Diophantine Equation Problem Set

February 8, 2026

P1 (O1) Find all integers $x, y \in \mathbb{Z}$ such that

$$x^3 - y^3 = 67.$$

P2 (O1) Find all integers $x, y \in \mathbb{Z}$ and all natural numbers $n \in \mathbb{N}$ such that

$$x + x^2 + x^3 = 67^n.$$

P3 (O1) Find all integers $x, y \in \mathbb{Z}$ and all natural numbers $n \in \mathbb{N}$ such that

$$x + y + xy = 67^n - 1.$$

P4 (O1) Find all natural numbers $x \in \mathbb{N}$ and primes p such that the number

$$x^p - x$$

has exactly two prime divisors.

P5 (O2) Show that for $x > 1$, the number

$$1 + x^2 + x^4$$

cannot be a prime.