Math 724, Fall 2021
Clarifications and Corrections to Bogart's Text

p.28, Supplementary Problem #1: Interpret "list" as ordered list. In other words, $4 = 2 + 1 + 1$ is *not* considered to be the same composition as $4 = 1 + 1 + 2$.

p.37, problem 86(g): change "reasonable" to "reasonably"

p.63, before problem 157: The symbol for partitions is backwards — it should be $\lambda \vdash 3$ (LaTeX: `\vdash`). Also, in what comes before, it is standard to put parentheses around partitions, e.g., $\lambda = (1, 1, 1)$ instead of $\lambda = 1, 1, 1$.

p.79, problem 192: In (a), k should be quantified, e.g., "...in which the coefficient of x^k is 1 <u>for all k</u>." In (b), I think it should read "...the number of multisets <u>of size k</u> chosen from n...". (The omission confused some students.)

p.80, problem 199 (d): in the third line, there should be a question mark after "what are the possible degrees of the other vertices"

p.81, problem 201: "When working with generating functions for partitions, it is becoming standard to use q rather than x as the variable in the generating function." That's debatable. I don't think there is a "standard" variable to use; it depends on the context and the author's whim. The main point is that students should not be too concerned with what letter is used.

p.84, problem 210 (c): One of the $\begin{bmatrix} m+n \\ n \end{bmatrix}_q$'s should be $\begin{bmatrix} m+n \\ m \end{bmatrix}_q$.

p.85, problem 211 (a): The first displayed equation should have summand $a_i x^i$, not x^i. (I would also suggest taking the parentheses bigger, e.g., with `\left(\sum ... \right)`.)

p.85, problem 211 (a): In the second displayed equation, the second sum should have a_{i-1} rather than a_i.

p.86, problem 213: a_n should really be the number of *pairs of rabbits* at the end of month n. (One of my students suggested using animals that reproduce asexually, such as amoebas, to avoid confusion between rabbits and pairs of rabbits.)

p.89, problem 224 (c): "number real number" should be "real number"

p.95, paragraph after Problem 231: "If is also" should be "It is also"

p.97, problems 236–7: The problem includes the unstated assumption that every couple includes one man and one woman. I would prefer a more neutral formulation, such as the following for #237:

> *At a conference, n students and their n advisors meet for dinner. In order to encourage mingling, they need to be seated at a round table so that no student sits next to a student (equivalently, no advisor sits next to an advisor) and no student-advisor pair is seated next to each other. In how many ways can this be done?*

In the instructor's version, in line 4 of the solution, I think $|\cup_{i:\, i \in S} A_i|$ should be $|\cap_{i:\, i \in S} A_i|$ (as in line 9).

p.99: "greek" should be capitalized in the footnote.

p.153, problem 374: $\frac{e^x - e^{-x}}{2}$ is $\sinh x$, not $\cosh x$. (Like their ordinary counterparts, $\sinh x$ is odd and $\cosh x$ is even.) Thus

$$\cosh x = \frac{e^x + e^{-x}}{2} = \sum_{n\geq 0} \frac{x^{2n}}{(2n)!}, \qquad \sinh x = \frac{e^x - e^{-x}}{2} = \sum_{n\geq 0} \frac{x^{2n+1}}{(2n+1)!}$$

which are the EGFs for $1, 0, 1, 0, \ldots$ and $0, 1, 0, 1, \ldots$ respectively. (Perhaps the problem should ask about both sinh and cosh.)

p.153, problem 375: I think the last sentence should read "We will then define $\ln\left(\frac{1}{1-x}\right)$ to be the power series you get." i note that the solution manual assumes $\ln(z) = -\ln(1/z)$, which is not (to me) obvious from the formal definition of the logarithm (but I was never very good at Calculus II).

p.154, problem 379: EFG should be EGF.

p.155, problem 384: First, y should be defined as $\sum_{i=0}^{\infty} a_i x^i / i!$. Second, the recurrence $a_n = na_{n-1} + n(n-1)$ implies in particular that $a_1 = a_0$. So, e.g., the LHS of the equation could be written as $y - a_0 - a_0 x$, and the solution (p.274 in the instructor version) can be written more simply as $y = \dfrac{a_0 + x^2 e^x}{1-x}$. Something is wrong with this problem, because the recurrence evidently produces integers, but the final expression for a_n does not. Possibly the answer should be multiplied by $n!$, but in any case I propose the following easier problem as a replacement:

Consider the recurrence defined by $a_0 = 1$ and $a_n = na_{n-1} + 1$ for $n \geq 1$. Let $A = A(x) = \sum_{n\geq 0} a_n x^n / n!$. Determine a closed form for A and use it to obtain a formula for a_n (which will include a summation).

Solution: Multiplying both sides of the recurrence by $x^n/n!$ and summing for $n \geq 1$ we get

$$\begin{aligned}
\sum_{n\geq 1} a_n \frac{x^n}{n!} &= \sum_{n\geq 1} na_{n-1} \frac{x^n}{n!} + \sum_{n\geq 1} \frac{x^n}{n!} \\
A - 1 &= \sum_{n\geq 1} a_{n-1} \frac{x^n}{(n-1)!} + (e^x - 1) \\
A &= e^x + x \sum_{n\geq 1} a_{n-1} \frac{x^{n-1}}{(n-1)!} = e^x + x \sum_{m\geq 0} a_m \frac{x^m}{m!} = e^x + xA
\end{aligned}$$

and solving for A gives $A = \frac{e^x}{1-x}$. Expanding this in turn, we get

$$A = \left(\sum_{k\geq 0} x^k\right)\left(\sum_{m\geq 0} \frac{x^m}{m!}\right) = \sum_{n\geq 0}\left(\sum_{m=0}^{n} \frac{1}{m!}\right) x^n = \sum_{n\geq 0}\left(\sum_{m=0}^{n} \frac{n!}{m!}\right)\frac{x^n}{n!}.$$

Therefore,

$$a_n = \sum_{m=0}^{n} \frac{n!}{m!}.$$

p.158, problem 394: "of set of books" should be "of a set of books"

p.159, problem 395: "whom" is misspelled "whome"

p.159, problem 396: I think "for a species" should be "form a species." Also, the sentence needs a period.

p.160, between C.4.1 and C.4.2: "not particular" should be "not particularly"

p.162: Three lines before problem 409, "exp" should be in roman type. In problem 409 itself, I think it should be specified that "block" means "nonempty block", so that $f(x) = e^x - 1$ and $g(x) = e^{f(x)}$.