Geometric representation of complex numbers — Problem Set

Warm-up

1. Convert the following into modulus-argument form:

   a) 8 + 3i
   b) 2 - 6i
   c) -1 - 2i
   d) -8 + 7i

2. Convert the following into the form $x + iy$:

   a) $2(cos(\frac{\pi}{4}) + isin(\frac{\pi}{4}))$
   b) $\sqrt{3}(cos(-\frac{\pi}{6}) + isin(-\frac{\pi}{6}))$
   c) $\sqrt{2}(cos(\frac{5\pi}{8}) + isin(\frac{5\pi}{8}))$
   d) $\sqrt{5}(cos(\frac{3\pi}{4}) + isin(\frac{3\pi}{4}))$

3. Find the modulus of $z \times w$ if:

   a) the modulus of z is 3 and the modulus of w is 7
   b) the modulus of z is 1 and the modulus of w is 4
   c) the modulus of z is 2 and the modulus of w is 6
   d) the modulus of z is $\sqrt{18}$ and the modulus of w is $\sqrt{2}$

4. Find the argument of $z \times w$ if:

   a) the argument of z is $\frac{\pi}{3}$ and the argument of w is $\frac{\pi}{3}$
   b) the argument of z is $\frac{\pi}{2}$ and the argument of w is $\frac{\pi}{6}$
   c) the argument of z is $\frac{\pi}{5}$ and the argument of w is $\frac{2\pi}{5}$
   d) the argument of z is $\frac{\pi}{3}$ and the argument of w is $\frac{\pi}{4}$

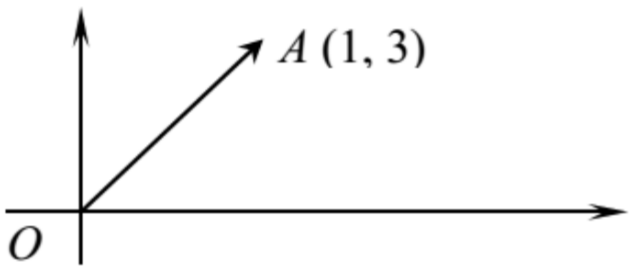
5. Find w (in modulus-argument form) so that $w \times z$ is :

   a) 3 times longer than z, and rotated 30 degrees anticlockwise from z
   b) The same length as z, and rotated 90 degrees anticlockwise from z
   c) Half as long as z, and rotated 45 degrees clockwise from z
   d) 4 times longer than z, and pointing the opposite direction to z

Skill-building

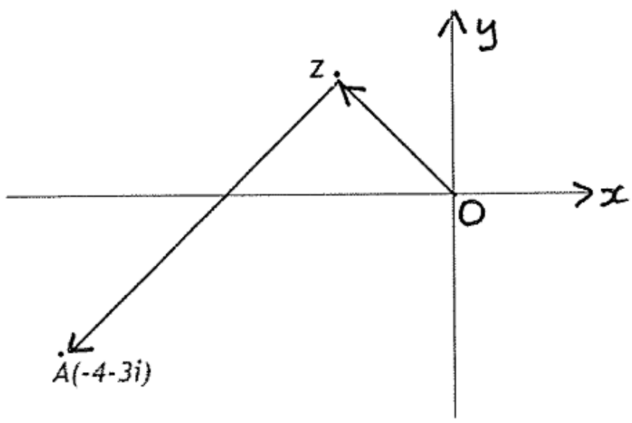
1. If $|z| = \sqrt{2}$ and $Arg(z) = \frac{\pi}{4}$,
   a) Find $|z^2|$ and $Arg(z^2)$
   b) For what values of n is z^n purely imaginary?
   c) For what values of n is z^n purely real?
   d) Sketch the first 4 powers of z on an Argand diagram

2. Let point A, below, represent the complex number 1+3i in the Argand diagram. $\vec{OA}$ is rotated

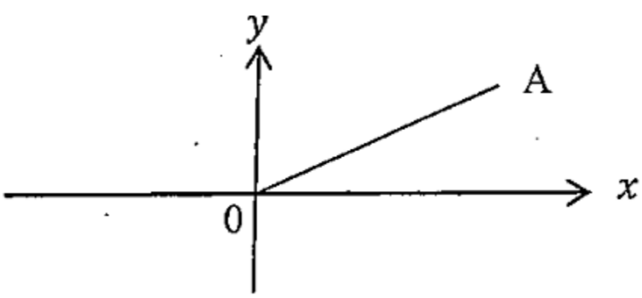


45 degrees clockwise about O and enlarged by a factor of $\sqrt{10}$. Find this new complex number in the form $a + bi$.

3. The point A represents the complex number -4-3i. $\angle OZA = 90°$ and $|\vec{ZA}| = 2|\vec{OZ}|$. find the complex number represented by Z



4. Z is an arbitrary point on the Argand diagram such that $\vec{OZ} = z$ (See diagram below). Copy the diagram and draw:



a) $z+i$

b) $z-(1+2i)$

c) iz

d) z^2

e) z^3

f) $\frac{1}{z}$

g) $-\bar{z}$

h) $z \times (cos(-\frac{\pi}{6}) + isin(-\frac{\pi}{6}))$

i) $\sqrt{z}$ (both square roots)

(Use multiple diagrams if required)

Easier Exam Questions

1. (Caringbah 2021 Q4) What is the modulus and argument of $-1-i$?

   (A) Modulus $\sqrt{2}$ and argument $-\frac{\pi}{4}$

   (B) Modulus $\sqrt{2}$ and argument $-\frac{3\pi}{4}$

   (C) Modulus 2 and argument $-\frac{\pi}{4}$

   (D) Modulus 2 and argument $-\frac{3\pi}{4}$

2. (Hornsby Girls 2022 Q11a)

   (i) Express each of the following complex numbers $z_1 = -\sqrt{2} + \sqrt{2}i$ and $z_2 = \sqrt{3} + i$ in modulus-argument form. **2**

   (ii) Represent the two complex numbers z_1 and z_2 as vectors on an Argand diagram. **1**

   (iii) Find the exact values of arg $\left(\frac{z_1}{z_2}\right)$ and $\arg(z_1 + z_2)$. **2**

3. (Manly 2020 Q7) Let $\arg(z) = \frac{\pi}{5}$ for a certain complex number z. What is $\arg(z^7)$?

   (A) $-\frac{7\pi}{5}$

   (B) $-\frac{3\pi}{5}$

   (C) $\frac{2\pi}{5}$

   (D) $\frac{3\pi}{5}$

4. (Normanhurst Boys 2025 Q11b) Given that $w = 2 - i(2\sqrt{3})$ and $\text{Arg}(zw) = \frac{\pi}{3}$, find the exact value of $\text{Arg}(z)$. **2**

5. (Penrith 2023 Q11b) Find the modulus and argument of $z = \frac{5-i}{2-3i}$. **3**

6. (St George Girls 2020 Q1) In modular-argument form, the complex number $i-1$ is:

   (A) $\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right)$

   (B) $\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$

   (C) $2\text{cis}\left(-\frac{5\pi}{4}\right)$

(D) $2\text{cis}\left(\frac{3\pi}{4}\right)$

7. (Sydney Tech 2025 Q11a) For the complex number $z = -\sqrt{3} + i$

   (i) Express z in the form $r(\cos\theta + i\sin\theta)$. **2**

   (ii) Hence, find n, the smallest positive integer such that $z^n \in \mathbb{R}$. **2**

Harder Exam Questions

1. (St George Girls 2024 Q3) For $z \in \mathbb{C}$, if $\text{Im}(z) > 0$, then $\arg\left(\frac{z\bar{z}}{z-\bar{z}}\right)$ is:

   (A) $-\frac{\pi}{2}$

   (B) 0

   (C) $\frac{\pi}{4}$

   (D) $\frac{\pi}{2}$

2. (Sydney Tech 2025 Q8) The square has vertex O at the origin. $\vec{OA}$ is represented by the complex number $2+3i$ and B is in the second quadrant. Which of the following is the complex number representing the diagonal $\vec{AC}$?

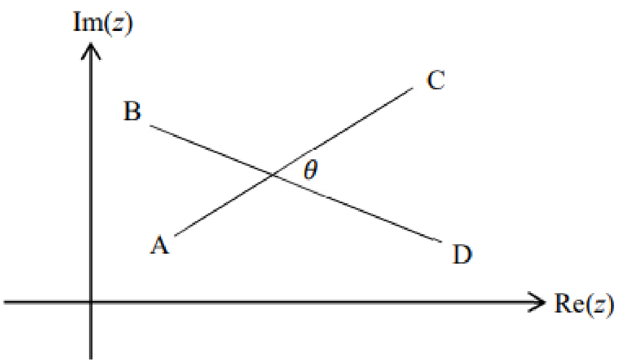
   (A) $-5-i$

   (B) $-1-5i$

   (C) $-5+i$

   (D) $1-5i$

3. (Gosford 2025 Q7) The points A, B, C and D on the Argand diagram below represent the complex numbers a, b, c, and d, respectively. The angle θ is the acute angle between AC and BD. Which of the following is a correct expression for θ?



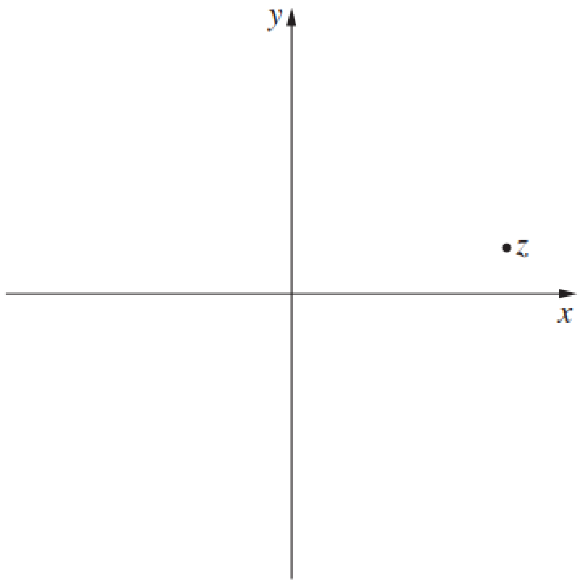
   (A) $\theta = \arg\left(\frac{c-d}{a-b}\right)$

   (B) $\theta = \arg\left(\frac{a-b}{c-d}\right)$

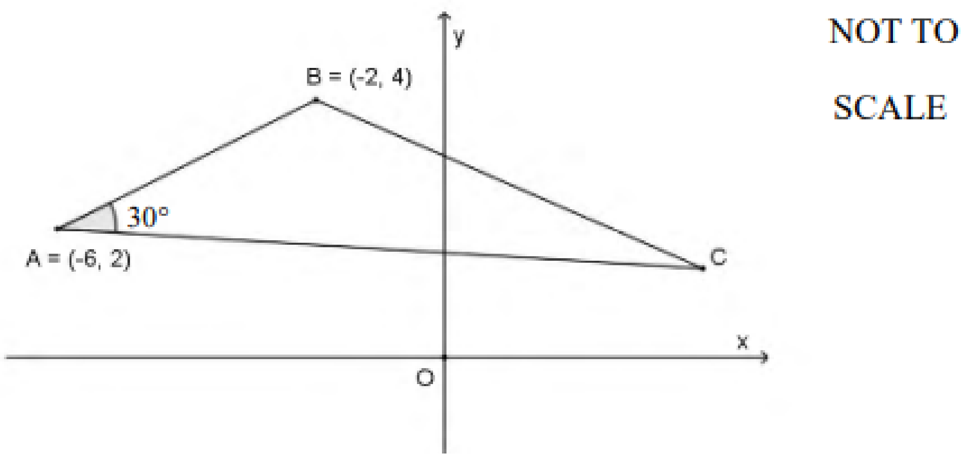
   (C) $\theta = \arg\left(\frac{c-a}{d-b}\right)$

   (D) $\theta = \arg\left(\frac{d-b}{c-a}\right)$

4. (HSC 2025 Q11a) The location of the complex number z is shown on the diagram. Indicate the locations of $\bar{z}$ and $i\bar{z}$. **2**



5. (Hornsby Girls 2022 Q12d) ΔABC is drawn in the Argand diagram above where $\angle BAC = 30°$,



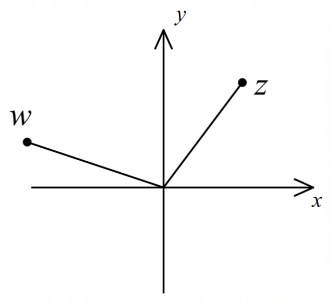
A and B are the points $(-6, 2)$ and $(-2, 4)$ respectively. The length of side AC is twice the length of side AB.

(i) Show that the complex number that represent vector $\vec{AB}$ is $4 + 2i$. **1**

(ii) Find the complex number that the point C represents. **2**

(iii) Find the complex number that the point D represents such that ΔABD is an equilateral triangle. **2**

6. (Manly 2023 Q8) The Argand diagram shows the complex numbers z and w, where z lies in the first quadrant and w lies in the second quadrant. Which complex number could lie in the



third quadrant?

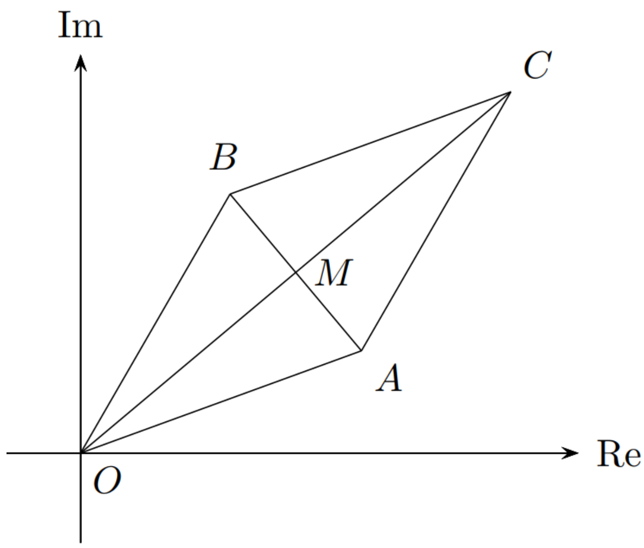
(A) $-w$

(B) $2iz$

(C) $\bar{z}$

(D) $w - z$

7. (Normanhurst Boys 2024 Q12a) Let $z_1 = \cos\alpha + i\sin\alpha$ and $z_2 = \cos\beta + i\sin\beta$ where $0 < \alpha < \beta < \frac{\pi}{2}$. The complex numbers z_1, z_2 and $(z_1 + z_2)$ are represented by the points A, B and C respectively in the Argand diagram.



(i) Give a brief reason why $OACB$ is a rhombus. **1**

(ii) Show that $r = 2\cos\left(\frac{\beta-\alpha}{2}\right)$ **2**

(iii) Show that $\theta = \frac{1}{2}(\alpha + \beta)$, and hence show that: **2**

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\beta - \alpha}{2}\right)\cos\left(\frac{\beta + \alpha}{2}\right)$$