17.4-3

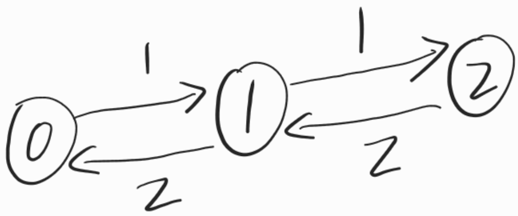
The expected pay is: $100P(T<2)+80(P\geq 2)=100-20P(T\geq 2)$.

$P_{ori}(T\geq 2) = e^{-\frac{1}{4}\times 2} = 0.607$, $P_{sp}(T\geq 2) = e^{-\frac{1}{2}\cdot 2} = 0.368$.

Hence the expected in pay is: $20(0.607-0.368) = 4.78$

17.5-2

(a)



(b)
$$\begin{cases} P_0 = 2P_1 \\ (2+1)P_1 = 2P_2 + P_0 \\ 2P_2 = P_1 \\ P_0+P_1+P_2 = 1 \end{cases}$$

(c) $P_1 = \frac{2}{7}$, $P_0 = \frac{4}{7}$, $P_2 = \frac{1}{7}$

(d) $P_1 = \frac{\lambda_0}{\mu_1}P_0 = \frac{1}{2}P_0$, $P_2 = \frac{\lambda_0\lambda_1}{\mu_1\mu_2}P_0 = \frac{1}{4}P_0$

$P_0+P_1+P_2 = (1+\frac{1}{2}+\frac{1}{4})P_0 = 1 \Rightarrow P_0 = \frac{4}{7}, P_1 = \frac{2}{7}, P_2 = \frac{1}{7}$

$L = \sum_{n=0}^{\infty} nP_n = 0P_0 + 1\cdot P_1 + 2\cdot P_2 = \frac{4}{7}$

$L_q = \sum_{n=1}^{\infty}(n-1)P_n = 0\cdot P_1 + 1\cdot P_2 = \frac{1}{7}$

$\bar{\lambda} = \sum_{n=0}^{\infty}\lambda_n P_n = 4\cdot P_0 + 2\cdot P_1 = \frac{16}{7} + \frac{4}{7} = \frac{20}{7}$

$W = \frac{L}{\bar{\lambda}} = \frac{1}{5}$, $W_q = \frac{L_q}{\bar{\lambda}} = \frac{1}{20}$

17.6-10

(a) $\lambda = 30$, $\mu = \frac{60}{1.5} = 40$

$L = \frac{\lambda}{\mu-\lambda} = \frac{30}{40-30} = 3$ customers, $W = \frac{1}{\mu-\lambda} = \frac{1}{40-30} = 0.1$ hours

$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L}{\mu} = \frac{3}{40}$ hours, $L_q = \lambda W_q = 30 \cdot \frac{3}{40} = \frac{9}{4}$ customes

$P_0 = 1-\rho = 1-\frac{3}{4} = \frac{1}{4}$, $P_1 = (1-\rho)\rho = \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16}$, $P_2 = (1-\rho)\rho^2 = \frac{9}{64}$

$1-P_0-P_1-P_2 = 1-\frac{1}{4}-\frac{3}{16}-\frac{9}{64} = 1-\frac{37}{64} = \frac{27}{64}$, hence the probability of have more than two customers at checkout stand is $\frac{27}{64}$.

(c) $\mu = 60$, $L = \frac{\lambda}{\mu-\lambda} = \frac{30}{60-30} = 1$ customer, $W = \frac{1}{\mu-\lambda} = \frac{1}{30}$ hours,

$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L}{\mu} = \frac{1}{60}$ hours, $L_q = \lambda \cdot W_q = 30 \times \frac{1}{60} = 0.5$ customers

$P_0 = 1-\rho = 1-\frac{1}{2} = 0.5$, $P_1 = (1-\rho)\rho = 0.25$, $P_2 = (1-\rho)\rho^2 = 0.125$.

Hence there is a 12.5% chance of having more than 2 customers at the checkout stand.

(e) The manager should hire another person to help the cashier by bagging the groceries.

17.9-5

(a) $\lambda_1 = a_1 + 0 \cdot a_1 + 0.3 \cdot a_2 + 0.4 a_3 = 15.7$

$\lambda_2 = a_2 + 0.5 a_1 + 0 \cdot a_2 + 0.5 a_3 = 21.5$

$$\lambda_3 = a_3 + 0.3a_1 + 0.2a_2 + 0a_3 = 9$$

(b) $$\rho_i = \frac{\lambda_i}{s_i \mu_i} = \begin{cases} 15.7/40 = 0.3925, & i=1 \\ 21.5/50 = 0.43, & i=2 \\ 9/30 = 0.3, & i=3 \end{cases}$$

$P_{n_1} = (1-0.3925)(0.3925)^{n_1} = 0.6075(0.3925)^{n_1}$ for facility 1

$P_{n_2} = (1-0.43)(0.43)^{n_2} = 0.57(0.43)^{n_2}$ for facility 2

$P_{n_3} = (1-0.3)(0.3)^{n_3} = 0.7(0.3)^{n_3}$ for facility 3

$$P\{(N_1, N_2, N_3) = (n_1, n_2, n_3)\} = P_{n_1} P_{n_2} P_{n_3}$$
$$= 0.2423925 (0.3925)^{n_1} (0.43)^{n_2} (0.3)^{n_3}$$

(c) $$P\{(N_1, N_2, N_3) = (0,0,0)\} = 0.2423925$$

(d) $$L_1 = \frac{\lambda_1}{\mu_1 - \lambda_1} = \frac{15.7}{40-15.7} = \frac{15.7}{24.3} = \frac{157}{243}$$

$$L_2 = \frac{\lambda_2}{\mu_2 - \lambda_2} = \frac{21.5}{50-21.5} = \frac{21.5}{28.5} = \frac{43}{57}$$

$$L_3 = \frac{\lambda_3}{\mu_3 - \lambda_3} = \frac{9}{30-9} = \frac{9}{21} = \frac{3}{7}$$

$$L = L_1 + L_2 + L_3 \approx 1.829$$

(e) $$W = \frac{L}{a_1 + a_2 + a_3} = \frac{1.829}{10+15+3} \approx 0.0653$$