

清华大学2021春季学期

电路原理C

第11讲 二阶动态电路

内容

1 **RLC**串联二阶电路

2 **RLC**并联二阶电路

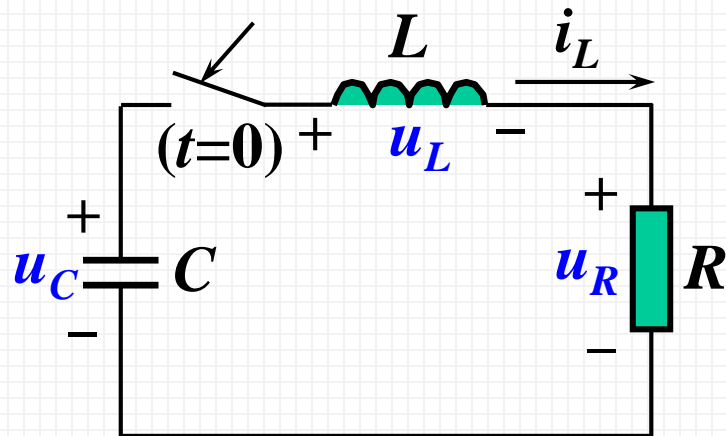
3 二阶电路的直觉解法

4 二阶电路的应用

1、RLC串联二阶电路

零输入RLC串联

(1) 列方程



i_L 代入上式

$$\begin{cases} u_C = L \frac{di_L}{dt} + Ri_L \\ i_L = -C \frac{du_C}{dt} \end{cases}$$

u_C 代入下式

$$\frac{d^2 u_C}{dt^2} + \frac{R}{L} \frac{du_C}{dt} + \frac{1}{LC} u_C = 0$$

衰减系数 α

2α

ω_0^2

自由振荡角频率/
自然角频率 ω_0

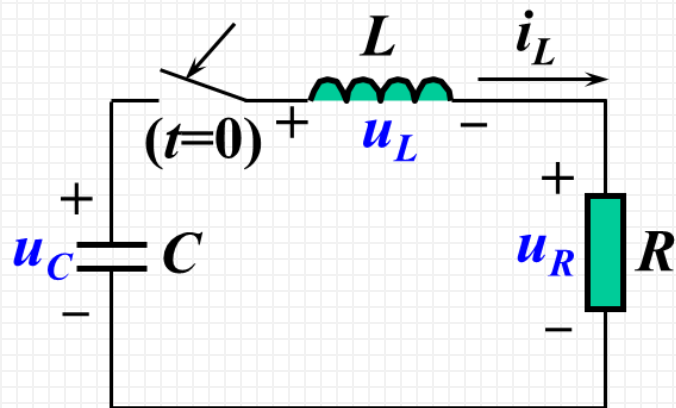
$$\frac{d^2 i_L}{dt^2} + \frac{R}{L} \frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

课外练习：

以 u_R 、 u_L 为变量列写微分方程。

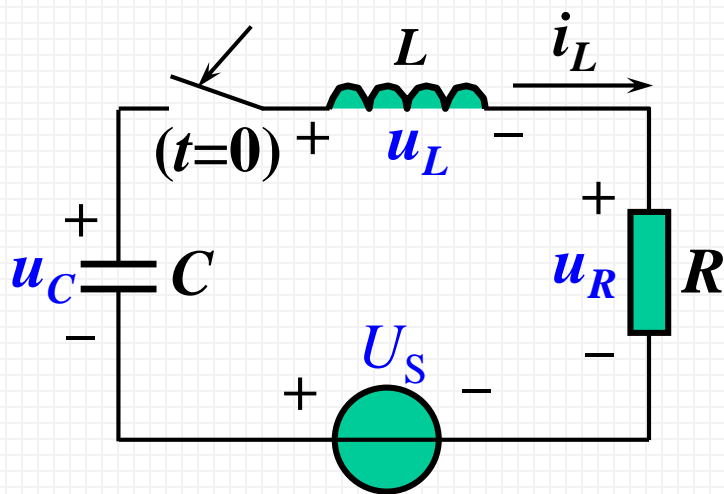
结果：特征方程都一样

零输入RLC串联



以不同的变量列写方程，
得到的特征方程相同。

有输入RLC串联



$$\frac{d^2 i_L}{dt^2} + 2\alpha \frac{di_L}{dt} + \omega_0^2 i_L = 0$$

$$\frac{d^2 u_C}{dt^2} + 2\alpha \frac{du_C}{dt} + \omega_0^2 u_C = 0$$

$$\frac{d^2 u_L}{dt^2} + 2\alpha \frac{du_L}{dt} + \omega_0^2 u_L = 0$$

$$\frac{d^2 u_R}{dt^2} + 2\alpha \frac{du_R}{dt} + \omega_0^2 u_R = 0$$

可先列写零输入电路方程，
求得特征根。

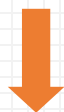
有独立源电路和零输入
电路的特征方程相同。

$$\frac{d^2 i_L}{dt^2} + 2\alpha \frac{di_L}{dt} + \omega_0^2 i_L = 0$$

(2) 求自由分量

*LC*参数不变, 随*R*增加, 状态怎么变?

$$\frac{d^2 u_C}{dt^2} + 2\alpha \frac{du_C}{dt} + \omega_0^2 u_C = 0$$



$$2\alpha = \frac{R}{L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

$$\alpha^2 > \omega_0^2$$

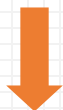


$$p_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

过阻尼

$$u_C = Ae^{p_1 t} + Be^{p_2 t}$$

$$\alpha^2 = \omega_0^2$$

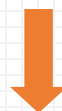


$$p_1 = p_2 = -\alpha$$

临界阻尼

$$u_C = Ae^{p_1 t} + Bte^{p_2 t}$$

$$\alpha^2 < \omega_0^2$$



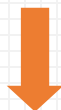
$$p_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

欠阻尼

$$u_C = Ke^{-\alpha t} \sin(\omega_d t + \theta)$$

$$u_C = K \sin(\omega_0 t + \theta)$$

$$\alpha = 0$$



$$p_{1,2} = \pm j\omega_0$$

无阻尼

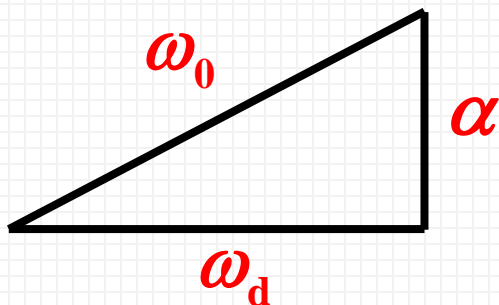
有关RLC串联欠阻尼3个参数的讨论

$$\frac{d^2 u_C}{dt^2} + \underbrace{\frac{R}{L}}_{2\alpha} \frac{du_C}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} u_C = 0$$

衰减系数 α

自由振荡角频率/
自然角频率 ω_0

$$\omega_0^2 = \omega_d^2 + \alpha^2$$



$$\frac{d^2 u_C}{dt^2} + 2\alpha \frac{du_C}{dt} + \omega_0^2 u_C = 0$$

$$b^2 - 4ac < 0$$



$$p_{1,2} = \frac{-2\alpha \pm j2\sqrt{\omega_0^2 - \alpha^2}}{2}$$

$$= -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

$$= -\alpha \pm j\omega_d$$

衰减振荡角频率 ω_d



数值例子 R 分别为 5Ω 、 4Ω 、 1Ω 、 0Ω 时求 $u_C(t)$ 、 $i_L(t)$ ， $t \geq 0$

$$\frac{d^2 u_C}{dt^2} + \frac{R}{L} \frac{du_C}{dt} + \frac{1}{LC} u_C = 0$$



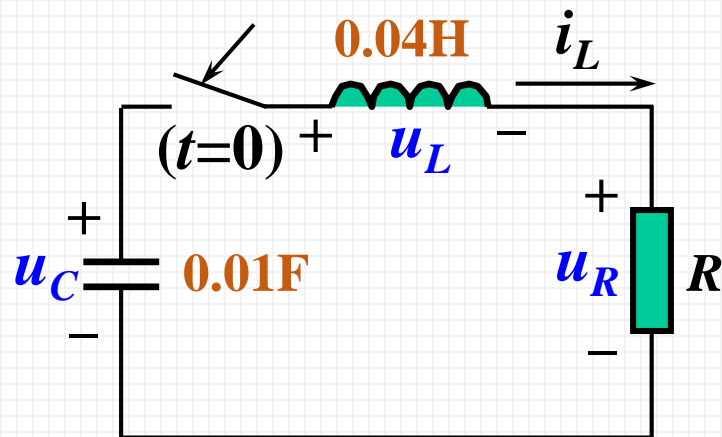
$$\frac{d^2 u_C}{dt^2} + 25R \frac{du_C}{dt} + 2500 u_C = 0$$



特征方程

$$p^2 + 25Rp + 2500 = 0$$

$$b^2 - 4ac = 625R^2 - 10000$$



$$u_C(0^-) = 3V, \quad i_L(0^-) = 0$$

$$R=5\Omega \rightarrow \begin{cases} b^2 - 4ac = 5625 > 0 \\ p_1 = -25 \quad p_2 = -100 \\ u_C(t) = A_1 e^{-25t} + A_2 e^{-100t} \end{cases}$$

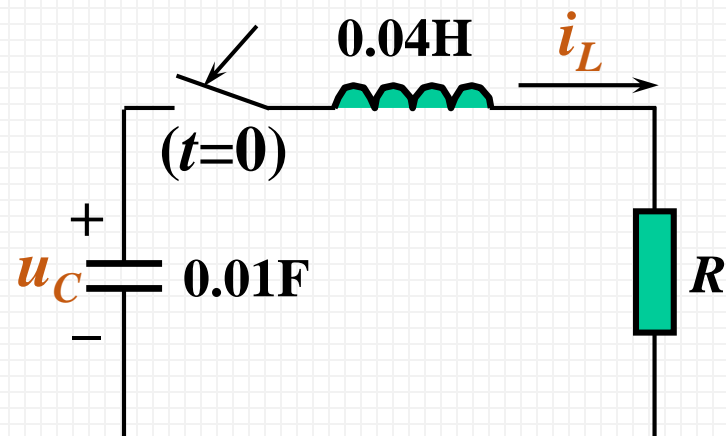
$$R=4\Omega \rightarrow \begin{cases} b^2 - 4ac = 0 \\ p_1 = p_2 = -50 \\ u_C(t) = A_1 e^{-50t} + A_2 t e^{-50t} \end{cases}$$

$$R=1\Omega \rightarrow \begin{cases} b^2 - 4ac = -9375 < 0 \\ p_{1,2} = -12.5 \pm j48.4 \\ u_C(t) = K e^{-12.5t} \sin(48.4t + \theta) \end{cases}$$

$$R=0\Omega \rightarrow \begin{cases} p_{1,2} = \pm j50 \\ u_C(t) = K \sin(50t + \theta) \end{cases}$$

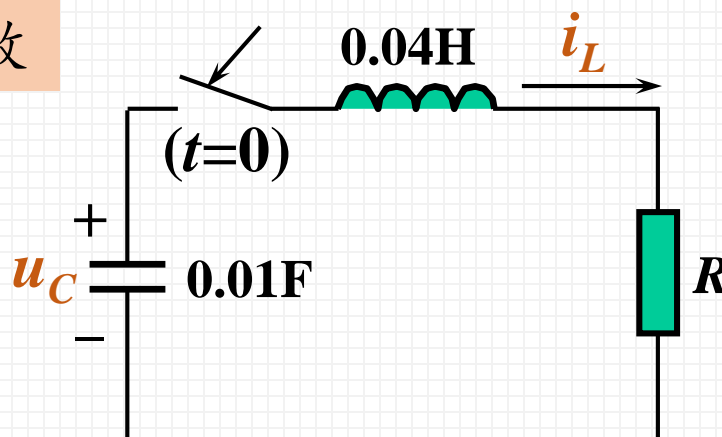
$$p^2 + 25Rp + 2500 = 0$$

$$b^2 - 4ac = 625R^2 - 10000$$



$$u_C(0^-) = 3V, i_L(0^-) = 0$$

(3) 用初值确定待定系数



$$u_C(0^-) = 3\text{V}$$

$$i_L(0^-) = 0$$

$$\left. \frac{du_C}{dt} \right|_{0^+} = 0$$

$$R=5\Omega \rightarrow \begin{cases} u_C(t) = A_1 e^{-25t} + A_2 e^{-100t} \\ A_1 + A_2 = 3 \\ -25A_1 - 100A_2 = 0 \end{cases} \rightarrow A_1 = 4 \quad A_2 = -1$$

$$u_C(t) = 4e^{-25t} - e^{-100t} \text{ V} \quad t > 0^+$$

如何直接求 i_L ?

$$C \frac{du_C}{dt} = -i_L$$

$$i_L(t) = e^{-25t} - e^{-100t} \text{ A} \quad t > 0^+$$

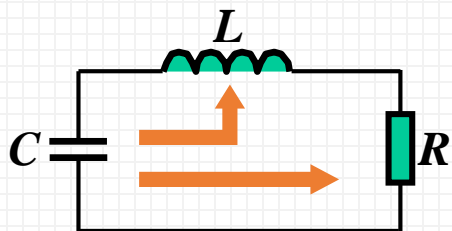
波形图和能量转换关系

$$R=5\Omega$$

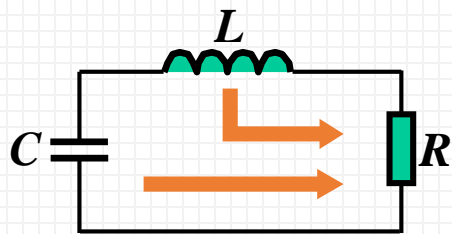
$$u_C(t) = 4e^{-25t} - e^{-100t} \text{ V} \quad t > 0^+$$

$$i_L(t) = e^{-25t} - e^{-100t} \text{ A} \quad t > 0^+$$

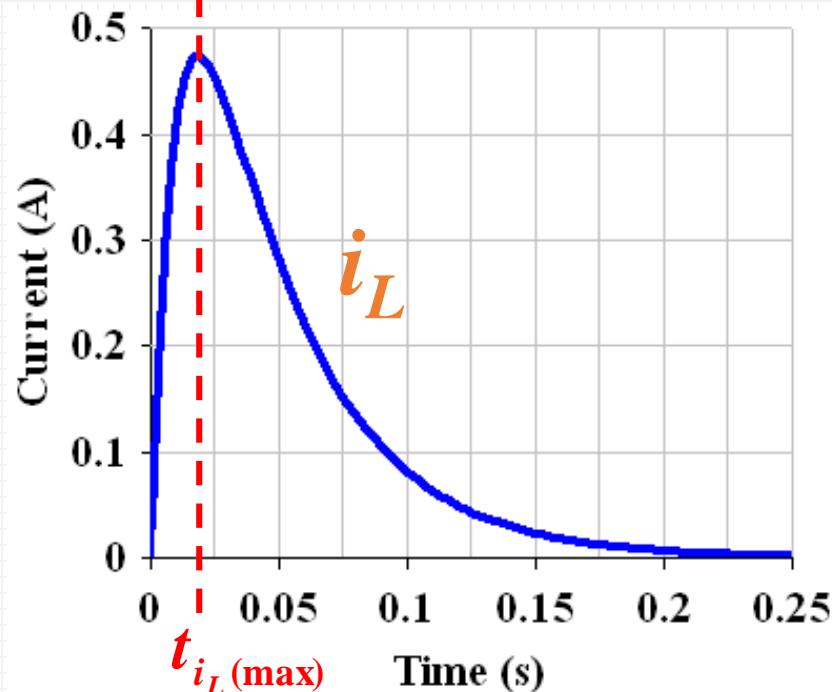
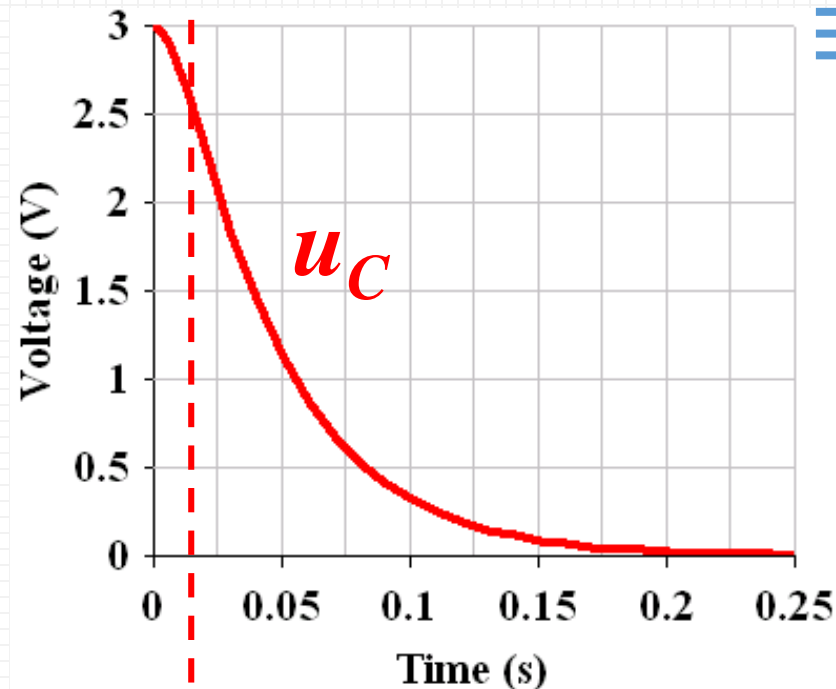
$0 < t < t_{i_L(\max)}$ u_C 减小, i_L 增大。

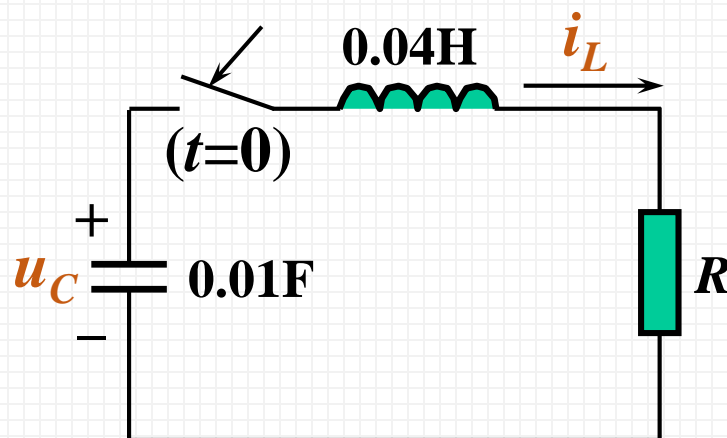


$t > t_{i_L(\max)}$ u_C 减小, i_L 减小。



非振荡放电 过阻尼





$$u_C(0) = 3\text{V}$$

$$\left. \frac{du_C}{dt} \right|_{t=0^+} = 0$$

$$R = 4\Omega$$



$$\begin{cases} u_C(t) = A_1 e^{-50t} + A_2 t e^{-50t} \\ A_1 = 3 \\ -50A_1 + A_2 = 0 \end{cases} \quad \rightarrow \quad A_1 = 3 \quad A_2 = 150$$

$$u_C(t) = 3e^{-50t} (1 + 50t) \text{V} \quad t > 0^+$$

$$C \frac{du_C}{dt} = -i_L$$

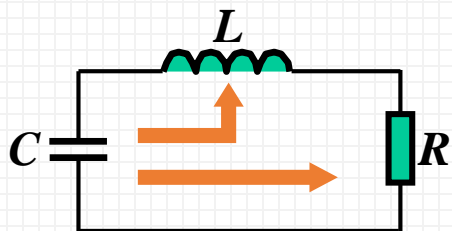
$$i_L(t) = 75te^{-50t} \text{A} \quad t > 0^+$$

$$R=4\Omega$$

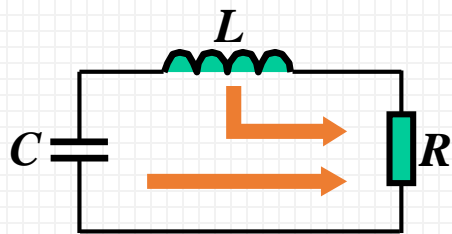
$$u_C(t) = 3e^{-50t}(1+50t)\text{V} \quad t > 0^+$$

$$i_L(t) = 75te^{-50t}\text{A} \quad t > 0^+$$

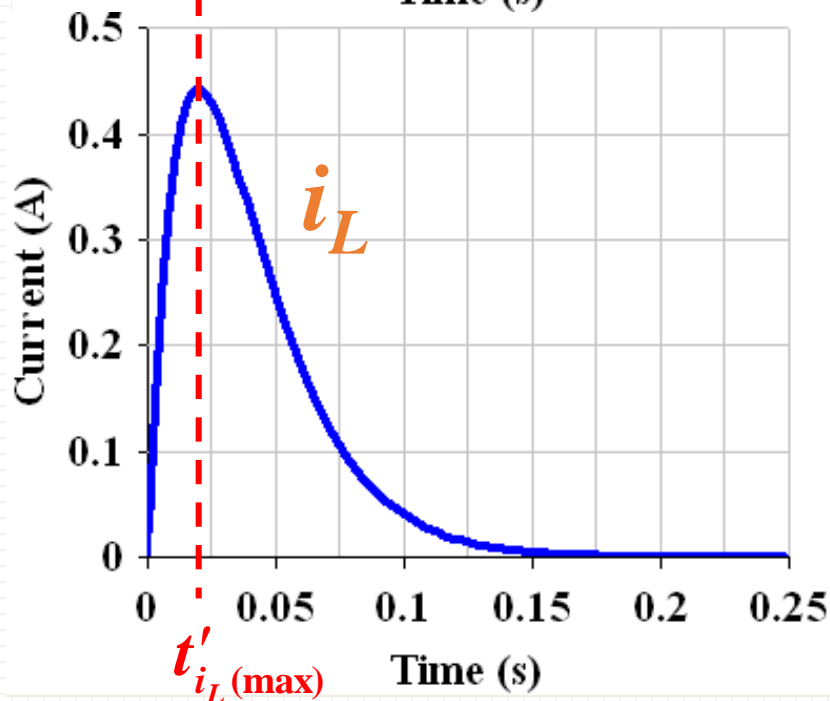
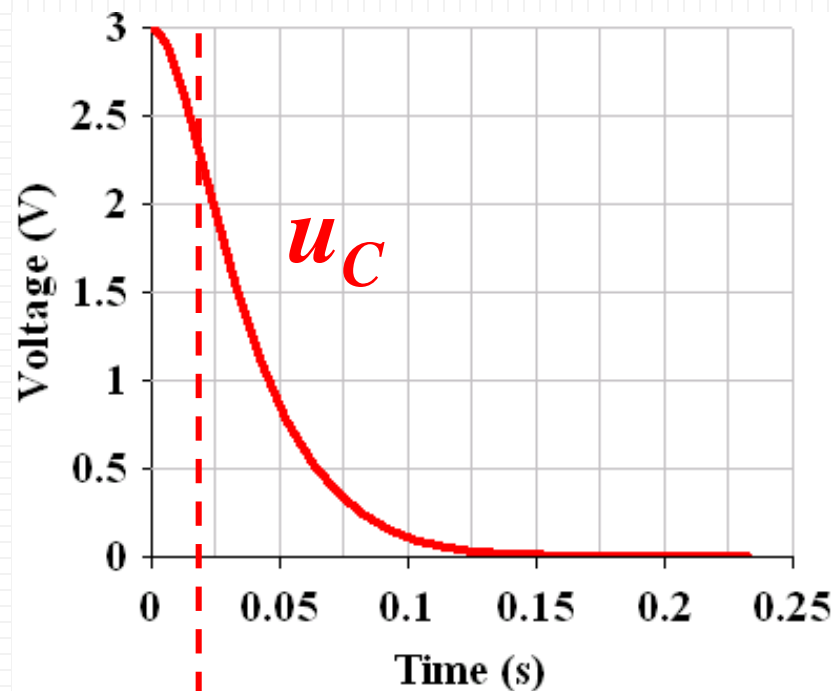
$0 < t < t'_{i_L(\max)}$ u_C 减小, i_L 增大。

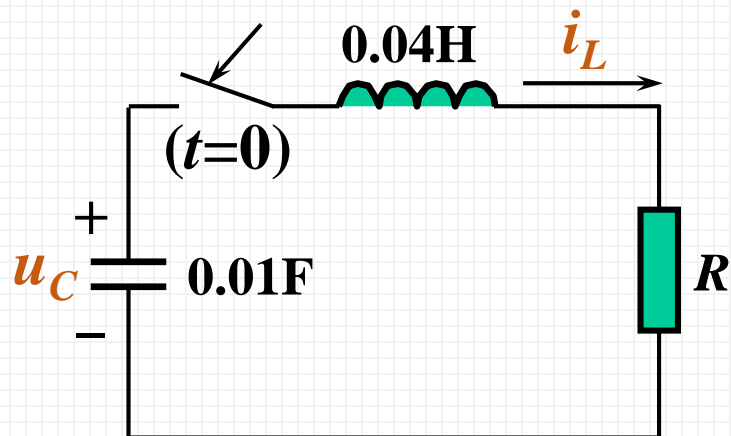


$t > t'_{i_L(\max)}$ u_C 减小, i_L 减小。



非振荡放电 临界阻尼





$$u_C(0) = 3\text{V}$$

$$\left. \frac{du_C}{dt} \right|_{t=0^+} = 0$$

$$R = 1\Omega \rightarrow \begin{cases} u_C(t) = Ke^{-12.5t} \sin(48.4t + \theta) \\ K \sin \theta = 3 \\ -12.5K \sin \theta + 48.4K \cos \theta = 0 \end{cases}$$

$$\rightarrow K = 3.1 \quad \theta = 75.5^\circ$$

$$u_C(t) = 3.10e^{-12.5t} \sin(48.4t + 75.5^\circ) \text{V} \quad t > 0^+$$

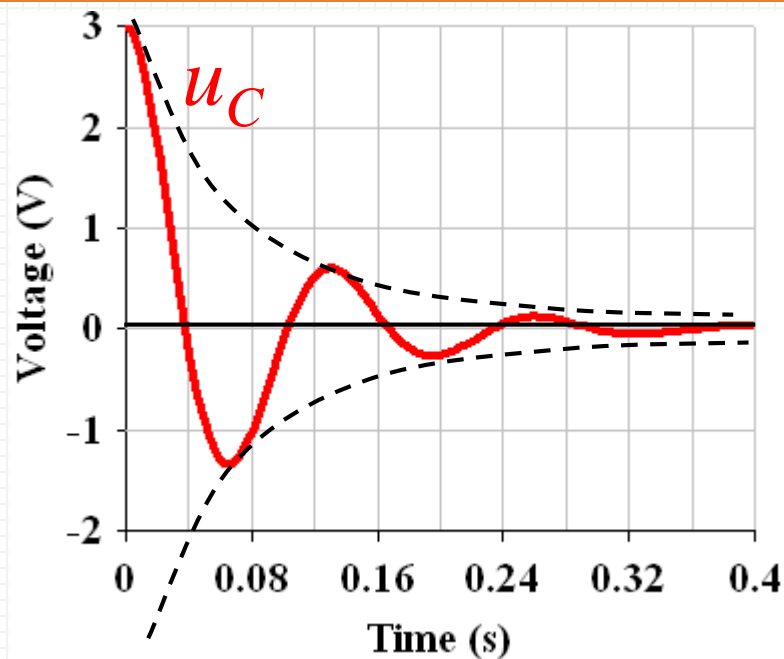
$$C \frac{du_C}{dt} = -i_L$$

$$i_L(t) = 1.55e^{-12.5t} \sin(48.4t) \text{A} \quad t > 0^+$$

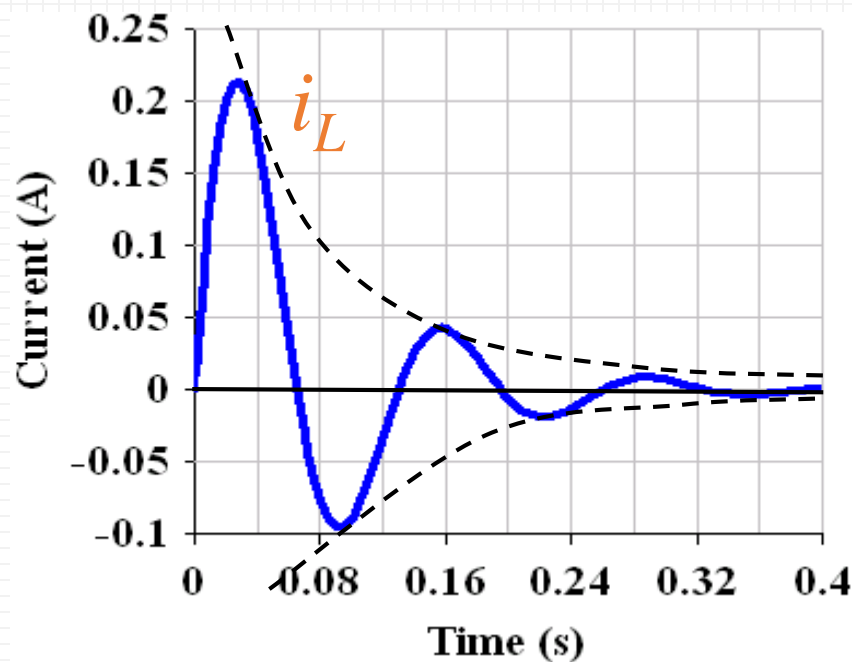


$$R=1\Omega$$

$$u_C(t) = 3.10e^{-12.5t} \sin(48.4t + 75.5^\circ) \text{ V} \\ t > 0^+$$

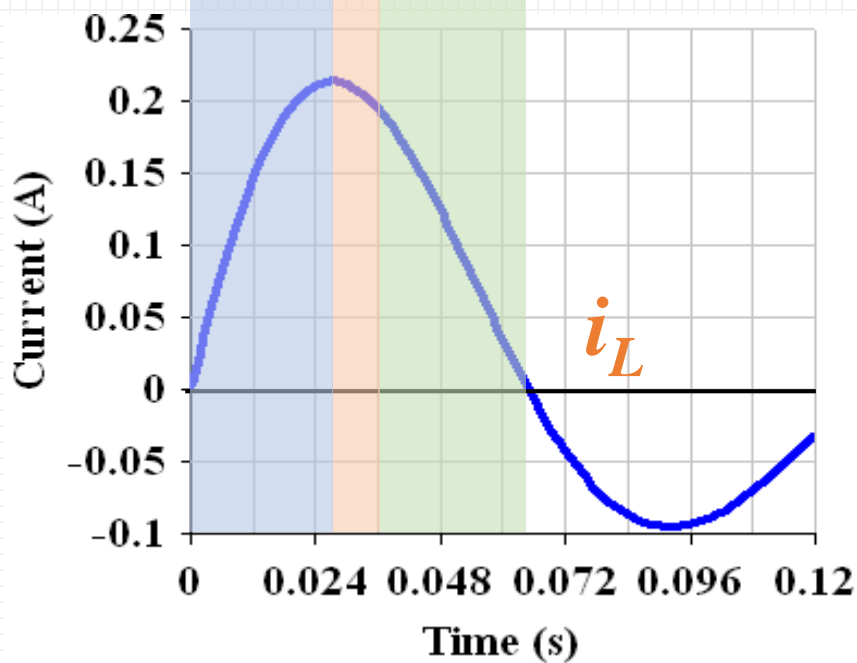
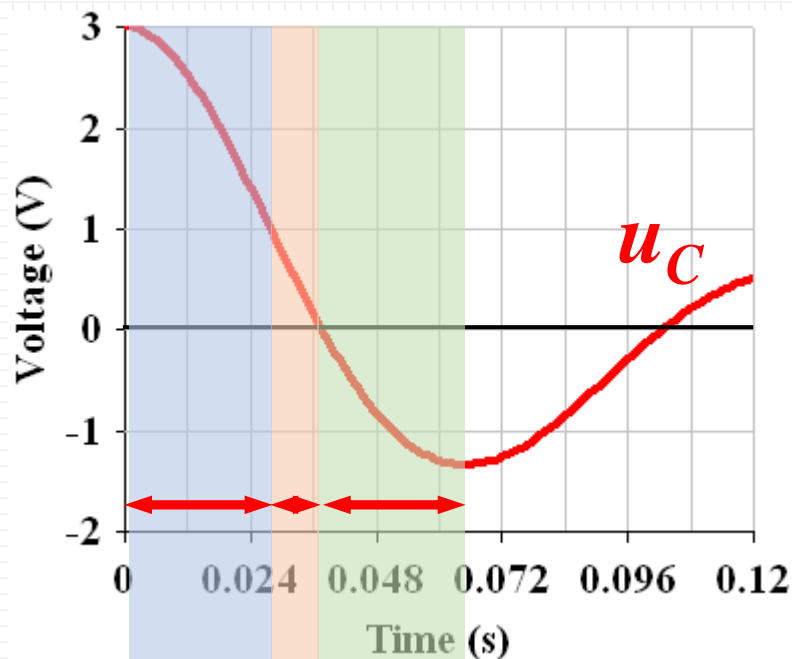


$$i_L(t) = 1.55e^{-12.5t} \sin 48.4t \text{ A} \\ t > 0^+$$



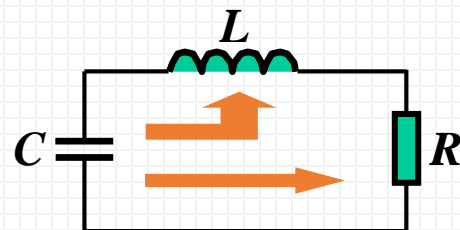
衰减振荡 欠阻尼

讨论半个周期中能量的关系



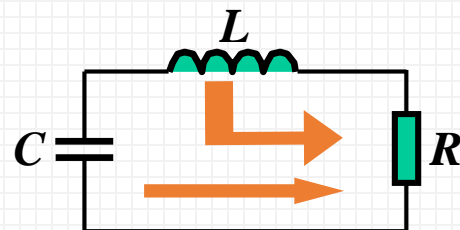
$$0 \leq 48.4t \leq 75.5^\circ$$

u_C 减小, i_L 增大



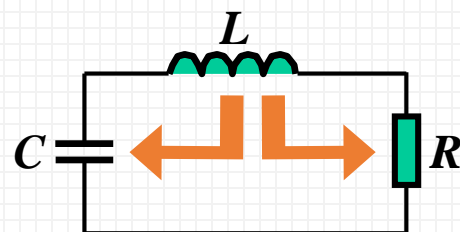
$$75.5^\circ \leq 48.4t \leq 104.5^\circ$$

u_C 减小, i_L 减小



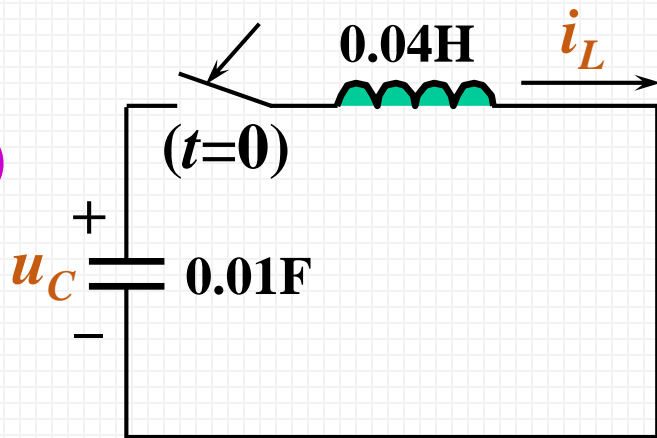
$$104.5^\circ \leq 48.4t \leq 180^\circ$$

$|u_C|$ 增大, i_L 减小



周而复始, 电阻不断消耗能量, u_C i_L 衰减到零

$$R=0$$



$$LC \frac{d^2 u_C}{dt^2} + u_C = 0$$

$$p^2 + 2500 = 0 \quad p = \pm j50$$

$$u_C(t) = K \sin(50t + \theta)$$

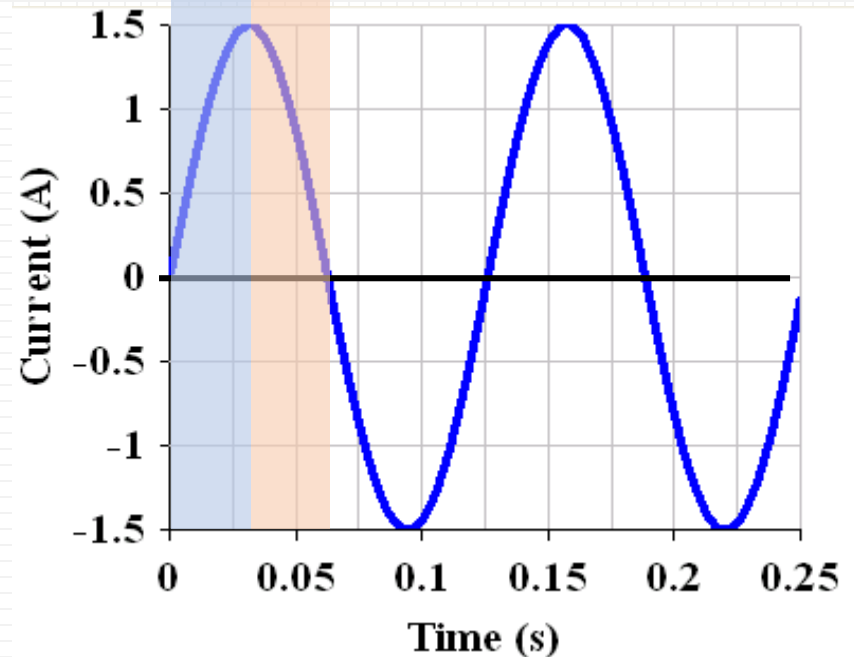
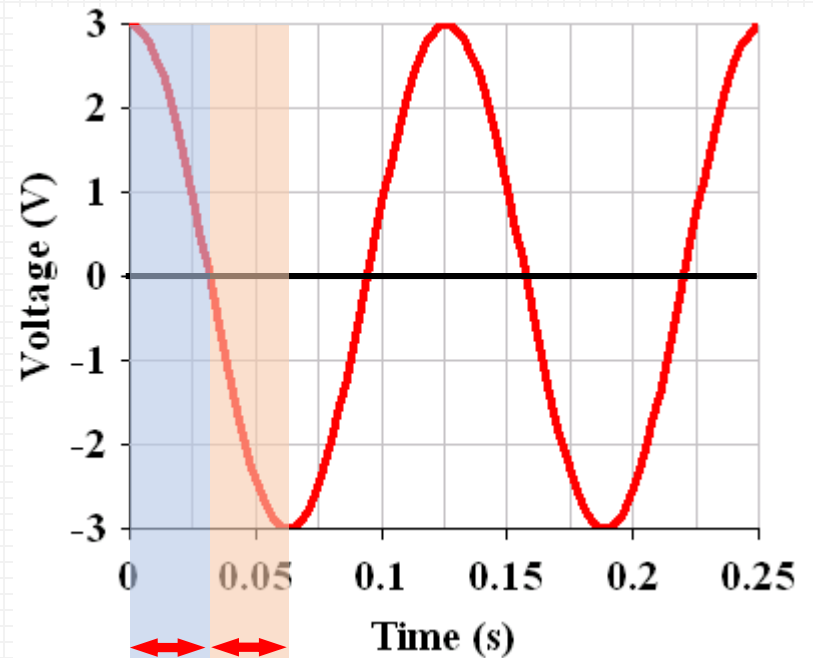
$$u_C(0) = 3 \quad \left. \frac{du_C}{dt} \right|_{t=0^+} = 0$$

$$K = 3 \quad \theta = 90^\circ$$

$$u_C(t) = 3 \cos 50t \text{ V} \quad t > 0^+$$

$$i_L(t) = 1.5 \sin 50t \text{ A} \quad t > 0^+$$

等幅振荡 无阻尼



RLC串联二阶

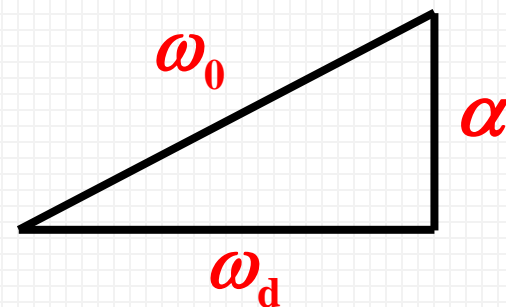
$$p^2 + 2\alpha p + \omega_0^2 = p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$

衰减系数

$$\alpha = \frac{R}{2L}$$

自由振荡角频率/
自然角频率

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



衰减振荡角频率

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$u_C = Ke^{-\alpha t} \sin(\omega_d t + \theta)$$

$$\alpha^2 > \omega_0^2$$

过阻尼

$$\alpha^2 = \omega_0^2$$

临界阻尼

$$\alpha^2 < \omega_0^2$$

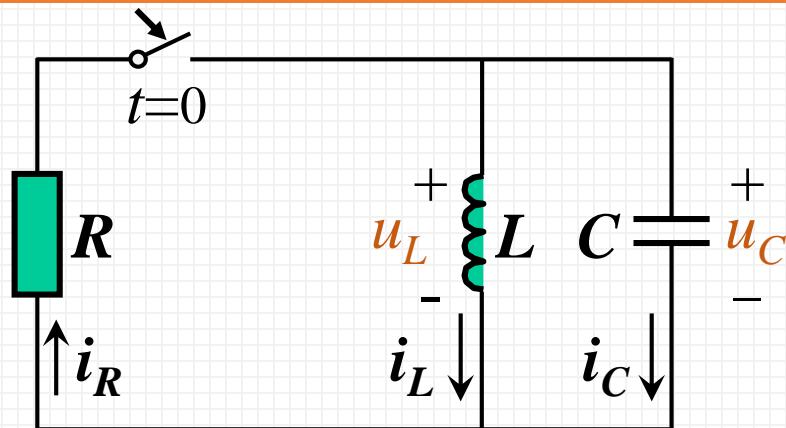
欠阻尼

$$\alpha = 0$$

无阻尼

2、RLC并联二阶电路

零输入RLC并联



$$\begin{cases} i_R = i_L + C \frac{du_C}{dt} \\ u_C = L \frac{di_L}{dt} \\ i_R = -\frac{u_C}{R} \end{cases}$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC} \frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\frac{d^2 i_L}{dt^2} + 2\alpha \frac{di_L}{dt} + \omega_0^2 i_L = 0$$

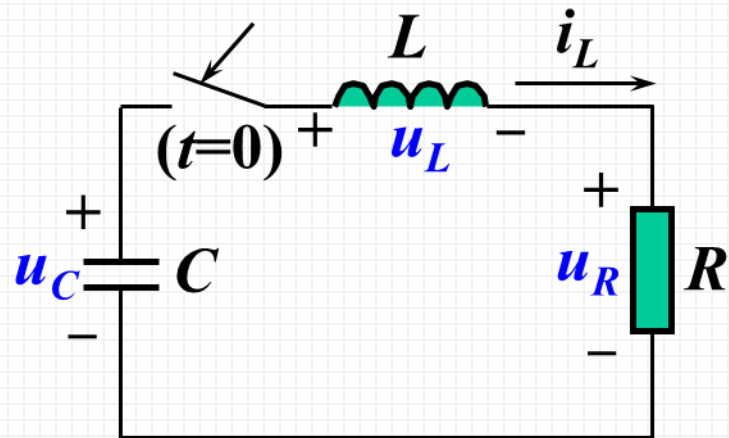
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RLC串联

$$2\alpha = \frac{R}{L}$$

$$\omega_0^2 = \frac{1}{LC}$$



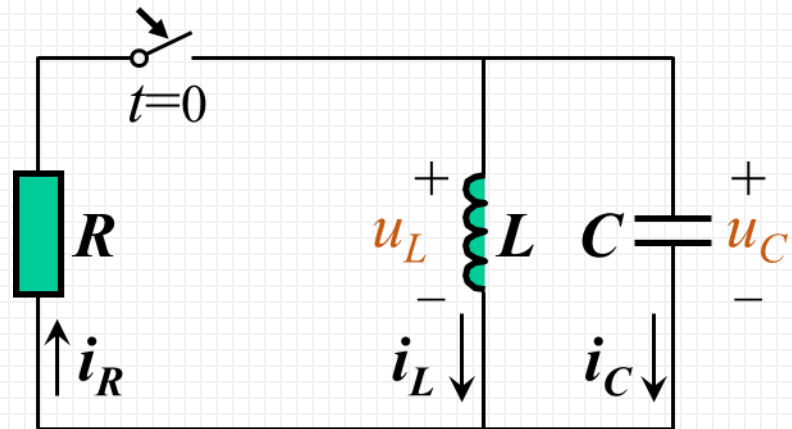
对偶!

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

RLC并联

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$



3、二阶电路的直觉解法

不求待定系数定性画支路量的变化曲线

(1) 过阻尼或临界阻尼（无振荡衰减）

以过阻尼为例

$$p_1 = -25 \quad p_2 = -100$$

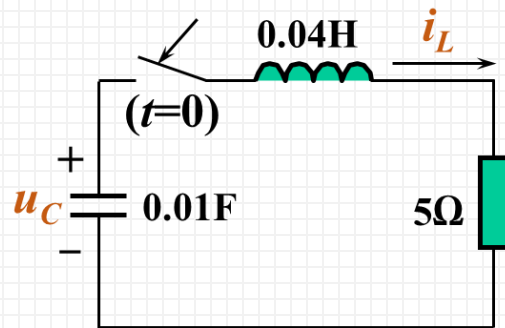
$$\begin{cases} u_C(0^+) = 3V \\ \left. \frac{du_C}{dt} \right|_{t=0^+} = 0 \end{cases}$$

- 初值
- 导数初值
- 终值

$$\begin{cases} i_L|_{0^+} = 0 \\ \left. \frac{di_L}{dt} \right|_{0^+} = \frac{1}{L} u_L|_{0^+} = \frac{3}{L} \end{cases}$$

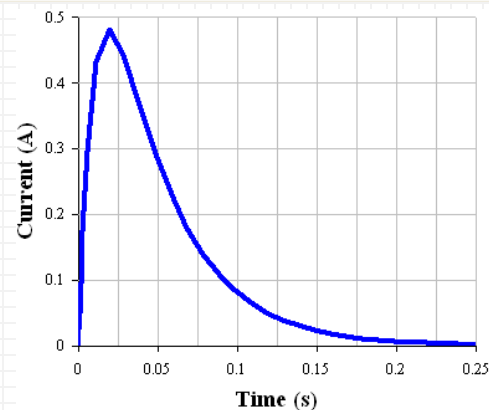
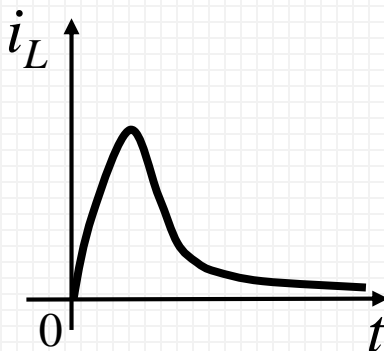
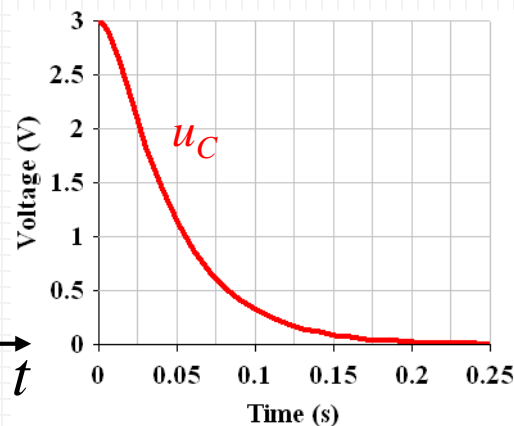
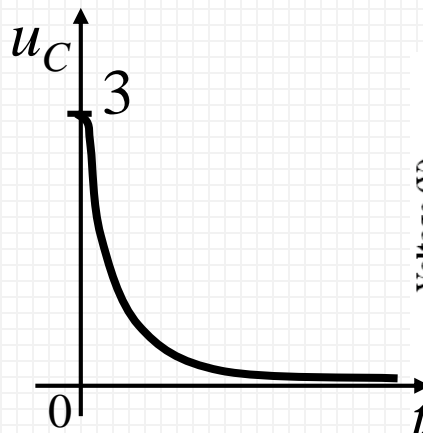
过（临界）阻尼，无振荡放电

$$p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$



$$u_C(0^-) = 3V$$

$$i_L(0^-) = 0$$



(2) 欠阻尼 (衰减振荡)

$$p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$

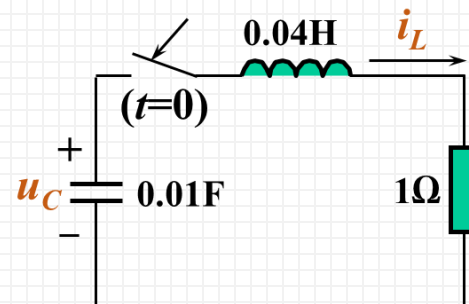
$$p_{1,2} = -12.5 \pm j48.4$$

衰减振荡角频率 ω_d

衰减系数 α

$$\begin{cases} u_C(0^+) = 3V \\ \left. \frac{du_C}{dt} \right|_{t=0^+} = 0 \end{cases}$$

- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕



$$u_C(0^-) = 3V$$

$$i_L(0^-) = 0$$

回忆一阶电路中的时间常数 τ : $3 \sim 5 \tau$ 后过渡过程结束

$$3 \times \frac{1}{\alpha} = \frac{3}{12.5} = 0.24 \text{ s}$$

$$5 \times \frac{1}{\alpha} = \frac{5}{12.5} = 0.4 \text{ s}$$

过渡过程结束

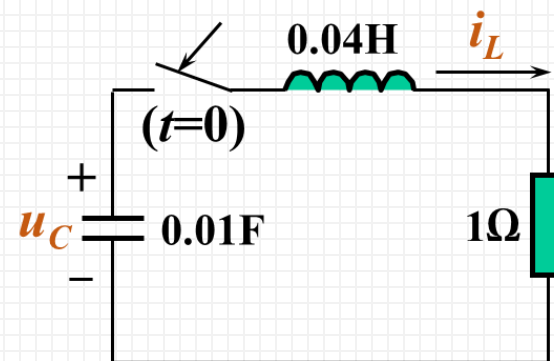
振荡周期为

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{48.4} = 0.13 \text{ s}$$

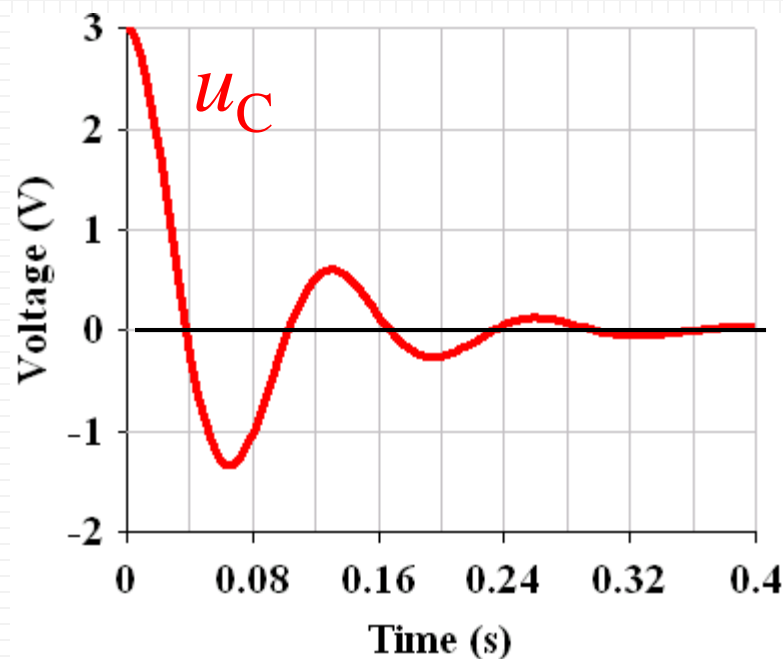
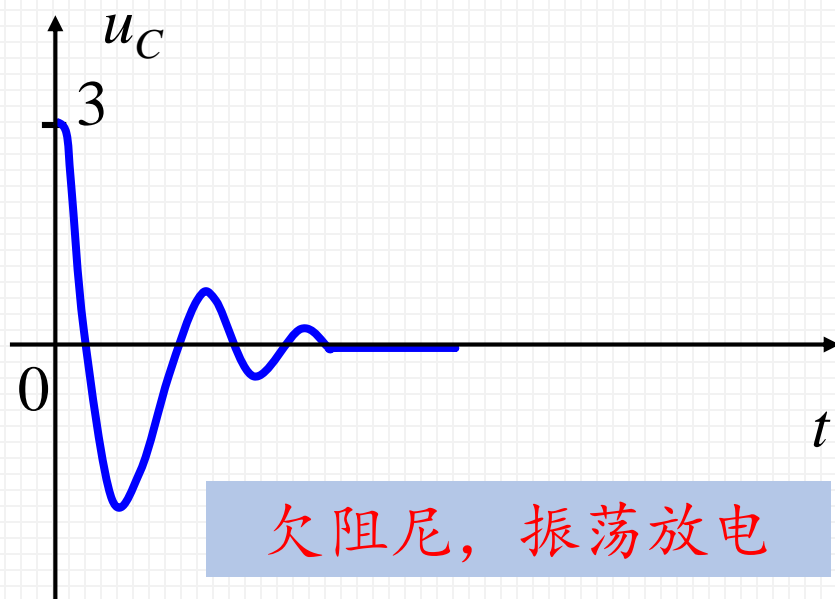
$$\lambda_{1,2} = -12.5 \pm j48.4$$

$$\begin{cases} u_C(0^+) = 3V \\ \left. \frac{du_C}{dt} \right|_{t=0^+} = 0 \end{cases}$$

衰减过程中有
 $0.24/0.13 \approx 2$ 次振荡
 或 $0.4/0.13 \approx 3$ 次振荡



- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕

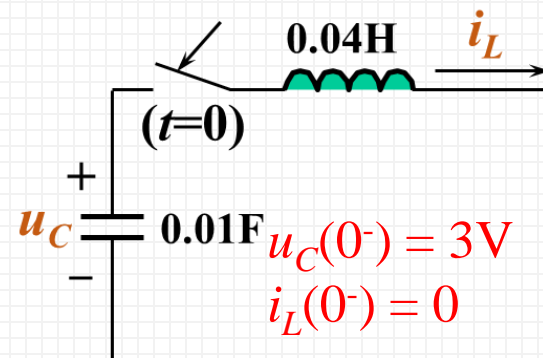


(3) 无阻尼

$$p_{1,2} = \pm j50$$

$$\begin{cases} i_L|_{0^+} = 0 \\ \frac{di_L}{dt}|_{0^+} = \frac{1}{L}u_L = \frac{3}{L} \end{cases}$$

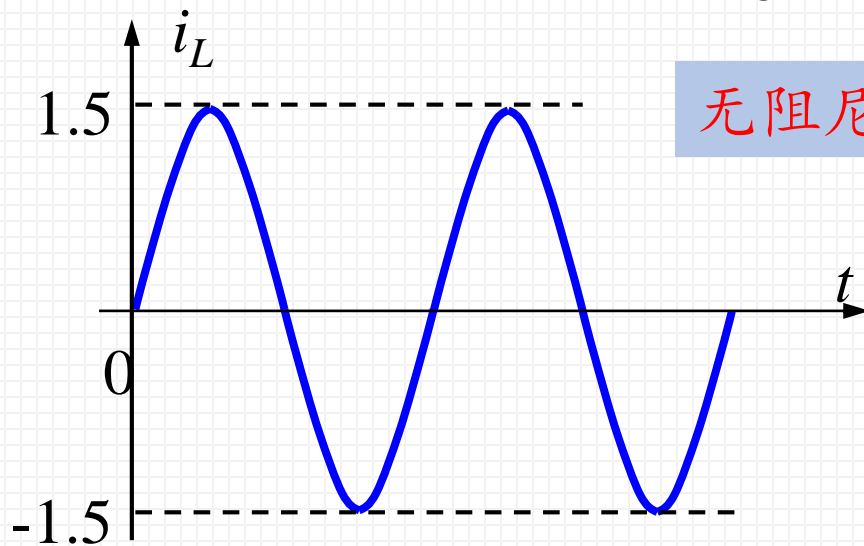
- 初值
- 导数初值
- 最大值



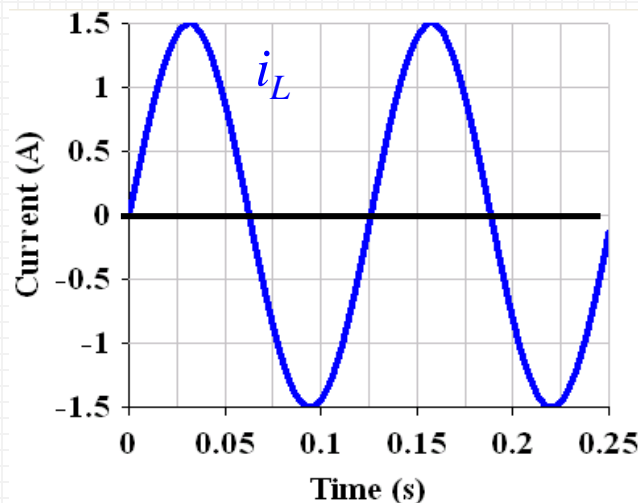
因为无阻尼，所以无能量损失

$$\frac{1}{2}Cu_C^2(0) + \frac{1}{2}Li_L^2(0) = \frac{1}{2}Cu_C^2(t) + \frac{1}{2}Li_L^2(t)$$

i_L 取最大值时, $u_C=0$, 因此 $i_{L,\max} = \sqrt{\frac{C}{L}}u_C(0) = 1.5\text{A}$



无阻尼振荡

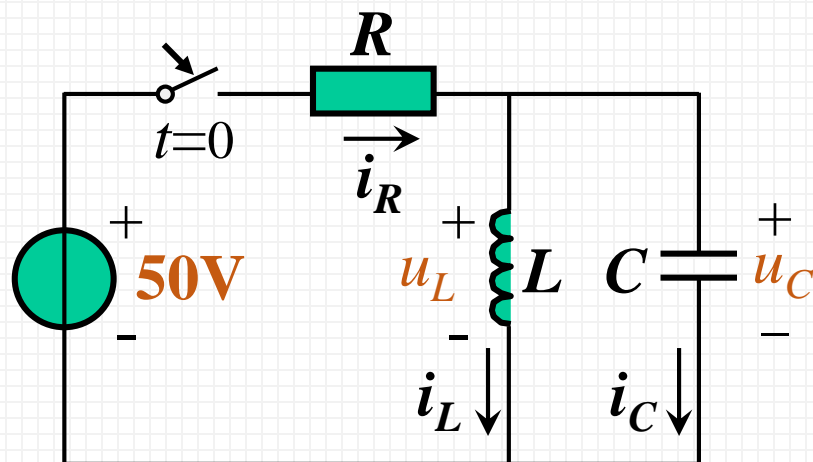


例 已知 $i_L(0)=2\text{A}$ $u_C(0)=0$
 $R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$ 。
 求: $i_R(t)$ 。

法1: 列 u_C 的微分方程先求 u_C 再求 i_R

法2: 列 i_R 的微分方程求解

法3: 通过求解一系列电阻电路求 i_R



Step 1 由零输入电路得响应形式

零输入RLC并联

$$p^2 + 2\alpha p + \omega_0^2 = 0 \quad 2\alpha = \frac{1}{RC} = 200 \quad \omega_0^2 = \frac{1}{LC} = 20000$$

$$p_{1,2} = -100 \pm j100$$

响应形式

$$i_R = i_R(\infty) + Ke^{-100t} \sin(100t + \theta)$$

已知 $i_L(0)=2\text{A}$ $u_C(0)=0$
 $R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$ 。
 求: $i_R(t)$ 。

Step2 求稳态解

$$i_R(\infty) = 1\text{A}$$

通解

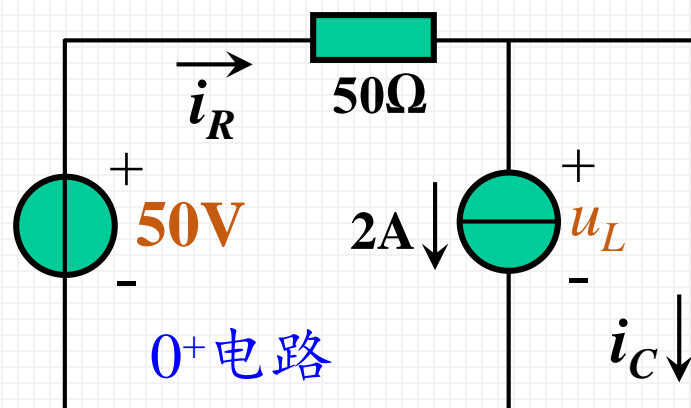
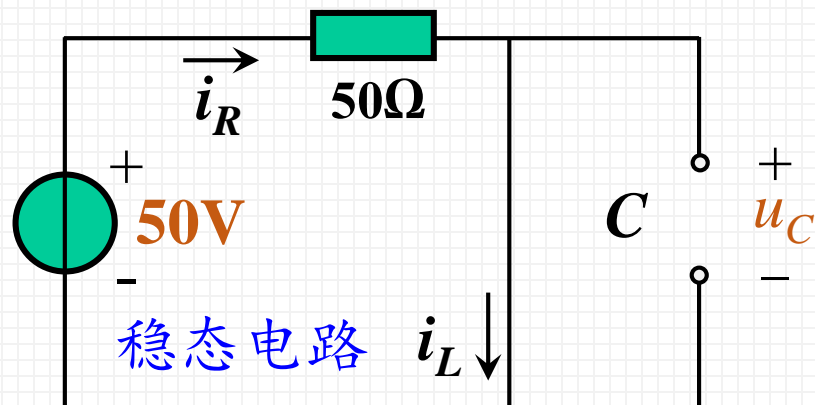
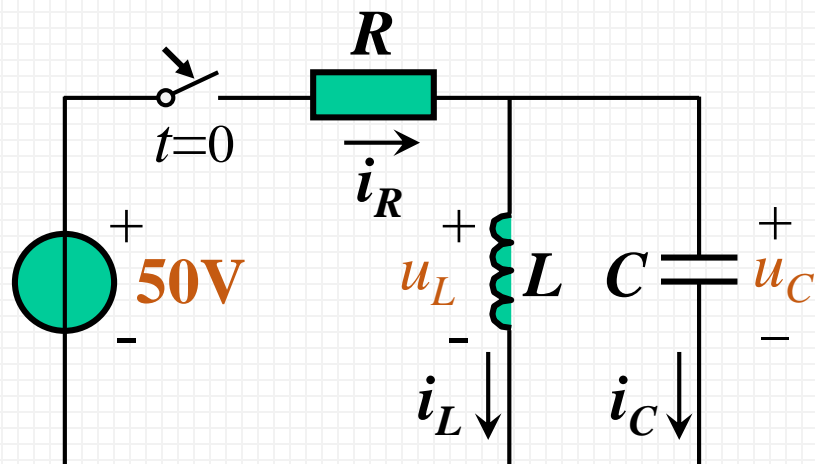
$$i_R = 1 + Ke^{-100t} \sin(100t + \theta)$$

Step3 求初值 $i_L(0)=2\text{A}$ $u_C(0)=0$

$$i_R(0^+) = \frac{50 - u_C(0^+)}{50} = 1\text{A}$$

怎么求？

$$\left. \frac{di_R}{dt} \right|_{t=0^+}$$





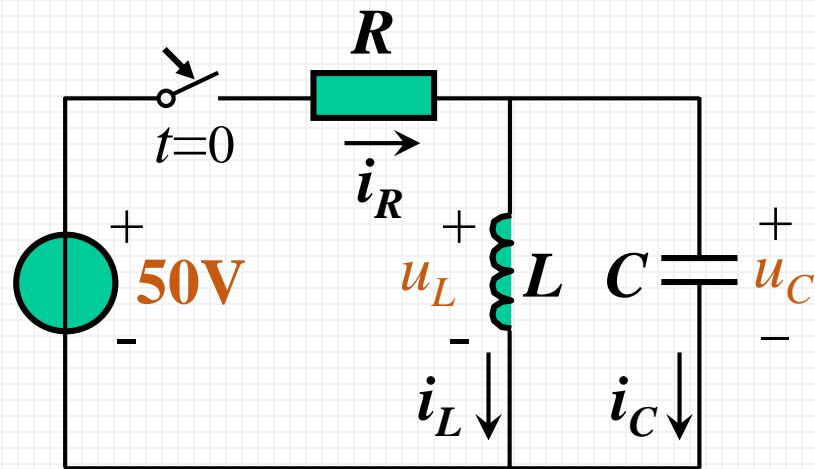
思路：用电源、 u_C 和 i_L 来表示 i_R

$$i_R = \frac{50 - u_C}{R}$$

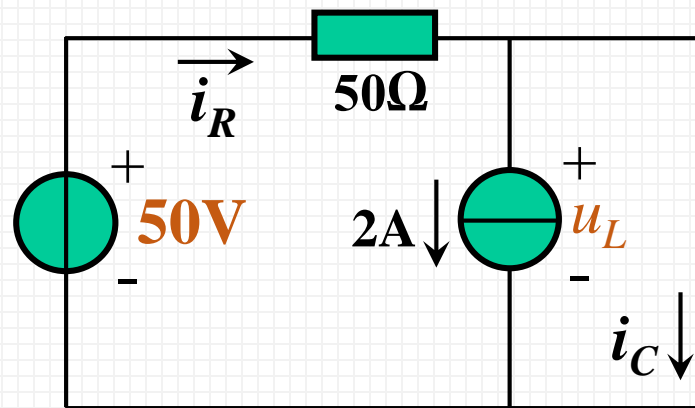
$$\left. \frac{di_R}{dt} \right|_{0+} = \left. \frac{d}{dt} \left(\frac{50 - u_C}{R} \right) \right|_{0+} = -\frac{1}{R} \left. \frac{du_C}{dt} \right|_{0+}$$

$$= -\frac{1}{RC} i_C(0^+)$$

$$= -\frac{-1}{50 \times 100 \times 10^{-6}} = 200 \text{ A/s}$$



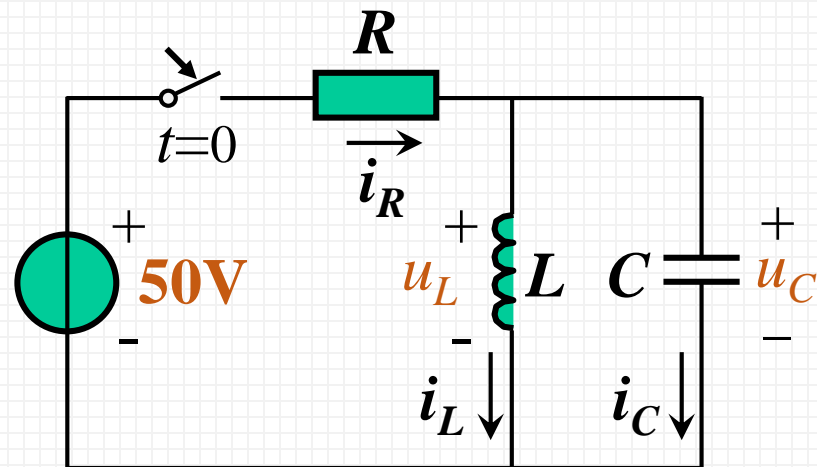
0⁺ 电路



$$i_C(0^+) = -1\text{A}$$



已知: $i_L(0)=2\text{A}$ $u_C(0)=0$
 $R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$ 。
求: $i_R(t)$ 。



Step4 求待定系数

通解 $i_R = 1 + Ke^{-100t} \sin(100t + \theta)$

$$\begin{cases} i_R(0^+) = 1\text{A} \\ \left. \frac{di_R}{dt} \right|_{0^+} = 200 \text{ A/s} \end{cases}$$

$$i_R(t) = 1 + 2e^{-100t} \sin 100t \text{ A} \quad t > 0^+$$



总结二阶电路的求解

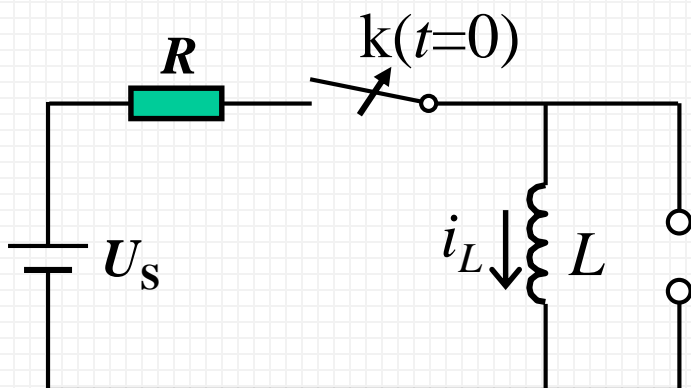
- 求响应形式
 - RLC串联、RLC并联 → 直接得到特征方程
 - ZIR非RLC串并联怎么办? → L12 (根据状态方程)
- 求稳态值 → 得通解表达式
 - 电阻电路
- 求初值
 - 0^- 电阻电路 → 换路定理得 0^+ 电阻电路 → 初值
- 求导数初值
 - 将支路量用独立源、 u_C 、 i_L 来表示 → 0^+ 电路求 i_C 、 u_L
 - L12 (根据输出方程和状态方程)
- 用初值和导数初值确定通解待定系数

为什么一个动态电路中任意支路量都有相同的变化性质? (L12)

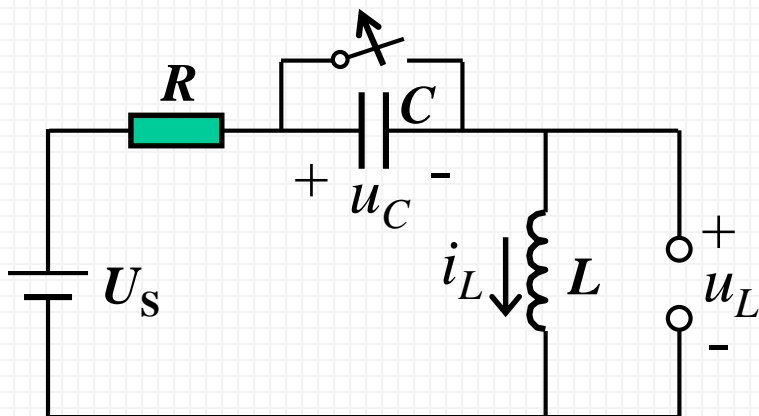
4、二阶电路的应用

(1) 汽车点火系统

开关和气隙在搏命



开关的设计耐压高于气隙的击穿电压



一阶点火电路的问题：

开路开关和火花塞承受相同的无穷大电压

二阶点火电路的好处：

开路开关的电压被电容钳位
可通过电路参数控制开关电压

(2) 电磁轨道炮电源

Dahlgren Surface Warfare Center

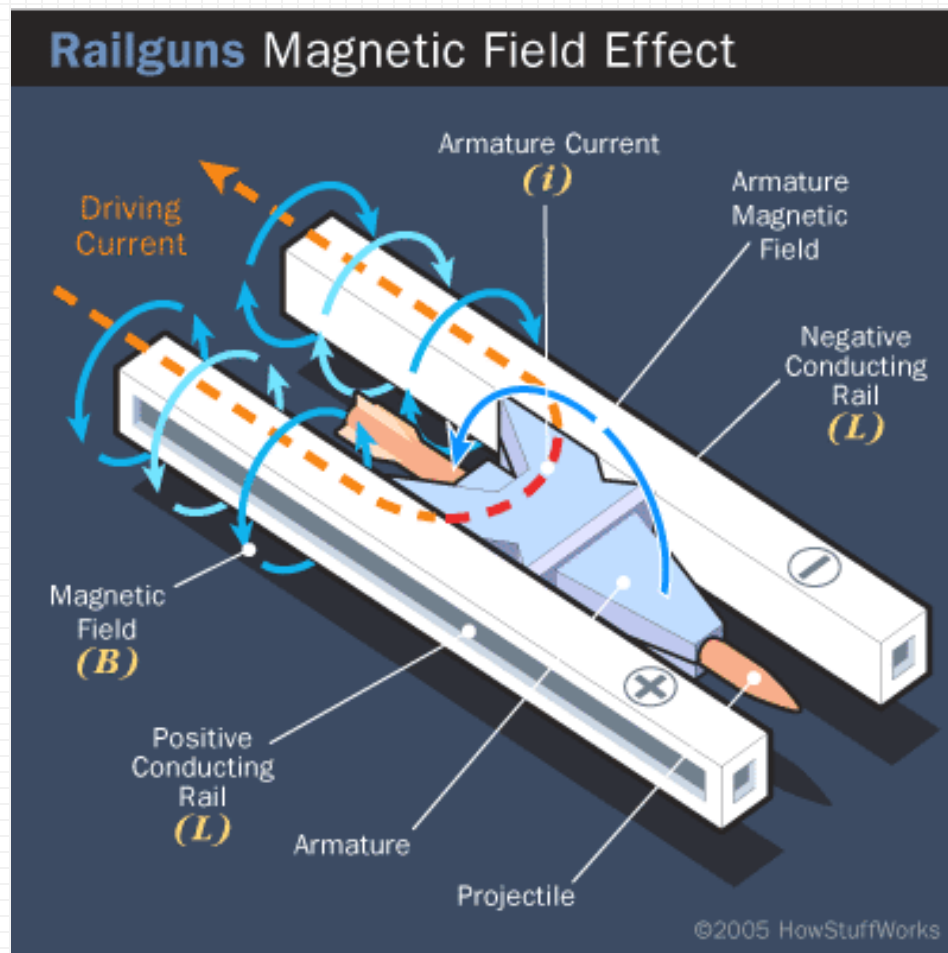
2012



电磁轨道炮原理

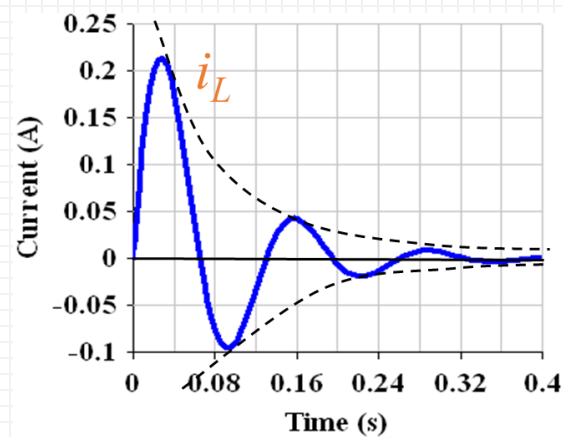
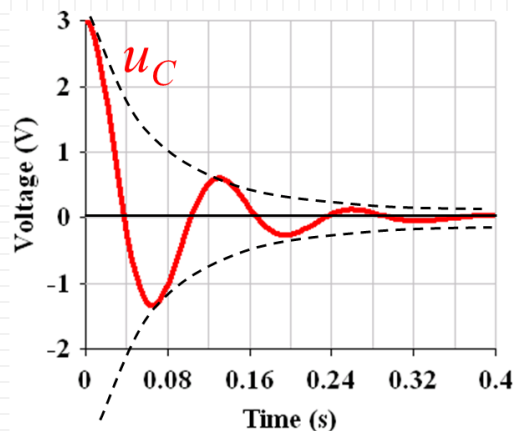
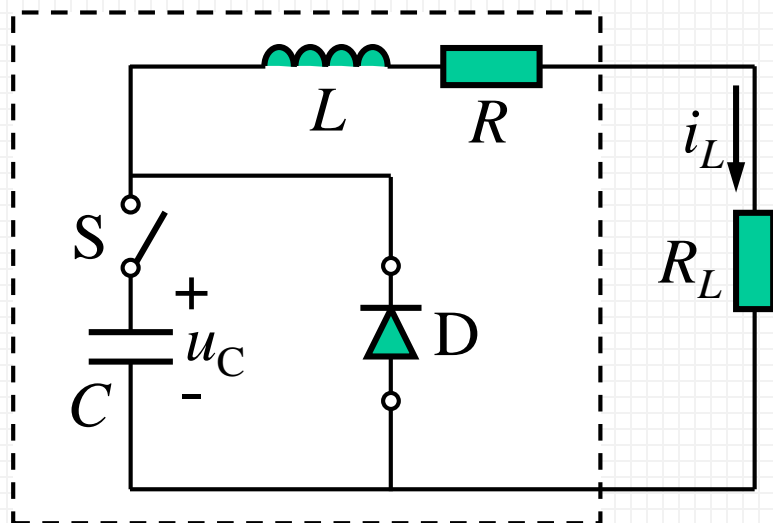
关键是
可控的脉冲大电流

$$F=0.5L'i^2$$



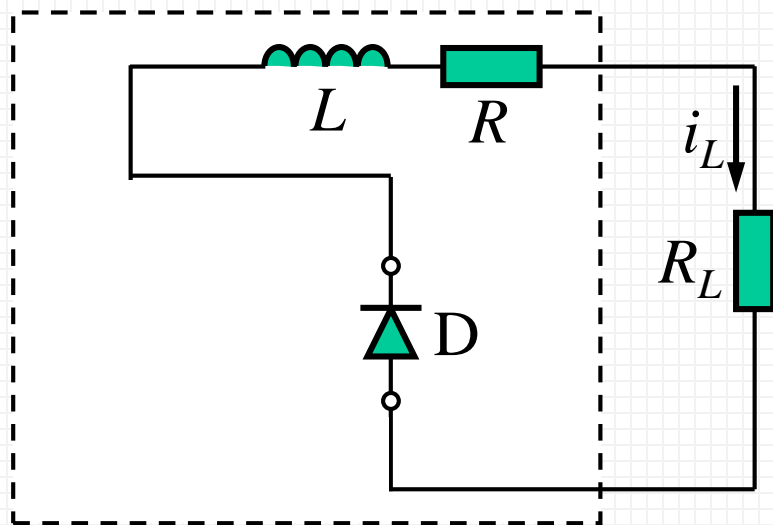


电磁轨道炮脉冲电源的基本电路 (PFU) (Pulse Forming Unit)



衰减振荡 欠阻尼

电磁轨道炮脉冲电源的基本电路 (PFU) (**P**ulse **F**orming **U**nit)



一阶RL放电电路

4个PFU并联

