作业1116

2020年11月16日 16:32

习题5.4：2.（1）（3）；3.（1）（4）（9）；4.（2）（11）（14）；5.（4）；6.（5）（6）.

2.(1) $f(x)=\begin{cases} x^2+1, & x\le 0 \\ \cos x, & x>0 \end{cases}$ $F(x)=\begin{cases} \frac{1}{3}x^3+x+C_1, & x\le 0 \\ -\sin x+C_2, & x>0 \end{cases}$

$F(x)$ 在 $(-\infty,+\infty)$ 连续 $F(0)=\lim\limits_{x\to 0^+}(-\sin x+C_2)$ $C_1=C_2$

故存在原函数 $F(x)=\begin{cases} \frac{1}{3}x^3+x+C, & x\le 0 \\ -\sin x+C, & x>0 \end{cases}$

(3) $f(x)=\begin{cases} -\sin x, & x\le 0 \\ \frac{1}{\sqrt{x}}, & x>0 \end{cases}$ $F(x)=\begin{cases} \cos x+C_1, & x\le 0 \\ 2\sqrt{x}+C_2, & x>0 \end{cases}$

$F(x)$ 连续 $\lim\limits_{x\to 0^+}F(x)=F(0)$ $C_2=1+C_1$

故存在原函数 $F(x)=\begin{cases} \cos x+C_1, & x\le 0 \\ 2\sqrt{x}+C_1+1, & x>0 \end{cases}$

$F(x)$ 在 $x=0$ 处不可导，则根据定义不存在原函数。

3.(1) $\int (x-x^{-2})\sqrt{x\sqrt{x}}\,dx=\int (x^{\frac{7}{4}}-x^{-\frac{5}{4}})dx=\frac{4}{11}x^{\frac{11}{4}}+4x^{-\frac{1}{4}}+C$

(4) $\int (x-1)(3x-2)\,dx=\int (3x^2-5x+2)dx=3\cdot\frac{1}{3}x^3-5\cdot\frac{1}{2}x^2+2x+C=x^3-\frac{5}{2}x^2+2x+C$

(9) $\int |(x-1)(3x-2)|\,dx=\begin{cases} x^3-\frac{5}{2}x^2+2x+C_1 & x\le\frac{2}{3} \\ -x^3+\frac{5}{2}x^2-2x+C_2 & \frac{2}{3}<x<1 \\ x^3-\frac{5}{2}x^2+2x+C_3 & x\ge 1 \end{cases}$

由连续性 $\begin{cases} (\frac{2}{3})^3-\frac{5}{2}(\frac{2}{3})^2+2\times\frac{2}{3}+C_1=-(\frac{2}{3})^3+\frac{5}{2}(\frac{2}{3})^2-2\times\frac{2}{3}+C_2 \\ 1-\frac{5}{2}+2+C_3=-1+\frac{5}{2}-2+C_2 \end{cases}$ $C_3=C_2-1$ $C_1=C_2-\frac{28}{27}$

$$\int |(x-1)(3x-2)|\,dx=\begin{cases} x^3-\frac{5}{2}x^2+2x-\frac{28}{27}+C & x\le\frac{2}{3} \\ -x^3+\frac{5}{2}x^2-2x+C & \frac{2}{3}<x<1 \\ x^3-\frac{5}{2}x^2+2x-1+C & x\ge 1 \end{cases}$$

4.(2) $\int \frac{x}{\sqrt{4-x^2}}dx=\int\frac{d(\frac{1}{2}x^2)}{\sqrt{4-x^2}}\overset{令u=\frac{1}{2}x^2}{=\!=}\int\frac{du}{\sqrt{4-2u}}=-\sqrt{4-2u}+C=-\sqrt{4-x^2}+C$

(11) $\int\frac{1}{e^x+e^{-x}}dx=\int\frac{e^x}{e^{2x}+1}dx\overset{令u=e^x}{=\!=}\int\frac{du}{u^2+1}=\arctan u+C=\arctan(e^x)+C$

(14) $\int\frac{2^x}{\sqrt{4-4^{x+1}}}dx=\int\frac{d(2^x)}{2\ln 2\sqrt{1-2^{2x}}}\overset{令u=2^x}{=\!=}\int\frac{1}{2\ln 2}\frac{du}{\sqrt{1-u^2}}=\frac{1}{2\ln 2}\arcsin u+C=\frac{1}{2\ln 2}\arcsin(2^x)+C$

5.(4) $\int\frac{x-1}{x^2-4x+8}dx=\int\frac{x-2}{(x-2)^2+4}dx+\int\frac{\frac{1}{4}dx}{(\frac{x-2}{2})^2+1}\ \overset{令u=(x-2)^2,\ 令t=\frac{x-2}{2}}{=\!=}\ \int\frac{1}{2}\frac{du}{u+4}+\int\frac{1}{2}\frac{dt}{t^2+1}$

$=\frac{1}{2}\ln(u+4)+\frac{1}{2}\arctan t+C=\frac{1}{2}\ln[(x-2)^2+4]+\frac{1}{2}\arctan\frac{x-2}{2}+C$

6.(5) $\int\frac{2x-1}{\sqrt{4x^2+4x+5}}dx=\int\frac{2x+1}{\sqrt{(2x+1)^2+4}}dx-\int\frac{d(2x+1)}{\sqrt{(2x+1)^2+4}}=\frac{\sqrt{4x^2+4x+5}}{2}-\ln(|\sqrt{4x^2+4x+5}+2x+1|)+C$

(6) $\int \frac{x^2}{\sqrt{3+2x-x^2}}dx = \int \frac{\frac{1}{2}x^2 dx}{\sqrt{1-(\frac{x-1}{2})^2}} \xlongequal[t\in(-\frac{\pi}{2},\frac{\pi}{2})]{令\sin t=\frac{x-1}{2}} \int \frac{(2\sin t+1)^2}{\cos t}\cos t\,dt$

$= \int (4\sin^2 t+4\sin t+1)dt = \int (-2\cos 2t+4\sin t+3)dt = -\sin 2t - 4\cos t+3t+C$

$= -\frac{x+3}{2}\sqrt{3+2x-x^2}+3\arcsin(\frac{x-1}{2})+C$

习题5.4：7.（1）（2）（4）（5）（10）.

习题5.5：1.（1）（3）（7）；2.（1）（2）（7）（9）.

习题5.4

7(1) $\int x\cos 2x\,dx = \int \frac{x}{2}d\sin 2x = \frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x\,dx$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$

(2) $\int xe^{-3x}dx = \int -\frac{x}{3}de^{-3x} = -\frac{x}{3}e^{-3x}+\frac{1}{3}\int e^{-3x}dx$

$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}+C$

(4) $\int x\arctan x\,dx = \int \arctan x\,d\frac{x^2}{2} = \frac{x^2}{2}\arctan x - \int \frac{x^2}{2}d\arctan x$

$= \frac{x^2}{2}\arctan x - \int \frac{x^2}{2}\frac{dx}{1+x^2} = \frac{x^2+1}{2}\arctan x - \frac{1}{2}x+C$

(5) $\int x\ln(x-1)dx = \int \ln(x-1)d\frac{x^2}{2} = \frac{x^2}{2}\ln(x-1) - \int \frac{x^2}{2}d\ln(x-1)$

$= \frac{x^2}{2}\ln(x-1) - \int \frac{x^2}{2}\frac{1}{x-1}dx = \frac{x^2}{2}\ln(x-1) - \frac{1}{2}\int[(x+1)+\frac{1}{x-1}]dx$

$= \frac{x^2-1}{2}\ln(x-1) - \frac{1}{4}x^2 - \frac{1}{2}x+C$

(10) $\int e^x\sin^2 x\,dx = \int \sin^2 x\,de^x = e^x\sin^2 x - \int e^x d\sin^2 x$

$= e^x\sin^2 x - \int e^x\sin 2x\,dx = e^x\sin^2 x - \int \sin 2x\,de^x = e^x\sin^2 x - (e^x\sin 2x - \int e^x d\sin 2x)$

$= e^x\sin^2 x - e^x\sin 2x + \int e^x 2\cos 2x\,dx = e^x\sin^2 x - e^x\sin 2x + 2e^x - 4\int e^x\sin^2 x\,dx$

$\int e^x\sin^2 x\,dx = \frac{1}{5}e^x(\sin^2 x - \sin 2x+2)+C$

习题5.5

1.(1) $\frac{1}{(x+1)(x+2)^2} = \frac{A_1}{x+1}+\frac{A_2}{x+2}+\frac{A_3}{(x+2)^2}$

比较系数 $A_1=1,\ A_2=-1,\ A_3=-1$

$\int \frac{1}{(x+1)(x+2)^2}dx = \int[\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{(x+2)^2}]dx = \ln|\frac{x+1}{x+2}|+\frac{1}{x+2}+C$

(3) $\frac{x^3+1}{\cdots} = 1+\frac{5x^2-6x+1}{\cdots} = 1+\frac{A_1}{\cdots}+\frac{A_2}{\cdots}\ \frac{A_3}{\cdots}$

$\int \frac{\ \ }{(x+1)(x+2)^2}\,dx = \int \left[\frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{(x+2)^2}\right] dx = \ln\left|\frac{x+1}{x+2}\right| + \frac{1}{x+2} + C$

(3) $\frac{x^3+1}{x^3-5x^2+6x} = 1 + \frac{5x^2-6x+1}{x(x-2)(x-3)} = 1 + \frac{A_1}{x} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$

$A_1 = \frac{1}{6} \quad A_2 = -\frac{9}{2} \quad A_3 = \frac{28}{3}$

$\int \frac{x^3+1}{x^3-5x^2+6x}dx = \int \left[1 + \frac{1}{6x} - \frac{9}{2(x-2)} + \frac{28}{3(x-3)}\right] dx$

$= x + \frac{1}{6}\ln|x| - \frac{9}{2}\ln|x-2| + \frac{28}{3}\ln|x-3| + C$

(7) $\int \frac{x^7 dx}{(1-x^2)^5} = \int \frac{\frac{1}{2}x^6 dx^2}{(1-x^2)^5} \xlongequal{令x^2=u} \int \frac{\frac{1}{2}u^3 du}{(1-u)^5} = \int \frac{\frac{1}{2}\frac{1}{u^2}du}{(\frac{1}{u}-1)^5} = \int \frac{-\frac{1}{2}d(\frac{1}{u})}{(\frac{1}{u}-1)^5} = -\frac{1}{2}\cdot(-\frac{1}{4})(\frac{1}{u}-1)^{-4} + C = \frac{1}{8}\frac{x^8}{(1-x^2)^4} + C$

或者 $\int \frac{x^7 dx}{(1-x^2)^5} = \int \frac{-\frac{1}{2}x^6 d(1-x^2)}{(1-x^2)^5} \xlongequal{令u=1-x^2} \frac{1}{2}\int \frac{(u-1)^3 du}{u^5} = \frac{1}{2}\int (\frac{1}{u^2} - \frac{3}{u^3} + \frac{3}{u^4} - \frac{1}{u^5}) du = \frac{1}{2}(-\frac{1}{u} + \frac{3}{2u^2} - \frac{1}{u^3} + \frac{1}{4}\frac{1}{u^4}) + C$

$= \frac{1}{2(x^2-1)} + \frac{3}{4(x^2-1)^2} + \frac{1}{2(x^2-1)^3} + \frac{1}{8(x^2-1)^4} + C$

2(1) $\int \frac{\sin^4 x}{\cos^3 x}dx = \int \frac{\sin^4 x}{\cos^4 x}\cos x\, dx = \int \frac{\sin^4 x}{\cos^4 x}\, d\sin x \xlongequal{令u=\sin x}$

$\int \frac{u^4}{(1-u^2)^2}du = \int \left[1 + \frac{2u^2-1}{(u-1)^2(u+1)^2}\right] du = \int \left[1 + \frac{A_1}{u-1} + \frac{A_2}{(u-1)^2} + \frac{B_1}{u+1} + \frac{B_2}{(u+1)^2}\right] du$

$\begin{cases} A_1+B_1=0 \\ A_1+A_2-B_1+B_2=2 \\ A_2=B_2 \\ -A_1+A_2+B_1+B_2=-1 \end{cases} \quad \begin{cases} A_1=\frac{3}{4} \\ B_1=-\frac{3}{4} \\ A_2=\frac{1}{4} \\ B_2=\frac{1}{4} \end{cases}$

$= u + \frac{3}{4}\ln|u-1| - \frac{1}{4}\frac{1}{u-1} - \frac{3}{4}\ln|u+1| - \frac{1}{4}\frac{1}{u+1} + C$

$= \sin x - \frac{1}{4}(\frac{1}{\sin x-1} + \frac{1}{\sin x+1}) + \frac{3}{4}\ln\frac{1-\sin x}{1+\sin x} + C$

(2) $\int \frac{1}{\sin x\cos^4 x}dx = \int \frac{-d\cos x}{\sin^2 x\cos^4 x} \xlongequal{令u=\cos x} \int \frac{-du}{(1-u^2)u^4} = \int \frac{1}{(u+1)(u-1)u^4}du$

$= \int (\frac{A_1}{u} + \frac{A_2}{u^2} + \frac{A_3}{u^3} + \frac{A_4}{u^4} + \frac{A_5}{u+1} + \frac{A_6}{u-1}) du = \int \left[-\frac{1}{u^2} - \frac{1}{u^4} - \frac{1}{2(u+1)} + \frac{1}{2(u-1)}\right] du = \frac{1}{u} + \frac{1}{3u^3} + \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C$

$\begin{cases} A_1+A_5+A_6=0 \\ A_2-A_5+A_6=0 \\ -A_1+A_3=0 \\ -A_2+A_4=0 \\ -A_3=0 \\ -A_4=1 \end{cases} \quad \begin{cases} A_1=0 \\ A_2=-1 \\ A_3=0 \\ A_4=-1 \\ A_5=-\frac{1}{2} \\ A_6=\frac{1}{2} \end{cases}$

$= \frac{1}{\cos x} + \frac{1}{3\cos^3 x} + \frac{1}{2}\ln\frac{1-\cos x}{1+\cos x} + C$

(7) $\int \frac{\sin x}{\sin x+\cos x}dx = \int \frac{\tan x}{\tan x+1}dx \xlongequal{令u=\tan x} \int \frac{u}{u+1}\, d\arctan u$

$= \int \frac{u}{(u+1)(u^2+1)}du = \int (\frac{A_1}{u+1} + \frac{B_1 u+C_1}{u^2+1}) du$

令 $u=0 \quad 0=A_1+C_1$

$u=1 \quad \frac{1}{2}=A_1+B_1+C_1$

$u=2 \quad 2=5A_1+3(2B_1+C_1)$

$\begin{cases} A_1=-\frac{1}{2} \\ B_1=\frac{1}{2} \\ C_1=\frac{1}{2} \end{cases}$

$= \int (-\frac{1}{2}\frac{1}{u+1} + \frac{1}{2}\frac{u+1}{u^2+1}) du$

$= \frac{1}{2}\int -\frac{du}{u+1} + \frac{\frac{1}{2}d(u^2)}{u^2+1} + \frac{du}{u^2+1} = \frac{1}{2}(-\ln|u+1| + \frac{1}{2}\ln|u^2+1| + \arctan u) + C$

$= \frac{1}{2}\ln\frac{\sqrt{\tan^2 x+1}}{|\tan x+1|} + \frac{1}{2}x + C = -\frac{1}{2}\ln|\sin x+\cos x| + \frac{1}{2}x + C$

(9) $\int \frac{\cos x}{\sin x+\cos x}dx \xlongequal{令\tan x=u} \int \frac{1}{u+1}\, d\arctan u = \int \frac{1}{(u+1)(u^2+1)}du$

$= \int (\frac{A_1}{u+1} + \frac{B_1 u+C_1}{u^2+1}) du \quad$ 令 $u=0 \quad 1=A_1+C_1 \quad u=1 \quad \frac{1}{2}=A_1+B_1+C_1$

$\int \quad \sin x+\cos x \qquad \int \quad u+1 \qquad \int (u+1)(u^2+1)$

$= \int \left(\frac{A_1}{u+1} + \frac{B_1 u + C_1}{u^2+1} \right) du$ 令 $u=0 \quad 1 = A_1 + C_1 \quad u=1 \quad \frac{1}{2} = A_1 + B_1 + C_1$

$u=2 \quad 1 = 5A_1 + 3(2B_1 + C_1) \quad B_1 = -\frac{1}{2} \quad A_1 = \frac{1}{2} \quad C_1 = \frac{1}{2}$

$$= \frac{1}{2} \int \left(\frac{1}{u+1} + \frac{1}{u^2+1} - \frac{1}{2} \cdot \frac{2u}{u^2+1} \right) du = \frac{1}{2} \left(\ln|u+1| + \arctan u - \frac{1}{2} \ln|u^2+1| \right) + C$$

$$= \frac{1}{2} \ln|\sin x + \cos x| + \frac{1}{2} x + C$$

$$\int \frac{\cos x}{\sin x + \cos x} dx = \frac{1}{2} \int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\sin x + \cos x} dx$$

$$= \frac{1}{2} x + \frac{1}{2} \int \frac{d(\sin x + \cos x)}{\sin x + \cos x}$$

$$= \frac{1}{2} x + \frac{1}{2} \ln |\sin x + \cos x| + C$$