Machine Learning In Physics Exam

First Author
Odysseas Pattas odysseas.pattas@ru.nl

Abstract

This assignment investigates the application of machine learning techniques to simulate and predict the trajectories in two-body and three-body systems governed by gravitational forces. Initially, simple regression networks are developed and their performance is compared with advanced autoregressive models such as Long Short-Term Memory (LSTM) networks. The primary goal is to evaluate the suitability of these neural network architectures in modeling complex orbital dynamics.

1 Simulation of the Two-Body Problem with a Circular Orbit

In this simulation, we investigate the motion of a light object (space ship) with mass $m = 10^4$kg rotating around a heavy object with mass $M = 10^9$ kg that is at rest. The radius between the objects is $r = 10.000$ m, and the gravitational force between them is governed by Newton's law of gravitation:

$$F = -\frac{GMm}{r^2} \tag{1}$$

The position and velocity of the light object are updated using the Euler method. For a detailed explanation, see Appendix A.

To mimic real-world conditions, a 1% relative uncertainty was introduced in the measurements of the angle, radius, and velocity of the orbiting object. The initial coordinates of the light object are $x = 0$ m and $y = 10.000$ m, while the heavy object, which remains fixed due to its larger mass, is positioned at $x = 0$ m and $y = 0$ m.

The simulation runs for 10,000 timesteps, with the total simulated time is covering 19 periods of the orbit. The period is calculated from :

$$\text{period} = \frac{2\pi r}{\text{Initial velocity}} \tag{2}$$

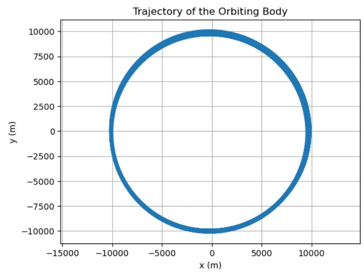

Figure 1: Trajectory of the space ship.

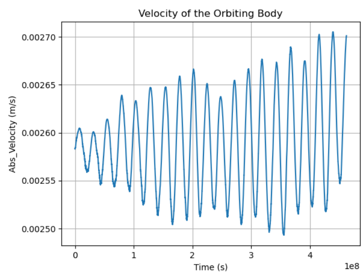

Figure 2: Absolute value velocity as a function of time.

2 Second force on the light object

Starting with the same initial velocity as before, a new force is introduced in the direction of the velocity. The absolute value of the acceleration α of this force is set to 5×10^{-9}. The x and y components of this additional force are given by:

$$f_X = \alpha \sin v_x \tag{3}$$

$$f_y = \alpha \sin v_y \tag{4}$$

This force depends on the velocity components of the light object and introduces a sinusoidal variation in the acceleration. To explore the impact of this force, several datasets were created by increasing the acceleration in steps of 10%.

For each step, the simulation was run to generate the trajectory and velocity data, which were then analyzed to observe the changes in the orbital dynamics of the light object. Only the plots with the starting acceleration are being shown here.

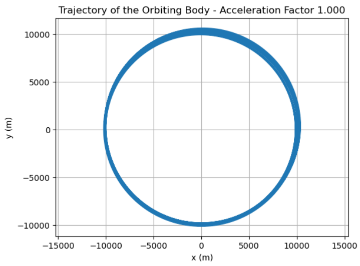

Figure 3: Trajectory of the spaceship With The Addition of Sinusoidal Force

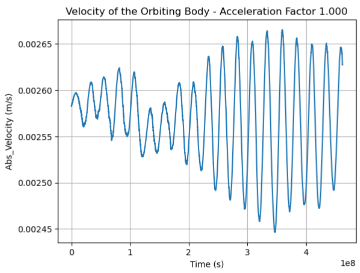

Figure 4: Absolute value velocity as a function of time

3 3-Body Problem in 2 Dimensions

In this case, two fixed heavy bodies are located at $x = 10.000\,\mathrm{m}$ and $x = -10.000\,\mathrm{m}$ respectively while the initial coordinates for the spaceship are x= 0 and $y = 29400\,\mathrm{m}$. Again, we simulate this setup with the initial velocity of the light object as 5 but at this time the radius is equal to the initial y coordinate. Then we simulate it for a slight change in the initial y position of the spaceship. Again, not all plots are being shown here.

4 Regression Networks for the prediction of the tajectory of the Spaceship

The regression networks were trained using the data generated from the previous simulations. These networks predict the position and velocity of the light object based on the last three positions and velocities. The trained models are then used to predict the next 10, 100, and 500 time steps of the trajectory, starting from the position and velocity at time step 5000 in the test data.

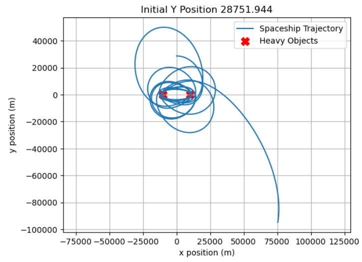

Figure 5: Trajectory of the space ship Around The Two Heavy Bodies

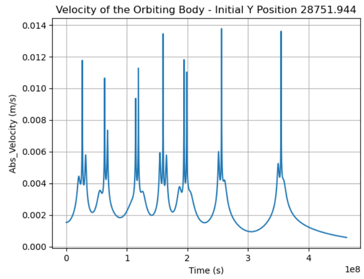

Figure 6: Absolute value velocity as a function of time.

4.1 Data preparation

The data preparation involved creating sequences of the spacecraft's positions and velocities for model training. Initially, sequences of three time steps of position and velocity data were created, with the subsequent time step used as the target for prediction. The features (X) and targets (y) were then normalized using StandardScaler to ensure all variables had a mean of zero and a standard deviation of one. StandardScaler was chosen because it standardizes the data, ensuring each feature contributes equally to the model's performance. Finally, the normalized data was split into training, validation, and test sets, with the test set starting from the 5000th index.

4.2 Trainning and Evaluation of Models

The training and evaluation of the models were performed over 200 epochs. During training, the model's performance was monitored using validation data, and training history plots were generated to visualize the loss over epochs. After training, the model's predictions on the test set were evaluated using mean absolute error (MAE) and root mean squared error (RMSE) metrics to assess accuracy.

5 Models and Predictions

5.1 Two-Body Problem with a Circular Orbit

A simple dense regression model was used to predict the positions and velocities of the spaceship. Given the circular nature of the spaceship's orbit, a complex model structure was unnecessary. The model consisted of an input layer, followed by two hidden layers with ReLU activation and dropout layers to prevent overfitting. The output layer predicted the x and y positions and velocities. The model was compiled using the Adam optimizer and mean squared error (MSE) as the loss function.

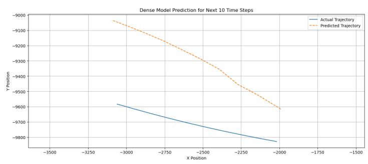

Figure 7: Prediction for Next 10 Time Steps

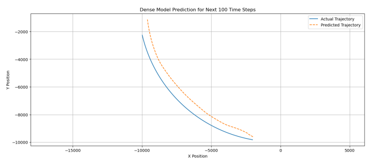

Figure 8: Prediction for Next 100 Time Steps

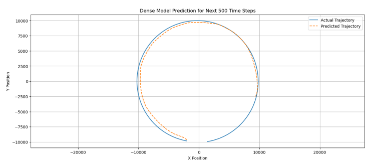

Figure 9: Prediction for Next 500 Time Steps

Figure 10: Predictions plot for the Two-body Problem with a Circular Orbit

5.2 Two-body Problem With Additional Sinusoidal Force

To account for the increased complexity of the data introduced by the additional sinusoidal force, a new layer with 128 neurons was added to the dense regression model.

5.3 Three-Body problem in Two Dimensions

For this scenario, multiple models were trained without any success. The model that produced

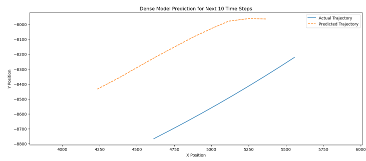

Figure 11: Prediction for Next 10 Time Steps

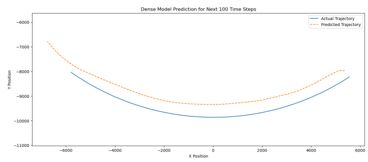

Figure 12: Prediction for Next 100 Time Steps

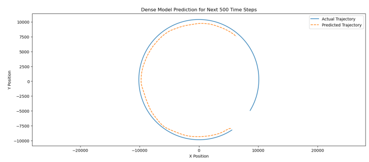

Figure 13: Prediction for Next 500 Time Steps

Figure 14: Predictions plot for the Two-body Problem with a Second Force

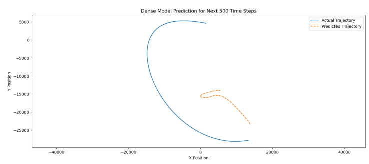

Figure 15: Three Body Problem.Predictions of the model for Next 500 time steps

these plots is similar to the one used in the previous section. An additional layer with 256 neurons was added, and there was an increase in the dropout rate to address the increase in the complexity of the system. A simple plot will be provided as a showcase of the inability of the model to predict the correct trajectory.

6 LSTM model for the prediction of Two-Body Problem with a Circular Orbit

The model architecture consists of two LSTM layers followed by a dense layer and an output layer. The LSTM layers use the tanh activation function to capture the oscillatory nature of the trajectories, while the dense layer employs the ReLU activation function to introduce non-linearity. Dropout layers with a rate of 0.3 are included to prevent over-

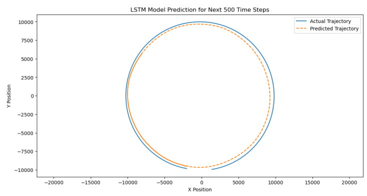

Figure 16: Prediction of the LSTM model for the next 500 timesteps. Number of input is 3.

fitting. The model is compiled using the Adam optimizer with a learning rate of 0.0001 and mean squared error (MSE) as the loss function.

To examine if the model works better, another model were trained with input number equal to 13, meaning that the model uses the last 13 positions and velocities to predict the next one.

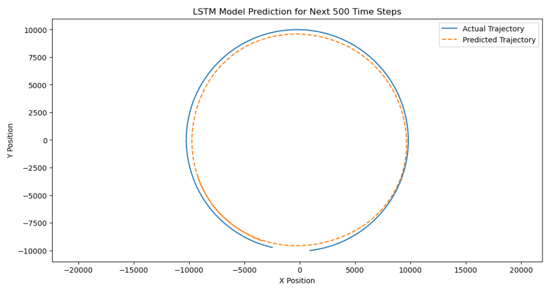

Figure 17: Prediction of the LSTM with increased input for the prediction of the next 500 timesteps. Number of input is 13.

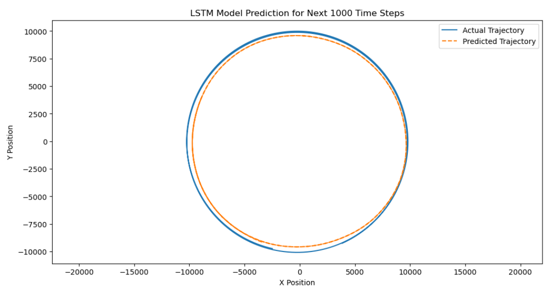

Figure 18: Prediction of the LSTM with increased input for the prediction of the next 1000 timesteps. Number of input is 13.

7 Conclusions

From the plots of the trajectories of the spaceship, we can conclude that the Euler method is an accurate way to simulate the orbit of light objects around heavy bodies.

For the regression models, using a higher number of future time steps for predictions resulted in greater accuracy. Specifically, the model's predictions for the next 500 time steps were the most accurate, while predictions for the next 10 time steps were the least accurate. This demonstrates the importance of considering a sufficient number of future time steps to enhance the accuracy of trajectory predictions. This is not the case for the Three-body system where the prediction was inaccurate in depended on the timesteps. A more complex model such as LSTM or Physics-Informed Neural Network is needed for this system.

Lastly, the LSTM model demonstrated accurate predictions for the circular orbit scenario. It was observed that increasing the number of input positions and velocities improved the accuracy of the model's predictions, especially over longer time steps.

Appendix A: Detailed Explanation of the Euler Method

The Euler method is a straightforward numerical procedure for solving ordinary differential equations (ODEs). It is used to update the position and velocity of the light object in the simulation.

Position Update

The position at the next time step $t+\Delta t$ is updated using the current position $r(t)$ and velocity $v(t)$:

$$r(t+\Delta t) = r(t) + v(t)\Delta t$$

Initial Velocity Calculation

The initial velocity v_0 is computed to ensure a circular orbit by setting the gravitational force equal to the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving for v:

$$v_0 = \sqrt{\frac{GM}{r}} \tag{5}$$

This method is iteratively applied to update the position and velocity at each timestep, simulating the orbit of the light object around the heavy object.