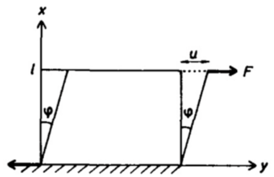
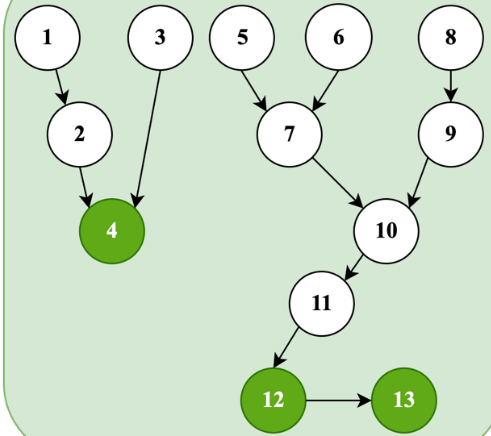
Diagram



Context

Consider a rectangular solid block with length l (along x-direction), cross-sectional area A (the area of face parallel to applied force), shear modulus n (for the block's material), and density $\\rho$. A tangential shearing force F is applied to one face, causing a tangential displacement u at the upper surface and producing a small shear angle $\\varphi$. For the analysis of elastic waves, let $u(x, t)$ denote the displacement in the y-direction at position x and time t. You should use F for the shearing force, A for the area, n for the shear modulus, l for the block length, u for the tangential displacement, $\\varphi$ for the shear angle, $\\rho$ for the density, and $\\upsilon$ for the speed of the transverse elastic wave.

DAG



Sub-questions

(a) A shearing force F is applied tangentially to a rectangular solid block as shown in Fig. Find, within elastic limits, the relation between the tangential displacement u at the upper surface and the applied force F....

(b) The elastic properties of the solid support elastic waves. Assume a transverse plane wave propagates in the x-direction, with oscillations in the y-direction. Derive the equation of motion for the y-direction displacement $u(x, t)$, where u is the displacement at ......

(c) Find, in terms of the shear modulus n and density $\\rho$, the speed $\\upsilon$ of the transverse elastic wave as described in part (b).

Final answer form: algebraic **Final answer instructions:** Your final answer should be given as a equation to reflect the relationship between... and use only...

Solutions

(a) Hooke's law for shearing yields **$$\\frac{F}{A} = n \\varphi$$ (1)** The tangential dusplacement at ... by **$$u = l \\varphi$$ (2)**. But the tangential ...by u, so$$u = l \\varphi$$ Solving for $\\varphi$ **$$\\varphi = \\frac{F}{A n}$$ (3)**. Substituting this results into the previous equation gives $$u = l \\varphi$$ **$$u = \\frac{l F}{A n}$$ (4)**

(b) The potential ... is **$$\frac{1}{2} n \\varphi^2$$ (5)**. For small ..., **$$\\varphi = \\frac{\\partial u}{\\partial x}$$ (6)** Thus ... **$$\\frac{1}{2} n \\left(\\frac{\\partial u}{\\partial x} \\right)^2$$ (7)** The kinetic energy ... is **$$\\int_{0}^{l} \\frac{1}{2} \\rho A \\left(\\frac{\\partial u}{\\partial t} \\right)^2 dx$$ (8)** According to ..., $$\\delta \\int_{t_1}^{t_2} L dt = 0$$, where L is ... , with kinetic energy **$$T = \\int_{0}^{l} \\frac{1}{2} \\rho A \\left(\\frac{\\partial u}{\\partial t} \\right)^{2} dx$$ (9)** and potential energy **$$V = \\int_{0}^{l} \\frac{1}{2} n A \\left(\\frac{\\partial u}{\\partial x} \\right)^{2} dx$$ (10)** So the action is **$$S = \\int_{t_1}^{t_2} \\int_0^l \\left[\\frac{n}{2} \\left(\\frac{\\partial u}{\\partial x} \\right)^2 - \\frac{\\rho}{2} \\left(\\frac{\\partial u}{\\partial t} \\right)^2 \\right] A dx dt$$ (11)** Applying the Euler-Lagrange equation leads to **$$\\frac{\\partial^2 u}{\\partial x^2} - \\frac{\\rho}{n} \\frac{\\partial^2 u}{\\partial t^2} = 0$$ (12)**

(c) From wave equation, $$\\frac{\\partial^2 u}{\\partial x^2}=\\frac{\\rho}{n}\\frac{\\partial^2 u}{\\partial t^2}$$ The speed of transverse elastic wave is **$$\\upsilon = \\sqrt{\\frac{n}{\\rho}}$$ (13)**

Difficulty

Medium

Physics Domain

Mechanics

Knowledge Concepts

Analytical Mechanics