MATH0461-2 Introduction to numerical optimization

Faculty of Applied Science

Randomized Condorcet Voting System

Teacher :
Quentin LOUVEAUX

Assistant :
Adrien BOLLAND

Students :
Romain LAMBERMONT, s190931
Arthur LOUIS, s191230

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1 Model

1.1 Implementation of the model in linear programming

In order to to compute the winning distribution law of the RCVS, one can implement a linear formulation of the model. The linear formulation is based on the following variables:

- A the voting matrix where $A_{i,j}$ represents the results of a duel between the i-th and j-th candidates. The elements of the matrix are computed following this rule : for each voter, if the i-th canditate is ranked higher than the j-th candidate the element $A_{i,j}$ is incremented and the element $A_{j,i}$ is decremented.
- p the probability vector where p_i represents the probability that the i-th candidate wins.
- e the vector of ones.

The linear formulation of the model is the following:

$$\begin{aligned} \min_p \ & \sum p^T A \\ \text{s.t. } & p^T e = 1 \\ & p \geq 0 \\ & p^T A \geq 0 \end{aligned}$$

This formulation was implemented in the file `q1_model.jl` using the `JuMP` and `Gurobi` packages.

1.2 Application of the RCVS to an example

We want to apply the Condorcet winning system to the following example where edges goes from loser to winner with relatives weights :

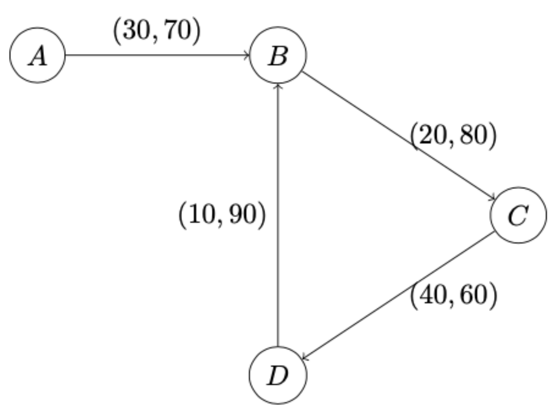

Figure 1: Example of voting graph

In this example, there will no be any Condorcet winner because the graph is not a directed acyclic graph, indeed there is a cycle between the candidates B, C and D. The RCVS is then needed to solve this problem. We simply need to compute the A matrix respecting the graph represented in figure 1 following the rule described in section 1.1:

$$A = \begin{pmatrix} 0 & -40 & 0 & 0 \\ 40 & 0 & -60 & 80 \\ 0 & 60 & 0 & -20 \\ 0 & -80 & 20 & 0 \end{pmatrix}$$

When lauching the `q1_model.jl` file with this matrix as input, we obtain the following lottery for the RCVS :

Candidate	A	B	C	D
Probability	0.0	0.125	0.5	0.375

Table 1: Lottery probabilities for each candidate

1.3 Discussion of the dual variables and optimal dual basis

1.4 Solution of the linear in a linear system

Assuming that there exists a RCVS winning lottery can be reformulated as saying that there won't be any tie between two candidates, an assumption that is probaliticly true, as when the number of voters gets bigger, the probability of a tie gets smaller.

1.5 *Bonus* : Comparaison of the RCVS with an alternative voting system

2 Linear Robust Formulation

3 Quadratic Robust Formulation