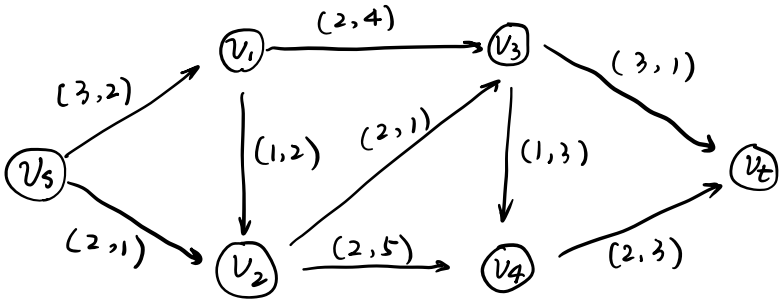
1.

(1)



(2). 根据 $L(x,z,\lambda,\mu) = c^T x + z^T(Ax-b) + \lambda^T(-x) - \mu^T(u-x)$

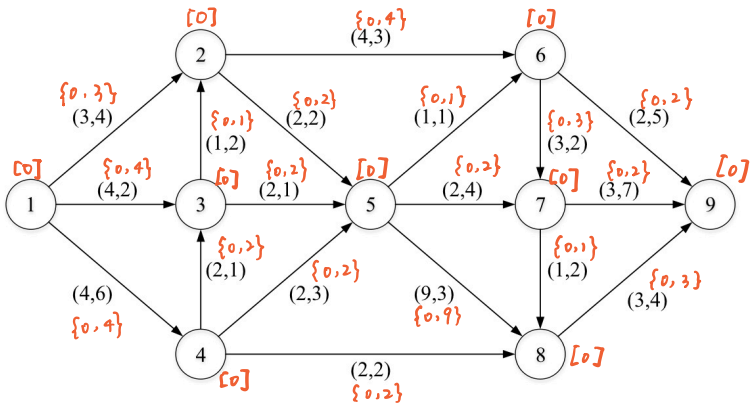
$= (c^T + z^T A + \mu^T - \lambda^T)x - z^T b - \mu^T u$

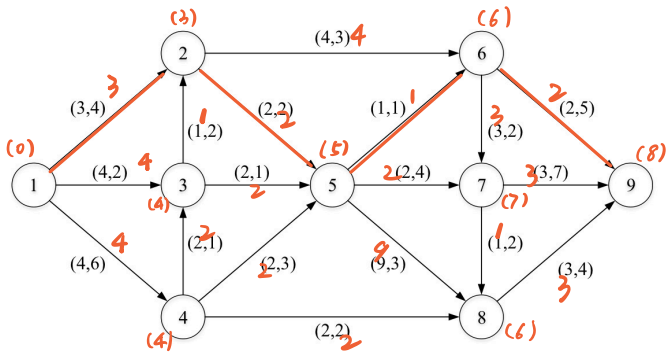
可知 Lagrange 对偶问题为：

$$\begin{cases} \max & -z^T b - \mu^T u \\ \text{s.t.} & c^T + z^T A - \lambda^T + \mu^T = 0 \\ & \lambda \geq 0,\ \mu \geq 0 \end{cases}$$

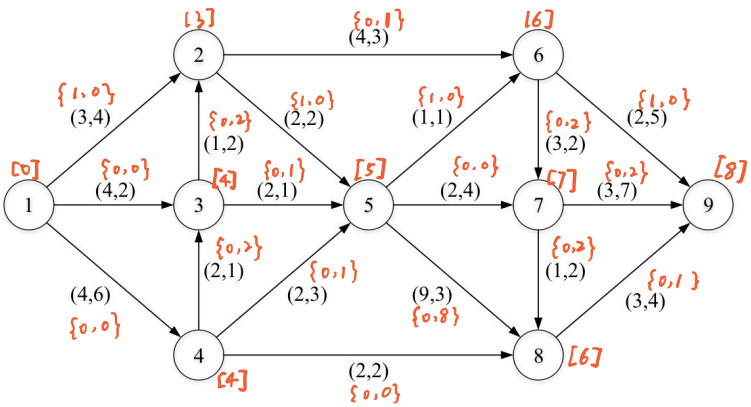
2.

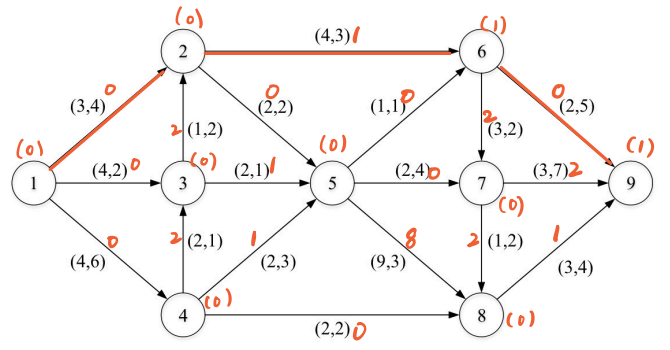
①





②





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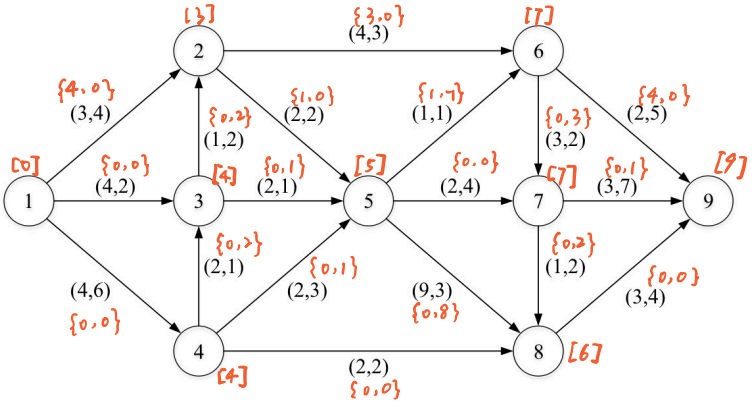

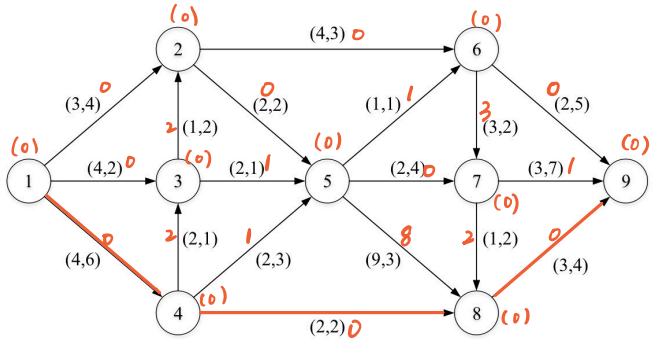

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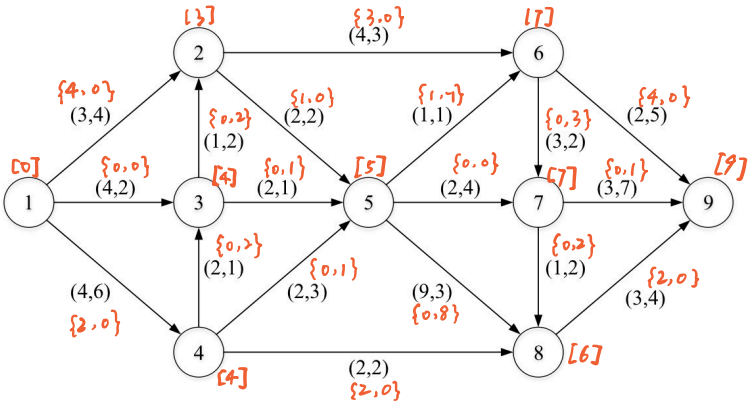

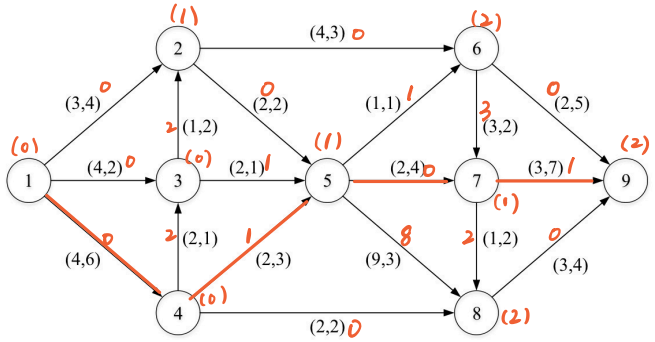

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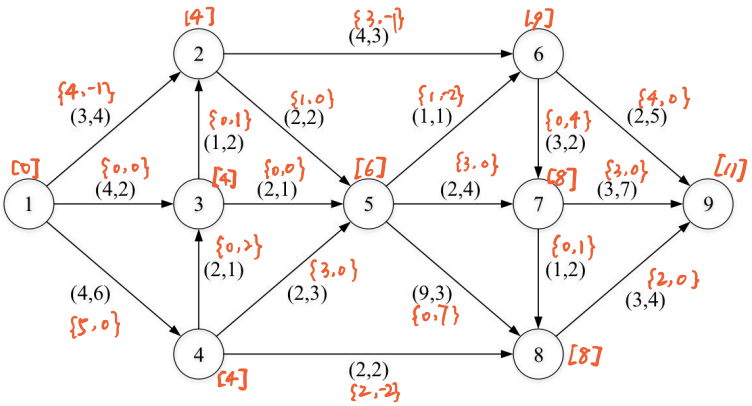

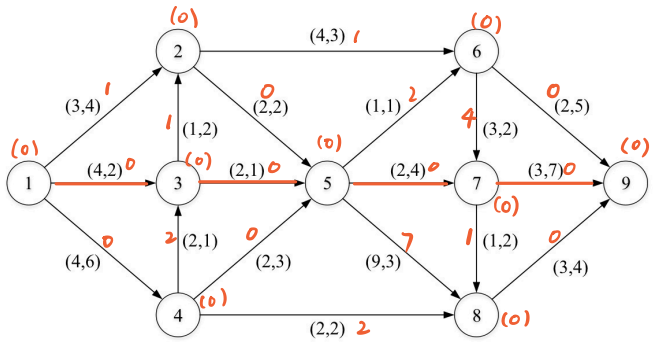

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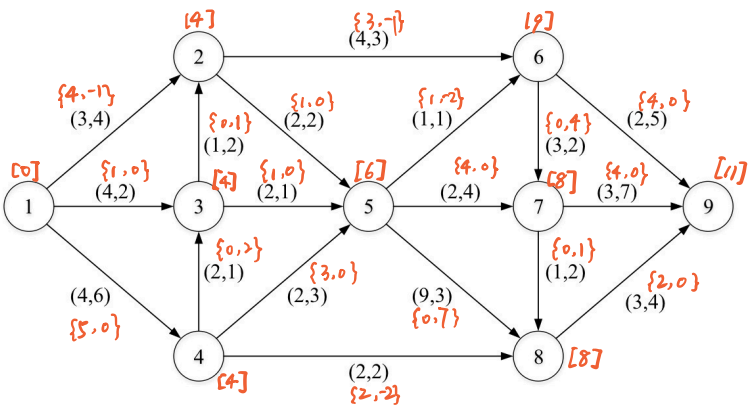

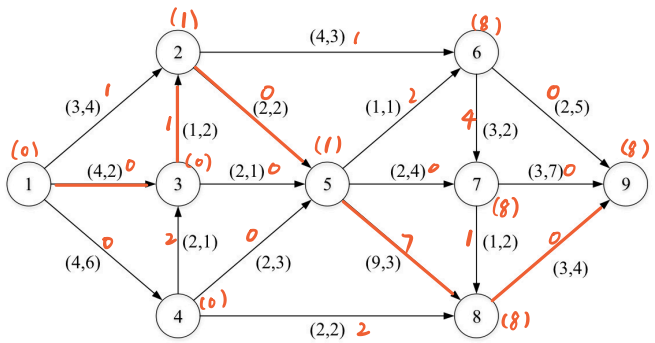

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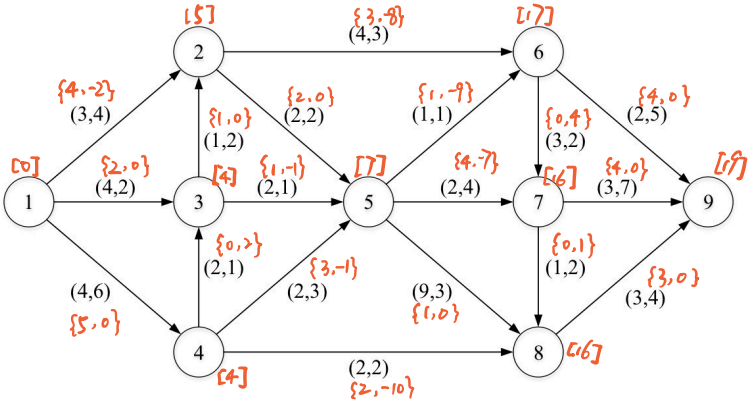
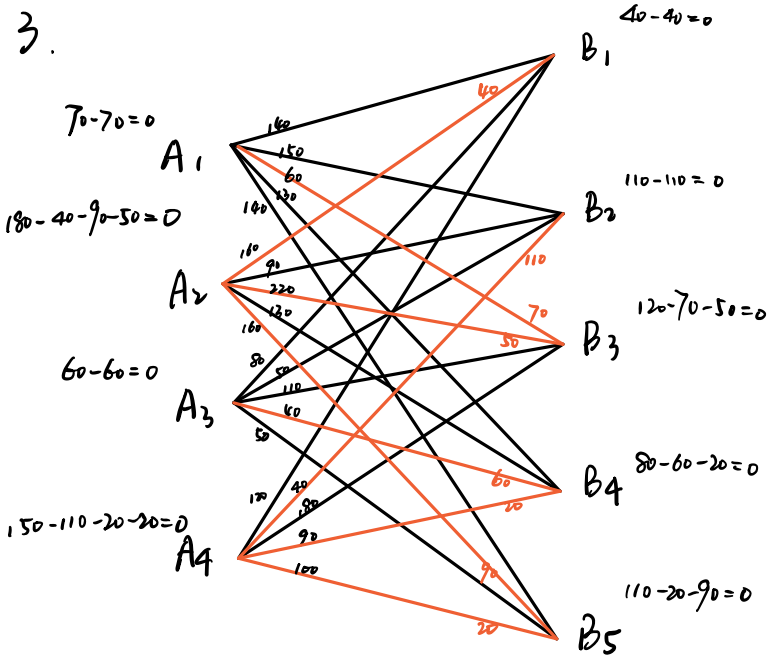


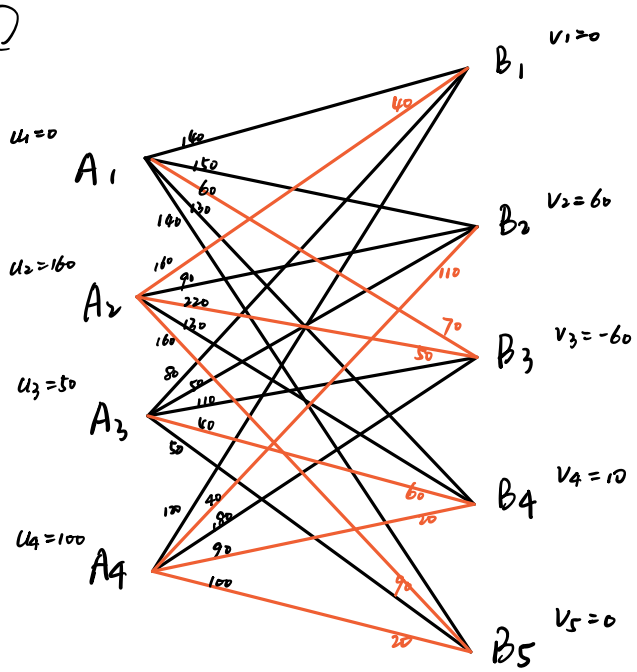
此时，w=11

求得最小费用为 116

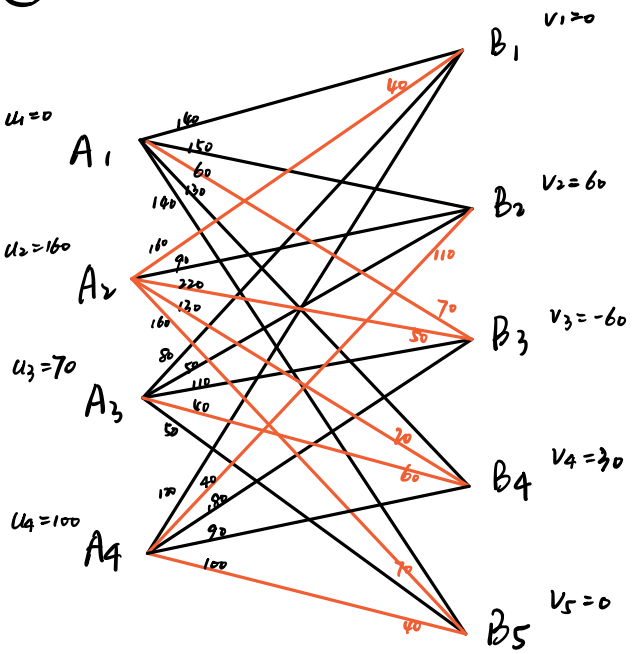
3.


40-40=0
B1
70-70=0
A1
110-110=0
B2
180-40-90-50=0
A2
120-70-50=0
B3
60-60=0
A3
80-60-20=0
B4
150-110-20-20=0
A4
110-20-90=0
B5

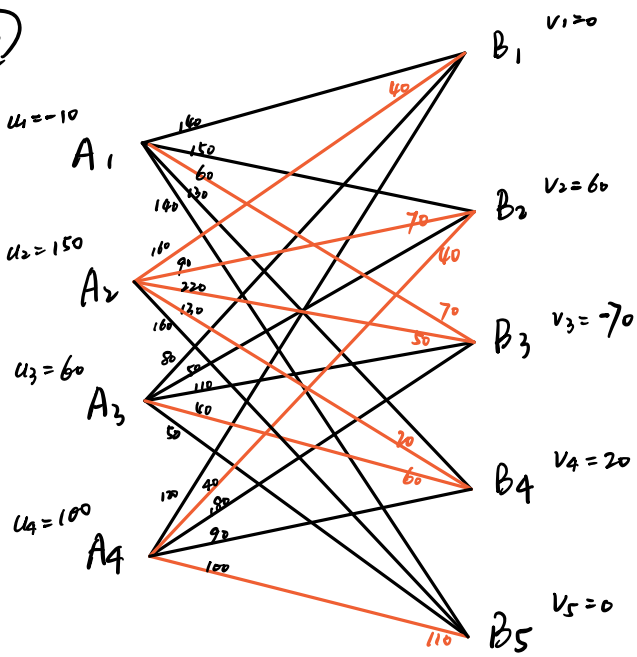
①


v1=0
B1
u1=0
A1
v2=60
B2
u2=160
A2
v3=-60
B3
u3=50
A3
v4=10
B4
u4=100
A4
v5=0
B5

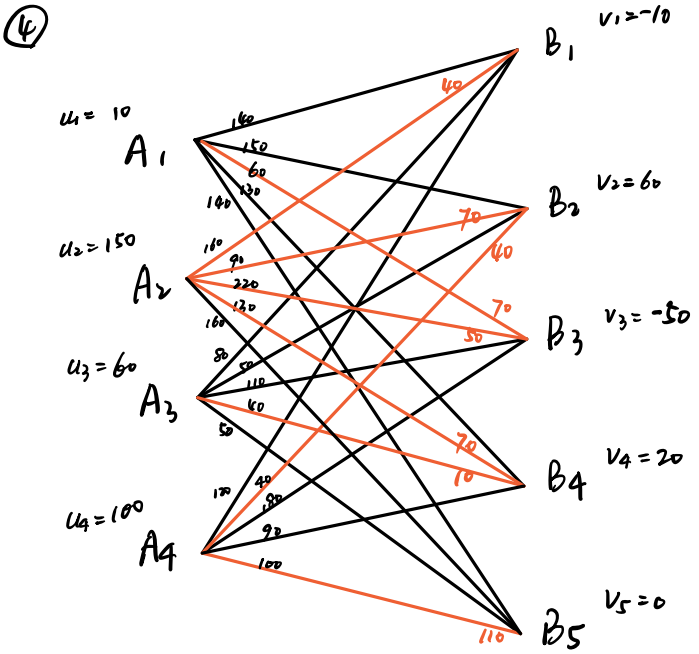
②


v1=0
B1
u1=0
A1
v2=60
B2
u2=160
A2
v3=-60
B3
u3=70
A3
v4=30
B4
u4=100
A4
v5=0
B5

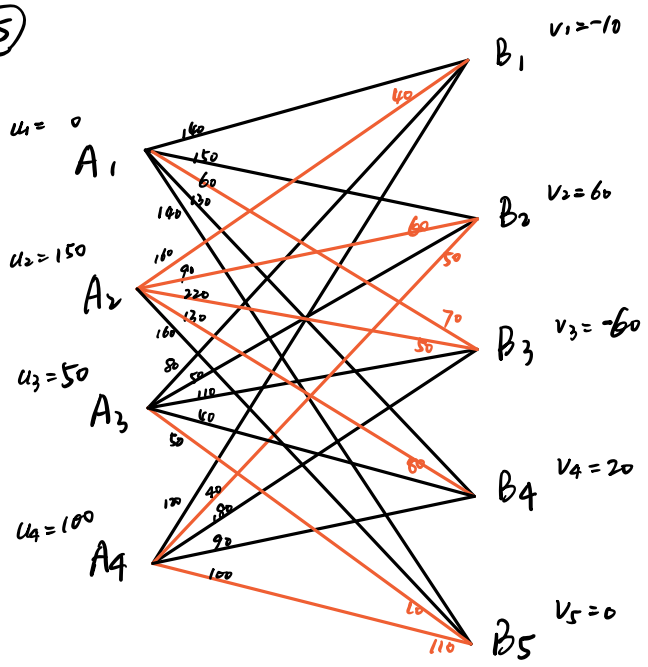
③


v1=0
B1
u1=-10
A1
v2=60
B2
u2=150
A2
v3=-70
B3
u3=60
A3
v4=20
B4
u4=100
A4
v5=0
B5

④


v1=-10
B1
u1=10
A1
v2=60
B2
u2=150
A2
v3=-50
B3
u3=60
A3
v4=20
B4
u4=100
A4
v5=0
B5

⑤



此时得最优解

	B_1	B_2	B_3	B_4	B_5
A_1	0	0	70	0	0
A_2	40	60	0	80	0
A_3	0	0	50	0	10
A_4	0	50	0	0	100

4. 该问题可表示为

$$\min \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij}x_{ij}$$

$$s.t. \sum_{j=1}^{5} x_{ij} = 1 \quad \sum_{i=1}^{5} x_{ij} = 1 \quad x_{ij} \geq 0 \quad \forall i,j$$

$$\mathcal{L}(w,x,u,v,\lambda) = \min \sum_{i=1}^{5}\sum_{j=1}^{5}(-w_{ij}x_{ij}) + \sum_{i=1}^{5} u_i\left(\sum_{j=1}^{5} x_{ij} - 1\right) + \sum_{j=1}^{5} v_j\left(\sum_{i=1}^{5} x_{ij} - 1\right) + \sum_{i=1}^{5}\sum_{j=1}^{5} \lambda_{ij}(-x_{ij})$$

化简可知，对偶问题可表示为

$$\max \sum_{i=1}^{5} u_i + \sum_{j=1}^{5} v_j$$

$$s.t. \ u_i + v_j \leq c_{ij} \quad j = 1,2,\cdots,5 \quad i = 1,2,\cdots,5$$