Lemma 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be a convex, twice differentiable function such that $f'(x) \geq 0$. Let $g : \mathbb{R} \to \mathbb{R}$ satisfy*

$$f(g(x)) \sim x$$

Then

$$g(x) \sim f^{-1}(x)$$

Proof. From $f(g(x)) \sim x$ follows

$$f(g(x)) = x + o(x)$$

Taking inverse of f

$$g(x) = f^{-1}(x + o(x))$$

Since f is convex and monotonic, we get

$$f^{-1}((1+\epsilon)x) \leq (1+\epsilon)f^{-1}(x)$$
$$f^{-1}((1-\epsilon)x) \geq (1-\epsilon)f^{-1}(x)$$

Hence,

$$g(x) = f^{-1}(x + o(x)) = (1 + o(1))f^{-1}(x) \sim f^{-1}(x)$$

$\square$