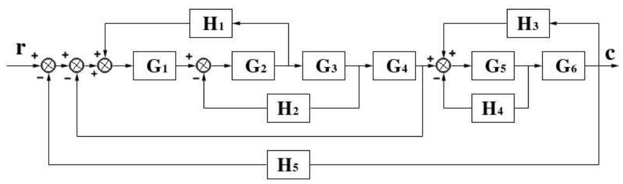
1、系统框图为

$$\frac{G_1(s)+G_2(s)}{1+G_2(s)G_3(s)}$$

2.



将系统变换框图逐次化简

得到传递函数为

$$\frac{G_{123456}}{(1+G_2G_3H_2-G_1G_2H_1+G_1G_2G_3G_4)(1+G_5H_4-G_5G_6H_3)+G_1G_2G_3G_4G_5G_6H_5}$$

3. 假定 $r(s)=0$

则 $[e(s)k(s)+p(s)]G(s)=-e(s)$

∴ 传递函数为 $\dfrac{-G(s)}{1+G(s)k(s)}$

4、(1) 系统框图可表示为

$$\begin{cases} e=u(s)-a_1x_3-a_2x_2-a_3x_1 \\ y(s)=b_0e+(b_1-a_1b_0)x_3+(b_2-a_2b_0)x_2+(b_3-a_3b_0)x_1 \\ x_3=\frac{1}{s}e \\ x_2=\frac{1}{s^2}e \\ x_1=\frac{1}{s^3}e \end{cases}$$

则传递函数 $G(s)=\dfrac{b_0s^3+(b_1-a_1b_0)s^2+(b_2-a_2b_0)s+b_3-a_3b_0}{s^3-a_1s^2-a_2s-a_3}$

(2) $\because \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u(s) - a_1 x_3 - a_2 x_2 - a_3 x_1 \\ y = b_0[u(s) - a_1 x_3 - a_2 x_2 - a_3 x_1] + (b_1 - a_1 b_0)x_3 + (b_2 - a_2 b_0)x_2 + (b_3 - a_3 b_0)x_1 \end{cases}$

则状态空间

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -b_0 a_3 + b_3 - a_3 b_0, & -b_0 a_2 + b_2 - a_2 b_0, & -b_0 a_1 + b_1 - a_1 b_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$+ b_0 u$$

5. (1) 化简系统框图

则 $G(s) = \dfrac{b_3 + s b_2 + s^2 b_1 + s^3 b_0}{s^3 + a_1 s^2 + a_2 s + a_3}$

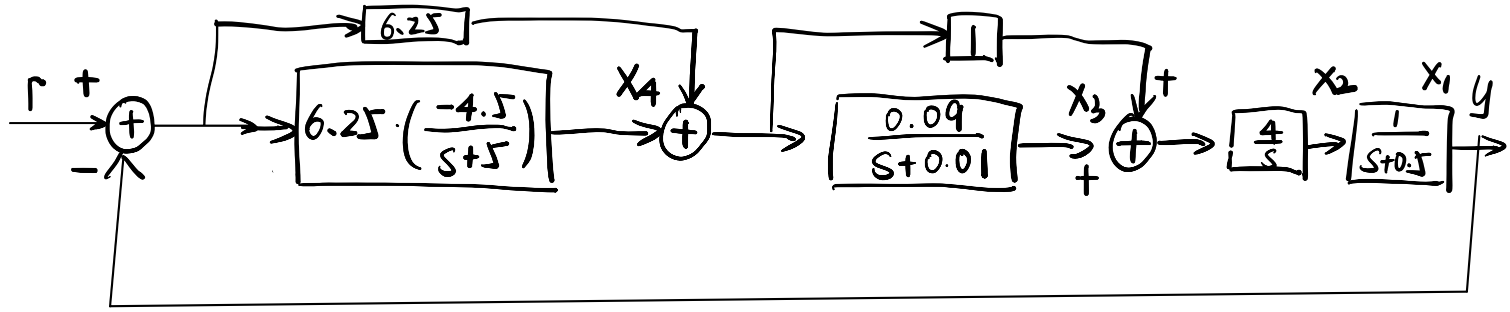
(2) $\begin{cases} \dot{x}_1 = -a_3 y(s) + b_3 u(s) \\ \quad = -a_3 x_3 - a_3 b_0 u(s) + b_3 u(s) \\ \dot{x}_2 = x_1 - a_2 x_3 + (b_2 - a_2 b_0) u(s) \\ \dot{x}_3 = x_2 + b_1 u(s) - a_1[x_3 + b_0 u(s)] \\ \quad = x_2 - a_1 x_3 + (b_1 - a_1 b_0) u(s) \end{cases}$

$y(s) = x_3 + b_0 u(s)$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_3 - a_3 b_0 \\ b_2 - a_2 b_0 \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$$\therefore y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_0 u$$

6、



则进而可得：

$$\begin{cases} \dot{x}_1 = -0.5x_1 + x_2 \\ \dot{x}_2 = 4x_3 + 4x_4 + 25r \\ \dot{x}_3 = -0.01x_3 + 0.09x_4 + 0.5625r \\ \dot{x}_4 = 28.125x_1 - 5x_4 - 28.125r \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -0.5 & 1 & 0 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -0.01 & 0.09 \\ 28.125 & 0 & 0 & -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \\ 0.5625 \\ -28.125 \end{bmatrix} r$$

状态空间习题

1、
$$\begin{cases} I = C \cdot \dot{V}_c + i_L \\ V_c = L\dot{i}_L + R i_L \end{cases}$$

$$\therefore \begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} I$$

2、
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 0 \\ \frac{1}{R} & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3. (1) 令 $z_1 = y$

$z_2 = \dot{y}$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{f}{m} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

(2) $G(s) = \dfrac{1}{ms^2 + fs + k}$

4. $\dfrac{1}{s^2 + 5s + 6}$

5. $sX(s) = AX(s) + bU(s)$

$\therefore X(s) = (sI - A)^{-1} bU(s)$

$Y(s) = C^T X(s) + dU(s)$

得 $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s^2 + 9s + 4}{(s-1)^2}$

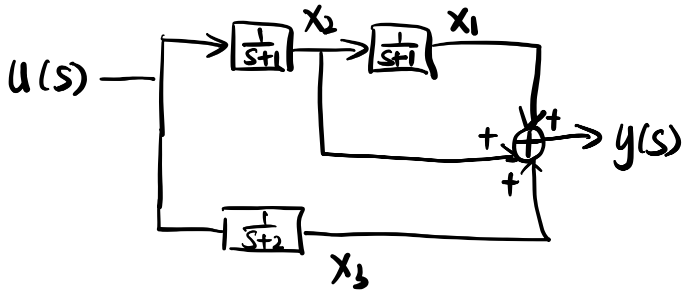
6. 同理:

$G(s) = \dfrac{Y(s)}{U(s)} = C^T(sI - A)^{-1}B$

$$= \begin{bmatrix} \dfrac{5s^2+26s+74}{s^3+9s^2+26s+24} & \dfrac{2s+2}{s^3+9s^2+26s+24} \\[2ex] \dfrac{-19s^2-96s-120}{s^3+9s^2+26s+24} & \dfrac{2s^2+2}{s^3+9s^2+26s+24} \end{bmatrix}$$

状态空间表达习题

1、 $g(s)=\frac{1}{s+1}+\frac{1}{(s+1)^2}+\frac{1}{s+2}$



则 $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 1\\ 1\end{bmatrix}u$

$\therefore y=\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$

2、 $g(s)=\frac{s^{-1}+8s^{-2}+5s^{-3}}{1+6s^{-1}+8s^{-2}}$

能控I型：

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -8 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}5 & 8 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

能观II型： $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & -8\\ 0 & 1 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}5\\ 8\\ 1\end{bmatrix}u$

$$\therefore y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. $$T^{-1} A T = \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix}$$

4. $$A = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -3 & & \\ & -2 & \\ & & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

$$\therefore \dot{x} = \begin{bmatrix} -3 & & \\ & -2 & \\ & & -1 \end{bmatrix} x + \begin{bmatrix} -2.5 \\ 3 \\ -0.5 \end{bmatrix}$$

$$y = [1, 1, 1]\, x$$