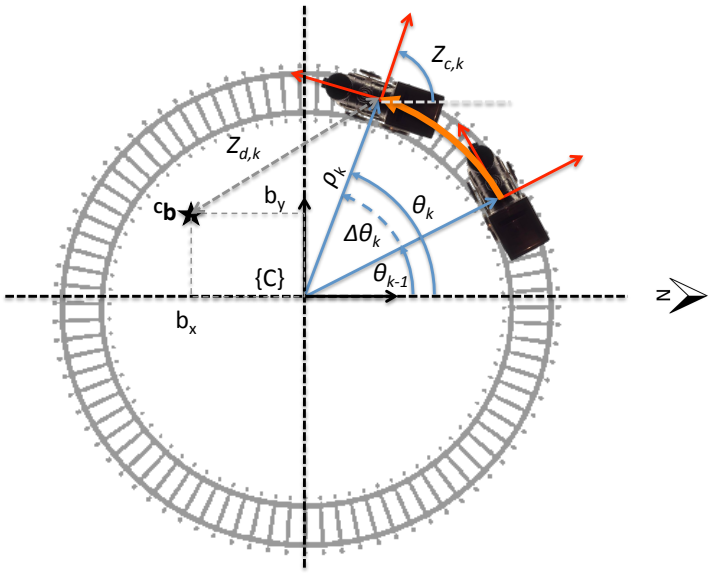
KF17. A toy train moves along a circular rail. The train is equipped with wheel encoders, which provide a measurement of the longitudinal displacement. It is also equipped with a compass which measures the north referenced orientation. There is a beacon located at a known position $^{C}\mathbf{b}$ in the plane, being {C} its reference frame being at the centre of the circular rail.



Answer the following questions:

1. If we want to estimate the train position in polar coordinates $[\rho\ \vartheta]^T$ using a KF/EKF, and taking into account that the radius of the rail is perfectly known a priori, what parameter should we place in the state vector?
2. Write the prediction equation if the train displacement, measured with the encoders, is given by the deterministic variable u_k and its uncertainty is given by the stochastic variable $w_k=N(0,\sigma_w^2{}_k)$. In particular you are requested to write first the stochastic equation and then equation for the mean and the equation for the covariance.
3. Write the observation equation for a train location measurement $z_{c,k}$ coming from a compass sensor assuming that the uncertainty in the compass measurement is given by an stochastic variable $v_k=N(0,\sigma^2{}_{vk})$. In particular you are requested to write the stochastic equation $z_k=h(x_k)+v_k$. Underline the involved stochastic.
4. Is the previous observation equation linear or nonlinear? Write the corresponding H_k matrix if it is linear or the linearizing Jacobian with respect to the state H_k otherwise.
5. Write the observation equation for a train-to-beacon distance measurement $z_{d,k}$, assuming that the uncertainty in the distance measurement is given by an stochastic variable $v_k=N(0,\sigma^2{}_{vk})$. In particular you are requested to write the stochastic equation $z_k=h(x_k)+v_k$. Underline the involved stochastic .