Review of Rings and Quotient Rings

The goal of this activity is to remind ourselves of basic but crucially important definitions we will need in our study of fields.

You will be asked to provide definitions. Some definitions will include terms that also should be defined. Make sure that everyone knows what every word in a definition means (if not, provide definitions for those words). For example, a **field** is a commutative division ring. If you have not yet defined commutative ring and division ring, you should say what these mean.

1. Give a definition of a **ring**.

2. What is a **commutative ring with unity**? How is this different from a ring? (Note, "unity" is also sometimes called "identity".)

3. What is a **commutative division ring**? What does "division" refer to here, and how is this different from a ring in general?

4. Give the definition of an **integral domain**. How does this relate to the other types of structures you defined above?

5. What is an **ideal**? What is the difference between an ideal and a **subring**?

6. Consider the integers $\mathbb{Z}$ (an integral doamin, right?). What does the notation $\langle 3\rangle$ mean? What sort of thing is this? What is $\langle r\rangle$ in general?

7. What is $\mathbb{Q}[x]$? Then give an example of an ideal in $\mathbb{Q}[x]$, using proper notation and by listing out some of the elements in the ideal.

8. Give the definition of a **quotient ring** (i.e. a **factor ring**). What do elements of a quotient ring look like? How are the operations defined?

9. Illustrate you you wrote about quotient rings above using two examples: First, $\mathbb{Z}/\langle 3\rangle$, and then $\mathbb{Q}[x]/\langle x^2+1\rangle$. How many elements are in each of these quotient rings? What do the elements look like? Show how to add/multiply elements.

If you have time.After completing the activity above, if you have time, you should look again at the activity distributed the first day of class, and try to answer the prompts on the second page.