

Integration with Partial Fractions — Problem Set

Warm-up

1. i) Find the values of A and B such that

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

ii) Hence find $\int \frac{1}{(x+1)(x+2)} dx$

2. i) Find the values of A and B such that

$$\frac{3}{(x+3)(x+6)} = \frac{A}{x+3} + \frac{B}{x+6}$$

ii) Hence find $\int \frac{3}{(x+3)(x+6)} dx$

3. i) Find the values of A and B such that

$$\frac{1}{(x+6)(2x+7)} = \frac{A}{x+6} + \frac{B}{2x+7}$$

ii) Hence find $\int \frac{1}{(x+6)(2x+7)} dx$

4. i) Find the values of A, B and C such that

$$\frac{1}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

ii) Hence find $\int \frac{1}{(x+2)(x^2+4)} dx$

5. i) Find the values of A, B and C such that

$$\frac{2x+5}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

ii) Hence find $\int \frac{2x+5}{(x+2)(x^2+4)} dx$

Skill-building

1. Find $\int \frac{1}{(x+2)(x-2)} dx$

2. Find $\int_1^2 \frac{1}{(2x+1)(2x-1)} dx$

3. Find $\int \frac{x}{(2x+3)(x+4)} dx$

4. Find $\int \frac{x+5}{(4x+1)(3x-1)} dx$

5. Find $\int \frac{1}{(x^2+1)(x-1)} dx$

6. Find $\int_1^2 \frac{x+1}{(2x^2+4)(x-3)} dx$

7. Find $\int_0^1 \frac{x^2+3x+4}{(x^2+9)(x+1)} dx$

Easier Exam Questions

1. (Blacktown Boys 2020 Q12c)

i) Find the values of A, B and C such that

$$\frac{6x^2 + 7x - 3}{(x-5)(x^2+1)} = \frac{A}{x-5} + \frac{Bx+C}{x^2+1}$$

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ii) Hence find $\int \frac{6x^2 + 7x - 3}{(x-5)(x^2+1)} dx$

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2. (Fort Street 2022 Q12a)

i) Find A and B such that $\frac{15}{x^2 - 2x - 15} = \frac{A}{x+3} + \frac{B}{x-5}$.

2

ii) Hence or otherwise find $\int \frac{x^2 - 2x}{x^2 - 2x - 15} dx$.

2

3. (Normanhurst 2024 Q13b)

i) Find real numbers a, b and c such that

2

$$\frac{5}{x^2(2-x)} \equiv \frac{ax+b}{x^2} + \frac{c}{2-x}$$

ii) Hence or otherwise, find

2

$$\int \frac{20}{x^2(2-x)} dx$$

4. (St George Girls 2024 a)

i) (i) Find the values for A, B and C such that $\frac{3x^2 + 4x + 12}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$.

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ii) Hence, find $\int \frac{3x^2 + 4x + 12}{x(x^2+4)} dx$.

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Harder Exam Questions

1. (Caringbah 2021 Q13b)

- i) (i) Find real numbers A, B and C such that

$$\frac{x^2}{4x^2 - 9} = A + \frac{B}{2x - 3} + \frac{C}{2x + 3}$$

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ii) Hence find $\int \frac{x^2}{4x^2 - 9} dx$

2

2. (Girraween 2022 Q12c)

i) Find A, B and C so that $\frac{A}{x-1} + \frac{Bx+C}{x^2+4} = \frac{-7x-18}{(x-1)(x^2+4)}$.

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ii) Hence find $\int \frac{-7x-18}{(x-1)(x^2+4)} dx$.

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3. (Gosford 2022 Q12c)

i) Express $\frac{2x}{(x+1)(x^2+1)}$ in the form $\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ where A, B and C are real.

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ii) Hence or otherwise, find $\int_0^1 \frac{2x}{(x+1)(x^2+1)} dx$.

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4. (Killara 2023 Q11b) Use partial fractions to find

$$\int \frac{(2x^2 + 5x + 9)}{(x-1)(x^2 + 2x + 5)} dx$$

5. (Manly 2023 Q11f) (Note this question also involves integration by parts)

i) (i) Show that $\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}$

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ii) Hence find $\int \frac{1}{x^2} \tan^{-1} x dx$

3

6. (Penrith 2024 Q16b)

- i) Find the real numbers a, b and c such that

$$\frac{5 - 5x^2}{(1 + 2x)(1 + x^2)} \equiv \frac{a}{1 + 2x} + \frac{bx + c}{1 + x^2}$$

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ii) Show that $\int_0^1 \frac{5 - 5x^2}{(1 + 2x)(1 + x^2)} dx = \frac{1}{2} \left(\pi + \ln \left(\frac{27}{16} \right) \right)$.

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iii) Hence, evaluate $\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1 + 2 \sin 2x + \cos 2x} dx$ using the substitution $t = \tan x$.

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