Resisted motion in a straight line — Problem Set

Warm-up

1. For the following scenarios, write an equation of the net force experienced by the object

   a) An object slides on a smooth frictionless surface and experiences air resistance equal to mkv Newtons.

   b) An object slides on a smooth frictionless surface and experiences air resistance equal to mkv^2 Newtons.

   c) An object slides on a surface and experiences air resistance equal to mkv Newtons. It also experiences a force of 5 Newtons due to friction on the surface.

   d) An object slides on a surface and experiences air resistance equal to mkv Newtons and an additional resistive force of mkv^2 Newtons due to a parachute being deployed. The object also experiences a frictional force of 100 Newtons.

2. For the scenarios in Question 1, find $\ddot{x}$ in terms of v. Assume the mass of the object (m) is 5kg.

3. For the following, find t in terms of v. Initially the object travelled at 10m/s.

   a) $\ddot{x} = v$

   b) $\ddot{x} = v^2$

   c) $\ddot{x} = 1 + v$

   d) $\ddot{x} = 1 + v^2$

   (Hint: $\ddot{x} = \frac{dv}{dt}$)

4. For the solutions in Question 3, rearrange to find $v(t)$ (v in terms of t).

5. For the following, find x in terms of v. Initially, the object travelled at 10m/s with a displacement of 0m.

   a) $\ddot{x} = 2v$

   b) $\ddot{x} = 4v^2$

   c) $\ddot{x} = 1 + 3v$

   d) $\ddot{x} = 1 + 9v^2$

   (Hint: $\ddot{x} = v\frac{dv}{dx}$)

6. For the solutions in Question 5, rearrange to find $v(x)$ (v in terms of x).

Skill-building

1. A particle of mass m is launched with a speed of u and moves in a straight line before coming to rest. The resultant force on the particle opposes its motion and has a magnitude of $m(1+v)$ Newtons.

   a) Show that $a = -(1+v)$
   b) Find x in terms of v
   c) Find v in terms of t
   d) Find x in terms of t
   e) Show that $x + v + t = u$
   f) Find the distance travelled and time taken for the particle coming to rest

2. A particle of mass 3kg moves in a line, experiences a force of 5 Newtons and a resistance of $3 + 3v$ Newtons. The initial velocity is v_0 m/s.

   a) Show that the acceleration is given by $\ddot{x} = 1 - v$
   b) Show that $v = 1 - e^{-t} + v_0 e^{-t}$
   c) Find the terminal velocity
   d) When the velocity increases from v_0 to v_1, show that the distance travelled, x, in that time is $x = (v_0 - v_1) + \ln\left(\frac{1-v_0}{1-v_1}\right)$

3. A particle of mass 2 kg is projected across a smooth horizontal surface with an initial velocity of 20 m/s. It experiences a resistive force of $0.5v^2$ Newtons.

   1. Show that the acceleration is given by $\frac{dv}{dt} = -0.25v^2$.
   2. Show that the velocity at time t is given by $v = \frac{20}{1+5t}$.
   3. Calculate the displacement x of the particle after 2 seconds.

4. A particle of mass 1 kg is projected with an initial velocity of 1m/s. It experiences a resistive force of $(1 + v^2)$ Newtons.

   1. Show that the time t taken to reach a velocity v is given by $t = \frac{\pi}{4} - \tan^{-1}(v)$.
   2. Find the time taken for the particle to come to rest.

5. A particle of uni t mass is released from the origin moving at $\sqrt{3}$ m/s. It faces a resistance of $v + v^3$ where v is the particle's velocity.

   a. Briefly explain why the acceleration of the particle is given by $\frac{dv}{dt} = -(v + v^3)$
   b. Show that the displacement x of the particle from the origin is given by

   $$x = \tan^{-1}\left(\frac{\sqrt{3} - v}{1 + v\sqrt{3}}\right)$$

   c. Show that the time t which has elapsed when the particle is travelling with velocity v is given by $t = \frac{1}{2}\ln\left(\frac{3(1+v^2)}{4v^2}\right)$
   d. Find V^2 as a function of t and hence find the limiting value of x

Easier Exam Questions

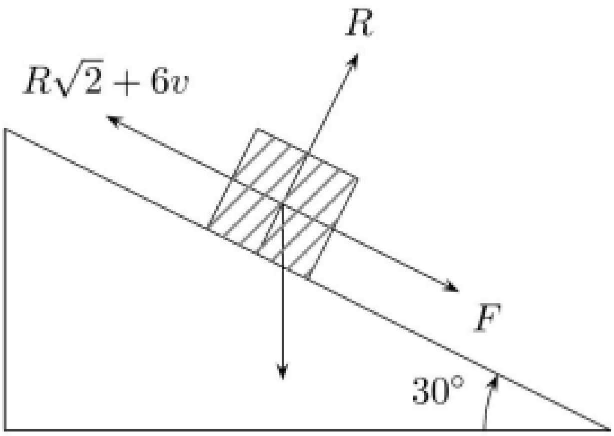
1. (HSC 2021 Q14b) An object of mass 5 kg is on a slope that is inclined at an angle of 60° to the horizontal. The acceleration due to gravity is g m s^{-2} and the velocity of the object down the slope is v m s^{-1}.
   As well as the force due to gravity, the object is acted on by two forces, one of magnitude $2v$ newtons and one of magnitude $2v^2$ newtons, both acting up the slope.

   (i) Show that the resultant force down the slope is $\frac{5\sqrt{3}}{2}g - 2v - 2v^2$ newtons. **2**

   (ii) There is one value of v such that the object will slide down the slope at a constant speed. Find this value of v in ms^{-1}, correct to 1 decimal place, given that $g = 10$. **2**

2. (Normanhurst Boys 2022 Q8) An object of mass 50 kg is pulled by a constant force of F newtons, down a long rough slope inclined at 30° to the horizontal.

   The object is met with a total resistive force of $R\sqrt{2}+6v$, where R is the normal reaction force exerted by the slope on the object and v is the velocity of the object in ms^{-1}. The acceleration of the object along the slope is a ms^{-2} and the acceleration due to gravity is g ms^{-2}.



   Which of the following is an expression for F?

   (A) $F = 50a - 25g + R\sqrt{2} + 6v$
   (B) $F = 50a - 25\sqrt{3}g + R\sqrt{2} + 6v$
   (C) $F = -R\sqrt{2} - 6v$
   (D) $F = 25g + R\sqrt{2} + 6v$

3. (Sydney Boys 2020 Q13e) A Formula 1 testing vehicle of mass M kg is capable of a top speed of 360 km/h. After it reaches this top speed, two different retarding forces combine to bring it to rest:

   First, a constant breaking force of $\frac{2}{5}M$ newtons due to the application of the brakes.

   Second, due to a parachute released from the back of the vehicle, a resistive force of $\frac{Mv^2}{200}$ newtons, where v is the speed of the car in metres per second.

   After the vehicle has reached top speed both of the above forces are applied and the vehicle eventually comes to a stop. Show that the distance travelled, x metres, after the application of the two forces, is given by $x = 100\ln\left(\frac{80+100^2}{80+v^2}\right)$. **2**

4. (Penrith 2023 Q10) When Mr Kim applies his brakes in his car, the change in velocity is given by $\frac{dv}{dt} = kv$, where k is a constant. Initially his car was travelling at 6 ms^{-1}. Ten seconds later, he was travelling at 2 ms^{-1}.

How fast was Mr Kim travelling five seconds after the brakes were applied? (answer to three significant figures)

(A) 3.29 ms^{-1}

(B) 4.82 ms^{-1}

(C) 4.72 ms^{-1}

(D) 3.46 ms^{-1}

Harder Exam Questions

1. (Girraween 2024 Q15a) A 100g ball is thrown horizontally into a headwind at 20 m/s. It experiences resistance from the wind of 0.5 Newtons, as well as air resistance of $0.05v$ Newtons, where v is the current velocity of the ball. Ignoring gravity(!!!)

(i) Show that the acceleration of the ball at time t is given by

$$\ddot{x} = -5 - 0.5v$$

1

(ii) Show that the time taken from when the ball is initially thrown to when it reaches a certain velocity is given by

$$t = -2\ln\left(\frac{10+v}{30}\right)$$

and find the time at which the ball stops moving forwards. **3**

(iii) Show that the ball's position at time t is given by

$$x = 60 - 60e^{-0.5t} - 10t$$

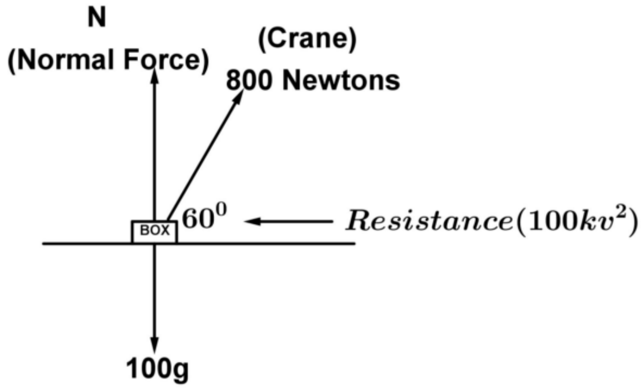
and find how far the ball has travelled before it stops moving forwards. **2**

2. (Sydney Tech 2023 Q15b) A particle of mass 4 kg moves in a straight line such that at time t seconds its displacement from a fixed origin is x metres and its speed is v ms^{-1}. The resultant force F is given as $F = 16 - 4v^2$ N.

(i) Find an expression for x as a function of v, given that the particle starts at the origin with $v = 0$ ms^{-1}. **2**

(ii) Find an expression for v as a function of t and hence find the displacement of the particle after 3 seconds. **4**

3. (Girraween 2021 Q14c) A box with a mass of 100kg is attached to a crane by a taut rope which is at an angle of 60 degrees to the horizontal. The box is initially stationary but then starts to move horizontally along the ground as the crane pulls it with an overall force of 800 Newtons. Once the box starts moving it experiences resistance in the form of friction of $100kv^2$ where v is its velocity (see diagram)
(i) Taking $g = 10$ m/s^2, by resolving forces vertically and assuming the box stays on the ground, find the magnitude of the normal force. **1**



(ii) By resolving forces horizontally, show that $\ddot{x} = 4 - kv^2$ and find the value of k if the box has a limiting horizontal velocity of 20 m/s. **2**

(iii) Show that $x = -\frac{1}{2k}\ln\left(1 - \frac{kv^2}{4}\right)$ and find how far the box has moved when it is moving at 10 m/s. **2**

4. (Killara 2021 Q14a) A vehicle of mass m kg moves in a straight line subject to a resistance $P + Qv^2$ Newtons, where v is the speed and P and Q are constants with $Q > 0$.

   (i) Form an equation of motion for the acceleration of the vehicle. **1**

   (ii) Hence, show that if $P = 0$, the distance required to slow down from the speed $\frac{3U}{2}$ to speed U is $\frac{m}{Q}\ln\left(\frac{3}{2}\right)$. **3**

   (iii) Also show that if $P > 0$, the distance required to stop from speed U is given by

$$D = \lambda \ln(1 + kU^2)$$

   Where k and λ are constants. **3**

5. (Barker 2025 Q14a) When an aircraft touches down on a runway, two different retarding forces combine to ultimately bring the aircraft to rest. There is a constant frictional force of $\frac{-1}{5}M$ newtons, where the mass of the aircraft is M kg. There is also a force of $\frac{-1}{125}Mv^2$ newtons, where v is the aircraft's velocity in m/s, due to the reverse thrust of the engines. The reverse thrust of the engines is applied from 10 seconds after touchdown until the plane stops.

   (i) The aircraft's speed at touchdown is 80 m/s.
   Show that, at the instant the reverse thrust of the engines takes effect, $v = 78$ m/s and the aircraft's displacement from its touchdown point $x = 790$ m. **3**

   (ii) Show that for $t > 10$, and until the aircraft stops, $\frac{d^2x}{dt^2} = \frac{-1}{125}(25 + v^2)$. **1**

   (iii) Show that when $t > 10$, $x = 790 + 62.5\,[\ln 6109 - \ln(25 + v^2)]$. **3**

   (iv) Calculate how far from the touchdown point the aircraft travels before it comes to rest. Answer to the nearest metre. **1**

6. (Killara 2020 Q16a) When a jet aircraft touches down, two different retarding forces combine to bring it to rest. If the jet has a mass of M kg and a speed of v m/s, there is a constant frictional force of $\frac{1}{4}M$ newtons and a force of $\frac{1}{108}Mv^2$ newtons due to the reverse thrust of the engines. The reverse thrust of the engines do not take into effect until 20 seconds after touch down.

   (i) Show that

$$\frac{d^2x}{dt^2} = -\frac{1}{4} \quad \text{for } 0 < t \leq 20$$

And that for $t > 20$, and until after the jet stops, **2**

$$\frac{d^2x}{dt^2} = -\frac{1}{108}(27 + v^2)$$

(ii) If the jet's speed at touch down is 60 m/s, show that $v = 55$ and $x = 1150$ at the instant the reverse thrust of the engines takes effect. **2**

(iii) Show that when $t > 20$,

$$x = 1150 + 54\{\ln(27 + 55^2) - \ln(27 + v^2)\}$$

2