

# Active Calculus Activities Book



Active Calculus: Matt Boelkins, Lead Author and Editor

Department of Mathematics

Grand Valley State University

[boelkinm@gvsu.edu](mailto:boelkinm@gvsu.edu)

<http://faculty.gvsu.edu/boelkinm/>

David Austin, Contributing Author

<http://merganser.math.gvsu.edu/david/>

Steven Schlicker, Contributing Author

<http://faculty.gvsu.edu/schlicks/>

Carroll College: Eric Sullivan, Contributing Author and Editor

[esullivan@carroll.edu](mailto:esullivan@carroll.edu)

<http://www.carroll.edu/esullivan>

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## Chapter 0

# Preliminaries

### 0.1 Functions, Slope, and Lines

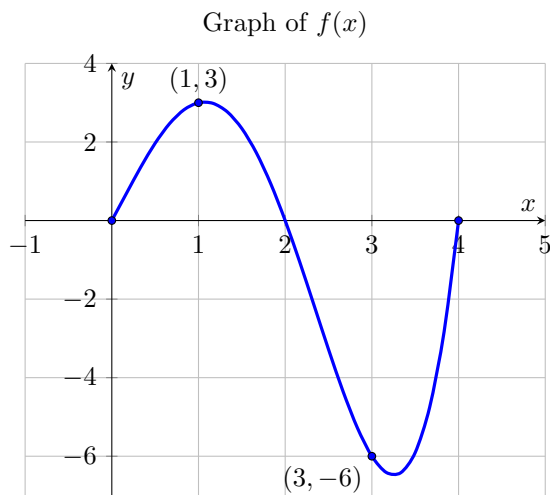
**Preview Activity 0.1.** This is the first Preview Activity in this text. Your job for this activity is to get to know the textbook.

- (a) Where can you find the full textbook?
- (b) What chapters of this text are you going to cover this semester. Have a look at your syllabus!
- (c) What are the differences between Preview Activities, Activities, Examples, Exercises, Voting Questions, and WeBWork? Which ones should you do before class, which ones will you likely do during class, and which ones should you be doing after class?
- (d) What materials in this text would you use to prepare for an exam and where do you find them?
- (e) What should you bring to class every day?



**Activity 0.1.**

The graph of a function  $f(x)$  is shown in the plot below.



- (a) What is the domain of  $f(x)$ ?
- (b) Approximate the range of  $f(x)$ .
- (c) What are  $f(0)$ ,  $f(1)$ ,  $f(3)$ ,  $f(4)$ , and  $f(5)$ ?

&lt;

**Activity 0.2.**

Find the equation of the line with the given information.

- (a) The line goes through the points  $(-2, 5)$  and  $(10, -1)$ .
- (b) The slope of the line is  $3/5$  and it goes through the point  $(2, 3)$ .
- (c) The  $y$ -intercept of the line is  $(0, -1)$  and the slope is  $-2/3$ .



**Activity 0.3.**

An apartment manager keeps careful record of the rent that he charges as well as the number of occupied apartments in his complex. The data that he has is shown in the table below.

Monthly Rent	\$650	\$700	\$750	\$800	\$850	\$900
Occupied Apartments	203	196	189	182	175	168

- (a) Just by doing simple arithmetic justify that the function relating the number of occupied apartments and the rent is linear.
- (b) Find the linear function relating the number of occupied apartments to the rent.
- (c) If the rent were to be increased to \$1000, how many occupied apartments would the apartment manager expect to have?
- (d) At a \$1000 monthly rent what net revenue should the apartment manager expect?

&lt;



**Activity 0.4.**

Write the equation of the line with the given information.

- (a) Write the equation of a line parallel to the line  $y = \frac{1}{2}x + 3$  passing through the point  $(3, 4)$ .
- (b) Write the equation of a line perpendicular to the line  $y = \frac{1}{2}x + 3$  passing through the point  $(3, 4)$ .
- (c) Write the equation of a line with  $y$ -intercept  $(0, -3)$  that is perpendicular to the line  $y = -3x - 1$ .



## Voting Questions

0.1.1 In the given equation, is  $y$  a function of  $x$ ?

$$y = x + 2$$

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).* Yes – one value of  $x$  gives one value of  $y$ .

by Carroll College MathQuest

SVC.01.01.001

CC HZ MA131 F06: **90**/10

CC KC MA121 F07: **97**/3 time 1:00

CC HZ MA131 F07: **100**/0 time 0:50

AS DH MA1561 010 F11: **85**/3/0/13 time 1:00

AS DH MA1561 030 F11: **97**/0/3/0 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **100**/0/0/0 time 0:30 Post small-group discussion vote

0.1.2 In the given equation, is  $y$  a function of  $x$ ?

$$x + y = 5$$

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).* Yes – one value of  $x$  gives one value of  $y$ .

by Carroll College MathQuest

SVC.01.01.002

CC LV MA121A F08: **63**/37 time 1:20

CC HZ MA131 F09: **90**/3

CC HZ MA121 F09: **81**/19



## 0.1. FUNCTIONS, SLOPE, AND LINES

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CC HZ MA131 F10: **64**/4/18/14  
CC KC MA121A F11: **80**/12/0/8  
CC KC MA121D F11: **50**/28/11/11  
AS DH MA1561 010 F11: **81**/12/0/7 time 1:40 "Individual, pre-discussion vote"  
AS DH MA1561 010 F11: **97**/3/0/0 time 1:00 Post small-group discussion vote  
AS DH MA1561 030 F11: **73**/9/3/15 time 2:00  
CC HZ MA131 F11: **30**/55/0/15 time 1:00  
CC HZ MA121 F12: **71**/0/24/5  
CC HZ MA131 F12: **41**/18/15/26 time 2:00

0.1.3 In the given equation, is  $y$  a function of  $x$ ?

$$x^3 + y = 5$$

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).* Yes – one value of  $x$  gives one value of  $y$ .

by Carroll College MathQuest

SVC.01.01.003

CC HZ MA131 F06: **80**/20  
CC HZ MA131 F07: **67**/33 time 0:25  
CC LV MA121A F08: **85**/15 time 0:55  
HC AS MA120 S12: **36.36**/40.91/13.64/9.09/0/0  
AS DH MA1561 010 F11: **97**/3/0/0 time 1:00  
AS DH MA1561 030 F11: **83**/11/3/3 time 0:50 "Individual, pre-discussion vote"  
AS DH MA1561 030 F11: **97**/3/0/0 time 0:30 Post small-group discussion vote  
AS DH MA1302 070 S12: **46**/17/12/25 time 3:00

0.1.4 In the given equation, is  $y$  a function of  $x$ ?

$$x^2 + y^2 = 5$$

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident

(d) No, and I am very confident

*Answer: (No).* No one value of  $x$  gives two different values of  $y$ . Or, graph is a circle and so doesn't pass vertical line test.

by Carroll College MathQuest

SVC.01.01.004

CC HZ MA131 F06: 20/**80**

CC KC MA121 F07: 61/**39** time 2:00

CC LV MA121A F08: 45/**55** time 2:00

CC HZ MA131 F09: 23/**70** time 1:00

CC HZ MA121 F09: 33/**67**

CC KC MA121A F11: 42/4/12/**42**

CC KC MA121D F11: 33/39/28/**0** "Individual, pre-discussion vote"

CC KC MA121D F11: 17/44/11/**28** Post small-group discussion vote

HC AS MA120 S12: 26.32/31.58/26.32/**15.79**/0/0

AS DH MA1561 010 F11: 73/10/7/**10** time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/3/**97** time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 3/0/0/**97** time 1:00

AS DH MA1302 070 S12: 8/8/12/**73** time 1:10

CC HZ MA131 F11: 25/10/15/**50** time 2:00

CC HZ MA121 F12: 18/27/27/**27** time 1:30

CC HZ MA131 F12: 4/7/15/**74** time 1:15

0.1.5 The set of points  $(x, y)$  which satisfy the equation  $(x - 1)^2 + (y + 3)^2 = 5^2$  can be represented via a mathematical function relating the  $x$  and  $y$  variables.

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (d).* This question is intended to reinforce student's abilities to graph circles and to use the horizontal line test. This set of points could be described with *two* functions. Students might be encouraged to give two such functions if time permits.

by Project MathVote

SVC.01.01.005

0.1.6 Does the table represent a function,  $y = f(x)$ ?

$x$	1	2	3	4
$f(x)$	2	3	2	4



## 0.1. FUNCTIONS, SLOPE, AND LINES

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- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).* There is one output for each input (okay that a  $y$ -value is repeated)

by Carroll College MathQuest

SVC.01.01.010

CC HZ MA131 F06: **70**/30

CC KC MA121 F07: **60**/40 time 2:00

CC HZ MA131 F07: **61**/39 time 0:35

CC LV MA121A F08: **36**/64 time 1:00

CC KC MA121A F11: **71**/8/8/12

CC KC MA121D F11: **39**/6/33/22 "Individual, pre-discussion vote"

CC KC MA121D F11: **78**/22/0/0 Post small-group discussion vote

HC AS MA120 S12: **50**/20/10/20/0/0

AS DH MA1561 010 F11: **46**/3/11/40 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: **69**/3/3/26 time 2:00 Post small-group discussion vote

AS DH MA1561 030 F11: **95**/0/0/5 time 4:00

0.1.7 Does the table represent a function,  $y = f(x)$ ?

$x$	1	2	2	4
$f(x)$	2	3	1	3

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (No).* There are two different outputs for input of 2.

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SVC.01.01.011

CC HZ MA131 F06: 10/**90**

CC LV MA121A F08: 11/**89** time :51

CC HZ MA131 F09: 0/**100** time 1:00

CC HZ MA121 F09: 32/**68**

CC HZ MA131 F10: 18/5/5/**73** time 0:45

HC AS MA120 S12: 0/0/5/**95**/0/0

AS DH MA1561 010 F11: 0/0/0/**100** time 1:00 "Individual, pre-discussion vote"  
 AS DH MA1561 010 F11: 0/0/0/**100** time 0:30 Post small-group discussion vote  
 AS DH MA1561 030 F11: 0/0/0/**100** time 0:30  
 CC HZ MA131 F11: 0/5/10/**85**  
 CC HZ MA121 F12: 4/13/30/**52** time 1:30  
 CC HZ MA131 F12: 26/0/7/**67**

0.1.8 Does this sentence describe a function? Wanda is two years older than I am.

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).* If I know my age, I can uniquely determine Wanda's age. Then ask students to write a formula for this function and give the domain and range. So if my age is  $m$  and Wanda's age is  $W$ , then Wanda's age is  $W = m + 2$ . Also, my age as a function of Wanda's age is  $m = f(W) = W - 2$ .

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SVC.01.01.012

CC HZ MA131 F06: **80**/20  
 CC LV MA121A F08: **90**/10 time 0:35  
 CC HZ MA131 F09: **97**/3 time 0:30  
 CC HZ MA121 F09: **95**/5  
 CC HZ MA131 F10: **100**/0/0/0 time 0:30  
 HC AS MA120 S12: **95.24**/4.76/0/0/0/0  
 AS DH MA1561 010 F11: **89**/6/3/3 time 1:00 "Individual, pre-discussion vote"  
 AS DH MA1561 010 F11: **100**/0/0/0 time 1:00 Post small-group discussion vote  
 AS DH MA1561 030 F11: **94**/6/0/0 time 2:00  
 CC HZ MA121 F12: **61**/36/4/0 time 1:00

0.1.9 The rule which assigns to each college student (at this exact point in time) a number equal to the number of college credits completed by that student is a function.

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.



## 0.1. FUNCTIONS, SLOPE, AND LINES

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*Answer: (a).* This question is intended to help broaden student notions of function. This is a function. Given the input of a college students, there is exactly one output corresponding to the number of credits that student has completed.

by Project MathVote

SVC.01.01.013

AU CS MA141 F11: 65/15/5/15

0.1.10 The rule which assigns to each car (at this exact point in time) the names of every person that has driven that car is a function.

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (d).* This question is intended to help broaden student notions of function. This is not a function. Given an input of a car, there could be multiple outputs corresponding to the names of all the people who have driven the car.

by Project MathVote

SVC.01.01.014

AU CS MA141 F11: 0/0/6/94

0.1.11 Could this table represent a linear function?

$x$	1	2	3	4
$f(x)$	1	2	4	8

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (No).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.015

CC HZ MA131 F05: 25/**75**  
 CC KC MA131 F05: 23/**77**  
 CC HZ MA131 F06: 40/**60**  
 CC KC MA121 F06: 68/**32**  
 CC KC MA121 F07: 13/**87** time 2:30  
 CC HZ MA131 F07: 33/**67** time 0:30  
 CC LV MA121A F08: 45/**55** time 1:03  
 CC HZ MA121 F09: 26/**70** Review  
 CC KC MA121A F11: 0/0/0/**100**  
 CC KC MA121D F11: 17/11/11/**61** "Individual, pre-discussion vote"  
 CC KC MA121D F11: 0/0/6/**94** Post small-group discussion vote  
 AU CS MA141 F11: 11/5/0/**84**  
 AS DH MA1561 010 F11: 8/3/5/**84** time 1:50  
 AS DH MA1561 030 F11: 58/11/3/**28** time 1:00 "Individual, pre-discussion vote"  
 AS DH MA1561 030 F11: 19/0/6/**75** time 1:00 Post small-group discussion vote

0.1.12 Could this table represent a linear function?

$x$	1	2	3	4
$f(x)$	-12	-9	-6	-3

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.020

CC HZ MA131 F05: **80**/20  
 CC KC MA131 F05: **100**/0  
 CC HZ MA131 F06: **100**/0  
 CC KC MA121 F06: **96**/4  
 CC HZ MA131 F09: **100**/0 time 0:30  
 CC HZ MA121 F09: **100**/0  
 CC HZ MA131 F10: **95**/5/0/0 time 0:40  
 HC AS MA120 S12: **86.96**/13.04/0/0/0/0  
 AU CS MA141 F11: **95**/0/0/5  
 AS DH MA1561 010 F11: **91**/9/0/0 time 1:00 "Individual, pre-discussion vote"  
 AS DH MA1561 010 F11: **100**/0/0/0 time 1:20 Post small-group discussion vote  
 AS DH MA1561 030 F11: **94**/6/0/0 time 3:00





## 0.1. FUNCTIONS, SLOPE, AND LINES

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CC HZ MA121 F12: **73**/23/4/0 time 0:45

0.1.13 Could this table represent a linear function?

$x$	1	2	4	8
$f(x)$	12	14	16	18

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (No).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.030

CC KC MA121 F07: 4/**96** time 1:00

CC HZ MA121 F09: 9/**91**

HC AS MA120 S12: 30.43/0/13.04/**56.52**/0/0

AU CS MA141 F11: 0/5/0/**95**/0/0

AS DH MA1561 010 F11: 59/7/7/**28** time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 37/0/4/**59** time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 27/15/0/**58** time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/8/**92** time 1:00 Post small-group discussion vote

CC HZ MA131 F11: 33/5/29/**33** time 1:40

0.1.14 Could this table represent a linear function?

$x$	1	2	4	8
$f(x)$	10	9	7	3

- (a) Yes, and I am very confident
- (b) Yes, but I am not very confident
- (c) No, but I am not very confident
- (d) No, and I am very confident

*Answer: (Yes).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.040

CC HZ MA131 F09: **47**/53 time 0:50  
 CC HZ MA131 F05: **60**/40  
 CC KC MA131 F05: **69**/31  
 CC HZ MA131 F06: **45**/55  
 CC KC MA121 F07: **90**/10 time 2:30  
 CC HZ MA131 F07: **55**/45 time 1:00  
 CC LV MA121A F08: **65**/35 time 1:20  
 CC HZ MA121 F09: **82**/18  
 CC HZ MA131 F10: **77**/14/0/9 time 0:55  
 CC KC MA121A F11: **62**/21/4/12  
 CC KC MA121D F11: **72**/17/6/6 "Individual, pre-discussion vote"  
 CC KC MA121D F11: **83**/17/0/0 Post small-group discussion vote  
 AU CS MA141 F11: **84**/5/0/11  
 AS DH MA1561 010 F11: **92**/0/0/8 time 1:30 "Individual, pre-discussion vote"  
 AS DH MA1561 010 F11: **92**/3/0/5 time 2:00 Post small-group discussion vote  
 AS DH MA1561 030 F11: **85**/6/3/6 time 3:30  
 CC HZ MA121 F12: **59**/18/9/14 time 2:00  
 CC HZ MA131 F12: **70**/4/4/22 time 1:00

0.1.15 True or False? All linear functions are examples of direct proportionality.

- (a) True, and I am very confident
- (b) True, but I am not very confident
- (c) False, but I am not very confident
- (d) False, and I am very confident

*Answer: (False).* The line must go through the origin.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.050

CC HZ MA131 F05: 95/**5**  
 CC KC MA131 F05: 77/**23**  
 CC MT MA131 F05: 69/**31**  
 CC HZ MA131 F06: 80/**20**  
 CC KC MA121 F07: 7/**93** time 1:45  
 CC HZ MA131 F07: 90/**10** time 0:20  
 CC LV MA121A F08: 83/**17** time 0:45  
 CC HZ MA131 F09: 82/**18** time 1:00  
 CC HZ MA121 F09: 82/**18**  
 CC HZ MA131 F10: 64/32/0/**0** time 0:40  
 CC KC MA121A F11: 58/8/8/**25**



## 0.1. FUNCTIONS, SLOPE, AND LINES

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AS DH MA1561 010 F11: 83/6/6/**6** time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 94/0/0/**6** time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: 85/10/5/**0** time 5:00

CC HZ MA121 F12: 38/48/5/**10**

0.1.16 Find the domain of the function  $f(x) = \frac{1}{x-2}$ .

- (a)  $x = 2$
- (b)  $x \neq 2$
- (c)  $x < 2$
- (d) all real numbers

*Answer: (b).*

by Project MathVote

SVC.01.01.051

STE.076

0.1.17 Find the domain of the function  $g(t) = \frac{2+t}{\sqrt{t-7}}$ .

- (a)  $t > 7$
- (b)  $t \geq 7$
- (c)  $t = 7$
- (d) all real numbers

*Answer: (a).*

by Project MathVote

SVC.01.01.052

STE.077

0.1.18 Which of the following functions has its domain identical with its range?

- (a)  $f(x) = x^2$
- (b)  $g(x) = \sqrt{x}$
- (c)  $h(x) = x^4$
- (d)  $i(x) = |x|$

*Answer: (b).* For  $g(x) = \sqrt{x}$  the domain and the range are all nonnegative numbers.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.060

CC KC MA121 F07: 0/**57**/0/43 time 2:30

HC AS MA120 S12: 0/**30**/0/70/0/0

AS DH MA1561 010 F11: 4/**50**/8/38 time 1:50 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 5/**57**/0/38 time 1:20 Post small-group discussion vote

AS DH MA1561 030 F11: 0/**25**/8/67 time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**30**/0/70 time 1:10 Post small-group discussion vote

0.1.19 The slope of the line connecting the points (1,4) and (3,8) is

(a)  $-\frac{1}{2}$

(b)  $-2$

(c)  $\frac{1}{2}$

(d)  $2$

*Answer: (d).* 2.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.070

CC HZ MA121 F09: 14/86/0/**0**

CC KC MA121A F11: 0/4/8/**88**

AS DH MA1561 010 F11: 0/0/11/**89** time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/0/**100** time 0:50 Post small-group discussion vote

AS DH MA1561 030 F11: 8/8/12/**71** time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/4/**96** time 0:50 Post small-group discussion vote

0.1.20 Which one of these lines has a different slope than the others?

(a)  $y = 3x + 2$

(b)  $3y = 9x + 4$

(c)  $3y = 3x + 6$

(d)  $2y = 6x + 4$



## 0.1. FUNCTIONS, SLOPE, AND LINES

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*Answer: (c).*

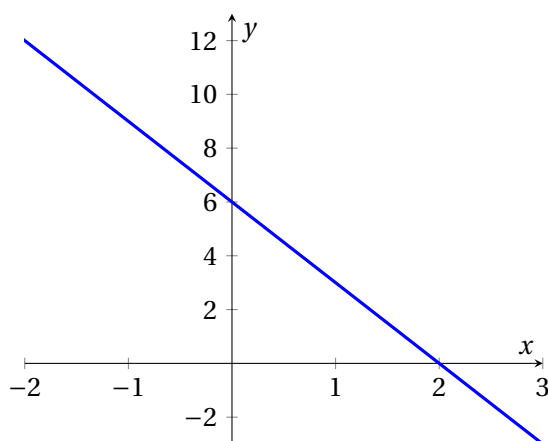
ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.080

CC KC MA121A F11: 0/4/8/88

0.1.21 The graph below represents which function?

- (a)  $y = 6x + 6$
- (b)  $y = -3x + 6$
- (c)  $y = -3x + 2$
- (d)  $y = -x + 6$
- (e)  $y = 6x - 2$
- (f)  $y = x - 2$



*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.090

CC KC MA121 F06: 0/93/4/4/0

HC AS MA120 S12: 4.55/77.27/0/9.09/9.09/0

AS DH MA1561 010 F11: 0/81/8/12 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/100/0/0 time 0:30 Post small-group discussion vote

AS DH MA1561 030 F11: 0/83/13/4 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/96/4/0 time 0:30 Post small-group discussion vote

CC HZ MA121 F12: 0/85/5/5/5/0 time 1:15

0.1.22 Which of the following functions is not increasing?

- (a) The elevation of a river as a function of distance from its mouth
- (b) The length of a single strand of hair as a function of time
- (c) The height of a person from age 0 to age 80
- (d) The height of a redwood tree as a function of time

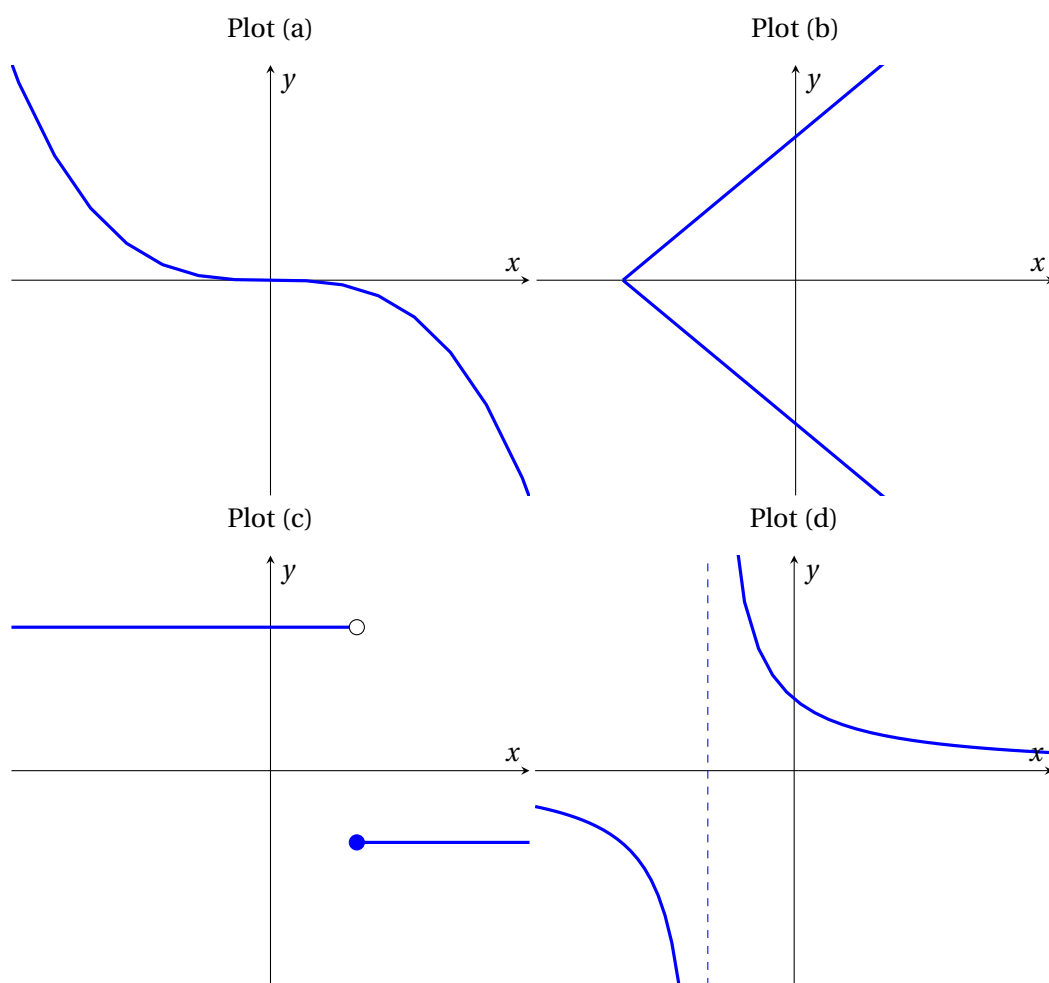
*Answer: (c).* In general, people stop growing when they are young adults and, before they are 80, they begin to lose height.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.100

CC KC MA121A F11: 80/0/8/12

0.1.23 Which of these graphs does not represent  $y$  as a function of  $x$ ?



*Answer: (b).*

For  $x > a$  there are two function values corresponding to the same value of  $x$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.01.110

## 0.1. FUNCTIONS, SLOPE, AND LINES

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CC LV MA121A F08: 0/**97**/0/3 time 0:40

CC HZ MA131 F09: 0/**97**/3/0 time 0:50

CC HZ MA121 F09: 9/**77**/9/5

AS DH MA1561 010 F11: 2/**95**/2/0 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/**100**/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/**97**/3/0 time 3:00

0.1.24 Calculate the average rate of change of the function  $f(x) = x^2$  between  $x = 1$  and  $x = 3$ .

- (a) 8
- (b) 4
- (c)  $\frac{1}{4}$
- (d) 0

*Answer: (b).*

by Project MathVote

SVC.01.01.120

STE.021

HC AS MA120 S12: 4.35/**91.3**/4.35/0

0.1.25 The EPA reports the total amount of Municipal Solid Waste (MSW), otherwise known as garbage, produced in the U.S. for the years 2005 through 2009:

Year	2005	2006	2007	2008	2009
Millions of tons	252.4	251.3	255	249.6	243

(source: <http://www.epa.gov/osw/nonhaz/municipal/>)

What are the appropriate units for the average rate of change in the amount of garbage produced between any two given years?

- (a) millions of tons
- (b) tons
- (c) millions of tons per year
- (d) tons per year

*Answer: (c)* This is the first of several questions about average rate of change of MSW.

by Project MathVote

SVC.01.01.130

STE.022

HC AS MA120 S12: 0/0/95.24/4.76

- 0.1.26 The EPA reports the total amount of Municipal Solid Waste (MSW), otherwise known as garbage, produced in the U.S. for the years 2005 through 2009:

Year	2005	2006	2007	2008	2009
Millions of tons	252.4	251.3	255	249.6	243

(source: <http://www.epa.gov/osw/nonhaz/municipal/>)

What is the average rate of change in the amount of MSW produced from 2005 to 2007?

- (a) 2.6 million tons per year
- (b) 2.6 million tons
- (c) 1.3 million tons
- (d) 1.3 million tons per year

*Answer: (d)* This is the second of several questions about average rate of change of MSW in the U.S. Students may still be confused about units, and may also forget to account for the fact that this average rate of change is calculated over two years.

by Project MathVote

SVC.01.01.140

STE.023

HC AS MA120 S12: 0/4.76/0/95.24

- 0.1.27 The EPA reports the total amount of Municipal Solid Waste (MSW), otherwise known as garbage, produced in the U.S. for the years 2005 through 2009:

Year	2005	2006	2007	2008	2009
Millions of tons	252.4	251.3	255	249.6	243





## 0.1. FUNCTIONS, SLOPE, AND LINES

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(source: <http://www.epa.gov/osw/nonhaz/municipal/>)

What is the average rate of change in the amount of MSW produced from 2007 to 2009?

- (a) -6 million tons per year
- (b) 6 million tons per year
- (c) -12 million tons per year
- (d) 12 million tons per year

*Answer: (a)* This is the third of several questions about average rate of change of MSW in the U.S. Students may be surprised that the result is negative, since the amount of garbage actually decreased over this period. Students may still forget to account for the fact that this average rate of change is calculated over two years.

by Project MathVote

SVC.01.01.150

STE.024 HC AS MA120 S12: **95.45**/0/4.55/0

0.1.28 Find the difference quotient  $\frac{f(x+h)-f(x)}{h}$  for the function  $f(x) = 2x^2 - x + 3$ . Simplify your answer.

- (a)  $\frac{2h^2-h+3}{h}$
- (b)  $2h - 1$
- (c)  $\frac{4xh+2h^2-2x+h+6}{h}$
- (d)  $4x + 2h - 1$

*Answer: (d).*

by Project MathVote

SVC.01.01.160

STE.025

0.1.29 When the temperature is  $0^\circ\text{C}$  it is  $32^\circ\text{F}$  and when it is  $100^\circ\text{C}$  it is  $212^\circ\text{F}$ . Use these facts to write a linear function to convert any temperature from Celsius to Fahrenheit.

- (a)  $C(F) = \frac{5}{9}F - \frac{160}{9}$
- (b)  $F(C) = C + 32$
- (c)  $F(C) = \frac{5}{9}C - \frac{160}{9}$
- (d)  $F(C) = \frac{9}{5}C + 32$

*Answer: (d).* This question gives students additional practice at finding the equation of a line given two points.

by Project MathVote

SVC.01.01.170

STE.072

0.1.30 Let  $f(x) = 1 + 4x^2$ . True or False:  $f(\frac{1}{2}) = \frac{f(1)}{f(2)}$ .

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (d) False.* This question checks student understanding of function notation.

by Project MathVote

SVC.01.01.180

STE.073 HC AS MA120 S12: 0/4.35/26.09/**69.57**

HC AS MA120 S12: 0/0/19.05/**80.95**

0.1.31 Let  $f(x) = 1 + 4x^2$ . True or False:  $f(a + b) = f(a) + f(b)$ .

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (d) False.* This question checks student understanding of function notation. It also can facilitate a discussion of what it means to be an additive function.

by Project MathVote

SVC.01.01.190

STE.074 HC AS MA120 S12: 4.35/0/30.43/**60.87**

0.1.32 Let  $f(x) = \frac{1}{x+2}$ . Find a value of  $x$  so that  $f(x) = 6$

- (a)  $-\frac{11}{6}$



## 0.1. FUNCTIONS, SLOPE, AND LINES

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- (b)  $\frac{13}{6}$
- (c)  $\frac{1}{8}$
- (d) none of the above

*Answer: (a).* This question checks understanding of input and output variables, and serves as an introduction to the idea of inverse functions.

by Project MathVote

SVC.01.01.200

STE.075 HC AS MA120 S12: **91.3**/0/0/8.7

0.1.33 True or False:  $\sqrt{x^2} = x$ .

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (d) False.* This is a common student misconception. It is false when  $x$  is negative.

by Project MathVote

SVC.01.01.210

STE.078 HC AS MA120 S12: 75/8.33/12.5/**4.17**

HC AS MA120 S12: 77.27/4.55/18.18/**0**

## 0.2 Exponential Functions

**Preview Activity 0.2.** Suppose that the populations of two towns are both growing over time. The town of Exponentia is growing at a rate of 2% per year, and the town of Lineola is growing at a rate of 100 people per year. In 2014, both of the towns have 2,000 people.

- (a) Complete the table for the population of each of these towns over the next several years.

	2014	2015	2016	2017	2018	2019	2020	2021	2022
Exponentia	2000								
Lineola	2000								

- (b) Write a linear function for the population of Lineola. Interpret the slope in the context of this problem.
- (c) The ratio of successive populations for Exponentia should be equal. For example, dividing the population in 2015 by that of 2014 should give the same ratio as when the population from 2016 is divided by the population of 2015. Find this ratio. How is this ratio related to the 2% growth rate?
- (d) Based on your data from part (a) and your ratio in part (c), write a function for the population of Exponentia.
- (e) When will the population of Exponentia exceed that of Lineola?



**Activity 0.5.**

Consider the exponential functions plotted in Figure 1

- (a) Which of the functions have common ratio  $r > 1$ ?
- (b) Which of the functions have common ratio  $0 < r < 1$ ?
- (c) Rank each of the functions in order from largest to smallest  $r$  value.

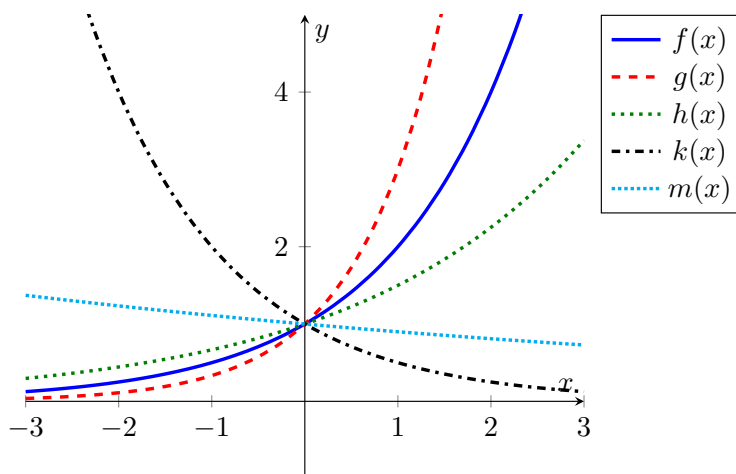


Figure 1: Exponential growth and decay functions

**Activity 0.6.**

A sample of  $\text{Ni}^{56}$  has a half-life of 6.4 days. Assume that there are 30 grams present initially.

- (a) Write a function describing the number of grams of  $\text{Ni}^{56}$  present as a function of time. Check your function based on the fact that in 6.4 days there should be 50% remaining.
- (b) What percent of the substance is present after 1 day?
- (c) What percent of the substance is present after 10 days?

&lt;

**Activity 0.7.**

Uncontrolled geometric growth of the bacterium *Escherichia coli* (*E. Coli*) is the theme of the following quote taken from the best-selling author Michael Crichton's science fiction thriller, *The Andromeda Strain*:

“The mathematics of uncontrolled growth are frightening. A single cell of the bacterium *E. coli* would, under ideal circumstances, divide every twenty minutes. That is not particularly disturbing until you think about it, but the fact is that that bacteria multiply geometrically: one becomes two, two become four, four become eight, and so on. In this way it can be shown that in a single day, one cell of *E. coli* could produce a super-colony equal in size and weight to the entire planet Earth.”

- (a) Write an equation for the number of *E. coli* cells present if a single cell of *E. coli* divides every 20 minutes.
- (b) How many *E. coli* would there be at the end of 24 hours?
- (c) The mass of an *E. coli* bacterium is  $1.7 \times 10^{-12}$  grams, while the mass of the Earth is  $6.0 \times 10^{27}$  grams. Is Michael Crichton's claim accurate? Approximate the number of hours we should have allowed for this statement to be correct?

## Voting Questions

0.2.1 The following table shows the net sales at Amazon.com from 2003 to 2010 (*source: Amazon.com quarterly reports*):

Year	2003	2004	2005	2006	2007	2008	2009	2010
Billions of dollars	\$5.26	\$6.92	\$8.49	\$10.72	\$14.84	\$19.15	\$24.51	\$34.21

If the net sales are modeled using an exponential function  $S(t) = a \cdot b^t$ , where  $S$  is the net sales in billions of dollars, and  $t$  is the number of years after 2003, which of the following is an appropriate value for  $a$ ?

- (a) 34.21
- (b) 1
- (c) 1.31
- (d) 5.26

*Answer: (d).* This is the third of three questions modeling Amazon's net sales with an exponential function.

by Project MathVote

SVC.01.02.103

STE.028

HC AS MA120 S12: 5.56/11.11/0/**83.33**

0.2.2 Which is better at the end of one year: An account that pays 8% annual interest compounded quarterly or an account that pays 7.95% interest compounded continuously?

- (a) 8% quarterly
- (b) 7.95% continuously
- (c) They are the same.
- (d) There is no way to tell.

*Answer: (b).* Students will need to first write an equation appropriate for continuously compounding interest,  $f(t) = Ae^{0.0795t}$ , and then find that the effective annual growth rate is  $e^{0.0795} = 1.0827$ , or 8.27% annual interest.

by Project MathVote

SVC.01.02.104





## 0.2. EXPONENTIAL FUNCTIONS

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STE.082

HC AS MA120 S12: 15.79/**84.21**/0/0

0.2.3 Caffeine leaves the body at a continuous rate of 17% per hour. How much caffeine is left in the body 8 hours after drinking a cup of coffee containing 100 mg of caffeine?

- (a) 389.62 mg
- (b) 22.52 mg
- (c) 25.67 mg
- (d) There is no way to tell.

*Answer: (c).* Students will need to first write an equation appropriate for a continuous decay rate,  $f(t) = Ae^{-.17t}$  and then evaluate it when  $A = 100$  and  $t = 8$ . Choice b is the value if the given rate is used as an annual decay rate. Choice a is the value if the value using continuous growth instead (omission of - sign).

by Project MathVote

SVC.01.02.105

STE.083 HC AS MA120 S12: 12.5/25/**62.5**/0

0.2.4 Caffeine leaves the body at a continuous rate of 17% per hour. What is the hourly growth factor?

- (a) .156
- (b) .17
- (c) .844
- (d) There is no way to tell.

*Answer: (c).* Students will need to first write an equation appropriate for a continuous decay rate,  $f(t) = Ae^{-.17t}$  and then note that  $e^{-.17} = .844$ .

by Project MathVote

SVC.01.02.106

STE.084

## 0.3 Transformations of Functions

**Preview Activity 0.3.** The goal of this activity is to explore and experiment with the function

$$F(x) = Af(B(x - C)) + D.$$

The values of  $A$ ,  $B$ ,  $C$ , and  $D$  are constants and the function  $f(x)$  will be henceforth called the *parent function*. To facilitate this exploration, use the applet located at <http://www.geogebraTube.org/student/m93018>.

- (a) Let's start with a simple parent function:  $f(x) = x^2$ .
- (1) Fix  $B = 1$ ,  $C = 0$ , and  $D = 0$ . Write a sentence or two describing the action of  $A$  on the function  $F(x)$ .
  - (2) Fix  $A = 1$ ,  $B = 1$ , and  $D = 0$ . Write a sentence of two describing the action of  $C$  on the function  $F(x)$ .
  - (3) Fix  $A = 1$ ,  $B = 1$ , and  $C = 0$ . Write a sentence of two describing the action of  $D$  on the function  $F(x)$ .
  - (4) Fix  $A = 1$ ,  $C = 0$ , and  $D = 0$ . Write a sentence of two describing the action of  $B$  on the function  $F(x)$ .
- (b) In part (a) you have made conjectures about what  $A$ ,  $B$ ,  $C$ , and  $D$  do to a parent function graphically. Test your conjectures with the functions  $f(x) = |x|$  (typed `abs(x)`),  $f(x) = x^3$ ,  $f(x) = \sin(x)$ ,  $f(x) = e^x$  (typed `exp(x)`), and any other function you find interesting.

▷◁

### 0.3. TRANSFORMATIONS OF FUNCTIONS

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#### Activity 0.8.

Consider the function  $f(x)$  displayed in Figure 2.

- (a) Plot  $g(x) = -f(x)$  and  $h(x) = f(x) - 1$ .
- (b) Define the function  $k(x) = -f(x) - 1$ . Does it matter which order you complete the transformations from part (a) to result in  $k(x)$ ? Plot the functions resulting from doing the two transformations in part (a) in opposite orders. Which of these functions is  $k(x)$ ?

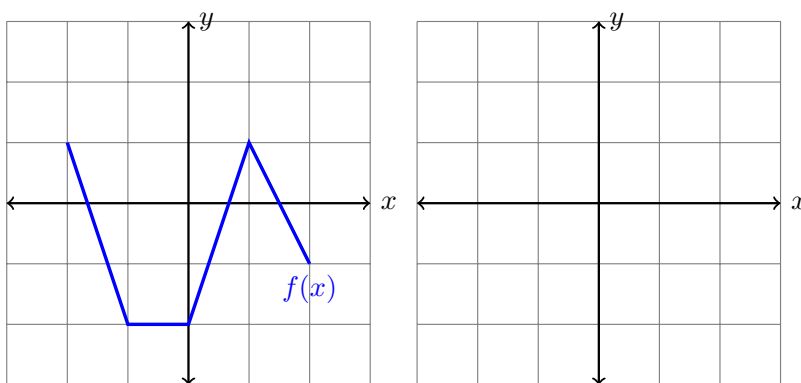


Figure 2: Function transformation for Activity 0.8

**Activity 0.9.**

- (a) Let  $f(x) = x^2$  and  $g(x) = x + 8$ . Find the following:

$$f(g(3)) = \underline{\hspace{2cm}}, \quad g(f(3)) = \underline{\hspace{2cm}}, \quad f(g(x)) = \underline{\hspace{2cm}},$$

$$g(f(x)) = \underline{\hspace{2cm}}, \quad f(x)g(x) = \underline{\hspace{2cm}}$$

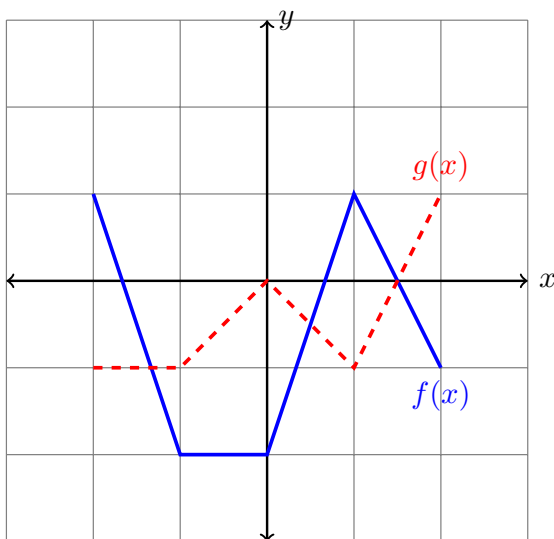
- (b) Now let  $f(x)$  and  $g(x)$  be defined as in the table below. Use the data in the table to find the following compositions.

$x$	-3	-2	-1	0	1	2	3
$f(x)$	3	1	-1	-3	-1	1	3
$g(x)$	-2	-1	0	1	0	1	2

$$f(-3) = \underline{\hspace{2cm}}, \quad g(3) = \underline{\hspace{2cm}},$$

$$f(g(-3)) = \underline{\hspace{2cm}}, \quad f(g(f(-3))) = \underline{\hspace{2cm}}$$

- (c) Now let  $f(x)$  and  $g(x)$  be defined as in the plots below. Use the plots to find the following compositions.



$$f(1) = \underline{\hspace{2cm}}$$

$$g(2) = \underline{\hspace{2cm}}$$

$$g(f(1)) = \underline{\hspace{2cm}}$$

$$f(g(1)) = \underline{\hspace{2cm}}$$

$$g(f(f(0))) = \underline{\hspace{2cm}}$$



### 0.3. TRANSFORMATIONS OF FUNCTIONS

#### Activity 0.10.

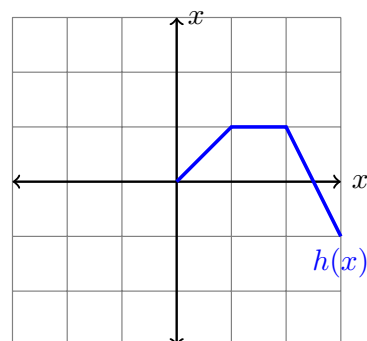
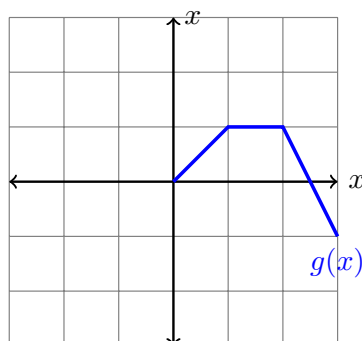
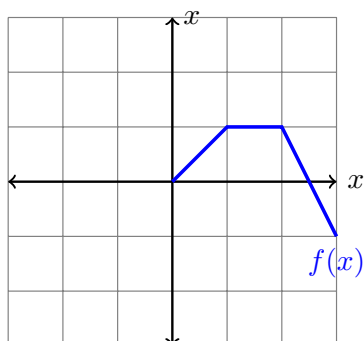
- (a) Based on symmetry alone, is  $f(x) = x^2$  an even or an odd function?
- (b) Based on symmetry alone, is  $g(x) = x^3$  an even or an odd function?
- (c) Find  $f(-x)$  and  $g(-x)$  and make conjectures to complete these sentences:
- If a function  $f(x)$  is even then  $f(-x) = \underline{\hspace{2cm}}$ .
  - If a function  $f(x)$  is odd then  $f(-x) = \underline{\hspace{2cm}}$ .

Explain why the composition  $f(-x)$  is a good test for symmetry of a function.

- (d) Classify each of the following functions as even, odd, or neither.

$$h(x) = \frac{1}{x}, \quad j(x) = e^x, \quad k(x) = x^2 - x^4, \quad n(x) = x^3 + x^2.$$

- (e) Each figure below shows only half of the function. Draw the left half so  $f(x)$  is even. Draw the left half so  $g(x)$  is odd. Draw the left half so  $h(x)$  is neither even nor odd.



**Activity 0.11.**

- (a) Find the inverse of each of the following functions by interchanging the  $x$  and  $y$  and solving for  $y$ . Be sure to state the domain for each of your answers.

$$y = \sqrt{x-1}, \quad y = -\frac{1}{3}x + 1, \quad y = \frac{x+4}{2x-5}$$

- (b) Verify that the functions  $f(x) = 3x-2$  and  $g(x) = \frac{x}{3} + \frac{2}{3}$  are inverses of each other by computing  $f(g(x))$  and  $g(f(x))$ .

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### 0.3. TRANSFORMATIONS OF FUNCTIONS

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#### Voting Questions

0.3.1 The functions  $f$  and  $g$  have values given in the table below. What is the value of  $f(g(0))$ ?

$x$	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

- (a) -2
- (b) -1
- (c) 0
- (d) 1
- (e) 2

*Answer: (b).*  $g(0) = 2$ , so  $f(g(0)) = f(2) = -1$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.010

CC KC MA121 F06: 39/**31**/4/8/15

CC KC MA121 F07: 14/**66**/0/10/10 time 2:45

CC HZ MA121 F09: 14/**50**/14/0/23

CC KC MA121A F11: 4/**75**/0/17/4

CC KC MA121D F11: 28/**33**/11/6/22 "Individual, pre-discussion vote"

CC KC MA121D F11: 11/**72**/11/0/6 Post small-group discussion vote

HC AS MA120 S12: 4.17/62.5/20.83/4.17/**4.17**/4.17

AS DH MA1561 010 F11: 8/**84**/3/5/0 time 2:30

AS DH MA1561 030 F11: 6/**29**/32/6/26 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 7/**80**/10/3/0 time 2:00 Post small-group discussion vote

CC HZ MA121 F12: 24/**24**/9/5/38 time 2:30

0.3.2 The functions  $f$  and  $g$  have values given in the table below. If  $f(g(x)) = 1$ , then what is  $x$ ?

$x$	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

- (a) -2
- (b) -1
- (c) 0
- (d) 1

(e) 2

Answer: (e). 2.  $f(-2) = 1$  and  $g(2) = -2$ , so  $x = 2$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.020

CC KC MA121 F06: 8/12/0/8/73

CC KC MA121 F07: 7/0/0/4/89 time 2:45

CC HZ MA121 F09: 5/0/23/5/68

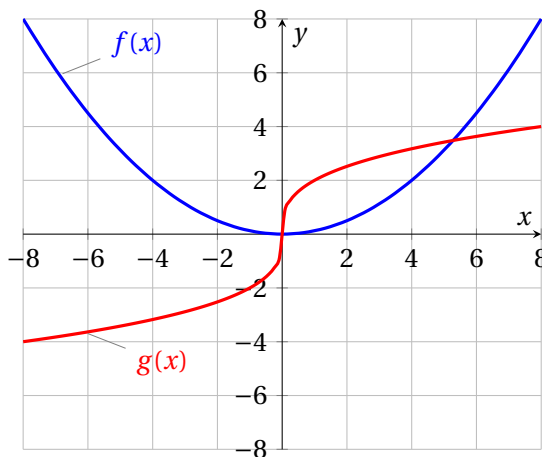
AU CS MA141 F11: 10/86/0/0/5

AU CS MA141 F11: 10/5/0/5/80 revote

AS DH MA1561 010 F11: 10/0/2/0/88 time 4:00

0.3.3 The graphs of  $f$  and  $g$  are shown in the figure below. Estimate the value of  $g(f(3))$ .

- (a) -1
- (b) 0
- (c) 1
- (d) 2
- (e) 3
- (f) 5



Answer: (d). 2.  $f(3) \approx 1$ , so  $g(f(3)) \approx g(1) \approx 2$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.030

CC KC MA121 F06: 0/4/19/77/0/0

CC KC MA121 F07: 0/7/0/93/0/0 time 2:30

CC HZ MA121 F09: 0/0/9/91/0/0

AS DH MA1561 010 F11: 3/3/0/92/0/3 time 4:30

CC HZ MA121 F12: 0/5/18/68/9/0 time 2:45

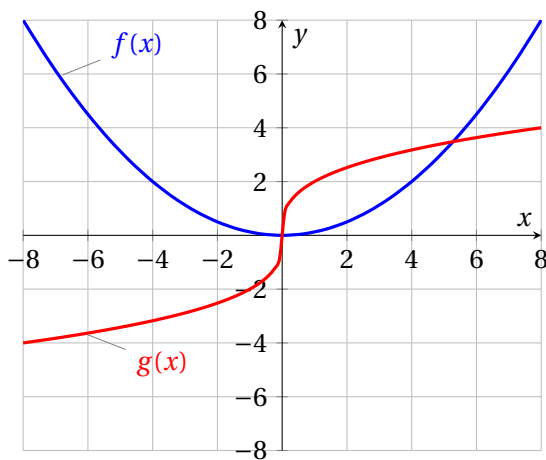
0.3.4 The graphs of  $f$  and  $g$  are shown in the figure below. Estimate the value of  $f(g(2))$ .





### 0.3. TRANSFORMATIONS OF FUNCTIONS

- (a) -1
- (b) 0
- (c) 1
- (d) 2
- (e) 3
- (f) 5



Answer: (c). 1.  $g(2) \approx 3$ , so  $f(g(2)) \approx f(3) \approx 1$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.040

CC HZ MA121 F09: 0/5/**95**/0/0/0

HC AS MA120 S12: 0/**8.7**/65.22/8.7/4.35/13.04

AU CS MA141 F11: 0/0/**100**/0/0/0

AS DH MA1561 010 F11: 4/0/**89**/7/0/0 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/**100**/0/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/9/**65**/13/9/4 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**92**/0/8/0 time 1:00 Post small-group discussion vote

0.3.5 If  $P = f(t) = 3 + 4t$ , find  $f^{-1}(P)$ .

- (a)  $f^{-1}(P) = 3 + 4P$
- (b)  $f^{-1}(P) = \frac{P-3}{4}$
- (c)  $f^{-1}(P) = \frac{P-4}{3}$
- (d)  $f^{-1}(P) = 4(P + 3)$
- (e)  $f^{-1}(P) = \frac{P+3}{4}$

Answer: (b).

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.050

CC KC MA121 F07: 14/**86**/0/0/0 time 3:00

CC KC MA121A F11: 0/**96**/0/0/4

CC KC MA121D F11: 17/**60**/0/6/17 "Individual, pre-discussion vote"

CC KC MA121D F11: 0/83/0/0/17 Post small-group discussion vote

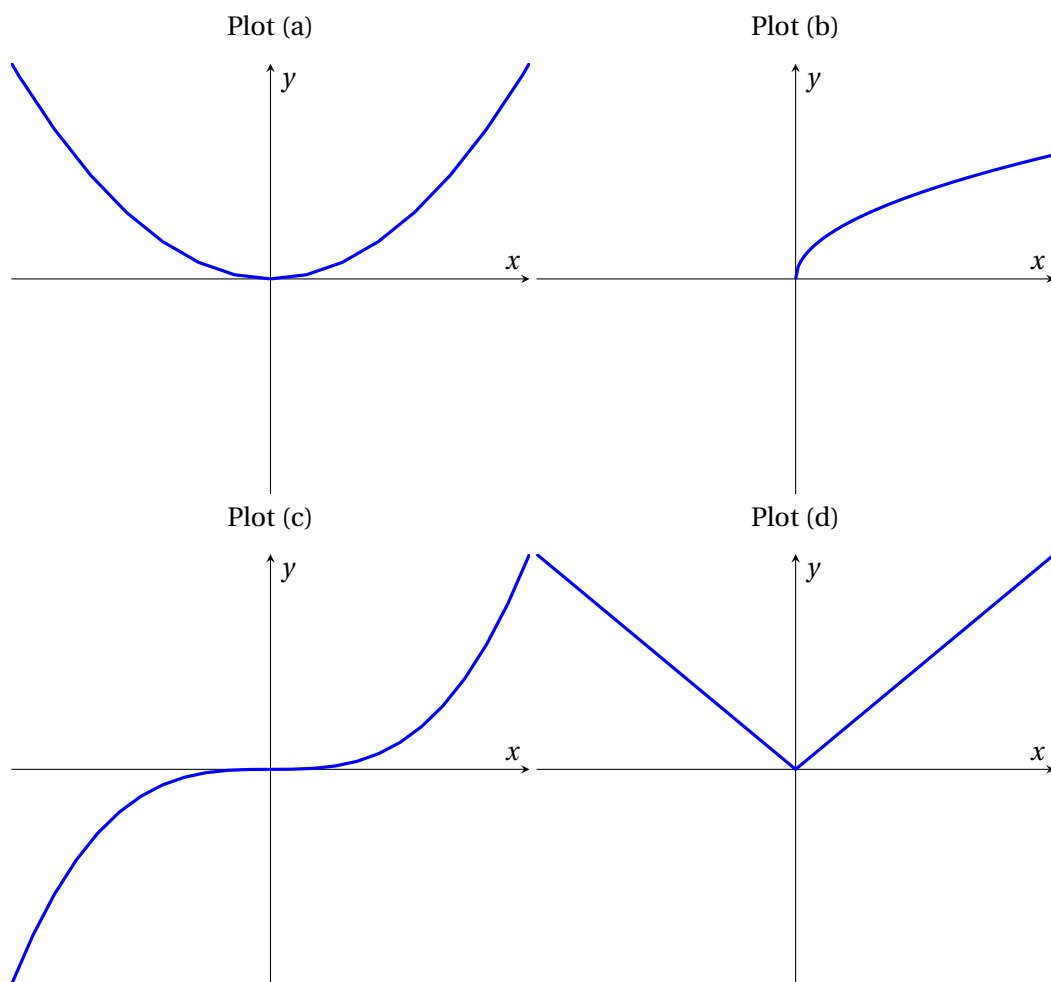
HC AS MA120 S12: 11.76/70.59/11.76/0/5.88/0

AS DH MA1561 010 F11: 2/95/2/0/0 time 2:30

AS DH MA1561 030 F11: 12/69/9/0/9 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/100/0/0/0 time 1:00 Post small-group discussion vote

0.3.6 Which of these functions has an inverse?



- (a) (a) only
- (b) (b) only
- (c) (c) only
- (d) (d) only
- (e) (a) and (b)

### 0.3. TRANSFORMATIONS OF FUNCTIONS

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(f) (b) and (c)

*Answer: (f).* Both (b) and (c) are one-to-one and therefore both have inverses.

by Project MathVote

SVC.01.03.051

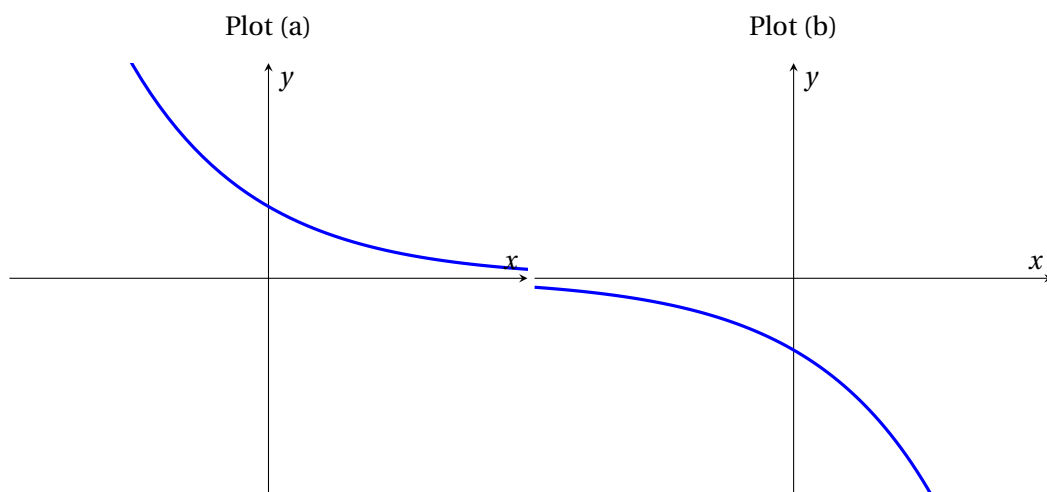
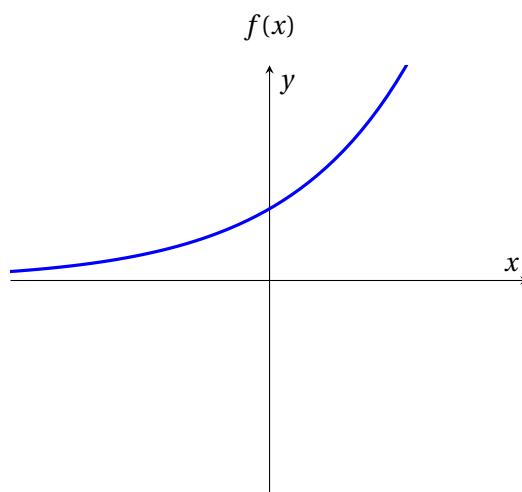
AU CS MA141 F11: 0/11/6/0/11/**72**

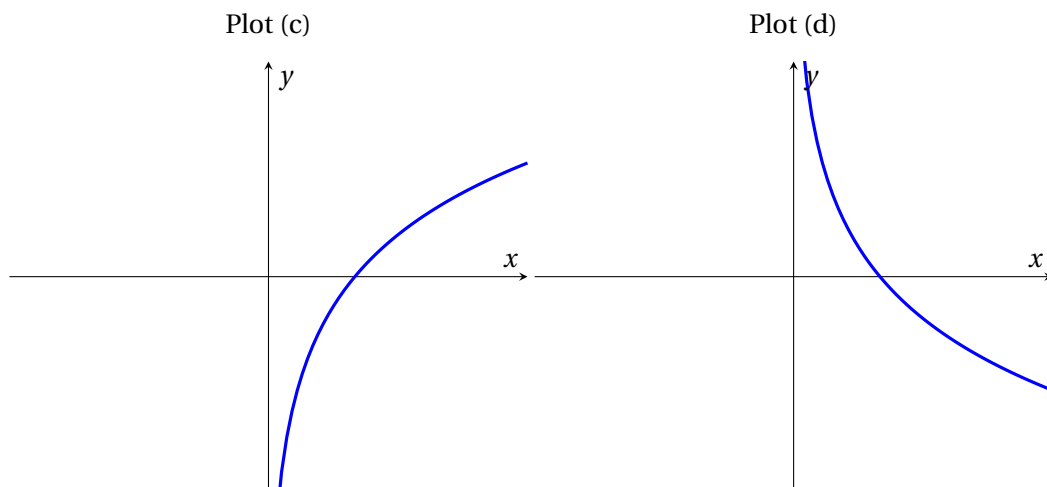
AS DH MA1561 010 F11: 0/8/0/0/17/**75** time 3:00

AS DH MA1561 030 F11: 16/19/6/3/6/**48** time 2:10 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/3/0/0/12/**85** time 1:50 Post small-group discussion vote

0.3.7 The following is a graph of  $f(x)$ . Which graph below is the inverse?





*Answer: (c).* This is a reflection of the original graph through the line  $y = x$ .

by Project MathVote

SVC.01.03.052

CC KC MA121A F11: 4/4/**92**/0

CC KC MA121D F11: 17/44/**39**/0 "Individual, pre-discussion vote"

CC KC MA121D F11: 0/28/**72**/0 Post small-group discussion vote

HC AS MA120 S12: 9.09/9.09/**81.82**/0/0/0

AU CS MA141 F11: 17/11/**67**/6

AS DH MA1561 010 F11: 8/5/**84**/3 time 2:00

AS DH MA1561 030 F11: 12/12/**70**/6 time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/4/**93**/4 time 0:40 Post small-group discussion vote

0.3.8 Given that  $f(x) = \sqrt[5]{\frac{x^3 - 72}{800}}$ , find  $f \circ f^{-1}(437)$ .

(a) 104,316.73

(b) 1671.2

(c) 437

(d) 10.08

*Answer: (c)* 437. This question makes use of the fact that  $f \circ f^{-1}(x) = x$ .

by Project MathVote

SVC.01.03.053

CC KC MA121A F11: 0/4/**79**/17

CC KC MA121D F11: 22/28/**17**/33 "Individual, pre-discussion vote"

### 0.3. TRANSFORMATIONS OF FUNCTIONS

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CC KC MA121D F11: 6/28/**28**/39 Post small-group discussion vote

AS DH MA1561 010 F11: 0/0/**97**/3 time 2:00

AS DH MA1561 030 F11: 4/8/**85**/4 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**100**/0 time 0:40 Post small-group discussion vote

0.3.9 If  $f(x) = \frac{x}{x^2 + 1}$ , what is  $f^{-1} \circ f(-2)$ ?

- (a)  $-\frac{2}{5}$
- (b)  $\frac{2}{3}$
- (c)  $-\frac{5}{2}$
- (d)  $-2$

*Answer:* (d)  $-2$ . This question makes use of the fact that  $f^{-1} \circ f(x) = x$ .

by Project MathVote

SVC.01.03.054

0.3.10 If  $(4, -2)$  is a point on the graph of  $y = f(x)$ , which of the following points is on the graph of  $y = f^{-1}(x)$ ?

- (a)  $(-2, 4)$
- (b)  $(-4, 2)$
- (c)  $(\frac{1}{4}, -\frac{1}{2})$
- (d)  $(-\frac{1}{4}, \frac{1}{2})$

*Answer:* (a)  $(-2, 4)$ .

by Project MathVote

SVC.01.03.055

HC AS MA120 S12: **55.56**/33.33/5.56/5.56

AU CS MA141 F11: **95**/5/0/0

AS DH MA1561 010 F11: **95**/5/0/0 time 0:50

AS DH MA1561 030 F11: **85**/9/0/6 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **96**/0/0/4 time 1:00 Post small-group discussion vote

0.3.11 Find the inverse of  $f(x) = \frac{1}{x}$ .

- (a)  $f^{-1}(x) = \frac{x}{1}$

(b)  $f^{-1}(x) = x$

(c)  $f^{-1}(x) = \frac{1}{x}$

(d)  $f^{-1}(x) = xy$

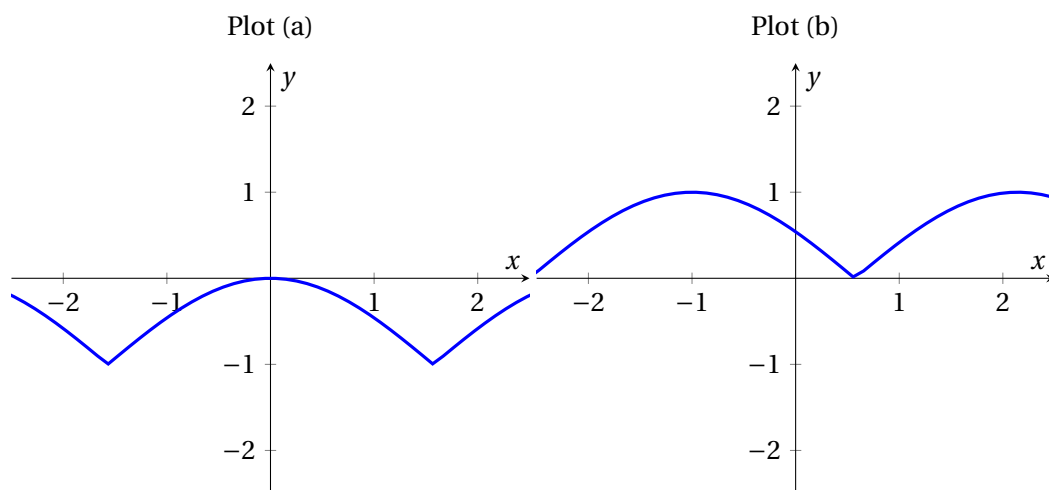
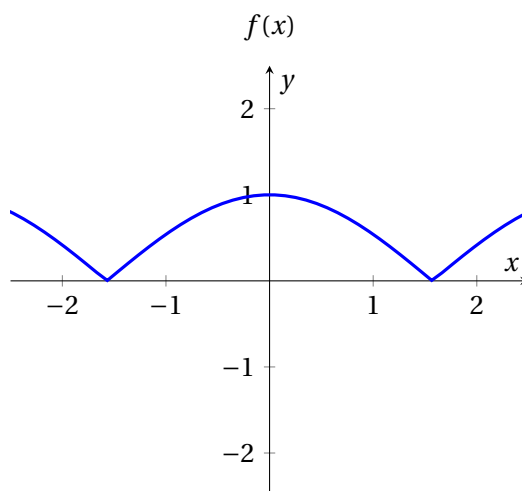
*Answer:* (c)  $f^{-1}(x) = \frac{1}{x}$ . The reciprocal function is its own inverse.

by Project MathVote

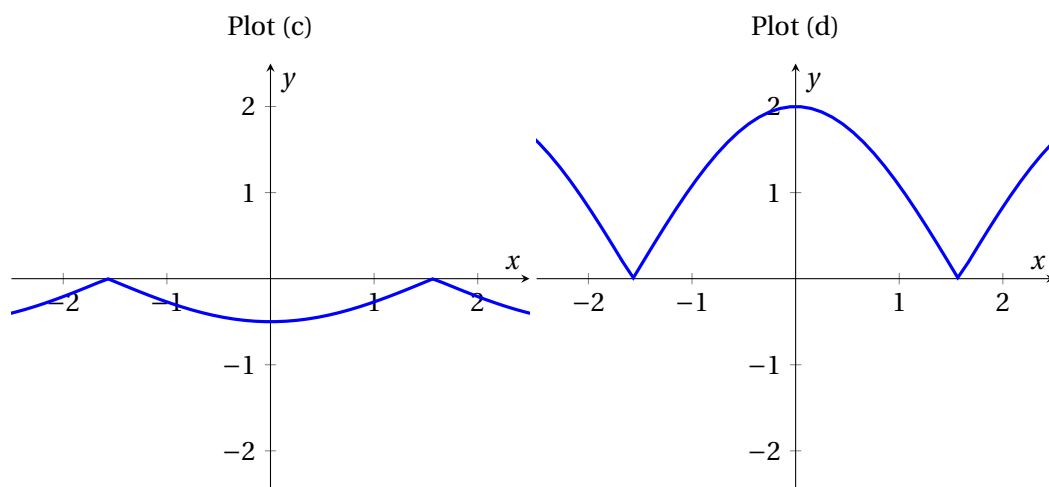
SVC.01.03.056

AU CS MA141 F11: 0/0/100/0

0.3.12 A function is given in Figure 1.10 below. Which one of the other graphs could be a graph of  $f(x+h)$ ?



### 0.3. TRANSFORMATIONS OF FUNCTIONS



*Answer: (b).* II. This graph appears shifted to the left by one unit and thus could be  $f(x + 1)$ . A good follow-up question is to ask what transformation could produce the other 3 graphs. Graph I is produced by  $f(x) - 1$ , graph III is produced by  $-\frac{1}{2}f(x)$ , and graph IV is produced by  $2f(x)$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.03.060

CC KC MA121 F06: 15/65/0/20

CC KC MA121 F07: 3/86/0/10 time 2:45

CC LV MA121A F08: 14/77/9/0 time 1:20

CC LV MA121B F08: 25/50/16/8 time 1:45

CC HZ MA121 F09: 14/86/0/0

CC KC MA121A F11: 0/100/0/0

AS DH MA1561 010 F11: 8/85/0/8 time 2:00

AS DH MA1561 030 F11: 0/97/0/3 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/100/0/0 time 1:00 Post small-group discussion vote

CC HZ MA121 F12: 10/90/0/0 time 3:30

0.3.13 How is the graph of  $y = 2^{x-1} + 3$  obtained from the graph of  $y = 2^x$ ?

- (a) Move 1 down and 3 right
- (b) Move 1 left and 3 up
- (c) Move 1 up and 3 right
- (d) Move 1 right and 3 up

*Answer: (d) Move 1 right and 3 up.*

by Project MathVote

SVC.01.03.065

AS DH MA1561 010 F11: 2/32/0/**65** time 2:30

AS DH MA1561 030 F11: 3/22/0/**75** time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/36/0/**64** time 1:00 Post small-group discussion vote

CC HZ MA121 F12: 0/16/0/**84** time 2:00

0.3.14 The function  $f(x)$  goes through the point A with coordinates (2,3).  $g(x) = 2f(\frac{1}{3}x - 2) + 4$ . What are the coordinates of point A in the function  $g(x)$ ?

- (a) (4, 10)
- (b)  $(4, -\frac{5}{2})$
- (c) (12, 10)
- (d)  $(-\frac{4}{3}, 10)$
- (e)  $(-\frac{4}{3}, -\frac{5}{2})$

*Answer:* (c). We know that  $f(2) = 3$ . We take the  $y$  coordinate and multiply it by 2 before adding 4, to shift  $y = 3$  to  $y = 10$ . We know that point A is where the input to the function  $f$  is 2, so  $2 = \frac{1}{3}x - 2$ , and thus  $x = 12$ .

by Carroll College MathQuest

SVC.01.03.070

CC KC MA121D F11: **25**/17/17/25/17 Review

0.3.15 The point (4, 1) is on the graph of a function  $f$ . Find the corresponding point on the graph of  $y = f(x - 2)$ .

- (a) (6, 1)
- (b) (2, 1)
- (c) (4, 3)
- (d) (4, -1)

*Answer:* (a).

by David A. Huckaby

SVC.01.03.071

AS DH 1561 010 F11: **68**/9/15/9 time 2:00 "Individual, pre-discussion vote"

AS DH 1561 010 F11: **94**/0/0/6 time 1:00 Post small-group discussion vote

AS DH 1561 030 F11: **41**/41/9/9 time 4:00





### 0.3. TRANSFORMATIONS OF FUNCTIONS

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0.3.16 The point  $(6, 1)$  is on the graph of a function  $f$ . Find the corresponding point on the graph of  $y = f(2x)$ .

- (a)  $(12, 1)$
- (b)  $(3, 1)$
- (c)  $(6, 2)$
- (d)  $(6, \frac{1}{2})$

*Answer: (b).*

by David A. Huckaby

SVC.01.03.072

AS DH 1561 010 F11: 13/**87**/0/0 time 3:00

AS DH 1561 030 F11: 44/**50**/6/0 time 1:30 "Individual, pre-discussion vote"

AS DH 1561 030 F11: 9/**88**/3/0 time 2:00 Post small-group discussion vote

0.3.17 Given the graph of a function  $f(x)$ , what sequence of activities best describes the process you might go through to graph  $g(x) = 5f(-x)$ ?

- (a) Expand the graph by a factor of 5, then reflect it across the  $y$ -axis.
- (b) Expand the graph by a factor of 5, then reflect it across the  $x$ -axis.
- (c) Reflect the graph across the  $y$ -axis, then expand it by a factor of 5.
- (d) Reflect the graph across the  $x$ -axis, then expand it by a factor of 5.
- (e) More than 1 of the above.
- (f) None of the above.

*Answer: (e).* Both (a) and (c) are correct. This question is intended to help students think about how operations on function can change the graph; in this case the operations being scaling and reflection. In this case, they also have to think about whether it makes a difference if you expand or reflect first. You can expand and reflect in any order without changing the result. This should be contrasted with the next question, where it does make a difference which operation is done first.

by Project MathVote

SVC.01.03.075

HC AS MA120 S12: 0/10.53/31.58/**21.05**/36.84/0

AS DH MA1561 010 F11: 13/8/18/36/**26**/0 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 3/0/11/16/**71**/0 time 2:00 Post small-group discussion vote

AS DH MA1561 030 F11: 24/3/32/11/**30**/0 time 3:30

0.3.18 Given the graph of a function  $f(x)$ , what sequence of activities best describes the process you might go through to graph  $g(x) = -f(x) + 2$ ?

- (a) Move the graph up 2 units, then reflect it across the  $x$ -axis.
- (b) Move the graph up 2 units, then reflect it across the  $y$ -axis.
- (c) Reflect the graph across the  $y$ -axis, then move it up by 2 units.
- (d) Reflect the graph across the  $x$ -axis, then move it up 2 units.
- (e) More than 1 of the above.
- (f) None of the above.

*Answer: (d).* This question is intended to help students think about how operations on function can change the graph; in this case the operations being translation and reflection. In this case, the order in which the actions are taken is important. Students might be encouraged to graph the results of doing the operations in the incorrect order and to give the equation which correctly describes doing operations in that order.

by Project MathVote

SVC.01.03.076

HC AS MA120 S12: 5.26/5.26/5.26/**42.11**/42.11/0

AS DH MA1561 010 F11: 12/5/2/**48**/32/0 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 5/0/0/**43**/51/0 time 2:00 Post small-group discussion vote

AS DH MA1561 030 F11: 18/0/0/**65**/15/3 time 4:00

0.3.19 Take the function  $f(x)$  and "Shift the function right  $h$  units. Reflect the result across the  $y$ -axis, then reflect the result across the  $x$ -axis. Finally shift the result up  $k$  units." The end result is:

- (a)  $f(x + h) + k$
- (b)  $f(x - h) + k$
- (c)  $-f(-x - h) + k$
- (d)  $-f(-x + h) + k$

*Answer: (d).*

by Lahna VonEpps

SVC.01.03.080

CC HZ MA121 F09: 0/0/100/**0**

CC KC MA121A F11: 0/0/62/**38**

AS DH MA1561 010 F11: 0/3/87/**10** time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/100/**0** time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 4/7/64/**25** time 1:30 "Individual, pre-discussion vote"



### 0.3. TRANSFORMATIONS OF FUNCTIONS

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AS DH MA1561 030 F11: 0/0/77/**23** time 1:00 Post small-group discussion vote

0.3.20 Given  $f(x) = x + 1$  and  $g(x) = 3x^2 - 2x$ , what is the composition  $g(f(x))$ .

- (a)  $3x^2 - 2x + 1$
- (b)  $(3x^2 - 2x)(x + 1)$
- (c)  $3x^2 + 4x + 1$
- (d)  $3(x + 1)^2 - 2x$

*Answer: (c).*

by Lahna VonEpps

SVC.01.03.090

CC HZ MA131 F09: 4/0/**84**/12

CC HZ MA121 F09: 10/0/**80**/10

HC AS MA120 S12: 10.53/5.26/**52.63**/31.58

AS DH MA1561 010 F11: 0/0/**100**/0 time 3:30

AS DH MA1561 030 F11: 12/19/**62**/6 time 3:10 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**97**/3 time 2:00 Post small-group discussion vote

CC HZ MA121 F12: 26/5/**68**/0 time 4:15

0.3.21 Write  $h(x) = e^{3x/2}$  as a composition of functions:  $f(g(x))$ .  $f(x) = \underline{\hspace{2cm}}$ ,  $g(x) = \underline{\hspace{2cm}}$ .

- (a)  $e^x, 3x/2$
- (b)  $3x/2, e^x$
- (c)  $x, e^{3x/2}$
- (d)  $x/2, 3e^x$

*Answer: (a).*

by Lahna VonEpps

SVC.01.03.100

CC HZ MA131 F09: **92**/0/0/8

HC AS MA120 S12: 100/**0**/0/0/0/0

AS DH MA1561 010 F11: **72**/16/8/4 time 2:40 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: **100**/0/0/0 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: **76**/12/8/4 time 2:40 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **100**/0/0/0 time 1:10 Post small-group discussion vote

0.3.22 If  $f(x) = x^2 + 6$  and  $g(x) = x - 3$ , what is  $f \circ g(x)$ ?

- (a)  $x^2 + 3$
- (b)  $x^2 - 6x + 15$
- (c)  $x^2 - 3$
- (d)  $x^3 - 3x^2 + 6x - 18$

*Answer:* (b)  $x^2 - 6x + 15$ . Note that  $f \circ g(x) = (x - 3)^2 + 6$ .

by Project MathVote

SVC.01.03.102

0.3.23 Which of the following functions IS invertible?

- (a)  $f(x) = -x^4 + 7$
- (b)  $g(x) = e^{3x/2}$
- (c)  $h(x) = \cos(x)$
- (d)  $k(x) = |x|$

*Answer:* (b).

by Lahna VonEpps

SVC.01.03.110

0.3.24 Let  $f(x) = x - 2$  and  $g(x) = 3 - x^2$ . Find  $g(f(2))$ .

- (a) -3
- (b) 0
- (c) 3
- (d) 2

*Answer:* (c). This function checks student understanding of function composition.

by Project MathVote

SVC.01.03.120

STE.079 HC AS MA120 S12: 4.17/0/95.83/0

0.3.25 If  $P = f(t) = 3 + 4t$ , find  $f^{-1}(7)$ .



### 0.3. TRANSFORMATIONS OF FUNCTIONS

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- (a) 31
- (b)  $\frac{1}{7}$
- (c) 0
- (d) 1

*Answer: (d).*

by Project MathVote

SVC.01.03.130

STE.080 HC AS MA120 S12: 5.88/5.88/0/**88.24**

0.3.26 Let  $f(x) = x^2$  and  $g(x) = x + 2$ . True or false? The domain of the function  $\frac{f}{g}$  is  $\mathbb{R}$ , all real numbers.

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: False.*  $\left(\frac{f}{g}\right)(-2)$  is undefined.

by David A. Huckaby

SVC.01.03.140

AS DH 1561 010 F11: 0/0/0/**100** time 1:40

AS DH 1561 030 F11: 3/6/26/**65** time 1:30 "Individual, pre-discussion vote"

AS DH 1561 030 F11: 3/3/3/**90** time 1:00 Post small-group discussion vote

0.3.27 Let  $f(x) = x^2 - 4$  and  $g(x) = \sqrt{x}$ . Find  $(g \circ f)(x)$  and the domain of  $g \circ f$ .

- (a)  $\sqrt{x^2 - 4}$ ; Domain:  $(-\infty, -2] \cup [2, \infty)$
- (b)  $x - 4$ ; Domain:  $\mathbb{R}$
- (c)  $x - 4$ ; Domain:  $[0, \infty)$
- (d)  $\sqrt{x^2 - 4}$ ; Domain:  $[0, \infty)$
- (e)  $\sqrt{x}(x^2 - 4)$ ; Domain:  $[0, \infty)$

*Answer: (a).*

by David A. Huckaby

SVC.01.03.150

AS DH 1561 010 F11: **97**/3/0/0/0 time PRE.00.40.025

AS DH 1561 030 F11: **58**/15/6/18/3 time 3:10 "Individual, pre-discussion vote"

AS DH 1561 030 F11: **87**/6/0/6/0 time 2:00 Post small-group discussion vote

## 0.4 Logarithmic Functions

**Preview Activity 0.4.** Carbon-14 ( $^{14}\text{C}$ ) is a radioactive isotope of carbon that occurs naturally in the Earth's atmosphere. During photosynthesis, plants take in  $^{14}\text{C}$  along with other carbon isotopes, and the levels of  $^{14}\text{C}$  in living plants are roughly the same as atmospheric levels. Once a plant dies, it no longer takes in any additional  $^{14}\text{C}$ . Since  $^{14}\text{C}$  in the dead plant decays at a predictable rate (the half-life of  $^{14}\text{C}$  is approximately 5,730 years), we can measure  $^{14}\text{C}$  levels in dead plant matter to get an estimate on how long ago the plant died. Suppose that a plant has 0.02 milligrams of  $^{14}\text{C}$  when it dies.

- (a) Write a function that represents the amount of  $^{14}\text{C}$  remaining in the plant after  $t$  years.
- (b) Complete the table for the amount of  $^{14}\text{C}$  remaining  $t$  years after the death of the plant.

$t$	0	1	5	10	100	1000	2000	5730
$^{14}\text{C}$ Level	0.02							

- (c) Suppose our plant died sometime in the past. If we find that there are 0.014 milligrams of  $^{14}\text{C}$  present in the plant now, estimate the age of the plant to within 50 years.



**Activity 0.12.**

Use the definition of a logarithm along with the properties of logarithms to answer the following.

- (a) Write the exponential expression  $8^{1/3} = 2$  as a logarithmic expression.
- (b) Write the logarithmic expression  $\log_2 \frac{1}{32} = -5$  as an exponential expression.
- (c) What value of  $x$  solves the equation  $\log_2 x = 3$ ?
- (d) What value of  $x$  solves the equation  $\log_2 4 = x$ ?
- (e) Use the laws of logarithms to rewrite the expression  $\log(x^3 y^5)$  in a form with no logarithms of products, quotients, or powers.
- (f) Use the laws of logarithms to rewrite the expression  $\log\left(\frac{x^{15} y^{20}}{z^4}\right)$  in a form with no logarithms of products, quotients, or powers.
- (g) Rewrite the expression  $\ln(8) + 5\ln(x) + 15\ln(x^2 + 8)$  as a single logarithm.

◁



**Activity 0.13.**

Solve each of the following equations for  $t$ , and verify your answers using a calculator.

(a)  $\ln t = 4$

(b)  $\ln(t + 3) = 4$

(c)  $\ln(t + 3) = \ln 4$

(d)  $\ln(t + 3) + \ln(t) = \ln 4$

(e)  $e^t = 4$

(f)  $e^{t+3} = 4$

(g)  $2e^{t+3} = 4$

(h)  $2e^{3t+2} = 3e^{t-1}$

**Activity 0.14.**

Consider the following equation:

$$7^x = 24$$

- (a) How many solutions should we expect to find for this equation?
- (b) Solve the equation using the log base 7.
- (c) Solve the equation using the log base 10.
- (d) Solve the equation using the natural log.
- (e) Most calculators have buttons for  $\log_{10}$  and  $\ln$ , but none have a button for  $\log_7$ . Use your previous answers to write a formula for  $\log_7 x$  in terms of  $\log x$  or  $\ln x$ .



**Activity 0.15.**

- (a) In the presence of sufficient resources the population of a colony of bacteria exhibits exponential growth, doubling once every three hours. What is the corresponding continuous (percentage) growth rate?
- (b) A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling so its temperature,  $T$  (measured in degrees Fahrenheit) after  $t$  minutes is given by

$$T(t) = 65 + 186e^{-0.06t}.$$

How long will it take from the time the food is served until the temperature is  $120^\circ\text{F}$ ?

- (c) The velocity (in ft/sec) of a sky diver  $t$  seconds after jumping is given by

$$v(t) = 80(1 - e^{-0.2t}).$$

After how many seconds is the velocity 75 ft/sec?



## Voting Questions

0.4.1 A logarithmic function of the form  $f(x) = \log_a x$  will always pass through the point  $(1, 0)$ .

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer: (a) True.* Students should be able to substitute the point  $(0, 1)$  to see that this would mean  $0 = \log_a 1$ , which is equivalent to  $a^0 = 1$ .

by Project MathVote

SVC.01.04.005

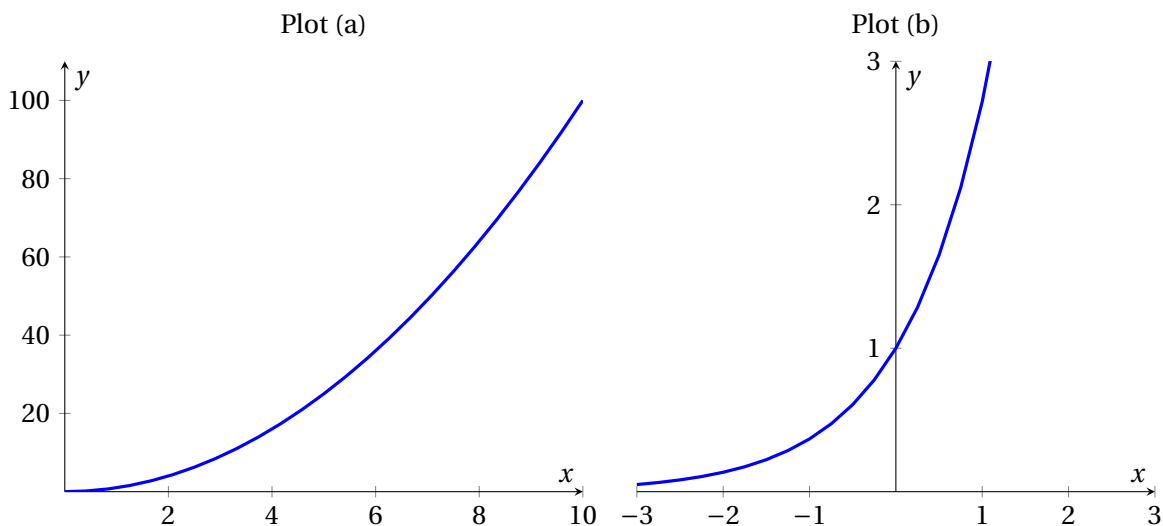
CC KC MA121A F11: **91**/9/0/0

CC KC MA121D F11: **45**/33/22/0 "Individual, pre-discussion vote"

CC KC MA121D F11: **66**/11/17/6 Post small-group discussion vote

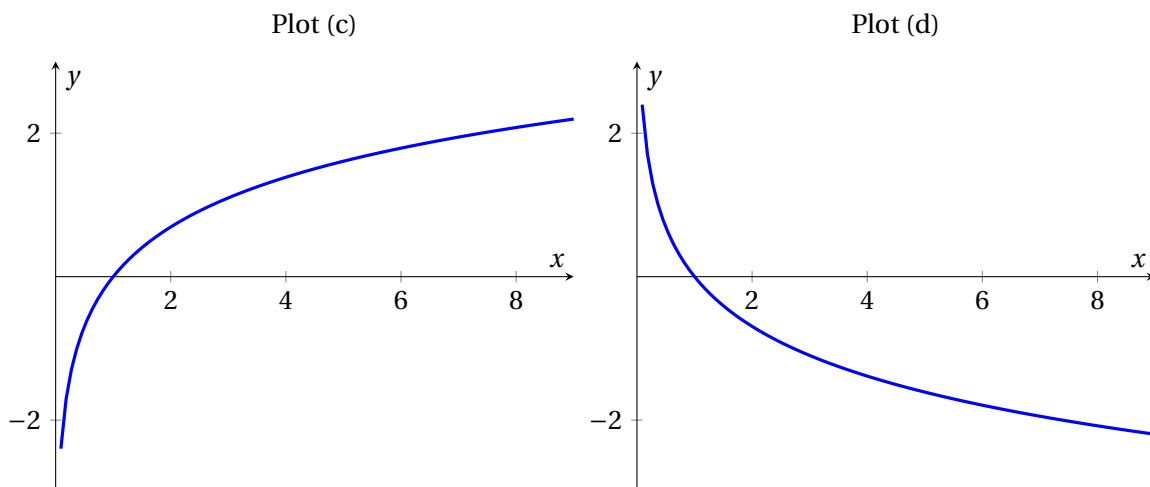
HC AS MA120 S12: 27.78/11.11/**33.33**/27.78/0/0

0.4.2 Which is a graph of  $y = \ln x$ ?



#### 0.4. LOGARITHMIC FUNCTIONS

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*Answer: (c).* It is an increasing function that passes through the point (1,0).

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.010

CC KC MA121 F06: 11/0/**89**/0

CC KC MA121 F07: 0/0/**100**/0 time 2:00

CC LV MA121A F08: 3/11/**85**/0 time 1:20

CC LV MA121B F08: 0/27/**72**/0 time 0:45

CC HZ MA121 F09: 9/13/**78**/0

HC AS MA120 S12: 16.67/16.67/**61.11**/5.56/0/0

AS DH MA1561 010 F11: 0/3/**97**/0 time 1:00

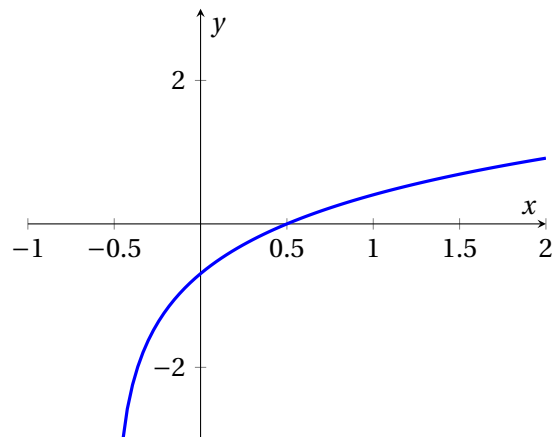
AS DH MA1561 030 F11: 0/9/**91**/0 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**100**/0 time 0:30 Post small-group discussion vote

CC HZ MA121 F12: 25/35/**40**/0 time 2:00

0.4.3 The graph below could be that of

- (a)  $y = \ln x + \frac{1}{2}$
- (b)  $y = \ln x - \frac{1}{2}$
- (c)  $y = \ln\left(x + \frac{1}{2}\right)$
- (d)  $y = \ln\left(x - \frac{1}{2}\right)$



*Answer: (c).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.020

CC KC MA121 F06: 11/11/75/4

CC KC MA121 F07: 3/0/97/0 time 1:40

AS DH MA1561 010 F11: 8/12/40/40 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 7/3/77/13 time 1:10 Post small-group discussion vote

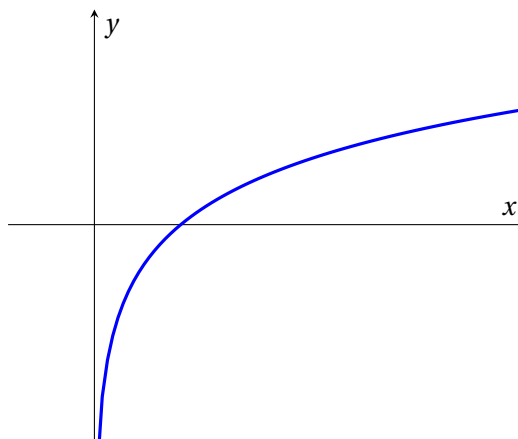
AS DH MA1561 030 F11: 4/4/46/46 time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/73/27 time 1:00 Post small-group discussion vote

CC HZ MA121 F12: 4/9/82/4 time 1:15

0.4.4 Which equation matches this graph?

- (a)  $y = b^x$  with  $b > 1$
- (b)  $y = b^x$  with  $0 < b < 1$
- (c)  $y = \log_b x$  with  $b > 1$
- (d)  $y = \log_b x$  with  $0 < b < 1$



*Answer: (c).*

by Project MathVote

SVC.01.04.024

HC AS MA120 S12: 0/11.76/76.47/11.76/0/0

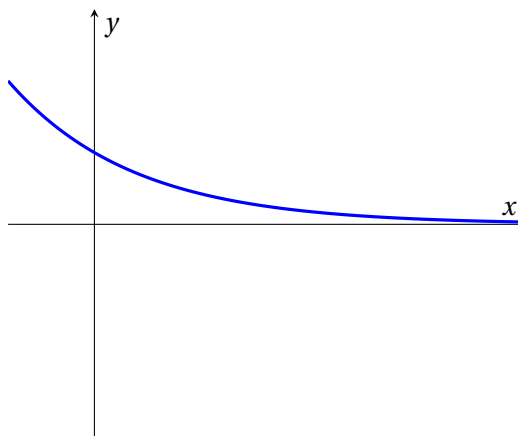
0.4.5 Which equation matches this graph?



#### 0.4. LOGARITHMIC FUNCTIONS

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- (a)  $y = b^x$  with  $b > 1$
- (b)  $y = b^x$  with  $0 < b < 1$
- (c)  $y = \log_b x$  with  $b > 1$
- (d)  $y = \log_b x$  with  $0 < b < 1$

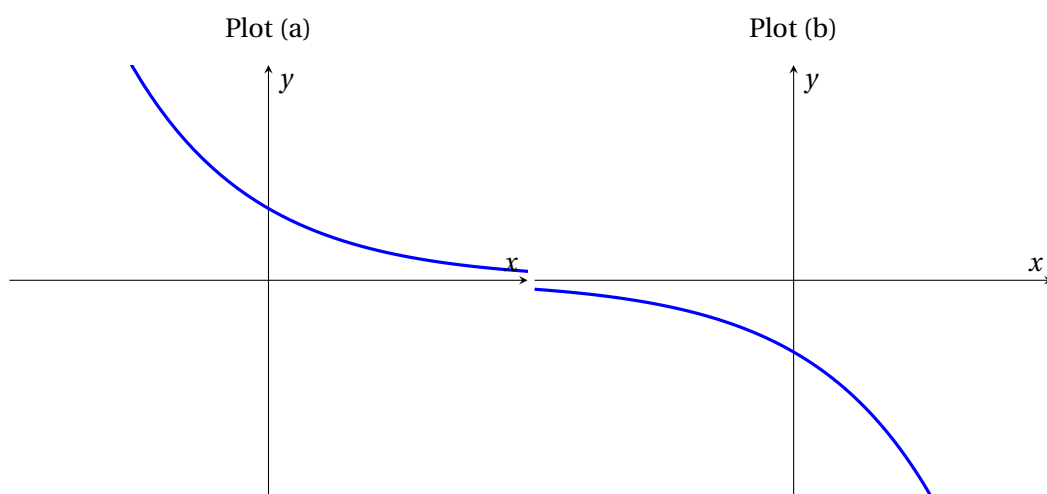


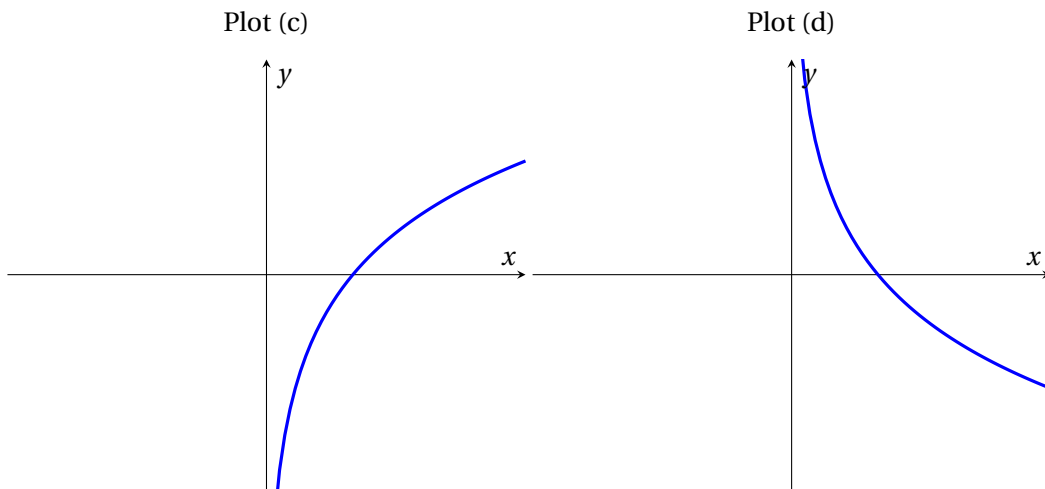
*Answer: (b).*

by Project MathVote

SVC.01.04.025

0.4.6 Which of the following is a graph of  $y = \log_2 x$ ?



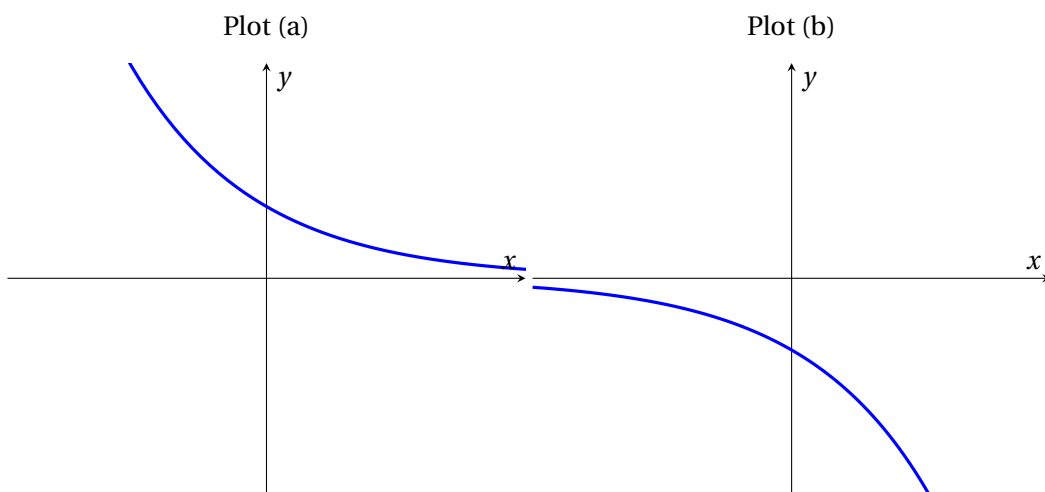


*Answer: (c).* This is a graph of a logarithmic function with base  $> 1$ .

by Project MathVote

SVC.01.04.026

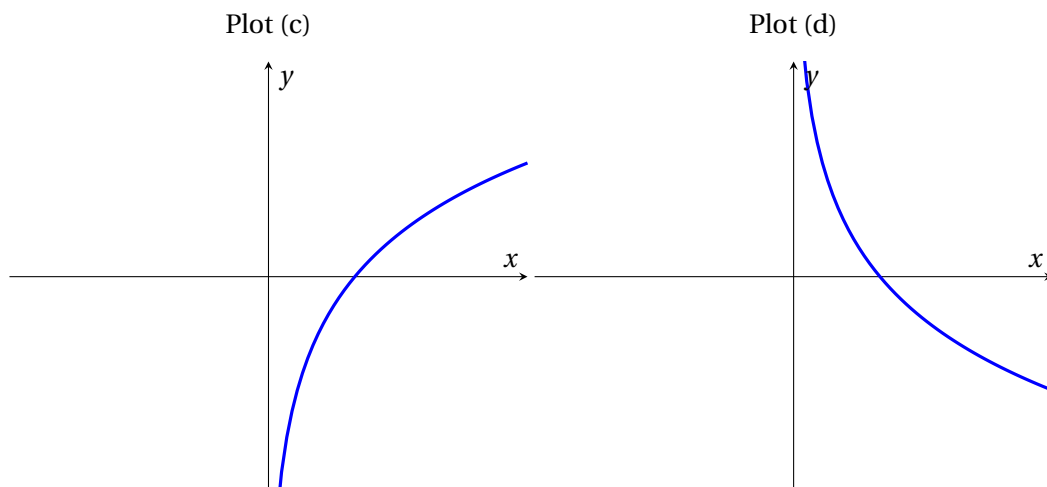
0.4.7 Which of the following is a graph of  $y = \log_{\frac{1}{2}} x$ ?





## 0.4. LOGARITHMIC FUNCTIONS

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*Answer: (d).* This is a graph of a logarithmic function with  $0 < \text{base} < 1$ .

by Project MathVote

SVC.01.04.027

HC AS MA120 S12: 5.56/**16.67**/22.22/55.56/0/0

0.4.8 Which of the following functions have vertical asymptotes of  $x = 3$ ?

- (a)  $y = \ln(x/3)$
- (b)  $y = \ln(x - 3)$
- (c)  $y = \ln(x + 3)$
- (d)  $y = 3 \ln x$

*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.030

CC KC MA121 F06: 0/**71**/7/21

CC KC MA121 F07: 3/**97**/0/0 time 1:30

CC LV MA121A F08: 9/**63**/18/9 time 1:45

CC HZ MA121 F09: 26/**65**/9/0 Review

CC KC MA121A F11: 0/**96**/0/4

CC KC MA121D F11: 11/**61**/11/17 "Individual, pre-discussion vote"

CC KC MA121D F11: 0/**94**/0/6 Post small-group discussion vote

AS DH MA1561 010 F11: 11/**83**/0/6 time 2:00

AS DH MA1561 030 F11: 3/**97**/0/0 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**100**/0/0 time 1:00 Post small-group discussion vote

0.4.9  $\log\left(\frac{M-N}{M+N}\right) =$

- (a)  $2\log M$
- (b)  $2\log N$
- (c)  $-2\log N$
- (d)  $\log(M - N) - \log(M + N)$

*Answer: (d).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.040

CC KC MA121 F06: 0/0/0/**100**

CC KC MA121 F07: 0/7/0/**93**

CC LV MA121A F08: 0/0/0/**100** time 1:00

CC LV MA121B F08: 0/0/9/**91** time 0:50

CC HZ MA121 F09: 0/0/0/**100**

AS DH MA1561 010 F11: 0/0/8/**92** time 3:00

AS DH MA1561 030 F11: 0/0/0/**100** time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/0/**100** time 1:00 Post small-group discussion vote

AS DH MA1302 070 S12: 5/0/0/**95** time 1:50

CC HZ MA121 F12: 0/0/10/**90** time 1:30

0.4.10 If  $\log_{10}(x - a) = n$ , then  $x =$

- (a)  $10^{a+n}$
- (b)  $a + 10^n$
- (c)  $n + 10^a$
- (d)  $n + a^{10}$

*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.050

CC KC MA121 F06: 7/**86**/4/4

CC KC MA121 F07: 10/**90**/0/0 time 3:00

CC LV MA121A F08: 22/**66**/7/4 time 1:30

CC LV MA121B F08: 0/**100**/0/0 time 1:00



#### 0.4. LOGARITHMIC FUNCTIONS

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CC HZ MA121 F09: 9/**83**/4/4

AS DH MA1561 010 F11: 3/**97**/0/0 time 2:00

AS DH MA1561 030 F11: 0/**100**/0/0 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**100**/0/0 time 0:30 Post small-group discussion vote

0.4.11 What is the exponential form of  $\log_r m = j$ ?

(a)  $r^j = m$

(b)  $j^r = m$

(c)  $m^j = r$

(d)  $r^m = j$

*Answer: (a)  $r^j = m$ .*

by Project MathVote

SVC.01.04.051

HC AS MA120 S12: 58.82/**29.41**/5.88/5.88/0/0

AS DH MA1561 010 F11: **100**/0/0/0 time 1:30

AS DH MA1561 030 F11: **97**/0/3/0 time 1:10 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **100**/0/0/0 time 0:30 Post small-group discussion vote

AS DH MA1302 070 S12: **85**/5/0/10 time 1:30

0.4.12 What is the logarithmic form of  $k^p = d$ ?

(a)  $\log_d k = p$

(b)  $\log_k d = p$

(c)  $\log_p d = p$

(d)  $\log_k p = d$

*Answer: (b)  $\log_k d = p$ .*

by Project MathVote

SVC.01.04.052

HC AS MA120 S12: 0/88.24/**5.88**/5.88/0/0

AS DH MA1302 070 S12: 0/**100**/0/0 time 1:00

0.4.13 What is the value of  $\log_{11} 86$ ? (Calculators are allowed.)

- (a) .4049
- (b) .5383
- (c) 1.8576
- (d)  $-2.0564$

*Answer: (c) 1.8576.* Students can use the base-changing formula to calculate  $\frac{\ln(86)}{\ln(11)}$ .

by Project MathVote

SVC.01.04.053

0.4.14 What is  $3 = \log_2 8$  in exponential form?

- (a)  $2^8 = 3$
- (b)  $3^2 = 8$
- (c)  $8^3 = 2$
- (d)  $2^3 = 8$

*Answer: (d)  $2^3 = 8$ .*

by Project MathVote

SVC.01.04.054

AS DH MA1302 070 S12: 0/6/0/94 time 2:00

0.4.15 What is  $k = \log_m q$  in exponential form?

- (a)  $m^k = q$
- (b)  $k^q = m$
- (c)  $m^q = k$
- (d)  $q^m = k$

*Answer: (a)  $m^k = q$ .*

by Project MathVote

SVC.01.04.055

0.4.16 What is  $4^2 = 16$  in logarithmic form?

- (a)  $\log_2 4 = 16$



#### 0.4. LOGARITHMIC FUNCTIONS

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(b)  $\log_4 16 = 2$

(c)  $\log_4 2 = 16$

(d)  $\log_{16} 4 = 2$

*Answer:* (b)  $\log_4 16 = 2$ .

by Project MathVote

SVC.01.04.056

HC AS MA120 S12: 0/94.12/5.88/**0**/0/0

AS DH MA1561 010 F11: 0/**100**/0/0 time 1:00

AS DH MA1561 030 F11: 3/**91**/3/3 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**97**/3/0 time 0:30 Post small-group discussion vote

AS DH MA1302 070 S12: 0/**94**/6/0 time 1:40

CC HZ MA121 F12: 0/**91**/9/0 time 1:00

0.4.17 What is  $3^{-1} = \frac{1}{3}$  in logarithmic form?

(a)  $\log_3(-1) = \frac{1}{3}$

(b)  $\log_{-1} \frac{1}{3} = 3$

(c)  $\log_{\frac{1}{3}} 3 = -1$

(d)  $\log_3 \frac{1}{3} = -1$

*Answer:* (d)  $\log_3 \frac{1}{3} = -1$ .

by Project MathVote

SVC.01.04.057

0.4.18 What is the inverse of the following function:

$$P = f(t) = 16 \ln(14t)$$

(a)  $f^{-1}(P) = \frac{1}{14} e^{16P}$

(b)  $f^{-1}(P) = \frac{1}{14} e^{P/16}$

(c)  $f^{-1}(P) = \frac{1}{14} \ln(P/16)$

(d)  $f^{-1}(P) = \frac{\ln 16}{14} P$

*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.060

CC KC MA121 F06: 20/**65**/10/5

CC KC MA121 F07: 10/**90**/0/0 time 4:00

CC KC MA121A F11: 4/**96**/0/0

AS DH MA1561 010 F11: 23/**70**/7/0 time 2:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 4/**96**/0/0 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: 58/**42**/0/0 time 2:40 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 33/**67**/0/0 time 2:00 Post small-group discussion vote

0.4.19 Solve for  $x$  if  $8y = 3e^x$ .

(a)  $x = \ln 8 + \ln 3 + \ln y$

(b)  $x = \ln 3 - \ln 8 + \ln y$

(c)  $x = \ln 8 + \ln y - \ln 3$

(d)  $x = \ln 3 - \ln 8 - \ln y$

*Answer: (c).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.070

CC KC MA121 F07: 0/0/**93**/7 time 3:00

CC HZ MA121 F09: 4/4/**91**/0

AS DH MA1561 010 F11: 4/0/**93**/4 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 4/0/**96**/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/21/**71**/8 time 2:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**100**/0 time 1:20 Post small-group discussion vote

0.4.20 Solve for  $x$  if  $y = e + 2^x$

(a)  $x = \frac{\ln y - 1}{\ln 2}$

(b)  $x = \frac{\ln(y-1)}{\ln 2}$

(c)  $x = \frac{\ln y}{\ln 2} - 1$

(d)  $x = \frac{\ln(y-e)}{\ln 2}$



#### 0.4. LOGARITHMIC FUNCTIONS

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*Answer: (d).* Good discussion during voting and good discussion regarding composition of function.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.04.080

CC KC MA121 F07: 54/0/0/**46** time 5:00

CC LV MA121A F08: 27/0/54/**18** time 2:10

CC KC MA121A F11: 61/0/9/**30**

CC KC MA121D F11: 17/17/6/**61** "Individual, pre-discussion vote"

CC KC MA121D F11: 39/0/0/**61** Post small-group discussion vote

AS DH MA1561 010 F11: 27/18/12/**42** time 2:50 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 21/6/0/**73** time 2:00 Post small-group discussion vote

AS DH MA1561 030 F11: 19/46/0/**35** time 4:10

0.4.21 Write the following expression using a single logarithmic function:

$$\ln(2x^3 + 1) + 5\ln(3 - x) - \ln(6x^5 + 2x + 1).$$

(a)  $\ln(-6x^5 + 2x^3 - 7x + 15)$

(b)  $\ln[(2x^3 + 1)(15 - 5x)(-6x^5 - 2x - 1)]$

(c)  $\ln\left(\frac{(2x^3 + 1)(3 - x)^5}{6x^5 + 2x + 1}\right)$

(d)  $\ln\left(\frac{(2x^3 + 1)(15 - 5x)}{6x^5 + 2x + 1}\right)$

*Answer: (c).*

by Project MathVote

SVC.01.04.085

AS DH MA1561 010 F11: 0/3/**97**/0 time 1:30

AS DH MA1561 030 F11: 0/0/**100**/0 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/0/**100**/0 time 0:30 Post small-group discussion vote

AS DH MA1302 070 S12: 0/0/**86**/14 time 3:00

CC HZ MA121 F12: 0/5/**90**/5 time 2:15

0.4.22  $\log\left(\frac{a^4 b^7}{c^5}\right) =$

(a)  $\log(a^4) + \log(b^7) + \log(c^5)$

(b)  $4\log a + 7\log b - 5\log c$

(c)  $28\log ab - 5\log c$

- (d)  $\frac{28}{5} (\log a + \log b - \log c)$   
 (e) None of the above

*Answer: (b).*

by Carroll College MathQuest

SVC.01.04.090

CC LV MA121A F08: 0/**62**/37/0 time 1:00

CC HZ MA121 F09: 0/**83**/13/0/4

CC KC MA121A F11: 0/**78**/9/4/9

HC AS MA120 S12: **0**/91.67/0/0/8.33/0

AS DH MA1561 010 F11: 3/**90**/7/0/0 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/**100**/0/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/**97**/3/0/0 time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**100**/0/0/0 time 0:50 Post small-group discussion vote

AS DH MA1302 070 S12: 0/**76**/24/0 time 2:10

0.4.23 Simplify the following expression:  $\ln \left( \frac{\sqrt{x^2 + 1}(x^3 - 4)}{(3x - 7)^2} \right)$ .

- (a)  $\frac{1}{2} \ln(x^2 + 1) + \ln(x^3 + 4) - 2 \ln(3x - 7)$   
 (b)  $\ln \left( \frac{1}{2}(x^2 + 1) \right) + \ln(x^3 + 4) - 2 \ln(3x - 7)$   
 (c)  $\ln(x^2 + 1) \ln(x^3 + 4) \ln(3x - 7)$   
 (d)  $\ln[(x^2 + 1)(x^3 + 4)(3x - 7)]$

*Answer: (a).*

by Project MathVote

SVC.01.04.095

AS DH MA1561 010 F11: **98**/0/0/2 time 4:00

AS DH MA1561 030 F11: **100**/0/0/0 time 2:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **100**/0/0/0 time 1:00 Post small-group discussion vote

AS DH MA1302 070 S12: **76**/14/5/5 time 2:40

0.4.24 25 rabbits are introduced to an island, where they quickly reproduce and the rabbit population grows according to an exponential model  $P(t) = P_0 e^{kt}$  so that the population doubles every four months. If  $t$  is in months, what is the value of the continuous growth rate  $k$ ?





#### 0.4. LOGARITHMIC FUNCTIONS

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- (a)  $k = \frac{1}{2} \ln 4$
- (b)  $k = \frac{1}{4} \ln 2$
- (c)  $k = \frac{1}{50} \ln \frac{4}{25}$
- (d)  $k = \frac{4}{25} \ln \frac{1}{50}$
- (e) None of the above

*Answer:* (b). We know that  $P(0) = 25$  and  $P(4) = 50$ .

by Carroll College MathQuest

SVC.01.04.100

CC HZ MA121 F09: 5/**95**/0/0/0

CC KC MA121A F11: 0/**100**/0/0/0

0.4.25 Simplify  $(\log_{16} 4) \left( \log_3 \frac{1}{9} \right)$ .

- (a)  $\frac{16}{3}$
- (b)  $\frac{4}{9}$
- (c) 1
- (d) -1

*Answer:* (d). Using the definition of the logarithm function, we know that  $\log_{16} 4 = x$  is equivalent to writing  $16^x = 4$ , and thus  $x = \frac{1}{2}$ . Similarly writing  $\log_3 \frac{1}{9} = y$  is equivalent to writing  $3^y = \frac{1}{9}$  and thus  $y = -2$ .

by Project MathVote

SVC.01.04.110

AS DH MA1561 010 F11: 0/3/0/**97** time 2:30

AS DH MA1561 030 F11: 13/37/3/**47** time 2:30 "Individual, pre-discussion vote"

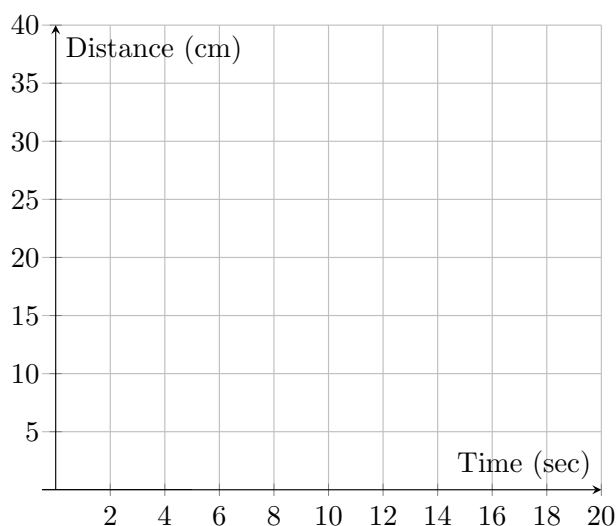
AS DH MA1561 030 F11: 0/0/0/**100** time 1:30 Post small-group discussion vote

## 0.5 Trigonometric Functions

**Preview Activity 0.5.** A tall water tower is swaying back and forth in the wind. Using an ultrasonic ranging device, we measure the distance from our device to the tower (in centimeters) every two seconds with these measurements recorded below.

Time (sec)	0	2	4	6	8	10	12	14	16	18	20
Distance (cm)	30.9	23.1	14.7	12.3	17.7	26.7	32.3	30.1	21.8	13.9	12.6

- (a) Use the coordinate plane below to create a graph of these data points.



- (b) What is the water tower's maximum distance away from the device?
- (c) What is the smallest distance measured from the tower to the device?
- (d) If the water tower was sitting still and no wind was blowing, what would be the distance from the tower to the device? We call this the tower's equilibrium position.
- (e) What is the maximum distance that the tower moves away from its equilibrium position? We call this the amplitude of the oscillations.
- (f) How much time does it take the water tower to sway back and forth in a complete cycle? We call this the period of oscillation.



## 0.5. TRIGONOMETRIC FUNCTIONS

### Activity 0.16.

In this activity we will review the trigonometry of the special angles  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ , and their multiples.

- (a) Use the fact that  $180^\circ$  is the same as  $\pi$  radians, convert each of the following angle measurements to radians.

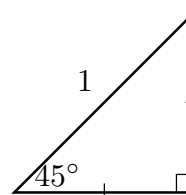
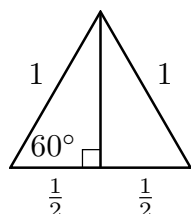
Degrees	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
Radians	0								$\pi$
Degrees	$210^\circ$	$225^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$315^\circ$	$330^\circ$	$360^\circ$	
Radians									

- (b) In part (a) of this problem there are several patterns that can help in remembering the radian conversions for certain angles. For example, you should have found that  $30^\circ$  converts to  $\frac{\pi}{6}$  radians. Therefore,  $60^\circ$  should be twice  $\frac{\pi}{6}$  which indeed it is:  $60^\circ = \frac{\pi}{3}$  radians. What other similar patterns can you find? What is the minimum number of radian measures that you need to memorize?
- (c) The sides of a 30–60–90 triangle follow well-known ratios. Consider the equilateral triangle on the left of the figure below. Fill in the rest of the sides and angles on the figure and use them to determine the trigonometric values of  $30^\circ$  and  $60^\circ$ .

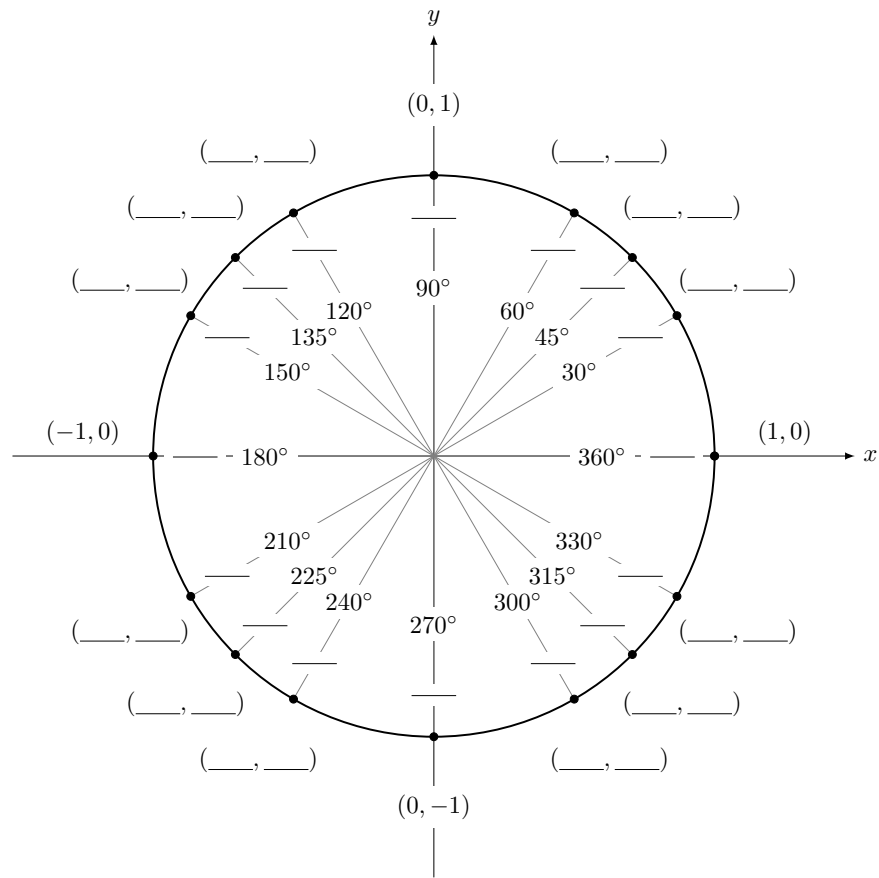
Angle (degrees)	Angle (radians)	Sine	Cosine	Tangent
$30^\circ$				
$60^\circ$				

- (d) The sides of a 45–45–90 triangle also follow well-known ratios. Consider the isosceles triangle on the right of the figure below. Fill in the rest of the sides and angles on the figure and use them to determine the trigonometric values of  $45^\circ$ .

Angle (degrees)	Angle (radians)	Sine	Cosine	Tangent
$45^\circ$				



- (e) Finally, we can organize all of the information about the special right triangles on a well-known organizational tool: the unit circle.



◁

**Activity 0.17.**

Figure 3 gives us the voltage produced by an electrical circuit as a function of time.

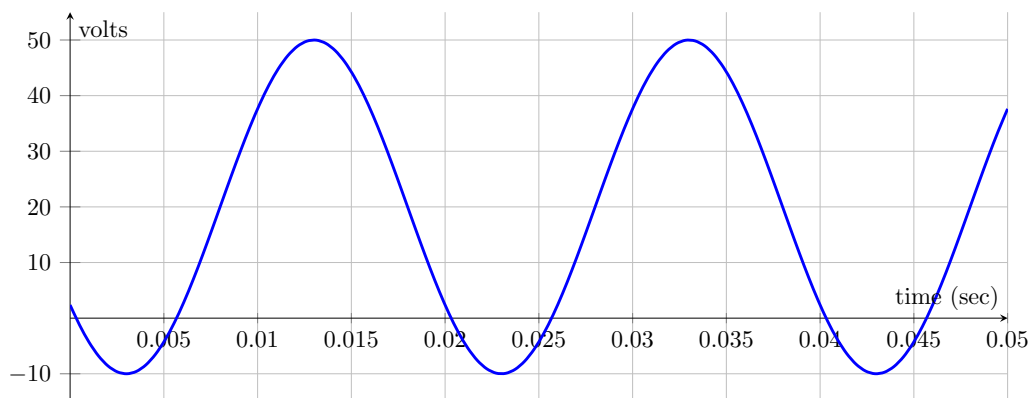


Figure 3: Voltage as a function of time.

- (a) What is the amplitude of the oscillations?
- (b) What is the period of the oscillations?
- (c) What is the average value of the voltage?
- (d) What is the shift along the  $t$  axis,  $t_0$ ?
- (e) What is a formula for this function?

**Activity 0.18.**

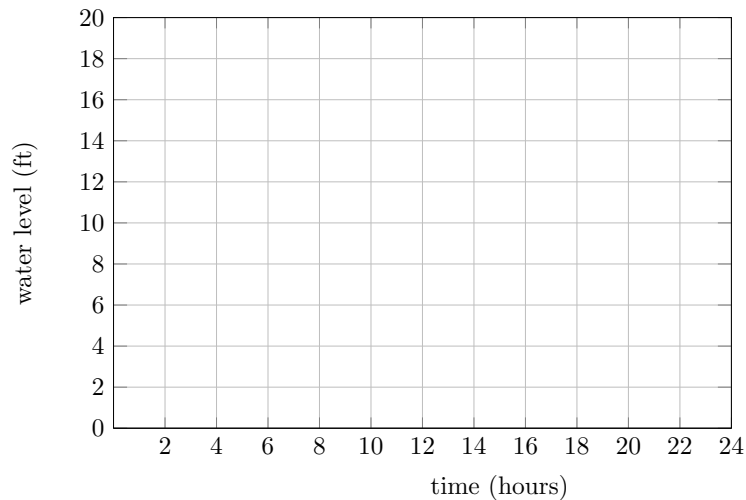
Suppose the following sinusoidal function models the water level on a pier in the ocean as it changes due to the tides during a certain day.

$$w(t) = 4.3 \sin(0.51t + 0.82) + 10.6$$

- (a) Using the formula above, make a table showing the water level every two hours for a 24 hour period starting at midnight.

time (hours)	0	2	4	6	8	10	12	14	16	18	20	22	24
water level (ft)													

- (b) Sketch a graph of this function using the data from your table in part (a).



- (c) What is the period of oscillation of this function?
- (d) What time is high tide?

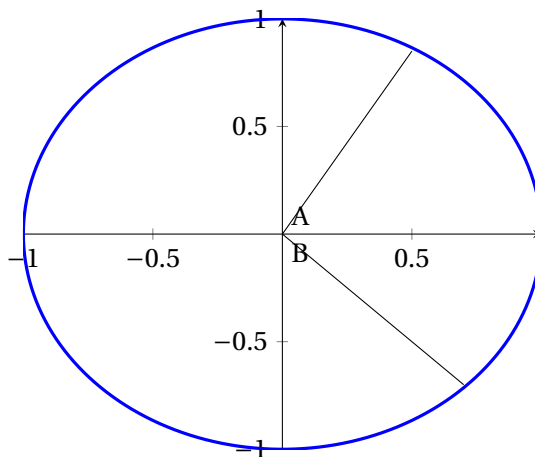
◁

## 0.5. TRIGONOMETRIC FUNCTIONS

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### Voting Questions

0.5.1 Which of the following is the approximate value for the sine and cosine of angles A and B in the figure below.



- (a)  $\sin A \approx 0.5$ ,  $\cos A \approx 0.85$ ,  $\sin B \approx -0.7$ ,  $\cos B \approx 0.7$
- (b)  $\sin A \approx 0.85$ ,  $\cos A \approx 0.5$ ,  $\sin B \approx -0.7$ ,  $\cos B \approx 0.7$
- (c)  $\sin A \approx 0.5$ ,  $\cos A \approx 0.85$ ,  $\sin B \approx 0.7$ ,  $\cos B \approx 0.7$
- (d)  $\sin A \approx 0.85$ ,  $\cos A \approx 0.5$ ,  $\sin B \approx 0.7$ ,  $\cos B \approx 0.7$

*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.05.005

CC KC MA121 F07: 0/96/0/4 time 3:30

CC LV MA121A F08: 6/65/6/20 time 2:10

CC LV MA121A F08: 18/72/9/0 time 2:05

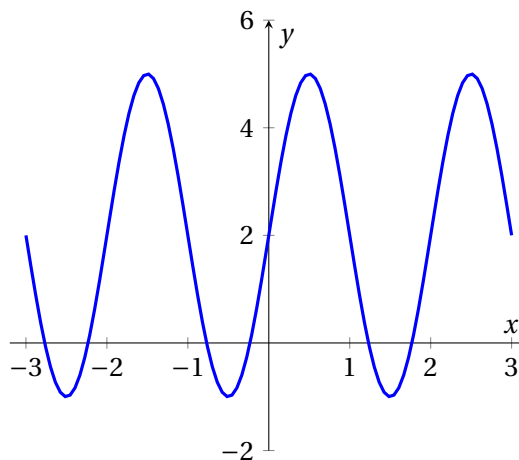
AS DH MA1561 010 F11: 3/89/3/6 time 1:50 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/100/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/100/0/0 time 3:00

0.5.2 The amplitude and period of the function below are

- (a) Amplitude = 2, Period = 2
- (b) Amplitude = 2, Period = 3
- (c) Amplitude = 2, Period =  $1/2$
- (d) Amplitude = 3, Period = 2
- (e) Amplitude = 3, Period =  $1/2$



Answer: (d).

ConceptTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.05.010

CC KC MA121 F06: 3/14/3/79/0

CC KC MA121 F07: 0/0/0/100/0 time 1:40

CC LV MA121A F08: 10/0/0/89/0 time 2:10

CC LV MA121B F08: 9/18/0/72/0 time 1:15

CC HZ MA121 F09: 15/0/0/85/0

HC AS MA120 S12: 13.33/26.67/0/53.33/6.67/0

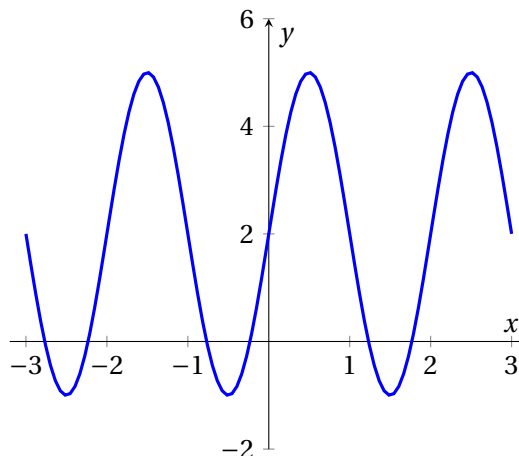
AS DH MA1561 010 F11: 0/8/3/89/0 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/0/100/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/0/0/100/0 time 2:00

0.5.3 What is the equation of the function shown in the graph?

- (a)  $y = 3 \sin(2x) + 2$
- (b)  $y = 3 \cos(2x) + 2$
- (c)  $y = 3 \sin(\pi x) + 2$
- (d)  $y = 3 \cos(\pi x) + 2$
- (e)  $y = 3 \sin\left(\frac{1}{\pi}x\right) + 2$
- (f)  $y = 3 \cos\left(\left(\frac{1}{\pi}\right)x\right) + 2$



Answer: (c). The period is 2, so the coefficient of  $x$  is  $2\pi/2 = \pi$ , and this is a sine function.



## 0.5. TRIGONOMETRIC FUNCTIONS

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by Project MathVote; graph from ConceptTests to accompany Calculus 4th Edition, Hughes-Hallet et al., John Wiley & Sons

SVC.01.05.015

CC HZ MA121 F09: 15/0/**80**/0/5

AS DH MA1561 010 F11: 45/10/**24**/14/7/0 time 3:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 42/0/**55**/0/3/0 time 1:40 Post small-group discussion vote

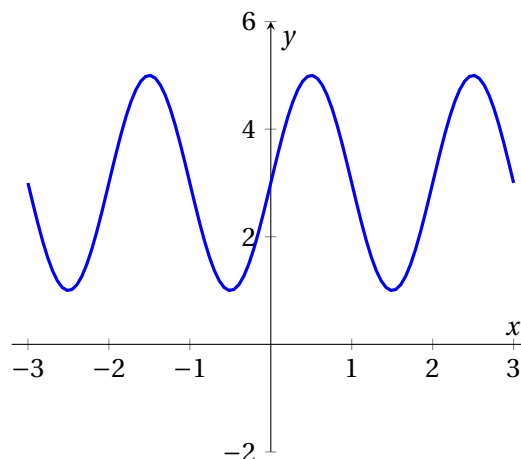
AS DH MA1561 030 F11: 61/4/**29**/7/0/0 time 3:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 50/0/**50**/0/0/0 time 1:40 Post small-group discussion vote

CC HZ MA121 F12: 50/28/**22**/0/0/0 time 3:00

0.5.4 The amplitude and period of the function below are

- (a) Amplitude = 2, Period = 2
- (b) Amplitude = 2, Period = 3
- (c) Amplitude = 2, Period =  $1/2$
- (d) Amplitude = 3, Period = 2
- (e) Amplitude = 3, Period =  $1/2$



*Answer: (c).*

ConceptTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.05.020

CC KC MA121 F06: 0/0/**100**/0/0

CC KC MA121 F07: 0/0/**100**/0/0 time 1:00

CC HZ MA121 F09: 5/0/**95**/0/0 Review

AS DH MA1561 010 F11: 3/0/**91**/0/6 time 1:20 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/**100**/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/0/**100**/0/0 time 2:00

CC HZ MA121 F12: 0/0/**94**/0/6 time 2:00

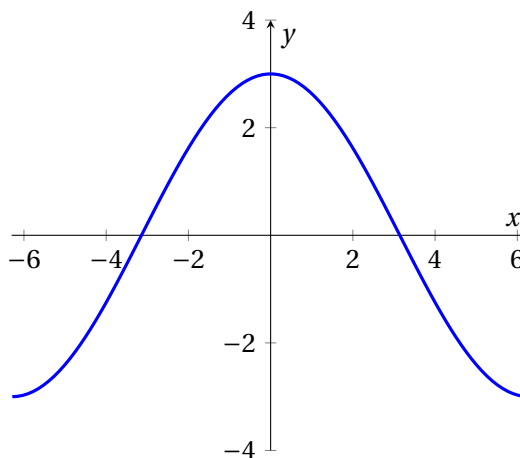
0.5.5 Which of the following could describe the graph below?

(a)  $y = 3 \cos(2x)$

(b)  $y = 3 \cos(x/2)$

(c)  $y = 3 \sin(2x)$

(d)  $y = 3 \sin(x/2)$



*Answer: (b).*

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.05.030

CC KC MA121 F06: 35/**55**/10/0

CC KC MA121 F07: 30/**70**/0/0 time 2:30

CC LV MA121A F08: 6/**89**/3/0 time 1:40

CC LV MA121B F08: 54/**45**/0/0 time 1:40

CC HZ MA121 F09: 16/**79**/0/5

CC KC MA121A F11: 43/**57**/0/0

CC KC MA121D F11: 33/**53**/7/7 "Individual, pre-discussion vote"

CC KC MA121D F11: 12/**88**/0/0 Post small-group discussion vote

AS DH MA1561 010 F11: 16/**78**/3/3 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 3/**97**/0/0 time 1:00 Post small-group discussion vote

AS DH MA1561 030 F11: 0/**100**/0/0 time 3:00

CC HZ MA121 F12: 6/**89**/6/0 time 4:00

0.5.6 The function  $f(x) = 3 \sin(2x + 4)$  is created when you take the function  $g(x) = 3 \sin(2x)$  and you...

- (a) shift it left by 4 units.
- (b) shift it right by 4 units.
- (c) shift it left by 2 units.
- (d) shift it right by 2 units.
- (e) shift it left by 8 units.

*Answer: (c).* First we must factor out the coefficient of  $x$  so we have  $3 \sin(2(x + 2))$ , so this is shifted left by 2 units.

## 0.5. TRIGONOMETRIC FUNCTIONS

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ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.05.040

CC KC MA121 F07: 97/3/0/0 time 2:00 (draft w/o e)

CC LV MA121A F08: 89/6/3/0 time 0:35. Follow up: what is the period and amplitude?

CC HZ MA121 F09: 81/0/10/0/0

CC KC MA121A F11: 95/5/0/0/0 "No one is right," rediscuss/revote 0/0/82/0/18

CC KC MA121D F11: 69/6/19/6/0 "Individual, pre-discussion vote"

CC KC MA121D F11: 88/0/12/0/0 Post small-group discussion vote

AS DH MA1561 010 F11: 11/3/84/0/3 time 1:30 "Individual, pre-discussion vote"

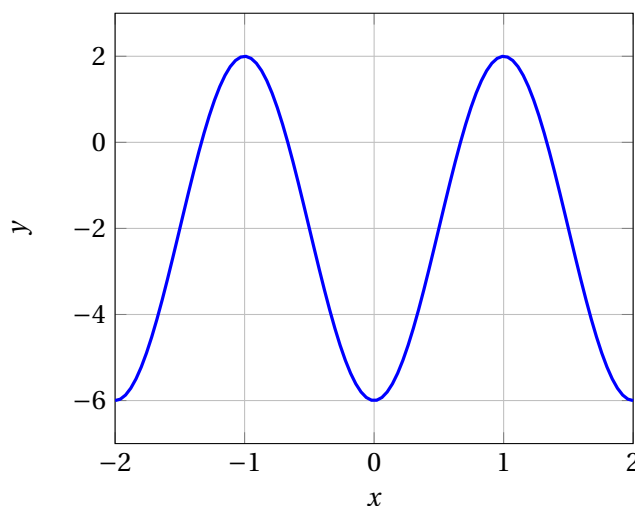
AS DH MA1561 010 F11: 0/0/97/3/0 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: 0/0/100/0/0 time 3:00

CC HZ MA121 F12: 82/9/9/0/0 time 1:00

0.5.7 Which of the following could describe the graph below?

- (a)  $y = 4 \sin\left(\pi x - \frac{\pi}{2}\right) - 2$
- (b)  $y = -4 \sin\left(\pi x + \frac{\pi}{2}\right) - 2$
- (c)  $y = -4 \cos(\pi x) - 2$
- (d)  $y = 4 \cos(\pi(x + 1)) - 2$
- (e) All of the above
- (f) More than one, but not all of the above



*Answer: (e).* All of these functions are equivalent.

by Carroll College MathQuest

SVC.01.05.050

CC HZ MA121 F09: 29/0/33/10/0/29

CC KC MA121A F11: 0/0/77/5/0/18

CC KC MA121D F11: 19/12/32/6/6/25 "Individual, pre-discussion vote"

CC KC MA121D F11: 0/0/19/25/0/56 Post small-group discussion vote

AS DH MA1561 010 F11: 6/16/29/6/3/39 time 3:00 "Individual, pre-discussion vote"

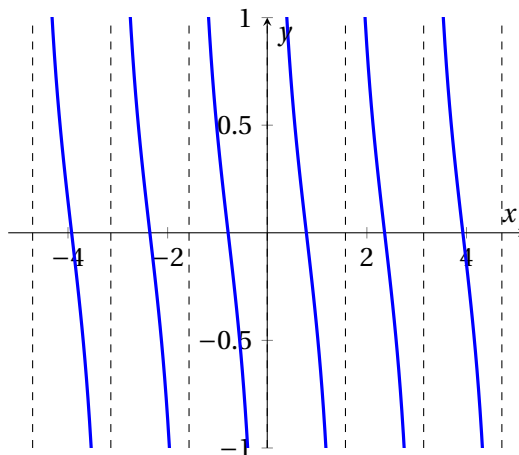
AS DH MA1561 010 F11: 0/0/22/3/5/70 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: 0/3/29/3/3/61 time 5:00

CC HZ MA121 F12: 21/0/42/32/0/5 time 4:00

0.5.8 What is an equation of the function whose graph is given below?

- (a)  $f(x) = \cot x$
- (b)  $f(x) = \cot 2x$
- (c)  $f(x) = \cot\left(x - \frac{\pi}{2}\right)$
- (d)  $f(x) = \cot\left(2x - \frac{\pi}{2}\right)$



*Answer: (b).*

by David A. Huckaby

SVC.01.05.055

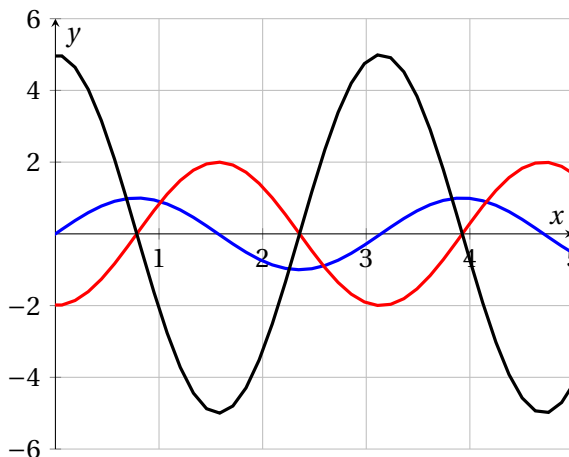
AS DH 1561 010 F11: 3/44/3/50 time 2:30 "Individual, pre-discussion vote"

AS DH 1561 010 F11: 0/72/3/26 time 2:00 Post small-group discussion vote

AS DH 1561 030 F11: 9/67/3/21 time 4:00

0.5.9 Three different functions of the form  $y = A\sin(Bx + C)$  are plotted below. Could these all have the same value of B?

- (a) Yes
- (b) No
- (c) Not enough information is given.



*Answer: (a).* All of these functions have the same period of oscillation, so they have the same B.

by Carroll College MathQuest

SVC.01.05.060

## 0.5. TRIGONOMETRIC FUNCTIONS

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CC LV MA121A F08: **71**/21/7 time 1:35

CC HZ MA121 F09: **95**/5/0

AS DH MA1561 010 F11: **48**/48/4 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: **50**/50/0 time 1:20 Post small-group discussion vote

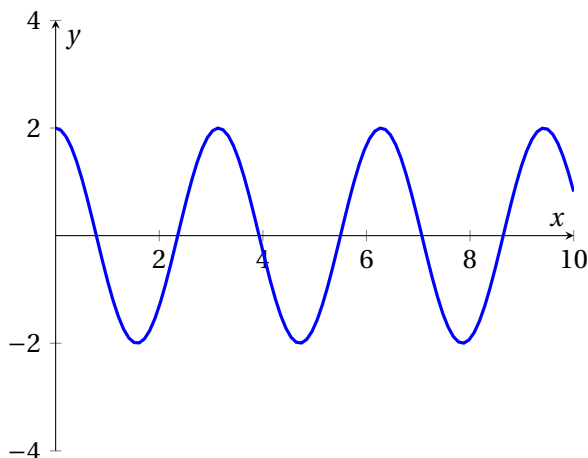
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AS DH MA1561 030 F11: **95**/5/0 time 1:00 Post small-group discussion vote

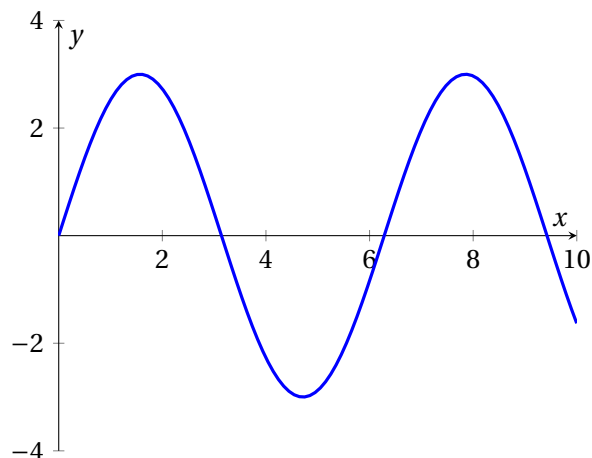
CC HZ MA121 F12: **58**/42/0 time 1:30

0.5.10 The functions plotted below are all of the form  $y = A\sin(Bx + C)$ . Which function has the largest value of  $B$ ?

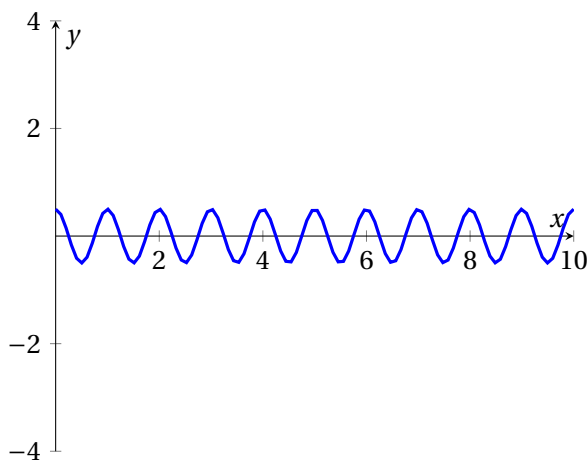
Plot (a)



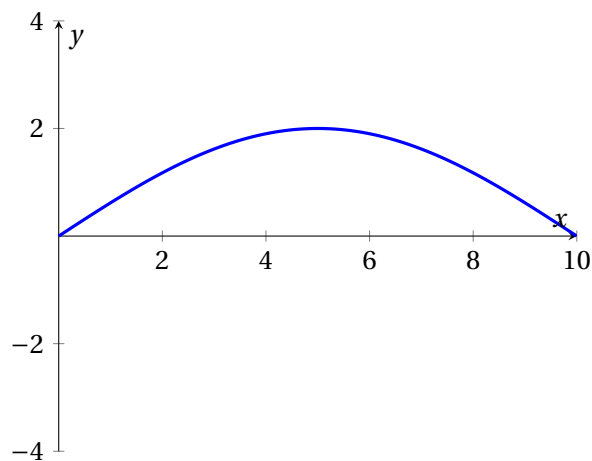
Plot (b)



Plot (c)



Plot (d)



*Answer: (c).* Graph (c) has the shortest period, so this function has the highest angular frequency, corresponding to the largest value of  $B$ .

by Carroll College MathQuest

SVC.01.05.070

CC HZ MA121 F09: 0/5/**59**/36

CC KC MA121A F11: 0/0/**73**/27

AS DH MA1561 010 F11: 4/7/**44**/44 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/4/**96**/0 time 1:20 Post small-group discussion vote

AS DH MA1561 030 F11: 0/27/**31**/42 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/4/**28**/68 time 1:20 Post small-group discussion vote

CC HZ MA121 F12: 0/5/**84**/11 time 1:45

0.5.11 What is the phase shift of  $f(x) = \frac{1}{5} \tan\left(2x + \frac{\pi}{2}\right)$ ?

- (a)  $2\pi$
- (b)  $\pi$
- (c)  $\frac{\pi}{2}$
- (d)  $\frac{\pi}{4}$
- (e)  $-2\pi$
- (f)  $-\pi$
- (g)  $-\frac{\pi}{2}$
- (h)  $-\frac{\pi}{4}$

*Answer: (h).*

by David A. Huckaby

SVC.01.05.075

AS DH 1561 010 F11: 0/3/0/5/0/14/11/**68** time 1:00 "Individual, pre-discussion vote"

AS DH 1561 010 F11: 0/0/0/0/0/0/0/**0** time 1:00 Post small-group discussion vote

AS DH 1561 030 F11: 0/0/4/0/0/4/0/**91** time 2:00

0.5.12 What is the amplitude of  $f(x) = -3 \sin(2x)$ ?

- (a) 3
- (b) -3
- (c)  $\pi$
- (d)  $2\pi$

## 0.5. TRIGONOMETRIC FUNCTIONS

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*Answer: (a) 3.*

by Project MathVote

SVC.01.05.080

0.5.13 What is the amplitude of  $f(x) = -2 \sin x$ ?

- (a) 1
- (b) 2
- (c)  $-2$

*Answer: (b).*

by David A. Huckaby

SVC.01.05.085

AS DH 1561 010 F11: 0/77/23 time 1:00 "Individual, pre-discussion vote"

AS DH 1561 010 F11: 0/100/0 time 1:00 Post small-group discussion vote

AS DH 1561 030 F11: 0/94/6 time 1:30

0.5.14 What is the period of  $f(x) = -3 \sin(2x)$ ?

- (a) 3
- (b)  $-3$
- (c)  $\pi$
- (d)  $2\pi$

*Answer: (c)  $\pi$ .*

by Project MathVote

SVC.01.05.090

CC HZ MA121 F12: 0/0/95/5 time 1:30

0.5.15 What is the period of  $f(x) = \frac{1}{5} \tan(2x)$ ?

- (a)  $\frac{1}{5}$
- (b)  $2\pi$
- (c)  $\pi$

(d)  $\frac{\pi}{2}$

(e)  $\frac{\pi}{4}$

*Answer: (d).*

by David A. Huckaby

SVC.01.05.095

AS DH 1561 010 F11: 0/8/19/**69**/3 time 1:20 "Individual, pre-discussion vote"

AS DH 1561 010 F11: 0/3/0/**97**/0 time 1:00 Post small-group discussion vote

AS DH 1561 030 F11: 0/0/3/**97**/0 time 2:00

0.5.16 Which of the basic trig functions below are odd functions?

(a)  $f(x) = \sin(x)$ .

(b)  $f(x) = \cos(x)$ .

(c)  $f(x) = \tan(x)$ .

(d) (a) and (b).

(e) (a) and (c).

(f) (b) and (c).

(g) (a), (b), and (c).

(h) None of the above.

*Answer: (e).* This question tests basic properties of trig functions and recalls even function / odd function language. As a followup question, students can note that  $f(x) = \cos(x)$  is an even function.

by Project MathVote

SVC.01.05.100

AS DH MA1561 010 F11: 0/0/12/3/**79**/3/3/0 time 0:30 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/3/0/**97**/0/0/0 time 0:30 Post small-group discussion vote

AS DH MA1561 030 F11: 21/0/3/0/**76**/0/0/0 time 1:00



## 0.6 Powers, Polynomials, and Rational Functions

**Preview Activity 0.6.** Figure 4 shows the graphs of two different functions. Suppose that you were to graph a line anywhere along each of the two graphs.

- 1 Is it possible to draw a line that does not intersect the graph of  $f$ ?  $g$ ?
- 2 Is it possible to draw a line that intersects the graph of  $f$  an even number of times?
- 3 Is it possible to draw a line that intersects the graph of  $g$  an odd number of times?
- 4 What is the fewest number of intersections that your line could have with the graph of  $f$ ? with  $g$ ?
- 5 What is the largest number of intersections that your line could have with the graph of  $f$ ? with  $g$ ?
- 6 How many times does the graph of  $f$  change directions? How many times does the graph of  $g$  change directions?

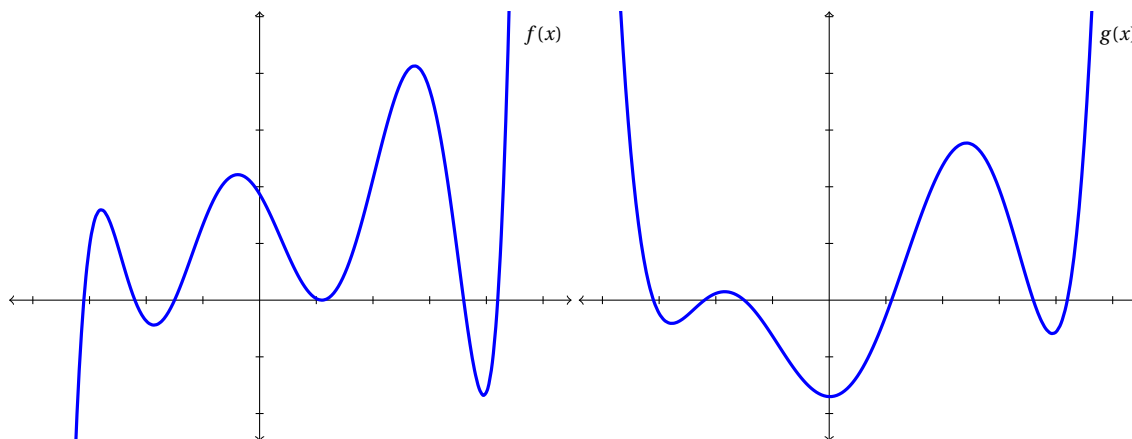


Figure 4:  $f(x)$  and  $g(x)$  for the preview activity.

**Activity 0.19.**

Power functions and exponential functions appear somewhat similar in their formulas, but behave differently in many ways.

- (a) Compare the functions  $f(x) = x^2$  and  $g(x) = 2^x$  by graphing both functions in several viewing windows. Find the points of intersection of the graphs. Which function grows more rapidly when  $x$  is large?
- (b) Compare the functions  $f(x) = x^{10}$  and  $g(x) = 2^x$  by graphing both functions in several viewing windows. Find the points of intersection of the graphs. Which function grows more rapidly when  $x$  is large?
- (c) Make a conjecture: As  $x \rightarrow \infty$ , which dominates,  $x^n$  or  $a^x$ ?
- (d) Suppose you are offered a job that lasts one month. You have the option of being paid in one of two ways: (1) One million dollars at the end of the month; or (2) One cent on the first day of the month, two cents on the second day, four cents on the third day, and, in general,  $2^{n-1}$  cents on the  $n^{\text{th}}$  day. Which option should you choose?
- (e) How much different (shorter or longer) would the work period need to be for your answer to the previous question change?

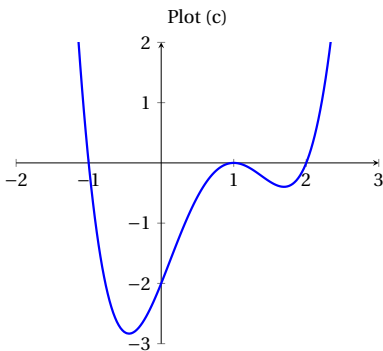
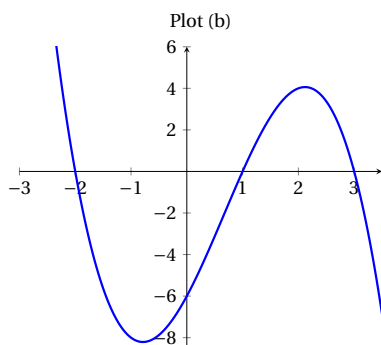
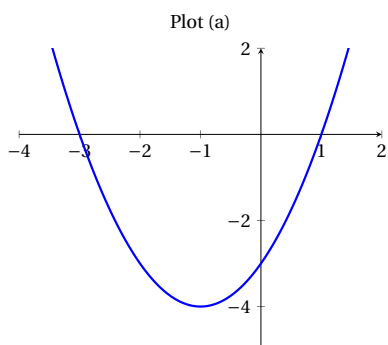
&lt;

## 0.6. POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

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### Activity 0.20.

For each of the following graphs, find a possible formula for the polynomial of lowest degree that fits the graph.



**Activity 0.21.**

(a) Suppose  $f(x) = x^2 + 3x + 2$  and  $g(x) = x - 3$ .

- 1 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  near  $x = -1$ ? (i.e. what happens to  $h(x)$  as  $x \rightarrow -1$ ?) near  $x = -2$ ? near  $x = 3$ ?
- 2 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  near  $x = -1$ ? near  $x = -2$ ? near  $x = 3$ ?

(b) Suppose  $f(x) = x^2 - 9$  and  $g(x) = x - 3$ .

- 1 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  near  $x = -3$ ? (i.e. what happens to  $h(x)$  as  $x \rightarrow -3$ ?) near  $x = 3$ ?
- 2 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  near  $x = -3$ ? near  $x = 3$ ?

(c) Suppose  $f(x) = \sin x$  and  $g(x) = x$ .

- 1 What is  $f(0)$ ? What is  $g(0)$ ?
- 2 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  near  $x = 0$ ? (i.e. what happens to  $h(x)$  as  $x \rightarrow 0$ ?)
- 3 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  near  $x = 0$ ?

&lt;

**Activity 0.22.**

- (a) Suppose  $f(x) = x^3 + 2x^2 - x + 7$  and  $g(x) = x^2 + 4x + 2$ .
- 1 Which function dominates as  $x \rightarrow \infty$ ?
  - 2 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  as  $x \rightarrow \infty$ ?
  - 3 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  as  $x \rightarrow \infty$ ?
- (b) Suppose  $f(x) = 2x^4 - 5x^3 + 8x^2 - 3x - 1$  and  $g(x) = 3x^4 - 2x^2 + 1$
- 1 Which function dominates as  $x \rightarrow \infty$ ?
  - 2 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  as  $x \rightarrow \infty$ ?
  - 3 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  as  $x \rightarrow \infty$ ?
- (c) Suppose  $f(x) = e^x$  and  $g(x) = x^{10}$ .
- 1 Which function dominates as  $x \rightarrow \infty$  as  $x \rightarrow \infty$ ?
  - 2 What is the behavior of the function  $h(x) = \frac{f(x)}{g(x)}$  as  $x \rightarrow \infty$ ?
  - 3 What is the behavior of the function  $h(x) = \frac{g(x)}{f(x)}$  as  $x \rightarrow \infty$ ?

**Activity 0.23.**

For each of the following functions, determine (1) whether the function has a horizontal asymptote, and (2) whether the function crosses its horizontal asymptote.

(a)  $f(x) = \frac{x+3}{x-2}$

(b)  $g(x) = \frac{x^2 + 2x - 1}{x - 1}$

(c)  $h(x) = \frac{x+1}{x^2 + 2x - 1}$

(d)  $k(x) = e^x \sin x$

&lt;

### Voting Questions

0.6.1 Which of the following is not a power function?

- (a)  $f(x) = 3x^2$
- (b)  $f(x) = x^{1.5}$
- (c)  $f(x) = 6 \cdot 2^x$
- (d)  $f(x) = -3x^{-\pi}$

*Answer:* (c). This is an exponential function. This question is intended to focus on student confusion about the structure of power versus exponential functions.

by Project MathVote

SVC.01.06.005

CC KC MA121A F11: 0/0/**100**/0

CC KC MA121D F11: 6/6/**64**/24 "Individual, pre-discussion vote"

CC KC MA121D F11: 0/7/**93**/0 Post small-group discussion vote

CC HZ MA121 F12: 0/15/**55**/30 time 2:00

0.6.2 As  $x \rightarrow \infty$ , which function dominates? That is, which function is larger in the long run?

- (a)  $0.1x^2$
- (b)  $10^{10}x$

*Answer:* (a). Power functions with the power greater than one and with a positive coefficient grow faster than linear functions.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.010

CC HZ MA121 F09: **73**/27

CC HZ MA121 F12: **84**/16 time 2:00

0.6.3 As  $x \rightarrow \infty$ , which function dominates?

- (a)  $0.25\sqrt{x}$
- (b)  $25,000x^{-3}$

*Answer: (a).*  $0.25\sqrt{x}$  is an increasing function, whereas  $25,000x^{-3}$  is a decreasing function.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.020

CC KC MA121 F06: **96**/4

CC HZ MA121 F09: **65**/35

CC KC MA121A F11: **91**/9

CC KC MA121D F11: **76**/24 "Individual, pre-discussion vote"

CC KC MA121D F11: **94**/6 Post small-group discussion vote

AS DH MA1561 010 F11: **61**/39 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: **71**/29 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: **77**/23 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **100**/0 time 1:00 Post small-group discussion vote

CC HZ MA121 F12: **90**/10 time 1:15

0.6.4 As  $x \rightarrow \infty$ , which function dominates?

(a)  $3 - 0.9^x$

(b)  $\log x$

*Answer: (b).*  $3 - 0.9^x$  levels out at 3, while  $\log x$  increases without bound, although slowly.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.030

CC KC MA121 F06: 35/**65**

CC KC MA121 F07: 36/**63** time 2:30

CC KC MA121 F07: 39/**61** time 1:00 (covering section 121D)

CC KC MA121A F11: 4/**96**

CC KC MA121D F11: 59/**41** "Individual, pre-discussion vote"

CC KC MA121D F11: 35/**65** Post small-group discussion vote

AS DH MA1561 010 F11: 19/**81** time 2:30

AS DH MA1561 030 F11: 9/**91** time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**100** time 1:00 Post small-group discussion vote

0.6.5 Which function dominates as  $x \rightarrow \infty$ ?

(a)  $x^2$

(b)  $e^x$





## 0.6. POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

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*Answer: (b).*

by David A. Huckaby

SVC.01.06.035

AS DH 2513 010 S12: 30/**70** time 1:10

0.6.6 As  $x \rightarrow \infty$ , which function dominates?

(a)  $x^3$

(b)  $2^x$

*Answer: (b).* Exponential growth functions grow faster than power functions.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.040

CC HZ MA121 F12: 32/**68** time 2:00

0.6.7 As  $x \rightarrow \infty$ , which function dominates?

(a)  $10(2^x)$

(b)  $72,000x^{12}$

*Answer: (a).* Exponential growth functions grow faster than power functions, no matter how large the coefficient.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.050

CC KC MA121 F07: **92**/8 time 2:20

CC LV MA121A F08:**46**/53 time 1:20

CC HZ MA121 F09: **73**/27

CC KC MA121A F11: **91**/9

CC KC MA121D F11: **65**/35 "Individual, pre-discussion vote"

CC KC MA121D F11: **94**/6 Post small-group discussion vote

AS DH MA1561 010 F11: **97**/3 time 1:30

AS DH MA1561 030 F11: **88**/12 time 1:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: **96**/4 time 0:30 Post small-group discussion vote

0.6.8 Which of these functions dominates as  $x \rightarrow \infty$ ?

- (a)  $f(x) = -5x$
- (b)  $g(x) = 10^x$
- (c)  $h(x) = 0.9^x$
- (d)  $k(x) = x^5$
- (e)  $l(x) = \pi^x$

*Answer:* (b). Exponential functions with larger bases grow faster.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.060

0.6.9 If  $f(x) = ax^2 + bx + c$  is a quadratic function, then the lowest point on the graph of  $f(x)$  occurs at  $x = -b/2a$ .

- (a) True, and I am very confident.
- (b) True, but I am not very confident.
- (c) False, but I am not very confident.
- (d) False, and I am very confident.

*Answer:* (d). This is only true if  $a > 0$ . If  $a < 0$ , then this will indicate the highest point. Students can be encouraged to give a counterexample.

by Project MathVote

SVC.01.06.065

0.6.10 Under what condition is the graph of the quadratic function described by  $f(x) = ax^2 + bx + c$  concave down?

- (a)  $a < 0$ .
- (b)  $b < 0$ .
- (c)  $c < 0$ .
- (d) More than one of the above.
- (e) None of the above.

*Answer:* (a). This question is intended to help students connect characteristics of the graph with properties of the coefficients of the function. In this case, as long as  $a$  is negative, the graph will be concave down.

by Project MathVote

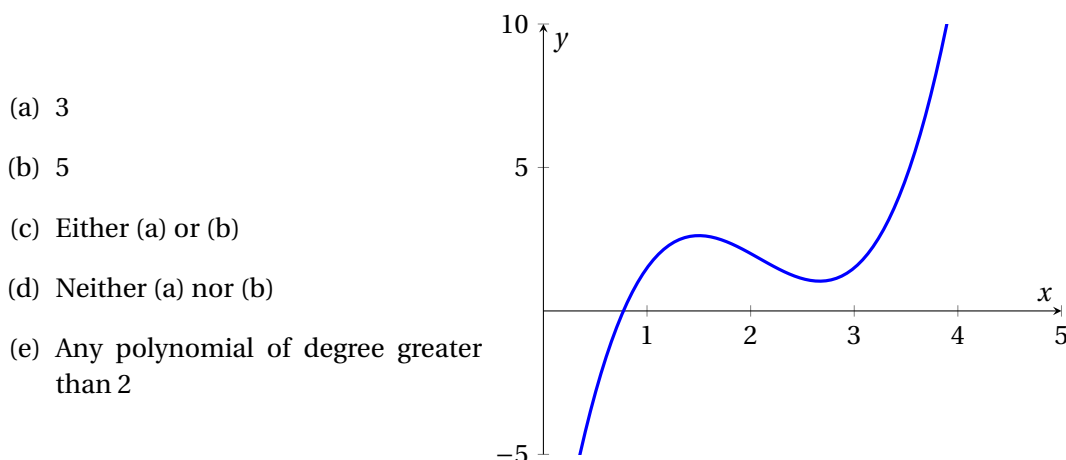
SVC.01.06.066



## 0.6. POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

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0.6.11 What is the degree of the graph of the polynomial in the figure below?



*Answer: (e).* The graph could represent many such polynomials. The window may not show all the crucial behavior of the polynomial.

ConceptTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.070

CC KC MA121 F06: 77/0/12/0/12

CC KC MA121 F07: 87/0/13/0/0 time 2:00

CC KC MA121 F07: 39/13/26/4/17 time 2:00 (covering section 121D)

CC LV MA121A F08: 33/5/57/5/0 time 1:10

CC KC MA121A F11: 61/0/17/0/22

CC KC MA121D F11: 64/6/12/6/12 "Individual, pre-discussion vote"

CC KC MA121D F11: 94/0/0/0/6 Post small-group discussion vote

AS DH MA1561 010 F11: 8/0/75/0/18 time 2:00

AS DH MA1561 030 F11: 21/0/36/6/36 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 9/0/12/0/78 time 1:30 Post small-group discussion vote

0.6.12 Which of the options below describes a function which is even?

- (a) Any polynomial of even degree.  
(b) Any polynomial of odd degree.  
(c)  $f(x) = 9x^6 - 3x^2 + 2$ .  
(d)  $f(x) = 3x^4 - 2x^3 + x^2$ .  
(e) More than 1 of the above.  
(f) None of the above.

*Answer: (c).* This question is intended to test the definition of even function. Here, only (c) is correct as it is a polynomial with all even powers. Answer (a) may be tempting, and students should be

encouraged to give a counter-example (such as  $f(x) = x^2 + x$ ) if this emerges as a serious contender in the discussion.

by Project MathVote

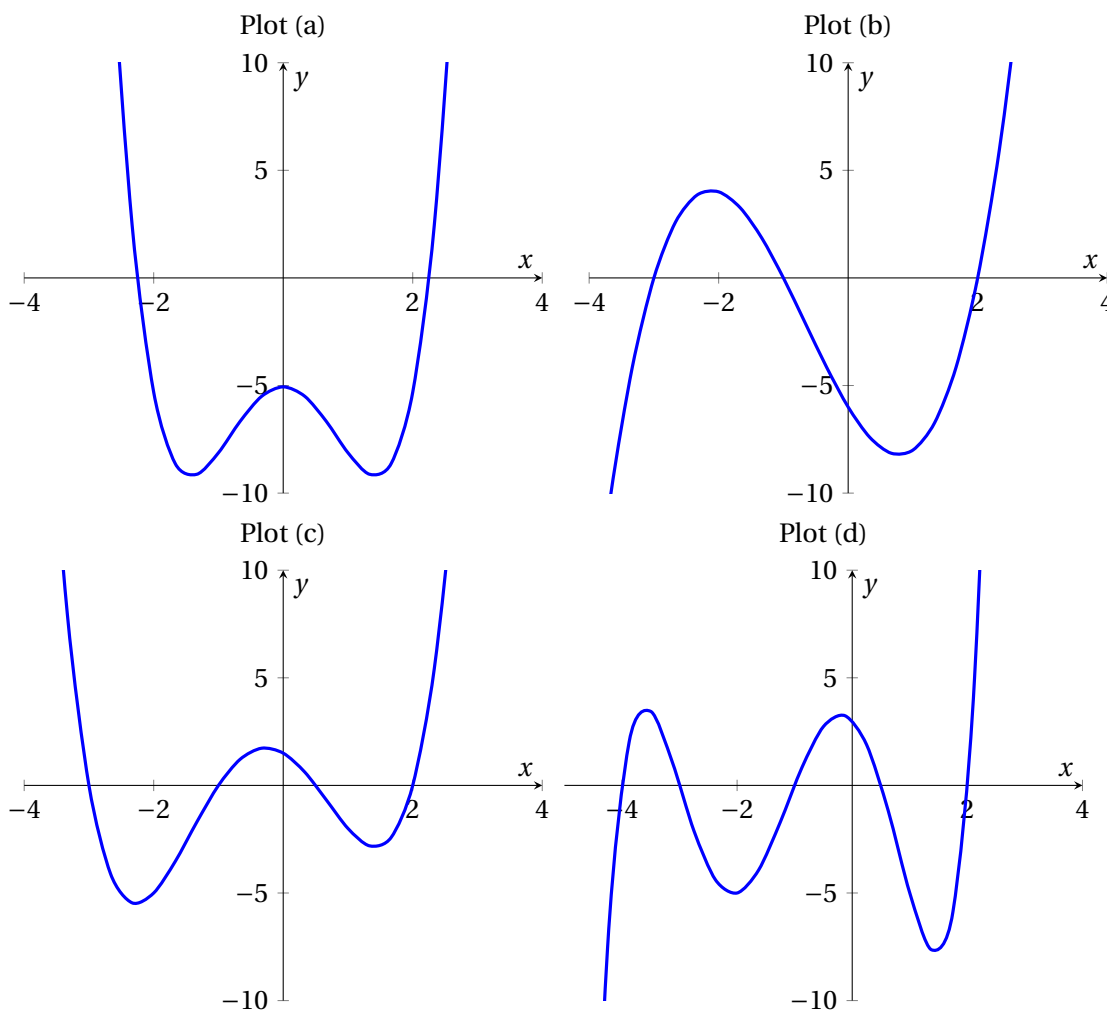
SVC.01.06.075

AS DH MA1561 010 F11: 3/0/45/3/45/5 time 1:50 "Individual, pre-discussion vote"

AS DH MA1561 010 F11: 0/0/45/0/55/0 time 1:30 Post small-group discussion vote

AS DH MA1561 030 F11: 6/0/0/0/94/0 time 2:00

0.6.13 The equation  $y = x^3 + 2x^2 - 5x - 6$  is represented by which graph?



*Answer: (b).* The graph must have a y-intercept of -6 and not be that of an even function.

ConceptTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

## 0.6. POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

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SVC.01.06.080

CC JO MA112 S08: 0/100/0/0 time 1:00

CC KC MA121 F07: 0/100/0/0 time 2:00

CC LV MA121A F08: 5/81/5/9 time 1:00

CC LV MA121B F08: 0/92/0/7 time 1:20

CC HZ MA121 F09: 0/68/27/5

AS DH MA1561 010 F11: 0/100/0/0 time 1:20

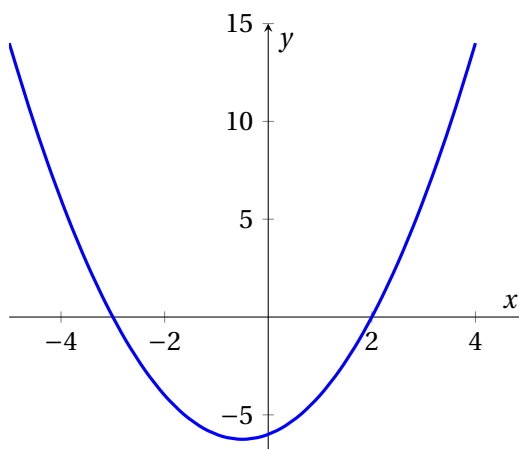
AS DH MA1561 030 F11: 0/88/0/12 time 1:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/100/0/0 time 1:00 Post small-group discussion vote

CC HZ MA121 F12: 10/58/10/21 time 2:30

0.6.14 The graph below is a representation of which function?

- (a)  $y = 3x + 2$
- (b)  $y = (x - 2)(x + 3)$
- (c)  $y = (x - 6)(x - 2)$
- (d)  $y = (x - 3)(x + 2)$
- (e) none of these



*Answer: (b).* The graph is a parabola with  $x$ -intercepts of 2 and -3.

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.090

CC JO MA112 S08: 0/100/0/0 time 1:10

CC KC MA121 F08: 0/100/0/0 time 2:30

CC KC MA121 F08: 0/87/4/9 time 2:30 (covering section 121D)

CC HZ MA121 F09: 0/55/5/41/0

AS DH MA1561 010 F11: 0/100/0/0/0 time 2:30

AS DH MA1561 030 F11: 0/76/12/9/3 time 2:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/93/3/3/0 time 1:30 Post small-group discussion vote

CC HZ MA121 F12: 0/88/0/12/0 time 2:30

0.6.15 Let  $f(x) = \frac{x^2 - 1}{x + 1}$  and  $g(x) = x - 1$ , then  $f(x) = g(x)$ .

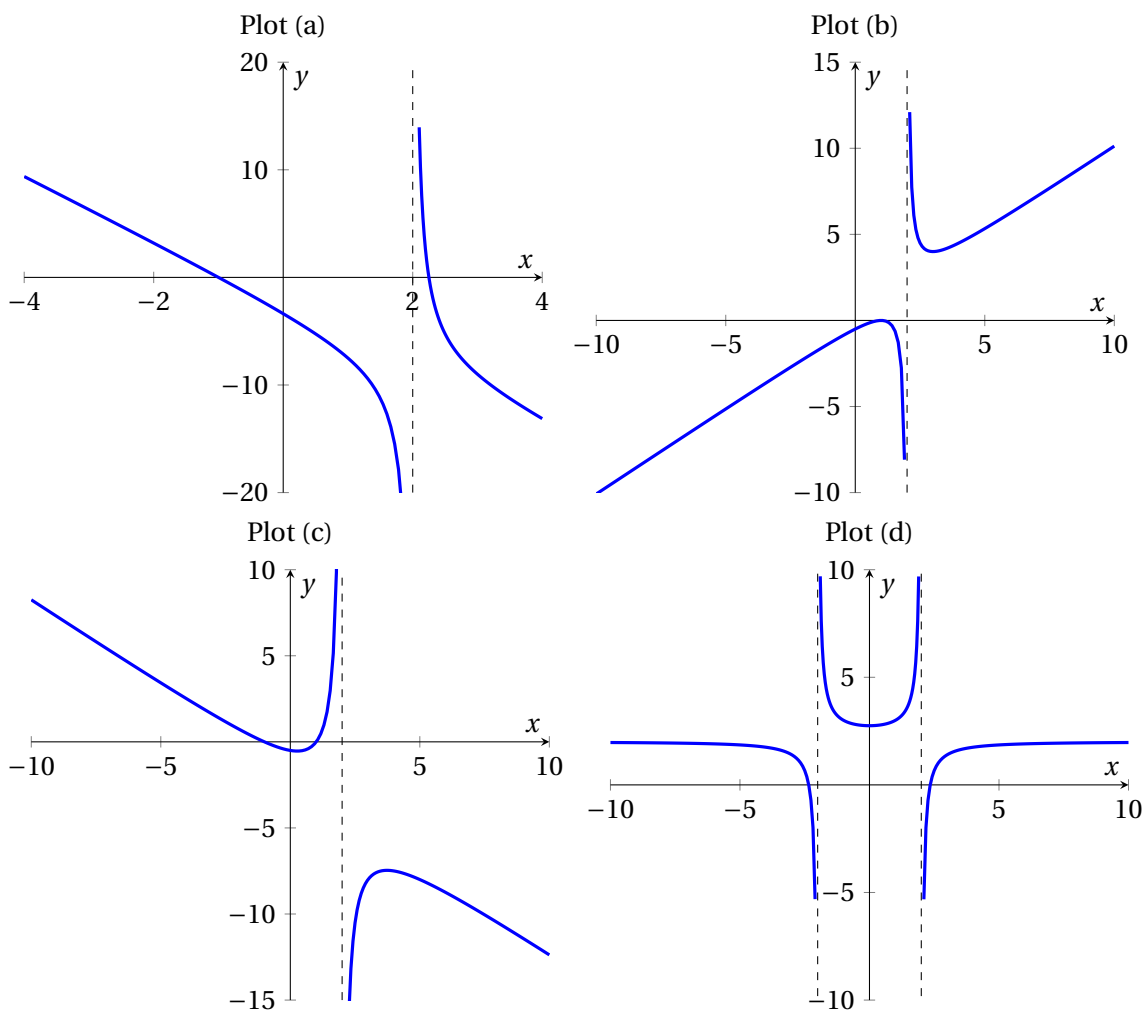
- (a) True, and I am very confident
- (b) True, but I am not very confident
- (c) False, but I am not very confident
- (d) False, and I am very confident

*Answer: (False).* These are only equal if  $x \neq -1$ .

ConcepTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.100

0.6.16 Which if the following is a graph for  $y = \frac{1-x^2}{x-2}$ . (No calculators allowed.)



*Answer: (c).* The equation indicates  $x$ -intercepts at  $\pm 1$  and a vertical asymptote at  $x = 2$ .

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## 0.6. POWERS, POLYNOMIALS, AND RATIONAL FUNCTIONS

SVC.01.06.110

CC JO MA112 S08: 17/50/**28**/6 time 3:15

CC KC MA121 F07: 0/24/**64**/12 time 5:00

CC HZ MA121 F09: 5/18/**73**/5

CC KC MA121A F11: 0/4/**96**/0

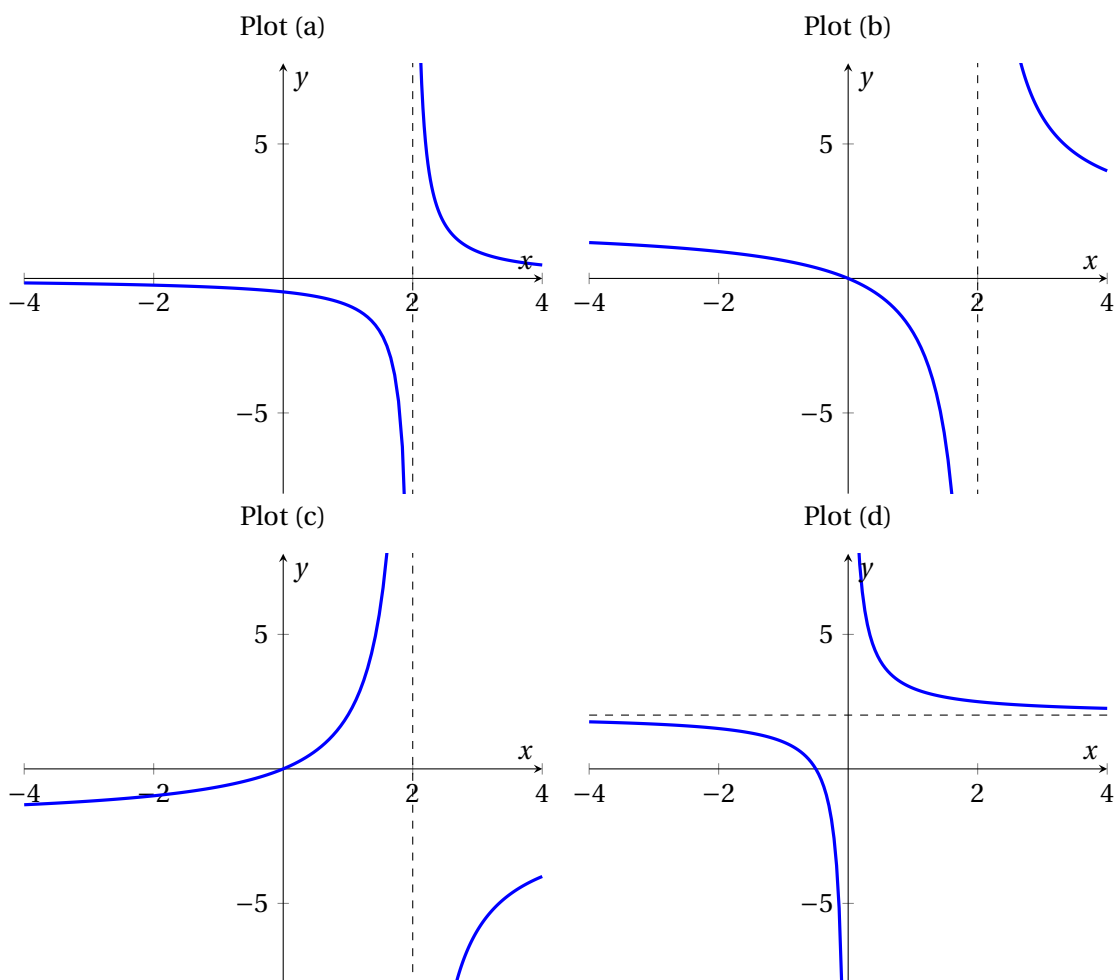
AS DH MA1561 010 F11: 2/2/**95**/0 time 5:00

AS DH MA1561 030 F11: 3/29/**61**/6 time 3:00 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 7/0/**93**/0 time 1:40 Post small-group discussion vote

CC HZ MA121 F12: 17/17/**67**/0 time 5:30

0.6.17 Which of the graphs represents  $y = \frac{2x}{x-2}$ ?



*Answer: (b).* The graph goes through the origin and is positive for  $x > 2$ .

ConceptTests - to accompany Calculus 4th Edition, Hughes-Hallet et al. John Wiley & Sons.

SVC.01.06.120

CC JO MA112 S08: 0/**72**/17/0 time 2:20

CC KC MA121 F07: 0/**84**/16/0 time 2:00

CC HZ MA121 F09: 0/**68**/9/23

AS DH MA1561 010 F11: 2/**82**/0/15 time 2:30

AS DH MA1561 030 F11: 8/**69**/11/11 time 2:30 "Individual, pre-discussion vote"

AS DH MA1561 030 F11: 0/**100**/0/0 time 2:30 Post small-group discussion vote