1、

由 $A=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$ $B=\begin{bmatrix}1\\ 1\end{bmatrix}$

则 $(sI-A)^{-1}=\begin{bmatrix}\frac{1}{s+1} & 0\\ 0 & \frac{1}{s+2}\end{bmatrix}$

又 $\because$ $x(0)=\begin{bmatrix}1\\ 1\end{bmatrix}$ $u(s)=\frac{1}{s}$ $Bu(s)=\begin{bmatrix}\frac{1}{s}\\ \frac{1}{s}\end{bmatrix}$

$\therefore$ $x(s)=(sI-A)^{-1}[x(0)+Bu(s)]$

$=\begin{bmatrix}\frac{1}{s}\\ \frac{s+1}{s(s+2)}\end{bmatrix}$

反 Laplace 变换

$\therefore$ $x(t)=\begin{bmatrix}1\\ \frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix}$

2、 $A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$ $B=\begin{bmatrix}0\\ 1\end{bmatrix}$

$\therefore$ $(sI-A)^{-1}=\begin{bmatrix}\frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)}\\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)}\end{bmatrix}$

又 $\because$ $x(0)=\begin{bmatrix}-1\\ 1\end{bmatrix}$ $u(s)=\frac{1}{s}$ $Bu(s)=\begin{bmatrix}0\\ \frac{1}{s}\end{bmatrix}$

$\therefore$ $x(s)=(sI-A)^{-1}[x(0)+Bu(s)]$

$=\begin{bmatrix}\frac{1}{2s}-\frac{3}{s+1}+\frac{3}{2(s+2)}\\ \frac{3}{s+1}-\frac{3}{s+2}\end{bmatrix}$

反 Laplace 变换

$\therefore$ $x(t)=\begin{bmatrix}\frac{1}{2}-3e^{-t}+\frac{3}{2}e^{-2t}\\ 3e^{-t}-3e^{-2t}\end{bmatrix}$

3. $e^{At} = I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\theta \\ 0 & \theta & 0 \end{bmatrix} t + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\theta^2 & 0 \\ 0 & 0 & -\theta^2 \end{bmatrix} \frac{t^2}{2} + \cdots$

$$= I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!}(\theta t)^{2k} & \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k+1)!}(\theta t)^{2k+1} \\ 0 & \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)!}(\theta t)^{2k+1} & \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!}(\theta t)^{2k} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta t & -\sin\theta t \\ 0 & \sin\theta t & \cos\theta t \end{bmatrix}$$

4. 由于 $\phi(t) = e^{At} = L^{-1}\{(sI-A)^{-1}\}$

$\therefore \phi(s) = (sI-A)^{-1}$

$$= \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & \frac{s}{(s+2)^2} & \frac{4}{(s+2)^2} \\ 0 & \frac{-1}{(s+2)^2} & \frac{s+4}{(s+2)^2} \end{bmatrix}$$

$\therefore \phi(s)^{-1} = sI - A = \begin{bmatrix} s+1 & 0 & 0 \\ 0 & s+4 & -4 \\ 0 & 1 & s \end{bmatrix}$

$\therefore A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 4 \\ 0 & -1 & 0 \end{bmatrix}$

5、

由 $x(t) = e^{At} x(0)$

$\therefore \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix} = e^{At} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} 2e^{-t} \\ -e^{-t} \end{bmatrix} = e^{At} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\therefore e^{At} = \begin{bmatrix} e^{-2t} & 2e^{-t} \\ -e^{-2t} & -e^{-t} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -e^{-2t} + 2e^{-t} & -2e^{-2t} + 2e^{-t} \\ e^{-2t} - e^{-t} & 2e^{-2t} - e^{-t} \end{bmatrix}$$

$$\therefore L(e^{At}) = \begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \\ \frac{-1}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}$$

求逆： $\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} 0 & 2 \\ -1 & -3 \end{bmatrix}$$