The Holocene 7,3 (1997) pp. 361–370

Calculating unbiased tree-ring indices for the study of climatic and environmental change

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Received 15 October 1996; revised manuscript accepted 29 January 1997

A HOLOCENE FORUM ARTICLE

Abstract: In dendroclimatology, tree-ring indices are traditionally calculated as part of the tree-ring chronology development process. This is accomplished by fitting a growth curve to the ring-width series and using it as a series of expectations for more or less well specified null conditions (uniform climate perhaps) of annual radial growth. The ratio of the actual ring widths to these expectations produces a set of dimensionless indices that can be averaged arithmetically with cross-dated indices from other trees into a mean chronology suitable for studies of climatic and environmental change. We show that tree-ring indices calculated in this manner can be systematically biased. The shape of this bias is defined by the reciprocal of the growth curve used to calculate the indices, and its magnitude depends on the proximity of the growth curve to the time axis and its intercept. The underlying cause, however, is lack of fit. To avoid this bias, residuals from the growth curve, rather than ratios, can be computed. If this is done, in conjunction with appropriate transformations to stabilize the variance, the resulting tree-ring chronology will not be biased in the way that ratios can be. This bias problem is demonstrated in an annual tree-ring chronology of bristlecone pine from Campito Mountain, which has been used previously in global change studies. We show that persistent growth increase since AD 1900 in that series is over-estimated by 23.6% on average when ratios are used instead of residuals, depending on how the ring widths are transformed. Such bias in ratios is not always serious, as it depends on the joint behaviour of the growth curve and data, particularly near the ends of the data interval. Consequently, ratios can still be used safely in many situations. However, to avoid the possibility of ratio bias problems, we recommend that variance-stabilized residuals be used.

Key words: Dendrochronology, dendroclimatology, tree-ring indices, tree growth, bias, variance-stabilized residuals, global change research.

Introduction

The use of tree-ring chronologies in studies of climatic and environmental change assumes that the chronologies being analysed are accurate reflections of the response of trees to current and past growing conditions. A number of issues related to this application of tree-ring chronologies have been discussed and debated. These include the assumed uniformitarian response of trees to climate over long time periods (Fritts 1976; Briffa *et al.*, 1996), and the resolvability and preservation of low-frequency climatic signals in the tree-ring series (Cook *et al.*, 1995; Briffa *et al.*, 1996). Here we describe a more basic problem that relates to the actual calculation of tree-ring indices as ratios of actual to expected growth, a method that has been traditionally used in the development of tree-ring chronologies since the early days of dendrochronology (Douglass, 1928; 1936). We will show that tree-ring indices calculated by the ratio method are potentially biased in a way that could lead to inflated estimates of recent growth increases in certain tree-ring chronologies. This bias could have serious consequences when issues regarding the significance of twentieth-century climatic and environmental change are being addressed using tree-ring analysis.

We use the term *bias* here in a way that is conceptually consistent with the definition of bias in statistical theory: an unbiased estimator is one with a mathematical expectation that equals the true parameter value (Judge *et al.*, 1988). Mathematically, this definition of unbiasedness for a single parameter can be expressed as $E[\hat{\theta}] = \theta$. So, a biased estimator is one in which $E[\hat{\theta}] \neq \theta$ and the bias is defined as $E[\hat{\theta}] - \theta = \delta$. In our study, the bias δ that we estimate is not based on a mathematical expectation *per se* because we do not have an *a priori* 'true parameter' to work with. The latter is never known precisely in dendrochronology. Therefore, the bias in tree-ring indices will be described in a purely statistical way: the 'true parameter' needed to demonstrate bias in tree-ring indices will be obtained from differences (or residuals) from the growth curve and the 'mathematical expectation' from

0959-6836(97)HL200FA

traditional tree-ring indices (or ratios), with the kind of fitted growth curve held constant in each case. Since the growth curve itself is an *a posteriori* estimate of expected growth (even if its mathematical form is defined *a priori*), our estimates of bias will clearly be conditioned by the properties of the fitted growth curves and the original tree-ring series themselves. Therefore, the bias we will describe is *conditional* because of its joint dependence on the data and the growth curves. Finally, the 'unbiased' procedure described here refers only to the elimination of the bias associated with tree-ring indices computed as ratios. Other biases may still be present in a chronology based on indices computed from residuals, such as those related to other properties of the growth curves used for detrending.

Obviously, any systematic bias in the calculation of tree-ring indices is undesirable. Interestingly, what we will suggest as a remedy (i.e. the use of residuals instead of ratios) is similar to what was once practised by some European dendrochronologists who first transformed their ring-width series logarithmically before detrending (e.g. Huber by 1943 in Høeg, 1956; Ruden, 1945; Høeg, 1956). The log transform served to stabilize the variance, thus allowing residuals from the growth curve to be used. So, in reality, we are recommending a return to this earlier method, with a needed refinement on how to transform the ring widths before detrending. In fairness to Douglass, he too considered working with logarithms of ring widths, but chose to use ratios because it was computationally easier to do so, and the frequent micro and locally absent rings encountered by him were easily handled by the ratio method (Douglass, 1928; 40–41).

In the following sections, we will first introduce tree-ring standardization and the calculation of tree-ring indices by the ratio method. Then, we will describe mathematically and by example the origin of the bias in tree-ring indices. This will be followed by a discussion of remedial measures to eliminate the potential for bias in tree-ring indices, including a method based on tree-ring residuals after power transformation of the ring widths.

Tree-ring standardization and the calculation of indices

Consider an idealized series of radial increment measurements R_t that is n years in length, collected from a tree growing in a disturbance-free, open-canopy forest. While somewhat unusual worldwide, such forests can be found in semi-arid regions, typically at the upper and lower elevational limits of tree growth. Now based on the allometry of tree growth and its effect on radial increment (and ignoring the early juvenile growth increase often found in such trees), it is usually the case that this 'raw' ring-width series will exhibit a decreasing trend with increasing age out to some positive, asymptotic limit k for over-mature trees. A useful model for this age-related trend in ring-width series is the modified negative exponential curve (Fritts *et al.*, 1969) of the form

$$G_t = ae^{-bt} + k \tag{1}$$

where a is the growth intercept at $t = 0$, b is the decay constant, k is the aforementioned asymptote, and t is time in years. Since this observed trend in ring widths is believed to be mostly non-climatic in origin (as it is related only to tree age and size), the usual practice is to remove it from the tree-rings by fitting some sort of smooth growth curve to the ring widths, like the modified negative exponential curve. The fitted annual growth curve can be thought of as a series of expected values of growth, $E[R_t]$, which would have occurred even in the absence of exogenous influences on ring width such as climate variability. That is,

$$E[R_t] = G_t \tag{2}$$

Taking the ratio of the actual-to-expected ring width for each year yields a set of dimensionless tree-ring 'indices' with a defined mean of 1.0 and a largely homogeneous variance, *viz.*

$$I_t = R_t/G_t \tag{3}$$

where R_t and G_t are the actual and expected ring widths and I_t is the resultant tree-ring index, all for years $t = 1,n$. An alternative way of expressing this relationship, and one that is much more informative, is due to Monserud (1986) who showed that if the relationship between R_t and G_t is expressed in the form of a simple regression equation with residuals E_t, *viz.*

$$R_t = G_t + E_t \tag{4}$$

then

$$I_t = 1.0 + E_t/G_t \tag{5}$$

In this form, tree-ring indices are seen to be one plus a series of residuals from G_t scaled by $1/G_t$ to make them homoscedastic, and the unit mean is explicitly derived. A somewhat different derivation that produces similar results can also be found in Matalas (1962). From equation (5), it is apparent that the usefulness of the $1/G_t$ transformation depends on details of the goodness-of-fit of G_t. This follows by noting that if the mean of R_t is used as an estimate of G_t, then the E_t will be scaled by a constant, which results in no variance stabilization at all (Cook *et al.*, 1995).

This process of detrending and transforming ring widths into dimensionless indices is known as 'standardization' because it tends to equalize the growth variations between cross-dated tree-ring series over time regardless of tree age or size (Fritts, 1976). In turn, this allows the cross-dated series from m individual trees to be properly averaged into a mean-value function, which more reliably reflects the high-frequency variations in growth presumably not related to the biological growth trends that were removed.

Tree-ring standardization, as described here, has been a fairly standard practice in dendrochronology since it was introduced by Douglass (1928). Since then, many changes and extensions have been made to the curve-fitting procedures used, including the use of orthogonal polynomials (Fritts, 1976), the cubic smoothing spline (Cook and Peters, 1981), and digital filters (Briffa *et al.*, 1987). Changes have even been made in how the mean value function is computed using robust estimation (Cook, 1985; 1987). Yet, as the science has evolved and matured, indices computed as ratios have remained a standard practice. Next, we will show by example why this too should probably change.

An example of biased tree-ring indices

Figure 1A shows the plot of annual ring widths from a white oak (*Quercus alba*) sampled in Kickapoo State Park in Iowa. This series covers the period 1670–1980. It shows clear evidence of a pulse-like disturbance occurring around 1730 and a long-term growth release beginning before 1870. Superimposed on this series is a fitted linear regression curve that models the overall trend in growth over time. This trend, which is positive due to the growth release, will be used to illustrate how tree-ring indices calculated by the traditional ratio method can be severely biased, in this case at the left end of the data interval. It is certainly arguable that a linear trend line is inappropriate for standardizing this series for dendroclimatic studies. However, the purpose of this example is to illustrate the potential for standardization bias and how it occurs, not to determine the 'best' growth curve model for standardizing this ring-width series.

Figure 1B shows the tree-ring indices estimated by the ratio

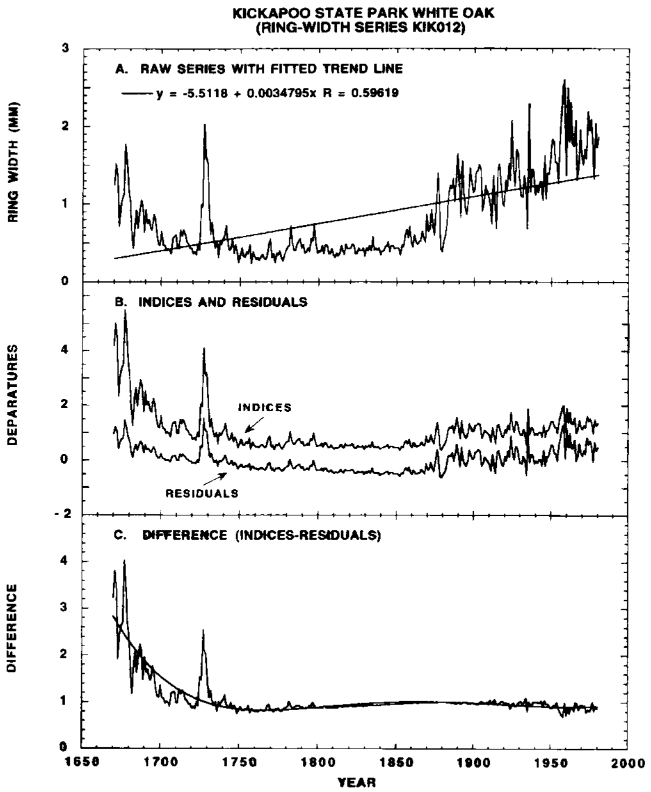

Figure 1 A white oak ring-width series from Kickapoo State Park, Iowa (A) with a superimposed linear growth trend used to illustrate bias in the use of ratios for tree-ring indices. The indices and residuals (B) were calculated from the linear trend fit to the ring widths. The difference between the indices and residuals (C) shows the bias caused by the ratio method of tree-ring index calculation.

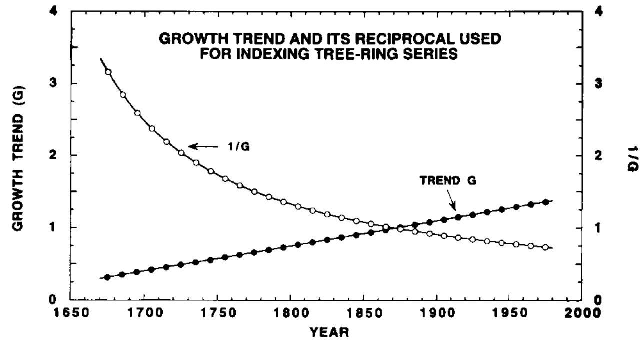

Figure 2 The linear growth trend from Figure 1A and its reciprocal used for computing tree-ring indices.

method using the fitted linear trend line. For comparison, the tree-ring residuals, or differences, from the trend line are plotted below the indices. As described above, the indices have an expected value of 1.0, while the residuals have an expected value of 0.0. Note that, ignoring the pulse, the two series parallel each other from 1980 back to about 1700. However, from 1700 to 1670 there is a systematic increase in the difference between the indices and residuals, with the indices becoming increasingly larger. These differences are clearly shown in Figure 1C. Back to about 1700, except for the pulse, the differences between the indices and residuals are nearly constant with an expected mean of 1.0. Prior to 1700, the indices are greatly inflated relative to the residuals. This trend is emphasized by the 5th order polynomial curve fitted to the systematic differences. Note that the inflated tree-ring indices have occurred even though the standardization curve in Figure 1A falls within the overall range of the data. Next, we will describe how this bias occurs by examining some of the properties of the linear trend line shown in Figure 1A.

Figure 2 shows the linear trend, G_t, and its reciprocal, $1/G_t$, that was used for calculating the tree-ring indices. Compared with G_t, $1/G_t$ is a distinctly non-linear, decreasing function of time, with most of the curvative occurring when G_t drops below 0.5 mm. The trend line intersects the time axis in 1584 when $G_t = 0$ and $1/G_t = \infty$. Division of the ring widths by G_t is equivalent to multiplication by $1/G_t$, which diverges at an increasing rate as G_t approaches the time axis intercept. In this example, the raw ring widths, which themselves increase from about 0.5 mm in 1700 to about 1.5 mm in 1670, are multiplied by numbers ($1/G_t$) that increase from 2.48 in 1700 to 3.34 in 1670. The slopes over this interval are about −0.033 for the residuals and −0.126 for the indices, the latter being almost 300% of the former! Most of this, about 200%, results from the magnitude of $1/G_t$, which is around 3, and the rest from the increase in $1/G_t$. If the raw ring widths had followed the growth line more closely and with decreasing variance in this interval, however, the indices here would have had both a stable mean near one and a stable variance as intended. If, on the other hand, this lack of fit had occurred where G_t was much larger there would have been no problem either. This example suggests that both a lack of fit of G_t to the ring widths, evident in the residuals, and small values of G_t are needed to cause divergent indices. While a lack of fit can occur anywhere in the data interval, the minimum of G_t must occur at $t = 1$ or $t = n$ for linear trend lines, so seriously divergent indices are generally an end-effect problem.

Equation (5) gives explicitly the relationship between the residuals and the indices. It shows that the residuals (shifted to zero mean) are scaled the same as the ring widths by indexing. In the example, E_t was positive and increasing between 1700 and 1670, showing a progressive lack of fit, while G_t was small, positive and decreasing, so the ratio E_t/G_t was larger by a factor of $1/G_t$, positive and increasing at a rate (slope) that was also larger by a factor of $1/G_t$. It is easy to see how the magnitude of G_t is related to the proximity of the time axis intercept, T, of G_t in this example.

$$G_t = a + b_t, \qquad (6)$$

and by definition T satisfies

$$G_T = a + bT = 0 \qquad (7)$$

so G_t can also be expressed as

$$G_t = b(t - T) \qquad (8)$$

Putting (8) into (5) gives

$$I_t = 1.0 + E_t/b(t - T) \qquad (9)$$

Equation (9) shows explicitly how the indices are related to the residuals, and thus how a systematic lack of fit evident in the residuals affects the indices for any value of t given b and T.

These results indicate that the *potential* for bias in tree-ring indices calculated by the ratio method is related to the divergence of the $1/G_t$ transformation, and this effect increases without bound as the trend line approaches zero. Thus, this *potential* appears to be greatest when standardizing ring-width series of slow-growing, over-mature trees, such as those commonly used in dendroclimatic studies. The trend line for these trees may have a negative slope that intersects the time axis some time after the last year of growth, rather than before the first year of growth as in the oak ring-width example, but the same effect will potentially occur. Note that we emphasize *potential* here because the standardization bias illustrated in our example is also a function of the divergence of the ring widths prior to 1750 from the linear trend line, i.e. the lack-of-fit of the growth curve. Had this divergence been less, the bias would have been less evident. Thus, the magnitude of the potential bias is clearly related to the *local* goodness-of-fit of the standardization curve as well.

To summarize, the potential for bias in tree-ring indices calculated as the ratios will increase as 1) the standardization curve decreases towards zero within the span of the data, especially below ~0.5 mm (1/0.5 = 2.0), and has a time-axis intercept near either end of the series; and 2) the standardization curve diverges negatively from the local ring widths near either end of the series, i.e. there is significant lack-of-fit between the curve and the data.

Neither of these problems is as evident when tree-ring residuals are calculated. This is due to the fact that only the trend line G_t is used, not $1/G_t$. In the example here, G_t is a linear trend line with a slope of 0.0034 mm/yr. Thus residuals are inherently bounded (even if the fitted growth curve is not linear), while ratios are inherently divergent as the growth curve approaches zero.

As remarked earlier, one of the cited benefits of calculating tree-ring indices as ratios is to correct for heteroscedastic variance in the raw ring-width series, such as that seen in Figure 1A. However, as noted by Cook (1985; 1987) and Cook *et al.* (1995), the degree to which the variance of the indices is made homoscedastic after transformation by $1/G_t$ is strongly dependent on the local goodness-of-fit of the fitted growth curve. This led Cook (1985) to describe the problem of simultaneously detrending and stabilizing the variance of tree-ring series in terms of an 'uncertainty principle', because the optimal solutions for each are not guaranteed to coincide. That is, in order to remove the dependence of the variance on the mean, a far more flexible curve may be needed than would be desirable for the preservation of all resolvable low-frequency changes due to climate. Thus, 'the preservation of long-period fluctuations of the mean may be antagonistic to the stabilization of the variance in tree rings' (Cook, 1985). This problem is clearly evident in Figure 1B. The large difference in variance seen in the raw ring-width series is essentially unchanged in the indices and, as it must be, in the residuals. This is due to the systematic local lack of fit of the trend line in Figure 1A. Unlike the potential bias in the indices, which is commonly (but not exclusively) restricted to the ends of the series, the lack of variance stabilization can occur anywhere in the series where there is a systematic local lack of fit of the fitted growth curve. This problem could significantly affect the interpretation of long tree-ring chronologies for changes in climatic variability.

Some remedial measures to reduce or eliminate bias

Given that tree-ring indices calculated as ratios may be seriously biased in some cases, how can we eliminate this potential bias from the general practice of tree-ring standardization? One possibility would be to add a positive constant to the ring widths prior to detrending in order to keep the fitted trend line out of the 'danger zone' (i.e. values $< \sim 0.5$ mm). This moves the time-axis intercept further away from the data interval. To illustrate how this might work, the series in Figure 1A was re-standardized after adding a constant of 1.0 mm to each ring width. The results are shown in Figure 3 for comparison to those in Figure 1. Instead of G_t in 1670 being ~ 0.3 mm, it is now ~ 1.3 mm, and the time-axis intercept has moved from 1584 to 1296. It is clear that the bias seen in the indices prior to 1700 in Figure 1B is no longer present in the new indices in Figure 3B.

This result confirms the way in which the proximity of the time-axis intercept influences the resulting indices in a highly non-linear way. It is also seen now that indexing has the effect of moderating the local ring-width divergence prior to 1700 because

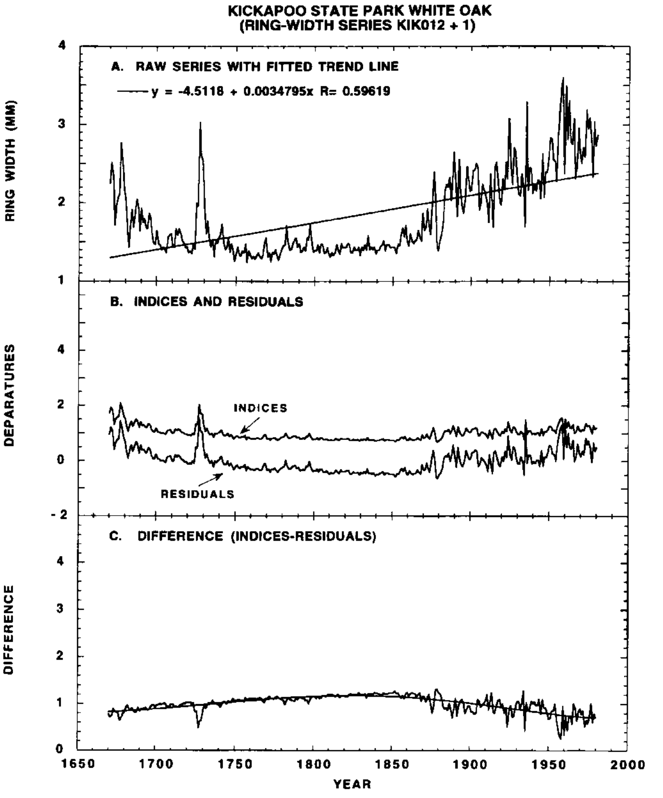

Figure 3 An experiment in adding a constant of 1.0 mm to the ring-width series in Figure 1A to reduce the bias caused by ratios. Otherwise, this figure follows the caption for Figure 1.

$1/G_t < 1$ instead of > 1. The proximity of the growth curve to zero is the critical factor. How near is too near depends on how well the trend line fits the raw data there, but unless the fit becomes 'exact' the indices will diverge. Roughly speaking, as $1/G_t$ decreases in magnitude going away from the time-axis intercept, it becomes flatter and distorts the data less.

Does adding a positive constant to the ring widths solve the bias problem? The answer appears to be a qualified 'yes'. By this we mean that the result shown in Figure 3 has yielded a positive result in this regard. However, there are some potential difficulties in proceeding this way. A comparison of the indices in Figures 1B and 3B indicate that the addition of the constant has resulted in a significant reduction in the standard deviation of the indices calculated over the relatively unbiased 1750–1980 period (from 0.372 in Figure 1B to 0.185 in Figure 3B). This need not be a problem since indices are relative numbers, e.g. the correlation between the two series exceeds 0.99 over the 1750–1980 period. However, it is clear that the choice of a constant to add to the ring widths will affect some of its statistics, which could make comparisons between chronologies difficult if the constant used is not *constant* across all data sets. It is also possible that the constant chosen may have its own biasing influence on the resulting indices. For example, the indices calculated from the 1730 pulse disturbance event appear to be positively biased in Figure 1C and negatively biased in Figure 3C. Thus, it is not clear that an arbitrary constant will produce unbiased tree-ring indices. Finally, even if an 'unbiased' constant were found, it still does not solve the lack of variance stabilization caused by the systematic lack of fit of the trend line (cf. Figures 1B and 3B).

As described earlier, tree-ring residuals from the fitted growth curve are not biased in the sense that ratios can be. Nor does the addition of a positive constant affect them in any way since the constant is directly removed as part of the growth-curve estimate. Therefore, it would appear that residuals are more robust than ratios as expressions of relative growth departures. This is a strong argument in favour of using residuals in tree-ring chronology development. The argument against using residuals has generally been that they contain the heteroscedastic variance found in the original ring widths. While this argument is certainly true in and of itself, it does not acknowledge the fact that the variance stabilizing property of $1/G_t$ is easily compromised by the way in which the growth curve fits the data. If a means could be found objectively to stabilize the variance in the residuals, the argument for using ratios would largely evaporate.

To this end, we have investigated the use of a data-adaptive power transformation method to stabilize the variance of ring-width series based on the relationship between level and spread (Emerson and Strenio, 1983). It is based on the premise that the local spread (in this case the standard deviation, S) is proportional to a power of the local level (in this case the arithmetic mean, M) of the ring widths. This model is expressed as

$$S = cM^b \tag{10}$$

or equivalently

$$\log S = \log c + b \log M \tag{11}$$

or, letting $k = \log c$

$$\log S = k + b \log M \tag{12}$$

The final equation has the form of a simple linear regression in logarithmic space. In dendrochronology, equation (11) was first explicitly derived by Matalas (1962) to describe the relationship between trend and variations about the trend in tree-ring widths. Now it turns out that if b is the slope of the spread-versus-level relationship, then $p = 1 - b$ is the appropriate power transformation (Emerson and Strenio, 1983). Note that for $b = 1$, the transform is logarithmic, so that the power transform can be considered as a generalization of that procedure, which makes it more useful.

Because tree-rings are time series, the estimation of the local mean and standard deviation requires a windowing procedure. Matalas (1962) used 10-year and 20-year windows in his study, depending on the length of the tree-ring series under investigation. After much experimentation, and in order to make this technique useful for relatively short series, we decided to use the denominator and numerator of the coefficient of mean sensitivity. Thus, the t_{th} local mean, M_t, and standard deviation, S_t, were computed as

$$M_t = (R_t + R_{t-1})/2 \tag{13}$$

and

$$S_t = | R_t - R_{t-1} | \qquad \text{for } t = 2 \ldots n \tag{14}$$

Thus, for each ring-width series of length n, there are $n - 1$ estimates of the local mean and standard deviation for estimating the regression slope and p.

Having estimated p by the above procedure, the raw ring widths are transformed as

$$R_t^* = R_t^p = R_t^{(1-b)} \tag{15}$$

where R_t is the original ring widths and R_t^* is the transformed ring widths. Then, the 'residual' indices, I_t^*, are computed as

$$I_t^* = R_t^* - G_t \tag{16}$$

To show how the power transformation works, we have applied the technique to the white oak ring widths in Figure 1A. Figure 4 shows the scatter plot of level versus spread estimated as described above. The linear regression fitted to this relationship has a slope of $b = 1.049$, which results in a power transformation $p = -0.049$. As this power is near zero (b is near 1), its effect on the variance approximates that of a logarithmic transformation (Emerson and Strenio, 1983). In this case, we have simply used the absolute value of p as our transformation. However, experience indicates that the optimal power transformation for the large majority of ring-width series falls in the 0.25–0.50 range. Interestingly, Matalas (1962) found that the slope b in equation (11) was approximately 1/2 in his test series. Since this is equivalent to $p = 0.50$, our experience is highly consistent with what Matalas (1962) found in his prescient paper.

Figure 5A shows the ring-width series after the power

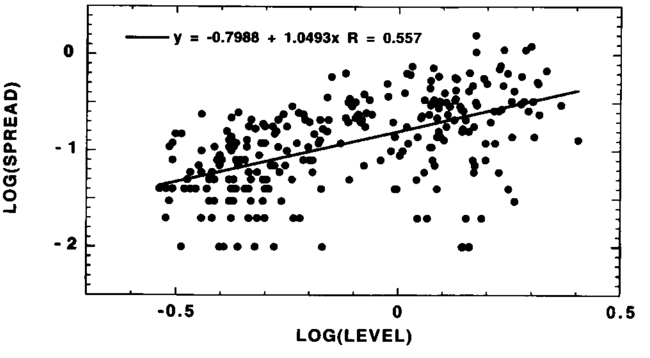

Figure 4 The spread-versus-level relationship in the white oak series. The estimators of level and spread are described in the text. The optimal power transformation is defined as $1 - b$, where b is the regression slope. See the text for more details.

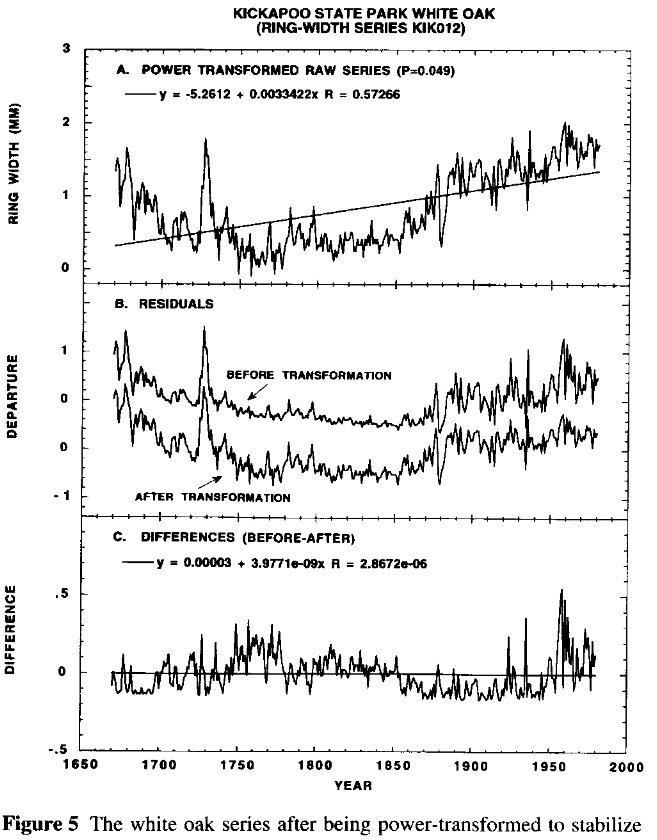

Figure 5 The white oak series after being power-transformed to stabilize the variance (A). The other plots are similar to those in Figures 1 and 3. Note how the variance in the residuals after transformation are much more homoscedastic than before transformation (B). The secular changes in the differences (C) are related to slight differences in the linear trend lines used.

transformation has been applied to it. The transformed series has also been rescaled to have the mean and standard deviation of the original series for comparison. A comparison of Figures 1A and 5A illustrates the much more homogenous variance of the transformed series. Figure 5B shows the residuals from the ring-width series, before and after transformation. Again, it is clear that the residuals from the transformed ring widths are more homoscedastic. In particular, the variance in the 1750–1869 period is now more similar to the variance of the 1870–1980 period, a result that was not achievable using the same growth-trend model and the ratio method. To do so with ratios would have required that the growth curve fit the data much more closely. In so doing, more low-frequency variance, perhaps related to climate, would have been lost.

Table 1 compares the variances calculated for the 1750–1869 (early) and 1870–1980 (late) periods of KIK012 using the different procedures described here. For the raw ring widths, the ratio of the late-to-early period variances is 19.05. After removal of the trend, the ratio of variances in the traditional tree-ring indices (Figure 1B) declines to 6.91. This is a large reduction in the ratio, but it is still high enough to indicate that variance stabilization has not been achieved very well. Surprisingly, the result for the indices after the addition of the 1.0 constant (Figure 3B) is even worse. In this case, the ratio of late-to-early variances actually increases from 6.91 to 9.03. This result argues once again against adding an arbitrary constant to ring-width series to avoid biasing the resulting indices. The variance ratio based on residuals from the trend line (Figure 1B) is 11.04, a poor result as expected. It is also worse than that based on indices from the same trend line, suggesting that at least some degree of variance stabilization has been achieved by the ratio method. Lastly, the variance ratio of the residuals after power transformation of the ring widths (Figure 5B) is the smallest at 2.47. While not perfect, the power transformation has produced tree-ring residuals that are much more homoscedastic than that achieved by the tree-ring indices.

Finally, Figure 5C shows the differences between the untransformed and transformed residuals. The differences show some secular variations that reflect slightly different linear trend-line estimates. In a least squares sense, the trend line fitted to the transformed series ought to be preferred since it adheres more closely to the assumption that the error variance is homoscedastic.

A practical example related to global change research: Campito Mountain bristlecone pine

The white oak ring-width series used above is clearly disturbed by non-climatic events. So, its relevance to the more general problem of using tree-ring chronologies in the study of climatic and environmental changes may seem elusive. To bring this problem into focus, we will illustrate how the potential bias in tree-ring indices computed by the ratio method can lead to inflated estimates of climatic/environmental change during the twentieth century. This issue has obvious relevance to the determination of how anomalous twentieth-century climate has been relative to the past few centuries.

The data set we use below is a suite of 23 annual ring-width series, collected by D.A. Graybill, from Great Basin bristlecone pines (*Pinus longaeva*) growing at an upper timberline site on Campito Mountain in the White Mountains of California. This site is relevant here because tree-ring chronologies developed from it have been used to study climatic change (LaMarche, 1974) and possible CO_2 fertilization effects on tree growth (Graybill, 1987). Figure 6A shows the mean ring-width chronology and its sample depth over the time interval AD 977–1983. The mean ring width of these series is 0.412 mm, with a range of 0.249–0.625 mm. So, on average, these series are potentially in the suggested 'danger zone' where the indices could be significantly biased by growth trends too close to zero.

In this example, the method of detrending used was as conservative as practical. Only negative exponential curves or linear regression with negative or zero slopes were used. The rationale for not allowing positive-slope linear regressions to detrend the ring widths was the belief that these trees have been responding to changing temperatures (or elevated CO_2 perhaps), which have been increasing over much of the twentieth century. If positive trends in growth were removed in the standardization process, trends in temperature and/or CO_2 fertilization might also be removed. This would be undesirable in either of these studies.

Figure 6B shows the mean standardized tree-ring chronology based on the detrending option described above and the calculation of indices as ratios. It shows a strong growth increase beginning around 1870 and sustained above-average growth since 1900 that appears to be unprecedented since the beginning of the record (where the sample size is only two, however). This increase is consistent with general summer warming over the Northern Hemisphere since the mid-1800s (Bradley and Jones, 1993). Figure 6C shows the mean chronology based on residuals after the variance of the individual series was stabilized with power transformations. In this case, the mean power transformation estimated from the spread-versus-level relationship was 0.614. This indicates that there was not much dependence between spread and level in the raw data. Indeed, the average spread-versus-level correlation in the raw ring widths is only 0.093, with a range of –0.169 to 0.430. The mean residual chronology has also been rescaled to have the same mean and standard deviation as the ratios over the AD 977–1899 period. That way, any divergence between the series after 1900 should be clearly evident and largely independent of scale.

A visual comparison of Figures 6B and 6C indicates that the tree-ring chronologies are extremely similar throughout. Yet, a close examination of the post-1899 period indicates that the twentieth-century growth increase is somewhat less in the residuals chronology. This difference is revealed more clearly in Figure 7A, which shows the differences between the ratios and residuals before and after power transformation of the ring widths. Both are shown to illustrate that the bias in the ratios is present whether or not the data are power transformed. Table 2 provides statistics on the average, maximum, and percent bias in the ratios relative to the residuals in the 1900–1983 period. The average absolute bias in the ratios is 0.071 and 0.082 index units compared to the untransformed and transformed residuals, respectively, with maxima of 0.202 and 0.236 index units. In terms of relative bias, the ratios are inflated 23.6 and 28.7% above the untransformed and transformed residuals on average. Clearly, this bias is appreciable. Yet, it is based on long accepted methods of tree-ring standardization.

Table 1 Change in variance in white oak ring-width series KIK012 between two time periods as a function of index calculation method. Raw is for unstandardized (raw) ring widths; Rat 1 is for tree-ring indices shown in Figure 1B calculated as ratios from the trend line; Rat 2 is for tree-ring indices shown in Figure 3B calculated after adding a constant of 1.0 to the raw ring widths; Res 1 is for the residuals shown in Figure 1B calculated as differences from the trend line; Res 2 is for the residuals shown in Figure 5B calculated after power transformation of the ring widths

Interval	Raw	Rat 1	Rat 2	Res 1	Res 2
1750–1869	0.0107	0.0145	0.0033	0.0133	0.0284
1870–1980	0.2039	0.1002	0.0298	0.1469	0.0704
Ratio	19.0561	6.9103	9.0303	11.0451	2.4789

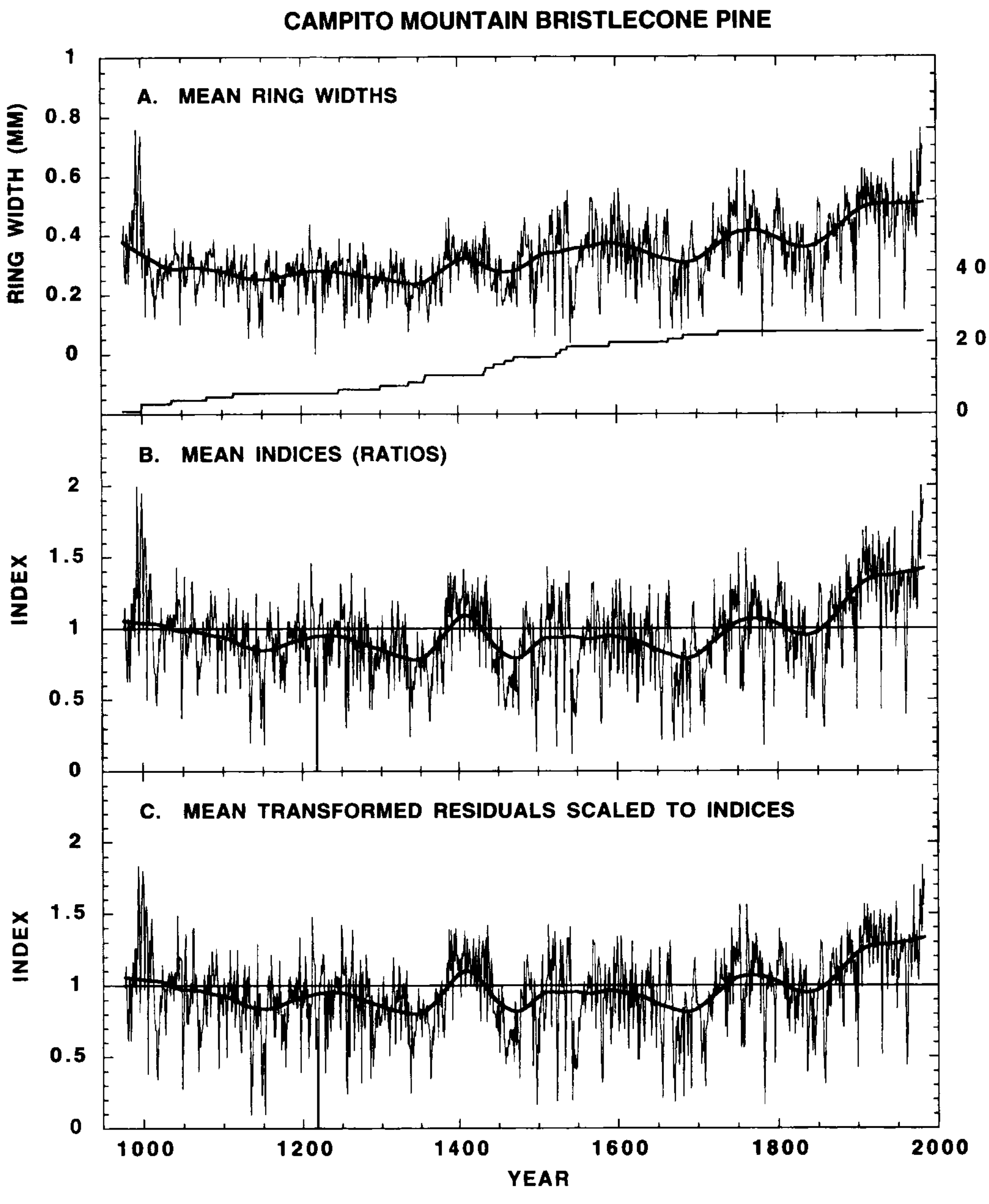

Figure 6 Tree-ring chronologies for Campito Mountain bristlecone pine. The first series (A) is the mean ring widths with sample depth information, followed by tree-ring indices based on the ratio method (B), and indices based on power transformed residuals (C).

The bias estimates in Table 2 are actually conservative because only eight out of the 23 ring-width series had trend lines with negative slopes fitted to them, based on the method of standardization chosen here. The remaining 15 series only had their arithmetic means removed, which cannot result in bias in the ratios as described here. Consequently, the bias estimates based on all series have been diluted by the 15 zero-slope series. Estimates of bias are provided for the eight series with negative slopes only in Figure 7B and in Table 2. The bias in the ratios is now considerably larger than before. In particular, the average absolute bias is now 0.116 and 0.136 index units, with maxima of 0.541 and 0.489 units. The relative bias in the ratios has likewise increased to 66.2 and 87.8% compared to the untransformed and transformed residuals, respectively.

Finally, for completeness, we show the bias results for the 15 zero-slope series. The ratios and residuals before transformation are identical here because nothing has been done to the data aside from rescaling the residuals to the mean and standard deviation of the ratios. Hence, the bias is identically zero everywhere. This result reinforces the fact that horizontal lines do not stabilize the variance in ring-width series, even if they need it. In contrast, some differences between the ratios and transformed residuals are evident, although these differences are generally quite small. Even so, the difference between the estimates in the twentieth century suggests that the power transformations used here might be slightly excessive. We are presently investigating refinements of the spread-versus-level method for estimating the power transformation. These refinements will include testing the significance of the spread-versus-level relationship and a direct test for heteroscedastic variance.

Discussion

The results shown in this study clearly indicate that the use of ratios in the calculation of standardized tree-ring chronologies should be used, if at all, with considerable caution. In the Campito Mountain example, the twentieth-century growth increase in both the ratios and residuals are both well above that which has happened in the past. So in this sense, either chronology in Figure 6 could be used to argue for unprecedented climatic warming during the twentieth century (assuming a pure temperature response by

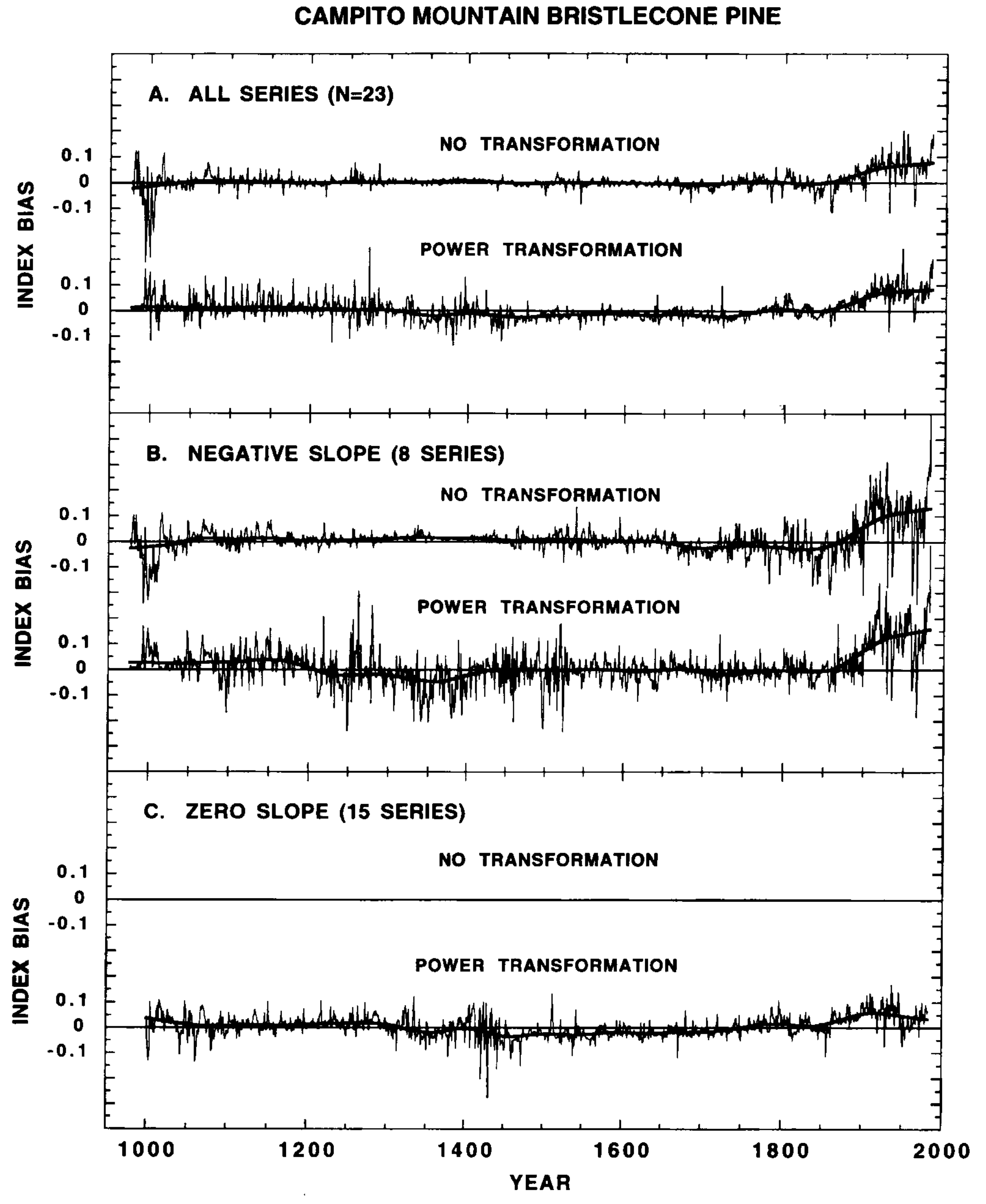

Figure 7 Examples of bias in the Campito Mountain tree-ring ratios during the twentieth century. The bias is estimated as the difference between the ratios and the residuals with or without power transformations. The expected value here is zero if no bias exists. This has been done for all 23 series (A) and for subsets determined by the presence (B) or absence (C) of negative growth trends in the ring-width series.

Table 2 Tests of index bias in Campito Mountain bristlecone pine tree-ring ratios, as a function of the number of series standardized with linear trend lines of negative or zero slope; the untransformed and transformed residual series have been rescaled to the mean and standard deviation of the ratios using the pre-1900 data as the base period

			Post-1899 bias in tree-ring ratios					
			Untransformed			Transformed		
Data Used	N	Interval	Avg1	Max1	Pct1	Avg2	Max2	Pct2
All Series	23	977–1983	0.071	0.202	0.236	0.082	0.241	0.287
Neg Slope	8	977–1983	0.116	0.541	0.662	0.136	0.489	0.878
No Slope	15	1000–1983	0.000	0.000	0.000	0.053	0.170	0.149

Avg1 = average bias in ratios compared to untransformed residuals; Max1 = maximum bias in ratios; Pct1 = percent average bias in ratios; Avg2, Max2, Pct2 are same statistics based on comparisons with the transformed residuals.

the trees), but the degree of unprecedented warming would be over-estimated by the ratios. If the inquiry were to move to CO_2 fertilization and its possible contribution to the overall growth increase (e.g. LaMarche *et al.*, 1984; Graybill and Idso, 1993), we may be in equally serious trouble using ratios. The bias in them increases in a way that could easily be misinterpreted as being due to the fertilization effect of elevated atmospheric CO_2 on tree growth, after climate effects have been taken into account by some means.

The example shown here illustrates the potential for bias in tree-ring indices based on ratios. Using the same testing procedures, biases of comparable magnitude have been found in several other tree-ring chronologies, including the St Anne River northern white cedar (*Thuja occidentalis*) chronology used in Northern-Hemisphere temperature reconstructions (Jacoby and D'Arrigo, 1989; D'Arrigo and Jacoby, 1993). Of course, this will not happen in every tree-ring data set, even including those with strong twentieth-century growth trends like that shown here. For example, the white spruce (*Picea glauca*) ring-width data set from the Twisted Tree-Heart Rot Hill site in the Yukon Territory (Jacoby and Cook, 1981), which has also been used to reconstruct past temperature changes (e.g. Jacoby and D'Arrigo, 1989; D'Arrigo and Jacoby, 1993), does not have any meaningful bias in its tree-ring ratios when tested the same way as here. This was because the growth curves were horizontal lines or negative exponential curves that had attained their asymptotic limiting values by the twentieth century. So, in this case, the right-hand time-axis intercepts for the negative exponential curves were effectively infinite. Thus, the bias in tree-ring ratios is not ubiquitous. It is simply a potentially serious artefact that can be easily avoided through the use of variance-stabilized tree-ring residuals.

The difficulty in replacing ratios with residuals in tree-ring chronology development relates more to abandoning tradition and experience than to the mechanics of the method itself. The only thing that is new in the residuals method is the power-transformation method recommended here. However, in abandoning ratios, something does appear to be lost. Ratios are easy to interpret, e.g. a tree-ring index of 2.0 indicates that growth for a given year was twice the expected value of the growth curve for that same year. The ratio method also handles zeroes (i.e. missing rings) in a very natural way. In contrast, the power-transformation method can be sensitive to zeroes, depending on the value of p. Ratios also provide a scale-free means of comparing tree-ring chronologies from any species or site in the world, while power transformed residuals are not scale-free as immediately calculated. This makes the comparison of tree-ring chronologies more difficult. One possible solution to this latter dilemma would be to rescale the residuals of the individual tree-ring series to have the same mean and standard deviation as the tree-ring indices (i.e. ratios) calculated from the same ring-width series and the same detrending method. This rescaling will usually not produce one-to-one agreement, however, because the growth curves estimated for the untransformed and power-transformed ring widths will differ slightly (cf. Figures 1A and 5A), and any bias in the ratios may distort the rescaling as well. In addition, if the ratios of the individual series remain distinctly heteroscedastic, as is likely to be the case in many situations, then the mean-value functions will also differ somewhat even if the residuals of the individual series are rescaled to look like indices before averaging. These relatively minor difficulties are more style than substance in our opinion.

Because the bias in ratios does not always occur (as it depends critically on the joint behaviour of the growth curve and data near the ends), we do not necessarily recommend that ratios be completely abandoned. On the other hand, there seems to be little to justify their use now, other than tradition. Therefore, we recommend that variance-stabilized residuals be considered seriously as a replacement for ratios in the development of annual tree-ring chronologies. The spread-versus-level method described here for estimating the power transformation is not foolproof. Experience indicates that the transformation estimated this way can be excessive, although rarely as excessive as a pure logarithmic transformation. We are presently investigating this and other ways of estimating to determine how best to transform the ring-width series. Results from that study will appear in a follow-up paper.

Acknowledgements

The writing of this paper was stimulated by discussions with Ricardo Villalba and Greg Wiles over issues of tree-ring standardization, after a very long incubation period of growing awareness about this problem. We also thank Gordon Jacoby, Ricardo Villalba, and Keith Briffa for valuable comments and criticisms on an earlier draft of this paper. Finally, we thank Dave Meko and an anonymous reviewer for excellent official reviews, and to Dave for pointing out the important paper by Matalas. Lamont-Doherty Earth Observatory Contribution No. 5700.

References

Bradley, R.S. and **Jones, P.D.** 1993: 'Little Ice Age' summer temperature variations: their nature and relevance to recent global warming trends. *The Holocene* 3, 367–76.

Briffa, K.R., Jones, P.D., Schweingruber, F.H., Karlén, W. and **Shiyatov, S.G.** 1996: Tree-ring variables as proxy-climate indicators: problems with low-frequency signals. In Jones, P.D., Bradley, R.S. and Jouzel, J., editors, *Climate variations and forcing mechanisms of the last 2000 years*, Berlin: Springer-Verlag, 9–41.

Briffa, K.R., Wigley, T.M.L. and **Jones, P.D.** 1987: Towards an objective approach to standardization. In Kairiukstis, L., Bednarz, Z. and Feliksik, E., editors, *Methods of dendrochronology – 1*, Proceedings of the Task Force Meeting on Methodology of Dendrochronology: East/West Approaches, June 2–6, 1986, Kraków, Poland. Polish Academy of Sciences, Warsaw, 69–86.

Cook, E.R. 1985: *A time series analysis approach to tree-ring standardization*. PhD dissertation, University of Arizona, Tucson.

—— 1987: The decomposition of tree-ring series for environmental studies. *Tree-Ring Bulletin* 47, 37–59.

Cook, E.R. and **Peters, K.** 1981: The smoothing spline: a new approach to standardizing forest interior tree-ring width series for dendroclimatic studies. *Tree-Ring Bulletin* 41, 45–53.

Cook, E.R., Briffa, K.R., Meko, D.M., Graybill, D.A. and **Funkhouser, G.** 1995: The segment length curse in long tree-ring chronology development for palaeoclimatic studies. *The Holocene* 5, 229–37.

D'Arrigo, R.D. and **Jacoby, Jr., G.C.** 1993: Secular trends in high northern latitude temperature reconstructions based on tree rings. *Climatic Change* 25, 163–77.

Douglass, A.E. 1928: *Climatic cycles and tree growth*, Vol. II. Washington: Carnegie Institution of Washington Publication 289.

—— 1936: *Climatic cycles and tree growth*, Vol. III. Washington: Carnegie Institution of Washington Publication 289.

Emerson, J.D. and **Strenio, J.** 1983: Boxplots and batch comparisons. In Hoaglin, D.C., Mosteller, F. and Tukey, J.W., editors, *Understanding robust and exploratory data analysis*, New York: John Wiley & Sons, 58–96.

Fritts, H.C. 1976: *Tree rings and climate*. London: Academic Press.

Fritts, H.C., Mosimann, J.E. and **Bottorff, C.P.** 1969: A revised computer program for standardizing tree-ring series. *Tree-Ring Bulletin* 29, 15–20.

Graybill, D.A. 1987: A network of high elevation conifers in the western US for detection of tree-ring growth response to increasing atmospheric carbon dioxide. In Jacoby, Jr., G.C. and Hornbeck, J.W., compilers, *Proceedings of the International Symposium on Ecological Aspects of Tree-Ring Analysis*, US Department of Energy, Publication CONF-8608144, 463–74.

Graybill, D.A. and **Idso, S.B.** 1993: Detecting the aerial fertilization effect of atmospheric CO_2 enrichment in tree-ring chronologies. *Global Biogeochemical Cycles* 7, 81–95.

Høeg, O.A. 1956: Growth-ring research in Norway. *Tree-Ring Bulletin* 21, 2–15.

Jacoby, Jr., G.C. and **Cook, E.R.** 1981: Past temperature variations inferred from a 400-year tree-ring chronology from Yukon Territory, Canada. *Arctic and Alpine Research* 13, 409–18.

Jacoby, Jr., G.C. and **D'Arrigo, R.** 1989: Reconstructed Northern Hemisphere annual temperature since 1671 based on high-latitude tree-ring data from North America. *Climatic Change* 14, 39–59.

Judge, G.G., Hill, R.C., Griffiths, W.E., Lütkepohl, H. and **Lee, T.-C.** 1988: *Introduction to the theory and practice of econometrics*. New York: John Wiley & Sons.

LaMarche, V.C., Jr. 1974: Paleoclimatic inferences from long tree rings records. *Science* 183, 1043–48.

LaMarche, V.C., Jr., Graybill, D.A., Fritts, H.C. and **Rose, M.R.** 1984: Increasing atmospheric carbon dioxide: tree-ring evidence for growth enhancement in natural vegetation. *Science* 225, 1019–21.

Matalas, N.C. 1962: Statistical properties of tree-ring data. *International Association of Scientific Hydrology* 7, 39–47.

Monserud, R.A. 1986: Time series analysis of tree-ring chronologies. *Forest Science* 32, 349–72.

Ruden, T. 1945: En vurdering av anvendte arbeidsmetoder innen trekronologi og årringanalyse ('A valuation of the methods employed in dendrochronology and annual ring analyses'). *Medd. Norske Skogforsøksvesen* 9(32), 181–267.