Simple Harmonic Motion — Problem Set

Warm-up

1. A particle travels in a straight line where its position is modelled by $x = \cos(t)$.
   Prove the particle is undergoing Simple Harmonic Motion by showing $\ddot{x} = -n^2(x - c)$ for some number n and c.

2. Show the following equations of motion are simple harmonic by showing showing $\ddot{x} = -n^2(x-c)$ for some number n and c.

   a) $x = 3\sin(t) + 2$
   b) $\dot{x} = 3\cos(t)$
   c) $x = 4\sin^2(t) + 5$

3. A particle moves according to the equation $x = 4\sin(t) + 3$. Find:

   a) The amplitude of motion
   b) The centre of motion (also called the equilibrium position or mean position)
   c) The extremities of the particle's position (the endpoints the particle moves to)
   d) The period of motion
   e) The time it first reaches $x = 1$
   f) The velocity at that time

4. A particle moves in simple harmonic motion with a period of 4π seconds, centered 2 metres from the origin with an amplitude of 5 metres. It begins motion from the centre, moving in the positive direction.

   a) Write the particle's motion in the form $\ddot{x} = -n^2(x - c)$
   b) Write the particle's motion in the form $v^2 = n^2(-x^2 + bx + c)$
   c) Further rewrite the particle's motion in the form $v^2 = n^2(A^2 - (x - c)^2)$ by completing the square.
   d) Write the particle's motion in the form $x = A\sin(nt + \theta) + c$

5. A particle moves according to the equation $x = 4\sin(t) + 3\cos(t) + 3$. Find:

   a) The amplitude of motion
   b) The extremities of the particle's position (the endpoints the particle moves to)
   c) The period of motion
   d) The time it first reaches $x = 1$
   e) The velocity at that time
   f) Graph the particle's position against time, labelling the period, amplitude, equilibrium position, and the extremities.

Skill-building

1. Show the following equations of motion are simple harmonic

   a) $x = 3\sin(t) + 2\cos(t) + 2$
   b) $\dot{x} = 3\cos(t) + \sin(t)$
   c) $x = 4\sin^2(t) + 2\cos^2(t) + 5$

2. A particle moves according to the equation $x = 6\sin^2(t) + 7$. Find:

   a) The amplitude of motion
   b) The centre of motion (also called the equilibrium position or mean position)
   c) The extremities of the particle's position (the endpoints the particle moves to)
   d) The period of motion
   e) The time it first reaches $x = 10$
   f) The velocity at that time

3. A particle moves in simple harmonic motion with a period of 2 seconds, centered 1 metre from the origin with an amplitude of 3 metres. It begins motion from the furthest point from the origin, moving back towards it.

   a) Write the particle's motion in the form $\ddot{x} = -n^2(x - c)$
   b) Write the particle's motion in the form $v^2 = n^2(A^2 - (x - c)^2)$. What do n, A, and c represent here?
   c) Write the particle's motion in the form $x = A\sin(nt + \theta) + c$

4. A particle moves according to the equation $x = 4\sin(t) + 3\cos(t) + 3$. Find:

   a) The amplitude of motion
   b) The extremities of the particle's position (the endpoints the particle moves to)
   c) The period of motion
   d) The time it first reaches $x = 1$
   e) The velocity at that time
   f) Graph the particle's position against time, labelling the period, amplitude, equilibrium position, and the extremities.

Easier Exam Questions

1. (Barker 2020 Q13c) A particle's motion is defined by the equation $v^2 = 12 + 4x - x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}. Initially the particle is 6 metres to the right of the origin.

   (i) Show that the particle is moving in Simple Harmonic Motion, stating the centre of motion. **2**

   (ii) Find the period and the amplitude of the motion. **2**

   (iii) The displacement of the particle at any time t is given by the equation

   $$x = a\sin(nt + \theta) + b.$$

   Find the values of θ and b, given $0 \leq \theta \leq 2\pi$. **2**

2. (Blacktown Boys 2023 Q13c) A particle moves in a straight line and at time t seconds, its velocity, v metres per second, is related to its displacement, x metres, by

$$v^2 = 189 - 42x - 7x^2$$

(i) Show that the motion is simple harmonic in the form of **1**

$$\ddot{x} = -n^2(x - c)$$

(ii) Find the period of the motion. **1**

(iii) Find the amplitude of the motion. **2**

(iv) What is the maximum speed of this particle and what is the value of x when the particle is at the maximum speed? **1**

3. (Caringbah 2021 Q13c) A particle is moving in Simple Harmonic Motion (SHM) with acceleration $\dfrac{d^2x}{dt^2} = -4x$ ms^{-2}. If the particle starts at the origin with a velocity of 3 ms^{-1}, find:

(i) the endpoints of its motion **3**

(ii) the exact speed when the particle is 1 m from the origin **1**

4. (Girraween 2025 Q13a) A particle moves so that its displacement at time t seconds is given by

$$x = 4\cos\frac{t}{2} - 4\sqrt{3}\sin\frac{t}{2}$$

(i) Show that the particle is moving in simple harmonic motion. **1**

(ii) By expressing x in the form $A\cos\left(\frac{t}{2} + \alpha\right)$ or otherwise, find the period and amplitude of the motion. **2**

5. (Hurlstone 2024 Q16c) An object is moving in simple harmonic motion along the x-axis. The acceleration of the object is given by $\ddot{x} = -4(x-3)$ where x is its displacement from the origin, measured in metres, after t seconds.

Initially, the object is 5.5 metres to the right of the origin and moving towards the origin. The object has a speed of 8 ms^{-1} as it passes through the origin.

(i) Between which two values of x is the particle oscillating? **2**

(ii) Find the first value of t for which $x = 0$, giving the answer correct to 2 decimal places. **2**

Harder Exam Questions

1. (Sydney Boys 2025 Q13a) A ball moves in a straight line according to

$$v^2 = 9(12 + 4x - x^2)$$

where v is the velocity of the ball in cm/s and x is the displacement of the ball from the origin in cm.

(i) Prove that the ball is moving in simple harmonic motion. **1**

(ii) By considering $v^2 \geq 0$, or otherwise, find the range of x values that the ball can move in. **1**

(iii) The ball is considered to be in the extreme zone when it deviates from its central position for more than 2 cm.

It completes one whole oscillation and returns to its starting position. For what percentage of time, correct to the nearest whole number, is the ball in the extreme zone? **3**

2. (Caringbah 2021 Q14b) A particle's motion is simple harmonic in a straight line. At time t seconds its displacement from a fixed point O in the line is x metres, given by:

$$x = 1 + \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$$

(i) Show that $\ddot{x} = -(x-1)$ **1**

(ii) Find the time taken for the particle to first pass through the point O. **2**

(iii) Find in simplest exact form, the average speed of the particle during one complete oscillation of its motion. **2**

3. (Cheltenham 2024 Q11e) A particle is moving along the x axis in simple harmonic motion centred at x = 2 and with amplitude 3. The velocity of the particle is given by $v^2 = p + qx - 6x^2$, where p and q are constants. Find the value of p and q, and the period of the motion. **4**

4. (Girraween 2025 Q13b) A particle moves in simple harmonic motion so that its acceleration at time t is given by $\ddot{x} = -9(x+2)$. If the particle passes through $x = 1$ with a speed of 12 m/s, show that $v^2 = 189 - 9x^2 - 36x$ and state the period, amplitude and centre of motion. **3**

5. (Penrith 2024 Q16a) The tide at a harbour can be modelled using simple harmonic motion. At the harbour, high tide is 12 metres and low tide is at 2 metres. It takes 4 hours to go from low tide to high tide. Initially at 2 am, it is at low tide. Let t be measured in hours since 2 am.

(i) Show that $x = 7 - 5\cos\left(\frac{t\pi}{4}\right)$. **3**

(ii) A ship needs at least 5 metres of depth of water to safely enter the harbour. Find the earliest time that the ship may enter the harbour. **3**