Interval sum is sum of consecutive terms in a sequence. An asymptotic formula is found for the number of interval sums for a class of sequences $\big(f(i)\big)_{i=1}^{\infty}$, where f is measurable, locally bounded away from 0 and ∞, eventually monotonic and regularly varying with index $\alpha > 1$. This extends results of O'Sullivan et al. on interval sums of prime powers.

The main theorem provides an asymptotic formula for the interval sum counting function

$$S(x) = \#\Big\{(i,j) \in \mathbb{N}^2 \mid i \leq j, f(i) + f(i+1) + \cdots + f(j) \leq x\Big\}. \tag{1}$$

That is

$$S(x) \sim n^2 C(\alpha) \qquad n = n(x) \to \infty \tag{2}$$

where $x \sim \frac{nf(n)}{\alpha+1}, x \to \infty$ and

$$C(\alpha) = \frac{1}{2(\alpha+1)} \frac{\Gamma(\frac{1}{\alpha+1})\Gamma(\frac{\alpha-1}{\alpha+1})}{\Gamma(\frac{\alpha}{\alpha+1})}. \tag{3}$$

The proof relies on classical results from the theory of regular variation, in particular the uniform convergence theorem and Potter's bound. The function $S(x)$ can be split into a sum of s_i that count the number of intervals of length i. The main idea in the proof is that s_i can be related to the inverse of

$$\phi(x) = (1 + x^{\alpha+1})^{\frac{1}{\alpha+1}} - x \tag{4}$$

which allows us to relate $S(x)$ asymptotically to

$$n^2 \int_0^1 \phi^{-1}(t)dt. \tag{5}$$

The integral can be calculated using standard substitution and using analytically extended beta-function, which completes the proof.