清华大学2021春季学期

电路原理C

第6讲

二端口网络

第6讲 二端口网络
(Two-port Network)

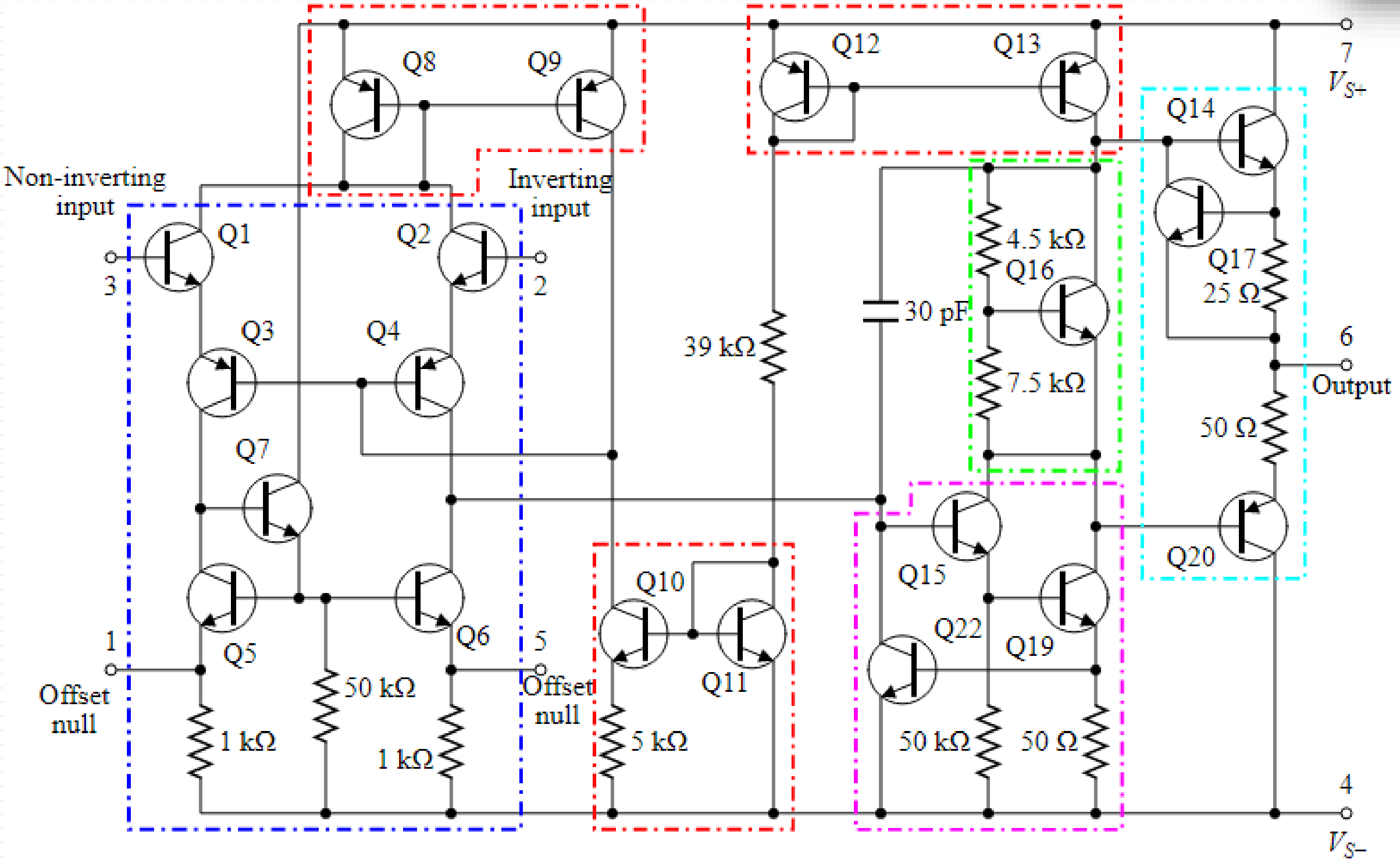
二端口网络的参数和方程

根据给定电路求二端口参数

二端口网络的等效电路

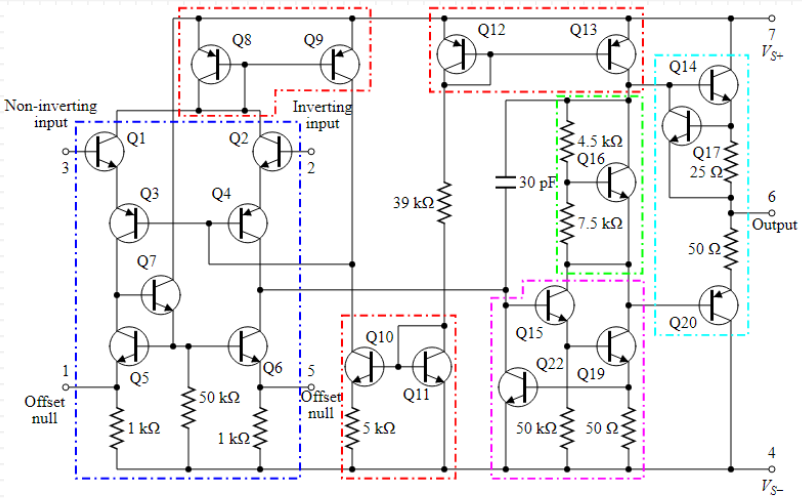
根据给定二端口参数求等效电路

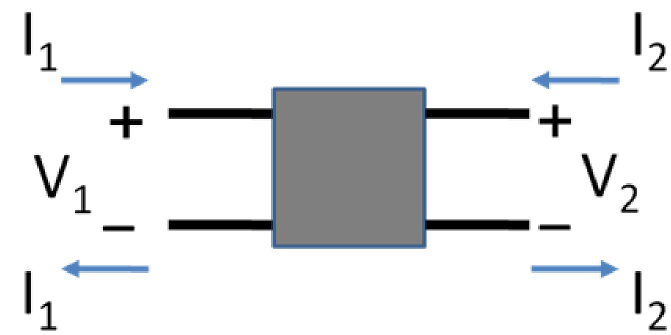
Why Two-port?



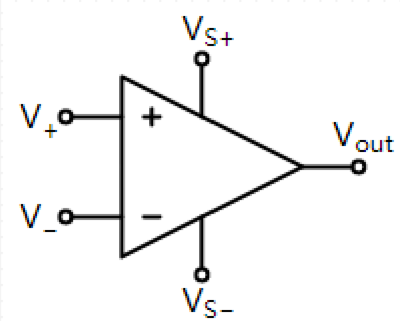
Q8
Q9
Q12
Q13
Q14
7
V_{S+}
Non-inverting input
Inverting input
Q1
Q2
3
2
Q3
Q4
4.5 kΩ
Q16
Q17
25 Ω
39 kΩ
30 pF
6
Output
7.5 kΩ
Q7
50 Ω
Q10
Q15
Q20
Q5
Q6
Q22
Q19
Q11
1
Offset null
5
Offset null
50 kΩ
1 kΩ
1 kΩ
5 kΩ
50 kΩ
50 Ω
4
V_{S-}

Why Two-port?

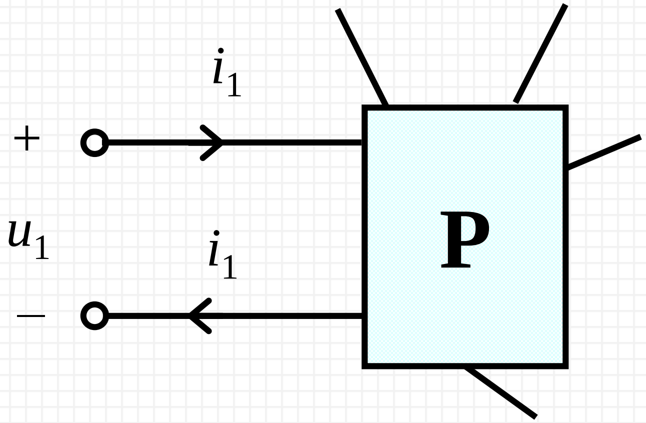




抽象的力量



(1) 端口 (port)



1. 定义

端口由两个接线端构成，且满足如下条件：**从一个接线端流入的电流等于从另一个接线端流出的电流。**

端口条件

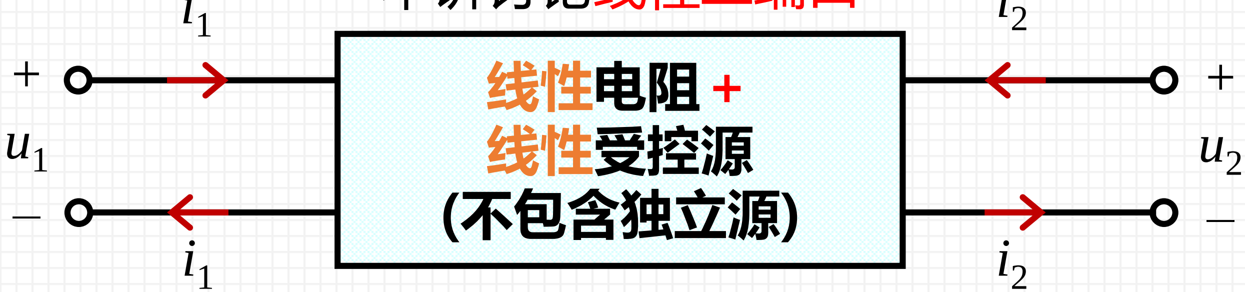
(2) 二端口 (two-port)

Franz Breisig 1920提出

当一个电路与外部电路通过两个端口连接时称此电路为**二端口网络**。

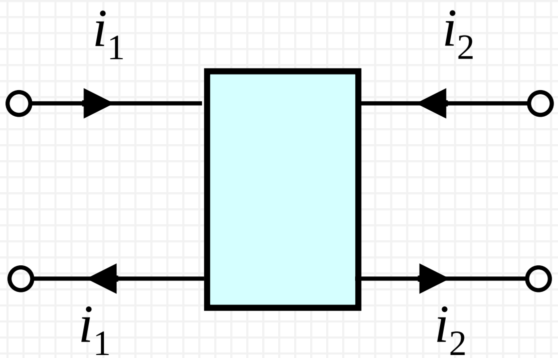
≠线性四端网络

本讲讨论线性二端口

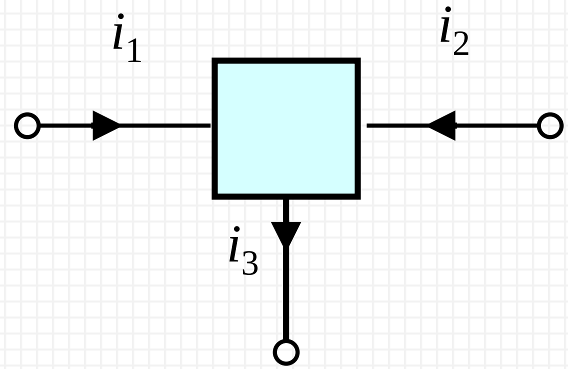


注意 参考方向：u上+下-，i 从 u 的+端流入。

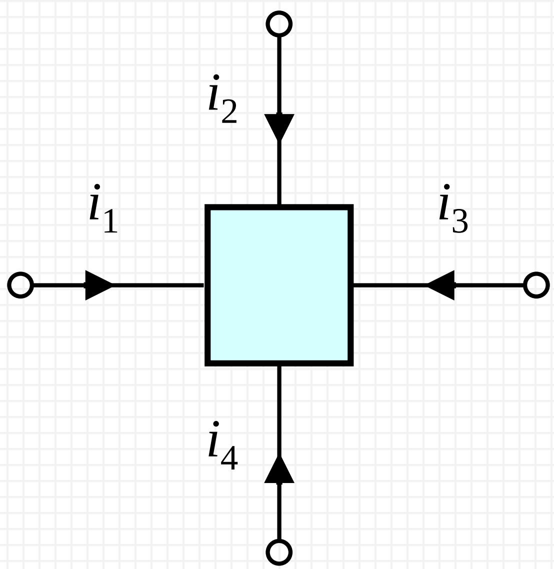
(3) 二端口网络与四端网络



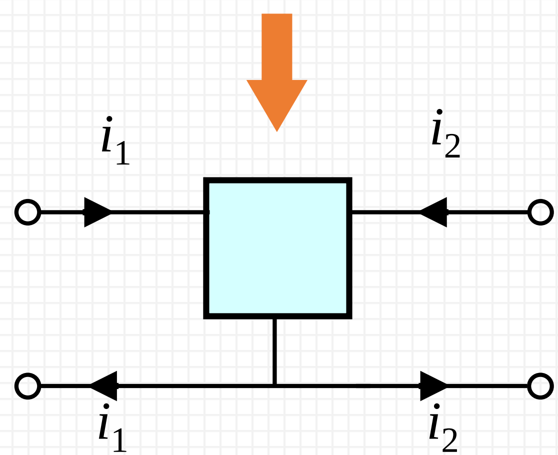
二端口



三端网络

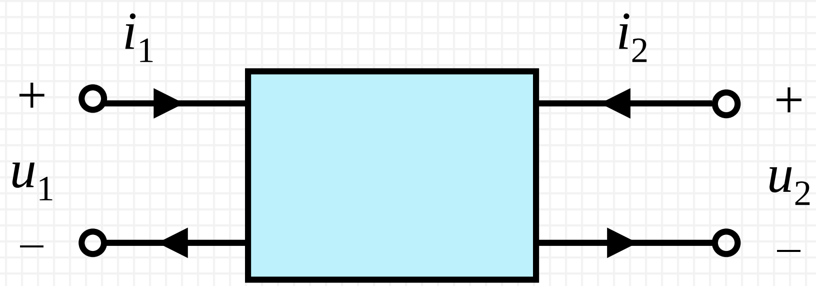


四端网络



具有公共端的二端口

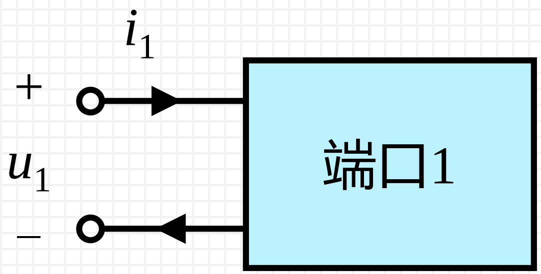
2 二端口的参数和方程

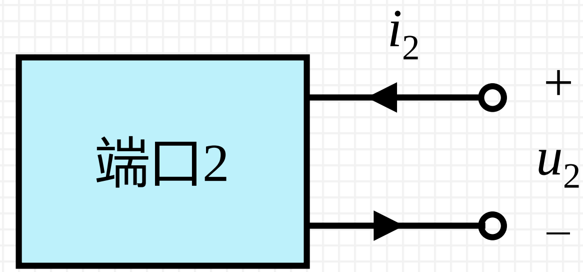


端口物理量4个 i_1 i_2 u_1 u_2

如何描述二端口网络的电压电流关系?

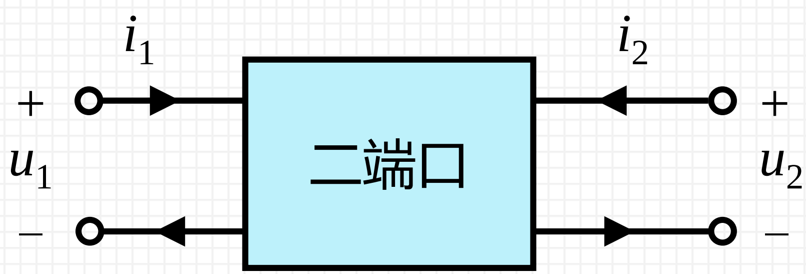
回忆一端口网络的电压电流关系





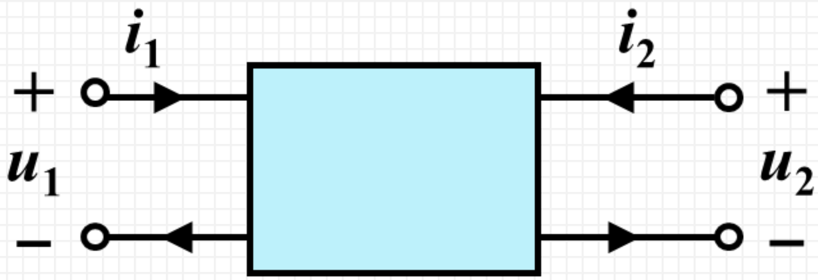
$$u_1 = f_1(i_1) \qquad u_2 = f_2(i_2)$$

或 $i_1 = g_1(u_1)$　　或 $i_2 = g_2(u_2)$



应该用两个电压电流关系方程来描述二端口网络

即：用两个物理量来表示另外两个物理量



端口物理量**4个**

i_1 i_2 u_1 u_2

共**6种**端口关系方程

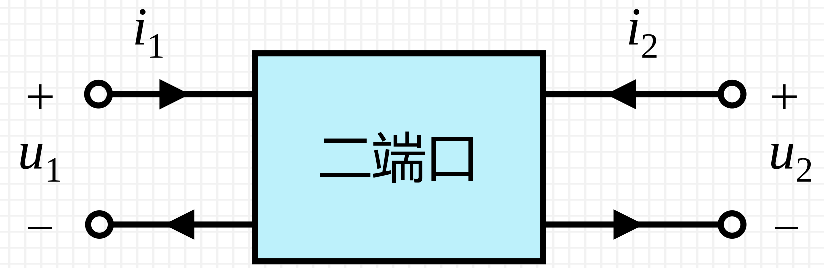
用		来表示	
u_1	u_2	i_1	i_2
i_1	i_2	u_1	u_2
i_2	u_2	i_1	u_1
i_1	u_2	i_2	u_1
i_1	u_1	i_2	u_2
i_2	u_1	i_1	u_2

(1) 用电压表示电流：*G* 参数和方程

再次强调：

二端口中没有独立源

$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$

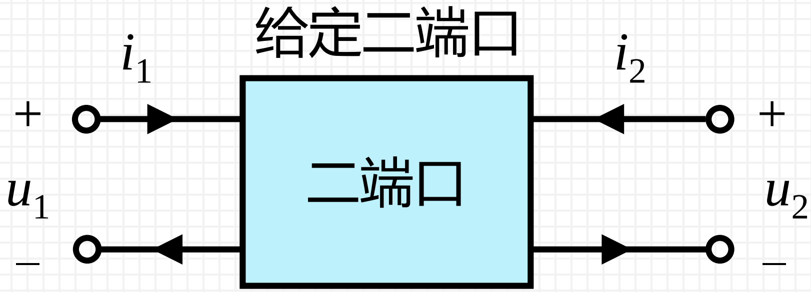


矩阵形式

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

令

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$



$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$

G参数的**实验测定**

对于某一**黑箱**二端口，
如何获得其G**参数**（不解方程）？

$$G_{11} = \left.\frac{i_1}{u_1}\right|_{u_2=0}$$ 自电导

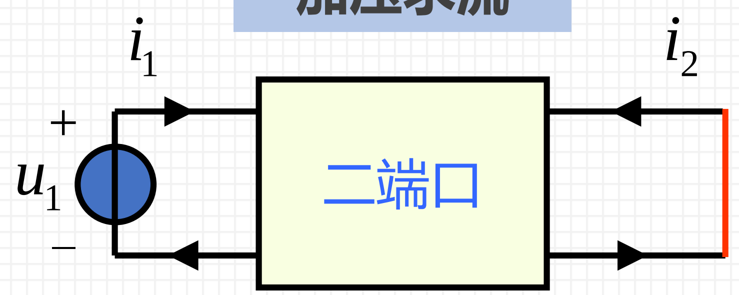
$$G_{21} = \left.\frac{i_2}{u_1}\right|_{u_2=0}$$ 转移电导

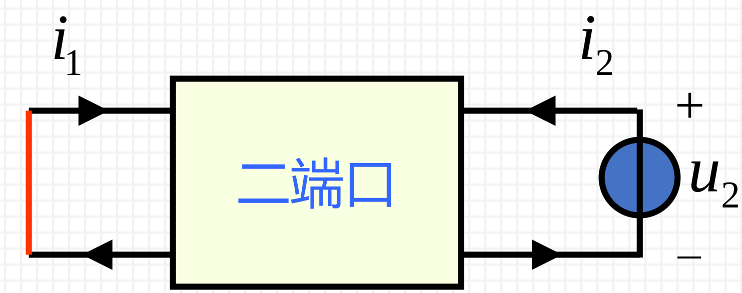
$$G_{12} = \left.\frac{i_1}{u_2}\right|_{u_1=0}$$ 转移电导

$$G_{22} = \left.\frac{i_2}{u_2}\right|_{u_1=0}$$ 自电导

类比一端口网络端口电导的求法

加压求流



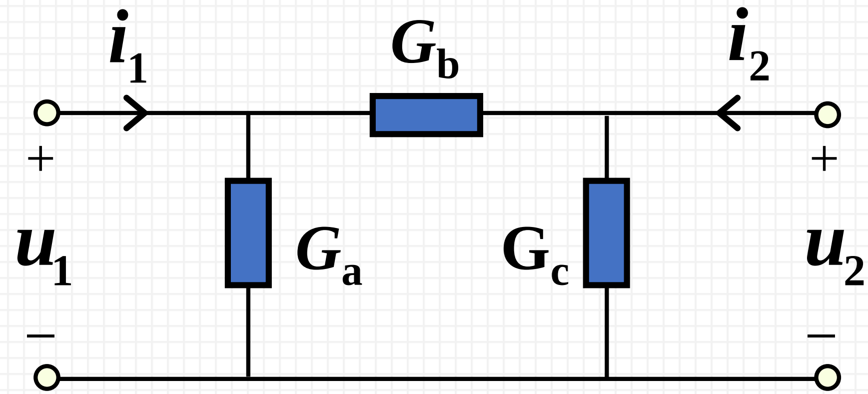


称G为**短路电导参数矩阵**

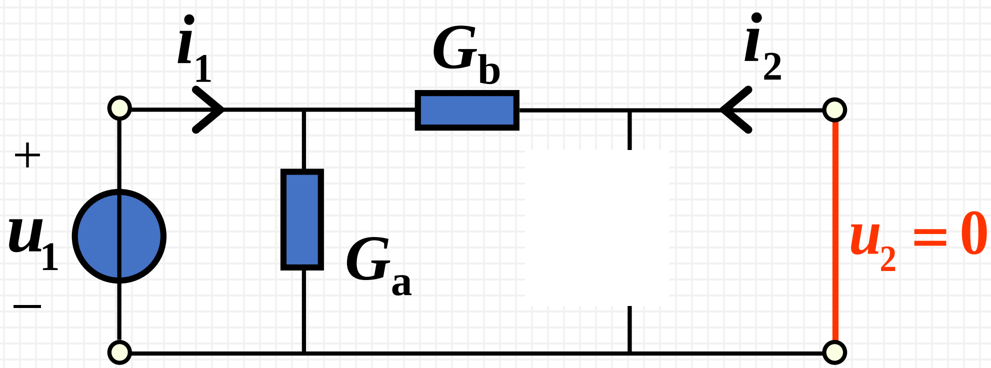
能这样做的前提是端口**能够被短路**！

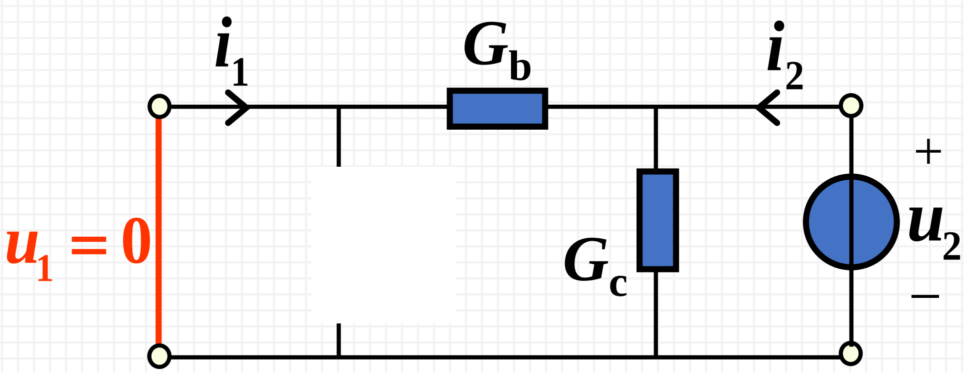
例1 求G参数。

$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$



解：法1（黑箱思路）





$$G_{11} = \left.\frac{i_1}{u_1}\right|_{u_2=0} = G_a + G_b$$

$$G_{21} = \left.\frac{i_2}{u_1}\right|_{u_2=0} = -G_b$$

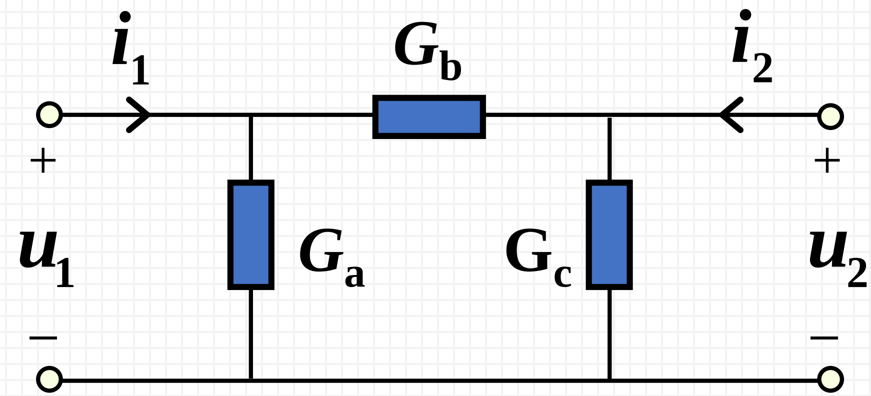
$$G_{12} = \left.\frac{i_1}{u_2}\right|_{u_1=0} = -G_b$$

$$G_{22} = \left.\frac{i_2}{u_2}\right|_{u_1=0} = G_b + G_c$$

$$\boxed{G_{12} = G_{21} = -G_b}$$

例1 求G参数。

$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$



解：法2 对**白箱**二端口，可直接求端口电压电流关系

$$i_1 = u_1G_a + (u_1 - u_2)G_b$$

$$i_2 = u_2G_c + (u_2 - u_1)G_b$$

$$G_{11} = G_a + G_b$$

$$G_{21} = -G_b$$

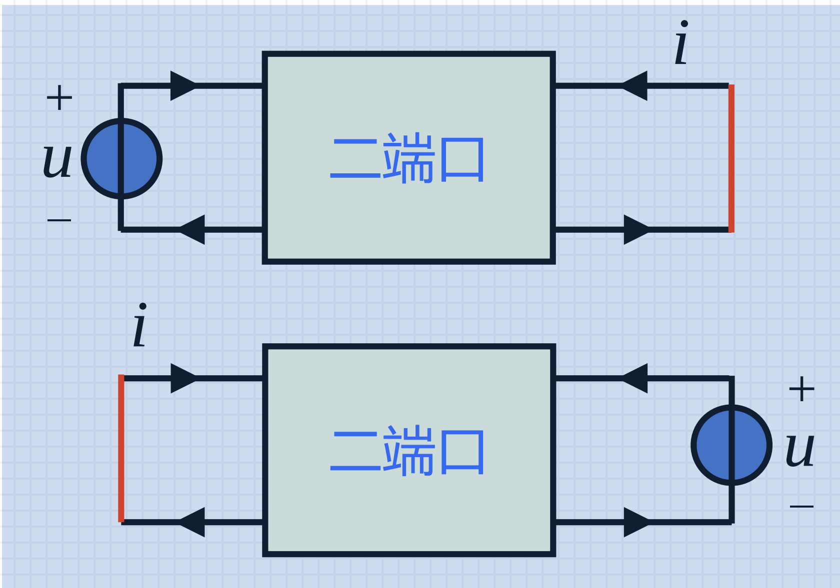
$$G_{12} = -G_b$$

$$G_{22} = G_b + G_c$$

互易二端口

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

激励无论加在哪侧，对侧产生的响应都一样



$i = G_{21} u$

$i = G_{12} u$

$G_{12} = G_{21}$

互易二端口网络四个参数中，

只有**三个**是**独立**的

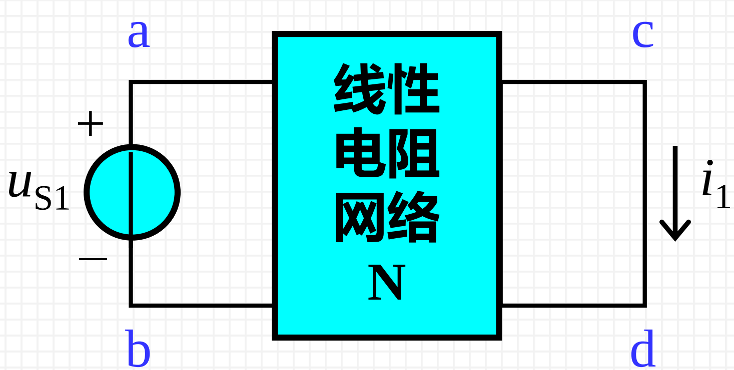
由线性电阻组成的二端口

互易定理

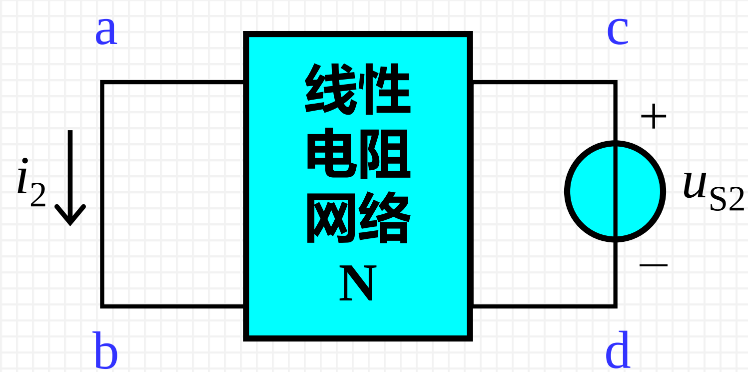
互易二端口

* 互易定理 (*Reciprocity Theorem*)

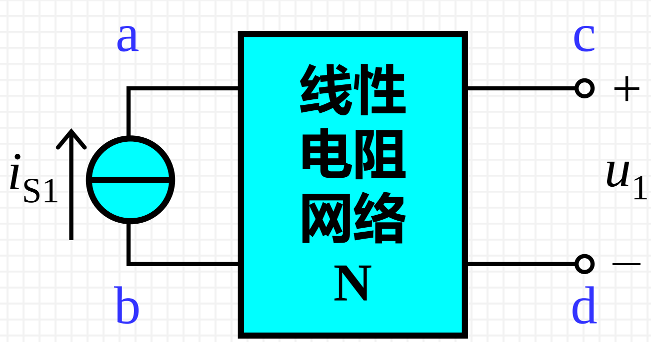
给定任一**仅由线性电阻构成的网络**…



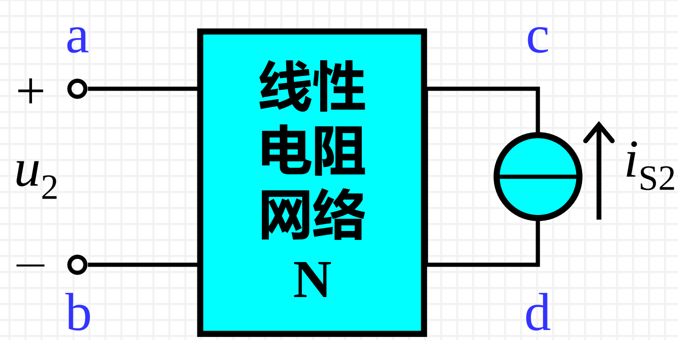
(1)



(2)



(3)

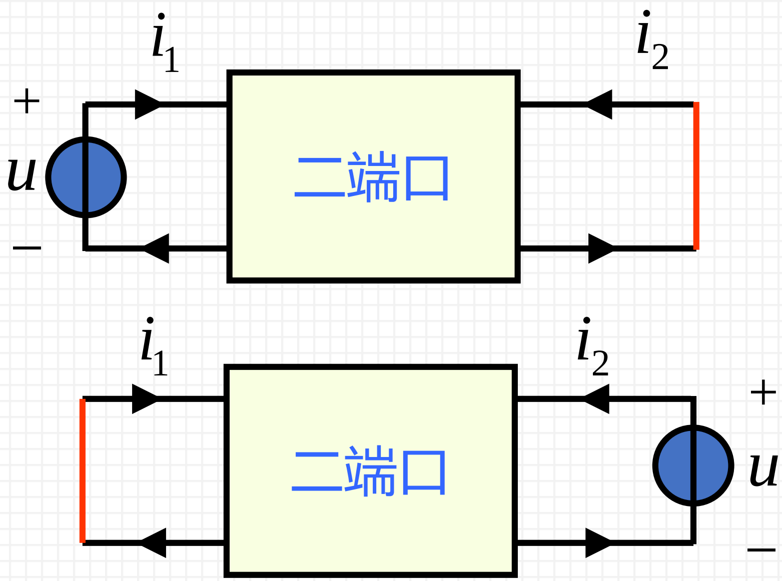


(4)

对称二端口

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

两个端口外特性(己侧/对侧)完全一样 → $G_{12}=G_{21}$, $G_{11}=G_{22}$



$i_2 = G_{21}u$

$i_1 = G_{11}u$

$i_2 = G_{22}u$

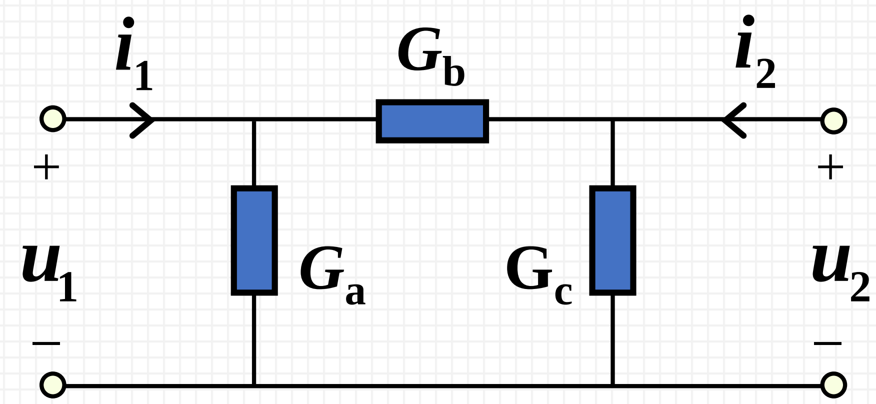
$i_1 = G_{12}u$

$G_{11}=G_{22}$

$G_{12}=G_{21}$

对称二端口只有两个参数是独立的。

$$G=\begin{bmatrix} G_a+G_b & -G_b \\ -G_b & G_b+G_c \end{bmatrix}$$



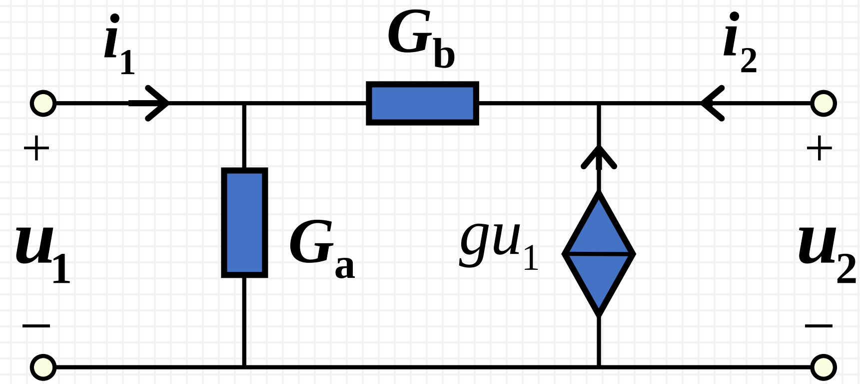
若 $G_a=G_c$

有 $G_{12}=G_{21}$，又$G_{11}=G_{22}$，为**对称二端口。**　　结构对称

结构对称的二端口 $\xrightarrow{\qquad}$ **对称二端口（电气对称）**

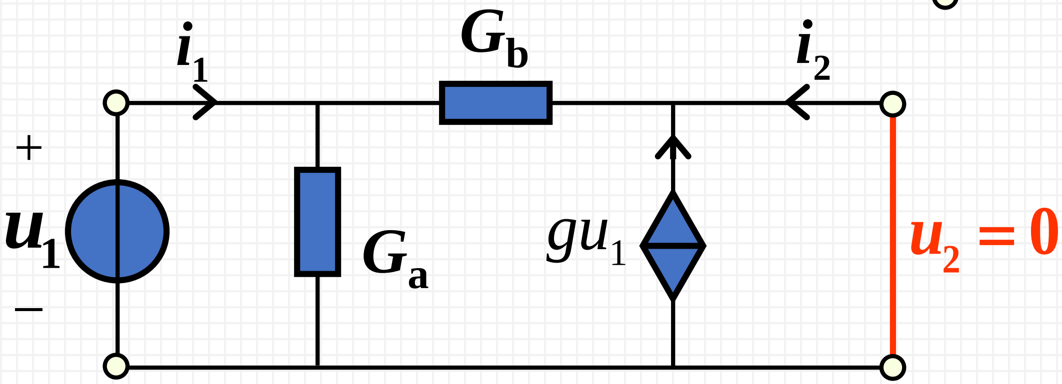
结构对称的二端口 $\cancel{\xleftarrow{\qquad}}$ **对称二端口（电气对称）**

例2 求所示电路的G参数



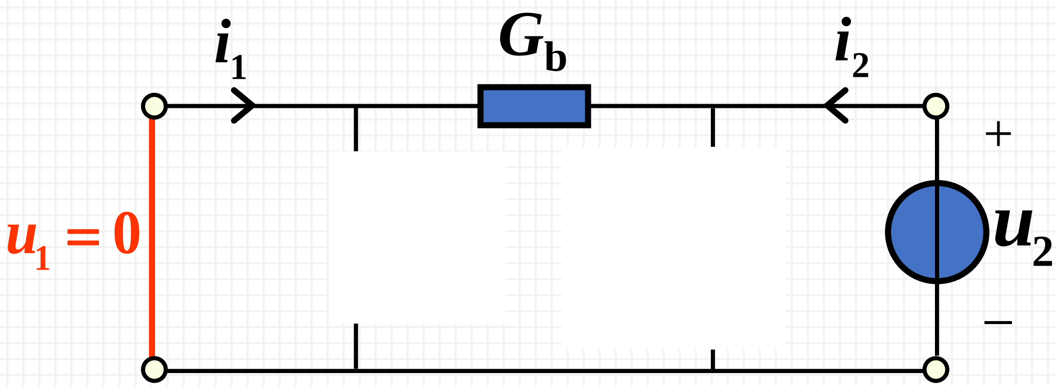
$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$

解: 法1（黑箱）



$$G_{11} = \left.\frac{i_1}{u_1}\right|_{u_2=0} = G_a + G_b$$

$$G_{21} = \left.\frac{i_2}{u_1}\right|_{u_2=0} = -G_b - g$$

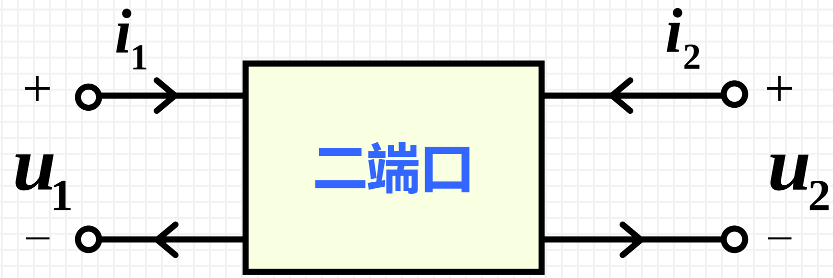


$$G_{12} = \left.\frac{i_1}{u_2}\right|_{u_1=0} = -G_b$$

$$G_{22} = \left.\frac{i_2}{u_2}\right|_{u_1=0} = G_b$$

法2（白箱） 复习时自行练习

(2) 用电流表示电压： R参数和方程



由G 参数方程 $\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$ $\xrightarrow{\text{解出}}$ u_1, u_2

即

$$\begin{cases} u_1 = \dfrac{G_{22}}{\Delta} i_1 + \dfrac{-G_{12}}{\Delta} i_2 = R_{11}i_1 + R_{12}i_2 \\ u_2 = \dfrac{-G_{21}}{\Delta} i_1 + \dfrac{G_{11}}{\Delta} i_2 = R_{21}i_1 + R_{22}i_2 \end{cases}$$

其中 $\Delta = G_{11}G_{22} - G_{12}G_{21} \neq 0$ **前提：G 非奇异**

其矩阵形式为

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

R 参数的实验测定（**黑箱**）

$$R_{11} = \frac{u_1}{i_1}\bigg|_{i_2=0} \qquad R_{12} = \frac{u_1}{i_2}\bigg|_{i_1=0}$$

$$R_{21} = \frac{u_2}{i_1}\bigg|_{i_2=0} \qquad R_{22} = \frac{u_2}{i_2}\bigg|_{i_1=0}$$

称 R 为**开路电阻参数矩阵**

能这样做的前提是**端口能够被开路**！

$$\begin{cases} u_1 = \dfrac{G_{22}}{\Delta} i_1 + \dfrac{-G_{12}}{\Delta} i_2 = R_{11} i_1 + R_{12} i_2 \\ u_2 = \dfrac{-G_{21}}{\Delta} i_1 + \dfrac{G_{11}}{\Delta} i_2 = R_{21} i_1 + R_{22} i_2 \end{cases}$$

互易二端口 $G_{12} = G_{21}$ → $R_{12} = R_{21}$

对称二端口

$G_{12} = G_{21}$

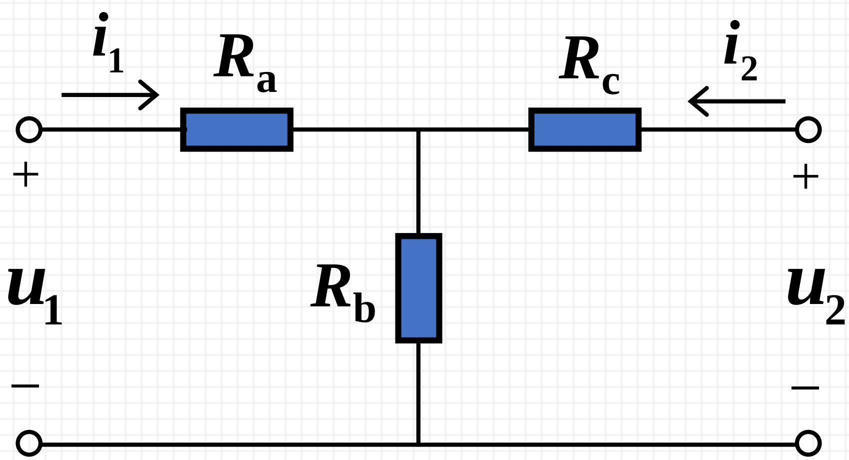
$G_{11} = G_{22}$

→

$R_{11} = R_{22}$

$R_{12} = R_{21}$

例3 求所示电路的 R 参数



$$u_1 = R_{11}i_1 + R_{12}i_2$$
$$u_2 = R_{21}i_1 + R_{22}i_2$$

法1（黑箱）

实验测定。自行完成

法2（白箱）

端口电压电流关系

$$u_1 = i_1 R_a + (i_1 + i_2) R_b$$

$$u_2 = i_2 R_c + (i_1 + i_2) R_b$$

互易二端口

(3) 用输出表示输入：T 参数和方程

如何用 u_2 和 i_2 来表示 u_1 和 i_1?

$$i_1 = G_{11}u_1 + G_{12}u_2 \qquad (1)$$

$$i_2 = G_{21}u_1 + G_{22}u_2 \qquad (2)$$

由(1)和(2)得:

$$u_1 = -\frac{G_{22}}{G_{21}}u_2 + \frac{1}{G_{21}}i_2 \qquad i_1 = \left(G_{12} - \frac{G_{11}G_{22}}{G_{21}}\right)u_2 + \frac{G_{11}}{G_{21}}i_2$$

令 $T_{11} = -\frac{G_{22}}{G_{21}}$ $\quad T_{12} = \frac{-1}{G_{21}}$ $\quad T_{21} = \frac{G_{12}G_{21} - G_{11}G_{22}}{G_{21}}$ $\quad T_{22} = -\frac{G_{11}}{G_{21}}$

即 $$u_1 = T_{11}u_2 - T_{12}i_2$$

$$i_1 = T_{21}u_2 - T_{22}i_2$$

(注意**负**号)

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \qquad \begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = T\begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$$

称为**传输参数 (T) 矩阵**

如何考虑 T 参数的**互易和对称**条件?　　　　基本思路：回归 G 参数

$$u_1 = T_{11}u_2 - T_{12}i_2 \qquad i_1 = T_{21}u_2 - T_{22}i_2$$

$$i_2 = -\frac{1}{T_{12}}u_1 + \frac{T_{11}}{T_{12}}u_2$$

$$= T_{21}u_2 + \frac{T_{22}}{T_{12}}u_1 - \frac{T_{11}T_{22}}{T_{12}}u_2$$

G_{21}　　G_{22}

$$i_1 = \frac{T_{22}}{T_{12}}u_1 + \frac{T_{12}T_{21} - T_{11}T_{22}}{T_{12}}u_2$$

G_{11}　　G_{12}

互易二端口　　$T_{11}T_{22} - T_{12}T_{21} = 1$

对称二端口　　$T_{11}T_{22} - T_{12}T_{21} = 1$　　$T_{11} = T_{22}$

T 参数的实验测定（黑箱）

$$u_1 = T_{11}u_2 - T_{12}i_2$$
$$i_1 = T_{21}u_2 - T_{22}i_2$$

$$T_{11} = \left.\frac{u_1}{u_2}\right|_{i_2=0} \qquad T_{21} = \left.\frac{i_1}{u_2}\right|_{i_2=0}$$

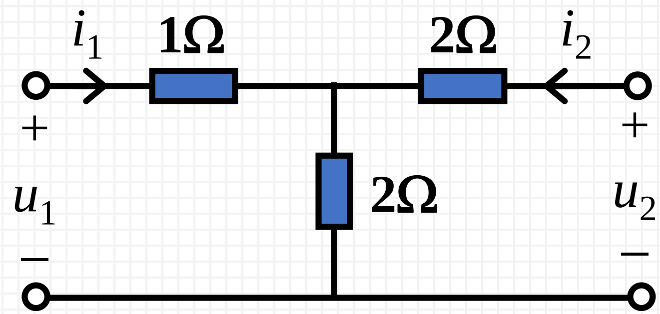
开路参数

$$T_{12} = \left.\frac{u_1}{-i_2}\right|_{u_2=0} \qquad T_{22} = \left.\frac{i_1}{-i_2}\right|_{u_2=0}$$

短路参数

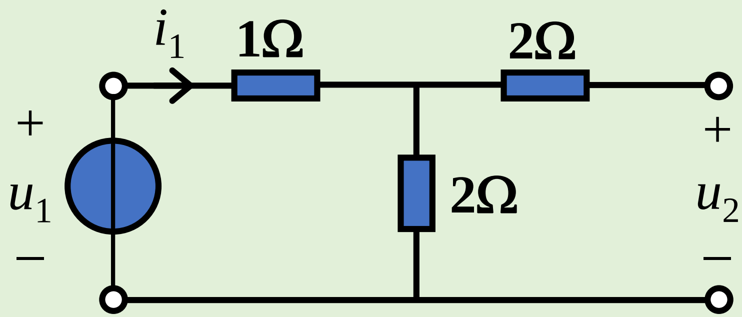
能这样做的**前提**是端口2能够**被开路和短路**！

例 求T参数



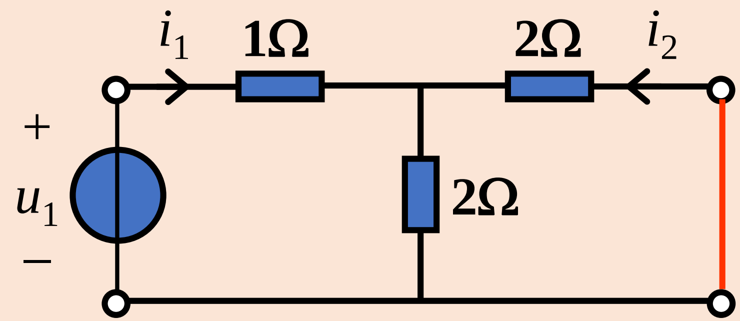
$$u_1 = T_{11}u_2 - T_{12}i_2$$
$$i_1 = T_{21}u_2 - T_{22}i_2$$

法1（黑箱）



$$T_{11} = \frac{u_1}{u_2}\bigg|_{i_2=0} = \frac{1+2}{2} = 1.5$$

$$T_{21} = \frac{i_1}{u_2}\bigg|_{i_2=0} = 0.5\ \mathrm{S}$$



$$T_{12} = \frac{u_1}{-i_2}\bigg|_{u_2=0} = \frac{i_1[1+(2//2)]}{0.5i_1} = 4\ \Omega$$

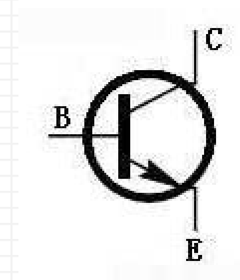
$$T_{22} = \frac{i_1}{-i_2}\bigg|_{u_2=0} = \frac{i_1}{0.5i_1} = 2$$

法2 先写出 G 或 R 参数，再解出 T 参数

法3（白箱） 根据 KCL、KVL 列方程并整理

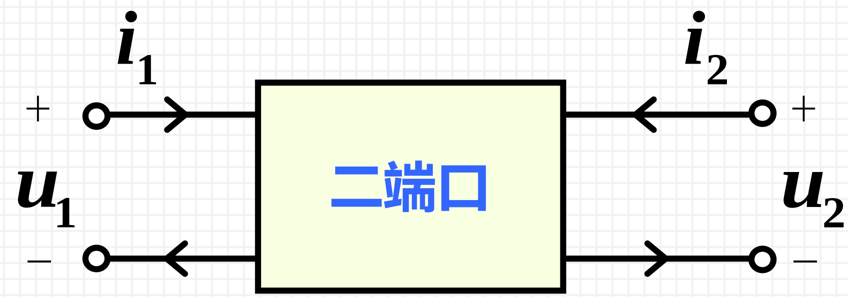
(4) *H* 参数和方程

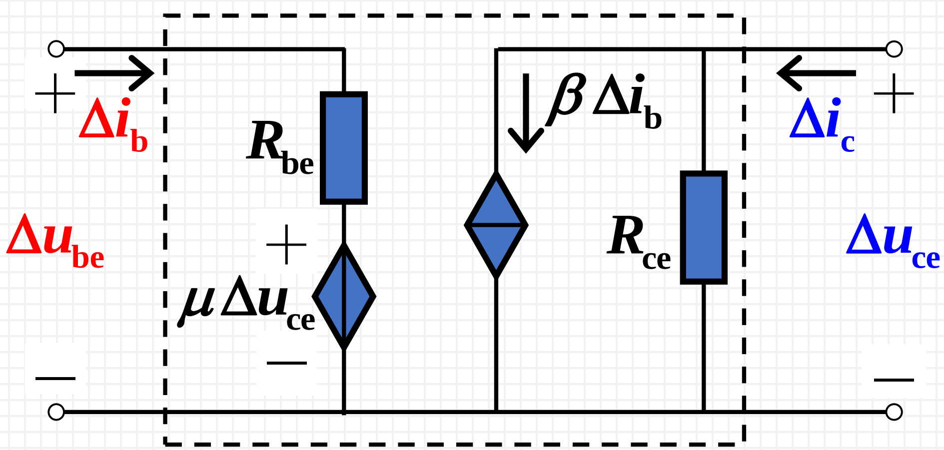
H 参数也称为**混合参数**，常用于**双极型晶体管**等效电路。



H 参数方程

$$u_1 = H_{11}i_1 + H_{12}u_2$$
$$i_2 = H_{21}i_1 + H_{22}u_2$$



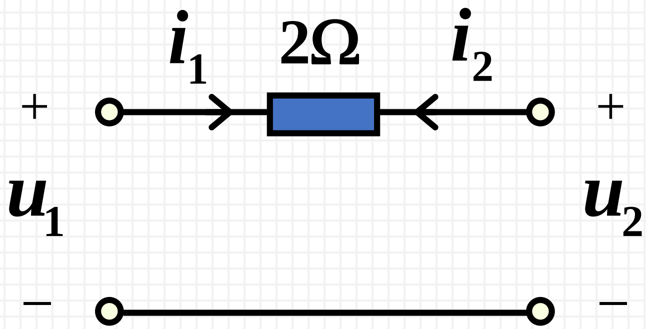


$$\Delta u_{be} = R_{be}\Delta i_b + \mu \Delta u_{ce}$$
$$\Delta i_c = \beta \Delta i_b + \frac{\Delta u_{ce}}{R_{ce}}$$

为什么用这么多参数表示?

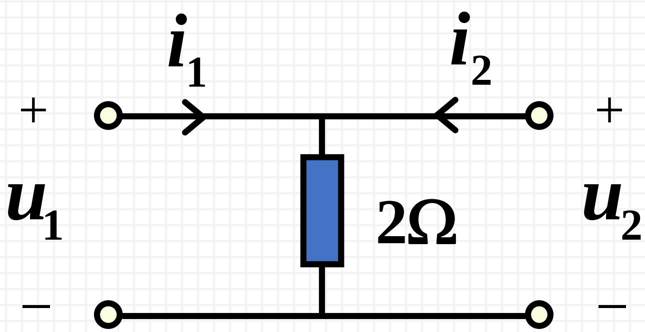
（1）为描述电路**方便**，测量方便(如H)。

（2）有些电路**只存在某几种参数**。



$$G=\begin{bmatrix}0.5 & -0.5\\ -0.5 & 0.5\end{bmatrix}\mathrm{S}$$

R 参数不存在



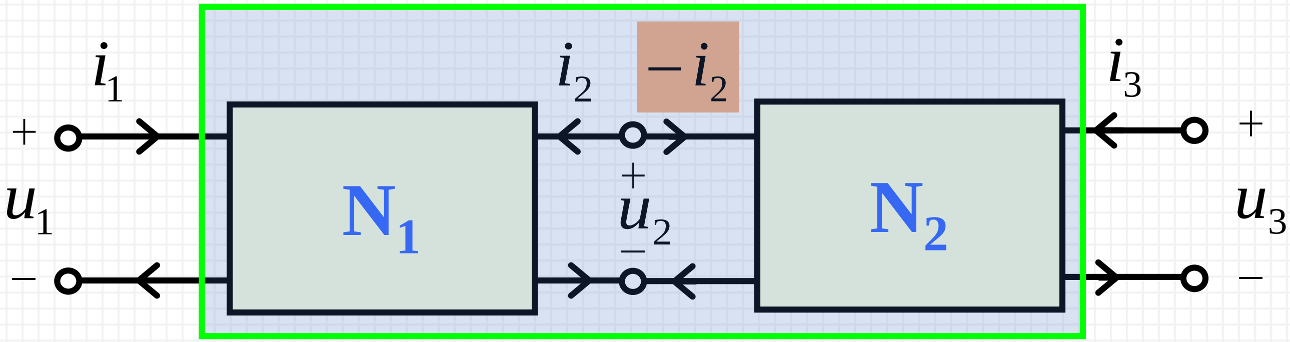
$$R=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\Omega$$

G 参数不存在

（3）有些电路**不能**端口**短路** / **开路**（黑箱法）。

$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = T \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$$

为什么T参数会有这么怪怪的定义?



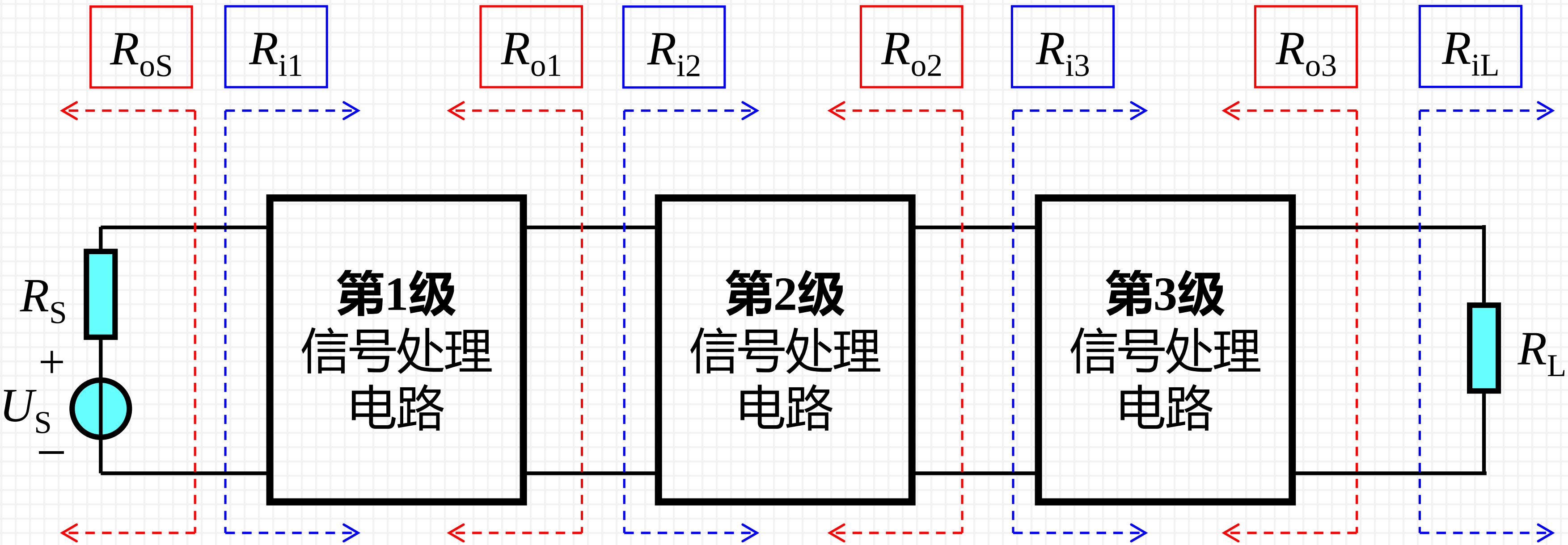
$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = T_1 \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix} = T_2 \begin{bmatrix} u_3 \\ -i_3 \end{bmatrix}$$

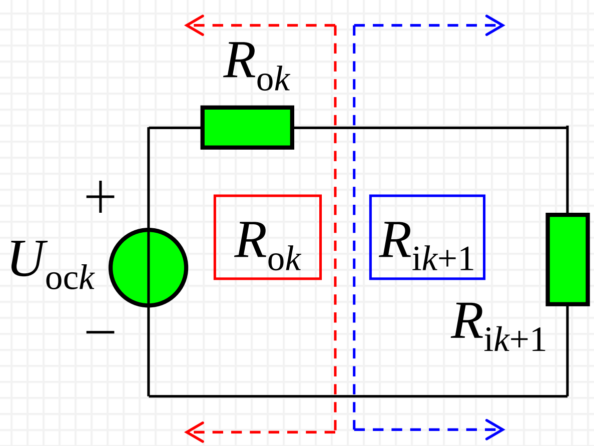
$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = T_1 T_2 \begin{bmatrix} u_3 \\ -i_3 \end{bmatrix}$$

级联

T 参数的定义方式，确保**级联**对外的 T 参数容易获取

级联的实际应用





每一级信号处理电路的$\boldsymbol{R_i}$为

从信号**输入端**向输出端看的**戴维南电阻**

每一级信号处理电路的$\boldsymbol{R_o}$为

从信号**输出端**向输入端看的**戴维南电阻**

R_i越大越好 ⟶ 从前一级信号处理电路获得的电压大 ⟶ 对前级影响小

R_o越小越好 ⟶ 给后一级信号处理电路的电压大 ⟶ 带载能力强

3 二端口的等效电路

❖ 两个二端口网络等效：

是指对外电路而言，端口的电压、电流关系相同。

❖ 求等效电路即根据给定的参数方程确定电路结构和参数。

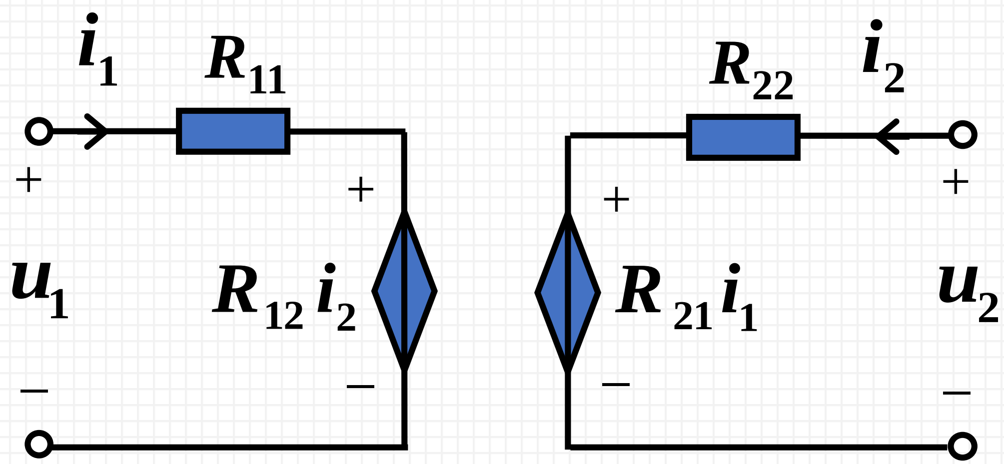
反向工程：

测量端口电压 - 电流关系 → 构造电路满足端口电压 - 电流关系

(1) 由R参数方程画等效电路

$$u_1 = R_{11}i_1 + R_{12}i_2$$

$$u_2 = R_{21}i_1 + R_{22}i_2$$



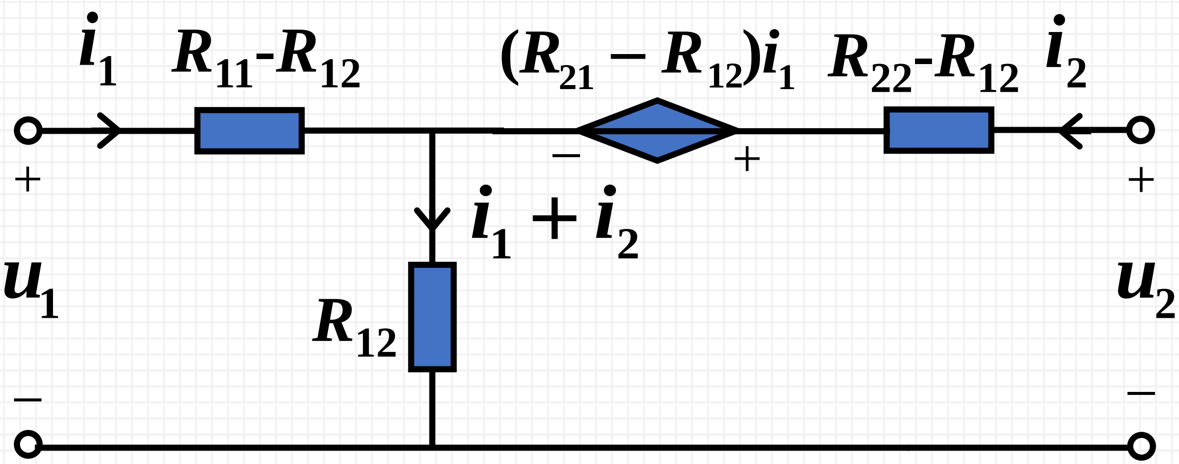
如果只用一个受控源

$$u_1 = R_{11}i_1 + R_{12}i_2$$
$$u_2 = R_{21}i_1 + R_{22}i_2$$

原方程改写为

$$u_1 = R_{11}i_1 + R_{12}i_2 + R_{12}i_1 - R_{12}i_1$$

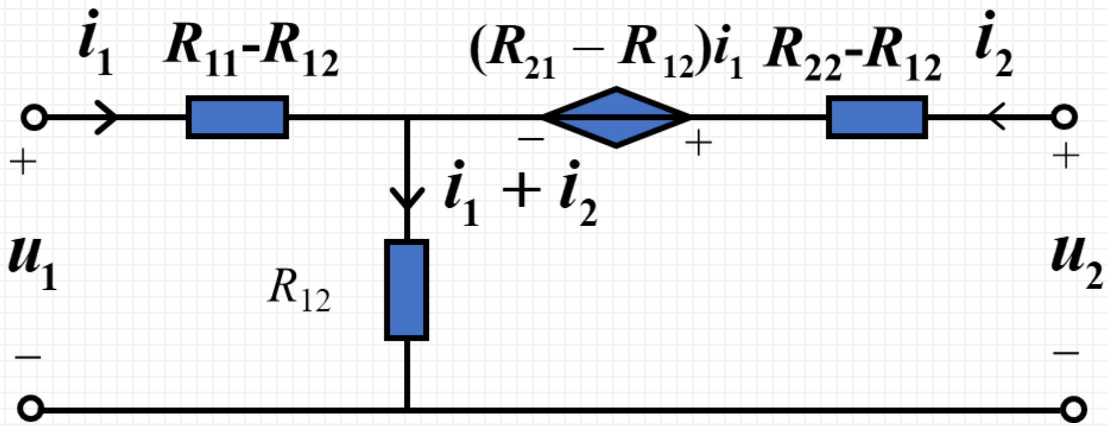
$$u_2 = R_{21}i_1 + R_{22}i_2 + R_{12}i_1 - R_{12}i_1 + R_{12}i_2 - R_{12}i_2$$



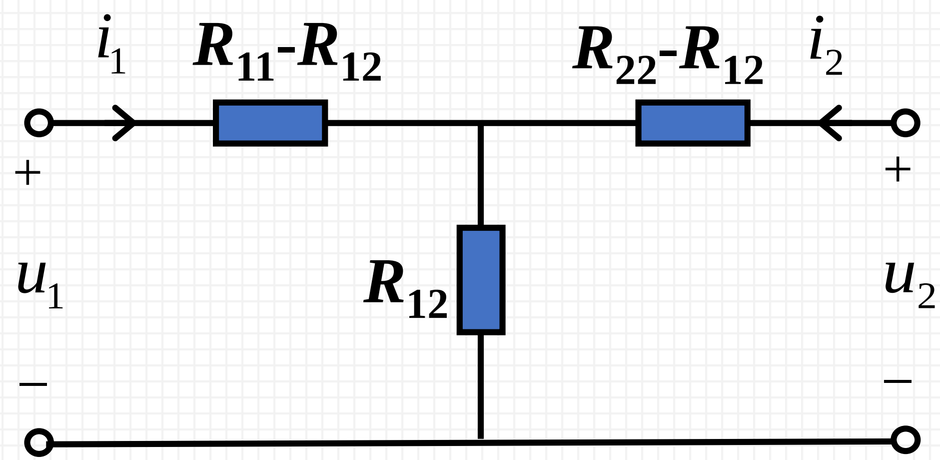
电路综合

同一个参数方程，可以画出结构**不同**的等效电路。 **等效电路不唯一。**

能不用受控源吗？为什么



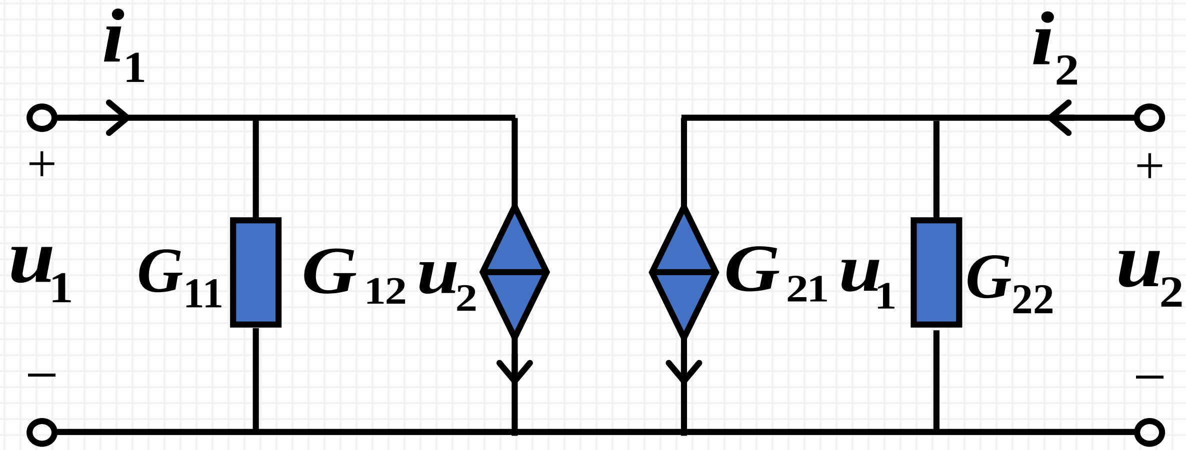
互易网络 $R_{12}=R_{21}$



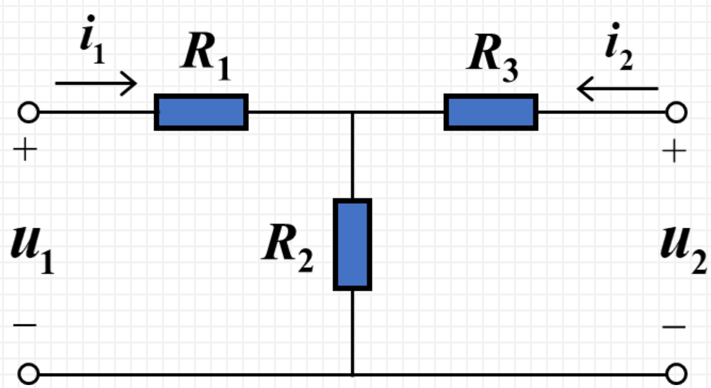
网络**对称**（$R_{11}=R_{22}$）则等效电路也对称

(2) 由G参数方程画等效电路（2个受控源）

$$\begin{cases} i_1 = G_{11}u_1 + G_{12}u_2 \\ i_2 = G_{21}u_1 + G_{22}u_2 \end{cases}$$



(3) T 参数的等效电路？教材例2.7.6



若二端口**互易**

$$R_2 = \frac{1}{T_{21}}$$

$$R_1 = \frac{T_{11} - 1}{T_{21}} \qquad R_3 = \frac{T_{22} - 1}{T_{21}}$$

记住这个有时候能占便宜!

小结

1) 等效只对两个端口的电压，电流关系成立。
   对端口内电路不一定成立。

2) 若网络对称则等效电路也对称。

3) 给定参数矩阵求等效电路。
   互易二端口: T 型或 Π 型电阻电路等效。
   非互易二端口：至少一个受控源。