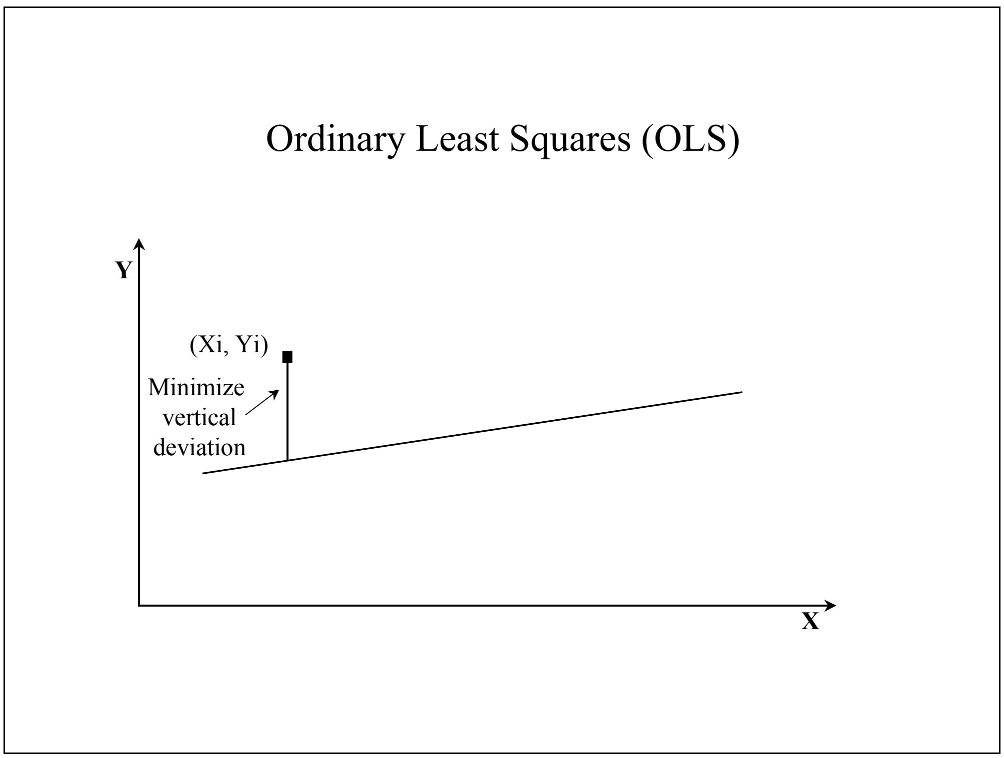
Command Reference: FillRegression()

Fill missing time series data using ordinary least squares regression

Version 10.20.00, 2013-04-21

Note: Some of the new features of this command are under development. In particular, output of statistics to the table is not completed. See highlighted comments below for additional notes about remaining enhancements.

The `FillRegression()` command fills missing data in a time series using ordinary least squares (OLS) regression and provides a variety of options for transforming the data and controlling the analysis. In OLS regression, the vertical distance from the data point to the regression line is minimized. OLS regression provides the minimum-variance estimate for a single value or observation. However, if an ensemble of points is estimated from OLS regression, the estimated values will have lesser variability than the true values.



See also the `FillMOVE2()` command, which utilizes additional variance from independent time series to determine the regression relationship, and the `FillMixedStation()` command, which automates the analysis of many time series to determine a "best estimate" filling approach. Regression can be applied only to regular interval time series. The dependent time series will be filled using the independent time series. The periods of record and output period for the time series should be verified to make sure that the time series periods overlap sufficiently. Regression relationships are developed using the analysis period for the time series and are applied to the fill period. Refer to the output statistics table, log file, and time series properties for analysis details. Several parameters are available to ensure that filling uses reasonable relationships. This command has functionality that may not be needed for simple analysis but which is useful for software testing and comparison with the `FillMixedStation()` command.

Important: TSTool does allow filled values to be flagged. However, other commands do not exclude these values from computations when determining relationships for subsequent fill steps. Therefore, it is important to perform regression data filling as early in data processing as possible so that data manipulation does not introduce derived values and bias.

The following OLS equation is used to estimate values for the dependent time series from the independent time series:

$$Y_i = \overline{Y}_1 + r\frac{S_{y1}}{S_{x1}}\left[X_i - \overline{X}_1\right]$$

or

$$Y_i = a + bX_i$$

where

N_1 = concurrent or overlapping period of record (the notation N_1 is used because the MOVE2 fill technique refers to N_2, which is the number of additional points outside of N_1 in the independent time series)

$\overline{X}_1$ = mean for independent variable for N_1 years = $\frac{\sum X_{1_i}}{N_1}$

$\overline{Y}_1$ = mean for dependent variable for N_1 years = $\frac{\sum Y_{1_i}}{N_1}$

S_{y1} = standard deviation for N_1 years = $\frac{1}{N_1 - 1}\sum\left(Y_{1i} - \overline{Y}_1\right)^2$

S_{x1} = standard deviation for N_1 years = $\frac{1}{N_1 - 1}\sum\left(X_{1_i} - \overline{X}_1\right)^2$

$r = R$ = correlation coefficient = $\frac{N_1\sum X_{1i}Y_{1i} - \sum X_{1i}\sum Y_{1i}}{\sqrt{\left[N_1\sum X_{1_i}{}^2 - \left(\sum X_{1_i}\right)^2\right]\cdot\left[N_1\sum Y_{1i}{}^2 - \left(\sum Y_{1_i}\right)^2\right]}}$

$$b = r\frac{S_{y1}}{S_{x1}}$$

$a = \overline{Y}_1 - b\overline{X}_1$

Note that the correlation coefficient, r, is used to compute the slope, b, of the line.

A number of statistics are computed and are available for output to a table, as described below (see the `TableID` and related command parameters for how to specify the table output). Creating a statistics table and then writing the table to a file is useful for checking the analysis and software. For example, the `CompareTables()` command can be used to compare this statistics table with a verification data set that is calculated by another tool. In the following descriptions, the statistic for one equation has a name like `Mean` and monthly statistics correspondingly have a name like `Mean_1`, where `1` corresponds to January and `12` to December.

In some cases, statistics are relevant in units of the raw values, in some cases statistics are relevant in transformed (log10) units, and in some cases both are relevant. For example, if the log10 transform is used to compute the relationship, then a and b are in transformed units. However, error computations between the original data values and values that would be computed by the relationship are in the raw units (regardless of whether the data were transformed) – this allows errors to be compared between relationships using raw and translated values (the FillMixedStation() command uses this information to compare relationships). Consequently, the third column of the following table indicates whether statistics are provided in raw (column name uses statistic only) or transformed units (additional _trans added to statistic for column name), and bold indicates that where both are available only the bold version is output). If the analysis does not use a transformation, then _trans will be omitted from column headings. In summary, if _trans is shown in a column heading, then the data have been transformed and the value in the column is relevant to transformed data.

Statistics From Regression Analysis

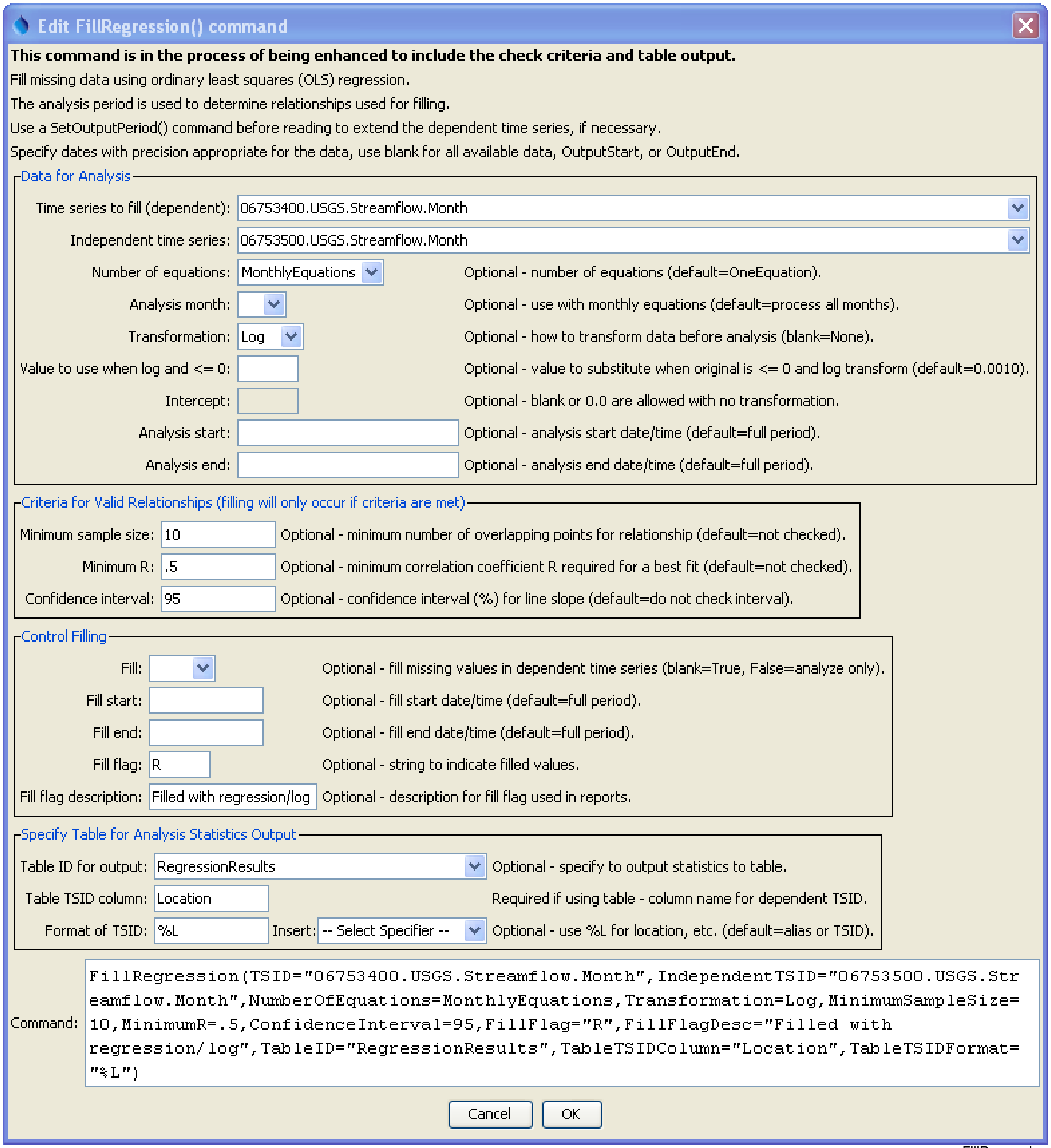
Statistic (Table Column Name)	Involves Dependent, Independent, or Both	Statistics Output as Raw and/or Transformed Values	Description
N1	Both		The number (count) of non-missing data values overlapping in the dependent and independent time series.
MeanX1	Independent	**raw**, transformed	The mean of the independent N1 data values.
SX1	Independent	**raw**, transformed	The standard deviation of the independent N1 values.
N2	Independent		The number (count) of non-missing independent values outside of N1.
MeanX2	Independent	**raw**, transformed	The mean of the independent N2 values.
SX2	Independent	**raw**, transformed	The standard deviation of the independent N2 values.
MeanY1	Dependent	**raw**, transformed	The mean of the dependent N1 values.
SY1	Dependent	**raw**, transformed	The standard deviation of the dependent N1 values.
NY	Dependent		The total number of non-missing dependent values.
MeanY	Dependent	**raw**, transformed	The mean of the dependent NY values.
SY	Dependent	**raw**, transformed	The standard deviation of the dependent NY values.
a	Both	transformed	The intercept for the relationship equation.
b	Both	transformed	The slope of the relationship equation.
R	Both	transformed	The correlation coefficient for N1 values.
R2	Both	transformed	R-squared, coefficient of determination for N1 values.
MeanY1est	Dependent	**raw**, transformed	The mean for N1 values computed from the relationship (estimate the dependent values where values were previously known).

Statistic (Table Column Name)	Involves Dependent, Independent, or Both	Statistics Output as Raw and/or Transformed Values	Description
`SY1est`	Dependent	**raw**, transformed	The standard deviation for `N1` values computed from the relationship (estimate the dependent at locations where values are known).
`RMSE`	Dependent	raw	The "room mean squared error" for `N1` overlapping values, which is a measure of the overall error of using the regression equation to estimate values, is calculated as: $$RMSE = \sqrt{\frac{\sum \left(Y_{1_i} - Y_{1_i}'\right)^2}{N_1}}$$ where Y_{1_i} is the original dependent value and Y_{1_i}' is the value estimated with the regression relationship.
`SEE`	Dependent	raw	The standard error of estimate for `N1` overlapping values, which is a measure of the overall error of using the regression equation to estimate values, calculated as: $$SEE = \sqrt{\frac{\sum \left(Y_{1_i} - Y_{1_i}'\right)^2}{N_1 - 2}}$$ where Y_{1_i} is the original dependent value and Y_{1_i}' is the value estimated with the regression relationship.
`SEP`	Both	raw	The standard error of prediction for each estimated value,calculated as: $$SEP = \sqrt{1 + \frac{1}{N_1} + \frac{(X_{1_i} - \overline{X}_1)^2}{\sum (X_{1_i} - \overline{X}_1)^2}} * SEE$$ where X_{1_i} is the original independent value and $\overline{X}_1$ is the mean of the `N1` independent values. Note when using the mixed station analysis in the `FillMixedStation()` command, this value may be used to determine the relationship.
`SESlope`	Both	transformed	The standard error (SE) of the slope (b) for `N1` overlapping values, calculated as:

Statistic (Table Column Name)	Involves Dependent, Independent, or Both	Statistics Output as Raw and/or Transformed Values	Description
			$$SE = \frac{\sqrt{\frac{\sum (Y_{1_i} - Y'_{1_i})^2}{N_1 - 2}}}{\sqrt{\sum (X_{1_i} - \overline{X}_1)^2}}$$ where X_{1_i} is the original independent value and $\overline{X}_1$ is the mean of the `N1` independent values; Y_{1_i} is the original dependent value and Y_{1_i}' is the value estimated with the regression relationship.
`TestScore`	Both	transformed	b/SE
`Test Quantile`	Both	transformed	From the Student's T-test, which is a function of the confidence interval and degrees of freedom (DF), where DF is the degrees of freedom equal to `N1` – 2 (corresponding to the intercept and the slope of the regression equation).
`Test Related`	Both	transformed	Will be `No` if `TestScore < TestQuantile`, indicating that the `b ≠ 0` data are related, and `Yes` if `TestScore >= TestQuantile`, indicating that the data are not related. If the data are not related, then the relationship between the dependent and independent time series will not be used for filling.

Need to include a description of the T-Test and confidence interval here if possible, with a graphic. In particular, we should reference the source of the table data and describe the approach/equations (if not clear in the referenced material) so that this whole process is transparent and can be revisited.

The following dialog is used to edit the command and illustrates the syntax of the command:



FillRegression

FillRegression() Command Editor

The command syntax is as follows:

```
FillRegression(Parameter=Value,…)
```

Command Parameters

Parameter	Description	Default
TSID	The time series identifier or alias for the time series to be filled.	None – must be specified.
Independent TSID	The time series identifier or alias for the independent time series.	None – must be specified.
NumberOf Equations	The number of equations to use for the analysis: `OneEquation` or `MonthlyEquations`.	`OneEquation`
AnalysisMonth	Indicate the month to process when using monthly equations. Currently only a single month can be specified.	Process all months.
Transformation	Indicates how to transform the data before analyzing. Specify as `None` (previously `Linear`) or `Log` (for Log_{10}). If the `Log` option is used, zero and negative values are replaced with the value specified by the `LEZeroLogValue` parameter value for analysis (missing data values are ignored in the analysis).	`None` (no transformation).
LEZeroLogValue	Value to use for data values less than or equal to zero when using a log transformation. The Log_{10} of this value will be used in calculations.	`.0010`
Intercept	Specify as `0` to force the intercept of the best-fit line through the origin (not available for log transformation).	Parameter is optional and if specified the default is to not force the intercept through zero.
AnalysisStart	The date/time to start the analysis – use to focus on only a period appropriate from analysis. For example specify the unregulated period for streamflow.	Analyze the full period.
AnalysisEnd	The date/time to end the analysis – use to focus on only a period appropriate from analysis.	Analyze the full period.
Minimum SampleSize	The minimum number of overlapping values required to use a relationship for filling.	No limit, other than imposed by calculation of statistics.
MinimumR	The minimum correlation coefficient required to use a relationship for filling.	No check is performed.
Confidence Interval	A confidence interval in percent (e.g., 95) required for the slope of the relationship. The T-test is performed to ensure that the independent and dependent time series are related. This is not fully enabled.	The T-test is not performed to evaluate the confidence interval.
Fill	Indicate whether fill should occur (`True`) or just analyze to compute statistics (`False`). The latter is useful for testing combinations of fill parameters prior to actually performing filling.	`True`
FillStart	The date/time to start filling, if other than the full time series period.	Fill the full period.
FillEnd	The date/time to end filling, if other than the full time series period.	Fill the full period.
FillFlag	A single character that will be used to flag filled data.	Filled values will not be flagged.

Parameter	Description	Default
FillFlagDesc	Description for the fill flag, used in reports.	Automatically generated.
TableID	A table identifier for a table to receive output of the regression analysis (statistics are described above).	Statistics are not written to the table. Refer to the log file for information.
TableTSIDColumn	The name of the column in the table that contains time series identifier information. This is used to match the table with time series being analyzed so that statistics can be written to the correct row.	Required if TableID is specified.
TableTSIDFormat	The specifier used to format the time series identifier in the TableTSIDColumn. The location part of the TSID, or the time series alias is typically used.	The alias will be used if available, or otherwise the full TSID will be used.
SEPTSID	The time series identifier of the SEP time series, calculated for ALL values in the analysis period. This parameter is not enabled but is envisioned to help evaluate filling and test FillMixedStation().	If not specified, no SEP time series will be generated.
SEPTSAlias	The alias to be assigned to the SEP time series. This parameter is not yet enabled.	No alias is assigned to the SEP time series.
FlagToWarn	A parameter is envisioned to warn the user if any values in the time series are flagged with a specific flag value. This will allow checks to ensure that FillRegression() is not used with data that have been filled in a previous step.	

The command logic is as follows, with reference to command parameters that control the process:

1. The dependent (TSID) and independent time series (IndependentTSID) are retrieved using the time series identifiers or aliases.
2. Data arrays of overlapping non-missing values are extracted from time series to be used as the samples for analysis, as specified by command parameters (analysis period specified by AnalysisStart and AnalysisEnd; transformation specified by Transformation, LEZeroLogValue, and Intercept; number of equations specified by NumberOfEquations and AnalysisMonth).
3. The independent and dependent statistics and relationships are calculated, computing as many of the statistics as possible (some are skipped if the sample size results in division by zero). Computing the statistics allows them to be saved in the output table for review, and is controlled by the TableID, TableTSIDColumn, and TableTSIDFormat parameters.
4. The statistics are analyzed to determine if the relationships are acceptable for filling by checking the minimum sample size (MinimumSampleSize), minimum correlation coefficient (MinimumR), and that the relationship meets the confidence interval (ConfidenceInterval). If monthly equations are used, then it is possible that some months can be filled but not others.
5. If Fill=True (the default), then the relationships that are acceptable from step 4 are used to fill the dependent time series for the period specified by the FillStart and FillEnd parameters, with FillFlag and FillFlagDesc optionally being used to indicate filled values.