Activity: Writing Cycles and Permutations

Goal: Understand how elements of S_n can be represented as cycles and products of cycles.

1. Can every permutation in S_n be represented using cycle notation? How could you represent the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 2 & 8 & 7 & 9 & 6 & 1 & 4 \end{pmatrix}$?

2. How should we define "cycle"? Write a definition. Some considerations: is (132)(45) a single cycle or two cycles? Is (3124) a cycle? What should we call the **length** of a cycle?

3. Here are a few permutations of S_7, written as products of cycles. Are there other ways to write each of these using cycle notation? What makes two products of cycles the *same*?

(142)(2534)(46)(135) (12546) (16)(14)(15)(12) (12)(37)(25)(37)(45)(46)

A *transposition* (or 2-cycle) is a cycle of *length* 2. That is, it swaps (transposes) a pair of elements.

4. Can the cycle (13254) be written as the product of transpositions? Can it be written as the product of transpositions in more than one way?

5. What about the product (124)(2354)?

6. Can you write (13)(24)(45)(15)(26)(13) as the product of more than 6 transpositions? Fewer than 6 transpositions? Can you write it as the product of an odd number of transpositions?

7. What numbers of transpositions can you write the identity (1) as?