微积分A2 的期中练习

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1. 解：∵ $f(x,y) = \dfrac{x-y}{x+y}$

$\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = 1$

$\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y) = -1$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)$ 不存在. 理由如下：

设 y 沿 $y=kx$ 趋近于 $(0,0)$

∴ $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \dfrac{x-kx}{x+kx}$

$= \dfrac{1-k}{1+k}$ 与k有关

故：$\lim\limits_{(x,y)\to(0,0)} f(x,y)$ 不存在

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$\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = 1$

$\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y) = -1$

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2. 证明：

$$\lim_{h\to 0^+} \frac{f(2h, e^{-\frac{1}{2h}}) - 2f(h, e^{-\frac{1}{h}}) + f(0,0)}{h^2}$$

$$= \frac{\partial^2 f}{\partial x^2}(0,0)$$

设 $g(h) =$ 左边

$= \dfrac{1}{h^2}[f(2h, e^{-\frac{1}{2h}}) - 2f(h, e^{-\frac{1}{h}}) + f(0,0)]$

∴ 分子、分母均 $\to 0$. 当 $h\to 0^+$

∴ 由洛必达法则

$\lim\limits_{h\to 0^+} g(h) = \dfrac{1}{2h} \times A(h)$
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设 $u=h$, $v=e^{-\frac{1}{2h}}$ $\Rightarrow f(2u, v^2) - 2f(u, v^2) + f(0,0)$

则 $A(h)$

$= f_{2u} \times 2 + f_v \times (e^{-\frac{1}{2h}}) \times (\frac{1}{2h^2})$

$-2 f_u - 2 f_{v^2} \times (e^{-\frac{1}{h}}) \times \frac{1}{h^2}$

∵ $h\to 0^+$, $-\frac{1}{2h} \to -\infty$.

$= 2(f_{2u} - f_u)$

∴ $\lim\limits_{h\to 0^+} g(h) = f_{uu}$

$= f_{xx} = \dfrac{\partial^2 f}{\partial x^2}(0,0)$

故：原命题得证.

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3. 解：充要条件：$(f(x,y)>0)$

$\because f(x,y)=g(x)\,h(y)$.

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4. 证明：$z=u(\sqrt{x^2+y^2})$.

令 $t=\sqrt{x^2+y^2}$

$\therefore t^2=x^2+y^2>0$.

$$\therefore \frac{\partial z}{\partial x}=\frac{\partial z}{\partial t}\frac{\partial t}{\partial x}$$

$$=u'(t)\times\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\times 2x$$

$$=u'(t)\times x(x^2+y^2)^{-\frac{1}{2}}$$

$$=\frac{1}{t}u'(t)\times x.$$

$$\therefore \frac{\partial^2 z}{\partial x^2}=\frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial x}$$

$$=\frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial t}\cdot\frac{\partial t}{\partial x}$$

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$$=x\cdot\left[\frac{u''(t)}{t}+\frac{u'(t)}{-t^2}\right]\times x(x^2+y^2)^{-\frac{1}{2}}$$

$$=x^2\cdot\left[\frac{u''(t)}{t^2}+\frac{u'(t)}{-t^3}\right]$$

同理：$\frac{\partial^2 z}{\partial y^2}=y^2\cdot\left[\frac{u''(t)}{t^2}+\frac{u'(t)}{-t^3}\right]$

又$\because$ $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=x^2+y^2$.

$$\therefore (x^2+y^2)\left[\frac{u''(t)}{t^2}+\frac{u'(t)}{-t^3}\right]=x^2+y^2.$$

$$\therefore u''(t)+\frac{1}{t}u'(t)=t^2$$

则 原命题得证.

解：$t>0$.

(1) $J(t)=\int_0^{+\infty} e^{-(x^2+\frac{t^2}{x^2})}dx$

$\therefore I'(t)$

$=\int_0^{+\infty} e^{-(x^2+\frac{t^2}{x^2})}\times(\frac{2t}{-x^2})dx$

故：$I'(t)=-2t\int_0^{+\infty} x^{-2}e^{-(x^2+\frac{t^2}{x^2})}dx$.

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(2) 由(1)可知：

$-2t\int_0^{+\infty} x^{-2}e^{-(x^2+\frac{t^2}{x^2})}dx=I'(t)$.

$I''(t)=-2\times\left[\int_0^{+\infty} x^{-2}e^{-(x^2+\frac{t^2}{x^2})}dx\right.$

$\left.+t\int_0^{+\infty} x^{-2}\times e^{-(x^2+\frac{t^2}{x^2})}\times\frac{t}{-x^2}dx\right]$

$=-2\int_0^{+\infty} x^{-2}e^{-(x^2+\frac{t^2}{x^2})}dx+$

$2t^3\int_0^{+\infty} x^{-4}\times e^{-(x^2+\frac{t^2}{x^2})}dx$

又$\because [e^{-(x^2+\frac{t^2}{x^2})}]'$

$=e^{-x^2-\frac{t^2}{x^2}}\times(-2x+\frac{2t^2}{x^3})$

$\therefore I'(t)=+2t\cdot\left\{\frac{1}{x}\cdot e^{-(x^2+\frac{t^2}{x^2})}\Big|_{x=0}^{x=+\infty}\right.$

$-\int_0^{+\infty}\frac{1}{x}\cdot(-2x+\frac{2t^2}{x^3})e^{-(x^2+\frac{t^2}{x^2})}$

$=2t\cdot(\int_0^{+\infty}-e^{-(x^2+\frac{t^2}{x^2})}+\frac{2t^2}{x^4}e^{\cdots})dx$

$=-2t\int_0^{+\infty}e^{-(x^2+\frac{t^2}{x^2})}$

$+4t^3\int_0^{+\infty}x^{-4}e^{\cdots}dx$

$=-2tI(t)+2I''(t)+4I'(t)$.

$\therefore 2tI(t)-3I'(t)+2I''(t)=0$.