作业四 参考答案

3.3 解：

系统微分方程为

$$T_aT_mK_3\frac{\mathrm{d}^3\varphi}{\mathrm{d}t^3}+T_mK_3\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+K_3\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\frac{K_1K_2}{K_d}\varphi=\frac{K_1K_2}{K_d}\psi-\frac{R_a}{K_d^2}\left(T_a\frac{\mathrm{d}M_L}{\mathrm{d}t}+M_L\right)$$

若忽略T_a，代入$T_m=0.4$，$K_d=0.2$，$K_1K_2=10$，$K_3=20$，系统微分方程在单位阶跃信号激励下为：

$$0.16\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+0.4\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\varphi=1(t)$$

因此

$$T=\sqrt{0.16}=0.4s$$

$$\zeta=\frac{0.4}{2T}=0.5$$

所以

$$t_r=\frac{T}{\sqrt{1-\zeta^2}}(\pi-\arccos\zeta)=\frac{4\sqrt{3}}{15}(\pi-\arccos 0.5)=0.97\mathrm{s}$$

$$t_p=\frac{\pi T}{\sqrt{1-\zeta^2}}=\frac{4\sqrt{3}\pi}{15}=1.45\mathrm{s}$$

利用图3.9.4的曲线，有

$$t_s(\Delta=5\%)=T\cdot\frac{t_s}{T}(5\%)=5T=2\mathrm{s}$$

$$t_s(\Delta=2\%)=T\cdot\frac{t_s}{T}(2\%)=8.2T=3.28s$$

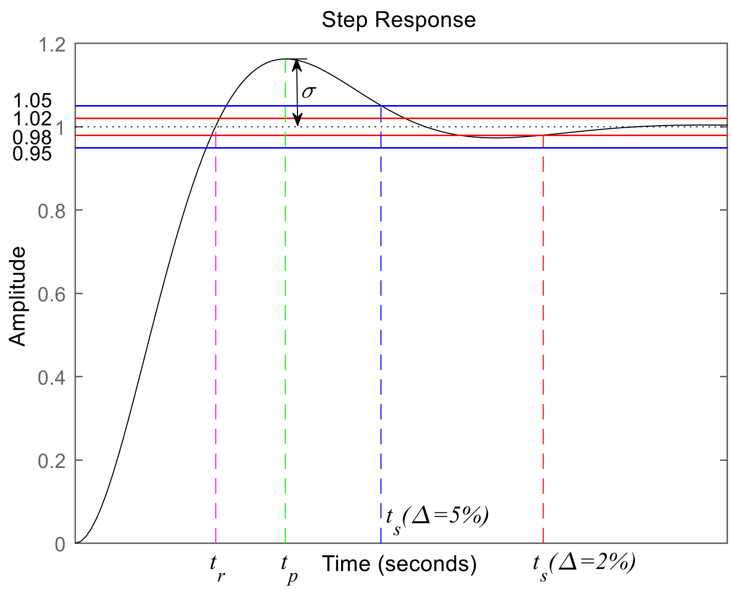
$$\sigma=18\%$$

若使用公式计算，则有

$$\sigma=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}=16.3\%$$

$$t_s=\frac{3\sim4}{\zeta}T=2.4\sim3.2s$$

$\varphi(t)$的单位阶跃曲线草图为



3.21 解：

由题意可知

$$u(s) = \frac{1}{s}$$

$$y(s) = \frac{1}{s} - \frac{1.2}{s+10} + \frac{0.2}{s+60}$$

则

$$H(s) = \frac{y(s)}{u(s)} = 1 - \frac{1.2s}{s+10} + \frac{0.2s}{s+60} = \frac{600}{s^2+70s+600}$$

因此阻尼系数为

$$\zeta = \frac{35}{600\sqrt{\frac{1}{600}}} = \frac{7\sqrt{6}}{12} = 1.43$$

4.4 解：

根据传递函数

$$G(s) = \frac{5.2}{0.1s^2+0.32s+1}$$

该二阶系统参数为

$$K = 5.2$$

$$20\lg K = 14.3\text{dB}$$

$$\mathrm{T} = \frac{\sqrt{10}}{10} = 0.316$$

$$\zeta = \frac{0.32}{2T} = 0.16 \times \sqrt{10} = 0.506$$

所以该系统属于欠阻尼二阶系统，使用振荡单元的对数频率特性图绘制方法

转折点频率为

$$\omega_0 = \frac{1}{T} = 3.16$$

$$\mu_0 = \lg\omega_0 = 0.5$$

截止频率为

$$\mu_c = lg\,\omega_c = \mu_0 + \frac{20\,lg\,K}{40} = 0.858$$

$$\omega_c = 7.21$$

因为$\zeta < 0.707$，所以频率特性图有谐振峰

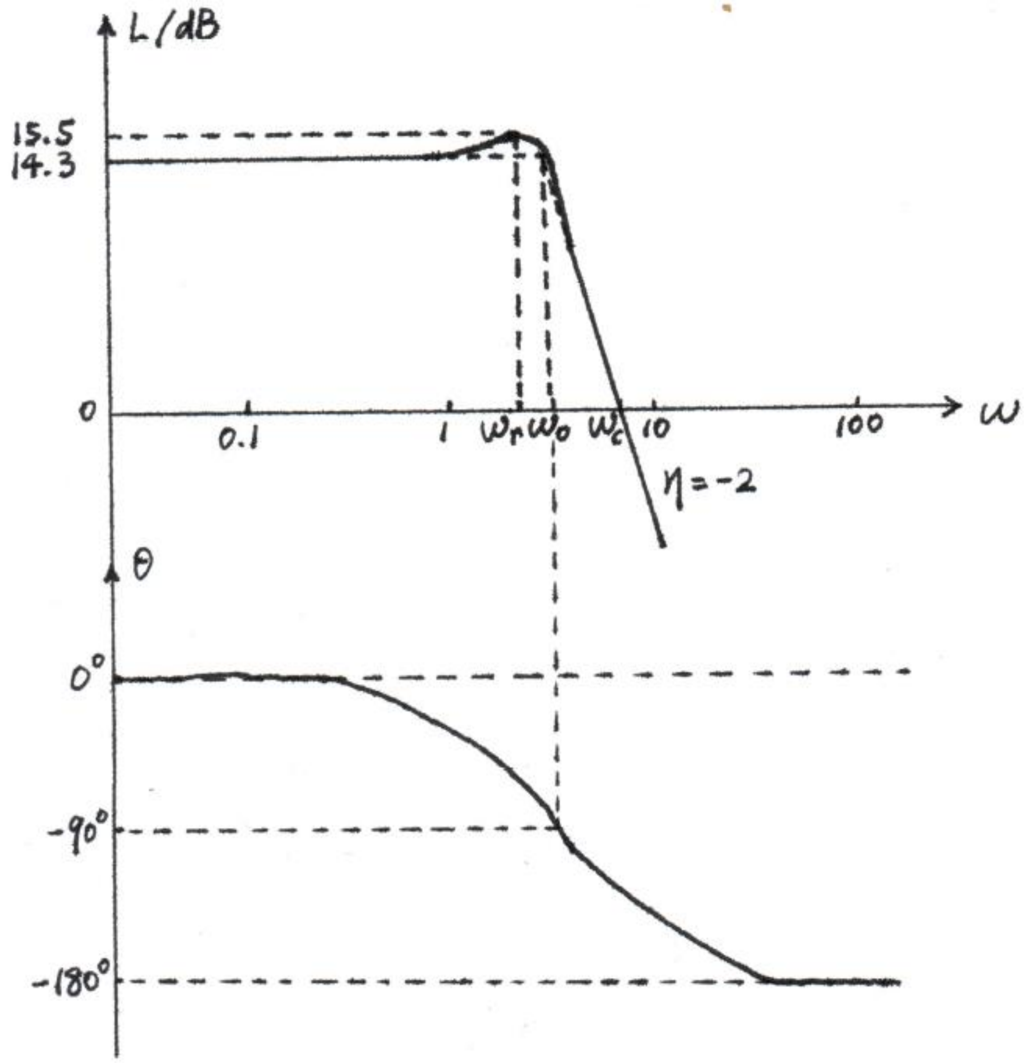
$$\omega_r = \frac{\sqrt{1-2\zeta^2}}{T} = 2.209$$

$$\Delta\mu_r = \lg\omega_r - \mu_0 = -0.156$$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.146$$

$$20\lg M_r = 1.2\text{dB}$$

因此对数频率特性图如下



4.7 解：

(a)传递函数为

$$G(s)=\frac{2.8(0.05s+1)}{s(0.15s+1)}$$

所以

$$\omega_z=\frac{1}{T_z}=20$$
$$\omega_p=\frac{1}{T_p}=\frac{20}{3}$$

估计$\omega_c<\omega_p<\omega_z$，则有

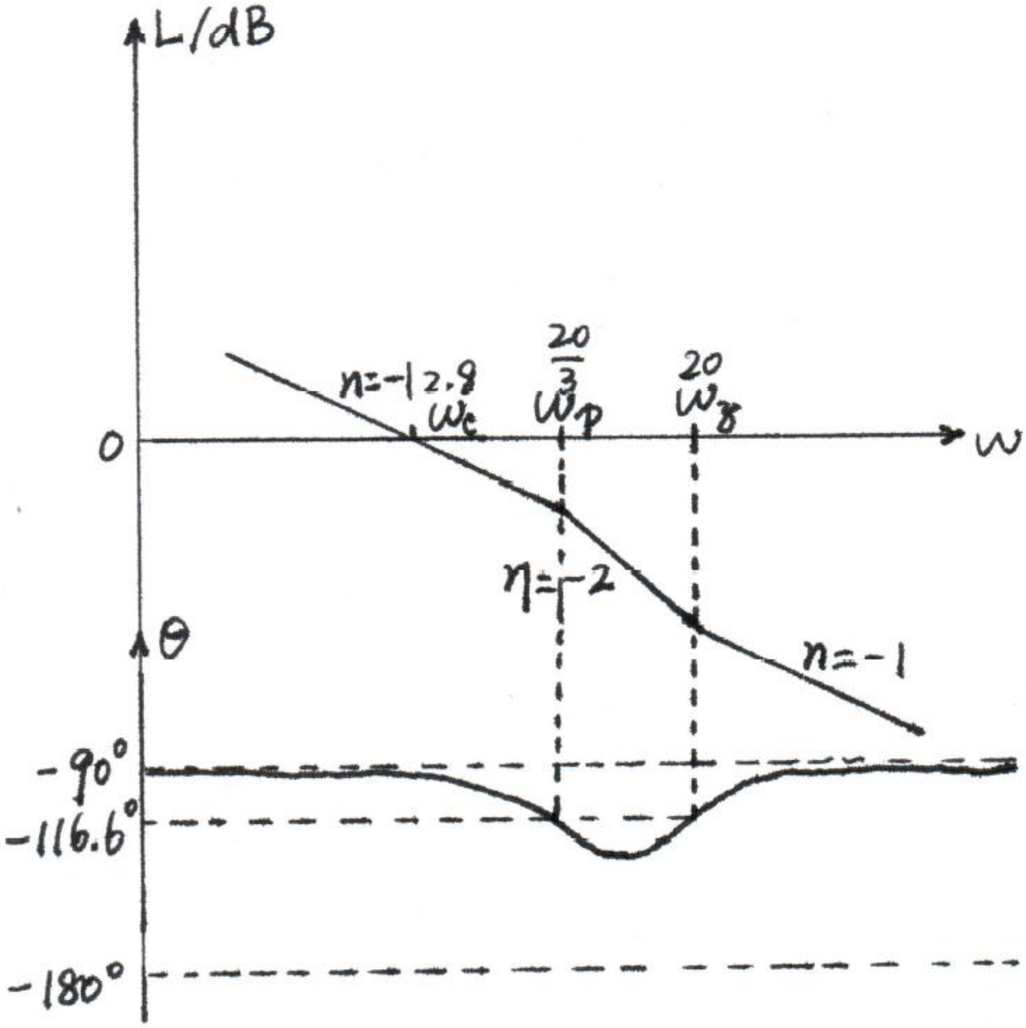
$$\omega_c=2.8$$

转折点处的相角为

$$\theta(\omega_c)=-90°+\arctan 0.05\omega_c-\arctan 0.15\omega_c=-104.8°$$
$$\theta(\omega_p)=-90°+\arctan 0.05\omega_p-\arctan 0.15\omega_p=-116.6°$$

$$\theta(\omega_z) = -90° + \arctan 0.05\omega_z - \arctan 0.15\omega_z = -116.6°$$

所以对数频率特性图如下



(b) 传递函数为

$$G(s) = \frac{2.8(0.5s+1)}{s(0.15s+1)}$$

所以

$$\omega_z = \frac{1}{T_z} = 2$$

$$\omega_p = \frac{1}{T_p} = \frac{20}{3}$$

估计$\omega_z < \omega_p < \omega_c$，则有

$$\omega_c * 0.15\omega_c = 2.8 * 0.5\omega_c$$

$$\omega_c = \frac{28}{3} = A9.33$$

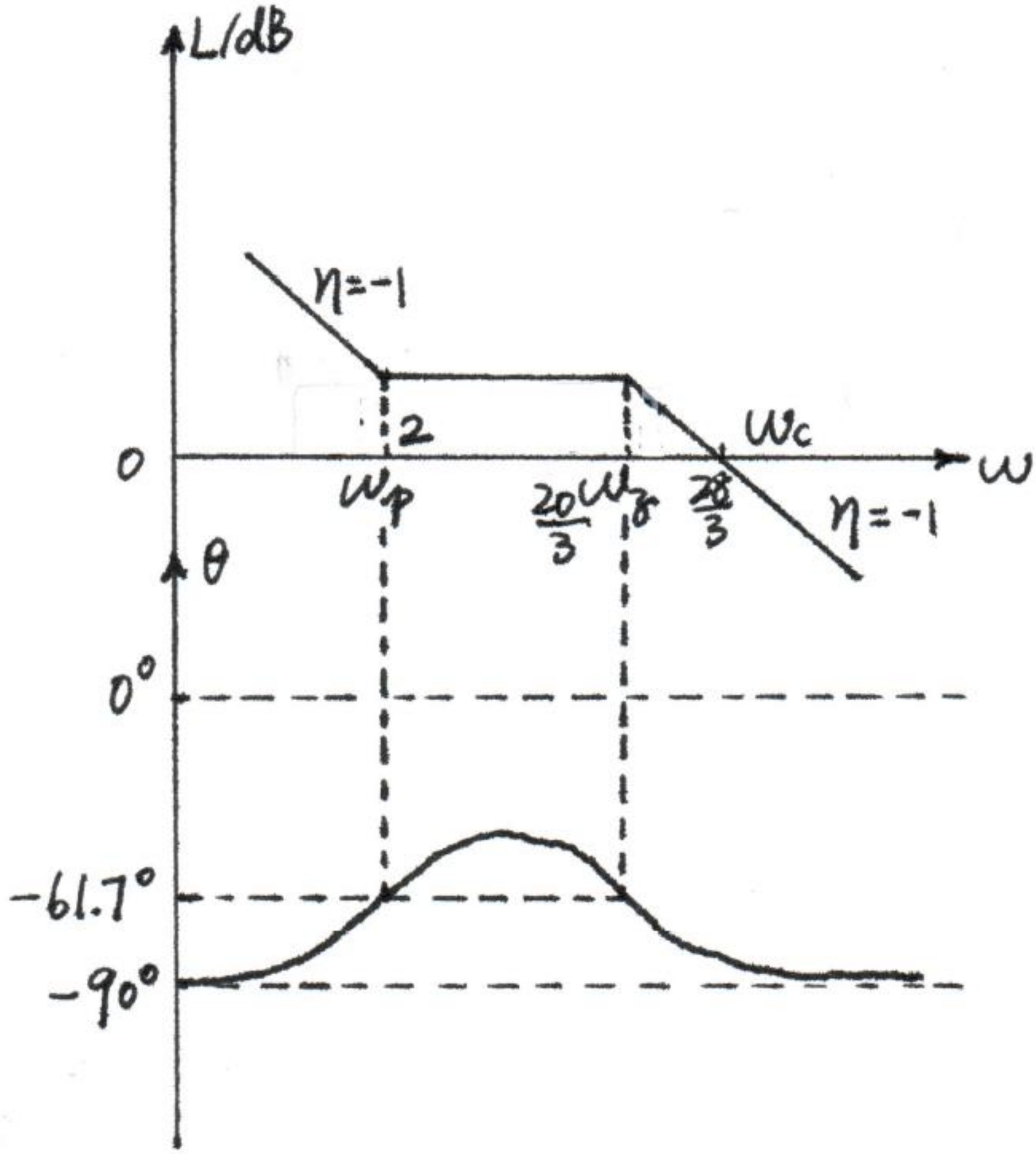
转折点处的相角为

$$\theta(\omega_c) = -90° + \arctan 0.5\omega_c - \arctan 0.15\omega_c = -66.6°$$

$$\theta(\omega_p) = -90° + \arctan 0.5\omega_p - \arctan 0.15\omega_p = -61.7°$$

$$\theta(\omega_z) = -90° + \arctan 0.5\omega_z - \arctan 0.15\omega_z = -61.7°$$

所以对数频率特性图如下



4.16 解：

由图可知，开环对象含有一重积分环节，且

$$\omega_{p1} = 0.002, \omega_{z1} = 0.02, \omega_{p2} = 0.2, \omega_{p3} = 1$$
$$T_{p1} = 500, T_{z1} = 50, T_{p2} = 5, T_{p3} = 1$$

由图像可知

$$\lg K = \frac{52}{20} + \lg \omega_{p1} = -0.1$$
$$K = 0.8$$

所以开环对象传递函数为

$$G_0(s) = \frac{0.8(50s+1)}{s(500s+1)(5s+1)(s+1)}$$

由对数幅频特性图可知

$$\lg\frac{\omega_c}{0.02} + 2\lg\frac{0.02}{0.002} = \frac{52}{20}$$

所以

$$\omega_c = 0.08$$

根据相角计算公式

$$\theta(\omega) = -90° + \arctan 50\omega - \arctan 500\omega - \arctan 5\omega - \arctan \omega$$

计算得到转折点处的相角为

$$\theta(\omega_c) = -129.0°$$

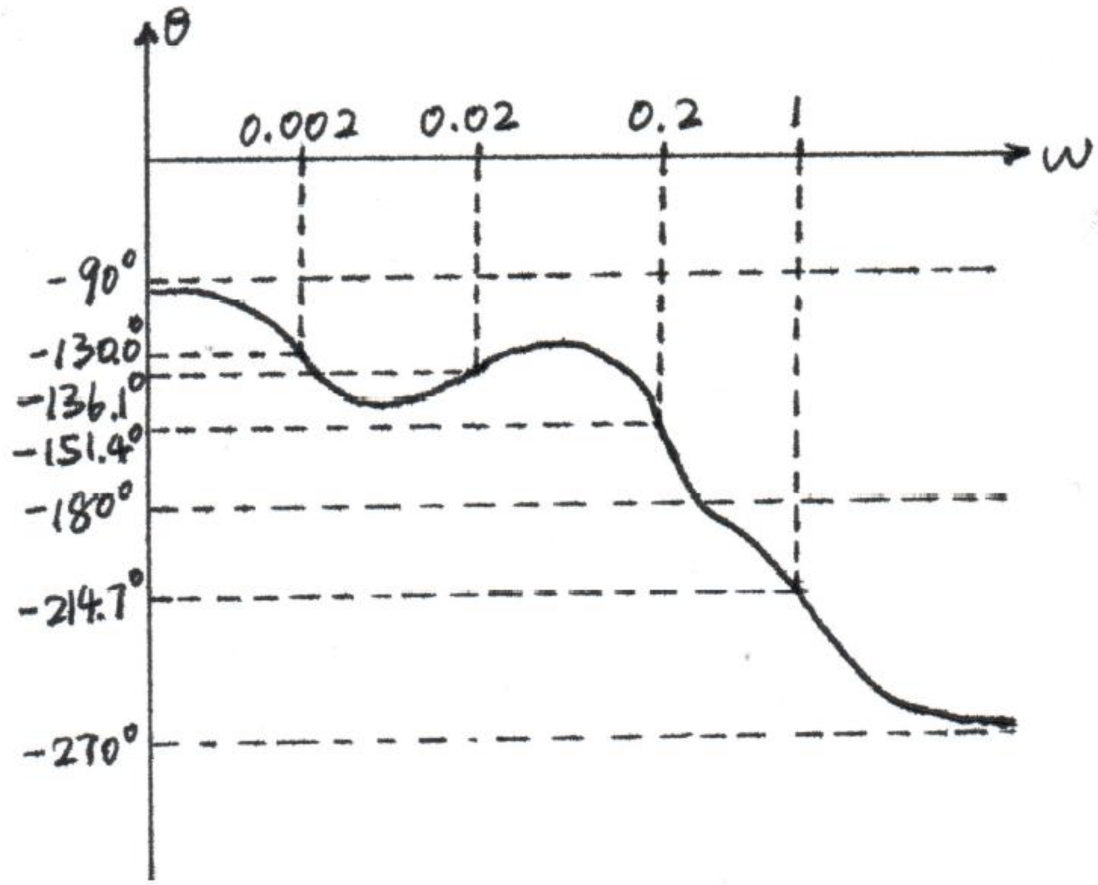
$$\theta(\omega_{p1}) = -130.0°$$

$$\theta(\omega_{z1}) = -136.1°$$

$$\theta(\omega_{p2}) = -151.4°$$

$$\theta(\omega_{p3}) = -214.7°$$

所以相角频率特性图如下



闭环传递函数为

$$G(s)=\frac{G_0(s)}{1+G_0(s)}=\frac{0.8(50s+1)}{0.8(50s+1)+s(500s+1)(5s+1)(s+1)}$$
$$=\frac{40s+0.8}{2500s^4+3005s^3+506s^2+41s+0.8}$$

系统微分方程为

$$2500\frac{\mathrm{d}^4y(t)}{\mathrm{d}t^4}+3005\frac{\mathrm{d}^3y(t)}{\mathrm{d}t^3}+506\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^3}+41\frac{\mathrm{d}y(t)}{\mathrm{d}t}+0.8y(t)$$
$$=40\frac{\mathrm{d}u(t)}{\mathrm{d}t}+0.8u(t)$$