Finite Sets Closed Under Algebraic Operations

February 18, 2026

Problem 1 (O1). Find all finite sets $S \subset \mathbb{N}$ such that

$$\forall a \in S : \quad \sqrt{a} \in S.$$

Problem 2 (O1). Find all finite sets $S \subset \mathbb{N}$ such that

$$\forall a, b \in S : \quad a^2 - b \in S.$$

Problem 3 (O2). Find all finite sets $S \subset \mathbb{Z}$ such that

$$\forall a, b \in S : \quad a^2 - b^2 \in S.$$

Problem 4 (O4). Does there exist arbitrarily large finite sets $S \subset \mathbb{N}$ such that
$$\forall a \neq b \in S : \quad (a - b)^2 \in S?$$
Either prove existence or show that the size of such sets is bounded.

Problem 5 (O5). Let $f : \mathbb{N} \to \mathbb{N}$ be a function such that the function

$$x \mapsto f(x) - x$$

is strictly increasing. Does there exist arbitrarily large finite sets $S \subset \mathbb{N}$ such that
$$\forall a \neq b \in S : \quad f(a - b) \in S?$$