$\Rightarrow I = \frac{\pi}{4}$

13 (1) 记 $F(x) = \int_0^x f(x)dx$. ~~记 $G(x) = \frac{1}{2}x^2$~~.

则 $\cancel{\frac{F(1)-F(0)}{G(1)-G(0)}}$ ~~Cauchy 中值定理~~.

需说明 $G(x)$ 在 $[0,1]$ 连续

在 $(0,1)$ 可导

记 $G(x) = xF(x)$

则 $G(0) = G(1) = 0$ 由罗尔定理

$\Rightarrow \exists \xi\ G'(\xi) = 0 \quad G'(\xi) = F(\xi) + \xi F'(\xi) = 0$

$\Rightarrow \int_0^\xi f(x)dx = -\xi f(\xi)$

(2). 记 $F(x) = \int_0^x f(x)dx$.

由于 $F(0) = F(1) = 0$ 由罗尔定理

$\exists\ \xi \in (0,1)\ F'(\xi) = 0 \Rightarrow \cancel{F(\xi)=0}\ f(\xi) = 0$

考虑 $G(x) = x^2 f(x)$

$G(0) = G(\xi) = 0$ 由罗尔定理

$\exists \eta \in (0,\xi) \subset (0,1)$. $G'(\eta) = 0$

$G'(\eta) = \eta^2 f'(\eta) + 2\eta f(\eta) = 0$

$\because \eta \neq 0$

$\Rightarrow 2f(\eta) + \eta f'(\eta) = 0$