BPMTimeline: Tempo Search and Time Mapping using an Analytical Solution

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ABSTRACT

Tempo maps is a common feature in many (commercial and/or open-source) Digital Audio Workstations, allowing the musician to automate tempo changes of a musical performance or work, as well as to visualize the relation between score time (beats) and real/performance time (seconds). Unfortunately, available music production, performance and remixing tools implemented with web technologies like JavaScript and Web Audio API do not offer any mechanism for flexible, and seamless, tempo manipulation and automation.

In this paper, we present BPMTimeline, a tempo mapping library, providing a seamless mapping between score and performance time. To achieve this, we model tempo changes as tempo functions (a well documented subject in literature) and realize the mappings through integral and the inverse of the integral of tempo functions.

1. INTRODUCTION

Tempo manipulation of a musical expression or works is used to (1) make a musical expression, or work, more lively (through a faster tempo) or more solemn (slower tempo); (2) allow a DJ to synchronize the tempo of several songs playing simultaneously, in order to align the beats; (3) create climaxes in a musical expression or work. Mainstream Digital Audio Workstations (DAWs), either commercial and/or open-source, like Ableton Live (see figure 1), Logic Pro and Reaper allow tempo automation through tempo maps, offering a seamless relation between performance and score time, using a tempo function. Unfortunately, there are no tempo mapping implementations in JavaScript.

In this paper, we present the theory supporting tempo maps as well as a JavaScript implementation, BPMTimeline. The development of this implementation followed three non-functional requirements:

- **No real impact on application performance:** the temporal overhead for time mappings and tempo search and time mapping should remain unnoticed;

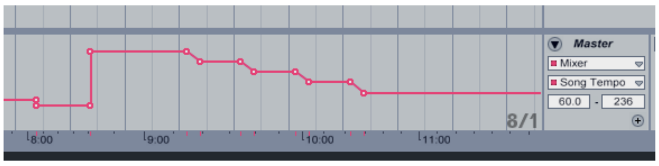


Figure 1: Tempo automation example in Ableton Live, using step and linear tempo functions.

- **Seamless tempo manipulation:** the developer should not be restricted to crips tempo changes (e.g.: step functions) nor should tempo changes be only allowed on restricted time marks (e.g.: the beginning of a beat or measure);

- **Easy to integrate:** the implementation should be self-contained, with no need to include external libraries.

In section 2, we describe essential theoretical definitions regarding the relation between score and real time. In section 3, we detail our implementation. Finally, in section 4, we describe some use cases for BPMTimeline.

Our implementation and demo pages are currently hosted at GitHub[1].

2. RELATED WORK

Tempo maps is a well document subject [2, 5, 4, 10, 3]. The main idea of a tempo map is to relate score time (beats) with real/performance time (seconds) through algebric manipulation of the closed form of tempo functions.

The available JavaScript live coding frameworks and DAWs do not offer tempo maps. In frameworks like Timbre.js [8] and Flocking [1], tempo manipulation is achieved through manual crisp changes, and tempo automation is achieved. In Tone.js [7], tempo manipulation and automation is achieved through both manual/crisp changes and through a Tone.Signal object, which offers a similar API to AudioParam. Still, Tone.js offers no tempo mapping facilities.

3. THEORETICAL BACKGROUND

We start by describing the basic relation between time, beats and tempo in order to infer the desired mappings. For additional background, please see [10, 4]. According to [10],

[1] https://github.com/echo66/bpm-timeline.js

music tempo T[2] is defined as the number of pulses/beats b per t time units,

$$T = \frac{b}{t} \tag{1}$$

the number of beats b, after t time units, with tempo T is given by

$$b = T * t \tag{2}$$

the duration t of b beats, with tempo T is defined as

$$t = \frac{b}{T} \tag{3}$$

and the *beat period*, which is the inverse of tempo T, is defined as

$$beatPeriod = \frac{1}{T} = \frac{t}{b} \tag{4}$$

the equations 1, 2, 3 and 4 assumes that T, t, b and $beatPeriod$ are constants.

To understand how do we map real time to score time using equations 1, 2, 3 and 4, we will use a set of examples. Consider the first one: how many minutes have passed when we reach beat 4, with a constant tempo of 120 bpm (beatPeriod = 0.5 secs)? According to equation 3, $t = \frac{4}{120} = 0.03\, min$, which is the same as the area under the curve in figure 2(a). If we decide to do a sudden change at beat 2, $t = \frac{2}{120} + \frac{2}{110} = 0.0348\, min$. But how do we use those equations to model a linear tempo change, beats 0 and 4, as depicted in 2(d)? One solution is to approximate the linear function through a sum of "steps" (figure 2(c)). If we use infinitesimally small steps, the mapping from score time to real time, $t(b)$, is defined as the integral of the beat period function. To map real time to score time, $b(t)$, we use the inverse of $t(b)$

$$t(b) = \int_0^b beatPeriod(\beta)\, d\beta \tag{5}$$

$$b(t) = t^{-1}(b) \tag{6}$$

According to the previous definitions, a tempo map can be defined by the closed form for $T(b)$, $beatPeriod(b)$, $t(b)$ and $b(t)$. For the current implementation, we provide three default closed forms for tempo mapping: linear, exponential and step tempo functions, all of them inspired in AudioParam automation functions.[3]

In [10], the authors used $T(t)$ instead of $T(b)$, resulting in a different integral and inverse of the integral for the tempo function. Despite the differences, the tempo map will produce the same results.

3.1 Supported Closed Forms

In the following three subsections, we present the three closed forms for tempo maps included in our implementation: step, linear and exponential forms. All three closed forms share a set of terms related to tempo and time:

[2]T can be interpreted as the frequency of the beat/pulse "train", in the score timeline [6].

[3]http://webaudio.github.io/web-audio-api/#the-audioparam-interface

- b_0, b_1: the initial and final score times;
- T_0, T_1: the initial and final tempos;
- bP_0, bP_1: the initial and final beat periods;
- t, b: the target time for tempo search and time mappings, t for real time and b for score time;
- t_s: the time offset, measure in seconds, for the function.

3.1.1 Linear

The linear closed form, based on the definition on the formula used in AudioParam *linearRampToValueAtTime* method

$$lin_{X_0,X_1,Y_0,Y_1}(x) = Y_0 + \Delta * (x - X_0),\ \Delta = \frac{Y_1 - Y_0}{X_1 - X_0} \tag{7}$$

this function is not defined for $X_0 = X_1$. Using this function, the tempo and beat period functions can be defined as

$$T(b) = lin_{b_0,b_1,T_0,T_1}(b),\ beatPeriod(b) = lin_{b_0,b_1,T_0,T_1}(b)$$

with both functions not defined for $b_0 = b_1$. This "corner case" occurs when there is a sudden jump in tempo values. In that case, one should model the tempo change with a step tempo change function. To obtain the mappings, we integrated the linear function and then obtained the inverse of the integral

$$L_{X_0,X_1,Y_0,Y_1,C}(x) = \int_0^x lin_{X_0,X_1,Y_0,Y_1}(x)\, d_x = \\ = \frac{\Delta}{2} * (x - X_0)^2 + Y_0 * (x - X_0) + C$$

$$L^{-1}_{X_0,X_1,Y_0,Y_1,C}(y) = \begin{cases} sol_1(y) + X_0, & if\ sol_1(y) > 0 \\ sol_2(y) + X_0, & otherwise \end{cases}$$

$$sol_1(y) = \frac{-Y_0 + k_1}{2 * \Delta}\ , sol_2(y) = \frac{-Y_0 - k_1}{2 * \Delta}$$

$$k_1 = Y_0^2 - \frac{4}{2} * \Delta * (C - y)$$

According to equations 5 and 6, the time mappings with the linear closed form are defined as

$$t(b) = L_{b_0,b_1,bP_0,bP_1,t_s}(b),\ b(t) = L^{-1}_{b_0,b_1,bP_0,bP_1,t_s}(t)$$

3.1.2 Exponential

The exponential closed form uses the exponential tempo function defined in [10]

$$exp_{X_0,X_1,Y_0,Y_1}(x) = Y_0 * e^{k_2 * (x - X_1)},\ k_2 = \frac{\log \frac{Y_1}{Y_0}}{X_1 - X_0}$$

and the tempo and beat period functions defined as

$$T(b) = exp_{b_0,b_1,T_0,T_1}(b),\ beatPeriod(b) = exp_{b_0,b_1,T_0,T_1}(b)$$

with both functions not defined for $b_0 = b_1$. This is the same "corner case" as in the linear closed form. Regarding the time mappings for the exponential closed form, they can be obtained in a similar fashion to the linear closed form with the integral of the exponential function

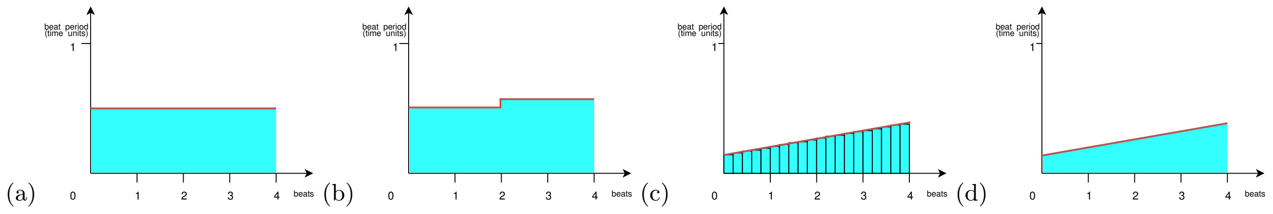


Figure 2: Examples of tempo functions, between beats 0 and 4. In (a), we have a constant tempo of 120 bpm *(beatPeriod = 0.5 secs)*. In (b), there is a sudden (step tempo function) change, at beat 2, from 120 to 110 bpm *(beatPeriod = 0.(54) secs)*. In (c), there is a sequence of step tempo change that are used to approximate a linear tempo change as seen in (d). In these examples, we chose to use beat period instead of BPM, for the y-axis, to make the section 2 explanation, regarding b(t) and t(b), more intuitive.

$$E_{X_0,X_1,Y_0,Y_1,C}(x) = \int_0^x exp_{X_0,X_1,Y_0,Y_1}(x)\,d_x =$$

$$= Y_0 * \frac{e^{k_3*(x-X_0)}-1}{k_3} + C$$

and the inverse of the integral

$$E^{-1}_{X_0,X_1,Y_0,Y_1,C}(y) = \frac{\log \frac{C*(y-C)+1}{Y_0}}{k_3} + X_0$$

and the final map between the generic functions and the time mapping functions

$$t(b) = E_{b_0,b_1,bP_0,bP_1,t_s}(b),\ \ b(t) = E^{-1}_{b_0,b_1,bP_0,bP_1,t_s}(t)$$

3.1.3 Step

The step closed form uses a similar tempo function to the AudioParam *setValueAtTime*,

$$step(x) = \begin{cases} Y_0, & if\ x < X_1 \\ Y_1, & otherwise \end{cases}$$

with the difference that the value jumps to Y_1 only for $x = X_1$ instead of $x = X_0$, like *setValueAtTime* formula.

$$beatPeriod(b) = \begin{cases} bP_0, & if\ b < b_1 \\ bP_1, & otherwise \end{cases} \tag{8}$$

$$t(b) = \frac{b - t_s}{bP_0} + b_0 \tag{9}$$

$$b(t) = bP_0 * (t - b_0) + t_s \tag{10}$$

3.2 Beats Per Minute (BPM) definition

T is (usually[4]) expressed in BPM. For example, $120\,bpm = \frac{120\,beats}{1\,min}$. But, usually, BPM is defined as

$$BPM = \frac{60\,(secs)}{beatPeriod} \tag{11}$$

with *beatPeriod* measured in seconds. How do we deduce equation 5 from equation 1? If music tempo T is measured as BPM, then $t = 1min$ and we can deduce from equation 1 that $BPM = \frac{b}{1} = b$. Due to the fact that $1\,min = 60\,secs$, we can make the following deduction

$$T = \frac{b}{t} \Leftrightarrow \frac{1}{T} = beatPeriod = \frac{t}{b}$$

$$t = 1\,min \Rightarrow T = \frac{b}{1} = b = BPM$$

$$t = 1\,min = 60\,secs \Rightarrow beatPeriod = \frac{60\,(secs)}{BPM} \Leftrightarrow$$

$$\Leftrightarrow BPM = \frac{60\,(secs)}{beatPeriod}$$

Through out the remainder of this paper, unless stated otherwise, T and t is measured as BPM and seconds. Additionally, we use equation 7 to relate BPM and beat period instead of using the generic tempo/time relations stated in equations 1 to 4.

4. IMPLEMENTATION

In this section, we describe our JavaScript implementation for tempo maps, as defined in section 2. The current implementation has seven main features:

- find tempo T at beat b (e.g.: what is the BPM T, at beat b);
- find tempo T at time t (e.g.: what is the BPM T, at t seconds);
- find what is the beat b at time t (i.e.: mapping time to beats);
- find what how much time t has passed at beat b. (i.e.: mapping beats to time);
- add, edit and remove tempo markers;
- add new closed forms for tempo functions;
- observe changes in a BPMTimeline instance through event listeners.

The first four features are related to tempo search and time mapping. The following two are related to tempo markers management. The final one offers a way to notify JavaScript components of changes in a BPMTimeline. A tempo marker is a JSON object that encodes the necessary information to evaluate local and global tempo functions:

[4]In many DAWs like Ableton Live, Logic Pro Tools, Reaper, Ardour and LMMS, music tempo is expressed in BPM. Still, it should be noted that music tempo can be expressed using, for example, italian tempo markings like *Largo*, *Adagietto*, *Andante moderato* and so on.

- *endBeat*: Number defining beat b_1, stated by the developer when inserting the tempo marker in the BPMTimeline instance;

- *endTime*: Number defining time t_1, measured in seconds, calculated using $t(b_1)$ when inserting the tempo marker in the BPMTimeline instance;

- *endTempo*: Number defining the final tempo T_1, stated by the developer when inserting the tempo marker in the BPMTimeline instance;

- *endPeriod*: Number defining the duration of a beat at the "end" of the corresponding tempo marker/function, measured in seconds, and calculated when inserting the tempo marker in the BPMTimeline instance;

- *type*: String identifying which closed form (step, linear, exponential or custom) is used to define the tempo and mapping functions, stated by the developer when inserting the tempo marker in the BPMTimeline instance.

The global tempo function, $T_g(b)$, is a non-continuous function, defined as

$$T_g(b) = \begin{cases} T_{l_1}(b), & if\ b_0 \leq b \leq b_1 \\ T_{l_2}(b), & if\ b_1 < b \leq b_2 \\ ... \\ T_{l_N}(b), & if\ b_{N-1} < b \end{cases}$$

where $T_{l_i}(b)$, $i = 1..N$, are (local) tempo functions. A global tempo function is represented as a sorted array of tempo markers, sorted by *endBeat.* Each BPMTimeline instance has only one global tempo function. Each tempo marker represents a local tempo function. The values T_0, t_0, b_0 and t_s for each tempo are obtained by accessing the *endTempo* (for T_0), *endTime* (for T_0) and *endBeat* of the previous marker in the global tempo function array.

As stated in section 1, (temporal) performance is a very important requirement. To fulfil this requirement, some trade-offs are needed for the different features available. As such, we make the following assumptions:

- in DAWs and DJ software, tempo maps are not changed very frequently during a live performance;

- as such, tempo search and time mapping are more important than tempo markers management.

We should note that the formulation, in section 2, for the tempo function $T(b)$ only mentions b. But, in practise, we could replace, in equations 8, 11, and 14 the terms b_0, b_1, bP_0, bP_1 and b for t_0, t_1, T_0, T_1 and t, the tempo would will be the same for both cases ($T(b) = T(t)$).

4.1 Insertion, Edition and Removal of Tempo Markers

There are three methods for tempo markers management:

- *add_tempo_marker(String type, Number endBeat, Number endTempo)*: performs a binary search in the tempo marker array to find the neighbour tempo markers A and B, where $A.endBeat < B.endBeat \wedge A.endBeat < endBeat < B.endBeat$. After finding those neighbours, insert the new tempo marker between them and update the *endTime* field of all markers for which $\forall_M A.endBeat \geq M.endBeat$. If the new marker is the last one, only its *endTime* field will be updated. If there is already a tempo marker with the same value for *endBeat*, an exception will be thrown.

- *remove_tempo_marker(Number endBeat)*: performs a binary search in the tempo marker array to find the tempo marker A for which $M.endBeat = endBeat$. After that, removes M from the tempo markers array and update the *endTime* field for all tempo markers that $\forall_M A.endBeat \geq M.endBeat$. If there is no tempo marker A for which $M.endBeat = endBeat$, then an exception will be thrown.

- *change_tempo_marker(Number oldEndBeat, Number newEndBeat, Number newEndTempo, String newType)*: removes the tempo marker from the array and adds the new version. If there is no tempo marker A for which $M.endBeat = endBeat$, then an exception will be thrown.

These four methods for tempo marker management have the temporal complexity is $O(N)$. This is due to the usage of a binary search over a sorted array, $O(log_2 N)$, and the calculation for the *endTime* field, $O(N)$. As such, we have $O(log_2 N) + O(N) = O(N)$.

To add a new closed form for tempo functions, the developer can use the *add_tempo_marker_type(String type, Function tempoFn, Function integralFn, Function inverseIntegralFn)* method. Parameters 2, 3 and 4 are explained in section 2 and their API must be defined as:

- *tempoFn(Number b_0, Number b_1, Number T_0, Number T_1, Number b)*: implementation of the tempo function $T(b)$, as defined in section 2;

- *integralFn(Number b_0, Number b_1, Number T_0, Number T_1, Number t_s, Number t)*: implementation of the mapping $t(b)$, as defined in section 2;

- *inverseIntegralFn(Number b_0, Number b_1, Number bP_0, Number bP_1, Number t_s, Number b)*: implementation of the mapping $b(t)$, as defined in section 2.

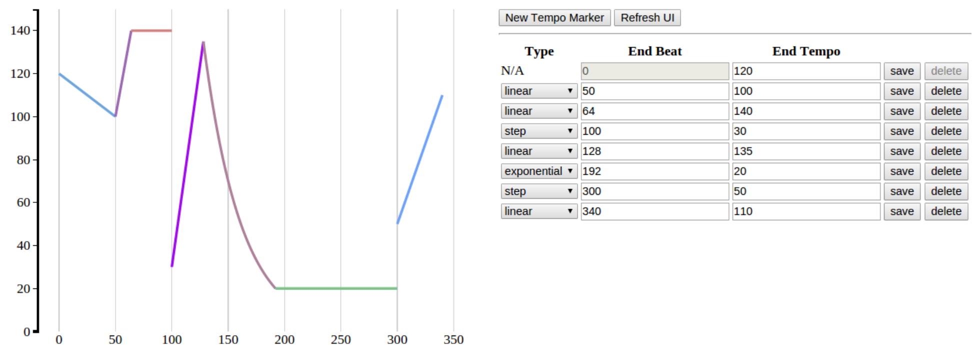
This method has temporal complexity of $O(1)$ (on average) because it only performs an insertion in an associative array.

4.2 Tempo Search and Time Mapping Methods

The current implementation has 4 methods for this task:

- *beat(Number t)*: Maps real time t to score time, $b(t)$. First, performs a binary search over the tempo markers array to find tempo markers A and B for which $A.endTime < B.endTime \wedge$ $A.endTime < t < B.endTime$. If the search returns both A and B, perform the mapping $b(t)$ using *inverseIntegralFn(A.endBeat, B.endBeat, A.endPeriod, B.endPeriod, B.endTime, t)*. If the search returns just a marker, then then $b(t) = A.endBeat$.

- *time(Number b)*: Maps score time b to real time, $t(b)$. First, performs a binary search over the tempo markers array to find tempo markers A and B for which



Type	End Beat	End Tempo		
N/A	0	120	save	delete
linear	50	100	save	delete
linear	64	140	save	delete
step	100	30	save	delete
linear	128	135	save	delete
exponential	192	20	save	delete
step	300	50	save	delete
linear	340	110	save	delete

Figure 3: A simple demo page with a graph plotting the output of BPMTimeline.tempo_at_beat, and a table listing the markers of the BPMTimeline instance.

$A.endBeat < B.endBeat \wedge A.endBeat < b < B.endBeat$. If the search returns both A and B, perform the mapping $b(t)$ using *integralFn(A.endBeat, B.endBeat, A.endPeriod, B.endPeriod, B.endTime, b)*. If the search returns just a marker, then then $t(b) = A.endTime$

- *tempo_at_time(Number t)*: Returns tempo at real time *t*. First, performs a binary search over the tempo markers array to find tempo markers A and B for which $A.endTime < B.endTime \wedge$ $A.endTime < t < B.endTime$. If the search returns both A and B, obtain the tempo through *tempoFn(A.endTime, B.endTime, A.endTempo, B.endTempo, t)*. If the search returns just a marker, then the tempo is equal to A.endTempo.

- *tempo_at_beat(Number b)*: Returns tempo at score time *b*. First, performs a binary search over the tempo markers array to find tempo markers A and B for which $A.endBeat < B.endBeat \wedge$ $A.endBeat < b < B.endBeat$. If the search returns both A and B, obtain the tempo through *tempoFn(A.endBeat, B.endBeat, A.endBeat, B.endBeat, b)*. If the search returns just a marker, then the tempo is equal to A.endTempo.

All four methods have temporal complexity of $O(log_2 N)$ due to the usage of binary search of a sorted array. If no marker is found in each of these four methods, an exception is thrown.

4.3 Event Listeners

Each BPMTimeline instance is observable: all changes (i.e. everytime a marker is added, edited or removed) in the instance results in an event being created and a set of event listeners will be invoked to deal with that event. In order to register/remove event listeners in a BPMTimeline instance, the class provides two functions:

- *add_event_listener(String observerId, String eventType, Function callback)*: register an event listener for events of the following types: *"add-tempo-marker"*, *"change-tempo-marker"* and *"remove-tempo-marker"*.

- *remove_event_listener(String observerId, String eventType)*: remove the listeners, that were registered by observerId, for events with eventType type.

The *observerId* argument is used to prevent conflicts between two observers using the same function as event listener. Each event object has the following schema:

```
{ type: String,
  oldMarker: MarkerDescription,
  newMarker: MarkerDescription }
```

The *type* field can have one of the following values: *"add-tempo-marker"*, *"remove-tempo-marker"* or *"edit-tempo-marker"*. When type is equal to *"add-tempo-marker"*, the *oldMarker* field does not exist. When *field* is equal to *"remove-tempo-marker"*, the *newMarker* field does not exit. Each *MarkerDescription* instance has the following schema:

```
{ startBeat: Number, endBeat: Number,
  startTime: Number, endTime: Number,
  startTempo: Number, endTempo: Number,
  type: String }
```

startBeat, *endBeat*: Numbers stating where does the marker tempo functions start and end in score time.

startTime, *endTime*: Numbers stating where does the marker tempo functions start and end in real time.

5. USE CASES

To date, we have explored three use case scenarios for BPMTimeline: (1) *event scheduling*, mapping time values in AudioContext.currentTime in order to schedule buffer and oscillator plays; (2) *auto-syncing* of several audio players to a dynamic master tempo, in a similar fashion to Ableton Live; (3) *time rulers*, for real and score times, related through a BPMTimeline instance.

5.1 Event Scheduling and Effect Automation

BPMTimeline can be used to control, through the mapping $t(b)$, the scheduling of Web Audio API (WAA) Audio Nodes (like Audio Buffer Source Nodes and Oscillator Nodes) and their Audio Parameters. Assuming that for $beat = 0 \Rightarrow AudioContext.currentTime = 0$, the developer can play a buffer source node and/or an oscillator node using the following code:

```
/** Initial tempo of 60 bpm. **/
var tl = new BPMTimeline(60);
var a_t_m = tl.add_tempo_marker;
```

```
a_t_m({type:"linear",endBeat:10,endTempo
    :200});
a_t_m({type:"linear",endBeat:15,endTempo
    :10});
a_t_m({type:"linear",endBeat:20,endTempo
    :400});
a_t_m({type:"linear",endBeat:60,endTempo
    :60});
var ctx = new AudioContext();
var osc = ctx.createOscillator();
/** Play 20 beats. **/
for (var i=0; i<20; i++) {
 scheduleNote(osc, 'G3', i, 0.5, ctx.
    currentTime);
}
osc.connect(ctx.destination); osc.start();
```

resulting in a half-beat pulse train, that increases and/or decreases its tempo throughout the time line. This code is available in our Git repository[5]. Additionally, this module could be used to control the tempo in libraries like Tuna.js[6] which, as far as we know, does not have any implementation for constant or dynamic tempo.

5.2 Automatic Audio Player Synchronization

Another use case is the synchronization of audio audio players to a master tempo time line, which is very common in live music performance applications like Ableton Live and Traktor Pro[7]. In one of our prototypes, we share a BPMTimeline instance (the master tempo) with several audio players. The stretching factor for each player is determined through the relation between the master tempo and the tempo of the audio segment being played/stretched[8].

5.3 Time Rulers

WAVES UI library [9] provides a set of music related UI components (automation lines/breakpoints, waveforms, time annotations, time rulers, etc...) for HTML5. One of those components, time axis[9], allows developers to present two time rulers: one for real time (seconds) and another for score time (beats), both related through a constant tempo (BPM). In order to use BPMTimline with WAVES UI library, the developer must decide what will be ”static” time line[10], either real time or score time, in the rulers. Traditionally, DAWs make score time as the ”static” time line. In order to do the same with WAVES UI, the developer will need to state the time values for segments, breakpoints, waveforms and traces in score time.

5.4 Evaluation

TODO

6. CONCLUSIONS AND FUTURE WORK

BPMTimeline provides an intuitive API for developers to relate time and beats, according to a custom tempo function.

[5]https://github.com/echo66/bpm-timeline.js/blob/master/demos/demo3.html
[6]https://github.com/Theodeus/tuna
[7]Traktor Pro has a master tempo clock to control effects and the time stretching factor of audio players. Still, that clock does not support tempo curves.
[8]https://github.com/echo66/OLA-TS.js/blob/master/SegmentProcessorV3.js#L121
[9]https://github.com/wavesjs/ui/tree/develop/es6/axis
[10]If we choose beat time to become ”static”, the tempo changes affect the real time ruler.

Due to the search method employed, we expect to minimize the (temporal) performance impact of time mapping and tempo search functions.

The next step will be the support for arbitrarily complex functions. Instead of specifying the closed form for all tempo functions (there are infinite tempo functions), one could sample an tempo function and interpolate the resulting sequence with a set of linear tempo functions.

After that, we plan to integrate BPMTimeline in browser live coding environments like Flocking [1], Timbre.js [8], Tone.js [7] and UI libraries/frameworks like Nexus UI [11] and WAVES UI [9].

Finally, we expect to port our code to C++ to include BPMTimeline in projects like QTractor [11], Ardour and LMMS.

7. ACKNOWLEDGMENTS

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