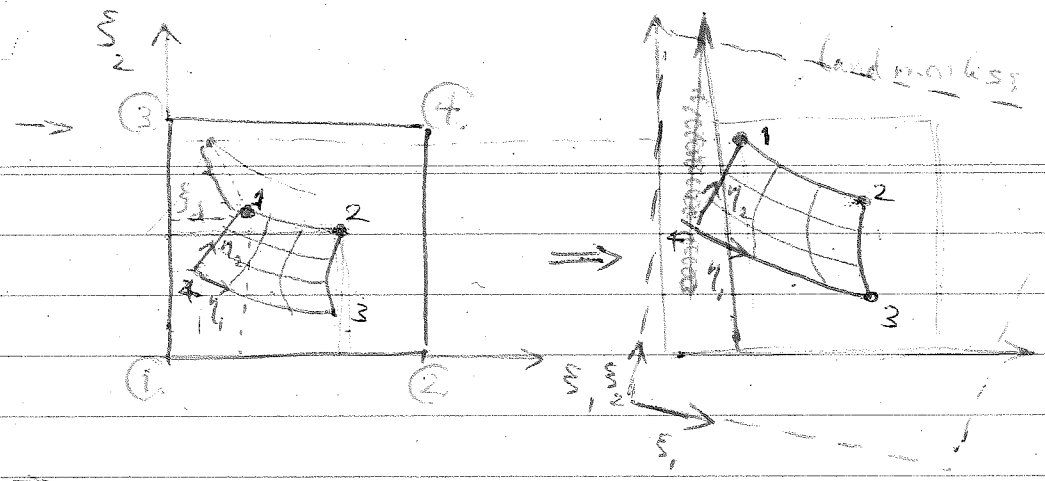


1: (0.3, 0.6) → (0.1, 0.3)

2: (0.7, 0.5) → (0.75, 0.6)

3: (0.7, 0.2) → (0.8, 0.25)

4: (0.4, 0.3) → (0.4, 0.4)

$$\Rightarrow U(\xi_1,\xi_2) = \sum_n \phi_n u_n = \xi_1\xi_2 u_1 + (1-\xi_1)\xi_2 u_2 + \xi_1(1-\xi_2)u_3 + (1-\xi_1)(1-\xi_2)u_4$$

$\Rightarrow$ landmark 1

$\Rightarrow \xi_1 = 0.3, \quad \xi_2 = 0.6$

$$U(0.3, 0.6) = (0.3)(0.6)u_1 + (0.7)(0.6)u_2 + (0.3)(0.4)u_3 + (0.7)(0.4)u_4$$

$$(0.18) \qquad (0.42) \qquad (0.12) \qquad (0.28)$$

$$\phi_1 = (1-\xi_1)(1-\xi_2) \qquad \phi_2 = \xi_1(1-\xi_2)$$

$$\phi_3 = (1-\xi_1)\xi_2 \qquad \phi_4 = \xi_1\xi_2$$

Landmark 1 $\Rightarrow \xi_1 = 0.3, \quad \xi_2 = 0.6$

$$\Rightarrow U_{\text{landmark 1}}(0.3, 0.6) = (0.7)(0.4)u_1 + (0.3)(0.4)u_2 + (0.7)(0.6)u_3 + (0.3)(0.6)u_4$$

$$= (0.28)u_1 + (0.12)u_2 + (0.42)u_3 + (0.18)u_4$$

$$U(\xi_1,\xi_2) = \sum \phi_n u_n = \phi_1 u_1 + \phi_2 u_2 + \phi_3 u_3 + \phi_4 u_4$$

Error Function $\left\| (0.28)u_1 + (0.12)u_2 + (0.42)u_3 + (0.18)u_4 - \xi_i \right\|^2$

$U(0.3, 0.6) = (0.7)(0.4)u_1 + (0.3)(0.4)u_2 + (0.7)(0.6)u_3 + (0.3)(0.6)u_4$

$= 0.28u_1 + 0.12u_2 + 0.42u_3 + 0.18u_4$

$U_2(0.7, 0.5) = (0.3)(0.5)u_1 + (0.7)(0.5)u_2 + (0.3)(0.5)u_3 + (0.7)(0.5)u_4$

$= 0.15u_1 + 0.35u_2 + 0.15u_3 + 0.35u_4$

$U(0.7, 0.2) = (0.3)(0.8)u_1 + (0.7)(0.8)u_2 + (0.3)(0.2)u_3 + (0.7)(0.2)u_4$

$= 0.24u_1 + 0.56u_2 + 0.06u_3 + 0.14u_4$

$U(0.1, 0.3) = (0.9)(0.7)u_1 + (0.1)(0.7)u_2 + (0.9)(0.3)u_3 + (0.1)(0.3)u_4$

$= 0.63u_1 + 0.07u_2 + 0.27u_3 + 0.03u_4$

$\Rightarrow$ Error function $= \sum \| \sum \phi_n u_n - \xi_d \|^2$

$\| x_1 - X_1 \|^2$

$\Rightarrow$

$X(0.3, 0.6) = 0.28X_1 + 0.12X_2 + 0.42X_3 + 0.18X_4$

$X(0.7, 0.5) = 0.15X_1 + 0.35X_2 + 0.15X_3 + 0.35X_4$

$X(0.7, 0.2) = 0.24X_1 + 0.56X_2 + 0.06X_3 + 0.14X_4$

$X(0.1, 0.3) = 0.63X_1 + 0.07X_2 + 0.27X_3 + 0.03X_4$

$Y(0.3, 0.6) = 0.28Y_1 + 0.12Y_2 + 0.42Y_3 + 0.18Y_4$

$Y(0.7, 0.5) = 0.15Y_1 + 0.35Y_2 + 0.15Y_3 + 0.35Y_4$

$Y(0.7, 0.2) = 0.24Y_1 + 0.56Y_2 + 0.06Y_3 + 0.14Y_4$

$Y(0.1, 0.3) = 0.63Y_1 + 0.07Y_2 + 0.27Y_3 + 0.03Y_4$

constraints

$\Rightarrow$

$u_1, v_1 \longrightarrow X_1, Y_1$ $\quad (0.3, 0.6) \longrightarrow (0.1, 0.9)$

$u_2, v_2 \longrightarrow X_2, Y_2$ $\quad (0.7, 0.5) \longrightarrow (0.75, 0.6)$

$u_3, v_3 \longrightarrow X_3, Y_3$ $\quad (0.7, 0.2) \longrightarrow (0.8, 0.25)$

$u_4, v_4 \longrightarrow X_4, Y_4$ $\quad (0.1, 0.3) \longrightarrow (-0.1, 0.5)$

$$\text{Error Function} = \|a_1 - X_1\|^2 + \cdots + \|a_4 - X_4\|^2$$

$$+ \|y_1 - Y_1\|^2 + \cdots + \|y_4 - Y_4\|^2$$

$$\Rightarrow Er = \sum_{d=1}^{4} \left\| \sum_{n=1}^{4} \phi_n a_n - X_d \right\|^2 + \left\| \sum_{n=1}^{4} \phi_n y_n - Y_d \right\|^2$$

$$\Rightarrow Er = \|0.28a_1 + 0.12a_2 + 0.42x_3 + 0.18a_4 - 0.1\|^2 + \|0.15x_1 + 0.35x_2 + 0.15x_3 + 0.35x_4 - 0.75\|^2$$

$$+ \|0.24x_1 + 0.56x_2 + 0.06x_3 + 0.14x_4 - 0.8\|^2 + \|0.63x_1 + 0.07x_2 + 0.27x_3 + 0.03x_4 + 0.1\|^2$$

$$+ \|0.28y_1 + 0.12y_2 + 0.42y_3 + 0.18y_4 - 0.9\|^2 + \|0.15y_1 + 0.35y_2 + 0.15y_3 + 0.35y_4 - 0.6\|^2$$

$$+ \|0.24y_1 + 0.56y_2 + 0.06y_3 + 0.14y_4 - 0.25\|^2 + \|0.63y_1 + 0.07y_2 + 0.27y_3 + 0.03y_4 - 0.5\|^2$$

$$\Rightarrow \begin{cases} \dfrac{\partial Er}{\partial a_n} = 0 \longrightarrow 4 \text{ Equations} \\ \dfrac{\partial E}{\partial y_n} = 0 \longrightarrow 4 \text{ Equation} \end{cases}$$

$$\frac{\partial Er}{\partial a_1} = 2(0.28)\| \textcircled{1} \| + 2(0.15)\| \textcircled{2} \| + 2(0.24)\| \textcircled{3} \| +$$

$$2(0.63)\| \textcircled{4} \| + 0 + 0 + 0$$

$$\frac{\partial Er}{\partial a_2} = 2(0.12)\| \textcircled{1} \| + 2(0.35)\| \textcircled{2} \| + 2(0.56)\| \textcircled{3} \| + 2(0.07)\| \textcircled{4} \| + 0 + 0 + 0$$

$$\frac{\partial Er}{\partial a_3} = 2(0.42)\| \textcircled{1} \| + 2(0.15)\| \textcircled{2} \| + 2(0.06)\| \textcircled{3} \| + 2(0.27)\| \textcircled{4} \| + 0 + 0 + 0$$

$$\frac{\partial Er}{\partial a_4} = 2(0.18)\| \textcircled{1} \| + 2(0.35)\| \textcircled{2} \| + 2(0.14)\| \textcircled{3} \| + 2(0.03)\| \textcircled{4} \| + 0 + 0 + 0$$

$$\frac{\partial Er}{\partial x_1} = \frac{\partial Er}{\partial x_2} = \frac{\partial Er}{\partial x_3} = \frac{\partial Er}{\partial x_4} = 0$$ Can be solved by a

$0.28\,\|\,①\,\| + 0.15\,\|\,②\,\| + 0.24\,\|\,③\,\| + 0.63\,\|\,④\,\| = 0$ A

$0.12\,\|\,①\,\| + 0.35\,\|\,②\,\| + 0.56\,\|\,③\,\| + 0.07\,\|\,④\,\| = 0$ B

$0.42\,\|\,①\,\| + 0.15\,\|\,②\,\| + 0.06\,\|\,③\,\| + 0.27\,\|\,④\,\| = 0$ C

$0.18\,\|\,①\,\| + 0.35\,\|\,②\,\| + 0.14\,\|\,③\,\| + 0.03\,\|\,④\,\| = 0$ D

A $\Rightarrow$

$(0.28)(0.28)x_1 + (0.28)(0.12)x_2 + (0.28)(0.42)x_3 + (0.28)(0.18)x_4$
$+ (0.15)(0.15)x_1 + (0.15)(0.35)x_2 + (0.15)(0.15)x_3 + (0.15)(0.35)x_4$
$+ (0.24)(0.24)x_1 + (0.24)(0.56)x_2 + (0.24)(0.06)x_3 + (0.24)(0.14)x_4$
$+ (0.63)(0.63)x_1 + (0.63)(0.07)x_2 + (0.63)(0.27)x_3 + (0.63)(0.03)x_4 =$
$= (0.28)(0.1) + (0.15)(0.75) + (0.24)(0.8) - (0.63)(0.1) =$
$= 0.5554x_1 + 0.2646x_2 + 0.3246x_3 + 0.1554x_4 = 0.2695$ ✓✓

B $\Rightarrow$

$(0.12)(0.28)x_1 + (0.12)(0.12)x_2 + (0.12)(0.42)x_3 + (0.12)(0.18)x_4$
$+ (0.35)(0.15)x_1 + (0.35)(0.35)x_2 + (0.35)(0.15)x_3 + (0.35)(0.35)x_4$
$+ (0.56)(0.24)x_1 + (0.56)(0.56)x_2 + (0.56)(0.06)x_3 + (0.56)(0.14)x_4$
$+ (0.07)(0.63)x_1 + (0.07)(0.07)x_2 + (0.07)(0.27)x_3 + (0.07)(0.03)x_4$
$= (0.12)(0.1) + (0.35)(0.75) + (0.56)(0.8) - (0.07)(0.1) =$
$0.2646x_1 + 0.4554x_2 + 0.1554x_3 + 0.2246x_4 = 0.7155$ ✓✓

C $\Rightarrow$ $(0.42)(0.28)x_1 + (0.42)(0.12)x_2 + (0.42)(0.42)x_3 + (0.42)(0.18)x_4$
$+ (0.15)(0.15)x_1 + (0.15)(0.35)x_2 + (0.15)(0.15)x_3 + (0.15)(0.35)x_4$
$+ (0.06)(0.24)x_1 + (0.06)(0.56)x_2 + (0.06)(0.06)x_3 + (0.06)(0.14)x_4$
$+ (0.27)(0.63)x_1 + (0.27)(0.07)x_2 + (0.27)(0.27)x_3 + (0.27)(0.03)x_4$
$= (0.42)(0.1) + (0.15)(0.75) + (0.06)(0.8) - (0.27)(0.1) =$
$0.3246x_1 + 0.1554x_2 + 0.2754x_3 + 0.1446x_4 = 0.1755$ ✓✓

D $\Rightarrow$ $(0.18)(0.28)x_1 + (0.18)(0.12)x_2 + (0.18)(0.42)x_3 + (0.18)(0.18)x_4$
$+ (0.35)(0.15)x_1 + (0.35)(0.35)x_2 + (0.35)(0.15)x_3 + (0.35)(0.35)x_4$
$+ (0.14)(0.24)x_1 + (0.14)(0.56)x_2 + (0.14)(0.06)x_3 + (0.14)(0.14)x_4$
$+ (0.03)(0.63)x_1 + (0.03)(0.07)x_2 + (0.03)(0.27)x_3 + (0.03)(0.03)x_4$
$= (0.18)(0.1) + (0.35)(0.75) + (0.14)(0.8) - (0.03)(0.1) =$
$0.1554x_1 + 0.2246x_2 + 0.1446x_3 + 0.1750x_4 = 0.3895$

$\Rightarrow$

$$\begin{bmatrix} 0.5554x_1 + 0.2646x_2 + 0.3246x_3 + 0.1554x_4 = 0.2695 \\ 0.2646x_1 + 0.4554x_2 + 0.1554x_3 + 0.2246x_4 = 0.7155 \\ 0.3246x_1 + 0.1554x_2 + 0.2754x_3 + 0.1446x_4 = 0.1755 \\ 0.1554x_1 + 0.2246x_2 + 0.1446x_3 + 0.1754x_4 = 0.3895 \end{bmatrix}$$

$$\frac{\partial Er}{\partial y_1} = \frac{\partial Er}{\partial y_2} = \frac{\partial Er}{\partial y_3} = \frac{\partial Er}{\partial y_4} = 0 \qquad A', B', C', D'$$

⇒ the left-Hand-Side Matrix would be the same as the matrix 4×4 and the RHS vector will be updated by the provided values..

So we will solve the above equation first.. and then we have:

$AX = B$

So matrix would be the same and we will have B as:

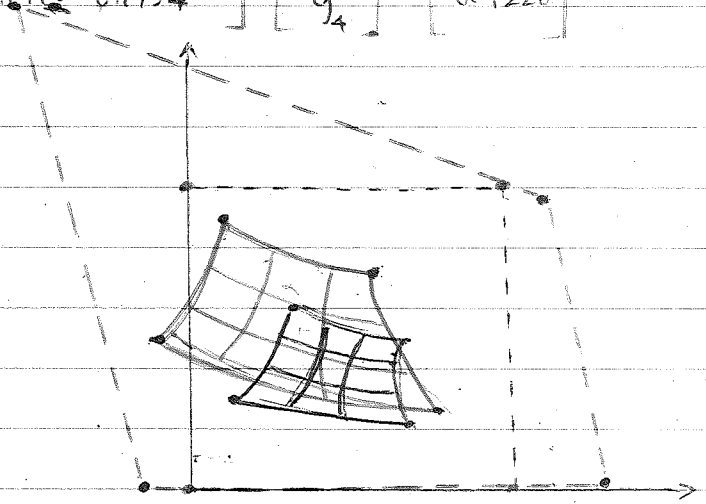
$$[B]_{4\times 1} = \begin{bmatrix} (0.28)(0.9) + (0.15)(0.6) + (0.24)(0.25) + (0.33)(0.5) \\ (0.12)(0.9) + (0.35)(0.6) + (0.56)(0.25) + (0.07)(0.5) \\ (0.42)(0.9) + (0.15)(0.6) + (0.06)(0.25) + (0.27)(0.5) \\ (0.18)(0.9) + (0.35)(0.6) + (0.14)(0.25) + (0.03)(0.5) \end{bmatrix} = \begin{bmatrix} 0.7170 \\ 0.4930 \\ 0.6180 \\ 0.4220 \end{bmatrix}$$

Now we ~~will have~~ can also solve them together or separately..

$$\begin{bmatrix} 0.5554 & 0.2646 & 0.3246 & 0.1554 & 0 & 0 & 0 & 0 \\ 0.2646 & 0.4554 & 0.1554 & 0.2246 & 0 & 0 & 0 & 0 \\ 0.3246 & 0.1554 & 0.2754 & 0.1446 & 0 & 0 & 0 & 0 \\ 0.1554 & 0.2246 & 0.1446 & 0.1754 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5554 & 0.2246 & 0.3246 & 0.1554 \\ 0 & 0 & 0 & 0 & 0.2646 & 0.4554 & 0.1554 & 0.2246 \\ 0 & 0 & 0 & 0 & 0.3246 & 0.1554 & 0.2754 & 0.1446 \\ 0 & 0 & 0 & 0 & 0.1554 & 0.2246 & 0.1446 & 0.1754 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0.2695 \\ 0.7155 \\ 0.1755 \\ 0.3895 \\ 0.7170 \\ 0.4930 \\ 0.6180 \\ 0.4220 \end{bmatrix}$$

Host:

$$\begin{cases} x_1 = -0.135 \\ x_2 = 1.25 \\ x_3 = -0.5143 \\ x_4 = 1.15 \end{cases} \qquad \begin{cases} y_1 = 0.007 \\ y_2 = 0.021 \\ y_3 = 1.725 \\ y_4 = 0.951 \end{cases}$$



Slave:

$$\begin{cases} (x_1, y_1)(0.3, 0.6) \Rightarrow (0.1, 0.9) \\ (x_2, y_2)(0.7, 0.5) \Rightarrow (0.75, 0.6) \\ (x_3, y_3): (0.7, 0.2) \Rightarrow (0.8, 0.25) \\ (x_4, y_4): (0.1, 0.3) \Rightarrow (-0.1, 0.5) \end{cases}$$

```
[hyou267@hpc5]bin/x86_64-linux/mpich2/gnu_4.6/HostMeshExample-debug
OpenCMISS(cm) version 0.3.0
Solving HostMesh Fitting Problem, Iteration:   1

Solver:
  Solver index = 1

Total user time for solver matrices assembly = 0.0000000
Total System time for solver matrices assembly = 0.0000000

Total user time for solver RHS assembly = 0.0000000
Total System time for solver RHS assembly = 0.0000000

Solver matrices:
Number of matrices = 1
Solver matrix : 1
Matrix Object: 1 MPI processes
  type: seqdense
5.5540000000000000e-01 2.6460000000000000e-01 3.2460000000000000e-01 1.5540000000000001e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
2.6460000000000000e-01 4.5539999999999992e-01 1.5540000000000001e-01 2.2459999999999997e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
3.2460000000000000e-01 1.5540000000000001e-01 2.7539999999999998e-01 1.4459999999999998e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
1.5540000000000001e-01 2.2459999999999997e-01 1.4459999999999998e-01 1.7540000000000000e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
5.5540000000000000e-01 2.6460000000000000e-01 3.2460000000000000e-01 1.5540000000000001e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
2.6460000000000000e-01 4.5539999999999992e-01 1.5540000000000001e-01 2.2459999999999997e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
3.2460000000000000e-01 1.5540000000000001e-01 2.7539999999999998e-01 1.4459999999999998e-01
0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00 0.0000000000000000e+00
1.5540000000000001e-01 2.2459999999999997e-01 1.4459999999999998e-01 1.7540000000000000e-01
Solver RHS vector:
Vector(:)          : 0.269500E+00  0.715500E+00  0.175500E+00  0.389500E+00  0.717000E+00  0.493000E
+00  0.618000E+00  0.422000E+00

Linear iterative solver parameters:
Final number of iterations = 4
Final residual norm =   4.6986158276600297E-014
Converged Reason = PETSc converged ATol

Solver solution vectors:
Number of solution vectors = 1
Solution vector for solver matrix : 1
Vector(:)          : -0.119444E+00  0.124167E+01 -0.545370E+00  0.118611E+01  0.694444E-02
0.208333E-01  0.172454E+01  0.951389E+00

Total user time for solve = 0.0000000
Total System time for solve = 9.99999978E-03
In direct memory block for handle type KEYVAL, 2 handles are still allocated
Program successfully completed.
```

✓ Results of the Computational Method.

✓ Numerical Solution.