Describing lines, curves and regions — Problem Set

Warm-up

1. Sketch the following on an Argand Diagram:

   a) $|z| = 1$
   b) $|z| = 2$
   c) $|z - 1| = 2$
   d) $|z + 1| = 3$
   e) $|z - i| = 1$
   f) $|z - 1 - i| = 2$

2. Sketch the following on an Argand Diagram:

   a) $arg(z) = \frac{\pi}{4}$
   b) $arg(z - 1) = \frac{\pi}{2}$
   c) $arg(z - i) = \frac{\pi}{2}$
   d) $arg(z + i + 1) = \frac{3\pi}{2}$
   e) $arg(z - 2 + 3i) = \frac{3\pi}{4}$
   f) $arg(z + 4 - 2i) = \frac{11\pi}{6}$

3. Sketch the following on an Argand Diagram:

   a) $|z| = |z - 1|$
   b) $|z| = |z - 2|$
   c) $|z - 1| = |z + 2i|$
   d) $|z + 1| = |z + 3 + 3i|$
   e) $|z - i| = |z - 1 + i|$
   f) $|z - 1 - i| = |z + 2|$

4. Sketch the following on an Argand Diagram:

   a) $Re(z) = 1$
   b) $Re(z) = -9$
   c) $Re(z) = 6$
   d) $Im(z) = 3$
   e) $Im(z) = -8$
   f) $Im(z) = -4$

Skill-building

1. Sketch the following regions on an Argand Diagram:

   a) $|z| < 3$
   b) $|z| > 7$
   c) $|z - 1| \leq 1$
   d) $|z + 1| \geq 2$
   e) $1 \leq |z - i| < 2$
   f) $1 \leq |z - 1 - i| < 6$

2. Sketch the following regions on an Argand Diagram:

   a) $arg(z) < \frac{\pi}{4}$
   b) $arg(z - 1) > \frac{\pi}{2}$
   c) $arg(z - i) \leq \frac{\pi}{2}$
   d) $arg(z + i + 1) \geq \frac{3\pi}{2}$
   e) $\frac{\pi}{2} < arg(z - 2 + 3i) < \frac{3\pi}{4}$
   f) $\frac{7\pi}{6} \leq arg(z + 4 - 2i) < \frac{11\pi}{6}$

3. Sketch the following regions on an Argand Diagram:

   a) $|z| < |z - 1|$
   b) $|z| > |z - 2|$
   c) $|z - 1| \leq |z + 2i|$
   d) $|z + 1| \geq |z + 3 + 3i|$
   e) $|z - i| < |z - 1 + i|$
   f) $|z - 1 - i| \geq |z + 2|$

4. Sketch the following regions on an Argand Diagram:

   a) $Re(z) > 1$
   b) $Re(z) \geq -9$
   c) $Re(z) < 6$
   d) $Im(z) \geq 3$
   e) $Im(z) \leq -8$
   f) $Im(z) > -4$

Easier Exam Questions

1. (Caringbah 2021 Q1) In the Argand diagram the locus of points representing the complex number z such that $|z-1+i|=9$ is a circle. What are the centre and radius of this circle?

   (A) Centre $(1,-1)$ and radius 3

   (B) Centre $(-1,1)$ and radius 3

   (C) Centre $(-1,1)$ and radius 9

   (D) Centre $(1,-1)$ and radius 9

2. (Blacktown Boys 2020 Q13c) Sketch the region in the complex plane showing the inequalities

$$1 \leq |z-1+i| \leq 2 \quad \text{and} \quad \frac{3\pi}{4} \leq \arg(z-1+i) \leq \pi$$

3

3. (Blacktown Boys 2022 Q13a) Shade on the Argand diagram the region given by **3**

$$|z-2| \leq 2 \quad \cap \quad -\frac{\pi}{4} \leq \arg z < \frac{\pi}{6}$$

4. (Cheltenham Girls 2024 Q13a) On the Argand diagram, sketch the region represented by the intersection of the inequalities: **3**

$$1 \leq |z+1| < 2 \quad \text{and} \quad \frac{\pi}{4} < \arg(z+1) < \frac{2\pi}{3}$$

5. (Fort St 2025 Q11c) Sketch the region defined by $Im(z) \geq 2$ and $\frac{\pi}{2} \leq arg(z) \leq \frac{3\pi}{4}$ **2**

6. (Girraween 2021 Q11d) Sketch on an Argand diagram: **2**

$$|z-i| \leq 5 \quad \text{and} \quad \frac{\pi}{4} < \text{Arg}(z) \leq \frac{\pi}{2}$$

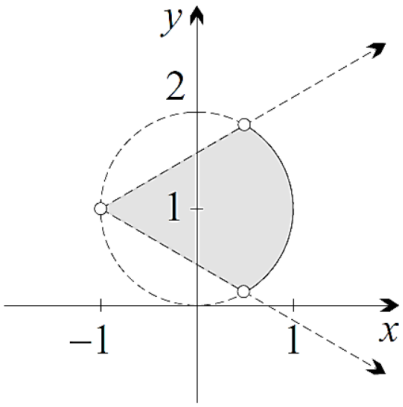
7. (Manly 2020 Q11d) A complex number z is defined by the equation $\arg(z-2-i) = \frac{3\pi}{4}$.

   (i) Plot on an Argand diagram all the points which satisfy this relationship. **2**

   (ii) What is the minimum value that $|z|$ can take? **1**

Harder Exam Questions

1. (Cheltenham Girls 2023 Q8) The shaded region below is constructed by taking the intersection of two other regions.



NOT TO SCALE

Which of the following best represents the two possible regions?

(A) $|z-i|<1$ and $-\frac{\pi}{6}\leq \text{Arg}(z-1+i)\leq \frac{\pi}{6}$

(B) $|z-i|<1$ and $-\frac{\pi}{6}\leq \text{Arg}(z+1-i)\leq \frac{\pi}{6}$

(C) $|z-i|\leq 1$ and $-\frac{\pi}{6}< \text{Arg}(z-1+i)< \frac{\pi}{6}$

(D) $|z-i|\leq 1$ and $-\frac{\pi}{6}< \text{Arg}(z+1-i)< \frac{\pi}{6}$

2. (Normanhurst Boys 2025 Q13a) Consider the region defined by

$$|z-2+2i|\leq 4 \quad \text{and} \quad -\frac{\pi}{3}\leq \text{Arg}(z-1+3i)\leq \frac{\pi}{4}$$

(i) On an Argand diagram, clearly sketch the region above. **3**

(ii) Let w represent any complex number that lies in the given region. Find the possible range of values for $\text{Arg}(w)$, giving your answer correct to two decimal places. **3**

3. (Killara 2023 Q5) The complex numbers $z=x+iy$ which satisfy the equation $\left|\frac{z-3i}{z+3i}\right|=1$ lie on

(A) circle with centre $(0,0)$ and radius 3

(B) a circle passing through the origin

(C) the straight line $y=3$

(D) the x-axis

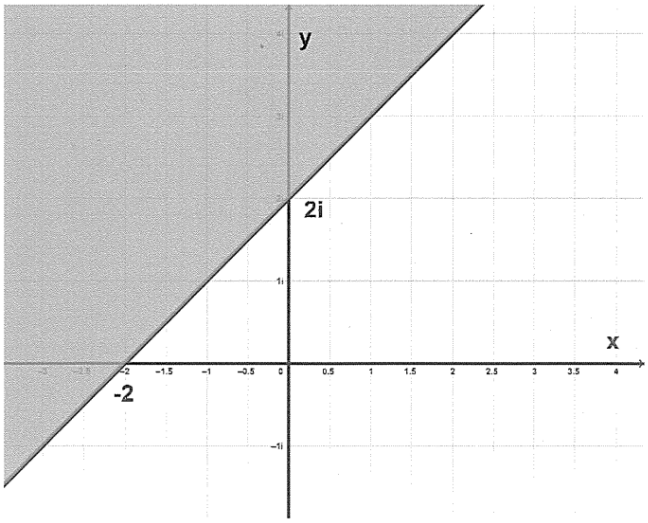
4. (Girraween 2024 Q6) The shaded area in the Argand diagram below is best represented by

(A) $|z+3+i|\geq |z+1-5i|$

(B) $|z-3-i|\geq |z+1-5i|$

(C) $|z+1-5i|\geq |z+3+i|$

(D) $|z+1-5i|\geq |z-3-i|$



5. (Girraween 2025 Q7) It is given for the complex number z that $|z-2-i|=1$. The maximum modulus for z is:

   (A) 1
   (B) $\sqrt{5}$
   (C) $\sqrt{5}-1$
   (D) $\sqrt{5}+1$

6. (Barker 2021 Q13a) On an Argand diagram, the point P representing the complex number z moves such that $|z-(1+i)|=1$.

   (i) Sketch the locus of P. **1**
   (ii) Find the greatest value of $|z|$. **1**
   (iii) Shade the region where $|z-(1+i)| \leq 1$ and $0 \leq \text{Arg}(z-1) \leq \frac{\pi}{4}$. **2**
   (iv) Find the area of the region in (iii). **1**