 10^7 cycles at a reliability of 99% (fewer than one failure in 100). If other values for design life or reliability are desired, the allowable stress can be modified using the procedure described in Section 9–9.

FIGURE 10–7
Geometry factor (J) for 22° normal pressure angle



(*a*) Geometry factor (J), for 22° normal pressure angle, standard addendum, and pre-shave hob



(*b*) J factor multipliers

10–4 PITTING RESISTANCE FOR HELICAL GEAR TEETH

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Pitting resistance for helical gear teeth is evaluated using the same procedure as that discussed in Chapter 9 for spur gears. Equation (9–25) is repeated here:

$$s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}} \qquad \textbf{(9–25)}$$

All of the factors are the same for helical gears except the geometry factor for pitting resistance, I. The values for C_p are found in Table 9–9. Note that the other K factors have the same values as the K factors discussed and identified in Section 10–3.

Because of the larger variety of geometric features needed to define the form of helical gears, it is not reasonable to reproduce all of the necessary tables of values or the complete formulas for computing I. Values change with the gear ratio, the number of teeth in the pinion, the tooth form, the helix angle, and the specific values for addendum, whole depth, and fillet radius. See References 6 and 13 for extensive discussions about the procedures. To facilitate problem solving in this book, Tables 10–1 and 10–2 give a few values for I.

TABLE 10–1 Geometry factors for pitting resistance, I, for helical gears with 20° normal pressure angle and standard addendum

A. Helix angle $\psi = 15.0°$

Gear teeth	Pinion teeth: 17	21	26	35	55
17	0.124				
21	0.139	0.128			
26	0.154	0.143	0.132		
35	0.175	0.165	0.154	0.137	
55	0.204	0.196	0.187	0.171	0.143
135	0.244	0.241	0.237	0.229	0.209

B. Helix angle $\psi = 25.0°$

Gear teeth	Pinion teeth: 14	17	21	26	35	55
14	0.123					
17	0.137	0.126				
21	0.152	0.142	0.130			
26	0.167	0.157	0.146	0.134		
35	0.187	0.178	0.168	0.156	0.138	
55	0.213	0.207	0.199	0.189	0.173	0.144
135	0.248	0.247	0.244	0.239	0.230	0.210

Source: Extracted from AGMA Standard 908-B89 (R 1995), *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

TABLE 10–2 Geometry factors for pitting resistance, I, for helical gears with 25° normal pressure angle and standard addendum

A. Helix angle $\psi = 15.0°$

Gear teeth	Pinion teeth 14	17	21	26	35	55
14	0.130					
17	0.144	0.133				
21	0.160	0.149	0.137			
26	0.175	0.165	0.153	0.140		
35	0.195	0.186	0.175	0.163	0.143	
55	0.222	0.215	0.206	0.195	0.178	0.148
135	0.257	0.255	0.251	0.246	0.236	0.214

B. Helix angle $\psi = 25.0°$

Gear teeth	Pinion teeth 12	14	17	21	26	35	55
12	0.129						
14	0.141	0.132					
17	0.155	0.146	0.135				
21	0.170	0.162	0.151	0.138			
26	0.185	0.177	0.166	0.154	0.141		
35	0.203	0.197	0.188	0.176	0.163	0.144	
55	0.227	0.223	0.216	0.207	0.196	0.178	0.148
135	0.259	0.258	0.255	0.251	0.246	0.235	0.213

Source: Extracted from AGMA Standard 908-B89, *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314.

For design, when the computed contact stress number is known, a material must be specified that has an allowable contact stress number, s_{ac}, greater than s_c. Design values for s_{ac} can be found from the following:

Figure 9–11: Steel, through-hardened, Grades 1 and 2

Table 9–3: Steel, case-hardened, Grades 1, 2, and 3; flame- or induction-hardened, carburized, or nitrided

Table 9–4: Cast iron and bronze

The data from these sources apply to a design life of 10^7 cycles at a reliability of 99% (fewer than one failure in 100). If other values for design life or reliability are desired, or if a service factor is to be applied, the allowable contact stress number can be modified using the procedure described in Section 9–11.

10–5 DESIGN OF HELICAL GEARS

The example problem that follows illustrates the procedure to design helical gears.

Example Problem 10–2 A pair of helical gears for a milling machine drive is to transmit 65 hp with a pinion speed of 3450 rpm and a gear speed of 1100 rpm. The power is from an electric motor. Design the gears.

Solution Of course, there are several possible solutions. Here is one. Let's try a normal diametral pitch of 12, 24 teeth in the pinion, a helix angle of 15°, a normal pressure angle of 20°, and a quality number of 8.

Now compute the transverse diametral pitch, the axial pitch, the transverse pressure angle, and the pitch diameter. Then we will choose a face width that will give at least two axial pitches to ensure true helical action.

$$P_d = P_{dn}\cos\psi = 12\cos(15^\circ) = 11.59$$

$$P_x = \frac{\pi}{P_d\tan\psi} = \frac{\pi}{11.59\tan(15^\circ)} = 1.012\text{ in}$$

$$\phi_t = \tan^{-1}(\tan\phi_n/\cos\psi) = \tan^{-1}[\tan(20^\circ)/\cos(15^\circ)] = 20.65^\circ$$

$$d = D_P/P_d = 24/11.59 = 2.071\text{ in}$$

$$F = 2P_x = 2(1.012) = 2.024\text{ in}\quad\text{(nominal face width)}$$

Let's use 2.25 in, a more convenient value. The pitch line speed and the transmitted load are

$$v_t = \pi D_P\,n/12 = \pi(2.071)(3450)/12 = 1871\text{ ft/min}$$

$$W_t = 33\,000\,(\text{hp})/v_t = 33\,000(65)/1871 = 1146\text{ lb}$$

Now we can calculate the number of teeth in the gear:

$$VR = N_G/N_P = n_P/n_G = 3450/1100 = 3.14$$

$$N_G = N_P\,(VR) = 24(3.14) = 75\text{ teeth}\quad\text{(integer value)}$$

The values for the factors in Equation (9–15) must now be determined to enable the calculation of the bending stress. The geometry factor for the pinion is found in Figure 10–6 for 24 teeth in the pinion and 75 teeth in the gear: $J_P = 0.48$. The value of J_G will be greater than the value of J_P, resulting in a lower stress in the gear.

The K factors are

K_o = overload factor = 1.5 (moderate shock)

K_s = size factor = 1.0

K_m = load-distribution factor = 1.26 for F/D_P = 1.09 and commercial-quality, enclosed gearing

K_B = rim thickness factor = 1.0 (solid gears)

K_v = dynamic factor = 1.35 for Q_v = 8 and v_t = 1871 ft/min

The bending stress in the pinion can now be computed:

$$s_{tP} = \frac{W_tP_d}{FJ_P}K_oK_sK_mK_BK_v$$

$$s_{tP} = \frac{(1146)(11.59)}{(2.25)(0.48)}(1.50)(1.0)(1.26)(1.0)(1.35) = 31\,400\text{ psi}$$

From Figure 9–10, a Grade 1 steel with a hardness of approximately 250 HB would be required. Let's proceed to the design for pitting resistance.

Use Equation (9–25):

$$s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{F D_P I}}$$

For two steel gears, $C_p = 2300$. Rough interpolation from the data in Table 10–1 for $N_P = 24$ and $N_G = 75$ gives $I = 0.202$. It is recommended that the computational procedure described in the AGMA standards be used to compute a more precise value for critical work. The contact stress is then

$$s_c = 2300\sqrt{\frac{(1146)(1.50)(1.0)(1.26)(1.35)}{(2.25)(2.071)(0.202)}} = 128\,200 \text{ psi}$$

Figure 9–11 indicates that a Grade 1 steel with a hardness of 310 HB would be recommended. Assuming that standard life and reliability factors are acceptable, we could specify AISI 5150 OQT 1000, which has a hardness of 321 HB as listed in Appendix 3.

Comments It is obvious that the contact stress governs this design. Let's adjust the solution for a higher reliability and to account for the expected number of cycles of operation. Certain design decisions must be made. For example, consider the following:

Design for a reliability of 0.999 (less than one failure in 1000): $K_R = 1.25$ (Table 9–8).

Design life: Let's design for 10 000 h of life as suggested in Table 9–7 for multipurpose gearing. Then, using Equation (9–18), we can compute the number of cycles of loading. For the pinion rotating at 3450 rpm with one cycle of loading per revolution,

$$N_c = (60)(L)(n)(q) = (60)(10\,000)(3450)(1.0) = 2.1 \times 10^9 \text{ cycles}$$

From Figure 9–24, we find that $Z_N = 0.89$.

No unusual conditions seem to exist in this application beyond those already considered in the various K factors. Therefore, we use a service factor, SF, of 1.00.

We can use Equation (9–27) to apply these factors. For the pinion, we use $C_H = 1.00$:

$$\frac{K_R(\text{SF})}{Z_N\, C_H}s_c = s_{ac} = \frac{(1.25)(1.00)}{(0.89)(1.00)}(128\,200 \text{ psi}) = 180\,000 \text{ psi}$$

Table 9–3 indicates that Grade 1 steel, case-hardened by carburizing, would be suitable. From Appendix 5, let's specify AISI 4320 SOQT 450, having a case hardness of HRC 59 and a core hardness of 415 HB. This should be satisfactory for both bending and pitting resistance. Both the pinion and the gear should be of this material. There is a modest difference in the Z_N factor, but it should not lower the required allowable contact stress number below that requiring case hardening. Also, when both the pinion and the gear are case-hardened, the hardness ratio factor, C_H, is 1.00.

10–6 FORCES ON STRAIGHT BEVEL GEARS

MDESIGN

Review Section 8–8 and Figure 8–22 for the geometry of bevel gears. Also see References 1, 5, and 12.

Because of the conical shape of bevel gears and because of the involute-tooth form, a three-component set of forces acts on bevel gear teeth. Using notation similar to that for helical gears, we will compute the tangential force, W_t; the radial force, W_r; and the axial force, W_x. It is assumed that the three forces act concurrently at the midface of the teeth and on the pitch cone (see Figure 10–8). Although the actual point of application of the resultant force is a little displaced from the middle, no serious error results.

The tangential force acts tangential to the pitch cone and is the force that produces the torque on the pinion and the gear. The torque can be computed from the known power transmitted and the rotational speed:

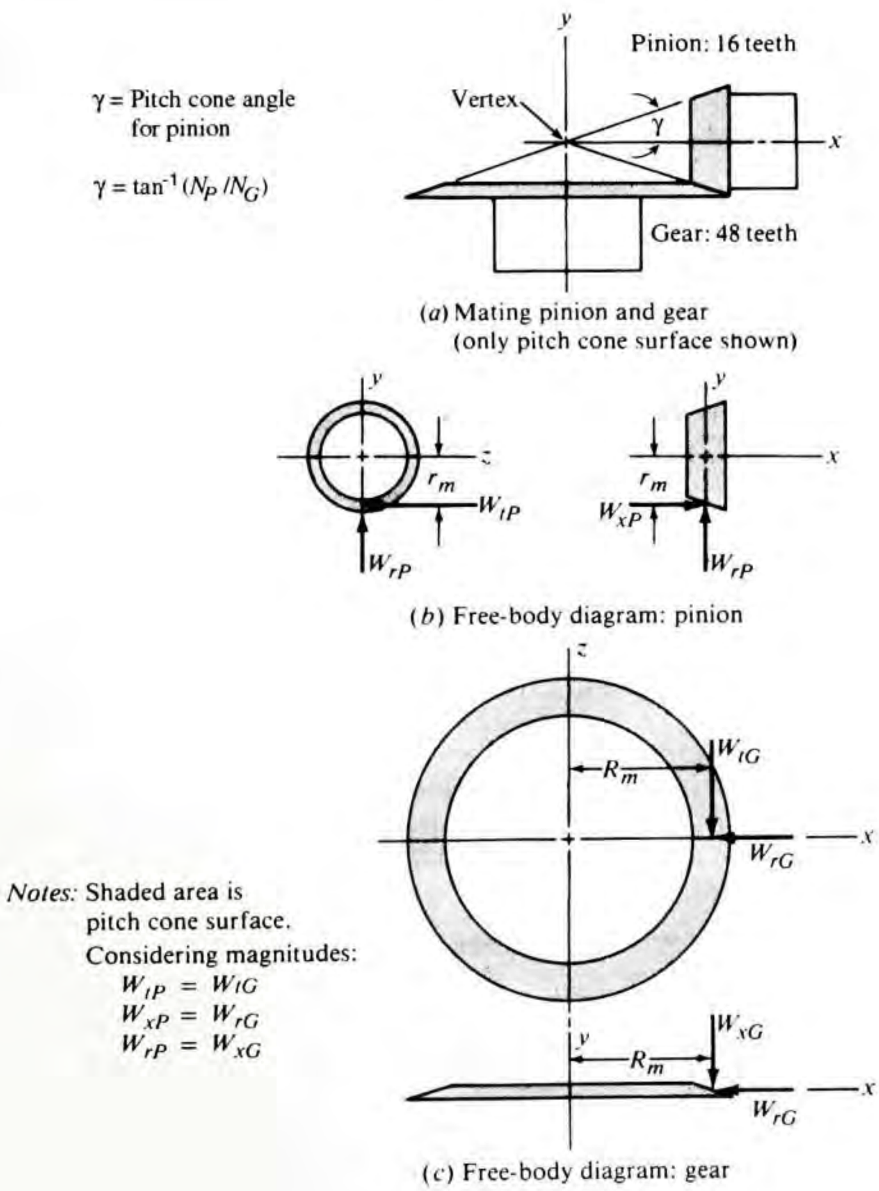
$$T = 63\,000\, P/n$$

Then, using the pinion, for example, the transmitted load is

$$W_{tP} = T/r_m \tag{10–9}$$

where r_m = mean radius of the pinion

FIGURE 10–8
Forces on bevel gears



The value of r_m can be computed from

$$r_m = d/2 - (F/2)\sin\gamma \tag{10–10}$$

Remember that the pitch diameter, d, is measured to the pitch line of the tooth at its large end. The angle, γ, is the pitch cone angle for the pinion as shown in Figure 10–8(a). The radial load acts toward the center of the pinion, perpendicular to its axis, causing bending of the pinion shaft. Thus,

$$W_{rP} = W_t \tan\phi \cos\gamma \tag{10–11}$$

The angle, ϕ, is the pressure angle for the teeth.

The axial load acts parallel to the axis of the pinion, tending to push it away from the mating gear. It causes a thrust load on the shaft bearings. It also produces a bending moment on the shaft because it acts at the distance from the axis equal to the mean radius of the gear. Thus,

$$W_{xP} = W_t \tan\phi \sin\gamma \tag{10–12}$$

The values for the forces on the gear can be calculated by the same equations shown here for the pinion, if the geometry for the gear is substituted for that of the pinion. Refer to Figure 10–8 for the relationships between the forces on the pinion and the gear in both magnitude and direction.

Example Problem 10–3 For the gear pair described in Example Problem 8–6, calculate the forces on the pinion and the gear if they are transmitting 2.50 hp with a pinion speed of 600 rpm. The geometry factors computed in Example Problem 8–6 apply. The data are summarized here.

Summary of Pertinent Results from Example Problem 8–6 and Given Data

Number of teeth in the pinion: $N_P = 16$

Number of teeth in the gear: $N_G = 48$

Diametral pitch: $P_d = 8$

Pitch diameter of pinion: $d = 2.000$ in

Pressure angle: $\phi = 20°$

Pinion pitch cone angle: $\gamma = 18.43°$

Gear pitch cone angle: $\Gamma = 71.57°$

Face width: $F = 1.00$ in

Rotational speed of pinion: $n_P = 600$ rpm

Power transmitted: $P = 2.50$ hp

Solution Forces on the pinion are described by the following equations:

$$W_t = T/r_m$$