Forces and Motion Without Resistance — Problem Set

Warm-up

1. For the following, find t in terms of x, then find x in terms of t

   a) $v = -x$

   b) $v = e^x$

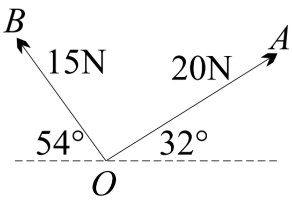
   c) $v = x^3$

   d) $v = cos^2 x$

2. The velocity of a particle satisfies $v = x^3$. If $x = 3$ when $t = 1$, find displacement (x) as a function of time.

3. A particle travels according to the equation $\ddot{x} = e^{-x}$. If its initial velocity is -1 and its initial displacement is 0, find v(t) and hence find x(t).

4. The diagram below shows two forces of magnitude 20 N and 15 N represented by the vectors $\vec{OA}$ and $\vec{OB}$.



   a) Express $\vec{OA}$ and $\vec{OB}$ as component vectors.

   b) Calculate the magnitude of the resultant of the two forces, correct to the nearest newton.

   c) Determine the direction of the resultant, correct to the nearest degree.

Skill-building

1. Objects falling under the influence of gravity experience a force in the form

$$F = \frac{mk}{x^2}$$

   A particle is projected up from a distance $x = R$, with an initial speed U.

   a) Show that the distance covered until the particle has lost half its initial speed is given by

$$x = \frac{8kR}{3UR + 8k}$$

   b) Show that the maximum height is given by $x = \frac{2R}{U^2+2}$

2. A particle starts at rest at the origin and moves to the left so that its acceleration is given by $a = 2x + 1\frac{m}{s^2}$. Find its velocity when the particle is 5 m to the left of the origin to 2 dp.

3. The acceleration of a particle is given by $\ddot{x} = 1/x \frac{cm}{s^2}$ and the particle is at rest when it is 3 cm to the right of the origin. If the particle is always moving to the right, find the velocity of the particle when its displacement is 10 cm (to 2 decimal places).

4. The acceleration of a particle is given by $a = x(x^2 - 3)^4 \frac{cm}{s^2}$. If the particle is initially 2 cm to the right of the origin, moving with a velocity of 4 cms^{-1}, find the velocity of the particle in terms of x.

5. A particle moves with a constant acceleration, a. Initially its velocity is u, and at a time t, its velocity is v and displacement is s. Prove that $v^2 = u^2 + 2as$.

Easier Exam Questions

1. (Blacktown Boys 2023 Q15b) The velocity of a particle at position $x \geq 0$ is given by $v = 4e^{-2x} - 3$, and initially the particle is at $x = 0$.

   i) Find the initial acceleration of this particle. **2**

   ii) Find the displacement of the particle x in terms of time t. **3**

   iii) Find the velocity of the particle v in terms of time t. **1**

2. (Cheltenham 2024 Q12e) A particle starts from $x = \ln(e^4 + 1)$ and moves with initial velocity e and acceleration given by $v^2(e^4 + \ln v)$, where v is the velocity of the particle.

   Find an expression for x, the displacement of the particle, in terms of v. **3**

3. (HSC 2025 Q14b) The acceleration of a particle is given by $\ddot{x} = 32x(x^2 + 3)$, where x is the displacement of the particle from a fixed-point O after t seconds, in metres. Initially the particle is at O and has a velocity of 12 ms^{-1} in the negative direction.

   i) Show that the velocity of the particle is given by $v = -4(x^2 + 3)$. **2**

   ii) Find the time taken for the particle to travel 3 metres from the origin. **2**

4. (Hurlstone 2022 Q12c) A particle moves in a straight line. At time t seconds, its displacement from a fixed origin is x metres and its velocity is v m/s. The acceleration of the particle is given by $\ddot{x} = x + 3$. At $t = 0$, the particle is at the origin and moving with velocity 3 m/s.

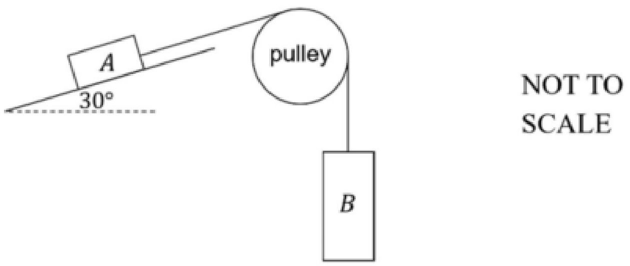
   i) Show that $v = x + 3$ **2**

   ii) Find an expression for x, the displacement of the particle, in terms of t. **2**

5. (Hurlstone 2025 Q16a) At time t the displacement, x, of a particle satisfies $t = 4 - e^{-2x}$.

   Find the acceleration of the particle as a function of x. **3**

6. (Caringbah 2024 Q4) Particle A and particle B, of masses M_A and M_B respectively, are connected by a light inextensible string passing over a frictionless pulley as shown. Particle A lies on a frictionless surface inclined at 30 degrees to the horizontal, while particle B hangs vertically from the string. The particles are initially at rest, and when they are released, neither particle moves.



Which expression shows the relationship between M_A and M_B?

(A) $M_A = M_B$
(B) $M_A = \sqrt{3}M_B$
(C) $M_A = 2M_B$
(D) $M_A = 2\sqrt{3}M_B$

Harder Exam Questions

1. (Barker 2020 Q13d) A particle of mass m kg is fired directly upwards with speed 200 ms^{-1} in a medium where the resistance is $\frac{1}{10}mv$ newtons when the speed is v ms^{-1}. Let $g = 10$ ms^{-2}.

   Hence, for the upward journey, $\ddot{x} = -\frac{1}{10}(100+v)$, where x metres is the vertical displacement from the point of projection.

   i) Show that the maximum height attained above the point of projection is $1000(2 - \log_e 3)$ metres. **3**

   ii) Show that the speed v ms^{-1} of the particle on return to its point of projection satisfies the equation $\frac{v}{100} + \ln\left|1 - \frac{v}{100}\right| + (2 - \ln 3) = 0$. **3**

2. (Hurlstone 2025 Q16d) A particle is moving horizontally. Initially, the particle is at the origin moving with velocity 1 m/s.

   The acceleration of the particle is given by

   $$a = x - 1$$

   where x is the displacement at time t.

   i) Show that the velocity of the particle is given by $v = 1 - x$. **3**

   ii) Find an expression for x as a function of t. **2**

   iii) Find the limiting position of the particle. **1**

3. (Penrith 2024 Q15d)' A car is initially at rest at a point A which is 1 km to the right of Woolworths in Seven Hills Plaza. The car then starts moving in a straight line towards Woolworths.

   For $x \neq 0$, the acceleration of the particle is given by $-\frac{k}{x^2}$, where x is the distance (in kilometres) from the shop and k is a positive constant.

i) Prove that $\dfrac{dx}{dt} = -\sqrt{\dfrac{2k(1-x)}{x}}$. **2**

ii) Use the substitution $x = \cos^2\theta$, show that the time taken to reach a distance D kilometres from Woolworths is given by

$$t = \sqrt{\frac{2}{k}} \int_0^{\cos^{-1}\sqrt{D}} \cos^2\theta \, d\theta$$

2

iii) Show that $t = \sqrt{\dfrac{1}{2k}} \left(\sqrt{D - D^2} + \cos^{-1}\sqrt{D}\right)$. **2**

4. (Sydney Boys 2023 Q13a) A particle is initially at $x = 1$ with a velocity of 2 m/s. The acceleration of the particle is given by

$$a = \frac{1}{2}\left(1 - \frac{1}{x^2}\right) \text{ m/s}^2$$

where x is the displacement of the particle from O.

i) Prove that $\dfrac{dx}{dt} = \dfrac{1+x}{\sqrt{x}}$. **3**

ii) Show that the time, in seconds, taken for the particle to reach $x = 3$ is

$$2\left(\sqrt{3} - \frac{\pi}{12} - 1\right)$$

4

5. (Sydney Tech 2021 Q15c) A particle moves along the x-axis, starting at $x = 0.1$ at time $t = 0$. The velocity of the particle is described by

$$v = \sqrt{2x}e^{-x^2}, \quad x \geq 0.1$$

where x is the displacement of the particle from the origin.

i) Show that the acceleration of the particle is given by **2**

$$a = e^{-2x^2}(1 - 4x^2)$$

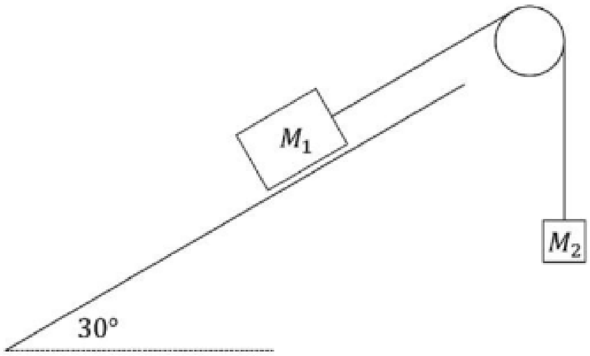
ii) Hence find the fastest speed attained by the particle. **2**

iii) Show that T, the time taken to travel from $x = 1$ to $x = 2$, can be expressed as **2**

$$T = \int_1^2 \frac{1}{\sqrt{2x}} e^{x^2} \, dx$$

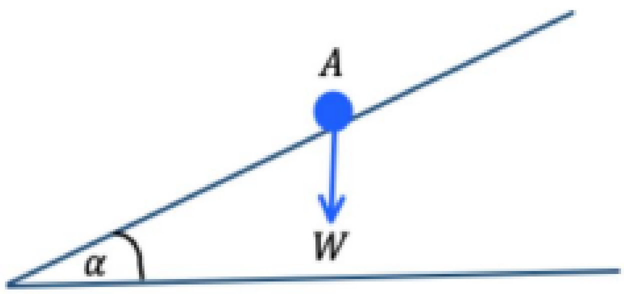
iv) Use the trapezoidal rule with three function values to give an approximate value of T correct to the nearest whole number. **2**

6. (Barker 2023 Q14a) Two masses, M_1 kilograms and M_2 kilograms, are attached by a light, inextensible string. The string is placed over a smooth pulley, and the mass M_1 rests on a smooth frictionless plane inclined at 30 degrees to the horizontal, as shown. The mass M_2 is suspended vertically by the string. The mass M_1 accelerates down the inclined plane at 2 m/s^2 and $g = 10$ m/s^2.



Given $M_1 = kM_2$, find the value of k. **3**

7. (Killara 2021 Q15b) The diagram below shows a plane inclined at an angle a to the horizontal. A body of weight W can be supported on this plane independently by either the force P that is acting parallel to the incline plane, or Q that is acting parallel to the horizontal plane.



By drawing free body diagrams in each case, Prove that **2**

$$\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{W^2}$$