A Simple Periodically-Forced Difference Equation Model for Mosquito Population Dynamics

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1 Introduction

We have developed a deterministic discrete time model for malaria in mosquitoes [4, 2] that we have integrated with a stochastic simulation model for malaria in humans [11]. We have used the model for malaria in mosquitoes to compare the effectiveness of vector control strategies that target adult mosquitoes, such as indoor residual spraying (IRS) and insecticide-treated nets (ITNs) [3]. However, the model did not include the dependence of the emergence rate of new mosquitoes on the number of eggs laid so it did not include the nonlinear effects of adulticides on reducing the population size. To include this effect and, also, to better model the effects of larval control we develop a model for mosquito population dynamics.

Most models of mosquito population dynamics have focussed on *Aedes* mosquitoes and have used ordinary differential equations (ODEs) [10], delay differential equations (DDEs) [5, 7, 9], or stochastic individual based models [1, 6, 8]. Dye [7] and Yakob *et al.* [12] developed difference equation models but only for successive generations.

However, to link it with our model for malaria in mosquitoes, we need a model for population dynamics with overlapping generations and a discrete time step of one day. We describe such a model here. We assume that the dynamics of the population are regulated by a periodically-varying larval carrying capacity. We use a simple model where we assume only one juvenile stage.

2 Model Description

We describe the parameters of the model in Table 1, derived parameters in Table 2, and additional parameters as in [2].

Table 1: Description of parameters for the model of mosquito population dynamics.

T_p:	Period of the system. Since we are using a daily time step, this period will usually be set to 365 days. However, we leave it as a parameter to allow other test values. Dimension: Time. $T_p \in \mathbb{N}$. [2]
τ:	Duration of resting period of mosquitoes. Dimension: Time. $\tau \in \mathbb{N}$. [2].
θ_j:	Duration of the juvenile stage. Dimension: Time. $\theta_j \in \mathbb{N}$.
b:	Number of female eggs laid by one female mosquito per oviposit. Dimensionless. $r > 1$.
ρ:	Survival probability of a mosquito from egg to emergence in the absence of density dependent mortality.
$\gamma(t)$:	Resource availability at time t. Dimension: 1/Animals. $\gamma(t) > 0$. $\gamma(t+T_p) = \gamma(t) \quad \forall t \in \mathbb{N}$.

In the model of malaria in mosquitoes, [2], the population of adult host-seeking mosquitoes was determined by,

$$x(t) = g\big(x(t-1), x(t-\tau), t\big) + E(t), \tag{1}$$

Table 2: Description of derived parameters for the model of mosquito population dynamics.

P_{df}:	Probability of finding a host and surviving the feeding cycle. Dimensionless. $0 < P_{df} < 1$. [2]
$E(t)$:	Emergence rate of adult mosquitoes at time t. Dimension: Animals. $E(t) > 0$. $E(t + T_p) = E(t) \quad \forall t \in \mathbb{N}$.

where $x(t)$ is the population of adult host-seeking mosquitoes at time t, $E(t)$ is a fixed periodic sequence of emerging mosquitoes, and $g\big(x(t-1), x(t-\tau), t\big)$ determines the survival of adult mosquitoes.

Here, we extend (1) to allow the emergence rate to depend on the adult population using a Beverton-Holt model,

$$x(t) = g\big(x(t-1), x(t-\tau), t\big) + \frac{\rho y(t-\theta_j)}{1 + \gamma(t-\theta_j) y(t-\theta_j)}, \tag{2a}$$

$$y(t) = b P_{df} x(t-\tau), \tag{2b}$$

where $y(t)$ is the population of juvenile stages at time t.

To initialize the system, we assume that the system (2) has a locally asymptotically stable periodic orbit that it has reached. In that case, the estimated periodic emergence rate is,

$$E(t) = \frac{\rho y(t-\theta_j)}{1 + \gamma(t-\theta_j) y(t-\theta_j)} \tag{3}$$

$$= \frac{\rho b P_{df} x(t-\theta_j-\tau)}{1 + \gamma(t-\theta_j) b P_{df} x(t-\theta_j-\tau)}. \tag{4}$$

Solving (4) for $\gamma(t)$ provides[1],

$$\gamma(t-\theta_j) = \frac{\rho b P_{df} x(t-\theta_j-\tau) - E(t)}{E(t) b P_{df} x(t-\theta_j-\tau)}. \tag{5}$$

To simulate larviciding, we can use

$$x(t) = g\big(x(t-1), x(t-\tau), t\big) + (1-c)\frac{\rho y(t-\theta_j)}{1 + \gamma(t-\theta_j) y(t-\theta_j)}, \tag{6a}$$

$$y(t) = b P_{df} x(t-\tau), \tag{6b}$$

where c is the proportional coverage of breeding sites treated with larvicides with $0 < c < 1$.

References

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[1] We again need to ensure that $\gamma(t) > 0$: I assume this will always be the case but there should be a check for it. Also, since $\gamma(t)$, $x(t)$, and $E(t)$ are periodic sequences, it is OK to solve both forwards and backwards in time.

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