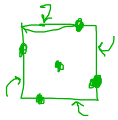


We will consider a new sort of operation: an interaction between a group and a set (which *could* be another group, but doesn't have to be). Here are some relevant definitions.

- We will call the group G and the set X. An *action* of G on X is a function that sends pairs in $G \times X$ to things in X. We write $(g, x) \mapsto gx$, satisfying

  $X = \{1,2,3,4\}$

  if $x \sim y$ then $gx = y$, $y \sim x$, $g^{-1}y = x$

  1. $ex = x$ for all $x \in X$
  2. $(g_1 g_2)x = g_1(g_2 x)$ for all $x \in X$ and all $g_1, g_2 \in G$.

  $g = (1432)$ $h = (12)$

  We call X a *G-set*. $(h,1) \mapsto 2$

  $g \cdot 1 = 4$ $(g,1) \mapsto 4$ $g \cdot 2 = 1$ $g \cdot 3 = 2$ $g \cdot 4 = 3$

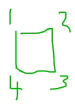
- Two elements $x, y \in X$ are *G-equivalent* (written $x \sim y$) provided there is some $g \in G$ such that $gx = y$.

  is $2 \sim 1$? yes b/c $g \cdot 1 = 2$ for some $g \in G$.

- The *orbit* of an element $x \in X$ (written $\mathcal{O}_x$) is the set of all elements $y \in X$ that are G-equivalent to x.

  $\mathcal{O}_x \subseteq X$

- The *fixed point set* of an element $g \in G$ (written X_g) is the set of all $x \in X$ such that $gx = x$.



- The *stabilizer subgroup* of an element $x \in X$ (written G_x) is the set of all $g \in G$ such that $gx = x$. $G_1 = \{(1), (24)\}$

To get used to these new definitions, let's work a few examples.

1. Let $X = \{1, 2, 3, 4, 5, 6\}$ and $G = \{(1), (12)(3456), (35)(46), (12)(3654)\}$ (a subgroup of S_6). G acts on X by $(\sigma, x) \mapsto \sigma(x)$.

   (a) For each $x \in X$, find $\mathcal{O}_x$, the orbit of x in G.

   $\mathcal{O}_1 = \{1, 2\}$ $\mathcal{O}_3 = \{3,4,5,6\} = \mathcal{O}_4 = \mathcal{O}_5 = \mathcal{O}_6$

   $\mathcal{O}_2 = \{1, 2\}$

   (b) For each $g \in G$, find X_g, the fixed point set of g in X.

   $X_{(1)} = X$ $X_{(35)(46)} = \{1, 2\}$

   $X_{(12)(3456)} = \emptyset = X_{(12)(3654)}$

   (c) For each $x \in X$, find G_x, the stabilizer subgroup of x in G.

   $G_1 = \{(1), (35)(46)\}$ $G_3 = \{(1)\} = G_4 = G_5 = G_6$

   $G_2 = \{(1), (35)(46)\}$

   (d) Do you notice anything about the sizes of the sets you found, how many different sets you found, etc?