3. $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}=\lim\limits_{h\to 0}\dfrac{f'(a+h)-f'(a-h)}{2h}$

$=\lim\limits_{h\to 0}\dfrac{f''(a+h)+f''(a-h)}{2}=f''(a)$ ~~f''不一定连续~~

二阶导函数存在 但不一定连续

6. $\ln(1+\sin^2 x)$ 　　$\sin x^2$ 不是 $\sin^2 x$

$= \sin^2 x - \frac{\sin^4 x}{2} + o(\sin^4 x)$

$= \left(x - \frac{x^3}{6} + o(x^4)\right)^2 - \frac{(x+o(x))^4}{2} + o(x^4)$

$= -\frac{1}{3}x^4 + x^2 + o(x^4) - \frac{1}{2}x^4 + o(x^4) + o(x^4)$

$= -\frac{5}{6}x^4 + x^2 + o(x^4), \quad x \to 0.$

$\alpha\left[(1+(1-\cos x))^{\frac{1}{3}} - 1\right]$

$= \alpha\left[1 + \frac{1}{3}(1-\cos x) - \frac{1}{9}(1-\cos x)^2 + o((1-\cos x)^2) - 1\right]$

$= \alpha\left[\frac{1}{3}\left(1-1+\frac{x^2}{2}-\frac{x^4}{24}+o(x^4)\right) - \frac{1}{9}\left(1-1+\frac{x^2}{2}+o(x^2)\right)^2 \ldots\right.$

8. 令 $a = x-1$, $b \ldots$

则

$\left|\frac{\ln x - \ln y}{x-y} - \frac{1}{y} \ldots\right.$

$= \left|\frac{\ln(1+a) - \ln \ldots}{a-b} \ldots\right.$

$= \left|\frac{a - \frac{a^2}{2} + \frac{a^3}{3(1 \ldots}}{\ldots} \ldots\right.$

$\left|\frac{b-a}{2} + \frac{a \ldots}{3(1+ \ldots} \ldots\right.$

两种函数写混了

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6. $\ln(1+\sin^2 x) = \sin^2 x + \cancel{\frac{1}{2}\sin^4 x} + o(\sin^2 x)$

$\sqrt[3]{2-\cos x} = 1 + \frac{1}{6}x^2 + o(x^2)$

$$\lim_{x\to 0}\frac{\ln(1+\sin^2 x) + a(\sqrt[3]{2-\cos x} - 1)}{x^2}$$

$$= \lim_{x\to 0}\frac{\sin^2 x + o(\sin^2 x) + a(1+\frac{1}{6}x^2 - 1 + o(x^2))}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2} + \frac{\frac{a}{6}x^2}{x^2}\right) + \lim_{x\to 0}\frac{o(\sin^2 x)}{x^2} + \frac{a\,o(x^2)}{x^2}$$

$= 1 + \frac{a}{6}$ $\therefore$ 阶为2

$a=-6$ 分类

利用泰勒展开式 只展到x的二阶项 没有对极限结果做讨论

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$= \lim_{x\to 0}\frac{x\cos x}{x^2}$

$\frac{0}{0}$型

$= \lim_{x\to 0}$

$\frac{0}{0}$型