MATH0461-2 Introduction to numerical optimization

Faculty of Applied Science

Randomized Condorcet Voting System

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Contents

List of Figures

List of Tables

1 Model

1.1 Implementation of the model in linear programming

In order to to compute the winning distribution law of the RCVS, one can implement a linear formulation of the model. The linear formulation is based on the following variables:

- A the voting matrix where $A_{i,j}$ represents the results of a duel between the i-th and j-th candidates. The elements of the matrix are computed following this rule : for each voter, if the i-th canditate is ranked higher than the j-th candidate the element $A_{i,j}$ is incremented and the element $A_{j,i}$ is decremented.
- p the probability vector where p_i represents the probability that the i-th candidate wins.
- e the vector of ones.

The linear formulation of the model is the following:

$$\begin{aligned} &\min_p \sum p^T A \\ \text{s.t. } & p^T e = 1 \\ & p \geq 0 \\ & p^T A \geq 0 \end{aligned}$$

This formulation was implemented in the file `q1_model.jl` using the `JuMP` and `Gurobi` packages.

1.2 Application of the RCVS to an example

We want to apply the Condorcet winning system to the following example where edges goes from loser to winner with relatives weights :

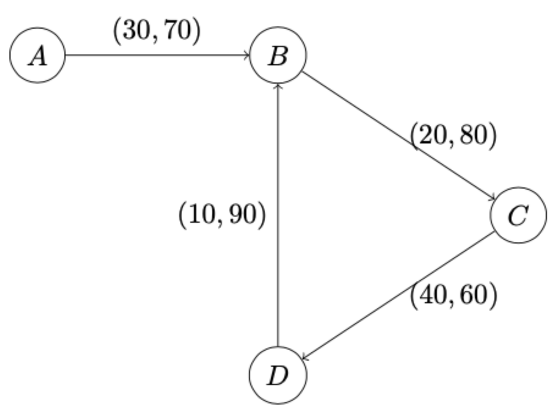

Figure 1: Example of voting graph

In this example, there will no be any Condorcet winner because the graph is not a directed acyclic graph, indeed there is a cycle between the candidates B, C and D. The RCVS is then needed to solve this problem. We simply need to compute the A matrix respecting the graph represented in figure 1 following the rule described in section 1.1:

$$A = \begin{pmatrix} 0 & -40 & 0 & 0 \\ 40 & 0 & -60 & 80 \\ 0 & 60 & 0 & -20 \\ 0 & -80 & 20 & 0 \end{pmatrix}$$

When lauching the `q1_model.jl` file with this matrix as input, we obtain the following lottery for the RCVS :

Candidate	A	B	C	D
Probability	0.0	0.125	0.5	0.375

Table 1: Lottery probabilities for each candidate

1.3 Discussion of the dual variables and optimal dual basis

1.4 Solution of the linear in a linear system

1.5 *Bonus* : Comparaison of the RCVS with an alternative voting system

2 Linear Robust Formulation

3 Quadratic Robust Formulation