Tsinghua-Berkeley Shenzhen Institute
OPERATIONS RESEARCH
Fall 2023

Homework 3

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- **Acknowledgments:** This assignment refers to the textbook: Introduction to Operations Research(10th).
- **Collaborators:** I finish this assignment by myself.

The answers below are arranged according to the order of the following questions in the textbook: 5.1-3 (a) (b) (c), 5.2-2, 5.3-3, 5.4-1.

3.1. 5.1-3

(a) The graph is shown as Figure 1.

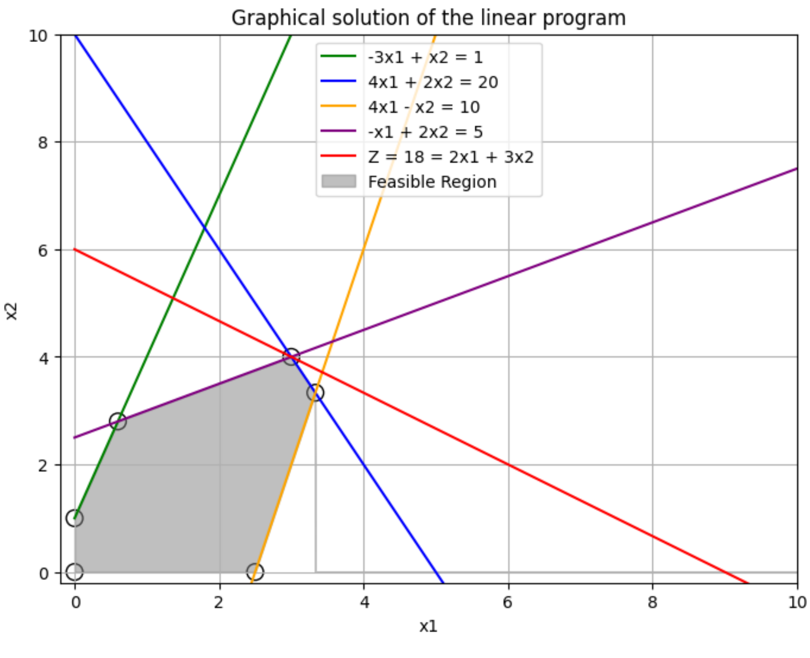

Figure 1: Graphical solution to Question 5.1-3 (a)

(b) The table is shown as Figure 2. We can see that the max value of z is 18, which means the optimal solution is $x_1 = 3, x_2 = 4$. And the Figure 1 also supports this result.

(c) The table is shown as Figure 3. As we can see, there is no set that can't yield a solution.

CPF	Df Eq	BF	None Basic	z
(0,0)	x1=0,x2=0	(0,0,1,20,10,5)	(x1,x2)	0
(0,1)	x1=0;-3x1+x2=1	(0,1,0,18,11,3)	(x1,x3)	3
(0.6,2.8)	-3x1+x2=1;-x1+2x2=5	(0.6,2.8,0,12,10.4,0)	(x3,x6)	9.6
(3,4)	-x1+2x2=5;4x1+2x2=20	(3,4,6,0,2,0)	(x4,x6)	18
(10/3,10/3)	4x1+2x2=20;4x1-x2=10	(10/3,10/3,23/3,0,0,5/3)	(x4,x5)	50/3
(2.5,0)	4x1-x2=10;x2=0	(2.5,0,8.5,10,0,7.5)	(x2,x5)	5

Figure 2: Table for Question 5.1-3 (b)(made in Excel)

CPIF	Df Eq	BF	None Basic
(1.8,6.4)	-3x1+x2=1;4x1+2x2=20	(1.8,6.4,0,0,9.2,-6)	(x3,x4)
(11,34)	-3x1+x2=1;4x1-x2=10	(11,34,0,-92,0,-52)	(x3,x5)
(-1/3,0)	-3x1+x2=1;x2=0	(-1/3,0,0,64/3,34/3,14/3)	(x2,x3)
(5,0)	4x1+2x2=20;x2=0	(5,0,16,0,-10,10)	(x2,x4)
(0,10)	4x1+2x2=20;x1=0	(0,10,-9,0,20,-15)	(x1,x4)
(25/7,30/7)	4x1-x2=10;-x1+2x2=5	(25/7,30/7,52/7,-20/7,0,0)	(x5,x6)
(0,-10)	4x1-x2=10;x1=0	(0,-10,11,40,0,25)	(x1,x5)
(-5,0)	-x1+2x2=5;x2=0	(-5,0,-14,40,30,0)	(x2,x6)
(0,2.5)	-x1+2x2=5;x1=0	(0,2.5,-1.5,15,12.5,0)	(x1,x6)

Figure 3: Table for Question 5.1-3 (c)(made in Excel)

3.2. 5.2-2 SOLUTION

$$c = [5, 8, 7, 4, 6, 0, 0]\,, A = \begin{bmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{bmatrix}, b = [20, 30]^T \tag{1}$$

Iteration 0:

$$B = B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{2}$$

$$x_B = [x_6, x_7] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [20, 30]^T = [20, 30]^T\,, \tag{3}$$

$$c_B = [0, 0]. \tag{4}$$

Hence x_2 will enter, and we can get the coefficients of x_2 in next iteration, which is:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [3, 5]^T = [3, 5]^T \tag{5}$$

Hence x_7 will leave because $20/3 > 30/5$.

Iteration 1:

$${B_1}^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -5/3 \\ 0 & 1/5 \end{bmatrix}, \tag{6}$$

$$x_B = [x_6, x_2] = \begin{bmatrix} 1 & -5/3 \\ 0 & 1/5 \end{bmatrix} [20, 30]^T = [2, 6]^T\,, \tag{7}$$

$$c_B = [0, 8]. \tag{8}$$

Hence row 0 is:

$$\begin{aligned}[0, 8/5] \begin{bmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{bmatrix} - [5, 8, 7, 4, 6, 0, 0] \\ = [-1/5, 0, -3/5, -4/4, -2/5, 0, 8/5]\end{aligned} \tag{9}$$

Hence x_4 will enter,and we can get the coefficients of x_4, which is:

$$\begin{bmatrix} 1 & -3/5 \\ 0 & 1/5 \end{bmatrix} [2, 2]^T = [4/5, 2/5]^T \tag{10}$$

Hence x_6 will leave because $2/0.8 < 6/0.4$.

Iteration 2:

$$\begin{gathered} {B_2}^{-1} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5/4 & -3/4 \\ -0.5 & 0.5 \end{bmatrix}, \\ x_B = [x_4, x_2] = \begin{bmatrix} 5/4 & -3/4 \\ -0.5 & 0.5 \end{bmatrix} [20, 30]^T = [2.5, 5]^T, \end{gathered} \tag{11}$$

$$c_B = [4, 8]. \tag{12}$$

Hence row 0 is:

$$\begin{aligned}[1, 1] \begin{bmatrix} 2 & 3 & 3 & 2 & 2 & 1 & 0 \\ 3 & 5 & 4 & 2 & 4 & 0 & 1 \end{bmatrix} - [5, 8, 7, 4, 6, 0, 0] \\ = [0, 0, 0, 0, 0, 1, 1]\end{aligned} \tag{13}$$

Hence the optimal solution is (0,5,0,2.5,0), and $z_{max} = 5 \times 8 + 2.5 \times 4 = 50$.

3.3. 5.3-3 SOLUTION

From the final simplex tableau, we have:

$$B^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix}, \tag{14}$$

$$c_B B^{-1} = [2, 0, 2], \tag{15}$$

Hence we have:

$$B^{-1}A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 0.5 \\ -4 & -2 & -1.5 \\ 1 & 2 & 0.5 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 4 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \tag{16}$$

$$\begin{aligned} c_B B^{-1} A - c = [2, 0, 2] \begin{bmatrix} 2 & 2 & 0.5 \\ -4 & -2 & -1.5 \\ 1 & 2 & 0.5 \end{bmatrix} - [6, 1, 2] \\ = [0, 7, 0], \end{aligned} \tag{17}$$

$$B^{-1}b = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 1 & 0 & -1 \end{bmatrix} [2,3,1]^T = [7,0,1]^T, \tag{18}$$

$$Z = [0,2,6][7,0,1]^T = 6. \tag{19}$$

We can see that all the coefficients in row 0 are not negative. Hence the final tableau is shown as Figure 4.

Basic Variable	Eq.	Coefficient of: z	x1	x2	x3	x4	x5	x6	Right Side
z	0	1	0	7	0	2	0	2	6
x5	1	0	0	4	0	1	1	2	7
x3	2	0	0	4	1	-2	0	4	0
x1	3	0	1	0	0	1	0	-1	1

Figure 4: Final tableau for Question 5.3-3 (made in Excel)

3.4. 5.4-1 SOLUTION With the information we already know, we have:
Iteration 0:

$$B = B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{20}$$

The coefficients of x_2 is:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [3,5]^T = [3,5]^T, \tag{21}$$

And x_2 will enter while x_7 will leave.
Iteration 1:

$$\begin{aligned} \eta &= [-3/5, 1/5]^T \\ {B_1}^{-1} &= \begin{bmatrix} 1 & -3/5 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3/5 \\ 0 & 1/5 \end{bmatrix} \end{aligned} \tag{22}$$

The coefficients of x_4 is:

$$\begin{bmatrix} 1 & 3/5 \\ 0 & 1/5 \end{bmatrix} [2,2]^T = [4/5, 2/5]^T, \tag{23}$$

And x_4 will enter while x_6 will leave.
Iteration 2:

$$\begin{aligned} \eta &= [-5/4, -1/2]^T \\ {B_2}^{-1} &= \begin{bmatrix} 5/4 & 0 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3/5 \\ 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 5/4 & -3/4 \\ -1/2 & 1/2 \end{bmatrix} \end{aligned} \tag{24}$$

A Relevant Files

Relevant files can be found in my GitHub repository:
`https://github.com/OpenGHz/TBSI-MyHomework.git`.