Probabilistic Robotics Exercise 1

Rodrigo Caye Daudt

April 17, 2016

1 Problem Description

A robot manipulator is equipped with a tactile sensor in its gripper. When the robot is commanded to grasp an object, if the object was not already grasped it succeeds with 0.7 probabilities but fails with a 0.3 probability, otherwise it continues grasping the object with 0.9 probability but can fail with 0.1 probability. Whenever an object is hold within the gripper, the tactile sensor detects the situation with a 0.6 probability but fails in the detection with a 0.4 probability. If the gripper is free, the tactile sensor detects the situation with 0.8 probability but fails with 0.2 probability.

The following sequence of actions/sensor readings have been executed:
1) The action GRASP is executed
2) The tactile sensor detects NO OBJECT GRASPED.

2 Solution

From the problem description we can organize the information as follows:

State transition probabilities:

$$p(X_t = ObjectGrasped | U_t = Grasp, X_{t-1} = NoObjectGrasped) = 0.7$$

$$p(X_t = NoObjectGrasped | U_t = Grasp, X_{t-1} = NoObjectGrasped) = 0.3$$

$$p(X_t = ObjectGrasped | U_t = Grasp, X_{t-1} = ObjectGrasped) = 0.9$$

$$p(X_t = NoObjectGrasped | U_t = Grasp, X_{t-1} = ObjectGrasped) = 0.1$$

Measurement probabilities:

$$p(Z_t = ObjectGrasped | X_t = ObjectGrasped) = 0.6$$

$$p(Z_t = NoObjectGrasped | X_t = ObjectGrasped) = 0.4$$

$$p(Z_t = ObjectGrasped | X_t = NoObjectGrasped) = 0.2$$

$$p(Z_t = NoObjectGrasped | X_t = NoObjectGrasped) = 0.8$$

Initial belief:

$$bel(X_0 = ObjectGrasped) = 0.5$$

$$bel(X_0 = NoObjectGrasped) = 0.5$$

2.1 Prediction

In this step we should apply the prediction step of the Bayes filter to calculate the new belief predictions, given that $U_1 = Grasp$.

$$\begin{aligned}
&\overline{bel}(X_1 = ObjectGrasped) = bel(X_0 = ObjectGrasped) \\
&\quad \cdot p(X_t = ObjectGrasped | U_t = Grasp, X_{t-1} = ObjectGrasped) \\
&\quad + bel(X_0 = NoObjectGrasped) \\
&\quad \cdot p(X_t = ObjectGrasped | U_t = Grasp, X_{t-1} = NoObjectGrasped) \\
&\quad = 0.5 \cdot 0.9 + 0.5 \cdot 0.7 = 0.8
\end{aligned}$$

$$\begin{aligned}
&\overline{bel}(X_1 = NoObjectGrasped) = bel(X_0 = ObjectGrasped) \\
&\quad \cdot p(X_t = NoObjectGrasped | U_t = Grasp, X_{t-1} = ObjectGrasped) \\
&\quad + bel(X_0 = NoObjectGrasped) \\
&\quad \cdot p(X_t = NoObjectGrasped | U_t = Grasp, X_{t-1} = NoObjectGrasped) \\
&\quad = 0.5 \cdot 0.1 + 0.5 \cdot 0.3 = 0.2
\end{aligned}$$

2.2 Correction

Then, we apply the correction step of the Bayes filter considering we obtained a measurement $Z_1 = NoObjectGrasped$:

$$\begin{aligned}
&bel(X_1 = ObjectGrasped) = \\
&\qquad \eta \cdot p(Z_t = NoObjectGrasped | X_t = ObjectGrasped) \\
&\qquad \cdot \overline{bel}(X_1 = ObjectGrasped) \\
&\qquad = \eta \cdot 0.4 \cdot 0.8 \\
&\qquad = \eta \cdot 0.32
\end{aligned}$$

$$
\begin{aligned}
&bel(X_1 = NoObjectGrasped) = \\
&\quad \eta \cdot p(Z_t = NoObjectGrasped | X_t = NoObjectGrasped) \\
&\quad \cdot \overline{bel}(X_1 = NoObjectGrasped) \\
&\quad = \eta \cdot 0.8 \cdot 0.2 \\
&\quad = \eta \cdot 0.16
\end{aligned}
$$

We then calculate η:

$$
\begin{aligned}
\eta &= \frac{1}{\frac{bel(X_1 = ObjectGrasped)}{\eta} + \frac{bel(X_1 = NoObjectGrasped)}{\eta}} \\
&= \frac{1}{0.32 + 0.16} = \frac{1}{0.48}
\end{aligned}
$$

Therefore:

$$
bel(X_1 = ObjectGrasped) = \frac{2}{3}
$$

$$
bel(X_1 = NoObjectGrasped) = \frac{1}{3}
$$

2.3 Final Results

The table can then be filled as follows:

Executed Action	Probability Object Grasped	Probability No Object Grasped
$U_t = Grasp$	0.8	0.2
$Z_t = NoObjectGrasped$	0.667	0.333