

**Interval sum** is sum of consecutive terms in a sequence. An asymptotic formula is found for the number of interval sums for a class of sequences  $(f(i))_{i=1}^{\infty}$ , where  $f$  is measurable, locally bounded away from 0 and  $\infty$ , eventually increasing, and regularly varying with index  $\alpha > 1$ . This generalizes and extends results of O'Sullivan et al. on interval sums of prime powers.

The main theorem provides an asymptotic formula for the interval sum counting function

$$S(x) = \# \left\{ (i, j) \in \mathbb{N}^2 \mid i \leq j, f(i) + f(i+1) + \cdots + f(j) \leq x \right\}.$$

That is

$$S(x) \sim n^2 C(\alpha) \quad n = n(x) \rightarrow \infty$$

where  $x \sim \frac{nf(n)}{\alpha+1}$ ,  $x \rightarrow \infty$  and

$$C(\alpha) = \frac{1}{2(\alpha+1)} \frac{\Gamma(\frac{1}{\alpha+1})\Gamma(\frac{\alpha-1}{\alpha+1})}{\Gamma(\frac{\alpha}{\alpha+1})}.$$

The proof relies on classical results from the theory of regular variation, in particular the uniform convergence theorem and Potter's bound. The function  $S(x)$  can be split into a sum of  $s_i$  that counts the number of intervals of length  $i$ . The main idea in the proof is that  $s_i$  can be related to the inverse of

$$\phi(x) = (1 + x^{\alpha+1})^{\frac{1}{\alpha+1}} - x$$

which allows us to relate  $S(x)$  asymptotically to

$$n^2 \int_0^1 \phi^{-1}(t) dt.$$

The integral can be calculated using standard substitution and using the analytically extended beta function, which completes the proof.