Tsinghua-Berkeley Shenzhen Institute
OPERATIONS RESEARCH
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Homework 4

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- **Acknowledgments:** This assignment refers to the textbook: Introduction to Operations Research(10th).
- **Collaborators:** I finish this assignment by myself.

The answers below are arranged according to the order of the following questions in the textbook: 6.1-3 (a) (b), 6.1-11, 6.3-6, 6.4-3.

4.1. 6.1-3 SOLUTION

(a) Since the primal problem has more functional constrains than variables, applying the simplex method to the dual problem may be more efficient, which has fewer functional constrains, i.e. fewer basic variables, and thus may have fewer iterations to solve.

(b) Since the primal problem has fewer functional constrains than variables, applying the simplex method to the primal problem may be more efficient, which has fewer functional constrains, i.e. fewer basic variables, and thus may have fewer iterations to solve.

4.2. 6.1-11 PROOF: From the weak duality property we know that if x is a feasible solution for the primal problem and y is a feasible solution for the dual problem, then $cx \leq yb$. If both the primal and the dual problem have feasible solutions, let x_0 be a feasible solution to the primal problem and its corresponding dual problem solution is y_0, we have $cx \leq y_0$ and $yb \geq cx_0$ for any x and y. Hence the primal and dual problem both are bounded, which means the both must have (at least) an optimal solution.

4.3. 6.3-6 SOLUTION

(a) Objective:

$$Minimize \quad W = 10y_1 + 10y_2, \tag{1}$$

subject to:

$$y_1 + 3y_2 \geq 2 \tag{2}$$
$$2y_1 + 3y_2 \geq 7 \tag{3}$$
$$y_1 + 2y_2 \geq 4, \tag{4}$$

and $y_1 \geq 0, y_2 \geq 0$.

(b) Since $cx \leq yb$ for any x and y, let $y_1 = 1, y_2 = 1.5$, which satisfies the constrains, and W = 10×1 + 10×1.5 = 25. Hence Z = cx can not exceed 25.

(c) Since $x_2 and x_3$ is conjectured to be the basic variables, the process of using Gaussian elimination is as below:

$$\begin{cases} -2x_1 - 7x_2 - 4x_3 = & 0 \\ x_1 + 2x_2 + x_3 + x_4 = & 10 \\ 3x_1 + 3x_2 + 2x_3 + x_5 = & 10 \end{cases}$$

$$\begin{cases} 2x_1 + x_2 + 4x_4 = & 40 \\ x_1 + 2x_2 + x_3 + x_4 = & 10 \\ x_1 - x_2 - 2x_4 + x_5 = & -10 \end{cases}$$

$$\begin{cases} 3x_1 + 2x_4 + x_5 = & 30 \\ 3x_1 + x_3 - 3x_4 + 2x_5 = & -10 \\ x_1 - x_2 - 2x_4 + x_5 = & -10 \end{cases}$$

Hence the basic solution is $(x_1, x_2, x_3, x_4, x_5) =$ (0,10,-10,0,0) with Z $=$ 30. And simultaneously the Eq.(0) for the primal problem is shown as Figure 1, from which we known that the basic solution for the dual problem is $(y_1, y_2, z_1 - c_1, z_2 - c_2, z_3 - c_3) = (2, 1, 3, 0, 0)$. And now we can draw conclusions that the basic solution for the primal problem is not feasible while that for the dual problem is feasible.

Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
z	0	1	3	0	0	2	1	30

Figure 1: Eq.(0) of primal tableau for Question 6.3-6(c) (made in Excel)

(d) The graph is shown as Figure 2, from which we can find the optimal feasible solution is $(y_1, y_2) = (0, 7/3)$ and $W_{min} = 70/3$. Hence, we can solve the equations below:

$$y_1 + 3y_2 + y_3 = 2 \tag{5}$$
$$2y_1 + 3y_2 + y_4 \geq 7 \tag{6}$$
$$y_1 + 2y_2 + y_5 \geq 4, \tag{7}$$

and the basic solution is $(y_1, y_2, y_3, y_4, y_5) =$ (0,7/3,-5,0, -2/3). Hence y_2, y_3, y_5 is basic solution and y_1, y_4 is nonbasic solution for the dual problem. From the relationship in row 0, we know that x_5, x_1, x_3 is nonbasic variables and x_2, x_4 is basic variables for the optimal solution of the primal problem. If we know which variables are basic for the optimal solution of the primal problem (actually we have known they are $x_2 and x_4$ just now), it will be easy to find the solution using Gaussian elimination. But for this question, I'm not sure whether this should be prior information, so I just assume nothing is known ahead. And thus the process is shown as Figure 3, which is more fussy than that in question (c). Hence the primal optimal basic solution is $(x_1, x_2, x_3, x_4, x_5) = (0, 10/3, 0, 10/3, 0)$ and $Z_{max} = 70/3$.

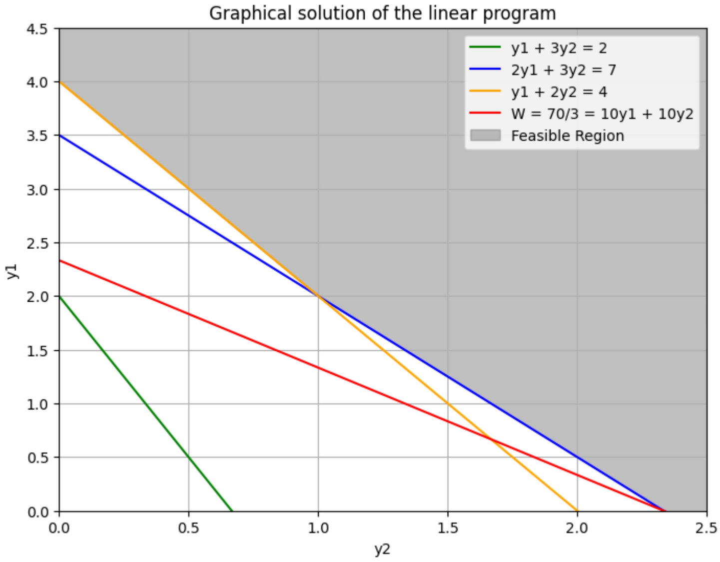

Figure 2: Graph solution for Question 6.3-6(d)

Iteration	Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
0	z	0	1	-2	-7	-4	0	0	0
	x4	1	0	1	2	1	1	0	10
	x5	2	0	3	3	2	0	1	10
0	z	0	1	5	0	2/3	0	7/3	70/3
	x4	1	0	-1	0	-1/3	1	-2/3	10/3
	x2	2	0	1	1	2/3	0	1/3	10/3

Figure 3: Gaussian elimination for Question 6.3-6(d)

4.4. 6.4-3 SOLUTION

The primal problem in Question 4.6.3 is:
Objective:

$$Minimize \quad Z = 2x_1 + 3x_2 + x_3, \tag{8}$$

subject to:

$$x_1 + 4x_2 + 2x_3 \geq 8 \tag{9}$$
$$3x_1 + 2x_2 \geq 6 \tag{10}$$
$$\tag{11}$$

and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.
Hence the dual problem is:
Objective:

$$Maximize \quad W = 8y_1 + 6y_2, \tag{12}$$

subject to:

$$y_1 + 3y_2 \leq 2 \tag{13}$$
$$4y_1 + 2y_2 \leq 3 \tag{14}$$
$$2y_1 \leq 1, \tag{15}$$

and $y_1 \geq 0, y_2 \geq 0$.

A Relevant Files

Relevant files can be found in my GitHub repository:
`https://github.com/OpenGHz/TBSI-MyHomework.git`.