Vector equations of lines and curves — Problem Set

Warm-up

1. A line passes through the point with position vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and has a direction vector $\mathbf{d} = \mathbf{i} - \mathbf{j}$. Write down the vector equation of this line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$.

2. Given the line $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$, state the coordinates of a specific point on the line and a direction vector for the line.

3. Write the vector equation for a line passing through the points (6,7) and (8,9).

4. Write the vector equation of a circle passing through the point (1,4) with radius 7.

5. Write the vector equation of a sphere passing through the point (1,4,5) with radius 7.

6. Find the cartesian equation of $\vec{r} = (4\cos(t) - 5)\hat{\imath} + (4\sin(t) + 6)\hat{\jmath}$.

Skill-building

1. The lines $\mathbf{r_1} = (1 + 2\lambda)\hat{\imath} + (4 - 8\lambda)\hat{\jmath} + (9 + 12\lambda)\hat{k}$ and $\mathbf{r_2} = (6 + 8\mu)\hat{\imath} + (2 - 6\mu)\hat{\jmath} + (u + 15\mu)\hat{k}$ intersect at a point. Find u and the intersection point

2. Find m so that the lines $\mathbf{r_1} = (1 + 2\lambda)\hat{\imath} + (8\lambda)\hat{\jmath}$ and $\mathbf{r_2} = (2 + \mu)\hat{\imath} + (1 + m\mu)\hat{\jmath}$ never intersect

3. Find the Cartesian equations of $\mathbf{r_1}$ and $\mathbf{r_2}$ in the last question.

4. A particle moves along a curve defined by $\mathbf{r}(t) = (t^2 - 4)\mathbf{i} + (t^3 - t)\mathbf{j}$. Determine the coordinates of the points where the curve intersects the y-axis and find the Cartesian equation of the curve by eliminating the parameter t

5. Consider the vector equation $\mathbf{r}(t) = 3\cos(2t)\mathbf{i} + 3\sin(2t)\mathbf{j}$.

   a) Show that this represents a circle and state its radius.

   b) How does the motion of a particle on this curve differ from a particle on the curve $\mathbf{r}(u) = 3\cos(u)\mathbf{i} - 3\sin(u)\mathbf{j}$?

6. A sphere is given by the equation $x^2 + y^2 + z^2 - 4x + 6y - 8z = 0$.

   1. Convert this into the vector form $|\mathbf{r} - \mathbf{c}| = a$.
   2. State the center and the radius of the sphere.

Easier Exam Questions

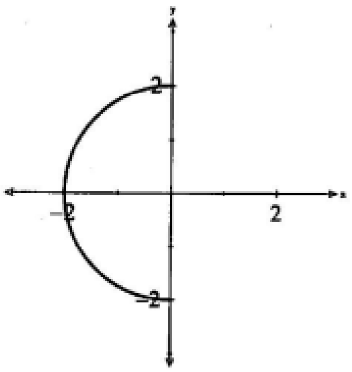
1. (Killara 2025 Q10) $\vec{r}_1(t) = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} \lambda \\ 1 \\ 2 \end{pmatrix}$ intersects $\left| \vec{r}_2(t) - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \right| = 3$ at exactly one point. Which one of the following is correct?

(A) $\lambda = -\frac{1}{4}$
(B) $\lambda = \frac{1}{4}$
(C) $t = -\frac{1}{4}$
(D) $t = \frac{1}{4}$

2. (Manly 2020 Q10) Which of the following graphs is correct for the following parametric function?

$$x = -|2\cos t| \qquad y = |2\sin t|$$

A



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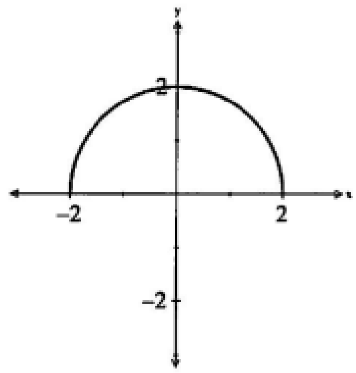

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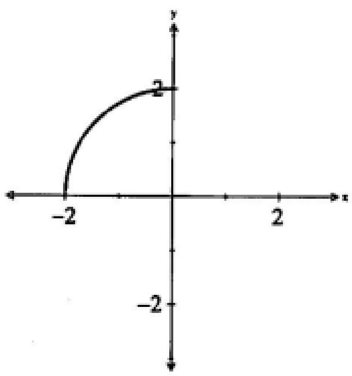

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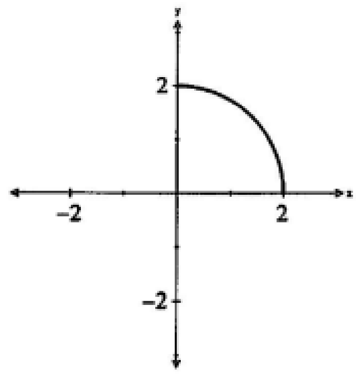


3. The vector equation of the line that passes through the points $A(1, 0, 2)$ and $B(3, 9, 6)$ is given by which of the following?

(A) $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + 9\mathbf{j} + 4\mathbf{k})$
(B) $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + 9\mathbf{j} + 8\mathbf{k})$
(C) $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(4\mathbf{i} + 9\mathbf{j} + 4\mathbf{k})$
(D) $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + 4\mathbf{k})$

4. (Manly 2022 Q6) The point $(-1, a)$ lies on the line with vector equation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, where $\mu \in \mathbb{R}$.

Which of the following is the correct value of a?

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $\frac{5}{4}$

(D) 2

5. (Manly 2024 Q7) Consider two spheres given by the equations $|\mathbf{r}_1| = 6$ and $\left|\mathbf{r}_2 - \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}\right| = 3$.

   What is the centre of the circle of intersection?

   (A) $(2, 0, 0)$

   (B) $\left(\frac{25}{8}, 0, 0\right)$

   (C) $\left(\frac{33}{8}, 0, 0\right)$

   (D) $\left(\frac{43}{8}, 0, 0\right)$

6. (Manly 2025 Q10) The point P with position vector $\overrightarrow{OP} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ lies on a sphere with centre O. Which one of the following points also lies on the sphere?

   (A) A with position vector $\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$

   (B) B with position vector $\overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$

   (C) C with position vector $\overrightarrow{OC} = 2\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$

   (D) D with position vector $\overrightarrow{OB} = 2\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$

Harder Exam Questions

1. (St George Girls 2024 Q12a) The point $(2, y, z)$ lies on the line $\mathbf{r} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$.

   Find the values of y and z. **2**

2. (Manly 2024 Q14a) Consider the lines

$$l_1 : x = y = z$$
$$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \text{ where } t \text{ is a parameter}$$

   (i) Show that l_1 and l_2 are non-parallel, non-intersecting lines. **3**

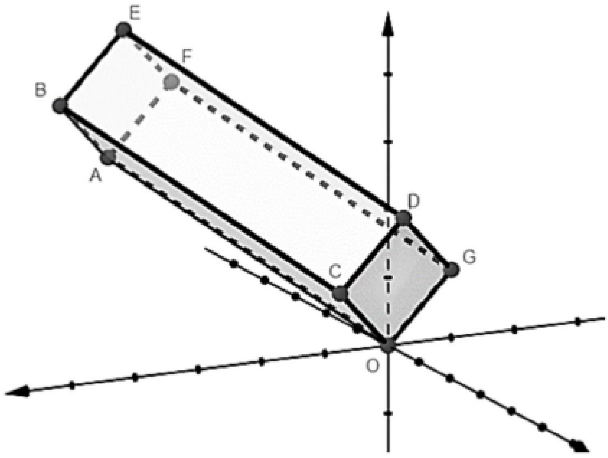
   (ii) Determine the angle between l_1 and l_2. **2**

3. (Sydney Tech 2020 Q13a) The equations of intersecting lines L and M are given below with respect to a fixed origin O.

$$L : \mathbf{r} = 11\mathbf{i} + 2\mathbf{j} + 17\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 4\mathbf{k})$$
$$M : \mathbf{r} = -5\mathbf{i} + 11\mathbf{j} + 1\mathbf{k} + \mu(p\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

where λ and μ are parameters and p is a constant.
If L and M are perpendicular, what is the value of p?

4. (Manly 2023 Q16a) The diagram shows a rectangular prism.



Let $\overrightarrow{OA} = 3\mathbf{i} - 12\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + \mathbf{j} + a\mathbf{k}$, and $\overrightarrow{OG} = -2\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(i) Show that $a = 2$ **1**

(ii) Hence or otherwise, show that $y = 0$ and $z = 2$. **3**

(iii) Calculate how high point E is above the x-y plane. **2**