Probabilistic Robotics Prelab 4 Extended Kalman Filter

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I. Transforming From World Coordinates To Robot Coordinates

THE transformations that can be used to obtain (ρ_r, ϕ_r) from (ρ_w, ϕ_w) given the robot's position (x_r, y_r, θ) are described below. When programming, the *angle_wrap* function should be used after the final calculation of ϕ_r.

$\rho_r' = \rho_w - x_r cos\phi_w - y_r sin\phi_w$;
$\phi_r' = \phi_w - \theta$;
if $\rho_r' >= 0$ **then**
  $\rho_r = \rho_r'$;
  $\phi_r = \phi_r'$;
else
  $\rho_r = -\rho_r'$;
  $\phi_r = \phi_r' + \Pi$;
end

II. Measurement Equations And Jacobian H

The system's state x is defined as the following vector:

$$x = \begin{bmatrix} x_r \\ y_r \\ \theta \end{bmatrix}$$

And the model we will use for the system dynamics is:

$$\hat{x_k} = \begin{bmatrix} \hat{x}_{r_{k-1}} + (\Delta x + w_x) \cdot cos\theta_{k-1} - (\Delta y + w_y) \cdot sin\theta_{k-1} \\ \hat{y}_{r_{k-1}} + (\Delta x + w_x) \cdot sin\theta_{k-1} + (\Delta y + w_y) \cdot cos\theta_{k-1} \\ \theta_{k-1} + (\Delta\theta + w_\theta) \end{bmatrix}$$

With the model above we can calculate the A_k and W_k matrices to be used for the EKF:

$$A_k = \begin{bmatrix} 1 & 0 & -\Delta x \cdot sin\theta_{k-1} - \Delta y \cdot cos\theta_{k-1} \\ 0 & 1 & \Delta x \cdot cos\theta_{k-1} - \Delta y \cdot sin\theta_{k-1} \\ 0 & 0 & 1 \end{bmatrix}$$

$$W_k = \begin{bmatrix} cos\theta_{k-1} & -sin\theta_{k-1} & 0 \\ sin\theta_{k-1} & cos\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As was stated in Section I, our definition of $h(x_k, v_k)$ has to be split in two cases. The case for $\rho_r' = 0$ since be treated with care since wither case could be valid.

If $\rho_r' >= 0$:

$$h(x_k, v_k) = \begin{bmatrix} \rho_r + v_\rho \\ \phi_r + v_\phi \end{bmatrix} = \begin{bmatrix} \rho_w - x_r cos\phi_w - y_r sin\phi_w + v_\rho \\ \phi_w - \theta + v_\phi \end{bmatrix}$$

$$H_k = \begin{bmatrix} -cos\phi_w & -sin\phi_w & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Else:

$$h(x_k, v_k) = \begin{bmatrix} \rho_r + v_\rho \\ \phi_r + v_\phi \end{bmatrix} = \begin{bmatrix} -\rho_w + x_r cos\phi_w + y_r sin\phi_w + v_\rho \\ \phi_w - \theta + \Pi + v_\phi \end{bmatrix}$$

$$H_k = \begin{bmatrix} cos\phi_w & sin\phi_w & 0 \\ 0 & 0 & -1 \end{bmatrix}$$