Lecture 9: Bagging, Random Forests, Boosting

Reading: Chapter 8

STATS 202: Data mining and analysis

Rajan Patel

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- Now, we will use the average of these predictions as an estimator with reduced variance:

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- **Classification:** To make a prediction for an input point x_0, take the majority vote from the set of predictions:

$$\hat{y}_0^{(1)}, \dots, \hat{y}_0^{(B)}.$$

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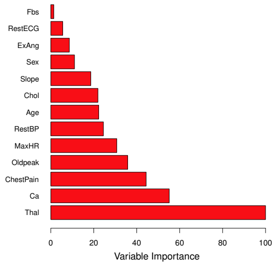

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- For B large, OOB error is virtually equivalent to LOOCV.

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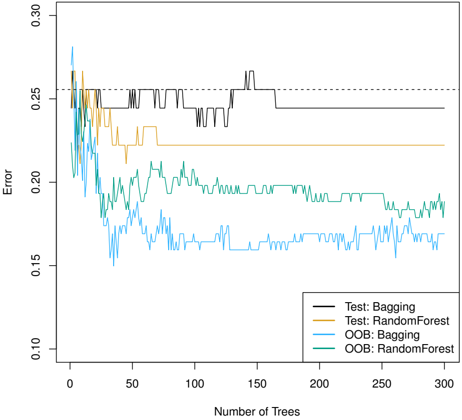

The test error decreases as we increase B
(dashed line is the error for a plain decision tree).

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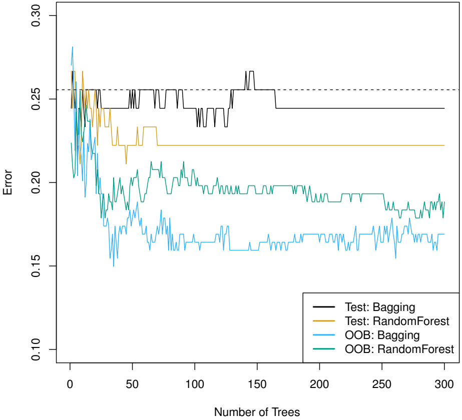
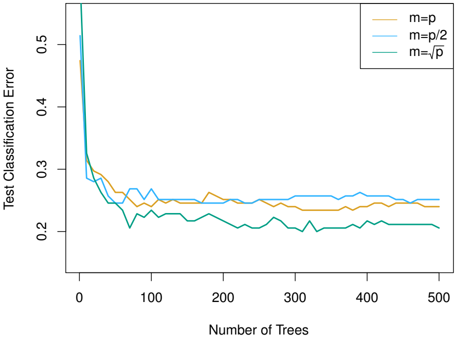
- We fit a decision tree to different Bootstrap samples.
- When growing the tree, we select a random sample of $m < p$ predictors to consider in each step.
- This will lead to very different (or "uncorrelated") trees from each sample.
- Finally, average the prediction of each tree.

Random Forests vs. Bagging



Random Forests, choosing m



The optimal m is usually around $\sqrt{p}$,
but this can be used as a tuning parameter.

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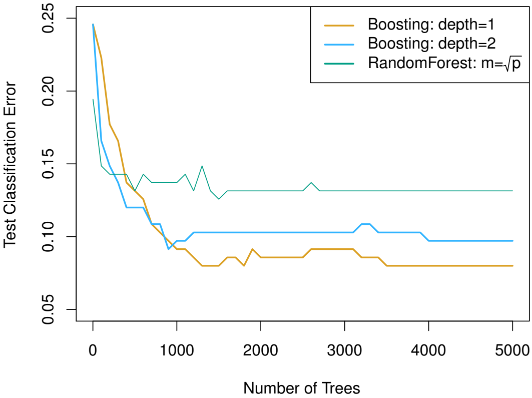
3. Output the final model:
$$\hat{f}(x) = \sum_{b=1}^{B} \lambda \hat{f}^b(x).$$

Boosting, intuitively

Boosting learns *slowly*:

We first use the samples that are easiest to predict, then slowly down weigh these cases, moving on to harder samples.

Boosting vs. random forests



The parameter $\lambda = 0.01$ in each case.
We can tune the model by CV using λ, d, B.