Activity: (Sub)Normal Series

Given a group, we can look at subgroups. We say that a sequence of subgroups

$$G = H_n \supset H_{n-1} \supset \cdots \supset H_1 \supset H_0 = \{e\}$$

is a **subnormal series** provided each H_i is normal in H_{i+1}, and a **normal series** if each H_i is normal in G.

A non-trivial group G is called **simple** provided it has no non-trivial normal subgroups. We say that a subnormal series is a **composition series** and that a normal series is a **principle series** if every quotient group H_{i+1}/H_i is simple.

1. Find a subnormal series for D_4. Is it a normal series?

2. Find two different normal series for $\mathbb{Z}_{60}$ of length 3 (length is the number of proper inclusions).

3. Find the quotient groups H_{i+1}/H_i for both series above. How are these related? Are the series composition series?

4. Find a composition series for $\mathbb{Z}_{60}$. Can you take it to be a **refinement** of the normal series you found above?