

**MIN-291 Engineering Analysis and Design Autumn 2023-24**  
**Tutorial 1**  
**Mathematical Preliminaries and Index Notation**  
**(All questions carry 2 marks)**

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- 1) Matrix **A** and Vector **t** are given below. Expand and find the following quantities:

$$A_{ii}, A_{ij}t_j$$

$$A = \begin{bmatrix} 10 & 5 & 7 \\ 5 & 3 & 9 \\ 7 & 9 & 0 \end{bmatrix} \quad t = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

- 2) Write the expression  $c_i = A_{ij}b_j - b_i$  in the form of  $c_i = B_{ij}b_j$  and hence find the expression for  $B_{ij}$

- 3) Discuss the validity of the following relations:

1)  $a_mb_s = c_m(d_r - f_r)$ ,

2)  $a_i = b_jc_id_i$

3)  $x_i x_i = r^2$

4)  $a_ib_jc_j = 3$

- 4) Components of a first order and second order tensor are given below. Determine the components of each tensor in a new coordinate system found through a rotation of  $60^\circ$  about  $x_3$  axis. Choose a counter-clockwise rotation when viewing along the negative  $x_3$  axis.

$$a_{ij} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & 4 \end{bmatrix} \quad a_i = \begin{Bmatrix} 1 \\ 4 \\ 2 \end{Bmatrix}$$

- 5) Matrix A is said to be symmetric if  $A_{ij} = A_{ji}$ . Matrix B is said to be antisymmetric/skew symmetric if  $B_{ij} = -B_{ji}$ . Prove that for the aforementioned pair of symmetric and antisymmetric matrices,  $A_{ij}B_{ij} = 0$ .