2 电路原理习题课

求初始值的步骤

1. 由换路前电路（稳定状态）求 $u_C(0^-)$ 和 $i_L(0^-)$。

电阻电路1(直流)

2. 由换路定律得 $u_C(0^+)$ 和 $i_L(0^+)$。

3. 画出0^+时刻的等效电路。

3.1 画换路后电路的拓扑结构；

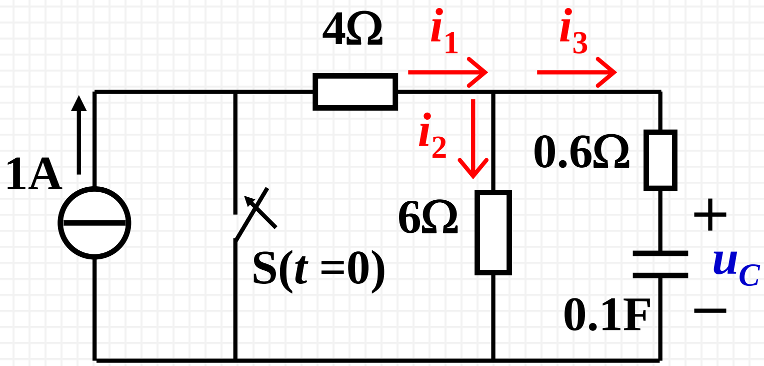
3.2 电容（电感）用电压源（电流源）替代。

取0^+时刻值，方向同原假定的电容电压、电感电流方向。

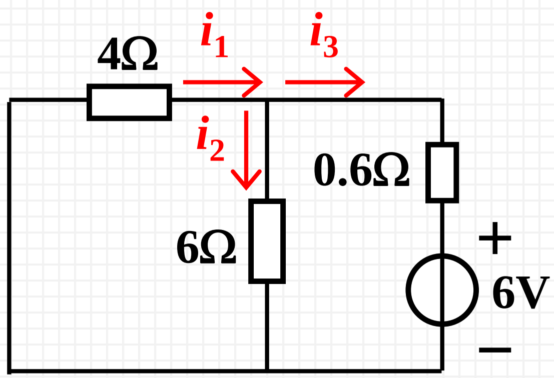
电阻电路2

4. 由0^+电路求其它各变量的0^+值。

1. 求: $i_1(0^+), i_2(0^+), i_3(0^+)$.



$t=0^+$时刻电路:



解:

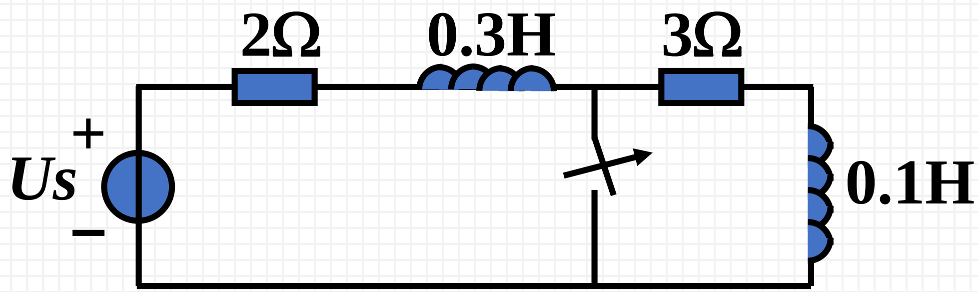
$$u_C(0^+)=u_C(0^-)=6\times1=6\text{ V}$$

$$i_3(0^+)=-\frac{6}{0.6+4//6}=-2\text{ A}$$

$$i_1(0^+)=\frac{i_3(0^+)\times6}{4+6}=-1.2\text{ A}$$

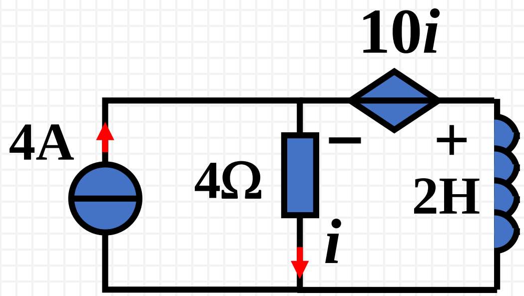
$$i_2(0^+)=-\frac{i_3(0^+)\times4}{4+6}=0.8\text{ A}$$

2. ①求时间常数.



$$\tau = \frac{L}{R} = \frac{0.3+0.1}{2+3} = 0.08\ \mathrm{s}$$

②求时间常数.

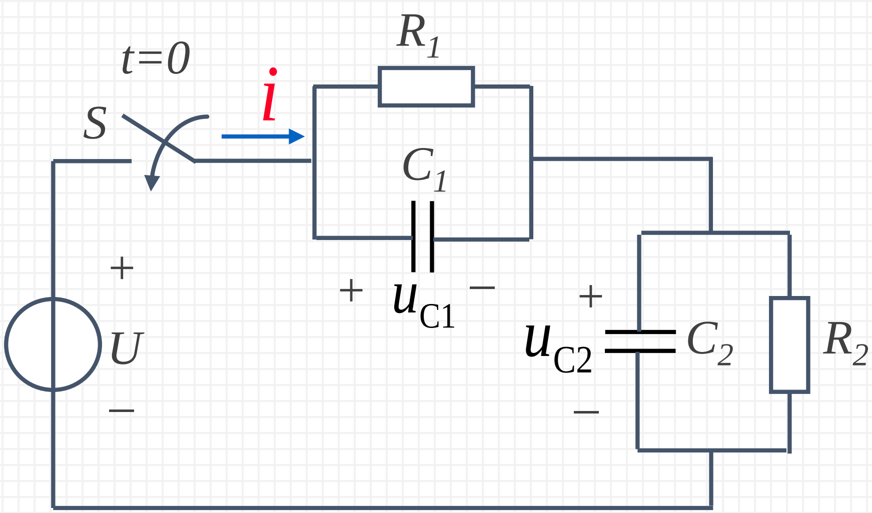


$$u=10i+4i=14i$$

$$R_{等}=14\Omega$$

$$\tau=2/R_{等}=2/14=0.143\text{s}$$

例： 两个**电容**虽不能等效成一个电容，但换路后与**恒压源**构成**回路**，所列**方程**是**一阶**的，所以仍是一阶电路。如：



$$\begin{cases} i = \dfrac{u_{C1}}{R_1} + C_1 \dfrac{\mathrm{d}u_{C1}}{\mathrm{d}t} = \dfrac{u_{C2}}{R_2} + C_2 \dfrac{\mathrm{d}u_2}{\mathrm{d}t} & (1) \\ u_{C1} + u_{C2} = U & (2) \end{cases}$$

将(2)代入(1)，消去u_{C2}, 得:

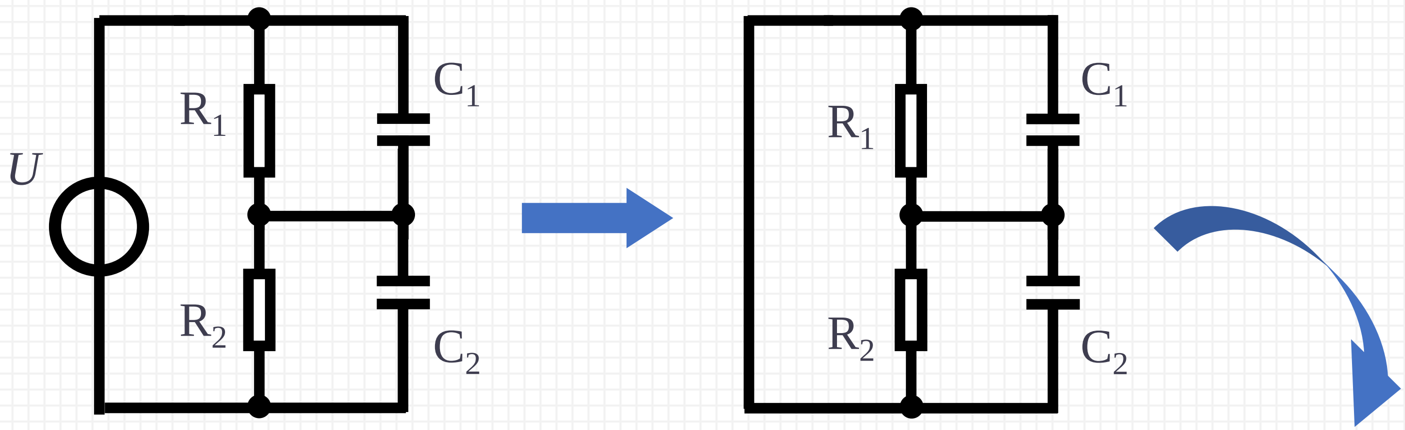
$$\frac{u_{C1}}{R_1} + C_1 \frac{\mathrm{d}u_{C1}}{\mathrm{d}t} = \frac{U - u_{C1}}{R_2} - C_2 \frac{\mathrm{d}u_{C1}}{\mathrm{d}t}$$

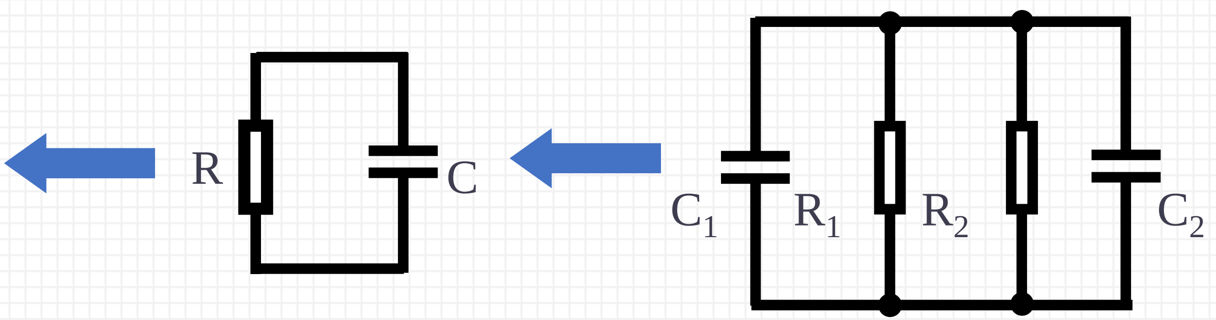
整理后得: $$(C_1 + C_2)\frac{\mathrm{d}u_{C1}}{\mathrm{d}t} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) u_{C1} = \frac{U}{R_2}$$

此方程为**一阶**微分方程，所以该电路是一阶电路，可用**三要素法**解。

判断含多个储能元件的电路,是否为一阶电路的**方法**

去除电路中的**独立源**（电压源短路、电流源开路），然后判断电路中的储能元件能否**等效**为**一个**。若能，则为一阶电路; 否则不是。如:





该电路是**一阶**电路

求解一阶电路的三要素法（适用直流与正弦激励）

$$i(t)=i(\infty)+[i(0^{+})-i(\infty)\big|_{t=0}]\mathrm{e}^{-\frac{t}{\tau}} \quad t>0$$

$t=0$对正弦激励而言

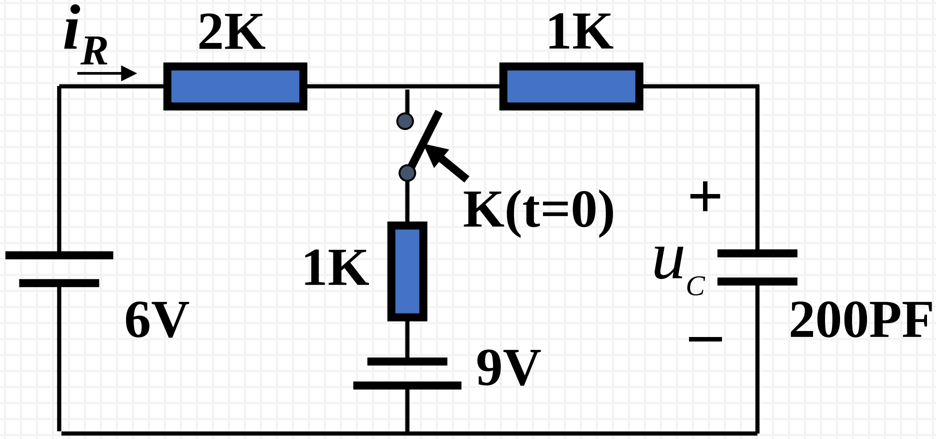
$i(\infty)$ 为 $i(t)\big|_{t\to\infty}$的简写

$$\text{三要素}\begin{cases} f(\infty) & \text{稳态解} \\ f(0^{+}) & \text{起始值} \\ \tau & \text{时间常数} \end{cases}$$

全响应 ＝ 零输入响应 ＋ 零状态响应

全响应	**＝**	**通解（自由分量）**	**＋**	**特解（强制分量）**

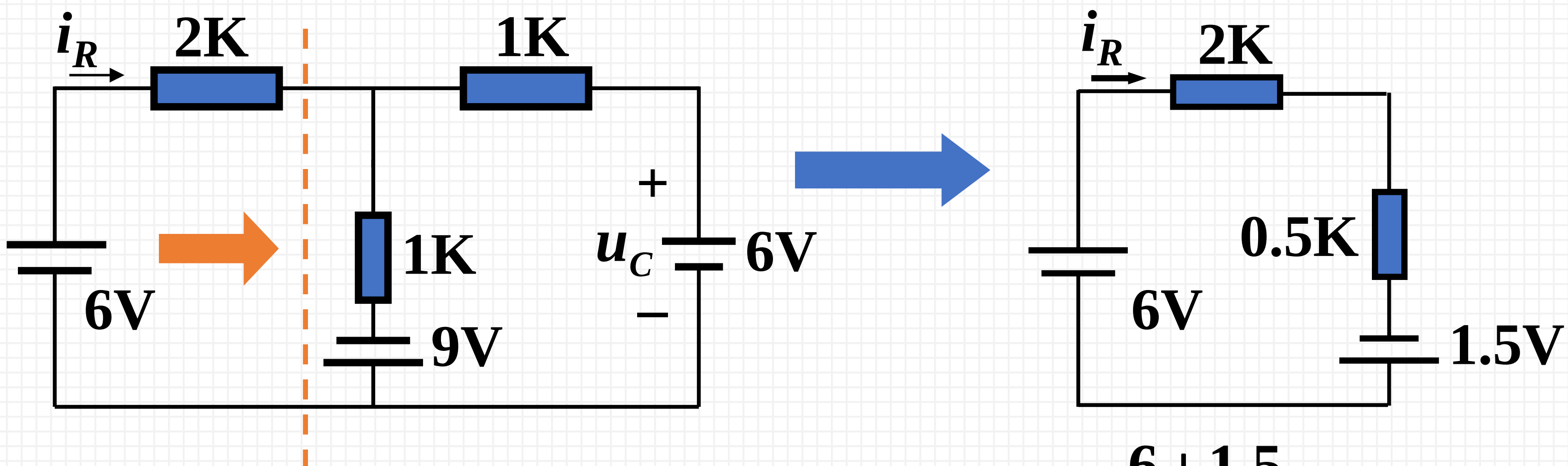
3.



K 在 t =0时闭合. 求 i_R 并定性画出其波形.

解: $u_C(0^+) = u_C(0^-) = 6\text{V}$

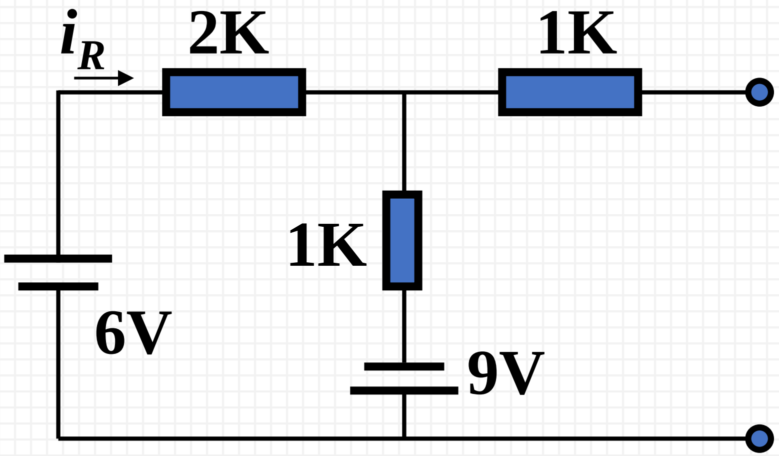
0^+时刻电路:



$$i_R(0^+) = \frac{6+1.5}{2.5} = 3\text{mA}$$

$$i_R(0^+) = 3\text{mA}$$

$t = \infty$时电路:

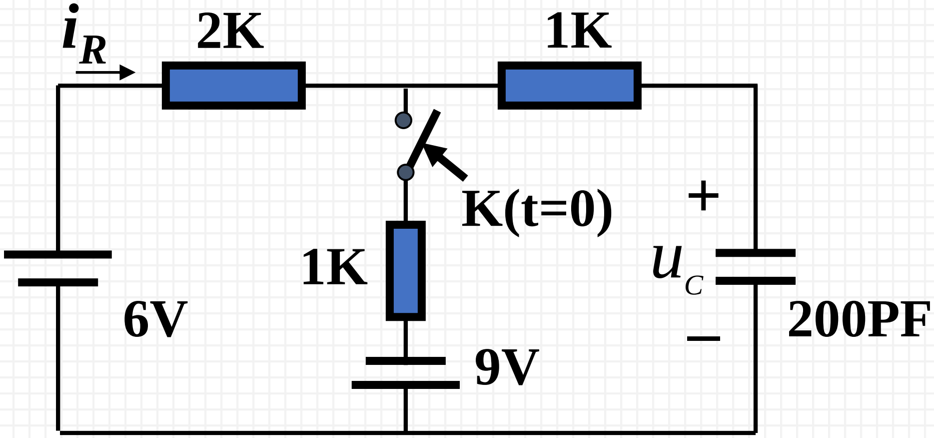


$$i_R(\infty) = \frac{6+9}{3} = 5\text{mA}$$

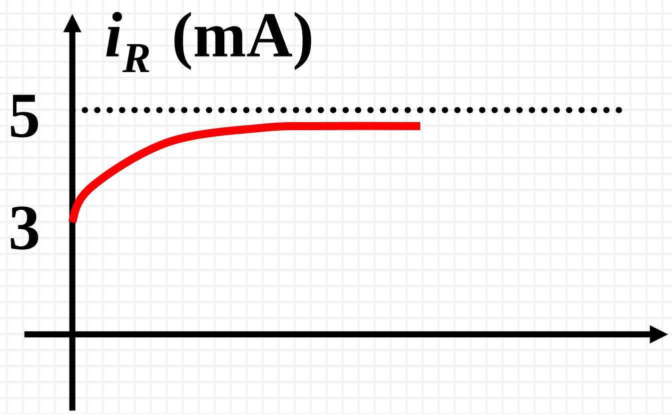
$$R_{等} = 1 + \frac{2\times1}{2+1} = \frac{5}{3}\text{K}\Omega$$

$$\tau = \frac{5}{3}\times10^3\times200\times10^{-12} = \frac{1}{3}\times10^{-6}\text{s}$$

$$i_R = 5 + (3-5)e^{-\frac{t}{\tau}} = 5 - 2e^{-\frac{t}{\tau}}\text{mA} \quad \underline{\underline{t \geq 0^+}}\ !$$



$$i_R = 5 + (3-5)e^{-\frac{t}{\tau}}\,\text{mA}$$



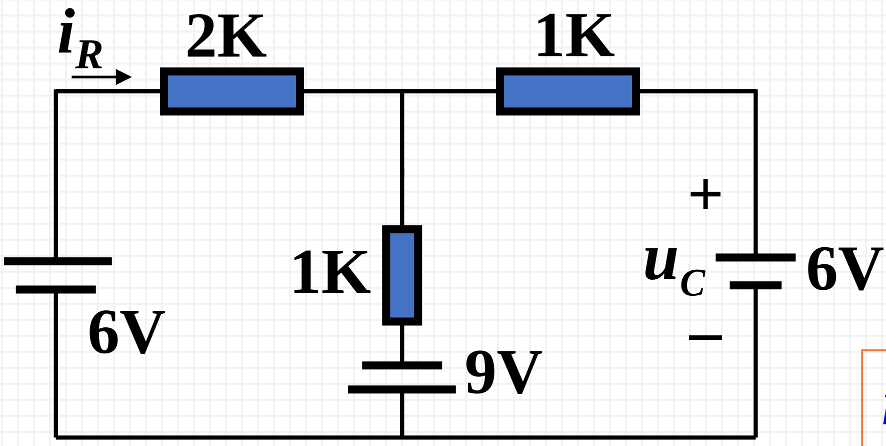
$$i_R(0^+) = 3\text{mA} \qquad i_R(\infty) = 5\text{mA}$$

i_R 的零输入响应：$i_{R\,ZIR} = 3e^{-\frac{t}{\tau}}$

i_R 的零状态响应：$i_{R\,ZSR} = 5\left(1-e^{-\frac{t}{\tau}}\right)\text{mA}$

?

0^+时刻电路:

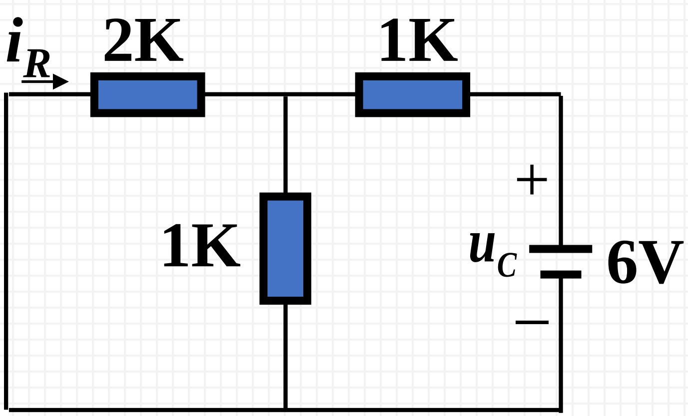


$$i_R(0^+)=3\text{mA}$$

i_L和 u_C在 $t=0^+$时的值由 $t=0^-$时刻的电路确定；而其它电量在$t=0^+$的值则由$t=0^+$时刻的电路确定，求全响应所用的0^+时刻电路与求零输入响应所用的0^+时刻电路时不一样的。

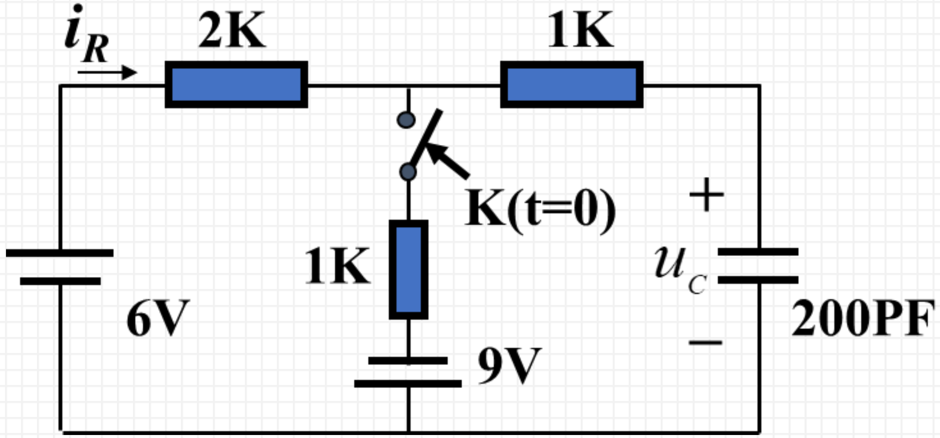
i_R 的零输入响应:

0^+时刻电路:



$$i_R(0^+)=-\frac{6}{1+2/3}\times\frac{1}{3}=-1.2\text{mA}$$

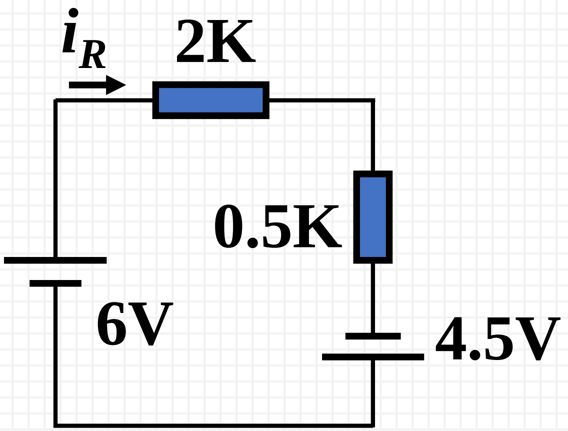
$$i_{R\,\text{ZIR}}=-1.2e^{-\frac{t}{\tau}}\text{mA}\quad t\geq 0^+$$



i_R 的零状态响应:

$$u_C(0^+) = 0$$

0⁺时刻电路:



$$i_R(0^+) = \frac{6+4.5}{2.5} = 4.2\text{mA}$$

$$i_R(\infty) = \frac{6+9}{3} = 5\text{mA}$$

$$i_{R\,\text{ZSR}} = 5 + (4.2-5)e^{-\frac{t}{\tau}} = 5 - 0.8e^{-\frac{t}{\tau}}\text{mA} \quad t \geq 0^+$$

$$i_R = i_{R\,\text{ZSR}} + i_{R\,\text{ZIR}}$$

$$= 5 - 0.8e^{-\frac{t}{\tau}} - 1.2e^{-\frac{t}{\tau}} = 5 - 2e^{-\frac{t}{\tau}}\text{mA} \quad t \geq 0^+$$

$$u_C = A + Be^{-\frac{t}{\tau}} \quad t > 0$$

$$u_C' = A(1 - e^{-\frac{t}{\tau}}) \quad t > 0$$

零状态响应

$$u_C'' = (A + B)e^{-\frac{t}{\tau}} \quad t > 0$$

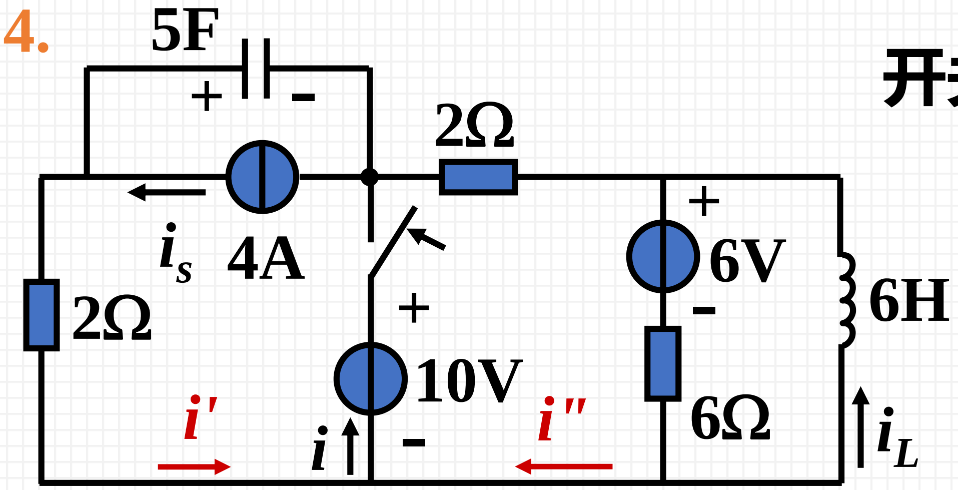
零输入响应

只有i_L和 u_C在 已知全响应的情况下，可以通过上述方法得到零状态响应和零输入响应。其他变量不能这么拆分。

如:

$$i_R = 5 - 2e^{-\frac{t}{\tau}}\text{mA} \begin{cases} i_{R零输入}(t) = -1.2e^{-\frac{t}{\tau}}\text{mA} \quad t > 0 \\ i_{R零状态}(t) = 5 - 0.8e^{-\frac{t}{\tau}}\text{mA} \quad t > 0 \end{cases}$$

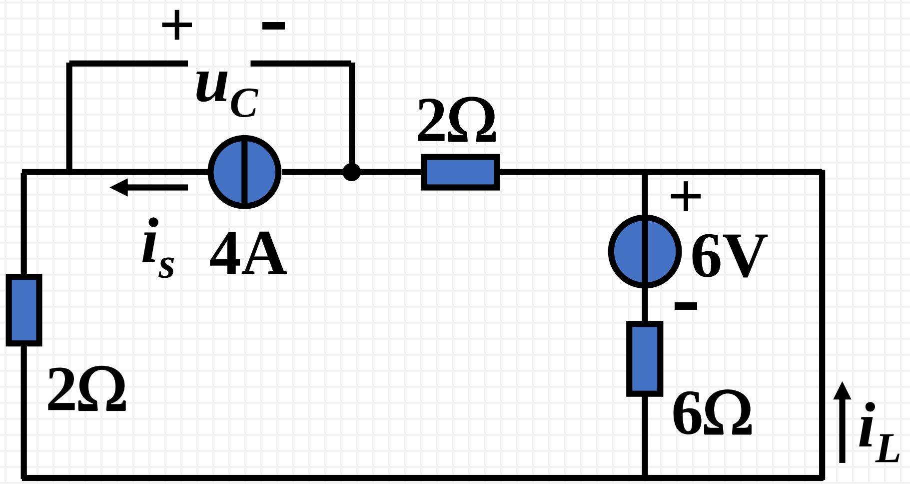
4.



开关在 $t=0$时刻闭合. 求 $i(t)$。

$i = i' + i''$

0⁻时刻电路:



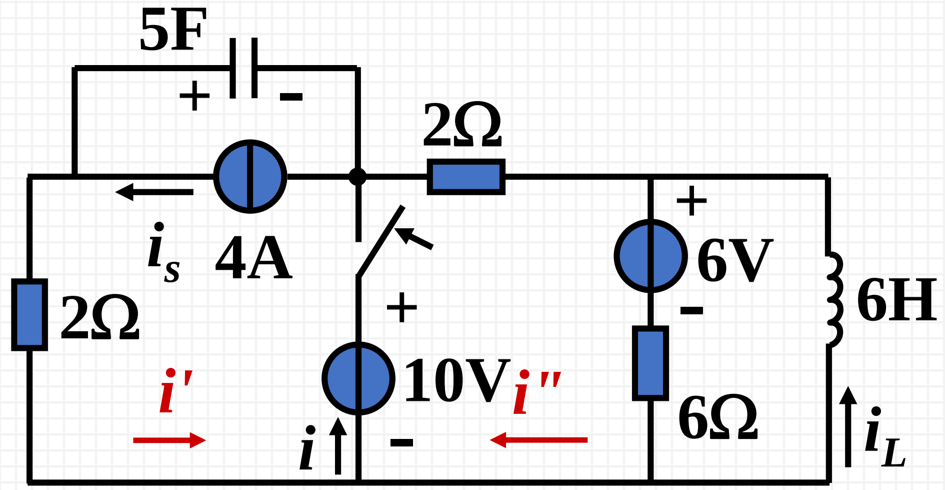
$u_C(0^-) = 16V$

$i_L(0^-) = 3A$

$u_C(0^+) = 16V$ $\quad$ $i_L(0^+) = 3A$

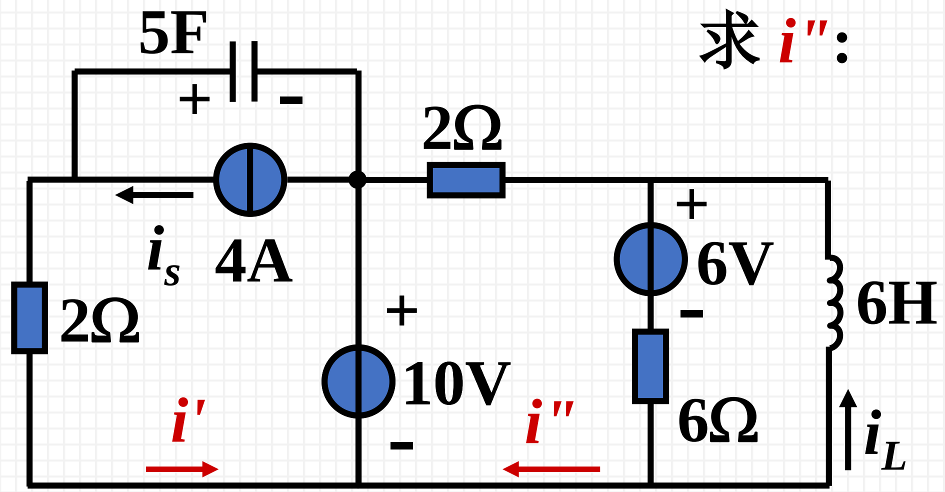
一阶还是二阶?

并联的恒压源



求 i':

求 i'':

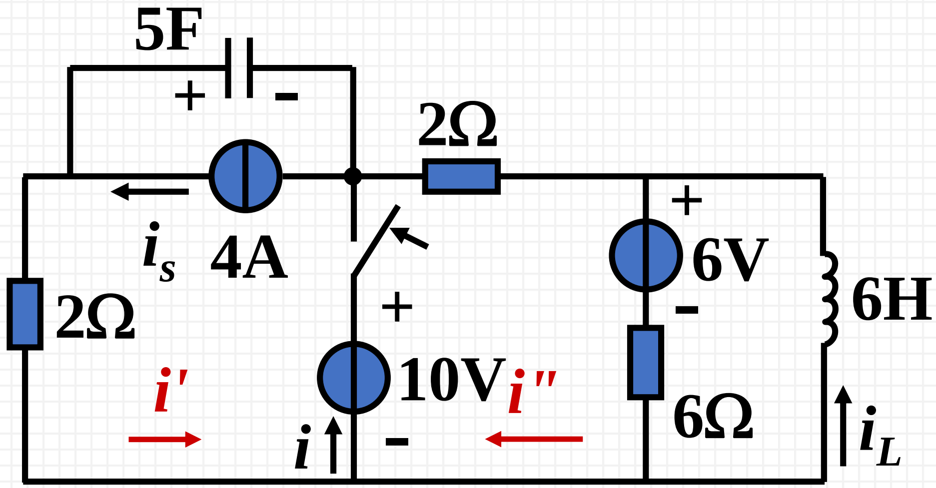


$u_C(0^+) = 16\text{V}$

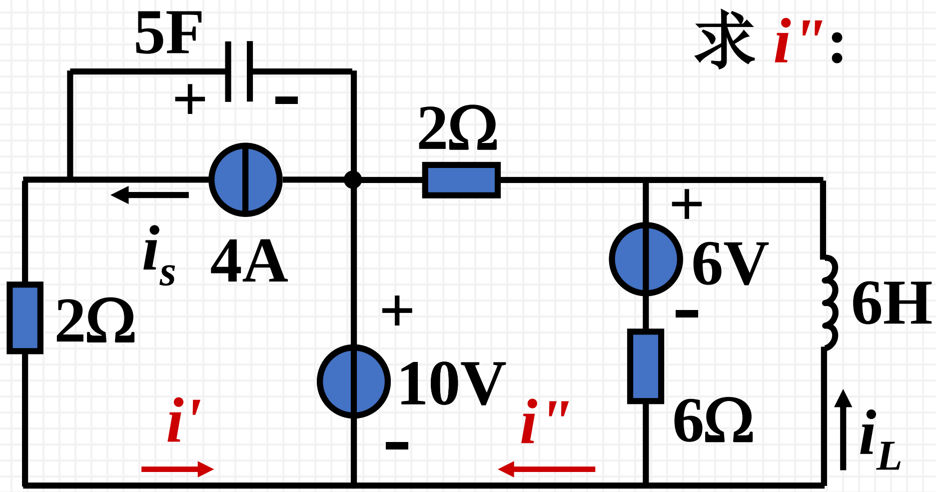
$i_L(0^+) = 3\text{A}$

一阶还是二阶?

并联的恒压源



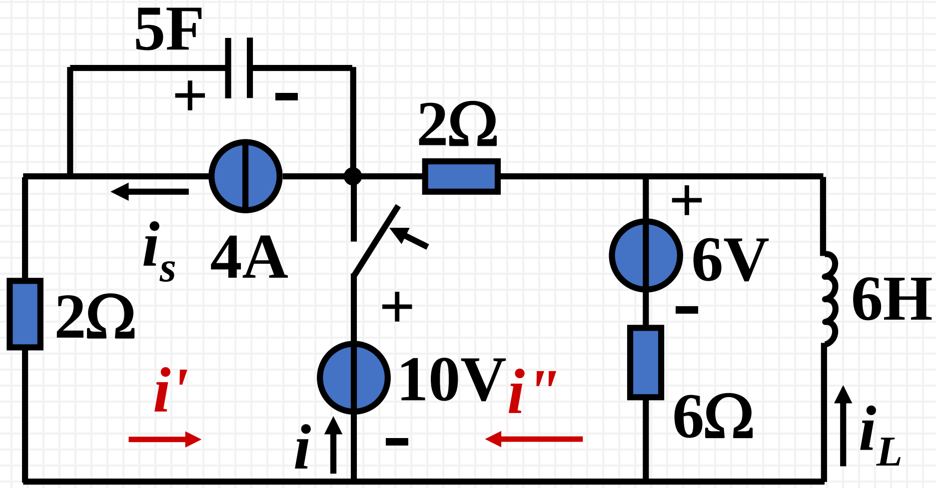
求 i':



求 i'':

$$u_C(0^+) = 16\text{V}$$

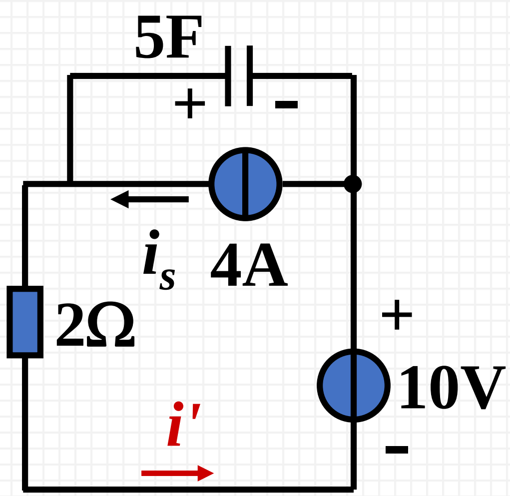
$$i_L(0^+) = 3\text{A}$$



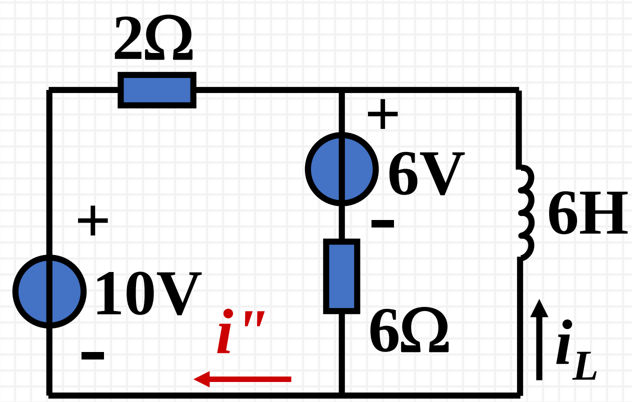
一阶还是二阶?

并联的**恒压源**

求 i':

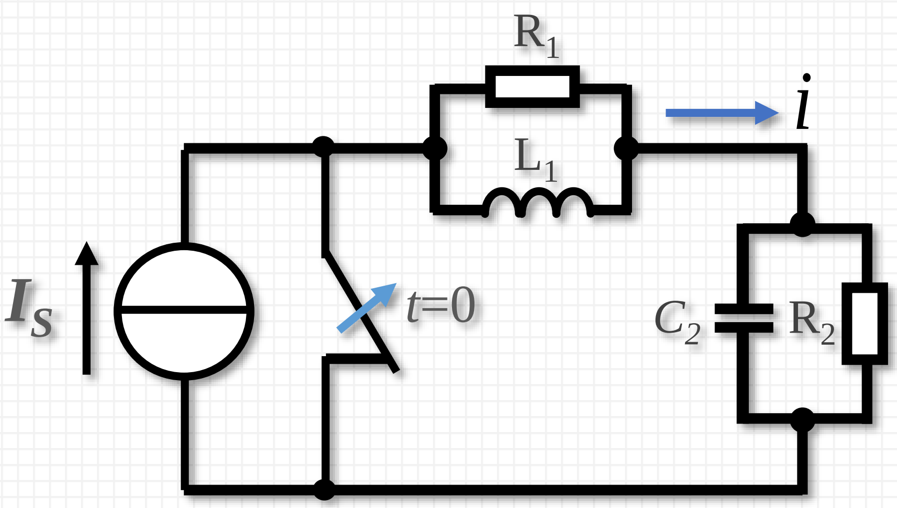


求 i'':



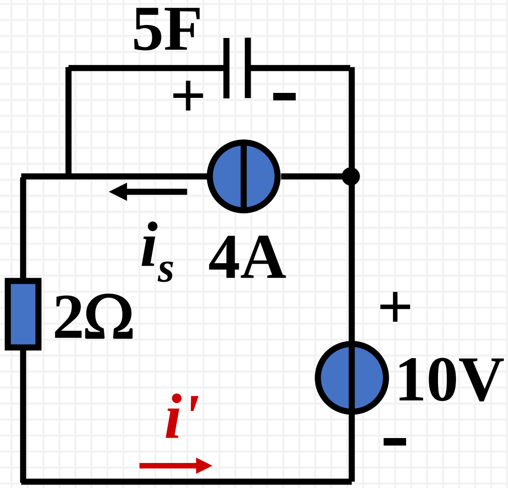
$$u_C(0^+) = 16\text{V}$$

$$i_L(0^+) = 3\text{A}$$



另外一种含LC的一阶

串联的恒流源



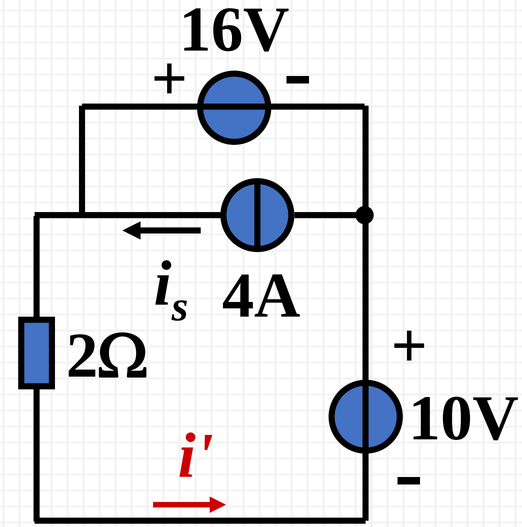
$$u_C(0^+) = 16\text{V}$$

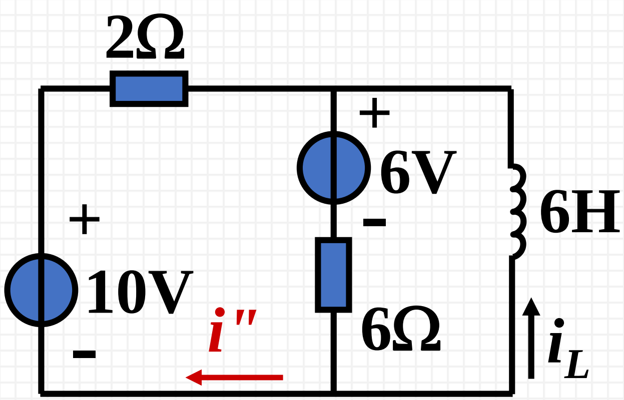
$$i'(0^+) = 13\text{A}$$

$$i'(\infty) = 4\text{A}$$

$$\tau_1 = 10\text{s}$$

$$i' = 4 + 9e^{-0.1t}\ \text{A}$$



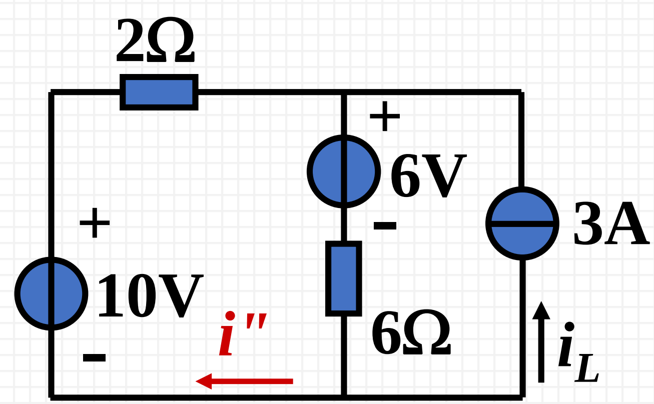


$$i_L(0^+) = 3\text{A}$$

$$i''(0^+) = \frac{10-6}{2+6} - 3\times\frac{6}{2+6} = -1.75\text{A}$$

$$i''(\infty) = 5\text{A}$$

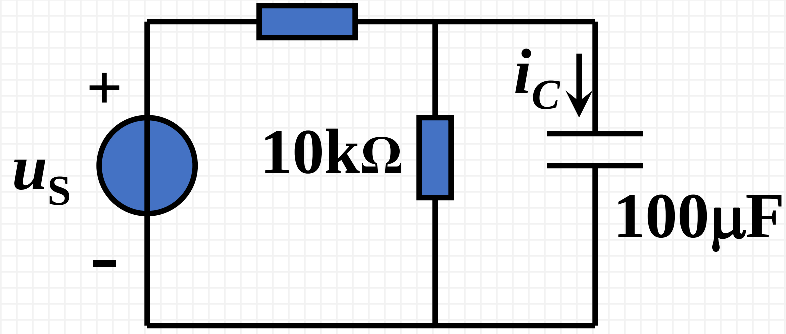
$$\tau_2 = 4\text{s}$$

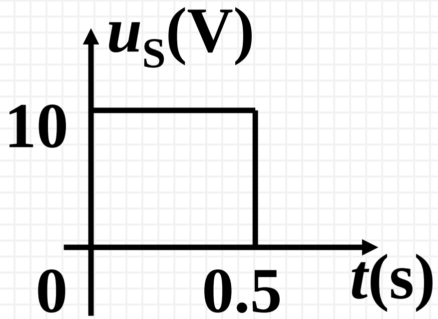


$$i'' = 5 - 6.75\text{e}^{-0.25t}\ \text{A}$$

$$i = i' + i'' = 9 + 9\text{e}^{-0.1t} - 6.75\text{e}^{-0.25t}\ \text{A} \quad t \ge 0^+$$

5. 求 $i_C(t)$.





$u_C(0^-)=0$

解： $0 \le t \le 0.5$

$$u_C(0^+) = u_C(0^-) = 0$$

$$u_C(\infty) = 5\text{V}$$

$$\tau = R_{eq}C = 0.5\text{s}$$

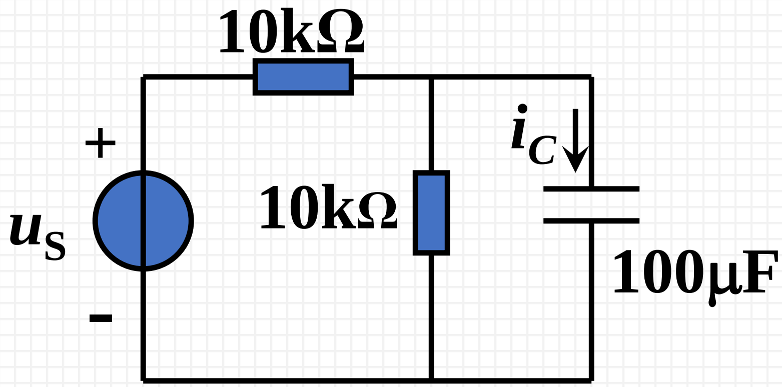
$$u_C(t) = 5\left(1 - e^{-2t}\right)\text{V}$$

$$i_C(t) = C\frac{du_C(t)}{dt} = e^{-2t}\text{mA}$$

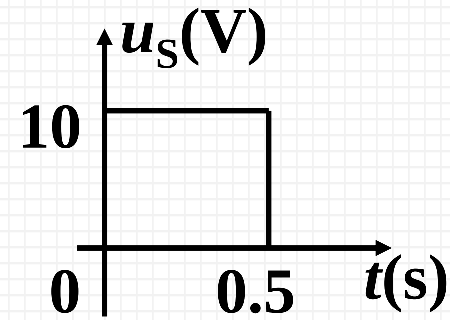
$$i_C(0^+) = 1\text{mA}$$

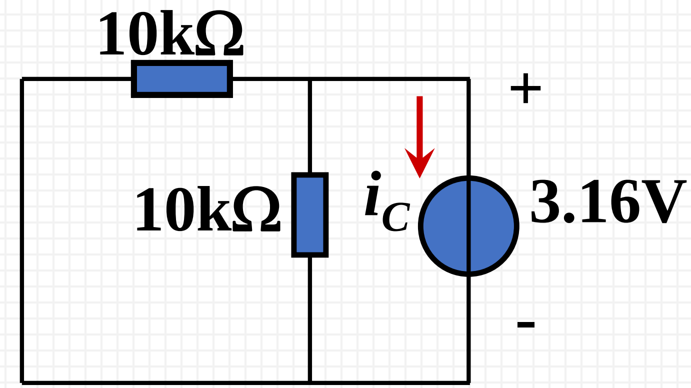
$$i_C(\infty) = 0$$

$$\tau = R_{eq}C = 0.5\text{s}$$



$u_C(0^-)=0$





0.5^+ 电路

$0 \le t \le 0.5 \quad u_C(t) = 5\left(1 - e^{-2t}\right)V$

$t \ge 0.5 \quad u_C(0.5^+) = u_C(0.5^-) = 3.16V$

$i_C(0.5^+) = -0.632mA$

$i_C(\infty) = 0$

$\tau = R_{eq}C = 0.5s$

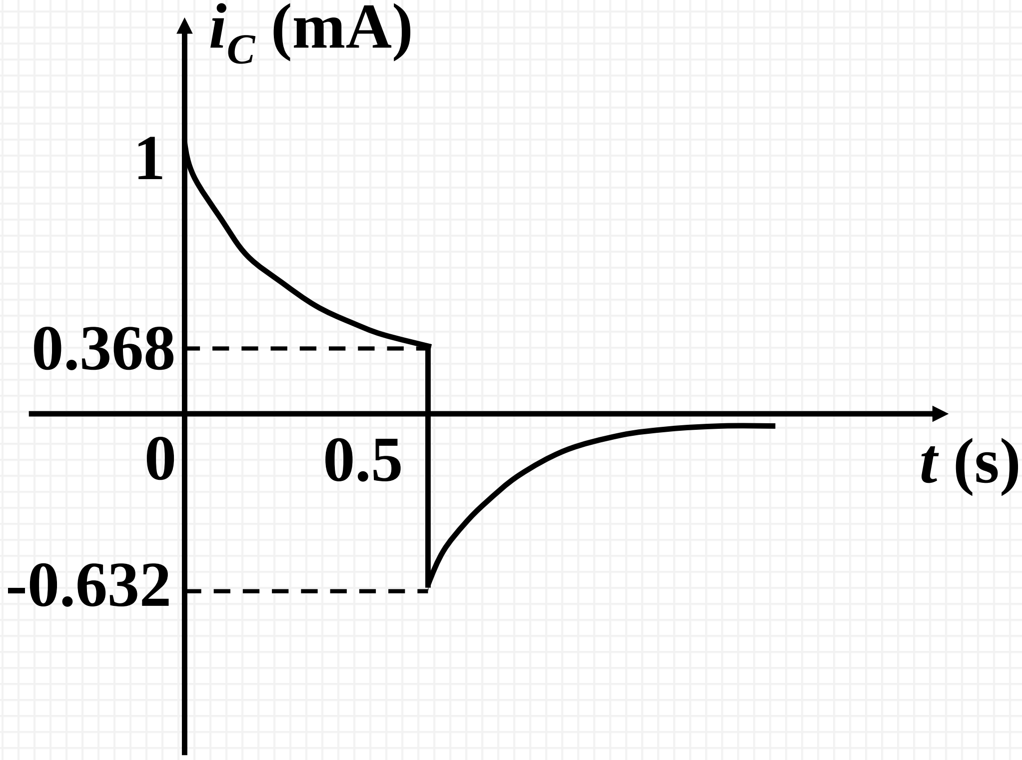
$i_C(t) = -0.632e^{-2(t-0.5)}mA \quad t > 0.5^+$

换路在 t_0 时刻发生：

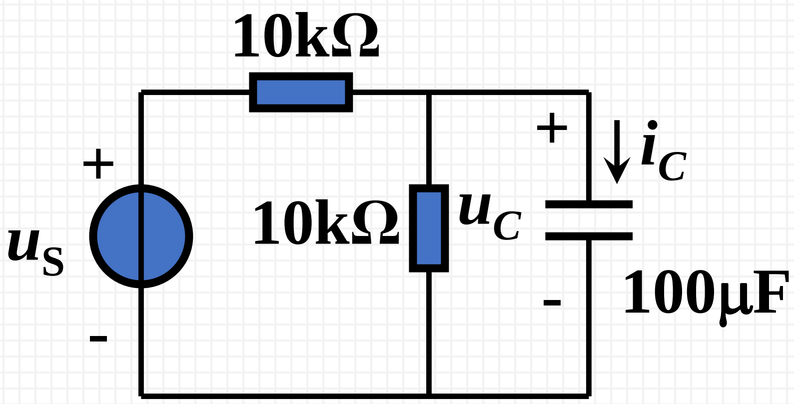
$$f(t) = f(\infty) + [f(t_0^+) - f(\infty)]e^{-\frac{t-t_0}{\tau}} \quad t > t_0$$

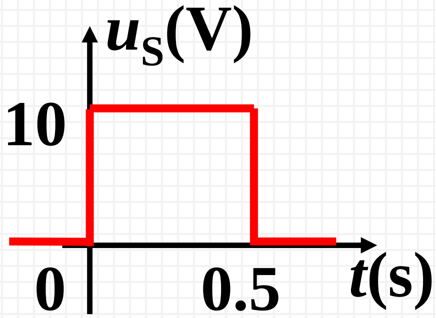
$0 \le t \le 0.5$ $\qquad$ $i_C(t) = e^{-2t}\,\text{mA}$

$t \ge 0.5$ $\qquad$ $i_C(t) = -0.632e^{-2(t-0.5)}\,\text{mA}$



求图示电路中电流 $i_C(t)$.





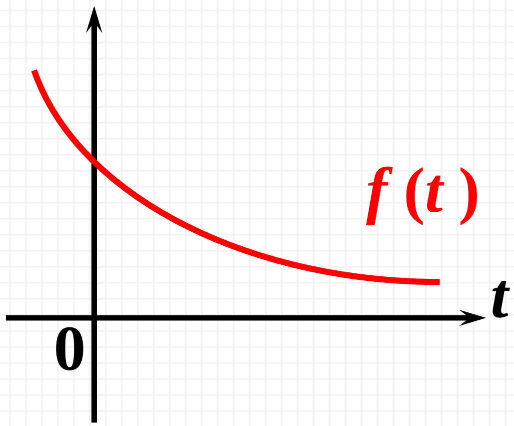
若$u_C(0^-)$ = 5V怎么求i_C?

$$u_C\left(0^+\right)=u_C\left(0^-\right)=5\text{V}$$

$$u_C\left(\infty\right)=5\text{V}$$

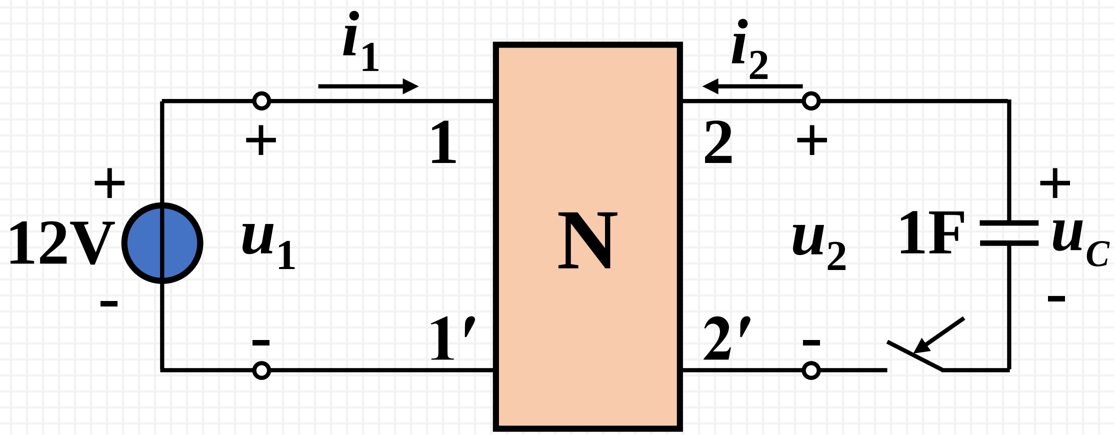
不存在过渡过程直接进入稳态

$$i_C\left(t\right)=0$$



6. 二端口N的传输参数矩阵为 $T=\begin{bmatrix}2 & 8\Omega\\ 0.5\,\mathrm{S} & 2.5\end{bmatrix}$。

t=0时刻闭合开关，已知$u_C(0^-)$=1V，求$u_C(t)$。



解法1： $u_C(0^+)=u_C(0^-)=1\text{V}$

$$\begin{cases}u_1=2u_2-8i_2\\ i_1=0.5u_2-2.5i_2\end{cases}$$

$u_1=12\text{V}\quad i_2=0$

$\Rightarrow u_C(\infty)=6\text{V}$

$$u_C(0^+) = u_C(0^-) = 1\text{V} \qquad u_C(\infty) = 6\text{V}$$

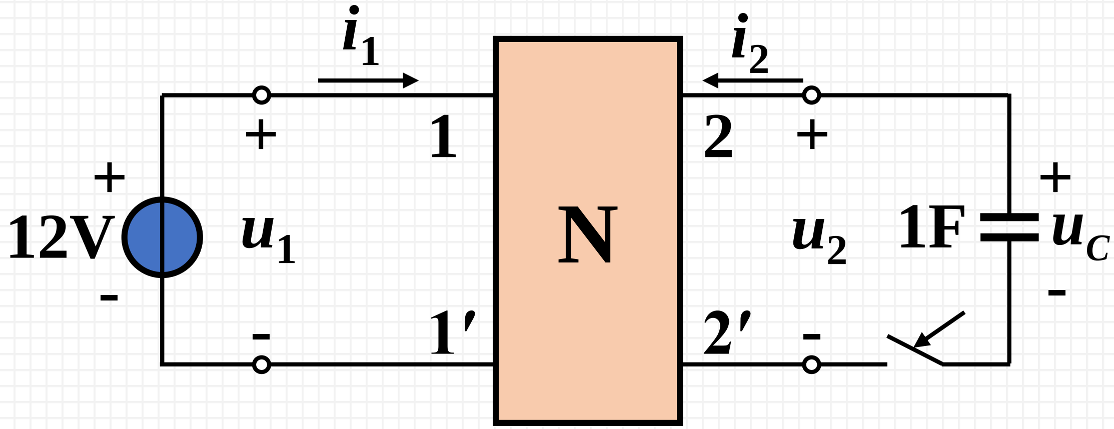
$$\begin{cases} u_1 = 2u_2 - 8i_2 \\ i_1 = 0.5u_2 - 2.5i_2 \end{cases}$$

$$u_1 = 0 \quad \Rightarrow \quad R_{\text{eq}} = \frac{u_2}{i_2} = 4\Omega$$

$$\tau = R_{\text{eq}}C = 4\text{s}$$

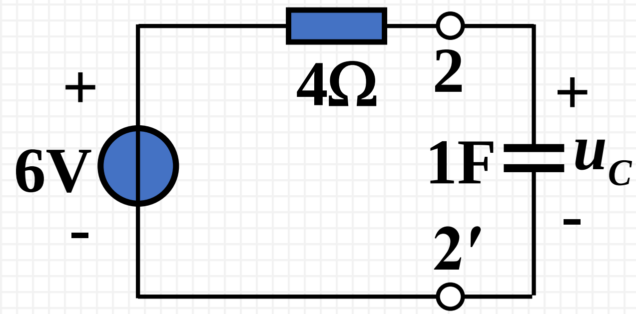
$$u_C(t) = 6 - 5\text{e}^{-0.25t} \quad \text{V} \quad t \geq 0$$

解法2



$$T=\begin{bmatrix} 2 & 8\Omega \\ 0.5\,\text{S} & 2.5 \end{bmatrix}$$

求2 - 2′左端的戴维南等效电路



$$\begin{cases} u_1 = 2u_2 - 8i_2 \\ i_1 = 0.5u_2 - 2.5i_2 \end{cases}$$

$$u_2\big|_{i_2=0} = u_1/2 = 6\text{V}$$

$$i_2\big|_{u_2=0} = -u_1/8 = -1.5\text{A}$$

$$R_{eq} = \frac{u_2\big|_{i_2=0}}{-i_2\big|_{u_2=0}} = 4\Omega$$

$$u_C(0^+) = u_C(0^-) = 1\text{V}$$

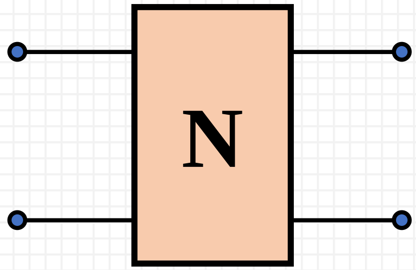
$$u_C(\infty) = 6\text{V}$$

$$\tau = R_{eq}C = 4\text{s}$$

$$u_C(t) = 6 - 5e^{-0.25t}\text{V} \quad t \ge 0$$

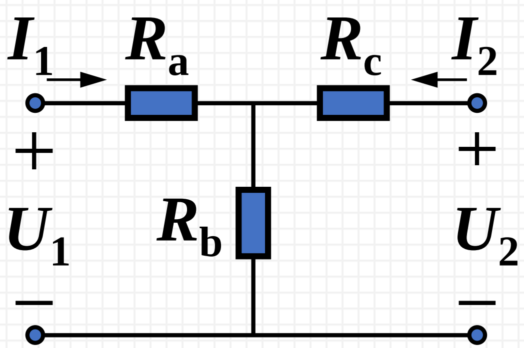
解法3 *T*参数满足$T_{11}T_{22}-T_{12}T_{21}=1$，因此，是互易二端口，不含受控源。

求等效电路：



$$T=\begin{bmatrix}2 & 8\Omega\\ 0.5\,\mathrm{S} & 2.5\end{bmatrix}$$

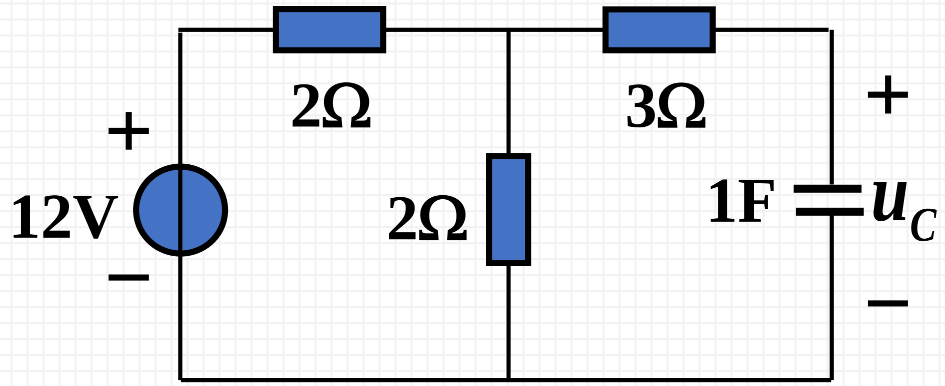
$$\begin{cases}U_1=T_{11}U_2-T_{12}I_2\\ I_1=T_{21}U_2-T_{22}I_2\end{cases}$$



$$\begin{cases}T_{11}=\dfrac{U_1}{U_2}\Big|_{I_2=0}=\dfrac{R_a+R_b}{R_b}=2\\ T_{21}=\dfrac{I_1}{U_2}\Big|_{I_2=0}=\dfrac{1}{R_b}=0.5\\ T_{22}=\dfrac{I_1}{-I_2}\Big|_{U_2=0}=\dfrac{R_c+R_b}{R_b}=2.5\end{cases}$$

解之得：

$$\begin{cases}R_a=2\Omega\\ R_b=2\Omega\\ R_c=3\Omega\end{cases}$$



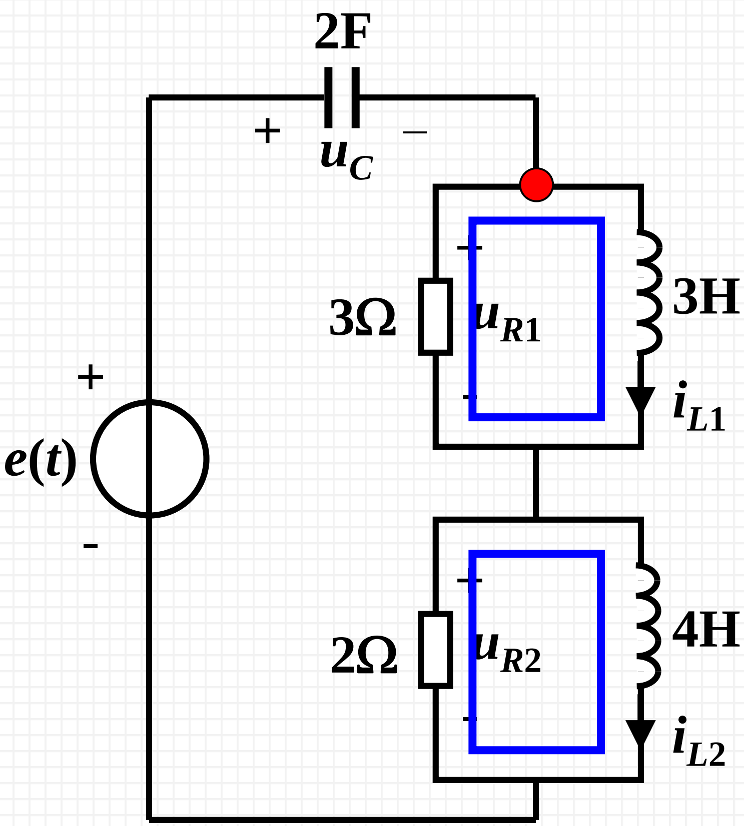
$$u_C(0^+) = u_C(0^-) = 1\text{V}$$

$$u_C(\infty) = 6\text{V}$$

$$\tau = R_{eq}C = 4\text{s}$$

$$u_C(t) = 6 - 5e^{-0.25t}\text{V} \qquad t \ge 0$$

7. 列状态方程



方法1

1)电容节点列KCL，电感回路列KVL

$$2\frac{\mathrm{d}u_C}{\mathrm{d}t}=i_{L1}+\frac{u_{R1}}{3} \qquad 3\frac{\mathrm{d}i_{L1}}{\mathrm{d}t}=u_{R1}$$

$$4\frac{\mathrm{d}i_{L2}}{\mathrm{d}t}=u_{R2}$$

2)消去非状态量u_{R1}，u_{R2}

$$\left.\begin{aligned}&u_C+u_{R1}+u_{R2}=e(t)\\&\frac{u_{R1}}{3}-\frac{u_{R2}}{2}=i_{L2}-i_{L1}\end{aligned}\right\}$$

含非状态量的两个独立方程有时较难一下子找到

$$2\frac{\mathrm{d}u_C}{\mathrm{d}t} = i_{L1} + \frac{u_{R1}}{3}$$

$$3\frac{\mathrm{d}i_{L1}}{\mathrm{d}t} = u_{R1}$$

$$4\frac{\mathrm{d}i_{L2}}{\mathrm{d}t} = u_{R2}$$

$$u_{R1} = -0.6\,u_C - 1.2\,i_{L1} + 1.2\,i_{L2} + 0.6\,e(t)$$

$$u_{R2} = -0.4\,u_C + 1.2\,i_{L1} - 1.2\,i_{L2} + 0.4\,e(t)$$

$$2\dot{u}_C = i_{L1} - 0.2\,u_C - 0.4\,i_{L1} + 0.4\,i_{L2} + 0.2\,e(t)$$

$$= -0.2\,u_C + 0.6\,i_{L1} + 0.4\,i_{L2} + 0.2\,e(t)$$

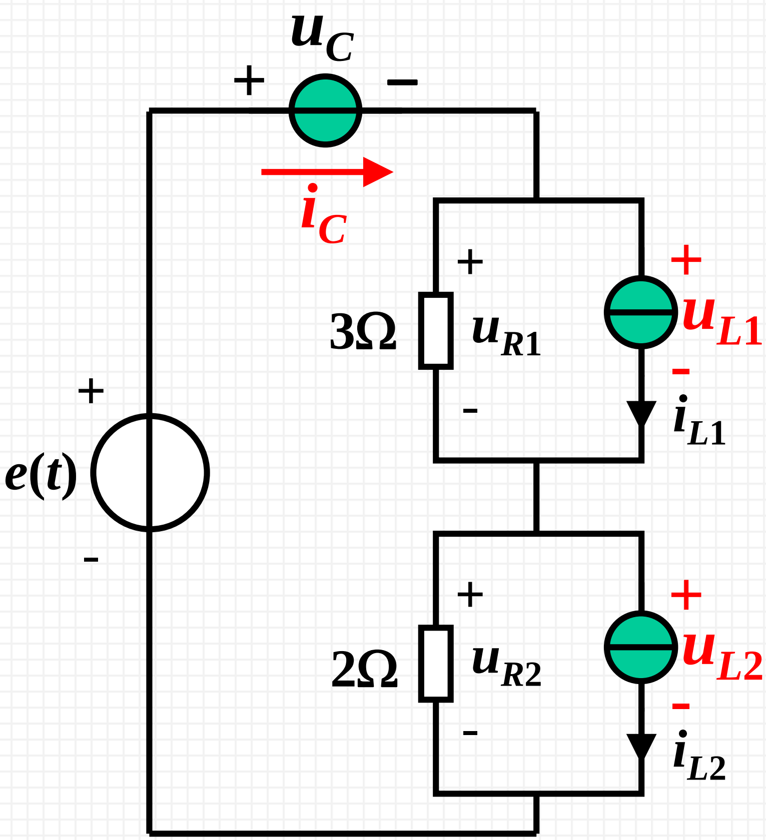
$$3\dot{i}_{L1} = -0.6\,u_C - 1.2\,i_{L1} + 1.2\,i_{L2} + 0.6\,e(t)$$

$$4\dot{i}_{L2} = -0.4\,u_C + 1.2\,i_{L1} - 1.2\,i_{L2} + 0.4\,e(t)$$

$$\begin{bmatrix} \dot{u}_C \\ \dot{i}_{L1} \\ \dot{i}_{L2} \end{bmatrix} = \begin{bmatrix} -0.1 & 0.3 & 0.2 \\ -0.2 & -0.4 & 0.4 \\ -0.1 & 0.3 & -0.3 \end{bmatrix} \begin{bmatrix} u_C \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix} e(t)$$

2、列状态方程

方法2



$$i_C = i_{L1} + \frac{u_{L1}}{3} = i_{L2} + \frac{u_{L2}}{2}$$

$$u_C + u_{L1} + u_{L2} = e(t)$$

$$u_C + 3i_C - 3i_{L1} + 2i_C - 2i_{L2} = e(t)$$

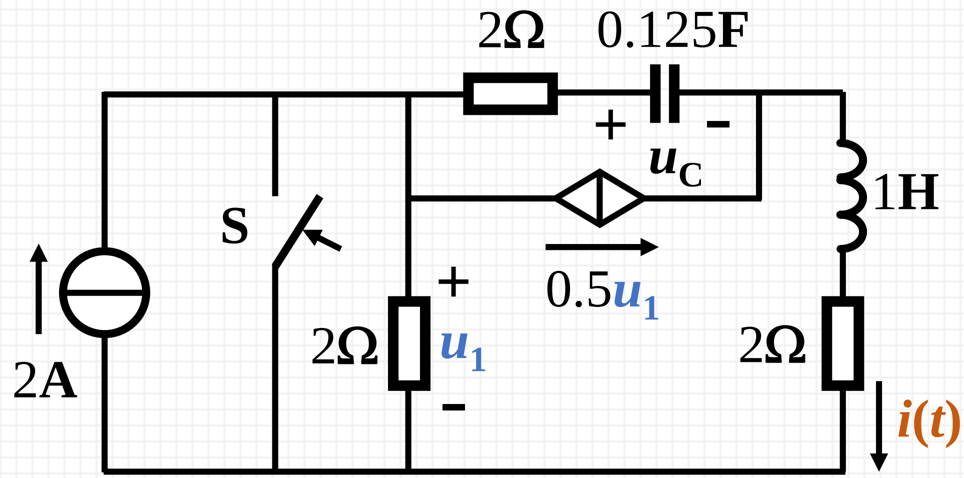
$$\Rightarrow 5i_C = -u_C + 3i_{L1} + 2i_{L2} + e(t)$$

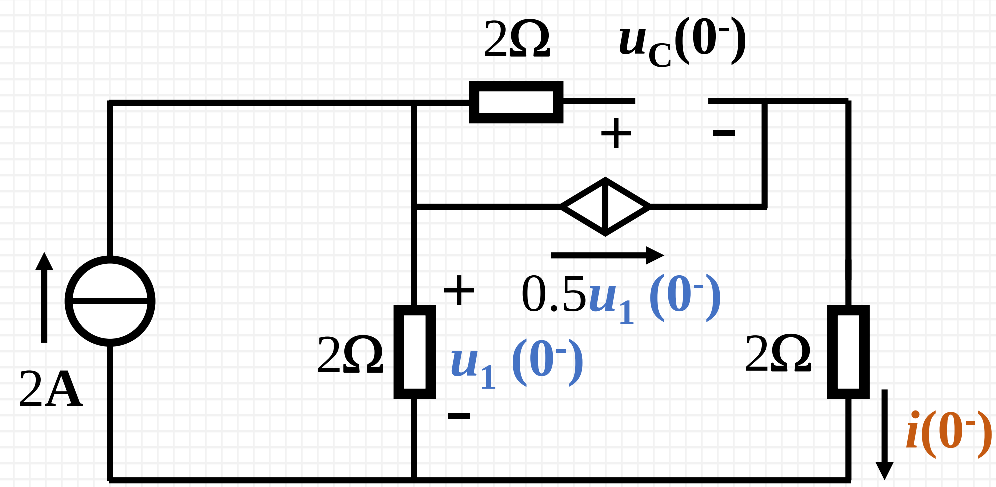
$$i_C = C\frac{du_C}{dt}$$

$$\Rightarrow \dot{u}_C = \frac{1}{C} i_C$$

同理，u_{L1}，u_{L2}.

8. **换路前电路已达稳态，在t=0时闭合开关S。求换路后的$i(t)$。用状态方程法求解。**

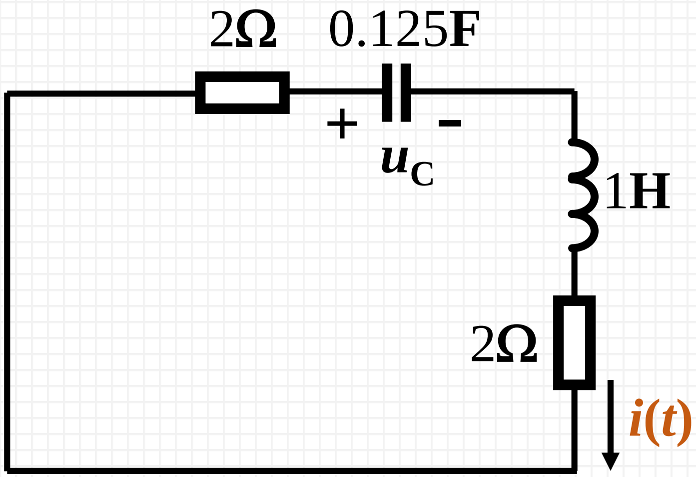




解：换路定理

$$i(0^+) = i(0^-) = 1\ A$$

$$u_C(0^+) = u_C(0^-) = 0$$



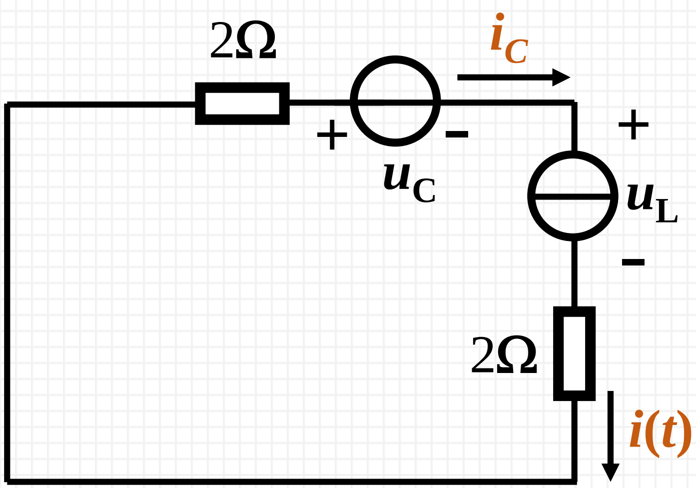
列状态方程和输出方程

$$i_C = i_L$$

$$2i_C + u_C + u_L + 2i_L = 0$$

$$u_L = -u_C - 4i_L$$

$$\begin{pmatrix} u_C' \\ i_L' \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ i_L \end{pmatrix}$$



$$i = i_L$$

求特征根

$$\begin{pmatrix} u_C' \\ i_L' \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ i_L \end{pmatrix}$$

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 2\alpha \frac{\mathrm{d}u_C}{\mathrm{d}t} + \omega_0^2 u_C = 0$$

$$\omega_0^2 = \omega_\mathrm{d}^2 + \alpha^2$$

$$u_C = K\mathrm{e}^{-\alpha t}\sin\left(\omega_\mathrm{d} t + \theta\right)$$

$$\begin{vmatrix} p & -8 \\ 1 & p+4 \end{vmatrix} = p^2 + 4p + 8 = 0$$

$$p_{1,2} = -2 \pm j2$$

$$\begin{cases} \alpha = 2 \\ \omega_d = 2 \end{cases}$$

$$i(t) = Ke^{-2t}\sin(2t + \theta)$$

$$i(t) = K e^{-2t} \sin(2t + \theta)$$

$$i_{0^+} = 1, \quad \left.\frac{di}{dt}\right|_{0^+} = -4$$

$$i(0^+) = i(0^-) = 1\ A$$

$$u_C(0^+) = u_C(0^-) = 0$$

$$\begin{pmatrix} u_C' \\ i_L' \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} u_C \\ i_L \end{pmatrix}$$

$$\begin{cases} 1 = K \sin\theta \\ -4 = -2K\sin\theta + 2K\cos\theta \end{cases}$$

$$K = -\sqrt{2}$$

$$\theta = -45^\circ$$

$$i(t) = -\sqrt{2} e^{-2t} \sin(2t - 45^\circ)\ A$$