清华大学2021春季学期

电路原理C

第3次应用介绍课

正弦稳态电路的应用

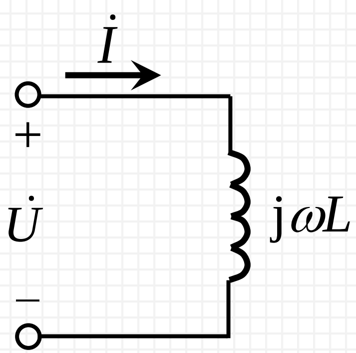
内容

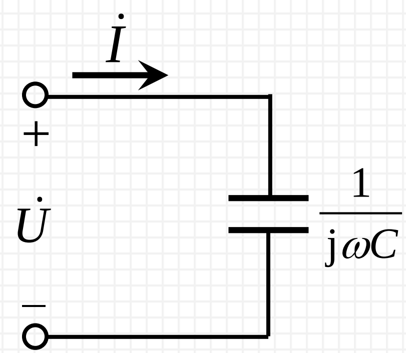
本讲重难点

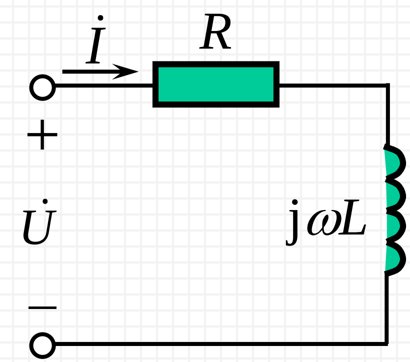
- 滤波器的**截止频率**
- 空心变压器的**引入阻抗**
- 理想变压器的 u-i **关系**
  - 双绕组
  - 三绕组

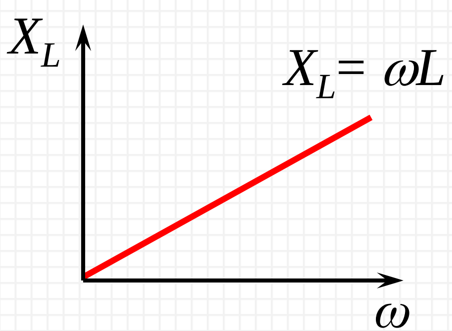
1、频率特性

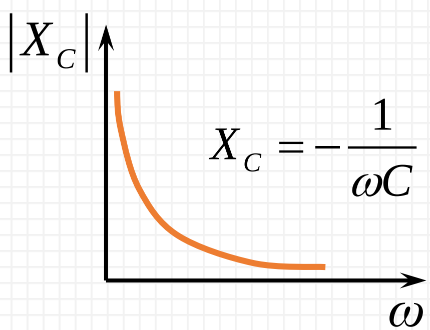
(1) 频率特性 (frequency characteristics)









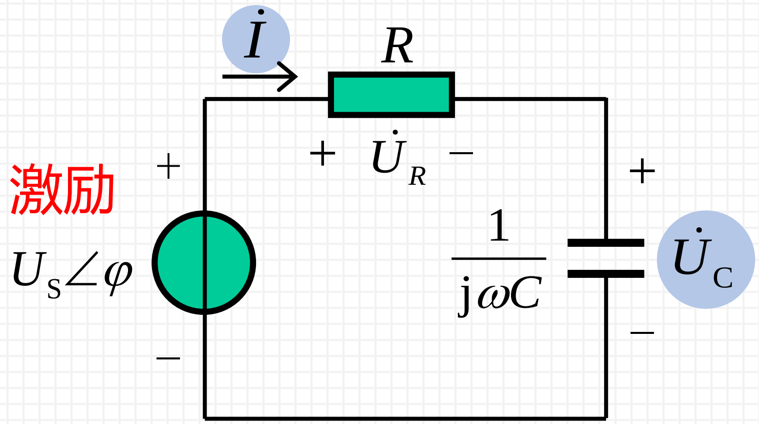


$$Z = R + \mathrm{j}\omega L$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\varphi = \arctan \omega L / R$$

端口入端**阻抗**的**幅值**和**相角**随**频率**的变化而变化

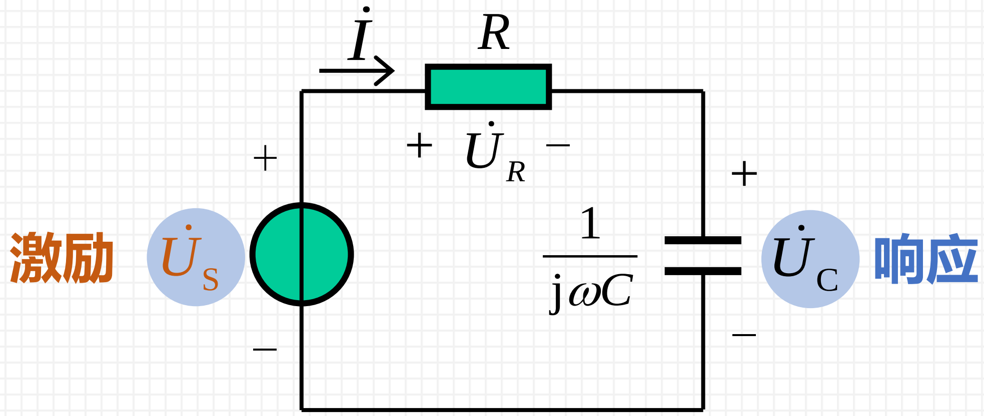


响应

$$\dot{I}=\frac{U_S\angle\varphi}{R-\mathrm{j}\frac{1}{\omega C}}=\frac{U_S}{\sqrt{R^2+({}^{1}\!/_{\omega C})^2}}\angle(\varphi+\arctan{}^{1}\!/_{\omega RC})$$

$$\dot{U}_C=\frac{1}{\mathrm{j}\omega C}\dot{I}=\frac{U_S}{\sqrt{(\omega RC)^2+1}}\angle(\varphi+\arctan{}^{1}\!/_{\omega RC}-90°)$$

电路响应的幅值和相角均随频率的变化而变化



$$H = \frac{\text{响应（相量）}}{\text{激励（相量）}}$$

网络函数
传递函数

$$H = \frac{\dot{U}_o}{\dot{U}_i} = \frac{\dot{U}_C}{\dot{U}_S} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\arctan(\omega RC)$$

网络函数的幅值和相角均随频率的变化而变化

电路的**频率响应** ⟷ 电路的**频率特性**

改变电路激励的频率（维持其幅值不变）对电路造成的影响

- 阻抗 → 阻抗频率特性；电抗频率特性
- 支路电压电流 → 电压频率特性；电流频率特性
- 输入输出关系 → 网络函数/传递函数

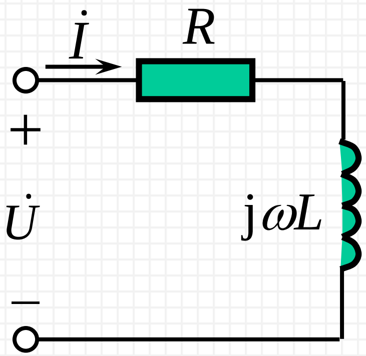
$$X_L = \omega L$$

$$\dot{I} = \frac{U_S}{\sqrt{R^2 + (1/\omega C)^2}} \angle(\varphi + \arctan 1/\omega RC)$$

$$\boldsymbol{H} = \frac{\text{响应}}{\text{激励}}$$

$$H = \frac{1}{\sqrt{1+(\omega RC)^2}} \angle -\arctan(\omega RC)$$

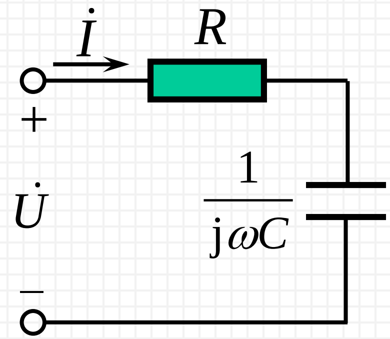
例1 画图示电路入端阻抗频率特性。



$$Z = R + \mathrm{j}\omega L$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

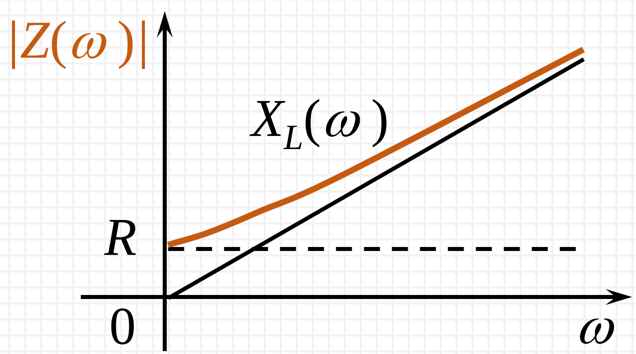
$$\varphi = \arctan\frac{\omega L}{R}$$

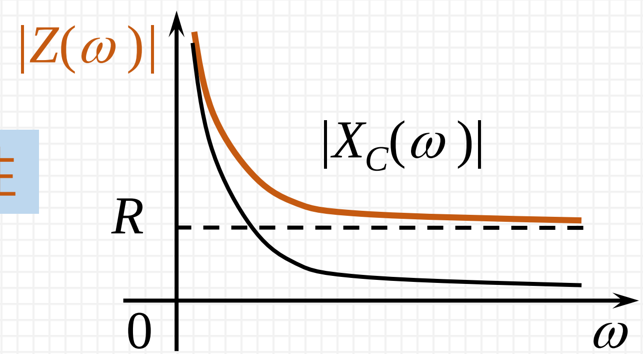


$$Z = R - \mathrm{j}\frac{1}{\omega C}$$

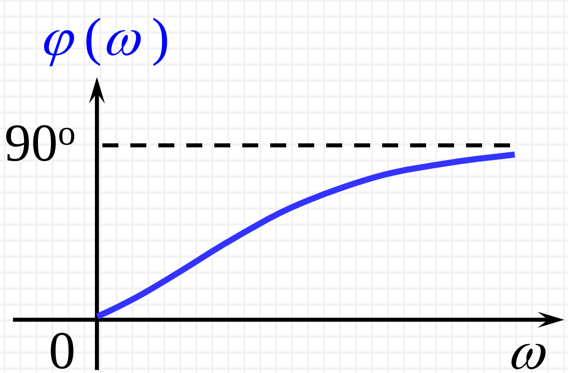
$$|Z| = \sqrt{R^2 + (\frac{1}{\omega C})^2}$$

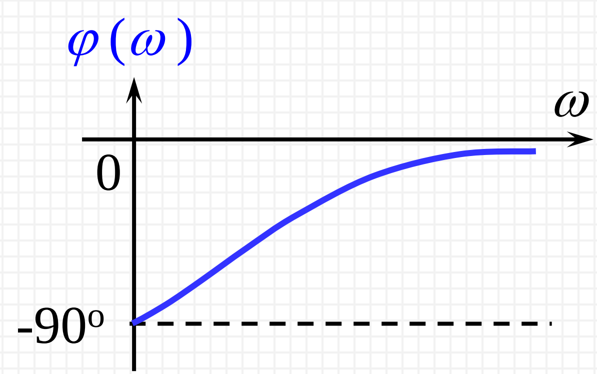
$$\varphi = -\arctan\frac{1}{\omega CR}$$





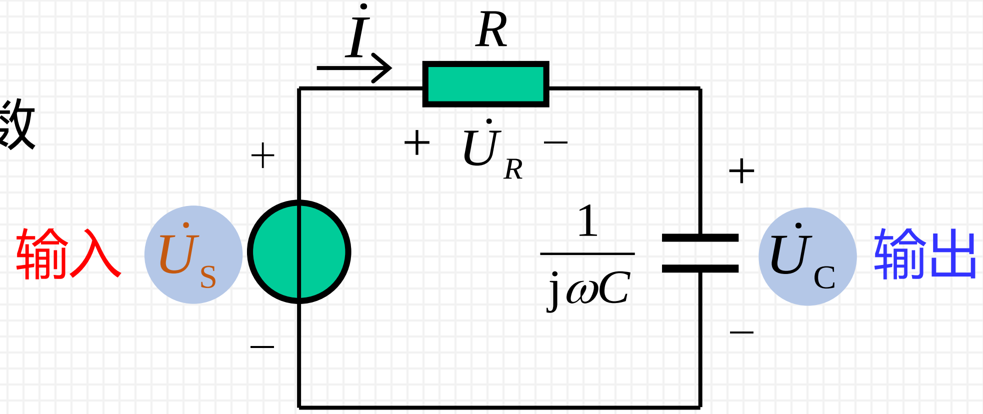
幅值频率特性



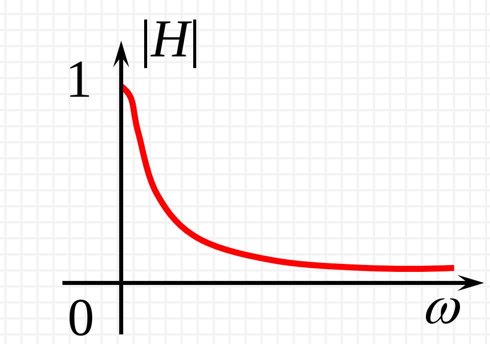


相位频率特性

例2 画图示电路网络函数的频率特性。

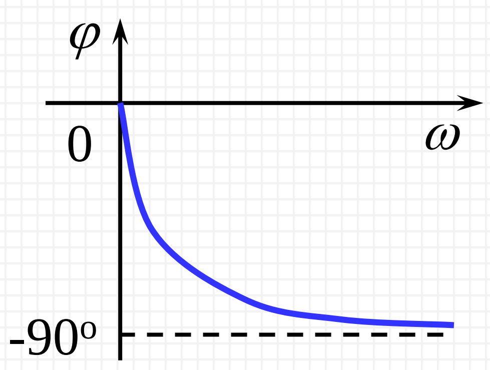


$$H(\omega) = |H| \angle \varphi = \frac{1}{\sqrt{1+(\omega CR)^2}} \angle -\arctan(\omega CR)$$



设 $RC=1$

ω /(rad/s)	$\|H\|$	φ (°)
0	1	0
1	0.707	-45
5	0.196	-78.7
10	0.1	-84.3
20	0.05	-87.1
100	0.01	-89.4



实际应用中，经常会出现

- **待考察量**的变化非常大
- **频率范围**非常宽

阻抗、电压电流、传递函数

$10^{-4} < |H| < 10^{+3}$

1mHz ~ 1GHz

幅值/相位频率特性的波形特点被掩盖

$-4 < \lg|H| < +3$

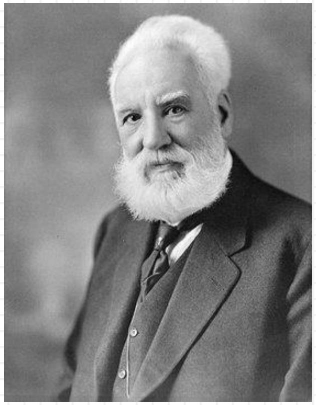
用对数表示

-3 ~ 9

分贝(decibel)

$$H_{dB} = 20\lg|H|$$

dB的由来和好处



Bell

- 人们在研究声音有多响亮的过程中，提出两种用log方式描述比例大小的手段
- 声音响亮的程度与功率成正比
- 声音的功率与声压平方成正比
- 好处
  - 从容地表示大比例
  - 对 >**1**和 <**1**区分明显

$$\eta = \lg \frac{A}{B} (\mathrm{Bell})$$

$$\mathrm{dB} = 10 \lg \frac{A}{B}$$

$$\mathrm{dB} = 10 \lg \frac{P_2}{P_1}$$

$$\mathrm{dB} = 10 \lg \frac{U_2^2}{U_1^2} = 20 \lg \frac{U_2}{U_1}$$

2、滤波器(Filter)

去除噪声的装置称为**滤波器**

噪声是无处不在的

假设信号与噪声的频带不重合 → 经典滤波器

估计信号特征，从而提高信噪比 → 现代滤波器

经典滤波器

用模拟系统实现 → 模拟滤波器

用数字系统实现 → 数字滤波器

现代信号处理

模拟滤波器

用无源元件实现 → 无源滤波器

用有源元件实现 → 有源滤波器

数字信号处理

电路与系统

模拟电子技术基础

电力电子技术基础

模拟滤波器

从功能上分类

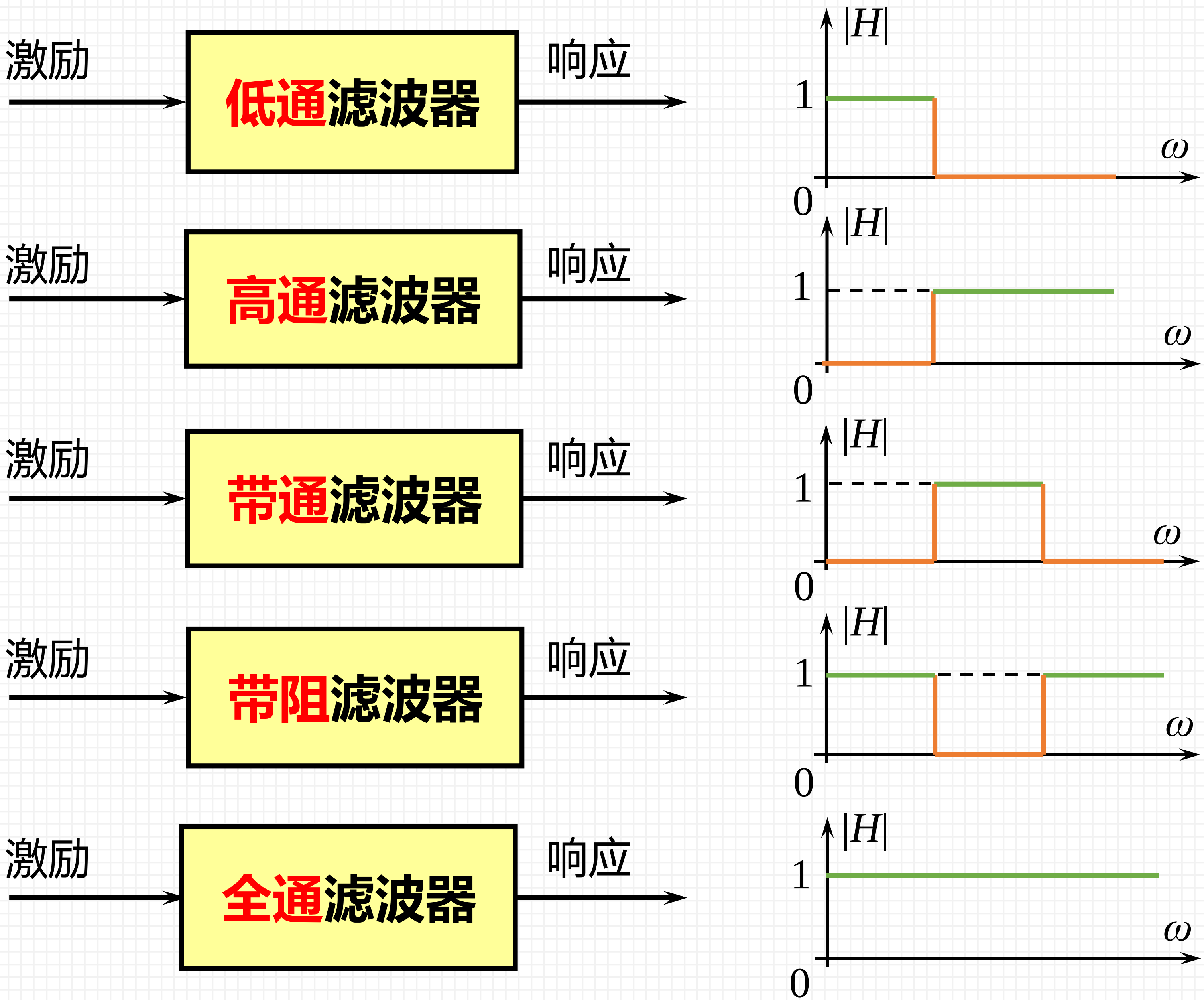
低通(LP)　高通(HP)　带通(BP)　带阻(BS, Notch)　全通(FP)

从实现方式上分类

无源滤波器　有源滤波器

RC　LC　Op Amp　电力电子器件

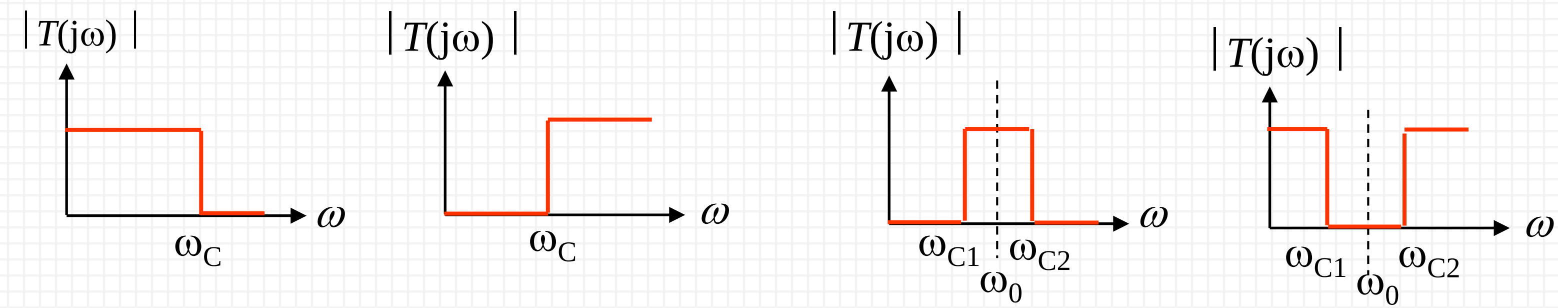
用无源元件实现　用有源元件实现


激励
低通滤波器
响应
激励
高通滤波器
响应
激励
带通滤波器
响应
激励
带阻滤波器
响应
激励
全通滤波器
响应
|H|
1
0
ω
|H|
1
0
ω
|H|
1
0
ω
|H|
1
0
ω
|H|
1
0
ω

无源滤波器

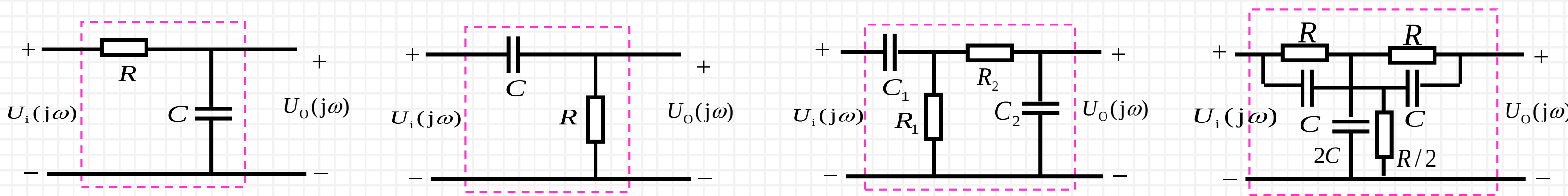
扩展知识

理想滤波器的幅频特性曲线



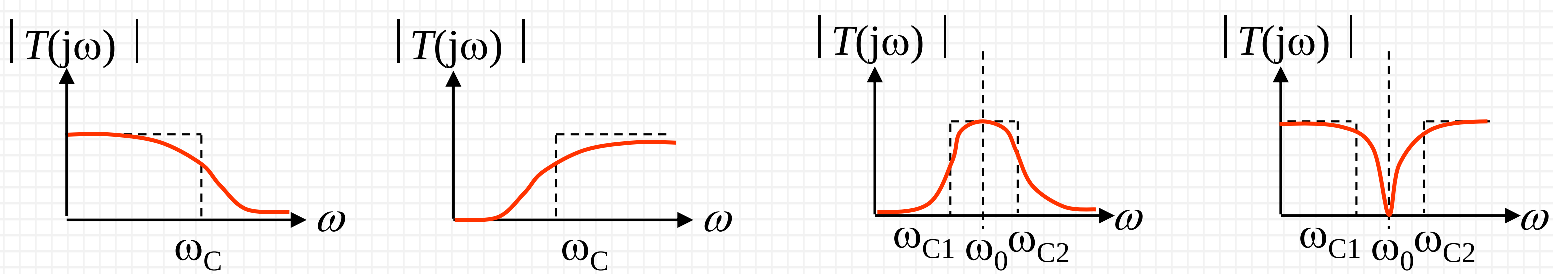
低通　　高通　　带通　　带阻

实际的滤波器



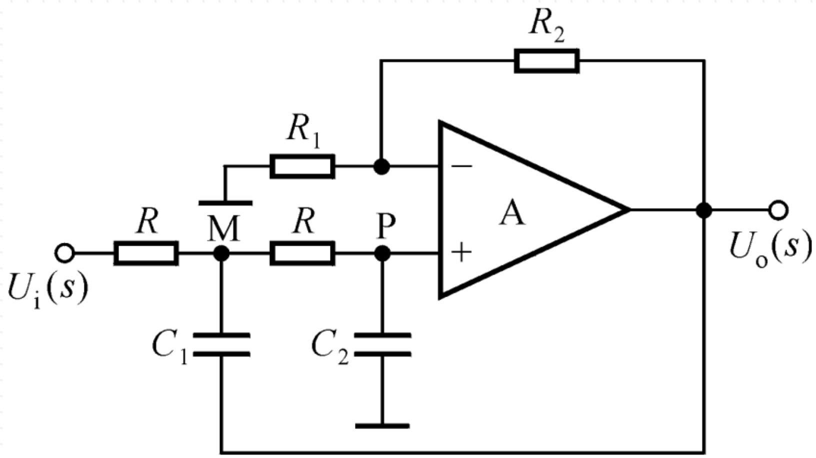
高通＋低通　　双T带阻滤波器

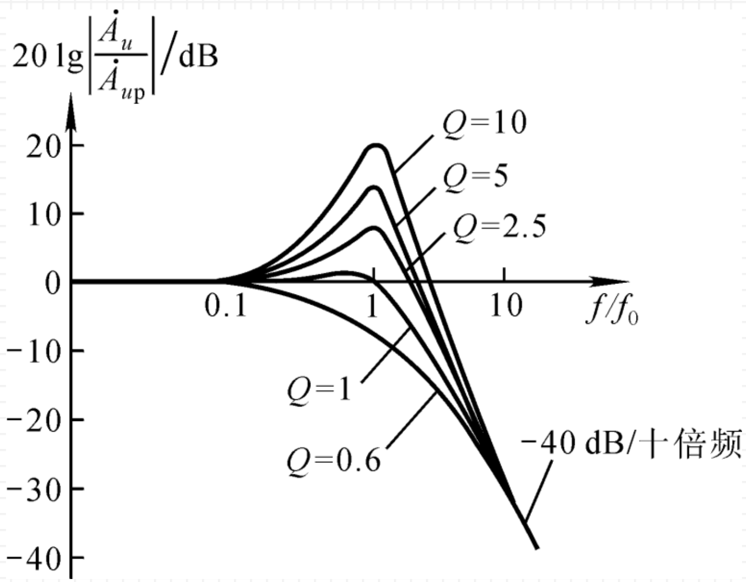
实际的滤波器幅频特性曲线



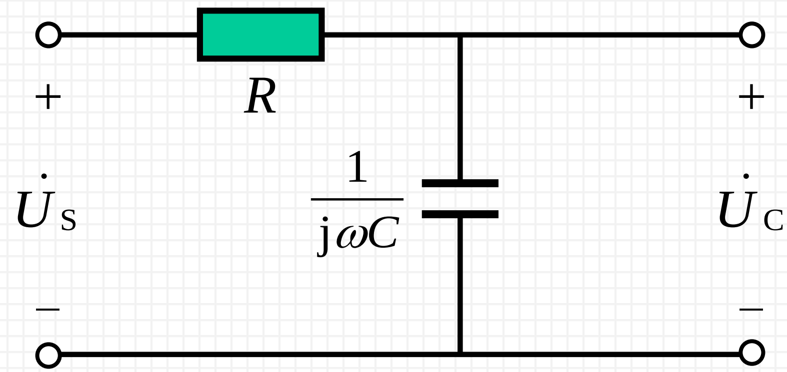
压控电压源二阶LPF(有源滤波器)

扩展知识

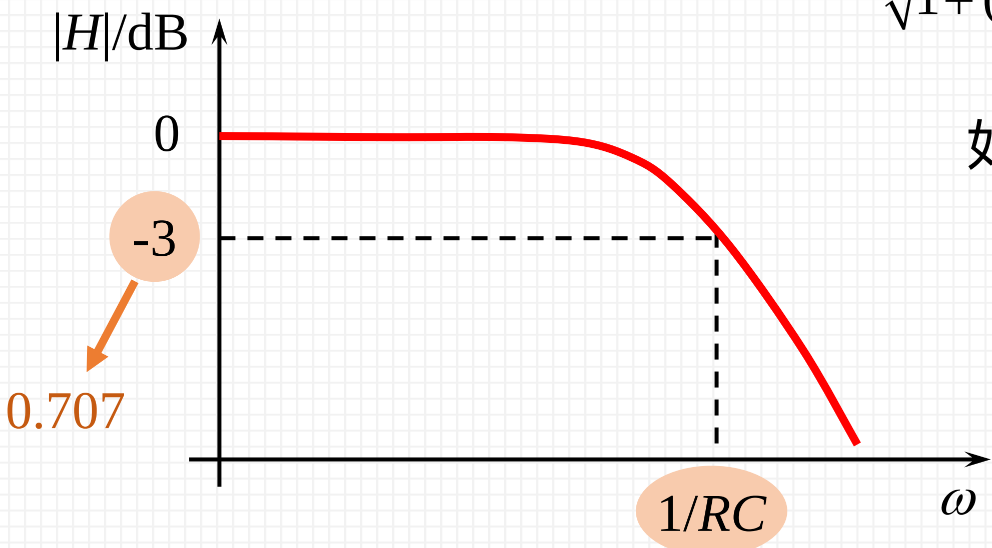




例1 RC低通滤波器



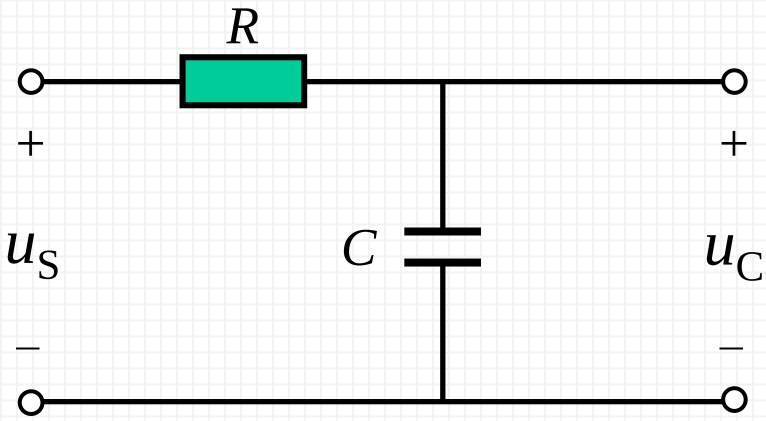
$$H(\omega)=|H|\angle\varphi=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan(\omega CR)$$



如果 $RC>>1$ 截止频率低

下面我们来讨论这个低通滤波器在时域是个什么

低通滤波器　截止频率　半功率频率



时域

低通滤波器

已知 $u_C(0^-)=0$，$u_S=10\,\mathrm{V}$

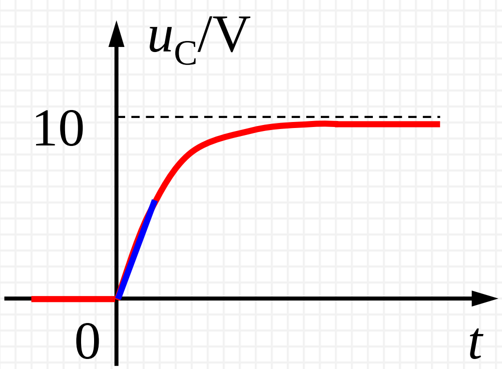
$$RC\frac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=u_S \quad\rightarrow\quad u_C(t)=10(1-\mathrm{e}^{-\frac{t}{RC}})$$

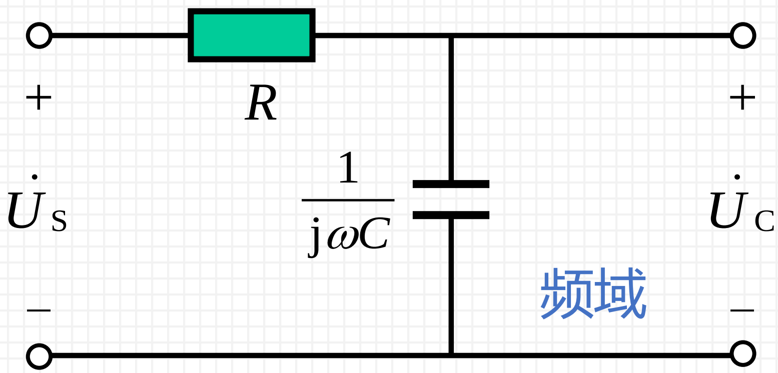
$RC>>1$

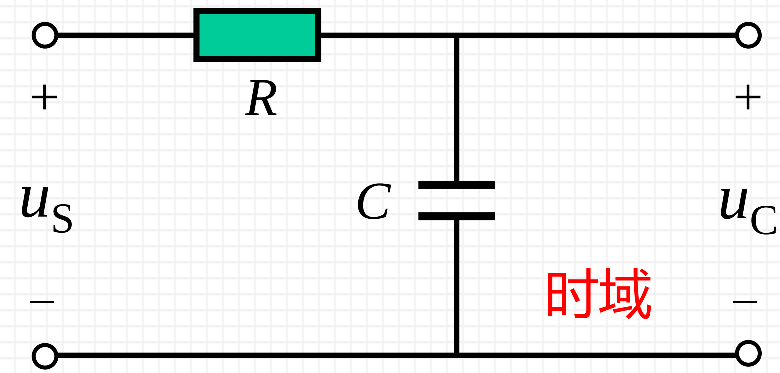
t 很小

$$u_C\approx\frac{1}{RC}\int u_S\mathrm{d}t \qquad u_C\approx 10\frac{t}{RC}$$

近似积分器







(截止频率比较低的)
低通滤波器

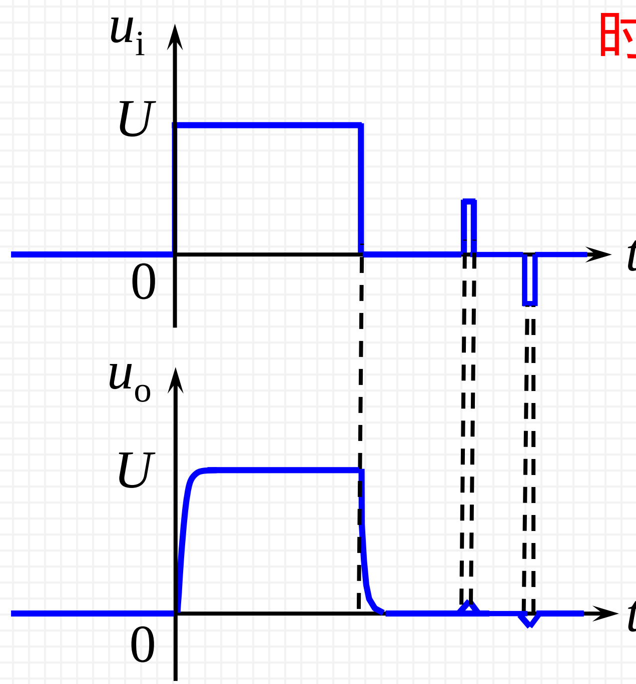
?

$RC >> 1$

(积分速度比较慢的)
积分器

频域

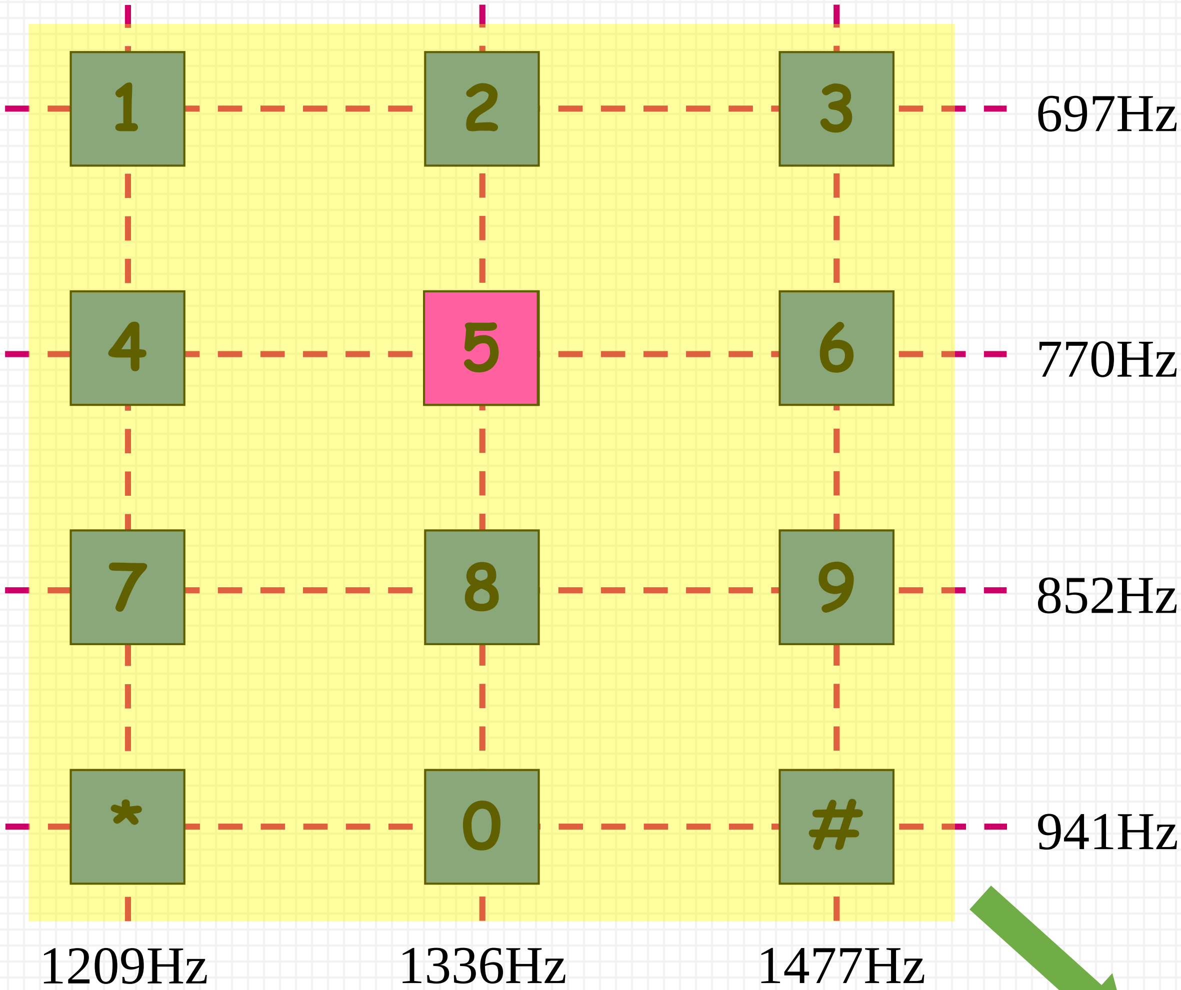
毛刺变化快
↓
高频成分多
↓
高频成分被滤除
↓
毛刺被抑制



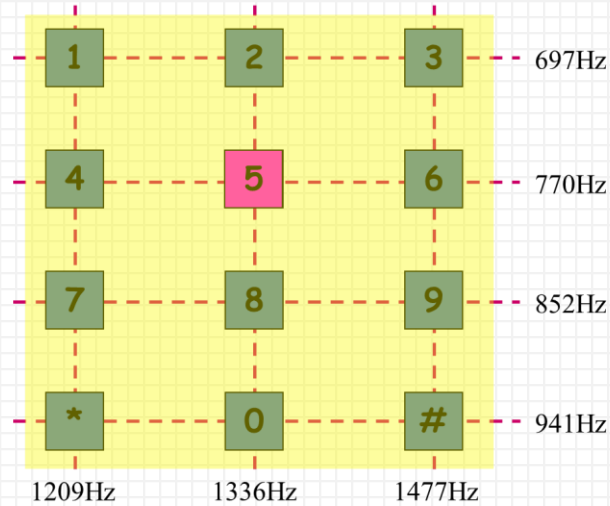
时域

毛刺时间短
↓
积分量小
↓
毛刺被抑制

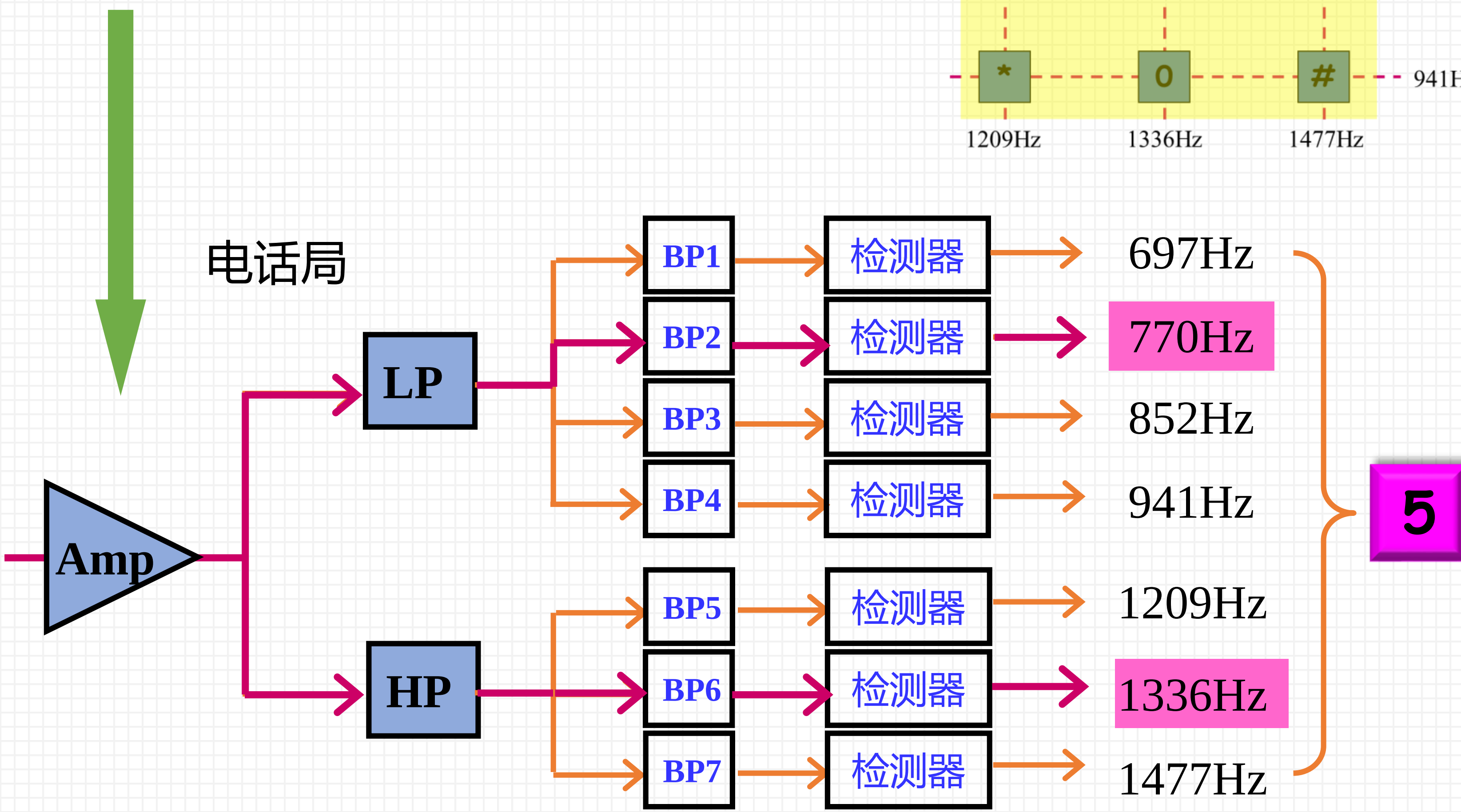
例2　按键式电话（tone，双音频电话）



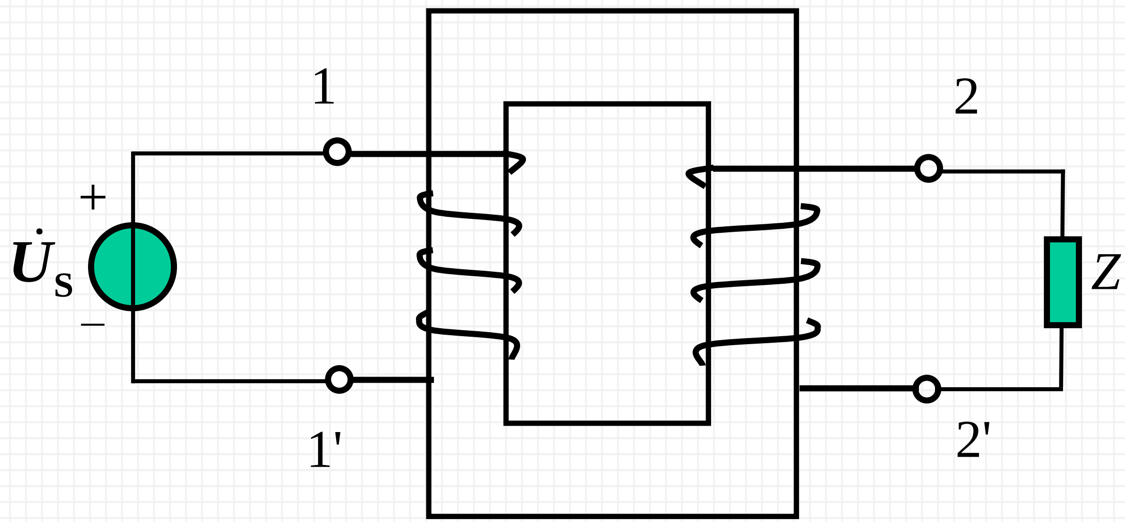
770Hz + 1336Hz



770Hz + 1336Hz



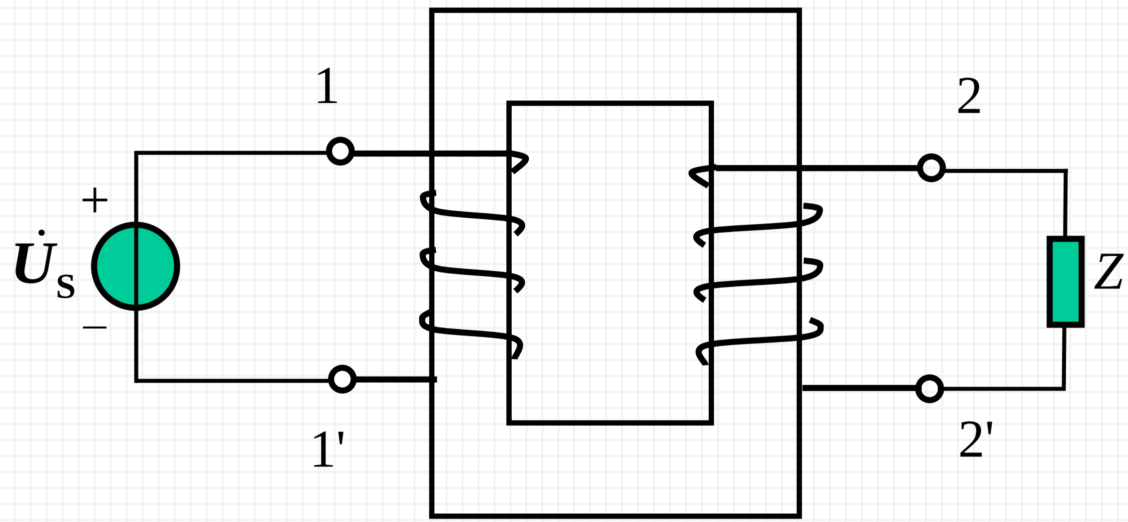
3、变压器 (Transformer)



利用**互感**的作用来**传递能量**

- 交流变压、变流
- 电隔离
- 传送功率
- 阻抗匹配

研究思路



1) 考虑线圈内阻，求从原边(副边)看的等效电路

空芯变压器模型

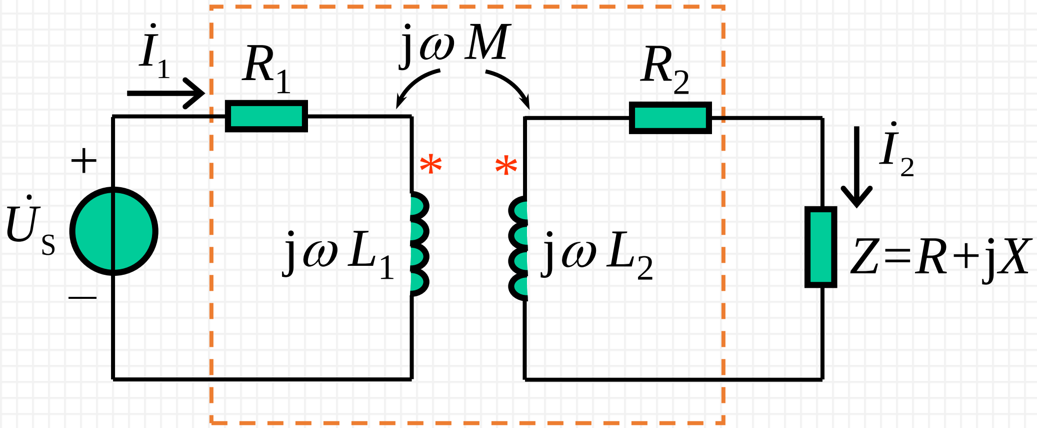
2) 忽略考虑线圈内阻，耦合系数为1

全耦合变压器模型

3) 感值趋向于无穷大

理想变压器模型

(1) 空心变压器



原边回路总抗阻

$Z_{11}=R_1+\mathrm{j}\omega L_1$

副边回路总阻抗

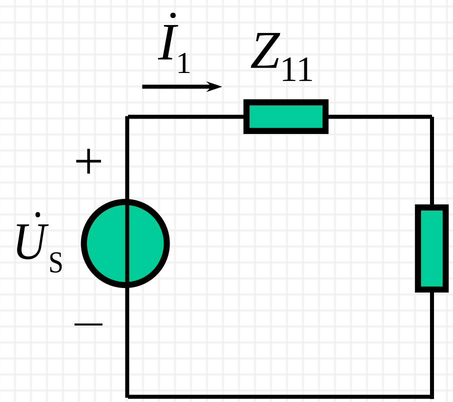
$Z_{22}=(R_2+R)+\mathrm{j}(\omega L_2+X)$

$$\begin{cases} Z_{11}\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 = \dot{U}_S \\ -\mathrm{j}\omega M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases}$$

$$Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_{11} + \frac{(\omega M)^2}{Z_{22}}$$

$$\dot{I}_2 = \frac{\mathrm{j}\omega M\dot{I}_1}{Z_{22}}$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$



$$Z_l \overset{\text{def}}{=} \frac{(\omega M)^2}{Z_{22}}$$

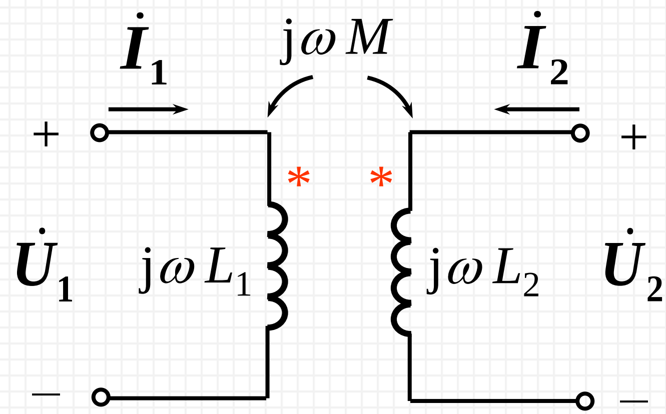
引入阻抗

原边等效电路

(2) 全耦合变压器 (unity-coupled transformer)

$$M = \sqrt{L_1 L_2}$$

忽略电阻



$$\dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2$$

$$= \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega \sqrt{L_1 L_2}\, \dot{I}_2$$

$$\dot{U}_2 = \mathrm{j}\omega M \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2$$

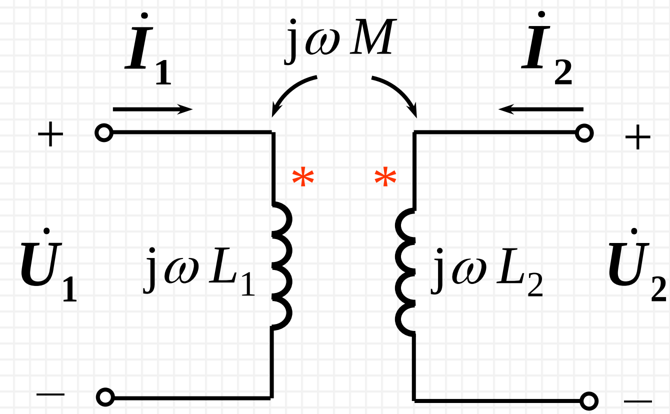
$$= \mathrm{j}\omega \sqrt{L_1 L_2}\, \dot{I}_1 + \mathrm{j}\omega L_2 \dot{I}_2$$

$$\dot{U}_1 \sqrt{L_2} = \dot{U}_2 \sqrt{L_1}$$

$$\frac{\dot{U}_1}{\dot{U}_2} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} = n \rightarrow \text{变比}$$

变压器实现变压

全耦合变压器电压、电流关系



$$M = \sqrt{L_1 L_2}$$

$$\boxed{\frac{\dot{U}_1}{\dot{U}_2} = n}$$

$$\dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2$$

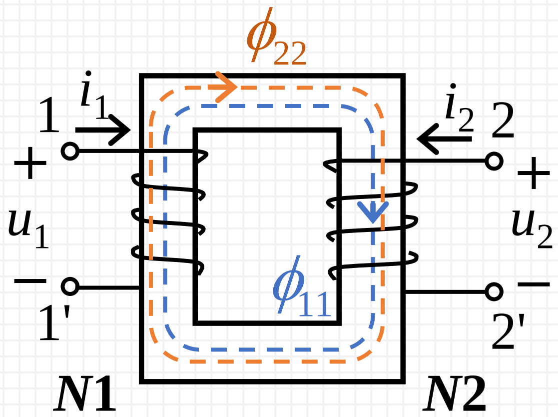
$$\dot{I}_1 = \frac{\dot{U}_1}{\mathrm{j}\omega L_1} - \frac{M}{L_1}\dot{I}_2 = \frac{\dot{U}_1}{\mathrm{j}\omega L_1} - \frac{\sqrt{L_1 L_2}}{L_1}\dot{I}_2 = \frac{\dot{U}_1}{\mathrm{j}\omega L_1} - \sqrt{\frac{L_2}{L_1}}\dot{I}_2$$

$$\boxed{\dot{I}_1 = \frac{\dot{U}_1}{\mathrm{j}\omega L_1} - \frac{1}{n}\dot{I}_2}$$

原边1000匝，副边5000匝的变压器和原边1匝，副边5匝的变压器，**有什么区别?**

全耦合变压器电压、电流关系
$$\begin{cases} \dfrac{\dot{U}_1}{\dot{U}_2} = n \\ \dot{I}_1 = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$$

(3) 理想变压器 (ideal transformer)

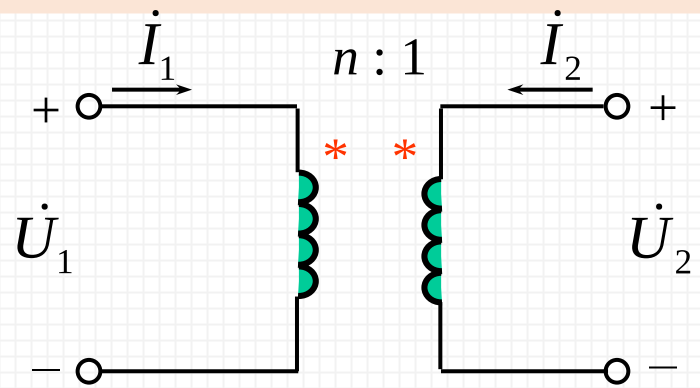


全耦合变压器 $\begin{cases} \dfrac{\dot{U}_1}{\dot{U}_2} = n \\ \dot{I}_1 = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$ $\quad n = \dfrac{N_1}{N_2}$

若 $L_1 \to \infty$ (原因可以是磁导率 $\mu \to \infty$ 或者匝数 $N_1 \to \infty$)，同时确保 L_1/L_2 **比值不变** ($L_2 \to \infty$)，且全耦合($M \to \infty$)，有

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$

理想变压器的元件特性

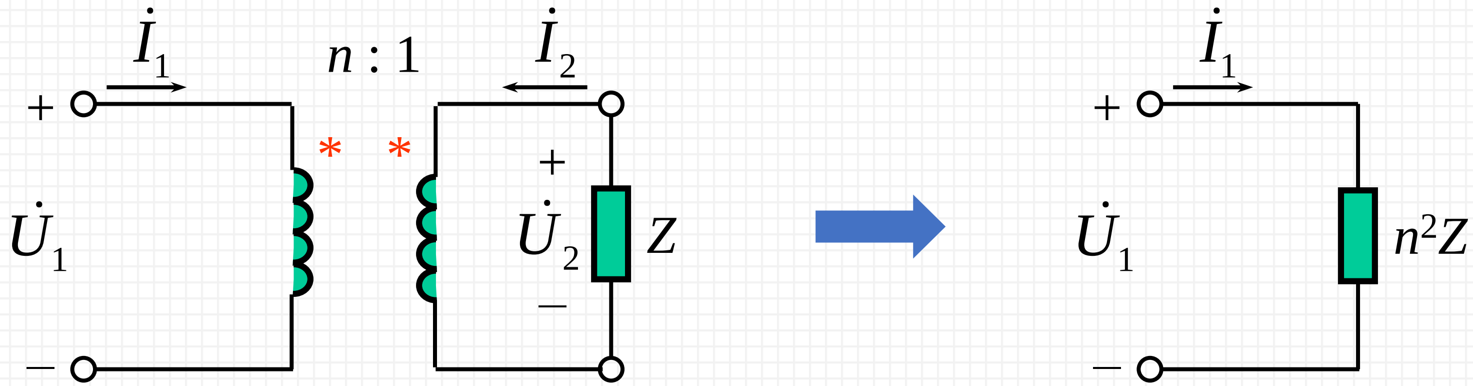


理想变压器的电路模型

理想变压器模型看不出电感！

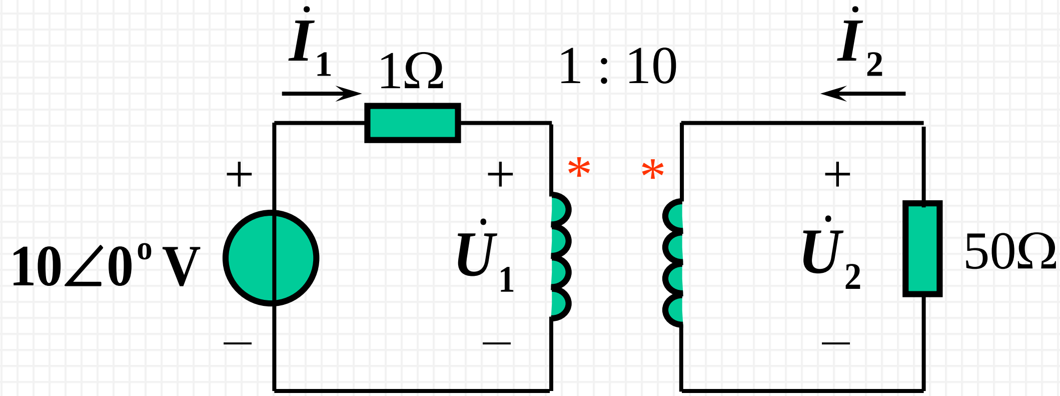
理想变压器的阻抗变换性质：

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$

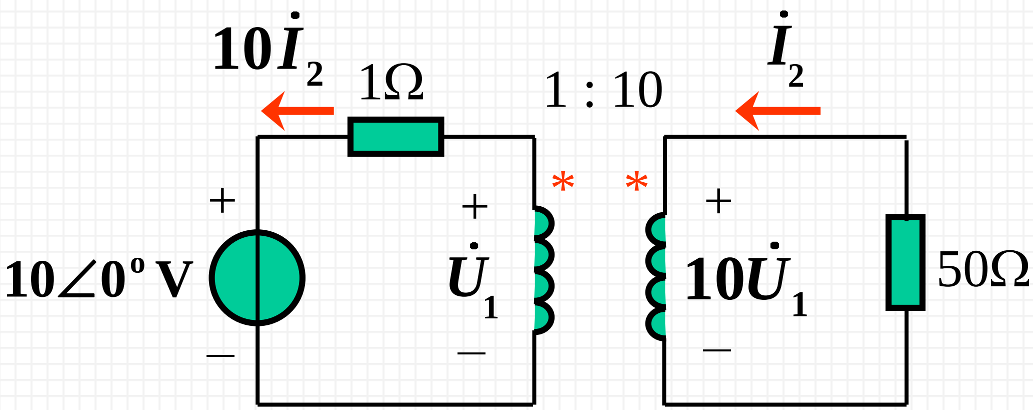


$$\frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-\frac{1}{n}\dot{I}_2} = n^2\left(-\frac{\dot{U}_2}{\dot{I}_2}\right) = n^2 Z$$

例 求图中电压 $\dot{U}_2$



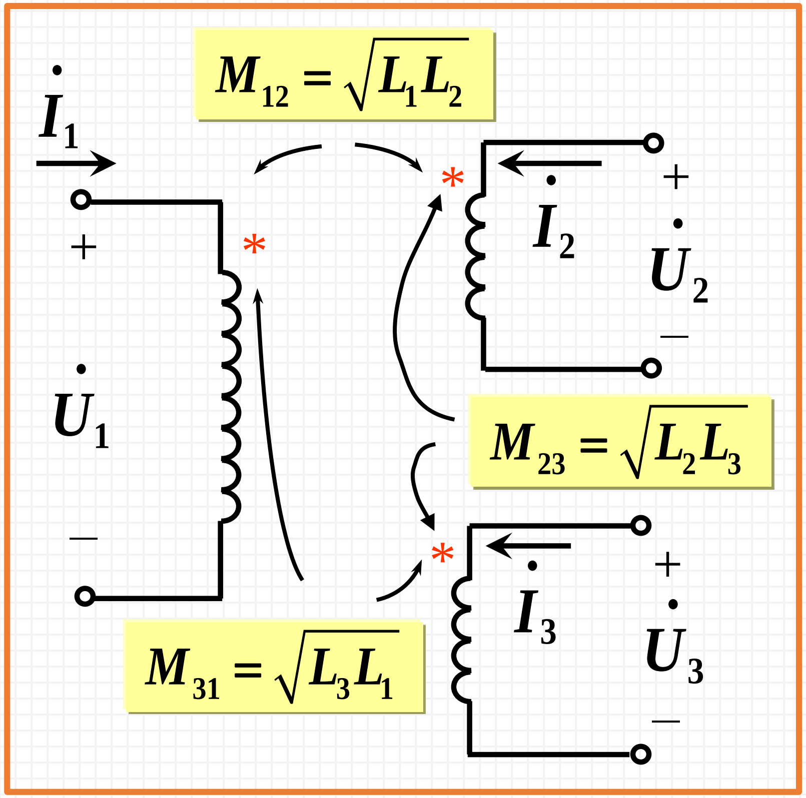
解:



$$\begin{cases} 10\dot{U}_1 = -50\dot{I}_2 \\ 10\dot{I}_2 + 10 = \dot{U}_1 \end{cases}$$

$$\boxed{\dot{U}_2 = 33.3\angle 0^\circ \text{ V}}$$

拓展：3绕组全耦合变压器的性质



$$\begin{cases} \dot{U}_1 = j\omega L_1\dot{I}_1 + j\omega\sqrt{L_1L_2}\,\dot{I}_2 + j\omega\sqrt{L_1L_3}\,\dot{I}_3 \\ \dot{U}_2 = j\omega\sqrt{L_1L_2}\,\dot{I}_1 + j\omega L_2\dot{I}_2 + j\omega\sqrt{L_2L_3}\,\dot{I}_3 \\ \dot{U}_3 = j\omega\sqrt{L_1L_3}\,\dot{I}_1 + j\omega\sqrt{L_2L_3}\,\dot{I}_2 + j\omega L_3\dot{I}_3 \end{cases}$$

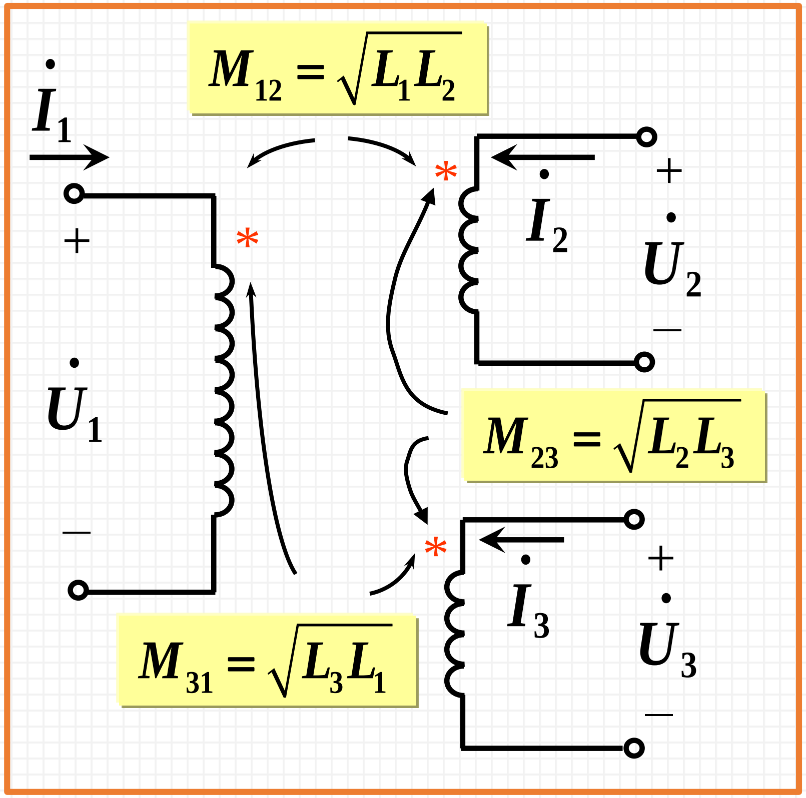
$$\begin{cases} \dot{U}_2 = \sqrt{\dfrac{L_2}{L_1}}\dot{U}_1 = \dfrac{N_2}{N_1}\dot{U}_1 \\ \dot{U}_3 = \sqrt{\dfrac{L_3}{L_1}}\dot{U}_1 = \dfrac{N_3}{N_1}\dot{U}_1 \\ \dot{I}_1 = \dfrac{\dot{U}_1}{j\omega L_1} - \sqrt{\dfrac{L_2}{L_1}}\dot{I}_2 - \sqrt{\dfrac{L_3}{L_1}}\dot{I}_3 \end{cases}$$

$$n_1 = \frac{N_1}{N_2}$$

$$n_2 = \frac{N_1}{N_3}$$

$$\begin{cases} \dot{U}_1 = n_1\dot{U}_2 \\ \dot{U}_1 = n_2\dot{U}_3 \\ \dot{I}_1 = \dfrac{\dot{U}_1}{j\omega L_1} - \dfrac{1}{n_1}\dot{I}_2 - \dfrac{1}{n_2}\dot{I}_3 \end{cases}$$

3绕组全耦合变压器的性质



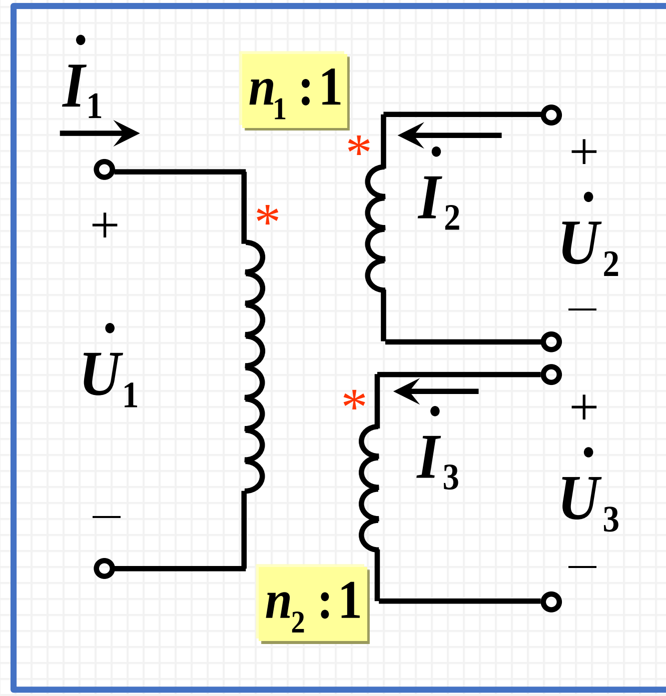
$$\begin{cases} \dot{U}_1 = n_1 \dot{U}_2 \\ \dot{U}_1 = n_2 \dot{U}_3 \\ \dot{I}_1 = \dfrac{\dot{U}_1}{j\omega L_1} - \dfrac{1}{n_1}\dot{I}_2 - \dfrac{1}{n_2}\dot{I}_3 \end{cases}$$

$$n_1 = \frac{N_1}{N_2}$$

$$n_2 = \frac{N_1}{N_3}$$

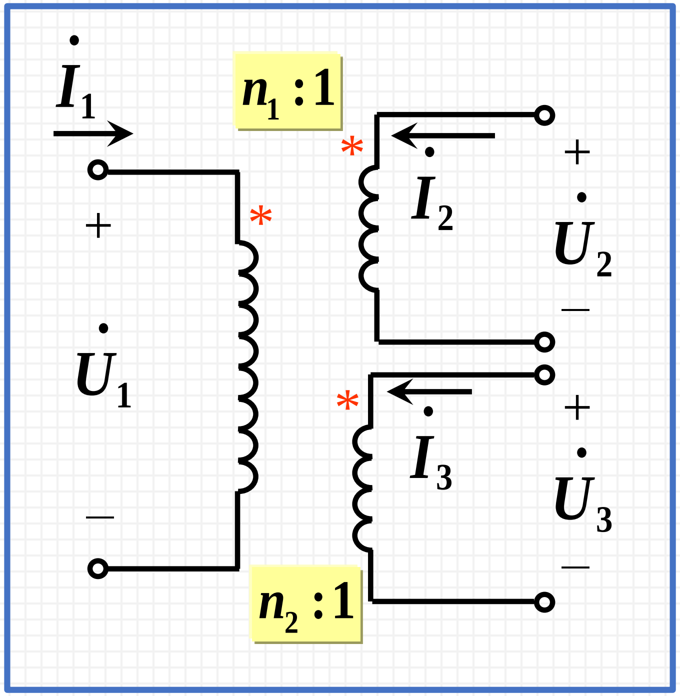
$L_1 \to \infty$，同时确保 L_1/L_2 和L_1/L_3比值不变，且全耦合

3绕组理想变压器的性质



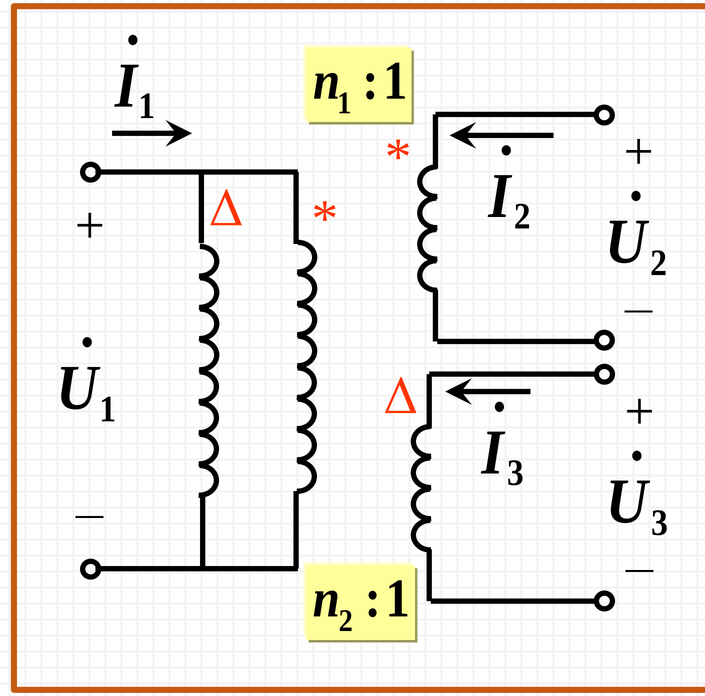
$$\begin{cases} \dot{U}_1 = n_1 \dot{U}_2 \\ \dot{U}_1 = n_2 \dot{U}_3 \\ \dot{I}_1 = -\dfrac{1}{n_1}\dot{I}_2 - \dfrac{1}{n_2}\dot{I}_3 \end{cases}$$

三绕组理想变压器的另一种观点



$$n_1 = \frac{N_1}{N_2}$$

$$n_2 = \frac{N_1}{N_3}$$



$$\begin{cases} \dot{U}_1 = n_1 \dot{U}_2 \\ \dot{U}_1 = n_2 \dot{U}_3 \\ \dot{I}_1 = -\dfrac{1}{n_1}\dot{I}_2 - \dfrac{1}{n_2}\dot{I}_3 \end{cases}$$