

MATH0461-2 INTRODUCTION TO NUMERICAL OPTIMIZATION



FACULTY OF APPLIED SCIENCE

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## Randomized Condorcet Voting System

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# 1 Model

## 1.1 Implementation of the model in linear programming

In order to compute the winning distribution law of the RCVS, one can implement a linear formulation of the model. The linear formulation is based on the following variables:

- $A$  the voting matrix where  $A_{i,j}$  represents the results of a duel between the  $i$ -th and  $j$ -th candidates. The elements of the matrix are computed following this rule : for each voter, if the  $i$ -th candidate is ranked higher than the  $j$ -th candidate the element  $A_{i,j}$  is incremented and the element  $A_{j,i}$  is decremented.
- $p$  the probability vector where  $p_i$  represents the probability that the  $i$ -th candidate wins.
- $e$  the vector of ones.

The linear formulation of the model is the following:

$$\begin{aligned} & \min_p \sum p^T A \\ & \text{s.t. } p^T e = 1 \\ & p \geq 0 \\ & p^T A \geq 0 \end{aligned}$$

This formulation was implemented in the file `q1_model.jl` using the JuMP and Gurobi packages.

## 1.2 Application of the RCVS to an example

We want to apply the Condorcet winning system to the following example where edges goes from loser to winner with relatives weights :

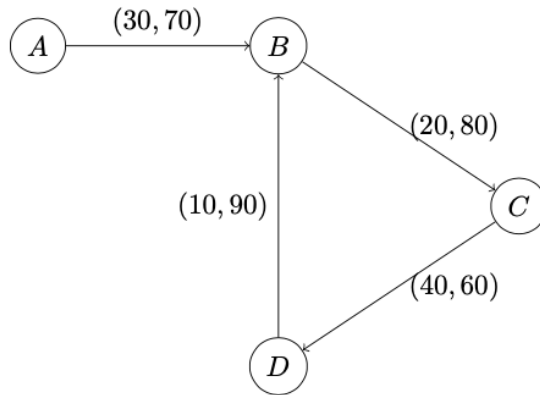


Figure 1: Example of voting graph

In this example, there will no be any Condorcet winner because the graph is not a directed acyclic graph, indeed there is a cycle between the candidates  $B$ ,  $C$  and  $D$ . The RCVS is then needed to solve this problem. We simply need to compute the  $A$  matrix respecting the graph represented in figure 1 following the rule described in section 1.1:

$$A = \begin{pmatrix} 0 & -40 & 0 & 0 \\ 40 & 0 & -60 & 80 \\ 0 & 60 & 0 & -20 \\ 0 & -80 & 20 & 0 \end{pmatrix}$$

When launching the `q1_model.jl` file with this matrix as input, we obtain the following lottery for the RCVS :

Candidate	$A$	$B$	$C$	$D$
Probability	0.0	0.125	0.5	0.375

Table 1: Lottery probabilities for each candidate

### 1.3 Discussion of the dual variables and optimal dual basis

### 1.4 Solution of the linear in a linear system

### 1.5 *Bonus* : Comparaison of the RCVS with an alternative voting system

## 2 Linear Robust Formulation

## 3 Quadratic Robust Formulation