Tsinghua-Berkeley Shenzhen Institute
Operations Research
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Homework 4

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- **Acknowledgments:** This assignment refers to the textbook: Introduction to Operations Research(10th).
- **Collaborators:** I finish this assignment by myself.

The answers below are about all the questions of problem 7.1-2 in the textbook.

4.1. SOLUTION

The linear programming problem in Question 7.1-1 is:
Objective:

$$Maximize \quad Z = 3x_1 + x_2 + 4x_3, \tag{1}$$

subject to:

$$6x_1 + 3x_2 + 5x_3 \leq 25 \tag{2}$$
$$3x_1 + 4x_2 + 5x_{\leq}20 \tag{3}$$

and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.
Hence we have:

$$c = [3, 1, 4],$$
$$A = \begin{bmatrix} 6 & 3 & 5 \\ 3 & 4 & 5 \end{bmatrix}. \tag{4}$$

And the corresponding final set of equations yielding the optimal solution is:

$$(0)Z + 2x_2 + 1/5x_4 + 3/5x_5 = 17 \tag{5}$$
$$(1)x_1 - 1/3x_2 + 1/3x_4 - 1/3x_5 = 5/3 \tag{6}$$
$$(2)x_2 + x_3 - 1/5x_4 + 2/5x_5 = 3 \tag{7}$$

Change these equations to tableau form in Figure 1. Hence we have:

Iteration	Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
Original	z	0	1	0	2	0	1/5	3/5	17
final	x1	1	0	1	-1/3	0	1/3	-1/3	5/3
tableau	x3	2	0	0	1	1	-1/5	2/5	3

Figure 1: Original final tableau for Question 7.1-2 (made in Excel)

$$y^* = [1/5, 3/5],$$
$$S^* = \begin{bmatrix} 1/3 & -1/3 \\ -1/5 & 2/5 \end{bmatrix}, \tag{8}$$

and the basic variables are x_1, x_3.

The answers are as follows:

(a) Since we have:

$$\begin{aligned} Z^* &= y^*\bar{b} = [1/5, 3/5]\,[10, 20]^T = 14, \\ b^* &= S^*\bar{b} = \begin{bmatrix} 1/3 & -1/3 \\ -1/5 & 2/5 \end{bmatrix} [10, 20]^T = [-10/3, 6]\,. \end{aligned} \tag{9}$$

Hence the final set of equations in tableau form for this question is shown as Figure 2. Since -10/3 $<$ 0, the solution is not feasible and thus not optimal.

	Basic		Coefficient of:						Right
Iteration	Variable	Eq.	z	x1	x2	x3	x4	x5	Side
Revised	z	0	1	0	2	0	1/5	3/5	14
final	x1	1	0	1	-1/3	0	1/3	-1/3	-10/3
tableau	x3	2	0	0	1	1	-1/5	2/5	6

Figure 2: Tableau for Question 7.1-2(a) (made in Excel)

(b) Since we have:

$$\begin{aligned} Z^* &= y^*\bar{b} = [1/5, 3/5]\,[25, 10]^T = 11, \\ b^* &= S^*\bar{b} = \begin{bmatrix} 1/3 & -1/3 \\ -1/5 & 2/5 \end{bmatrix} [25, 10]^T = [5, -1]\,. \end{aligned} \tag{10}$$

Hence the final set of equations in tableau form for this question is shown as Figure 3. Since -1$<$ 0, the solution is not feasible and thus not optimal.

	Basic		Coefficient of:						Right
Iteration	Variable	Eq.	z	x1	x2	x3	x4	x5	Side
Revised	z	0	1	0	2	0	1/5	3/5	11
final	x1	1	0	1	-1/3	0	1/3	-1/3	5
tableau	x3	2	0	0	1	1	-1/5	2/5	-1

Figure 3: Tableau for Question 7.1-2(b) (made in Excel)

(c) Since we have:

$$z_2^* - \bar{c_2} = y^* A_2 - \bar{c_2} = 3 - 3 = 0, \tag{11}$$

Hence the final set of equations in tableau form for this question is shown as Figure 4. It shows that the solution is feasible and optimal.

	Basic		Coefficient of:						Right
Iteration	Variable	Eq.	z	x1	x2	x3	x4	x5	Side
Revised	z	0	1	0	0	0	1/5	3/5	17
final	x1	1	0	1	-1/3	0	1/3	-1/3	5/3
tableau	x3	2	0	0	1	1	-1/5	2/5	3

Figure 4: Tableau for Question 7.1-2(c) (made in Excel)

(d) Since we have:

$$z_3^* - \bar{c_3} = y^* A_3 - \bar{c_3} = 4 - 2 = 2, \tag{12}$$

Iteration	Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
Revised	z	0	1	0	2	2	1/5	3/5	17
final	x1	1	0	1	-1/3	0	1/3	-1/3	5/3
tableau	x3	2	0	0	1	1	-1/5	2/5	3
Converted	z	0	1	0	0	0	3/5	-1/5	11
to proper	x1	1	0	1	-1/3	0	1/3	-1/3	5/3
form	x3	2	0	0	1	1	-1/5	2/5	3

Figure 5: Tableau for Question 7.1-2(d) (made in Excel)

Iteration	Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
Revised	z	0	1	0	4/5	0	1/5	3/5	17
final	x1	1	0	1	1/3	0	1/3	-1/3	5/3
tableau	x3	2	0	0	1/5	1	-1/5	2/5	3
Converted	z	0	1	0	2	0	1/5	3/5	17
to proper	x1	1	0	1	-1/3	0	1/3	-1/3	5/3
form	x3	2	0	0	1	1	-1/5	2/5	3

Figure 6: Tableau for Question 7.1-2(e) (made in Excel)

Hence the final set of equations in tableau form for this question is shown as Figure 5. It shows that the solution is feasible but not optimal because -1/5 $<$ 0 in row 0.

(e) Since we have:

$$\begin{aligned} z_2^* - c_2 &= y^* \bar{A}_2 - c_2 = 9/5 - 1 = 4/5, \\ A_2^* = S^* \bar{A}_2 &= \begin{bmatrix} 1/3 & -1/3 \\ -1/5 & 2/5 \end{bmatrix} [3,2]^T = [1/3, 1/5]\,. \end{aligned} \tag{13}$$

Hence the final set of equations in tableau form for this question is shown as Figure 6. It shows the solution is feasible and optimal.

(f) Since we have:

$$\begin{aligned} z_1^* - c_1 &= y^* \bar{A}_1 - c_1 = 17/5 - 3 = 2/5, \\ A_1^* = S^* \bar{A}_1 &= \begin{bmatrix} 1/3 & -1/3 \\ -1/5 & 2/5 \end{bmatrix} [3,2]^T = [5/3, -2/5]\,. \end{aligned} \tag{14}$$

Hence the final set of equations in tableau form for this question is shown as Figure 7. It shows the solution is feasible and optimal.

Iteration	Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	Right Side
Original	z	0	1	2/5	2	0	1/5	3/5	17
final	x1	1	0	5/3	-1/3	0	1/3	-1/3	5/3
tableau	x3	2	0	-2/5	1	1	-1/5	2/5	3
Converted	z	0	1	0	52/25	0	3/25	17/25	16.6
to proper	x1	1	0	1	-1/5	0	1/5	-1/5	1
form	x3	2	0	0	23/25	1	-3/25	8/25	3.4

Figure 7: Tableau for Question 7.1-2(f) (made in Excel)

A Relevant Files

Relevant files can be found in my GitHub repository:
`https://github.com/OpenGHz/TBSI-MyHomework.git`.