Orbits, Stabilizers, and Burnside

Wednesday, April 15

How many orbits 6

How many ways can you place blue and gold placemats around a 4 seat round table?

$X =$ {BBBB}, {GGGG}, {BBBG, BBGB, BGBB, GBBB}, {GGGB, GGBG, GBGG, BGGG}, {BBGG, BGGB, GGBB, GBBG}, {BGBG, GBGB}

$G = \{r_0, r_1, r_2, r_3\} \leq D_4$ $\quad$ X is a G-set.

$O_{BBBG} = \{BBBG, BBGB, BGBB, GBBB\}$

$O_{BBBB} = \{BBBB\}$

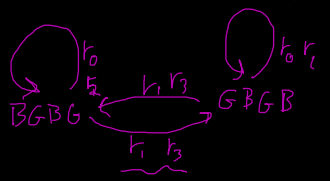
$O_{BGBG} = \{GBGB, BGBG\}$

$G_{BBBG} = \{r_0\} = G_{BBGB} = G_{GGGG}$

$G_{BBBB} = \{r_0, r_1, r_2, r_3\}$

$G_{BGBG} = \{r_0, r_2\}$

$G/G_{BGBG} = \{\{r_0, r_2\}, \{r_1, r_3\}\}$



Orbit–Stabilizer Theorem.

$|G| = |O_x| \cdot |G_x|$ for all $x \in X$.

$[G : G_x] = \frac{|G|}{|G_x|} = |O_x|$

X_g for $g \in G$

$X_{r_0} = X$

$X_{r_1} = \{BBBB,\ GGGG\}$

$X_{r_2} = \{BBBB,\ GGGG,\ BGBG,\ GBGB\}$

$X_{r_3} = \{BBBB,\ GGGG\}$

$\sum_{g \in G} |X_g| = 16 + 2 + 4 + 2$

$= 24$

$\frac{1}{|G|} \sum_{g \in G} |X_g| = \frac{24}{4} = 6$ ← # of orbits!!

$\sum_{g \in G} |X_g| = \{(g, x) \text{ s.t. } g \text{ fixes } x\} = \sum_{x \in X} |G_x|$

$\sum_{g \in \mathcal{O}_x} |G_g| = |\mathcal{O}_x| \, |G_x| = |G|$

$\sum_{x \in X} |G_x| = |G| \cdot K,$ K is the number of orbits

$\sum_{g \in G} |X_g| = |G| \cdot K$

$\frac{1}{|G|} \sum |X_g| = K.$ Burnsides Lemma.

$(r_0, BGGB)$ r_1

$(r_0, BBBB)$ $(r_1, BBBB)$

$(r_0, GGGG)$ $(r_1, GGGG)$