Galois (sub)Groups

Consider the polynomial $a(x) = x^4 - 2$ in $\mathbb{Q}[x]$.

1. Find the splitting field E for $a(x)$ over $\mathbb{Q}$.

2. Describe the Galois group G for $a(x)$ over $\mathbb{Q}$. How big is it? Does it look like any group we have studied (a lot) before?

3. Find all the subgroups of G. Find all the subfields of E above $\mathbb{Q}$. What do you notice? Draw the lattice of subgroups and subfields to illustrate the connections.