Learning Objectives   Chapter 3: Probability

LO 1. Define trial, outcome, and sample space.

LO 2. Explain why the long-run relative frequency of repeated independent events settle down to the true probability as the number of trials increases, i.e. why the law of large numbers holds.

LO 3. Distinguish disjoint (also called mutually exclusive) and independent events.

  - If A and B are independent, then having information on A does not tell us anything about B.

  - If A and B are disjoint, then knowing that A occurs tells us that B cannot occur.

  - Disjoint (mutually exclusive) events are always dependent since if one event occurs we know the other one cannot.

LO 4. Draw Venn diagrams representing events and their probabilities.

LO 5. Define a probability distribution as a list of the possible outcomes with corresponding probabilities that satisfies three rules:

  - The outcomes listed must be disjoint.

  - Each probability must be between 0 and 1, inclusive.

  - The probabilities must total 1.

LO 6. Define complementary outcomes as mutually exclusive outcomes of the same random process whose probabilities add up to 1.

  - If A and B are complementary, $P(A) + P(B) = 1$.

LO 7. Distinguish between union of events (A or B) and intersection of events (A and B).

  - Calculate the probability of union of events using the (general) addition rule.

    + If A and B are not mutually exclusive, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

    + If A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$, since for mutually exclusive events $P(A \text{ and } B) = 0$.

  - Calculate the probability of intersection of independent events using the multiplication rule.

    + If A and B are dependent, $P(A \text{ and } B) = P(A) \times P(B|A)$

    + If A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$, since for independent events $P(B|A) = P(B)$.

 * *Reading: Section 3.1 of Advanced High School Statistics*

 * *Videos:*

   – *Basics of probability, YouTube (1:42)*

   – *Union of events and the addition rule, YouTube (3:37)*

   – *Independent events, intersection of events, multiplication rule, and Bayes' Theorem, YouTube (3:25)*

 * *Test yourself:*

   1. *What is the probability of getting a head on the 6th coin flip if in the first 5 flips the coin landed on a head each time?*

2. *True / False: Being right handed and having blue eyes are mutually exclusive events.*

3. *P(A) = 0.5, P(B) = 0.6, there are no other possible outcomes in the sample space. What is P(A and B)?*

LO 8. Distinguish marginal and conditional probabilities.

LO 9. Construct tree diagrams to calculate conditional probabilities and probabilities of intersection of non-independent events using Bayes' theorem.

* *Reading: Section 3.2 of Advanced High School Statistics*

* *Videos:*

  – *Probability trees, Dr.Çetinkaya-Rundel (8:23)*

  – *Conditional probability, YouTube (8:59 - watch from 3:33 onwards)*

  – *Bayes' Theorem worked out example, YouTube, (9:20, somewhat lengthy)*

  – *Another example of conditional probabilities using Bayes' Theorem, YouTube (7:20)*

* *Test yourself: 50% of students in a class are social science majors and the rest are not. 70% of the social science students and 40% of the non-social science students are in a relationship. Create a contingency table and a tree diagram summarizing these probabilities. Calculate the percentage of students in this class who are in a relationship.*

LO 10. Determine if a random variable is binomial using the four conditions:

- The trials are independent.

- The number of trials, n, is fixed.

- Each trial outcome can be classified as a success or failure.

- The probability of a success, p, is the same for each trial.

LO 11. Calculate the number of possible scenarios for obtaining k successes in n trials using the choose function: $\binom{n}{k} = \frac{n!}{k! \ (n-k)!}$.

LO 12. Calculate probability of a given number of successes in a given number of trials using the binomial distribution: $P(K = k) = \frac{n!}{k! \ (n-k)!} \ p^k \ (1-p)^{(n-k)}$.

* *Reading: Section 3.3 of Advanced High School Statistics*

* *Test yourself: A bin contains 30 balls. 15 are Red, 10 are Green and 5 are Blue. 12 draws are made with replacement from the bin. Verify that the binomial formula can be used, and then calculate the probability that exactly 6 of the 12 draws are Red.*

LO 13. Carry out a simulation using a random digit table. This involves:

- Assigning digits to represent outcomes

- Simulating many repetitions using the random digit table, stopping when appropriate.

- Using results to estimate desired quantity.

* *Reading: Section 3.4 of Advanced High School Statistics*

* *Test yourself: A bin contains 30 balls. 15 are Red, 10 are Green and 5 are Blue. 12 draws are made with replacement from the bin. Use a random digit table to estimate the probability that exactly 6 of the 12 draws are Red. Repeat your simulation for 20 trials. What is your estimate? How does this estimate compare to the true probability calculated in the "Test yourself" question from the previous section on the Binomial Formula?*

LO 14. Random Variables

* *Reading: Section 3.5 of Advanced High School Statistics*

* *Test yourself:*