

### Activity: Writing Cycles and Permutations

Goal: Understand how elements of  $S_n$  can be represented as cycles and products of cycles.

- Can every permutation in  $S_n$  be represented using cycle notation? How could you represent the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 2 & 8 & 7 & 9 & 6 & 1 & 4 \end{pmatrix}$ ?
- How should we define “cycle”? Write a definition. Some considerations: is  $(132)(45)$  a single cycle or two cycles? Is  $(3124)$  a cycle? What should we call the **length** of a cycle?
- Here are a few permutations of  $S_7$ , written as products of cycles. Are there other ways to write each of these using cycle notation? What makes two products of cycles the *same*?

$$(142)(2534)(46)(135) \quad (12546) \quad (16)(14)(15)(12) \quad (12)(37)(25)(37)(45)(46)$$

4. Can the cycle  $(13254)$  be written as the product of transpositions? Can it be written as the product of transpositions in more than one way?

- What about the product  $(124)(2354)$ ?
- Can you write  $(13)(24)(45)(15)(26)(13)$  as the product of more than 6 transpositions? Fewer than 6 transpositions? Can you write it as the product of an odd number of transpositions?
- What numbers of transpositions can you write the identity  $(1)$  as?