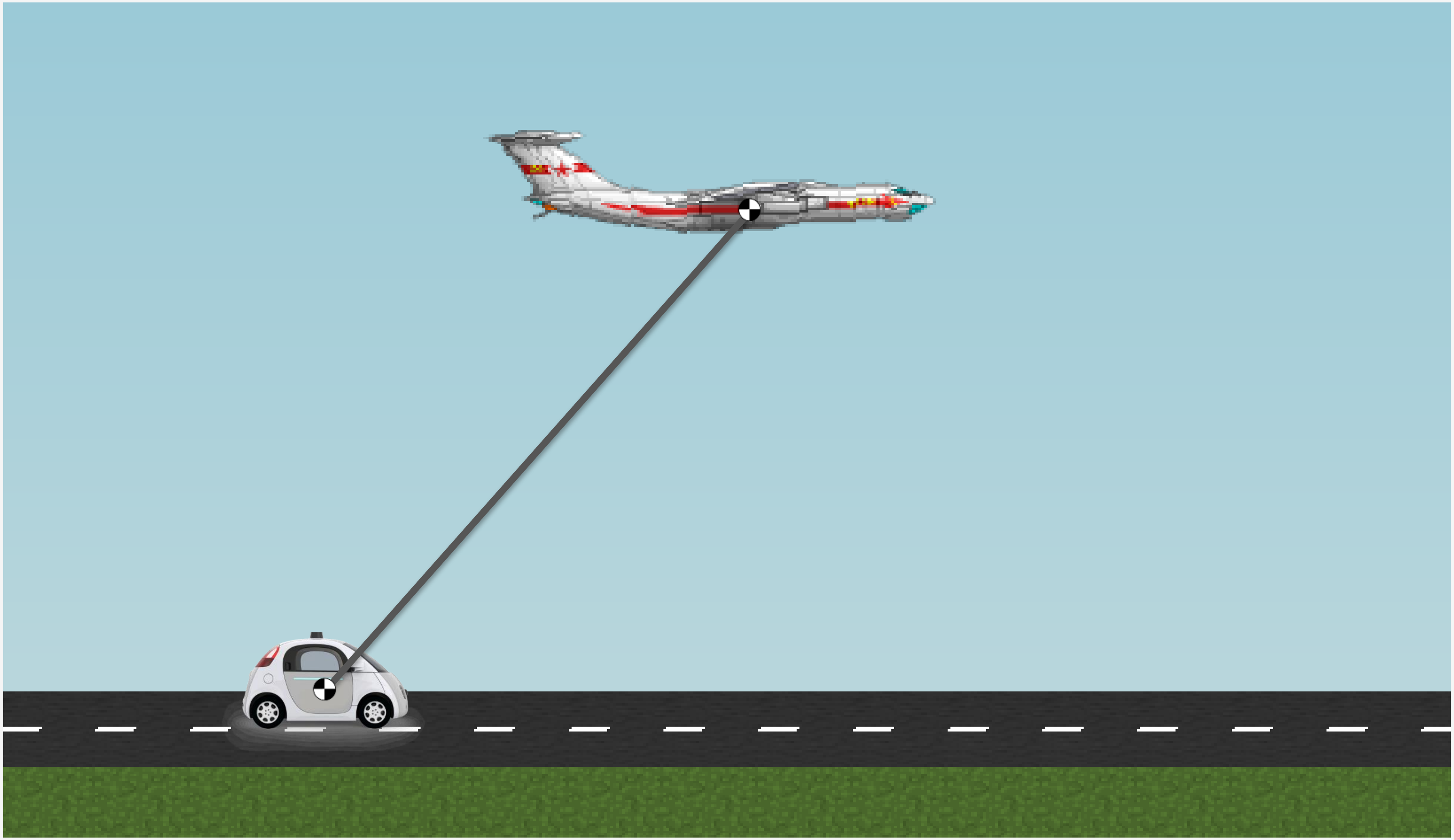
Optimal Control Problem of Cart-Plane model via DAE

Marie Curie PhD student Giovanni Licitra & Jonas Koenemann

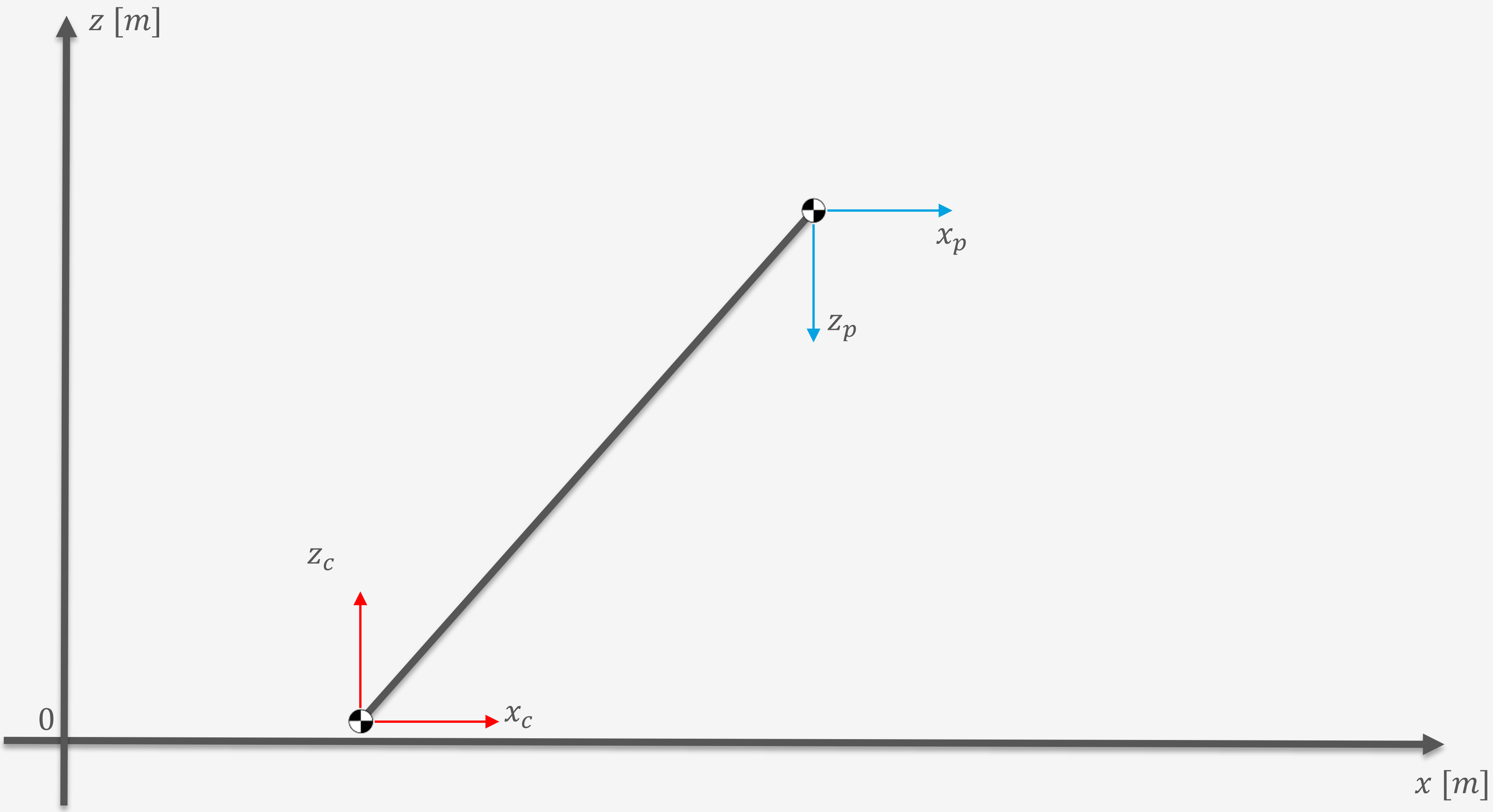
Optimal Control Problem of Cart-Plane model via DAE

Marie Curie PhD student Giovanni Licitra & Jonas Koenemann

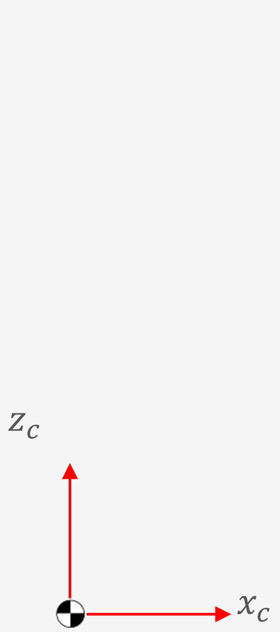
model description – Cart&Plane



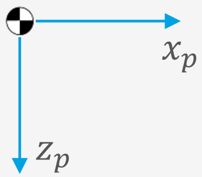
model description – Cart&Plane

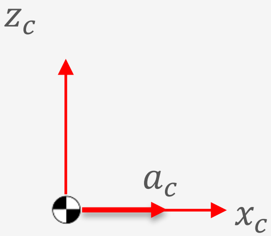


model description – Cart&Plane



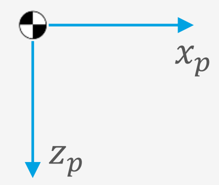
model description – Cart&Plane

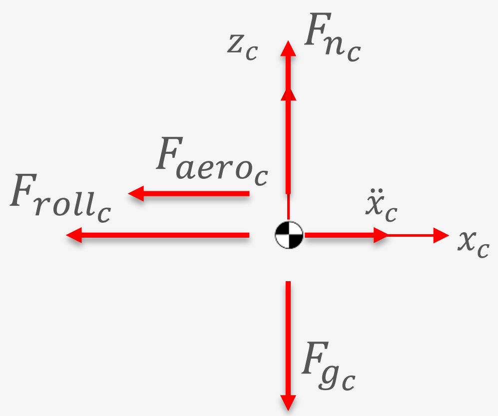




$$\begin{matrix} x_c : \\ z_c : \end{matrix} \begin{cases} F_{c_x} = m_c\,\ddot{x}_c \\ F_{c_z} = 0 \end{cases}$$

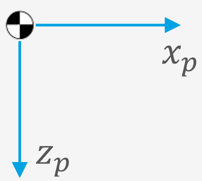
model description – Cart&Plane

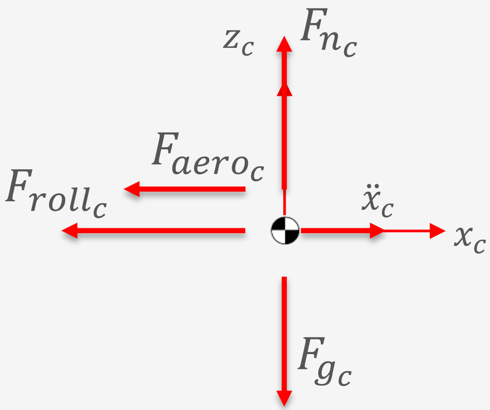




$$\begin{matrix} x_c: \\ z_c: \end{matrix} \begin{cases} F_{c_x} = m_c\,\ddot{x}_c \\ F_{c_z} = 0 \end{cases} \rightarrow \begin{cases} -F_{aero_c} - F_{roll_c} = m_c\,\ddot{x}_c \\ F_{n_c} - F_{g_c} = 0 \end{cases}$$

model description – Cart&Plane





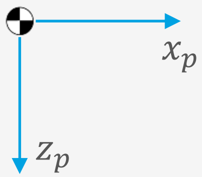
$$x_c : \begin{cases} F_{c_x} = m_c \, \ddot{x}_c \\ F_{c_z} = 0 \end{cases} \quad z_c : \quad \rightarrow \begin{cases} -F_{aero_c} - F_{roll_c} = m_c \, \ddot{x}_c \\ F_{n_c} - F_{g_c} = 0 \end{cases}$$

$$F_{n_c} = m_c \, g \qquad \textit{Normal force}$$

$$F_{aero_c} = 0.5 \, \rho \, C_{D_{aero_c}} A_c v_{x_c}^2 \qquad \textit{Aerodynamic drag}$$

$$F_{roll_c} = C_{D_{roll_c}} F_{n_c} \qquad \textit{Roll Drag [asphalt-tire]}$$

model description – Cart&Plane



model description – Cart&Plane

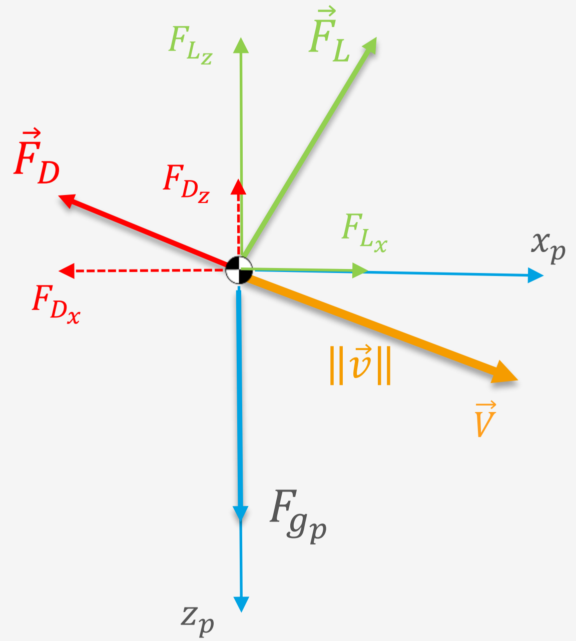
$\|\vec{v}\|$ *Airspeed*

$\hat{e}_L = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ v_{p_z} \end{bmatrix}$ *Lift direction*

$\hat{e}_D = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ -v_{p_z} \end{bmatrix}$ *Drag direction*

$$\vec{F}_L = \frac{1}{2}\rho\|\vec{v}\|^2 C_L S_{ref}\hat{e}_L$$

$$\vec{F}_D = \frac{1}{2}\rho\|\vec{v}\|^2 C_D S_{ref}\hat{e}_D$$

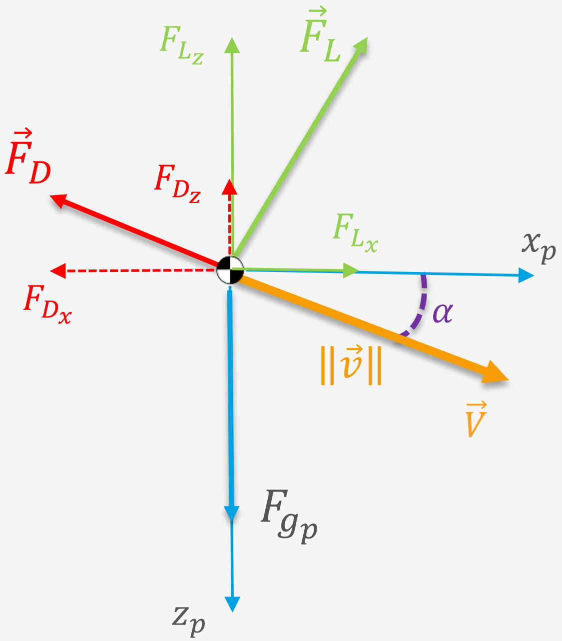


model description – Cart&Plane

$\|\|\vec{v}\|\|$	*Airspeed*
$\hat{e}_L = \frac{1}{\|\|\vec{v}\|\|}\begin{bmatrix}-v_{p_x}\\ v_{p_z}\end{bmatrix}$	*Lift direction*
$\hat{e}_D = \frac{1}{\|\|\vec{v}\|\|}\begin{bmatrix}-v_{p_x}\\ -v_{p_z}\end{bmatrix}$	*Drag direction*

$$\vec{F}_L = \frac{1}{2}\rho\|\vec{v}\|^2 C_L S_{ref}\hat{e}_L$$

$$\vec{F}_D = \frac{1}{2}\rho\|\vec{v}\|^2 C_D S_{ref}\hat{e}_D$$



$input\colon \alpha = angle\ of\ attack\ [rad]$

model description – Cart&Plane

$\|\vec{v}\|$ *Airspeed*

$\hat{e}_L = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ v_{p_z} \end{bmatrix}$ *Lift direction*

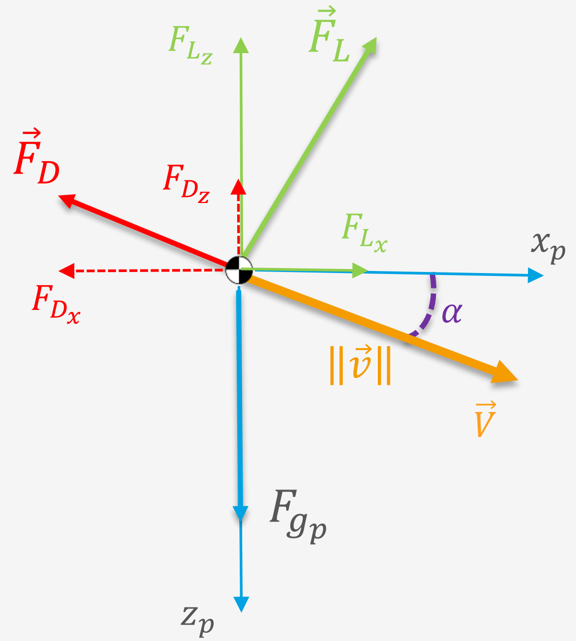
$\hat{e}_D = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ -v_{p_z} \end{bmatrix}$ *Drag direction*

$$\vec{F}_L = \frac{1}{2}\rho\|\vec{v}\|^2 C_L(\alpha) S_{ref}\hat{e}_L$$

$$\vec{F}_D = \frac{1}{2}\rho\|\vec{v}\|^2 C_D(\alpha) S_{ref}\hat{e}_D$$

$$C_L(\alpha) = 2\pi\frac{10}{12}\alpha$$

$$C_D(\alpha) = C_{D_0} + \frac{C_L(\alpha)^2}{AR\,\pi}$$



$input\!: \alpha = angle\ of\ attack\ [rad]$

model description – Cart&Plane

$\|\vec{v}\|$ *Airspeed*

$\hat{e}_L = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ v_{p_z} \end{bmatrix}$ *Lift direction*

$\hat{e}_D = \frac{1}{\|\vec{v}\|}\begin{bmatrix} -v_{p_x} \\ -v_{p_z} \end{bmatrix}$ *Drag direction*

$$\vec{F}_L = \frac{1}{2}\,\rho\|\vec{v}\|^2 C_L(\alpha) S_{ref}\,\hat{e}_L$$

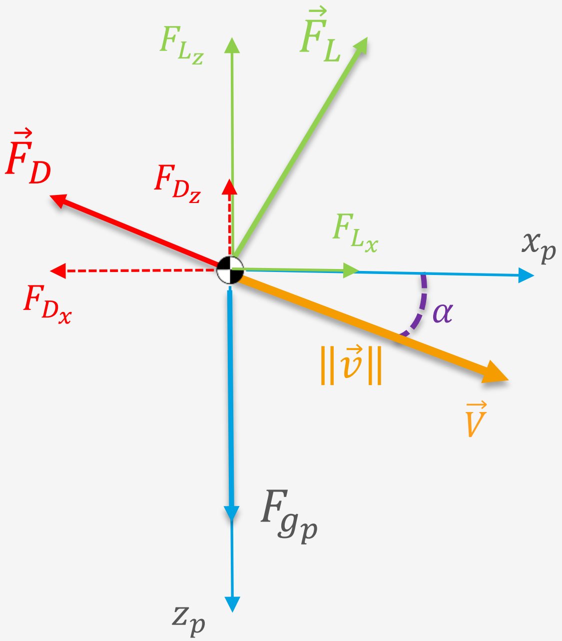
$$\vec{F}_D = \frac{1}{2}\,\rho\|\vec{v}\|^2 C_D(\alpha) S_{ref}\,\hat{e}_D$$

$$C_L(\alpha) = 2\pi\frac{10}{12}\alpha$$

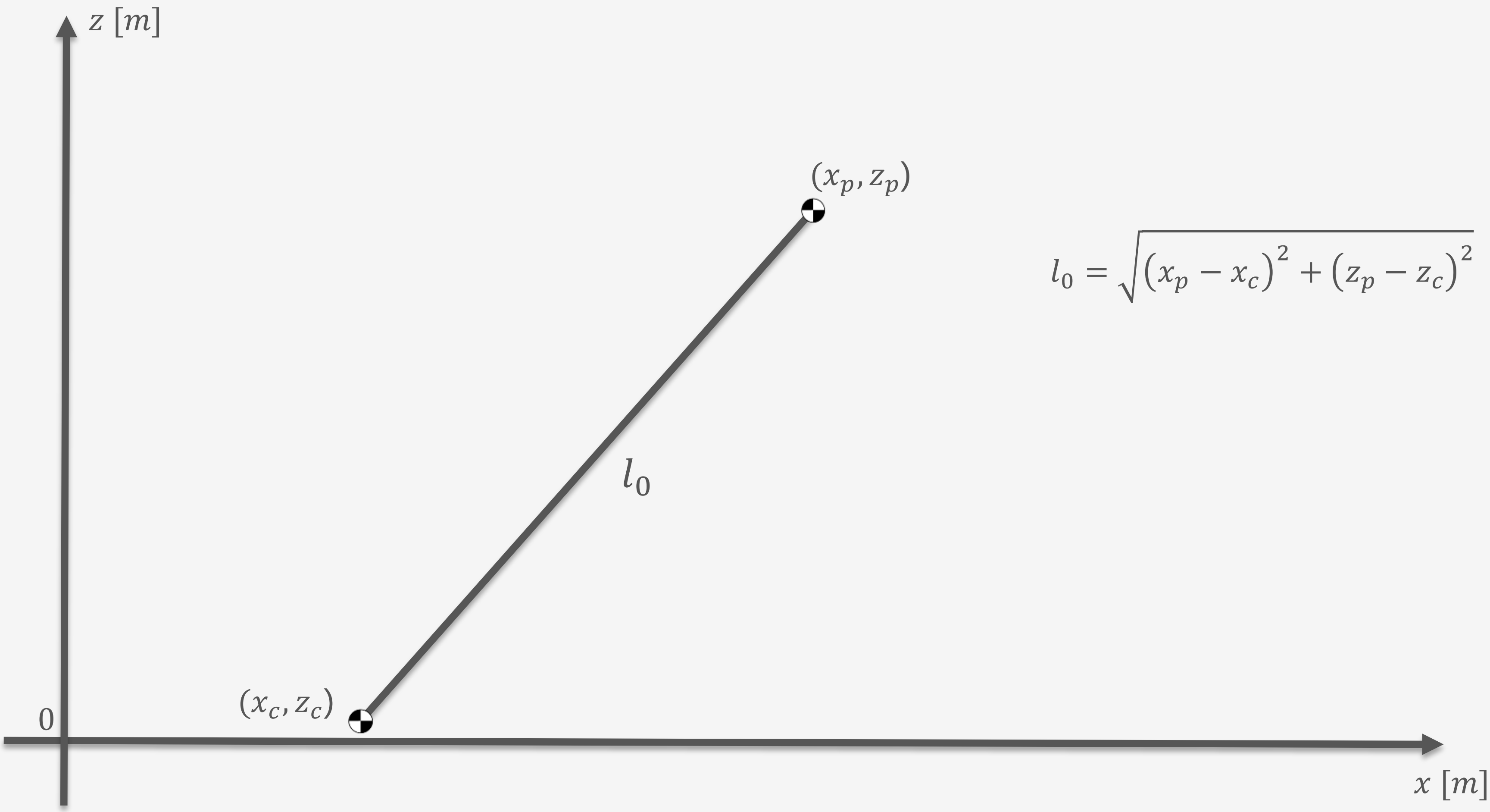
$$C_D(\alpha) = C_{D_0} + \frac{C_L(\alpha)^2}{AR\,\pi}$$

$$\vec{F}_L + \vec{F}_D + \vec{F}_g = m_p\vec{a}_p$$

$input:\ \alpha = angle\ of\ attack\ [rad]$

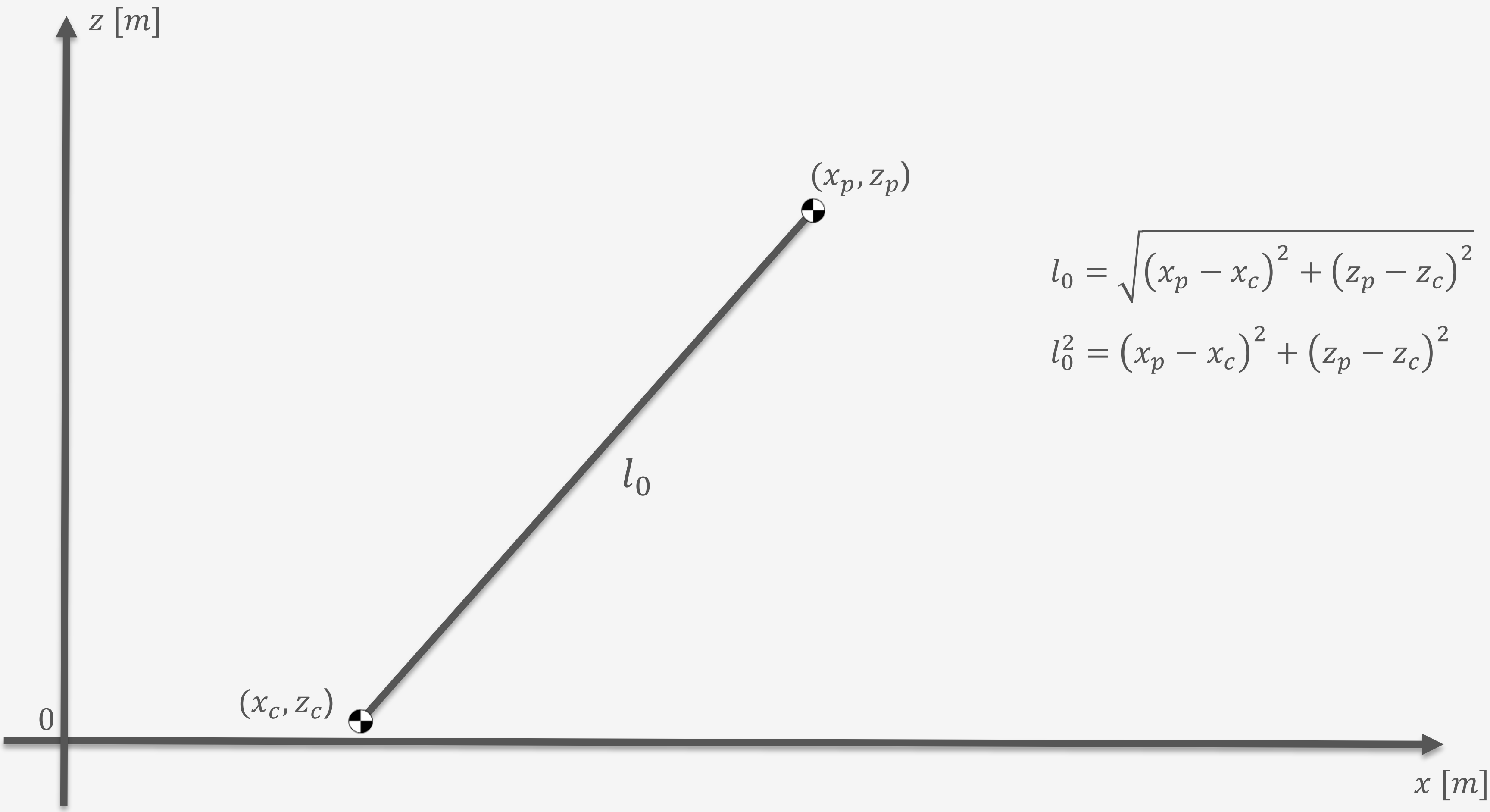


model description – Cable



$$l_0 = \sqrt{(x_p - x_c)^2 + (z_p - z_c)^2}$$

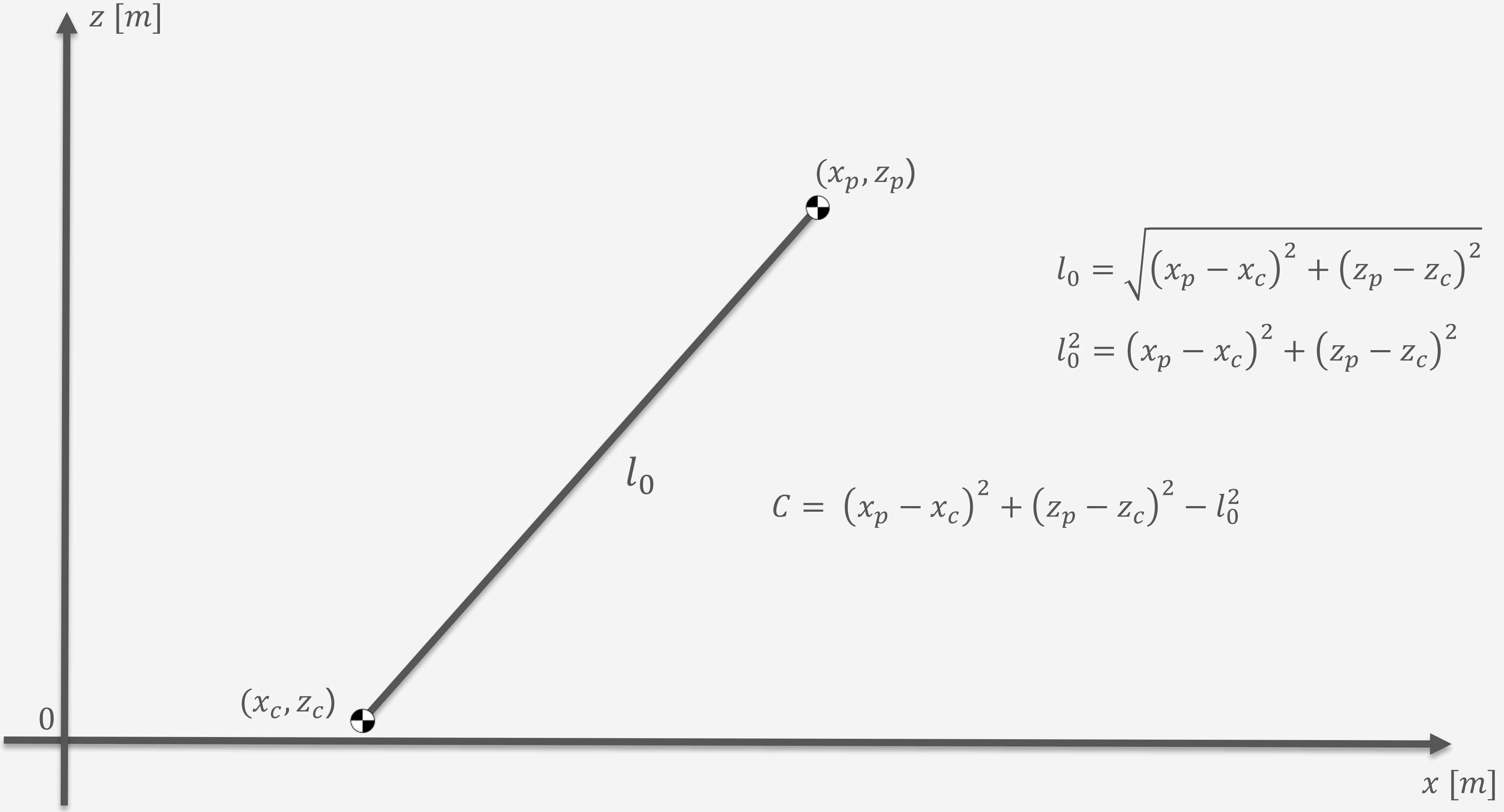
model description – Cable



$$l_0 = \sqrt{\left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2}$$

$$l_0^2 = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2$$

model description – Cable

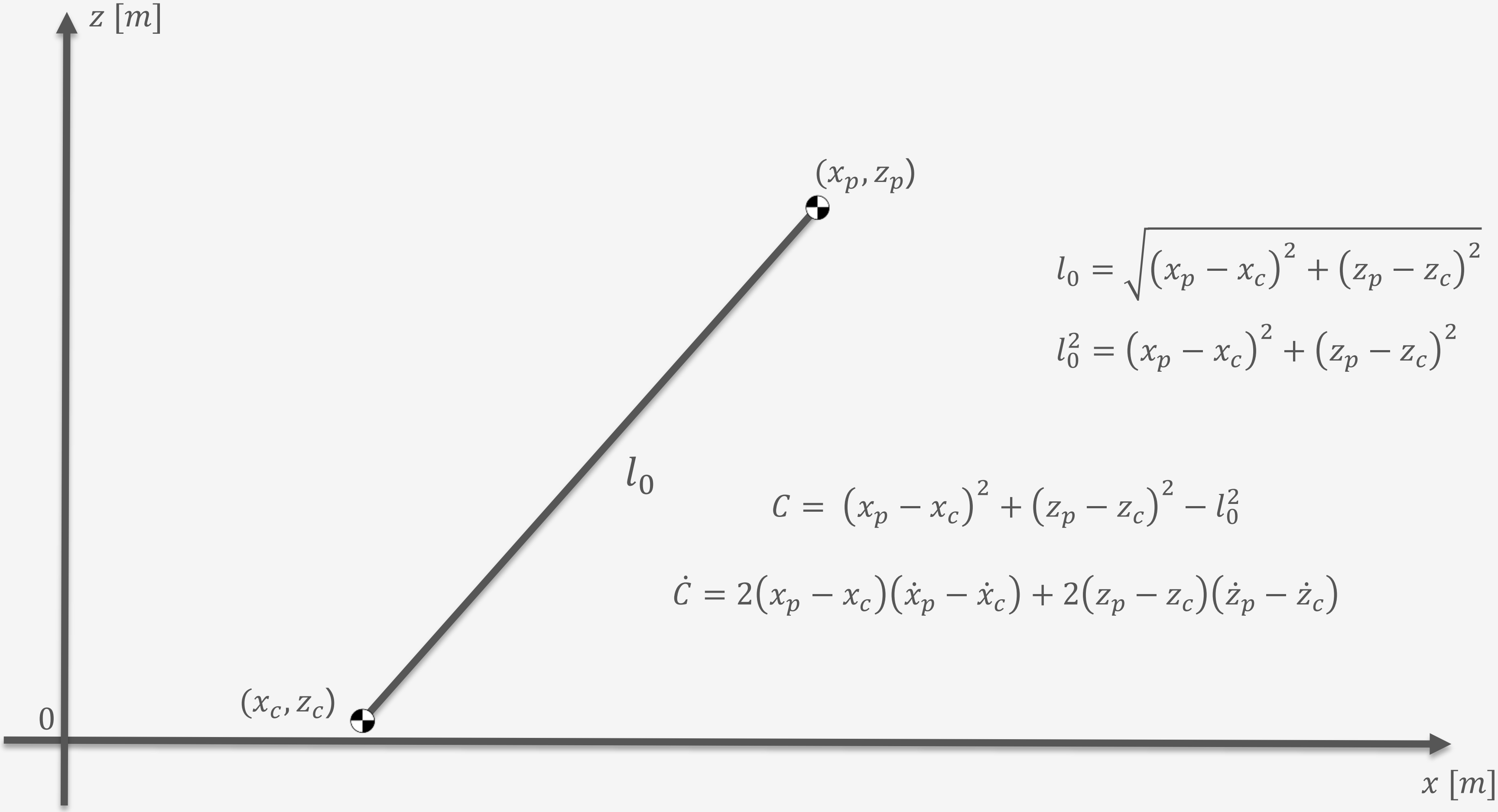


$$l_0 = \sqrt{\left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2}$$

$$l_0^2 = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2$$

$$C = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2 - l_0^2$$

model description – Cable



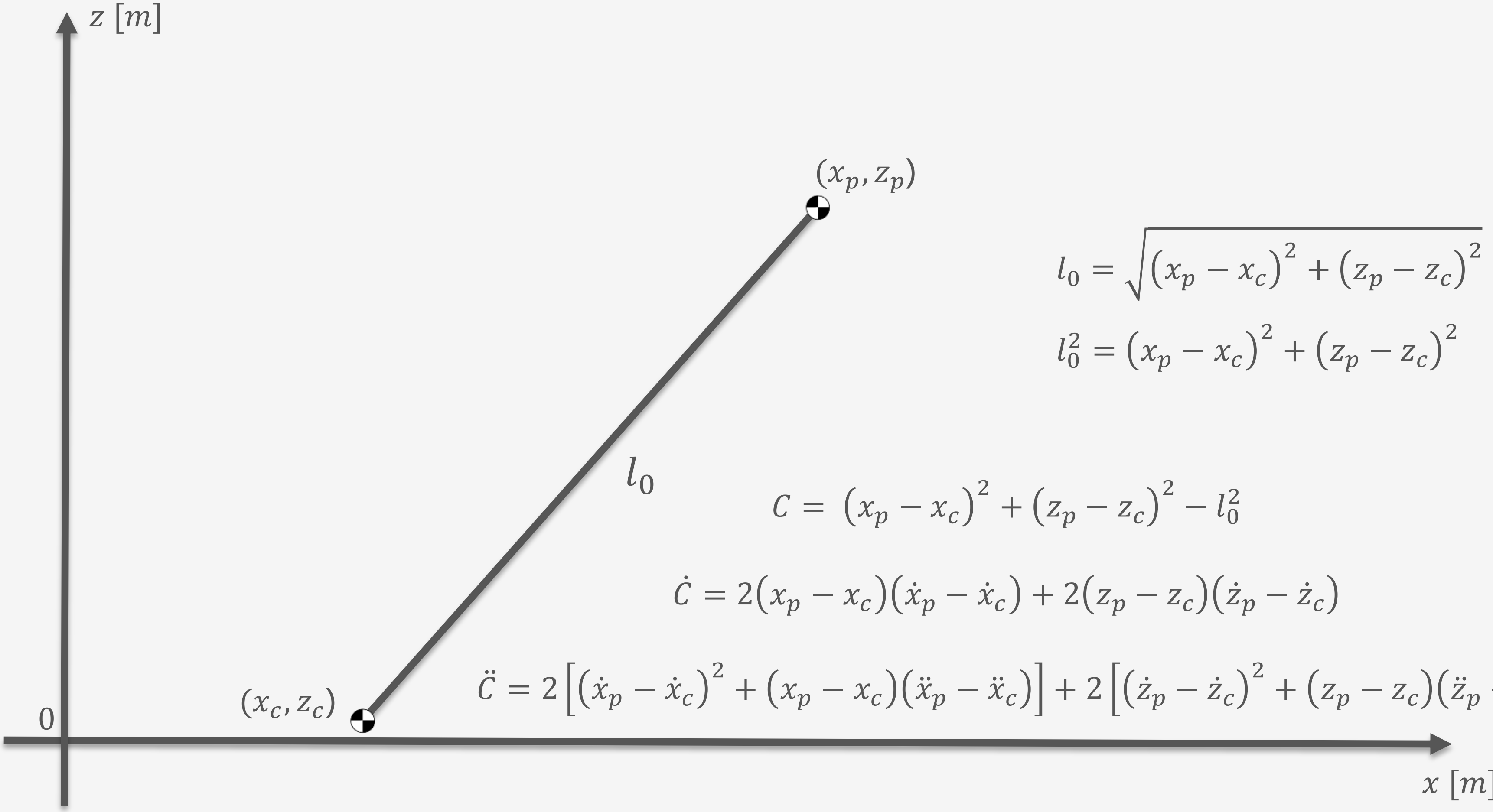
$$l_0 = \sqrt{\left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2}$$

$$l_0^2 = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2$$

$$C = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2 - l_0^2$$

$$\dot{C} = 2\left(x_p - x_c\right)\left(\dot{x}_p - \dot{x}_c\right) + 2\left(z_p - z_c\right)\left(\dot{z}_p - \dot{z}_c\right)$$

model description – Cable



$$l_0 = \sqrt{\left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2}$$

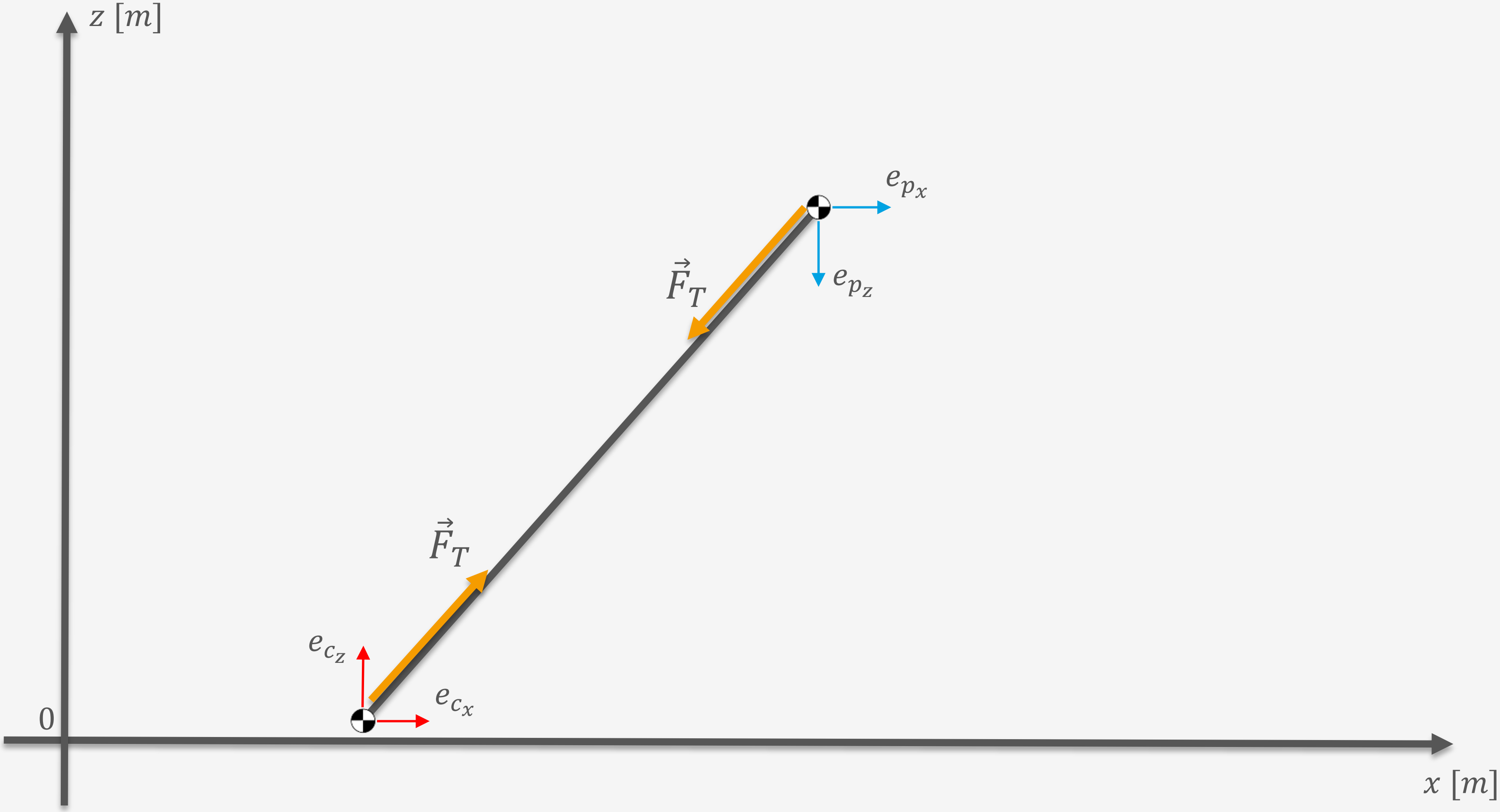
$$l_0^2 = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2$$

$$C = \left(x_p - x_c\right)^2 + \left(z_p - z_c\right)^2 - l_0^2$$

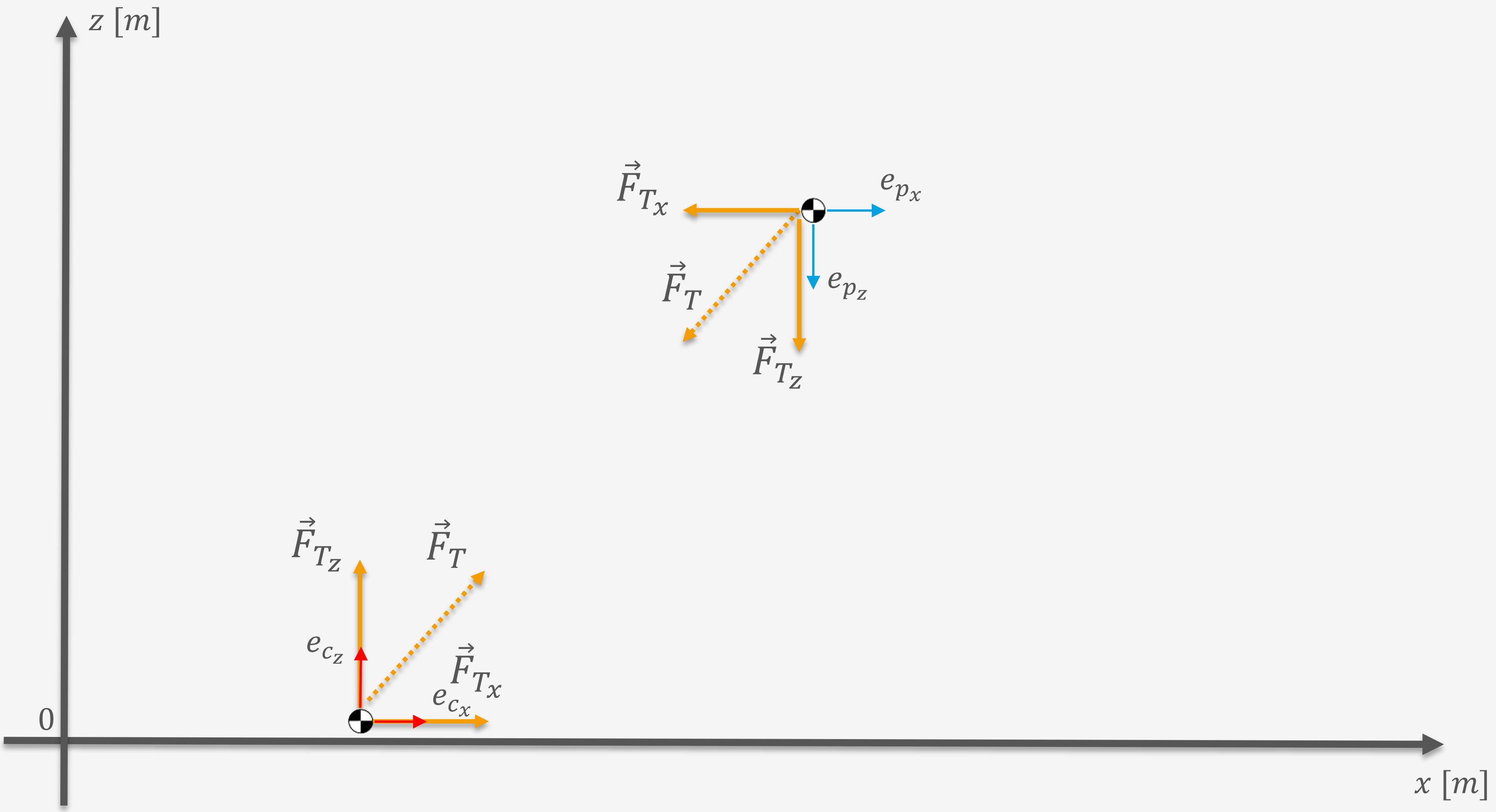
$$\dot{C} = 2\left(x_p - x_c\right)\left(\dot{x}_p - \dot{x}_c\right) + 2\left(z_p - z_c\right)\left(\dot{z}_p - \dot{z}_c\right)$$

$$\ddot{C} = 2\left[\left(\dot{x}_p - \dot{x}_c\right)^2 + \left(x_p - x_c\right)\left(\ddot{x}_p - \ddot{x}_c\right)\right] + 2\left[\left(\dot{z}_p - \dot{z}_c\right)^2 + \left(z_p - z_c\right)\left(\ddot{z}_p - \ddot{z}_c\right)\right]$$

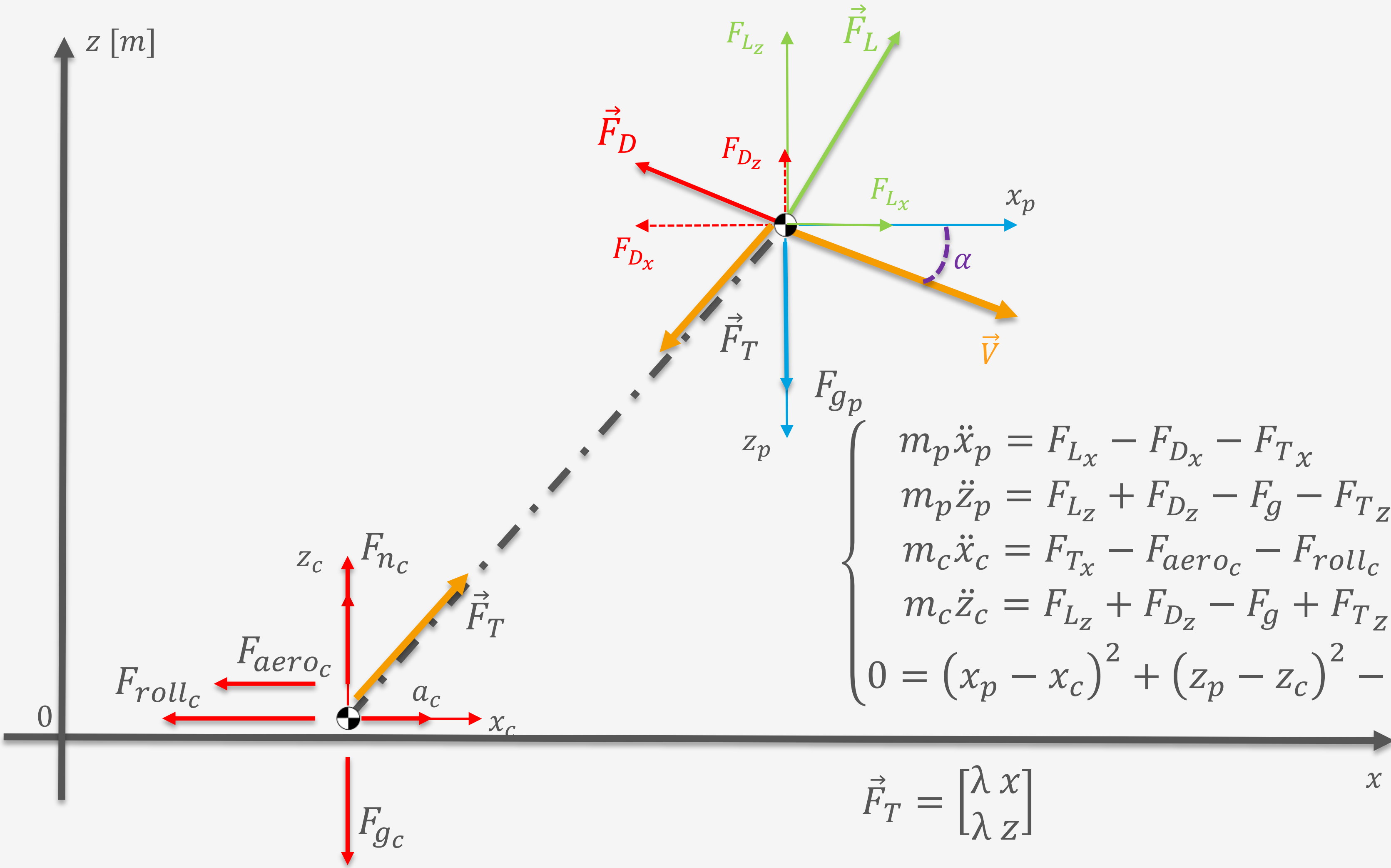
model description – Cable



model description – Cable



model description – Cable



$$
\begin{cases}
m_p\ddot{x}_p = F_{L_x} - F_{D_x} - F_{T_x} \\
m_p\ddot{z}_p = F_{L_z} + F_{D_z} - F_g - F_{T_z} \\
m_c\ddot{x}_c = F_{T_x} - F_{aero_c} - F_{roll_c} \\
m_c\ddot{z}_c = F_{L_z} + F_{D_z} - F_g + F_{T_z} \\
0 = (x_p - x_c)^2 + (z_p - z_c)^2 - l_0^2
\end{cases}
$$

$$\vec{F}_T = \begin{bmatrix} \lambda\, x \\ \lambda\, z \end{bmatrix}$$

DAE Formulation

Index-1 DAE

$$
\begin{cases}
\ddot{x}_p = \dfrac{1}{m_p}\left[F_{L_x} - F_{D_x} - \lambda\, x_p\right] \\[2ex]
\ddot{z}_p = \dfrac{1}{m_p}\left[F_{L_z} + F_{D_z} - F_{g_p} - \lambda\, z_p\right] \\[2ex]
\ddot{x}_c = \dfrac{1}{m_c}\left[\lambda\, x_c - F_{aero_c} - F_{roll_c}\right] \\[2ex]
\ddot{z}_c = \dfrac{1}{m_c}\left[F_{n_c} - F_{g_c} + \lambda\, z_c\right] \\[2ex]
0 = \left[\left(\dot{x}_p - \dot{x}_c\right)^2 + \left(x_p - x_c\right)\left(\ddot{x}_p - \ddot{x}_c\right)\right] + \left[\left(\dot{z}_p - \dot{z}_c\right)^2 + \left(z_p - z_c\right)\left(\ddot{z}_p - \ddot{z}_c\right)\right]
\end{cases}
$$

DAE Formulation

Index-1 DAE

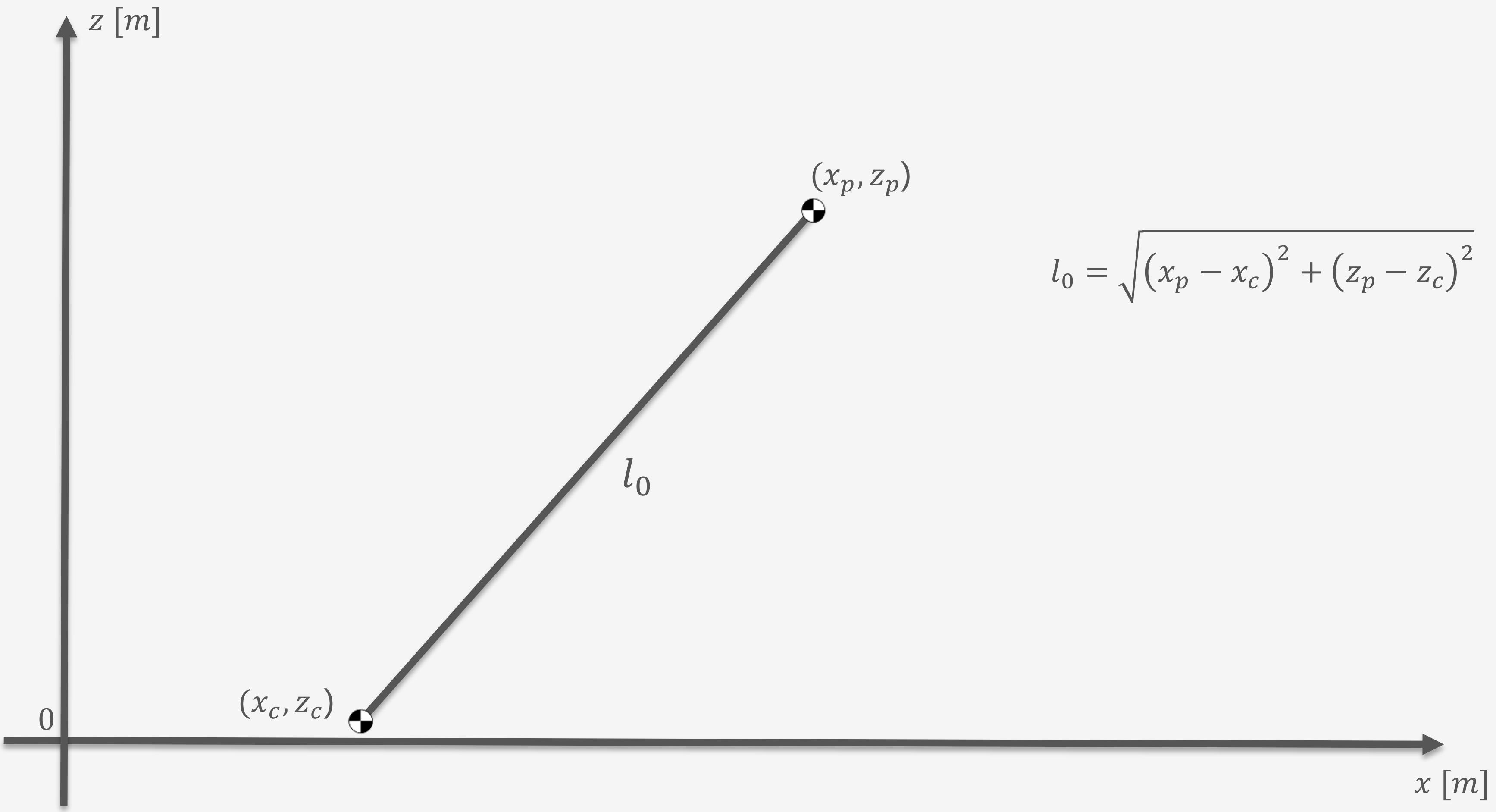
$$
\begin{cases}
\ddot{x}_p = \dfrac{1}{m_p}\left[F_{L_x} - F_{D_x} - \dfrac{\lambda}{l_0}\,(x_p - x_c)\right] \\
\ddot{z}_p = \dfrac{1}{m_p}\left[F_{L_z} + F_{D_z} - F_{g_p} - \dfrac{\lambda}{l_0}\,(z_p - z_c)\right] \\
\ddot{x}_c = \dfrac{1}{m_c}\left[\dfrac{\lambda}{l_0}\,(x_p - x_c)\; - F_{aero_c} - F_{roll_c}\right] \\
\ddot{z}_c = \dfrac{1}{m_c}\left[F_{n_c} - F_{g_c} + \dfrac{\lambda}{l_0}(z_p - z_c)\right] \\
0 = (\dot{x}_p - \dot{x}_c)^2 + (x_p - x_c)(\ddot{x}_p - \ddot{x}_c) + (\dot{z}_p - \dot{z}_c)^2 + (z_p - z_c)(\ddot{z}_p - \ddot{z}_c)
\end{cases}
$$

$$
\begin{bmatrix} \lambda\, x_j \\ \lambda\, z_j \end{bmatrix} = cable\ tension
$$

$$
C(t=0) = 0 \rightarrow \left(x_p(0) - x_c(0)\right)^2 + \left(z_p(0) - z_c(0)\right)^2 - l_0^2 = 0
$$

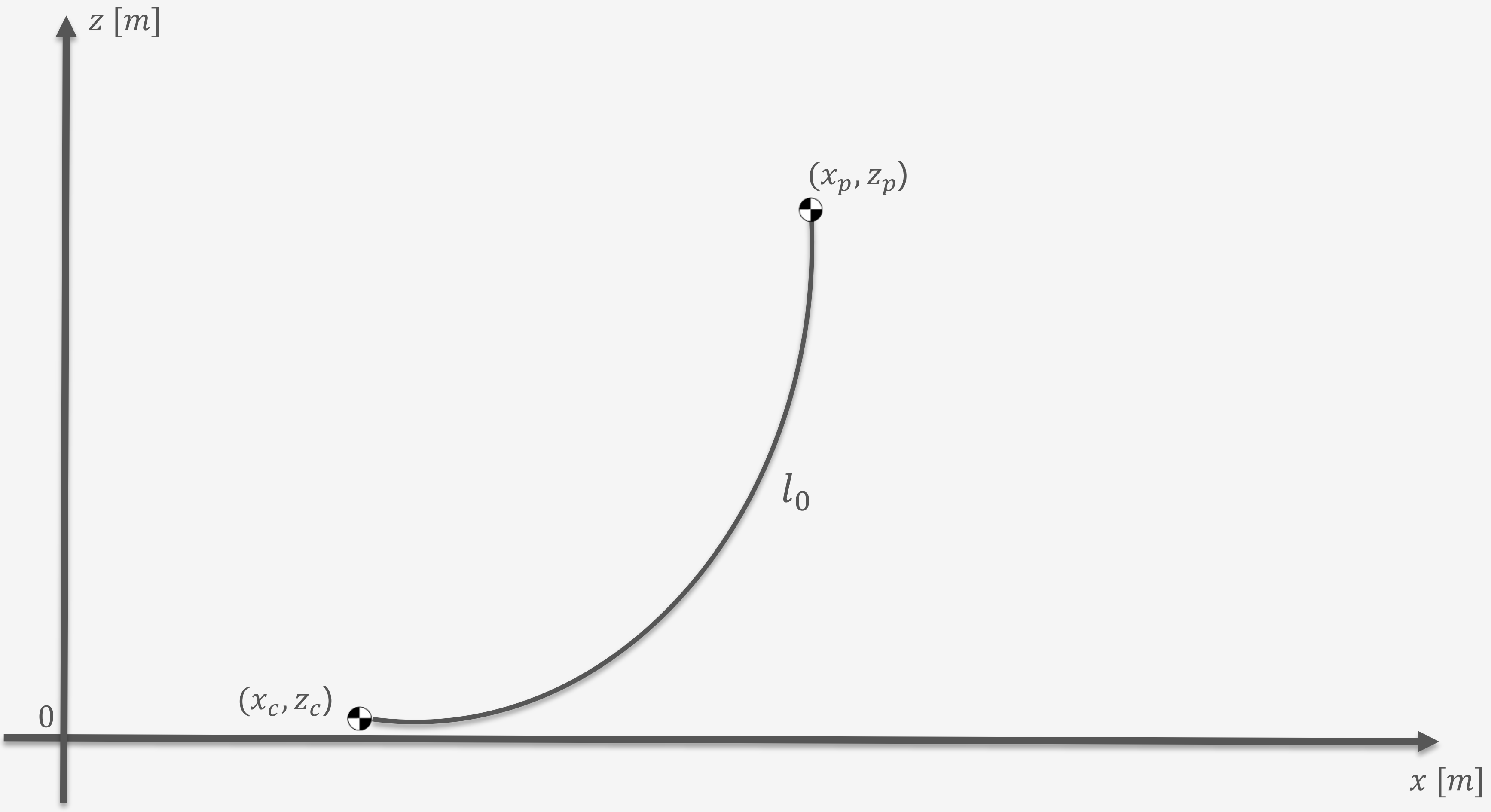
$$
\dot{C}(t=0) = 0 \rightarrow \left(x_p(0) - x_c(0)\right)\left(\dot{x}_p(0) - \dot{x}_c(0)\right) + \left(z_p(0) - z_c(0)\right)\left(\dot{z}_p(0) - \dot{z}_c(0)\right) = 0
$$

Something more challenging…

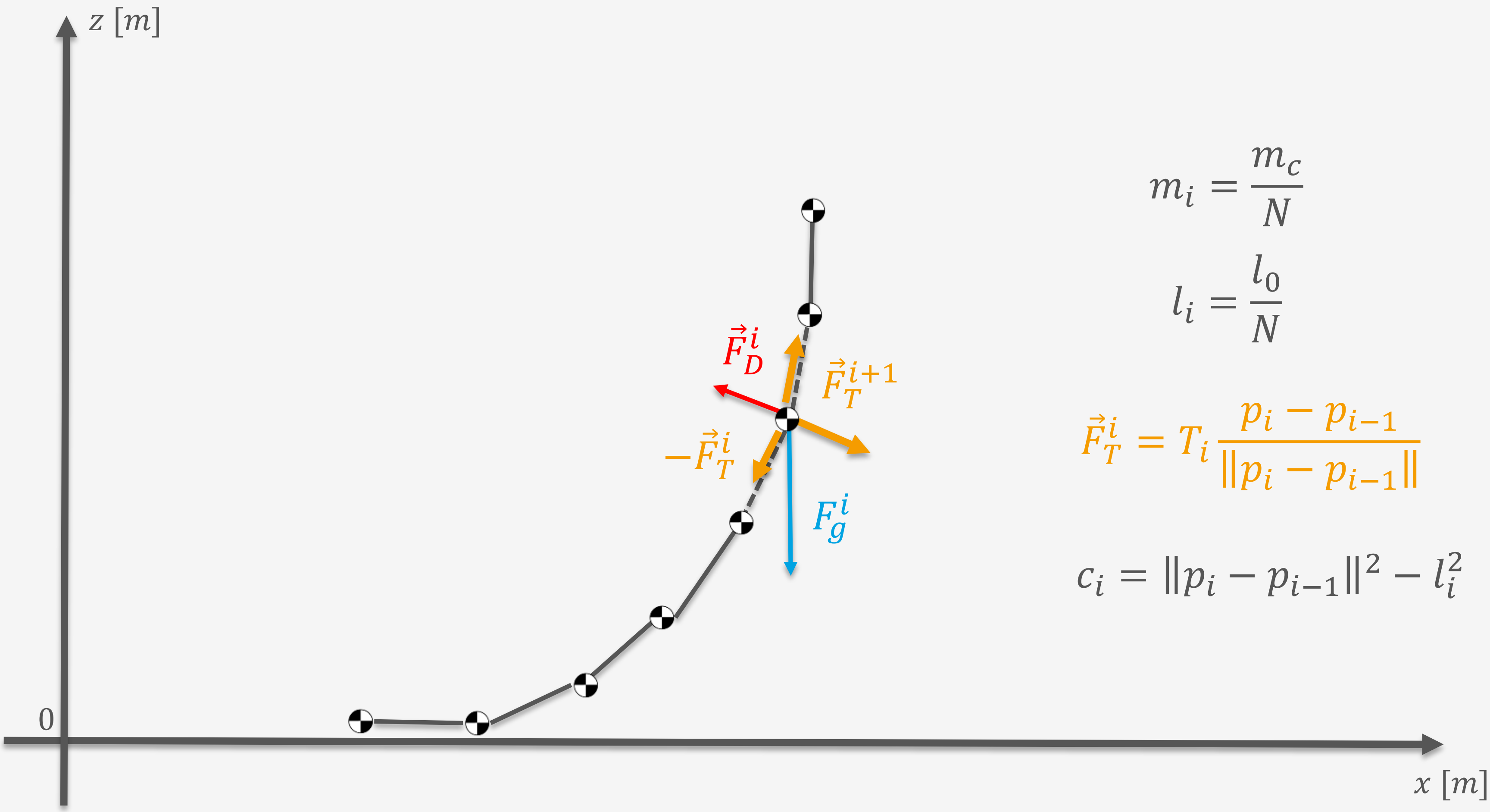


$$l_0 = \sqrt{(x_p - x_c)^2 + (z_p - z_c)^2}$$

Cable shape + tether drag



Cable shape + tether drag



$$m_i = \frac{m_c}{N}$$

$$l_i = \frac{l_0}{N}$$

$$\vec{F}_T^i = T_i \frac{p_i - p_{i-1}}{\|p_i - p_{i-1}\|}$$

$$c_i = \|p_i - p_{i-1}\|^2 - l_i^2$$

OCP via Direct Collocation

minimize $\quad U^T U$

$w = \{\theta_0, \theta_{0,1}, \dots, \theta_{0,K}, z_{0,1}, \dots, z_{0,K}, u_0, \dots \dots, \theta_N\}$

s.t.

$$\theta_0 = \bar{x}_0 \qquad \textit{initial constraint}$$

$$0 = \theta_{k,K} - \theta_{k+1,0} \qquad \textit{continuity constraint}$$

$$\frac{\partial x(\theta_k, t_{k,i})}{\partial t} = F(\theta_{k,i}, z_{k,i}, u_k) \qquad \textit{dynamics (collocation)}$$

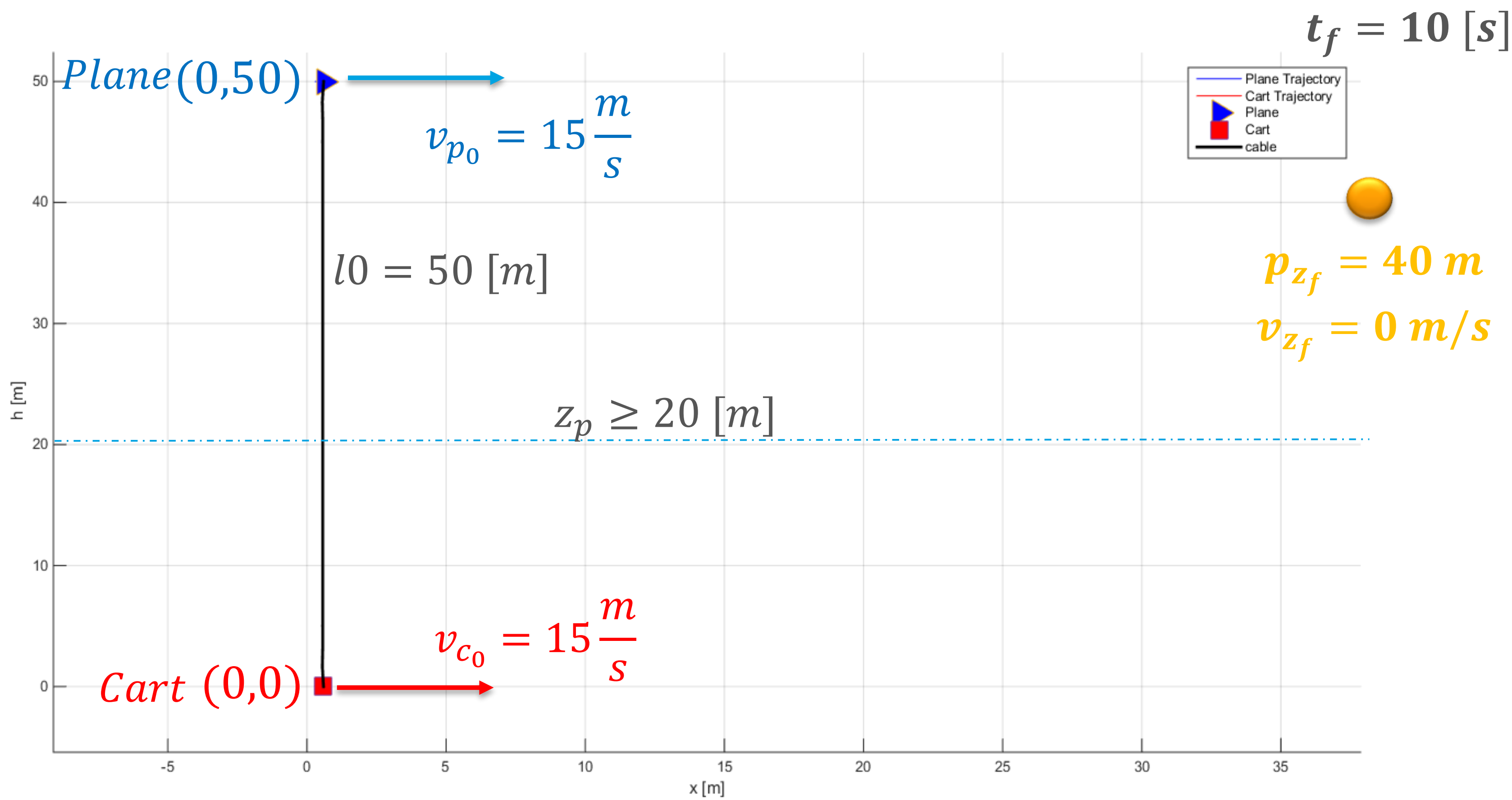
$$0 = G(\theta_{k,i}, z_{k,i}, u_k) \qquad \textit{algebraic constraint}$$

$$\theta_N = \bar{x}_f \qquad \textit{terminal constraint}$$

for $k = 0, \dots, N-1, \quad i = 1, \dots, K$

Radau Collocation Points, K=4

Boundary Conditions



Results [with and without control]

ampyx power

Thanks for your attention

UNI FREIBURG

TEMPO
Training in Embedded Predictive Control and Optimization
A Marie Curie Initial Training Network (ITN)

AWESCO