A concise course in Probability Theory

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This is a recounting of the contents I learned through the reading of Y.A. Rozanov's Probability Theory: a concise course.

To purchase this book see http://store.doverpublications.com/0486635449.html

1 Chapter 1: Basic Concepts

In this chapter of our concise course in Probability Theory, we will be covering the following concepts:

1. Probability and Relative Frequency

1.1 Probability and Relative Frequency

We will first consider an experiment whose outcomes are equally probable and whose possibility space or total set of outcomes, Ω, is finite. Think of the flipping of a coin or tossing of a dice. The probability of an event in that space ($A \subseteq \Omega$) is defined as the fraction:

$$P(A) = \frac{N(A)}{N(\Omega)}$$

where N(*) denotes the norm or count of outcomes leading to *.

Example 1. Consider an experiment where one tosses a n-sided dice whose sides are numbered. We want to know what the measure of probability is with respect to an event A defined as: A = The event that our dice lands on an even number.

In order to compute our probability, we must first take into consideration what the possible outcomes can be. In this instance, the best way to represent Ω will be with:

$$\Omega = \{\omega | \omega \in [1, n]\}$$

Our event A will then be defined with respect to Ω

$$A(\Omega) = \{\omega | (\omega \in \Omega) \wedge (\omega \bmod 2 \equiv 0)\}$$

With events Ω and $A(\Omega)$ defined, we now apply the definition of our function N(*) to each element to extract P(N(*)).

$$P(A) = \frac{N(A)}{N(\Omega)} = \frac{\frac{n}{2}}{n} = \frac{1}{2}$$

Example 2. Consider a darts player about to throw at a circular dartboard with a radius of 10 inches and is divided into 5 rings, labeled (0, 4) whose depth are 2 inches each. In order for this player to win the game, they must hit an odd labeled ring in their next shot. Assuming that the player will hit the dartboard and that it is equally probable for the dart to land at any location on the dartboard, what is the probability that the player wins the game?

We will define N(*) as the area of the region on the dartboard that's associated to to each event. In the case of our current problem's Ω, we will define

$$N(\Omega) = \pi 10^2$$

Next, we will notice that the event A, landing in an odd numbered region, defined in the problem can be considered a composition of sub-events (A_1, A_3) corresponding to the dart landing in region 1 or region 3. Because these events are mutually exclusive, we may write:

$$N(A) = N(A_1 \cup A_3) = N(A_1) + N(A_3)$$
$$= (\pi(4)^2 - \pi(2)^2) + (\pi(8)^2 - \pi(6)^2) = \pi 40$$

Finally, we apply the definition of probability:

$$P(A) = \frac{N(A)}{N(\Omega)} = \frac{40\pi}{100\pi} = \frac{2}{5}$$

The frequentist approach.

Another meas to define the Probability of an event to take the frequentist approach. This approach is to, like before, define events Ω and A of an experiment and then carry out that experiment α times. Having carried out the experiment α times, we will define a function β_α where $\beta_\alpha(*)$ = a measure of * in our α experiments. Our probability then then approximated by:

$$P(A) \sim \frac{\beta_\alpha(A)}{\beta_\alpha(\Omega)}$$

as the number of experiments conducted increases without bound, the above measure will increasingly converge upon the true value of the probability of that event. More exactly,

$$P(A) = \lim_{\alpha \to \infty} \frac{\beta_\alpha(A)}{\beta_\alpha(\Omega)}$$

2 Practice Problems

The following are a collection of practice problems of my own design. Please email me at jacob.vartuli.schonberg@gmail.com to discuss solutions.

1. Given a painted cube, which has since been divided into n^3 pieces, what is the probability that you randomly select a piece which is painted on one side as a function of n?

2. Having arbitrarily chosen a three digit number, α,what is the probability of its cube ending in 11.

$$A = \{\alpha | \alpha^3 = 11 + \sum_{i=3}^{6} q_i 10^i\}$$