Chapter 18 Reference Frames and Coordinate Systems

Reference Frames and Coordinate Systems

19.1 **OBJECTIVES AND INTRODUCTION**

Objectives

1. Understand the significance of the following terms when considering a reference frame: origin, lines, and planes.

2. Understand the property of reference-frame stability with respect to the description of target motion.

3. Understand the term inertial as it applies to reference frames.

4. Know the parameters of the following coordinate systems and their applications: spherical, cylindrical, and rectangular.

5. Be acquainted with the process and basic purpose of each of the following: coordinate conversion, coordinate transformation, and reference-frame translation (parallax).

6. Understand the need for weapon system alignment.

Introduction

The solution to the weapon control problem involves the gathering and processing of information relating to the position and motion of a target. In order to be useful, position and motion data must be related to a known reference. For example, an aircraft's velocity does not provide any meaningful information unless it is known whether the velocity is relative to the air in which if is flying, the ground over which it travels, or perhaps even to the distant fixed stars.

19.2 **REFERENCE FRAMES**

Frames of reference are characterized by several major properties: an origin, reference lines, reference planes, and stability.

19.2.1 **Origin.**

To be useful for measuring linear distances, a reference frame must have a reference point of origin that establishes the zero point for the frame. A distance of 100 yards has meaning within the frame since it is understood that the measured distance is relative to the frame's origin.

19.2.2 **Reference Lines.**

Simply measuring distance is not sufficient to establish position within a particular reference frame. In weapon systems, target direction from the origin is measured in angular quantities. To give meaning and a zero reference to angular quantities, reference lines are established. Reference lines pass through the origin and establish the axes of the reference frame.

19.2.3 **Reference Planes.**

In addition to reference lines, angular measurements require the definition of planes in which angles are to be measured. For example, an object located a distance of 100 yards from the origin and 45o from a reference line has virtually an infinite possibility of actual locations unless the plane in which the angle is measured is defined. Figure 19-1 illustrates two possible locations based upon a distance and an angle measured in two different planes.

A general reference frame can now be constructed based upon the three properties described above. Since it is desired to express position both with linear and angular quantities, the general reference frame is comprised of three reference axes (X,Y,Z), with the azimuth reference line arbitrarily chosen as the X axis. A horizontal reference plane is established and is that plane that contains the X and Y axes. A vertical reference plane is also employed and is defined as that plane that contains the target and the Z axis. Figure 19-2 illustrates this general reference frame.

19.2.4 **Stability.**

The property of reference frame stability directly affects the description of target motion within the reference frame. A truly stable or inertial reference frame is one that is held fixed or nonrotating with respect to the distant stars. In this type of reference frame, target motion is very easily described since all changes in target position within the frame are due only to the motion of the target itself. An unstable or non-inertial reference frame is one that is constantly rotating in a random manner, and therefore changes in target position within such a reference frame are the result of *both* target motion *and* reference frame rotation.

A reference frame may have virtually any point in the universe as its origin. Reference frames useful in weapons control originate either at the center of the earth or at the weapon station itself. In this text, attention will be concentrated on reference frames that originate at the weapon station.

19.3 **WEAPON STATION REFERENCE FRAMES**

Weapon station reference frames are fixed to, and move with, weapon platforms, such as ships, aircraft, submarines, and controlled missiles. The frames of reference used in weapon control are of two general types: those that are unstable and thus non-inertial; and those that are stabilized and considered inertial for the computation of target motion.

19.3.1 **The Unstabilized Weapon Station Frame.**

This reference frame has its origin one of the sensors of the fire control system. The reference axes are defined by the architecture of the weapon platform. One axis is oriented fore and aft, another horizontally and perpendicular to the fore-and-aft axis, and a third is oriented vertically, perpendicular to the other two. The major characteristic of the unstabilized reference frame is that it rotates about all three of its axes as the weapon station rotates in pitch, roll, and yaw. For this reason, it is considered non-inertial and thus not suitable for the description of target motion.

19.3.2 **The Stabilized Weapon Station Frame.**

The origin of this reference frame is again at the weapon station. However, the reference axes are no longer coincident with the structure of the vehicle. This is due to the incorporation of inertial elements (gyroscopes within the weapon station, which establish the axes for this frame of reference. One such element is made to track the axis of rotation of the earth, thus establishing a permanent north-south horizontal reference line. A second horizontal reference line is established by the perpendicular east-west line derived from the same inertial element as the N-S line. Another inertial element is made to track the center of the earth, and thus establish a fixed vertical axis. Because the axes of this reference frame are fixed to true north and the center of the earth, the stabilized weapon station frame is considered inertial during the flight times of most weapons. For this reason, the stabilized reference frame is employed for the description of target motion and the computation of lead angles.

Figure 19-3 illustrates that in any given weapon system there may be several functional reference frames. A surface combatant is used here as an example because it is on this type of weapon station that this concept of multiple reference frames is most pronounced. The sensor and tracking systems may gather target information in one weapon station frame (unstable) while the computational system computes lead angles in another (stable), and the launching system is positioned in still a third (unstable). Note that these related frames of reference are displaced from one another and will rotate with respect to one another.

19.4 **COORDINATE SYSTEMS**

A system of coordinates must be established to uniquely describe a position in space or to describe the magnitude and direction of target velocity with respect to a specified reference frame. For the location to have meaning we must designate a starting point or a reference point and a system of measure, such as length, time, weight, etc. In order to give the measuring system a zero point and to describe more easily the coordinate system, a reference frame will be established, as previously mentioned, consisting of three mutually perpendicular lines. The point from which these lines originate is the starting point, a zero position. In actual weapon systems, this zero position is generally the pivotal axes of the sensor and tracking subsystem since all initial target data is generated there. A coordinate, then, is one of a set of values used to locate a point relative to a reference system of lines or surfaces. (See figure 19-4).

Any weapon system can be subdivided by function into three general categories: data measurement, computation, and data utilization. By the nature of the hardware design constraints, each functional category employs a coordinate system that allows the simplest expression of the problem and the easiest method of actual implementation.

19.4.1 **Spherical Coorindate System**

A spherical coordinate system is ideal for use in the process of actually measuring target parameters. This is due to the fact that sensor and tracking systems rotate about their coordinate axes and measure distance along a straight line to the target. A generalized spherical coordinate system consists of two angles and a linear distance. This system can therefore be represented by an ordered triple of characters (, , R), which describe any point in the reference frame (figure 19-4). Specific examples of the use of the spherical coordinate system on two diverse real-world weapons platforms is illustrated in figure 19-5.

19.4.2 **Cylindrical Coordinate System**

Most weapon control computation is accomplished using a rectangular coordinate system, although some older gun fire control systems remain that use a cylindrical coordinate system. A cylindrical coordinate system employs an angle of azimuth, a vertical linear distance, and a horizontal linear distance (figure 19-6).

19.4.1 **Rectangular Coordinate System**

The rectangular or Cartesian coordinate system employs the target's range in the true north/south direction as the Y-axis distance, the target's range in the east/west direction as the X-axis distance, and the vertical height as the Z-axis distance. The rectangular coordinate system is illustrated in figure 19-7. Calculation of target velocity and prediction of future target position is made easier in cartesian coordinates, thus reducing the time required for computer solution of the fire control problem. For this reason, and the ease with which data can be shared by widely separated forces, the cartesian coordinate system is used by most modern fire control systems and command and control facilities.

19.5 **STABILIZATION**

To obtain a complete, stabilized fire control solution, usually three processes must be accomplished: conversion, transformation, and translation.

19.5.1 **Coordinate Conversion**

Coordinate conversion is the process of changing from one system of coordinates that describe a point within a reference frame to another system of coordinates describing the same point in the same reference frame. Figure 19-8 illustrates the relationship between the spherical coordinate system and the rectangular coordinate system. The equations that govern the conversion are as follows:

X = R cos cos (19-1)

Y = R cos sin (19-2)

Z = R sin (19-3)

19.5.2 **Coordinate Transformation (Reference Frame Rotation)**

The next step in the process of stabilization of the target coordinates is one of transforming the target coordinates from one reference frame to another using the same coordinate system. The reference frame rotation results from the roll and pitch of the weapon station. The process of transformation then involves the rotation of the coordinates in the unstabilized frame through the angles of pitch and roll to obtain the coordinates in a stabilized frame. Figure 19-9 provides a synopsis of coordinate conversion and transformation.

Consider first a simplified two-dimensional case where the unstabilized coordinates (R,) and the angle of pitch, , have been measured. The task is to first convert the measured coordinates into unstabilized rectangular coordinates and then rotate the unstabilized reference frame through the angle of pitch to obtain rectangular coordinates in a stabilized reference frame.

The basic relationships governing this transformation are:

X' = R cos (19-4)

Z' = R sin (19-5)

X = R cos (' +) (19-6)

Z = R sin (' +) (19-7)

Using the formulas for the cosine and sine of the sum of the two angles we have:

X = R cos (' +) = R (cos ' cos - sin ' sin)

= (R cos) cos - (R sin) sin

= X' cos - Z' sin (19-8)

Z = R sin (' +)= R (sin cos ' + cos sin ')

= (R cos ') sin + (R sin) cos

= X' sin + Z' cos (19-9)

<u>Example</u>: (TWO-DIMENSIONAL CASE)

<u>Target Parameters</u>

R = 20,000 meters

xi' = 35o (.6108 radians)

<u>Angle of Pitch</u>

= +8o (.1396 radians)

(1) Convert unstable spherical coordinates to unstable rectangular coordinates:

X' = R cos xi'

= 20,000 cos (35o)

= 20,000 (.819)

= 16,383.04 meters

Z' = R sin '

= 20,000 sin (35o)

= 20,000 (.573)

= 11,471.53 meters

Note: A starboard roll and down-pitch angle will both be considered as negative angles.

(2) Rotate the unstable coordinates through the angle of pitch using equations (19-8) and (19-9):

$\underline{X} = \underline{X}'$ cos - Z' sin

= 16,383.04 cos (8o) - 11,471.53 sin (8o)

= 16,383.04 (.99) - 11,471.53 (.139)

= 14,627.07 meters

$\underline{Z} = \underline{X}'$ sin - Z' cos

= 16,383.04 sin (8o) + 11,471.53 cos (8o)

= 16,383.04 (.139) + 11,471.53 (.99)

= 13,639.96 meters

Equations (19-8) and (19-9) represent the rectangular coordinates in the stabilized reference frame in terms of the coordinates in the unstabilized reference frame. These equations are general in nature and serve to illustrate a fundamental concept in analytic geometry, which can be stated as follow:

Rectangular coordinates in a reference frame that is rotated with respect to a second reference frame, can be expressed in terms of the rectangular coordinates of the second reference frame and *sine* and *cosine* functions of the angle of rotation.

For our purposes the angles of rotation are the angles of pitch and roll of the weapon station. Once rectangular coordinates are generated for the unstable system, direct applications of the above principle and the general forms of equations (19-8) and (19-9) for the angles of pitch and roll will yield rectangular coordinates in the stable system. Figure 19-11 illustrates the relationship between the stable and unstable coordinates when only the angle of pitch is considered.

In the general case where roll is non-zero, the coordinates (X, Y, Z) derived above would be an intermediate set of coordinates and would be used to rotate about the X axis or the roll axis as follows:

= Roll angle

X stable = X (rotation is about axis of roll) (19-10) Y stable = Y cos (- Z sin) (19-11)

Z stable = Y sin () + Z cos ()

The general scheme for reference frame rotation is as follows:

Convert unstable spherical coordinates

to unstable rectangular coordinates

X' = R cos ' cos'

Y' = R cos ' sin'

Z' = R sin '

<u>Rotate through the angle of pitch</u>

X = X' cos - Z' sin

= (R cos cos) cos - (R sin ') sin

Y = Y'(rotation is about the Y axis)

Z = X' sin + Z' cos

= (R cos cos) sin + (R sin) cos

Rotate through the angle of roll

X stable = X (Rotation is about axis or roll)

Y stable = Y cos - Z sin

Z stable = Y sin + Z cos

19.5.3 **Reference Frame Translation**

Reference frame translation can be defined as the location of a point (target) with respect to two different reference frames whose origins are physically displaced from one another. Figure 19-13 illustrates this relationship for two coplanar reference frames, A and B, which are displaced linearly by distance *h.* The point T (target) may be located in direction and distance from reference frame A by the angle Ph and the range R1. In order to transform target coordinates from reference frame A to reference frame B, the angle Ph must be taken into consideration to provide the coordinate 2 and R2.

This displacement of reference frames is common aboard ships. It is necessary, for example, to describe a target's location accurately in both the reference frame of a missile director and in the frame of its associated missile launcher. The term parallax correction is used to describe the data conversion needed to compensate for this translation of referenceframes. In addition to the horizontal displacement, illustrated in figure 19-13, a vertical displacement also exists due to the general configuration of directors and launchers (or guns) being mounted on different deck levels. (See figure 19-14)

19.6 **WEAPONS SYSTEM ALIGNMENT**

Modern weapons systems ashore and afloat include many elements that are physically displaced from one another. To function as an effective system, these elements must be aligned to one another in azimuth and elevation and must rotate in parallel planes. When this is accomplished, the following requirements will be met:

(1) All weapons bores, launcher rails, sight telescopes, and radar beams are parallel and remain parallel throughout any operating motion (when no parallax or ballistic corrections are made).

(2) All related readouts of movements are correct with respect to the established reference.

(3) All intra-element angle transmissions are correct.

To meet these requirements the following specifics must be accomplished:

(1) All readouts and transmission systems are aligned so that the zero-degrees train reference lines of the elements are parallel.

(2) The planes of rotation (roller-path planes) of the elements are parallel or effectively parallel.

(3) All readouts and transmission systems are aligned so that when the pointing lines are parallel in train, the zero elevation references are parallel.

19.6.1 **Alignment.**

Alignment of weapons system elements begins with establishment of parallel planes of rotation of the system elements. Ashore, in Marine systems, surveyors and combat engineers will determine the suitability of the chosen site with reference to slope of the ground, physical space available, and the presence of obstructions. Tuck or trailer mounted elements must be firmly emplaced and then leveled using adjustable braces provided for that purpose. Once this has been done, the only error would result from defects in the machined surface of the truck or trailer frame upon which the element rotates.

Because a ship changes shape when afloat, due to loading, and is in constant motion, due to wind and wave action, the initial phase of alignment of launchers, sensors, and navigational references is performed in dry dock. Once parallel roller-path planes are established with the ship constrained by keel blocks, the ship is floated, loaded to specified displacement, and the data re-checked. Sometimes machining of the roller path is required afloat due to the ships's bending or sagging when waterborne.

Whether aboard ship or ashore, one element is selected as a reference, and all others are aligned to it. Aboard ship, a forward gun director or, in the absence of a gun director, the lowest forward missile director is selected. Ashore, the element with greatest precision (such as the CW acquisition radar in a Hawk missile battery) is made the reference. In any case, it is helpful if the reference element is in visual sight of the other elements for train and elevation alignment.

Once parallel planes of rotation have been established, the zero degrees train and elevation reference must be established for each element, and display of angular measurements must be calibrated. Transmission of this data between elements must be verified for accuracy, and a correction applied if necessary. Ships or shore-based elements will employ calibrated metal bars or yokes called tram bars as gauges to verify the accuracy of angular measurement in periodic re-checks after alignment has been performed. The tram bar is placed between metal pads, one of which is located on the stationary base and the other on the rotating part of the element. This establishes a calibrated angle of rotation that is compared with readouts and transmission systems are adjusted to read the calibrated angle that should correct the error throughout the range of motion of the element.

One of the best ways to verify alignment between elements is by performing the star check. In this case a star is selected of proper elevation at a convenient bearing, and all system elements are trained and elevated to it. Launchers and guns will have temporary boresights installed so that they can be checked also. Stars are such a great distance away that the parallax between elements is essentially zero; therefore, all elements should have the same measurement of train and elevation when sighted on the same star (figure 19-15).

19.6.2 **Collimation.**

Many system elements such as missile fire control radars may employ several different radar beams plus an optical line-of-sight (LOS) reference. The parallelism between beams radiated by a single element is achieved by the process of collimation. At a specified distance, the element is sighted in on an optical target that has radar-receiving antennas displaced from it such that each will be located exactly in the center of the beam that antenna was designed to receive if that beam is properly aligned (figure 19-16). The element is trained and elevated back and forth across the target while power measurements are taken. If maximum signal strength is measured at some time other than when the optical target is in the center of the boresight telescope, a mechanical or electrical adjustment is made to the antenna to correct it.

19.6.3 **Aircraft Boresighting.**

The boresighting process accomplishes the same function for aircraft as battery alignment and collimation do for ship and land-based systems. Its purpose is to establish a relationship between the flight attitude of the aircraft, the bore axes of guns and weapon-releasing equipment, radar beam axes, and those of optical sights and Head-Up Displays (HUD). The process is designed to employ pertinent ballistics and mechanical data in establishing the relationship between the sight line of the optical or radio frequency aiming device and the trajectory of the weapon being fired or ejected, in order to predict its mean point of impact at predetermined ranges. The boresighting procedure is carried out in two phases: electrical boresighting and harmonization. Electrical boresighting will require that RF energy be radiated, but harmonization can be accomplished without radiation if the antenna's optical axis is in proper agreement with its RF axis (collimated).

19.6.3.1 Electrical boresighting. This procedure is the equivalent of collimation as presented previously. The aircraft's installed radar or radars are made parallel to the optical axis of the antenna. A portable frame is attached to the aircraft (figure 19-17), which incorporates radar and optical targets for various armament and sensors. This frame is employed much like the collimation tower in figure 19-16.

19.6.3.2 Harmonization. This procedure brings the radar optical axis into a parallel relationship with a baseline in the aircraft structure called the armament datum line. This is the equivalent of the process of roller-path plane-of-rotation adjustment for ships in addition to the establishment of zero train and elevation. Harmonization also includes alignment of the weapon-releasing equipment and optical sights or head-up displays (HUD).

19.7 **SUMMARY**

In this chapter we have presented the concepts of reference frames and coordinate systems as applied to the field of weapon control. Various types of reference frames were investigated with emphasis upon the weapon station reference frames. Non-inertial reference frames, those that partake in the rotational motions of the weapon station, are suitable for target data measurement and utilization but extremely cumbersome for computation. Target motion is most easily described and computations are facilitated by the employment of an inertial (stable) reference frame.

The transition from the non-inertial to the inertial frame necessitated the use of several different coordinate systems. These coordinate systems, spherical, cylindrical, and rectangular facilitate the description of target position and velocity. Finally the concepts of reference frame rotation and translation were investigated and examples given.

19.8 **REFERENCES/BIBLIOGRAPHY**

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