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Technical note

Using simulations and data to evaluate mean sensitivity (ζ) as a useful statistic in dendrochronology

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ABSTRACT

Mean sensitivity (ζ) continues to be used in dendrochronology despite a literature that shows it to be of questionable value in describing the properties of a time series. We simulate first-order autoregressive models with known parameters and show that ζ is a function of variance and autocorrelation of a time series. We then use 500 random tree-ring data sets with unknown parameters and show that ζ is at best equivalent to the standard deviation of a time series in cases without high autocorrelation and is an inefficient estimator of the coefficient of variation. It is hard to justify the use of ζ as a useful, descriptive statistic in dendrochronology on theoretical or empirical grounds. It is better to make a thorough evaluation of the time series properties of a data set and we suggest various avenues for doing so including some that are maybe unfamiliar to most dendrochronologists including generalized autoregressive conditional heteroscedasticity (GARCH) models.

1. Introduction

Mean sensitivity has a long use in dendrochronology going back to the foundation of the discipline in work by Douglass (1920) and built on by other noted pioneers in the field such as Schulman (1956) and Fritts (2001). Mean sensitivity was conceived as a statistic that would indicate if a series was useful for crossdating or responsive to climate. But mean sensitivity as a useful statistic is suspect. Strackee and Jansma (1992) provided a thorough investigation of the statistical properties of mean sensitivity and conclude that the statistic is ambiguous in describing tree-ring time series. Biondi and Qeadan (2008) voiced similar concerns that mean sensitivity includes only interannual variation and proposed using the Gini coefficient to integrate all possible lags in a mean-sensitivity function. Those authors also discuss the evolution of the formulas for ζ from Douglass (1920) onward. Despite the well-established problems with mean sensitivity, it is still being used in the literature. Searching the archives of *Dendrochronologia* turns up 24 papers with the phrase "mean sensitivity" from Volume 28, Issue 1 in 2010 through Volume 30, Issue 2 in 2012 (e.g., Dittmar et al., 2012; O'Donnell et al., 2010; Panyushkina et al., 2010) with some relying on mean sensitivity as an important parameter in the analyses.

We define mean sensitivity as ζ:

$$\zeta = \frac{2}{n-1} \sum_{t=2}^{n} \frac{|y_t - y_{t-1}|}{y_t + y_{t-1}} \tag{1}$$

where y is a measure of growth (e.g., ring-width or maximum latewood density), n is length, $t = 1 \ldots n$. The bounds of ζ are theoretically from 0 to 2 but values above 0.6 are rare in the literature. We note that there are other formulations of ζ some of which are equivalent and some of which differ slightly mathematically. However, we found that all variations of the mean sensitivity statistic suffer from the problems discussed by Strackee and Jansma (1992) and that we further illustrate below.

The primary theoretical concern with mean sensitivity is that of variance. This main point is that ζ, which expresses the year-to-year variability of the values in a time series, is dependent on the variance of the specific variable measured (e.g., ring width or cell-wall thickness). In living matter the magnitude of the variance is typically related to the nature of the variable itself. With tree growth, a series representing cell-wall thickness for example shows lower variance than a series of ring widths whether or not

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standardization is performed. Similarly the variance of ring-width series of evergreen tree species is higher than the variance of series from deciduous species, since the minimum ring-width value of evergreen species is zero whereas the minimum value of deciduous species such as oak (*Quercus* spp.) and ash (*Fraxinus* spp.) is equal to the width of their spring vessels (ca. 0.1 mm). This dependency on variance often remains implicit when the values of ζ for different tree-ring variables and/or tree species are compared.

Our first objective in this paper is to demonstrate that ζ can be decomposed by knowing the standard deviation (σ) and first-order autocorrelation (ϕ) structure of a time series. Our second objective is to show that in many cases where the actual parameters of a tree-ring time series are unknown ζ is effectively proportional to σ except in cases with strong autocorrelation.

2. Tree-ring data simulation

We explore ζ by simulating data similar to standard tree-ring chronologies (i.e., mean-value chronologies from detrended ring widths). We parametrized our simulations using publicly-available data as described below. The advantage of these simulations is that the statistical properties can be specified and we can explore ζ in a systematic way. We have also performed similar analyses on raw ring widths and seen similar results. All analysis was done in the R programming environment (R Development Core Team, 2012) using the dplR library (Bunn, 2008; Bunn et al., 2012).

2.1. Simulation of first-order autoregressive data

Tree growth is typically autocorrelated in some manner. Many tree-ring data sets can be approximated using a first-order autoregressive process where a year's growth is partially a function of growth from the prior year. The simplest of these processes is:

$$y_t = \phi y_{t-1} + \epsilon_t \tag{2}$$

where y and t are as in Eq. (1), ϕ is a coefficient ranging from -1 to 1, and ϵ is some other growth process (e.g., plant physiology, canopy dynamics, climate, etc.). This AR(1) model is familiar to most dendrochronologists as is an extended model as AR(p) where p indicates the order of an autoregressive model and other terms are as in Eq. (2):

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t \tag{3}$$

We randomly selected 500 data sets with at least 20 series and spanning at least 100 years from the International Tree-Ring Data Bank and the Digital Collaboratory for Cultural Dendrochronology (Jansma et al., 2012) and built a mean-value chronology using Tukey's biweight robust mean after detrending each series with a smoothing spline with frequency response of 0.50 at a wavelength of 0.67 of the series length (Cook and Peters, 1981; Cook and Kairiukstis, 1990). We fit each of these 500 chronologies with an autoregressive time-series model. The order ($p \leq 4$) was chosen using the Akaike Information Criterion. Over half of these chronologies (270) followed an AR process with $p = 1$ or $p = 2$ with the most-common model being a first-order process. Our characterization of these random tree-ring data sets is similar to the finding of Monserud (1986) who analyzed the time-series properties of 33 data sets in terms of p, σ, and ϕ in a classic paper on the time-series characteristics of tree-ring data. We found that these parameters were insensitive to change on repeated random grabs of the 500 data sets which indicated that species, time period, and other variables are largely irrelevant for values of p, σ, and ϕ. For this work, we

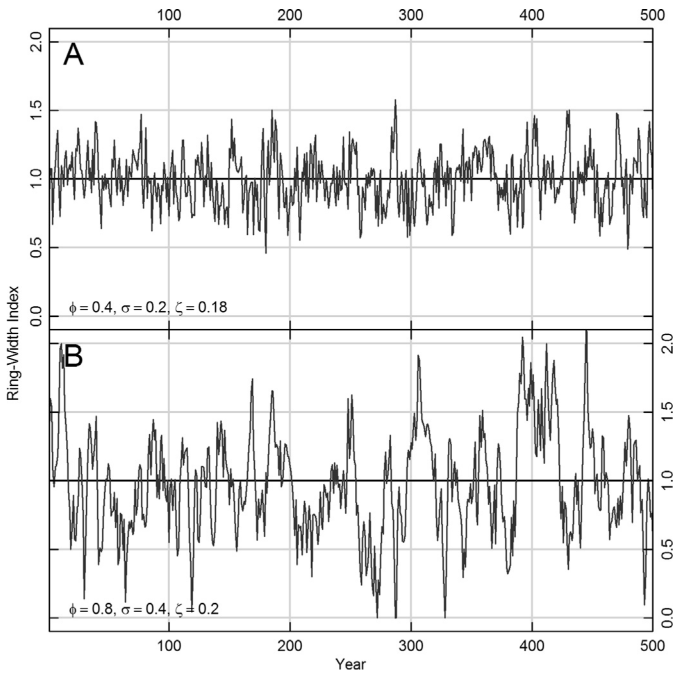

Fig. 1. Two examples of simulated tree-ring index data spanning 500 years using AR(1) models with relatively low autocorrelation and variance (A) and relatively high autocorrelation and variance (B). Note the similarity in ζ for A and B.

simulated AR(1) time series similar to these 500 tree-ring chronologies but with known statistical processes (Fig. 1). See Appendix A for further details on the simulations.

2.2. ζ as a function of ϕ and σ

We show that ζ varies with ϕ and σ by simulating AR(1) time series (each 500 years long) with ϕ ranging from -0.9 to 0.9 and σ ranging from 0.05 to 0.5. Each series had a mean of one ($\mu = 1$). That parameter space was uniformly filled with 10^6 simulated time series. Fig. 2 shows that ζ varies linearly with σ and non-linearly with ϕ. Indeed, the highest values of ζ are found when ϕ is negative and σ is high. There are few, if any, biological mechanisms that can consistently create a tree-ring chronology with strong negative autocorrelation in an AR(1) process. As an aside, while this is true for AR(1) processes, it may not be for AR(2+) processes. For example, pines (*Pinus* spp.) can produce mast seed crops causing resource depletion and subsequent negative values of ϕ although those growth processes would no longer fit an AR(1) model (see Kelly and Sork, 2002 for a review of masting).

3. ζ, ϕ, and σ using 500 data sets

We then used the 500 data sets described above and calculated ζ, ϕ, and σ for each. Many of these followed an AR(1) process as simulated above but others did not. Some showed no substantial autocorrelation structure while others followed more complex models including autoregressive moving-average models (ARMA). See the section below for further discussion. We also found that the data taken as a whole support the primary finding by Strackee and Jansma (1992) and shown above in Fig. 2: ζ varies mostly linearly with σ but has a compressed range and is therefore difficult to interpret (Fig. 3A). The first-order autocorrelation is not a good way of interpreting ζ (Fig. 3B), unless the two variables ϕ and σ are taken together (Fig. 4). When σ and ϕ are used in conjunction to interpret ζ in the 500 random chronologies (only some of which follow an

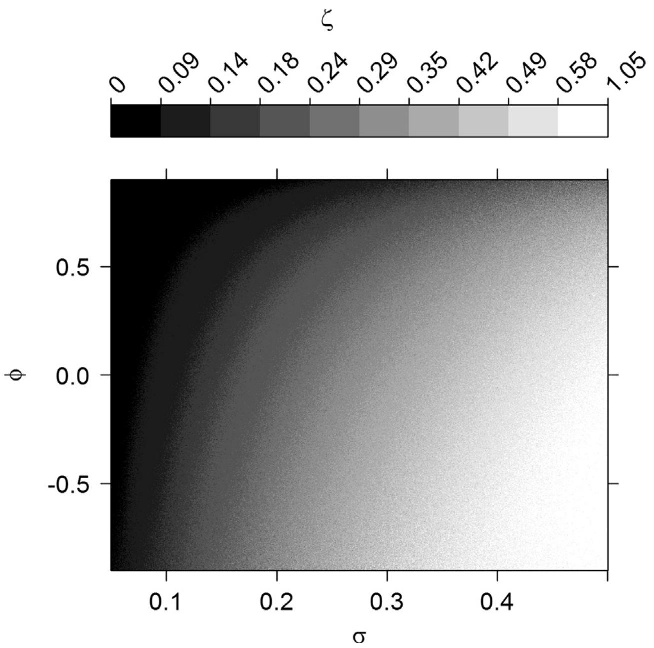

Fig. 2. The gray shades show ζ as a function of ϕ and σ from simulated AR(1) models. Note that the color scale is by 10% quantiles.

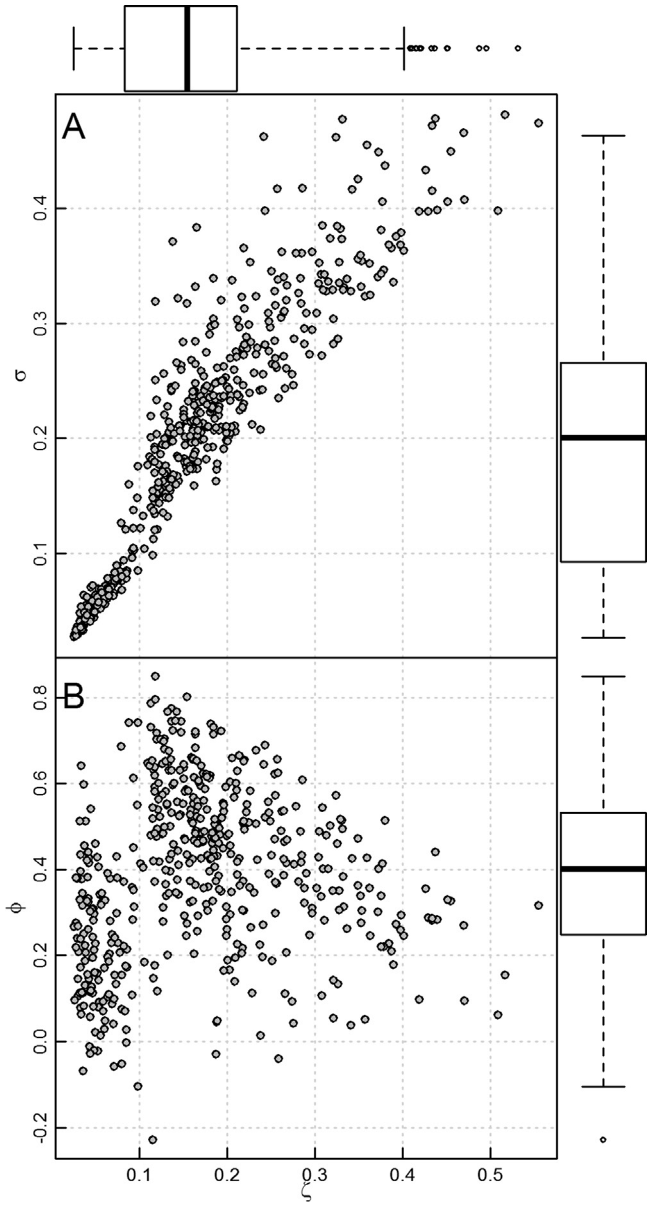

Fig. 3. ζ is plotted against σ (A) and ϕ (B) for 500 random tree-ring data sets. The marginal boxplots show the shape of the distributions for each variable.

AR(1) process) a pattern emerges consistent with the simulations shown in Fig. 2: the range of ζ becomes compressed relative to σ with high autocorrelation. Also, unlike the simulations in Fig. 2, none of the 500 random chronologies showed strongly negative values of ϕ. Finally, we note for higher values of ϕ (ca. >0.4), the slope of the relationship between σ and ζ is greater than one while for low values of ϕ (ca. <0.2) the slope is less than one (Fig. 4). In other words, when ϕ is high, ζ increases slightly more rapidly with increasing σ than it does when ϕ is low.

4. Implications and refinements

We have shown that mean sensitivity is a function of the standard deviation and autocorrelation of a series. Despite a theoretical range of zero to two, the practical range of ζ is much smaller only reaching values near one in series with very high variance and strong *negative* first-order correlation. This is not a situation that occurs often (if ever) in nature and the realized range of ζ is greatly reduced (Strackee and Jansma, 1992).

We have also shown that mean sensitivity is nearly proportional to the standard deviation of a time series using random data sets from the ITRDB and DCCD. The exceptions are when ζ is compressed in cases of strong autocorrelation. Given that parameters like σ and ϕ are standard descriptive statistics whose behavior is easily understood it seems that using ζ in their place is not advisable under many circumstances. We note that this is true even when the actual time-series properties of a tree-ring time series are unknown in the case of our 500 real-world examples.

Given that Figs. 3 and 4 show that ζ is a relatively well-behaved statistic in terms of σ and ϕ it is worth taking a deeper look at Eq. (1) algebraically. By doing so we can show that ζ is an inefficient estimator of the coefficient of variation ($CV = \sigma/\mu$) and express an estimation of ζ from σ and ϕ. Rearranging Eq. (1):

$$\zeta = \frac{1}{n-1}\sum_{t=2}^{n}\frac{\sqrt{(y_t - y_{t-1})^2}}{(y_t + y_{t-1})/2} \tag{4}$$

$$= \frac{1}{n-1}\sum_{t=2}^{n}\frac{\sqrt{2}\times s(y_t, y_{t-1})}{(y_t + y_{t-1})/2} \tag{5}$$

where $\sqrt{2}(s)$ in the numerator of Eq. (5) can be calculated as the standard deviation of y_t and y_{t-1} because for two values the estimator of the variance of n numbers is:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \tag{6}$$

$$= \frac{\left(y_{t-1} - \frac{(y_t + y_{t-1})}{2}\right) + \left(y_t - \frac{(y_t + y_{t-1})}{2}\right)}{2-1} \tag{7}$$

$$= \frac{(y_t - y_{t-1})^2}{2} \tag{8}$$

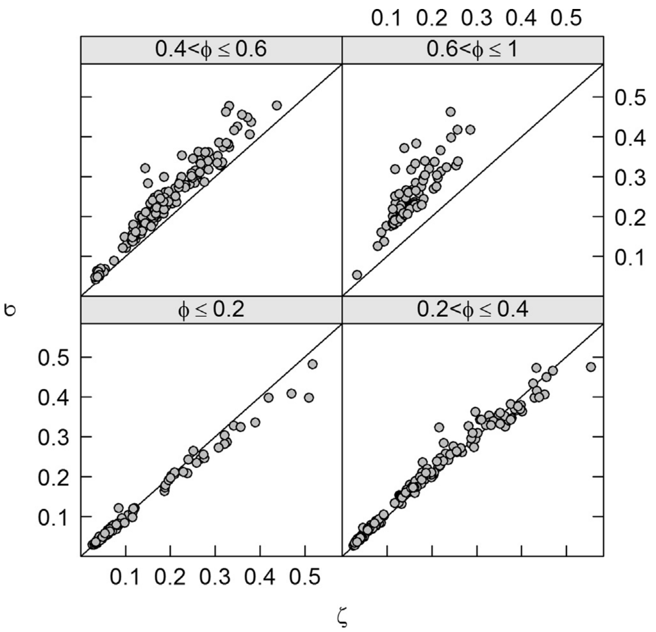

Fig. 4. ζ is plotted against σ and ϕ for the 500 random chronologies plotted in Fig. 3. Each panel shows a 1:1 line.

so that:

$$s = \frac{|y_1 - y_2|}{\sqrt{2}} \rightarrow |y_t - y_{t-1}| = s\sqrt{2} \tag{9}$$

and giving a final expression of ζ as a function of the mean coefficient of variation:

$$\zeta = \sqrt{2}\left(\sum_{t=2}^{n} \frac{s(y_t, y_{t-1})}{\mu(y_t, y_{t-1})}\right)(n-1)^{-1} \tag{10}$$

where $\mu(y_t, y_{t-1})$ is $(y_t + y_{t-1})/2$.

Thus, ζ is an estimator of the mean coefficient of variation for a time series y as the right side of Eqs. (4) and (5) gives us the mean pairwise *CV* for *t* and *t-1* multiplied by $\sqrt{2}$, where the numerator, *s*, is defined by Eqs. (6) to (9). Interpreting ζ as a function of the coefficient of variation is helpful because it allows us to recast the statistic in a more general framework in terms of the variance of a time series. However, we have argued that a powerful and intuitive way to understand tree-ring data is by thinking in the terms of their autocorrelation structure and Figs. 3 and 4 graph the relationship between ζ, σ, and ϕ. We can further extend our understanding of ζ as an estimator of mean *CV* by casting it in terms of an AR(1) process. If y is autocorrelated and stationary, which is the case for some but not all tree-ring data, ζ can be expected to be proportional to the following function of σ, ϕ, and μ:

$$\hat{\zeta} \approx \sqrt{2}\frac{\sigma\sqrt{1-\phi}}{\mu} \tag{11}$$

where terms are as above. In fact, we find a correlation coefficient of $r = 0.99$ between $\hat{\zeta}$ and ζ when using the 500 random chronologies from Fig. 4, only some of which follow an AR(1) process and some of which are potentially non-stationary. So far, we have shown that mean sensitivity as a statistic is essentially a function of variance unless autocorrelation is very strong (note the inverse relationship between ϕ and $\sqrt{1-\phi}$). Our simulations (e.g., Fig. 2) and estimate of ζ (Eq. (11)) all arrive at that same conclusion. One further refinement of $\hat{\zeta}$ is that we found, via simulation, that the fit between $\hat{\zeta}$ and ζ improved slightly by using the constant 1.14 in place of $\sqrt{2}$:

$$\hat{\zeta} \approx 1.14\frac{\sigma\sqrt{1-\phi}}{\mu} \tag{12}$$

The reasons why $\sqrt{2}$ was a good starting point for a multiplicative constant in Eq. (11) but inferior to using a final estimation for $\hat{\zeta}$ as in Eq. (12) are given in Appendix B.

4.1. Prospectus

We suggest that the important part about trying to understand sensitivity, writ large, in tree-ring data is to understand its variance and autocorrelation structure and do so explicitly. There are a number of tools and approaches to do so. The simplest conceptual way to model variance and autocorrelation in a stationary time series is to do AR(p) modeling as a standard part of the description and analysis of tree-ring data. AR modeling is a regular part of standardization software such as ARSTAN (Cook and Peters, 1981) and dplR (Bunn, 2008). AR models are perhaps the simplest models to understand the behavior of a tree-ring series but might not fully capture the dynamics of growth especially if the external controls on growth are not normally-distributed white noise. For instance, if the error term (ϵ_t) is autocorrelated the way growth (y_t) is in the AR(p) model we refer to this as an autoregressive-moving-average (ARMA) model that takes the form:

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t \tag{13}$$

where θ is a coefficient ranging from -1 to 1, q indicates the order of the moving average, and other terms are as in Eq. (3). These ARMA(p,q) models are outstanding for characterizing regular past events that might affect current year's ring widths. ARMA modeling is used by dendrochronologists but probably not as much as it could be given that the technique is powerful for unmixing different growth mechanisms and as such helping to answer important biological questions about tree growth. For instance, Millar et al. (2012) found two-year lags in negative correlations of water-year precipitation and climatic water deficit with ring widths in whitebark pine. Consequently, the ring width data best fit a ARMA(1,2) process.

In stationary processes (e.g., when the mean and variance do not change over time) a combination of AR(p) and ARMA(p,q) are good tools for understanding the sensitivity of tree-ring data. And heretofore, we have assumed variance (σ^2) to be constant over time. If not, variances are considered to be heteroscedastic. A further refinement in modeling tree-growth for assessing sensitivity is to account for potential heteroscedasticity using the ARCH (autoregressive conditional heteroscedasticity) test, often done under the Lagrange multiplier criterion (see Chatfield, 2003). If the variance is not stable over time, then serial changes in variance should be modeled according to the order of the ARCH test. In the basic ARCH model, variances are short-memory, dependent on recent squared deviations. But if the ARCH tests are significantly high-order, a generalized ARCH (GARCH) model (p>0) is indicated, where variances are long-memory or dependent on past variances (Chatfield, 2003). These are potentially powerful models for understanding growth and can provide good inference to the physical system but are not widely used in dendrochronology. One of the few examples is Millar et al. (2007) who modeled inter-annual variability in limber-pine ring widths using a GARCH(1,1) model. They found tree growth (and mortality) was an interaction with temperature and precipitation that affected the mean but also the variance of the growth signal. E.g., the tree-ring data indicated an increase in variance following a low ring-width event, followed by a slow decrease in variance, suggesting persistence in effect (see Figs. 4 and 5 in Millar et al., 2007).

A principal goal of dendrochronology is to better understand tree growth. For many in the field this means understanding how a tree responds to internal (e.g., stored carbohydrates) and external (e.g., climate) forcings. It is tempting and logical to try to simplify the sensitivity of growth with a simple metric - hence the common use of ζ. This inclination should be avoided for at least two reasons. First, we have shown here that ζ is a confusing and ambiguous statistic for describing the variations in tree growth. Second, values for ζ are often interpreted as reflecting the influence of growth-limiting phenomena, which ignores the dependency of ζ upon the variance of the biological variable that is studied. In addition, the exact nature of these growth-limiting factors (whose impacts may vary during a tree's life) cannot be deduced from the value of ζ. If one is seeking the simplest possible way to describe sensitivity in growth we recommend that ϕ and σ be used instead as better capturing "sensitivity" as it is usually conceptualized. However, we do not ultimately recommend any simple statistic for capturing the variations in growth that are seen in so many records but rather use the increasingly available suite of tools (ARMA, GARCH, etc.) to come to a more nuanced understanding of tree-ring data.

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Appendix A.

The simulations used followed an AR(1) process as shown in Eq. (2) of the body of the paper. Generating normally-distributed noise for the error term and keeping the correlation of y_t and y_{t-1} equal to ϕ required a slightly different approach than simply setting ϵ_t in Eq. (2) to $N(0, \sigma)$. Thus for the simulations, we produced time series y as $y_t = \phi(y_{t-1} - \mu) + \epsilon_t\sqrt{1 - \phi 2} + \mu$ where ϵ is normally distributed: $N(0, \sigma^2)$ and $y_1 = \epsilon_1$. The other terms are as defined in the paper (e.g., $\mu = 1$).

Appendix B.

In calculating $\hat{\zeta}$, Eq. (11) in the body of the paper uses $\sqrt{2}$ as a starting point as a multiplicative constant based on Eq. (5) which is merely a rearrangement of Eq. (1). However the use of $\sqrt{2}$ is not strictly theoretical. We do have $E\{(y_t - y_{t-1})^2\} = 2\sigma^2$ but $E\{\sqrt{(y_t - y_{t-1})^2}\} \neq \sqrt{2}\sigma$. Also, we have for the denominator $E\{(y_t + y_{t-1})/2\} = \mu$ but $E\{\frac{1}{(y_t+y_{t-1})/2}\} \neq \frac{1}{\mu}$. Put more simply, the expectation of the square root is not the same as the square root of the expectation and the expectation of the reciprocal is not the same as the reciprocal of the expectation. However, using the expectations in the "wrong" manner can give us a very good approximation to a better expression of $\hat{\zeta}$. Thus we used simulations (see Appendix A) to determine the best approximation for ζ using σ and ϕ. We found that a value of 1.14 slightly outperforms $\sqrt{2}$ as an accurate multiplicative constant that better accounts for the lack of the exact functional form for the expectation of ζ.

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