Theorem 1. *(Uniform Convergence Theorem) If l is slowly varying then*

$$\frac{l(\lambda x)}{l(x)} \to 1 \quad (x \to \infty)$$

uniformly in each compact λ-set in $(0, \infty)$.

Theorem 2. *(Potters Theorem)*

(i) If l is slowly varying function then for any chosen constants $A > 1$, $\delta > 0$ there exits $X = X(A, \delta)$ such that

$$\frac{l(y)}{l(x)} \leq A \max \left\{ \left(\frac{y}{x}\right)^{\delta}, \left(\frac{x}{y}\right)^{\delta} \right\} \quad (x \geq X, y \geq X)$$

(ii) If further, l is bounded away from 0 and ∞ in every compact subset of $[0, \infty)$, then for every $\delta > 0$ there exists $A' = A'(\delta) > 1$ such that

$$\frac{l(y)}{l(x)} \leq A' \max \left\{ \left(\frac{y}{x}\right)^{\delta}, \left(\frac{x}{y}\right)^{\delta} \right\} \quad (x \geq 0, y \geq 0)$$

(iii) If f is regularly varying of index ρ then for any chosen $A > 1$, $\delta > 0$ there exists $X = X(A, \delta)$ such that

$$\frac{f(y)}{f(x)} \leq A \max \left\{ \left(\frac{y}{x}\right)^{\rho+\delta}, \left(\frac{x}{y}\right)^{\rho+\delta} \right\} \quad (x \geq 0, y \geq 0)$$

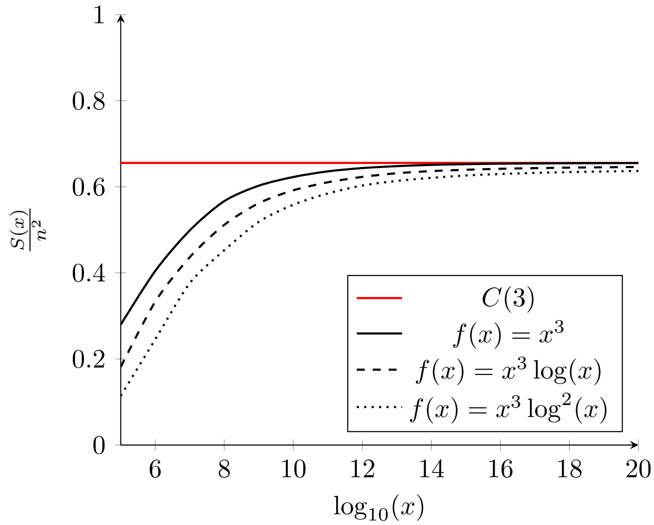

Figure 1: Example on how the slowly varying part affects convergence

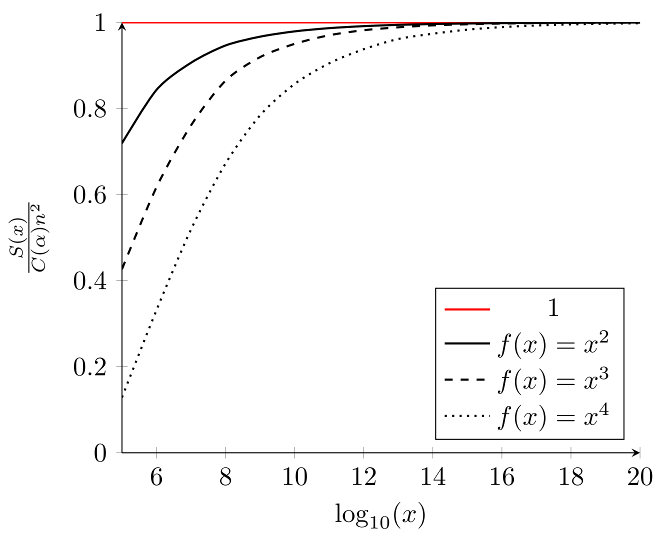

Figure 2: Example on how the index affects convergence