**LO 1.** Use the $t$-distribution for inference on a single mean, mean of paired difference, and difference of independent means.

**LO 2.** Explain why the $t$-distribution helps make up for the additional variability introduced by using $s$ (sample standard deviation) in calculation of the standard error, in place of $\sigma$ (population standard deviation).

**LO 3.** Describe how the $t$-distribution is different from the normal distribution, and what heavy tail means in this context.

**LO 4.** Note that the $t$-distribution has a single parameter, degrees of freedom, and as the degrees of freedom increases this distribution approaches the normal distribution.

**LO 5.** Calculate the required sample size to obtain a given margin of error at a given confidence level by working backwards from the given margin of error. Use $\sigma$ if it is given, along with the Z-score that corresponds to the confidence level.

**LO 6.** Use a $t$-statistic, with degrees of freedom $df = n - 1$ for inference for a population mean:

- Standard error: $SE = \frac{s}{\sqrt{n}}$
- Confidence interval: $\bar{x} \pm t^\star_{df} SE$
- Hypothesis test: $T_{df} = \frac{\bar{x} - \text{null value}}{SE}$

**LO 7.** Describe how to obtain a p-value for a $t$-test and a critical $t$-score ($t^\star_{df}$) for a confidence interval.

∗ *Reading: Section 7.1 of Advanced High School Statistics*

∗ *Test yourself:*

1. *What is the $t^\star$ for a 95% confidence interval for a mean, where the sample size is 13.*
2. *In a random sample of 1,017 Americans 60% said they do not trust the mass media when it comes to reporting the news fully, accurately, and fairly. The standard error associated with this estimate is 0.015 (1.5%). What is the margin of error a 95% confidence level? Calculate a 95% confidence interval and interpret it in context.*
3. *What is the p-value for a hypothesis test where the alternative hypothesis is two-sided, the sample size is 20, and the test statistic, T, is calculated to be 1.75?*

**LO 8.** Define observations as paired if each observation in one dataset has a special correspondence or connection with exactly one observation in the other data set.

**LO 9.** Carry out inference for paired data by first subtracting the paired observations from each other, and then treating the set of differences as a new numerical variable on which to do inference (such as a confidence interval or hypothesis test for the average difference).

**LO 10.** Calculate the standard error of the difference between two paired (dependent) samples as

$$SE = \frac{s_{diff}}{\sqrt{n_{diff}}}$$

and use this standard error in hypothesis testing and confidence intervals for the mean of paired differences.

**LO 11.** Use a $t$-statistic, with degrees of freedom $df = n_{diff} - 1$ for inference for a population mean:

- Standard error: $SE = \frac{s}{\sqrt{n}}$
- Confidence interval: $\bar{x}_{diff} \pm t^{\star}_{df} SE$
- Hypothesis test: $T_{df} = \frac{\bar{x}_{diff} - \text{null value}}{SE}$. Note that null value is often 0, since often $H_0 : \mu_{diff} = 0$.

∗ *Reading: Section 7.2 of Advanced High School Statistics*

∗ *Test yourself:*

1. *20 cardiac patients' blood pressure is measured before taking a medication, and after. For a given patient, are the before and after blood pressure measurements dependent (paired) or independent?*
2. *A random sample of 100 students were obtained and then randomly assigned into two equal sized groups. One group went on a roller coaster while the other in a simulator at an amusement park. Afterwards their blood pressure measurements were taken. Are the measurements dependent (paired) or independent?*

**LO 12.** Calculate the standard error of the difference between means of two independent samples as $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$, and use this standard error in hypothesis testing and confidence intervals comparing means of independent groups.

**LO 13.** Use a $t$-statistic, with degrees of freedom $df = min(n_1 - 1, n_2 - 1)$ or as provided by calculator for inference for the difference between two means:

- Standard error: $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- Confidence interval: $(\bar{x}_1 - \bar{x}_2) \pm t^{\star}_{df} SE$
- Hypothesis test: $T_{df} = \frac{(\bar{x}_1 - \bar{x}_2) - \text{null diff}}{SE}$. Note that null diff is often 0, since often $H_0 : \mu_1 - \mu_2 = 0$.

**LO 14.** Recognize that a good interpretation of a confidence interval for the difference of means between two parameters includes a comparative statement (mentioning which group has the larger parameter).

**LO 15.** Recognize that a confidence interval for the difference between two parameters that doesnt include 0 is in agreement with a hypothesis test where the null hypothesis (that the two parameters equal each other) is rejected.

∗ *Reading: Section 7.3 of Advanced High School Statistics*

∗ *Test yourself:*

1. *Describe how the two sample means test is different from the paired means test, both conceptually and in terms of the calculation of the standard error.*
2. *A 95% confidence interval for the difference between the number of calories consumed by mature and juvenile cats ($\mu_{mat} - \mu_{juv}$) is (80 calories, 100 calories). Interpret this interval, and determine if it suggests a significant difference between the two means.*