

2. The identity can be written as $\varepsilon = (13)(24)(35)(14)(12)(15)(34)(45)$. Mimic the proof that ε must be even and show how to eliminate x = 5 from the product of transpositions and write ε as the product of 2 fewer transpositions in the process. Show all intermediate steps.

3.	Suppose	the	group	G	has	subnormal	series

$$G\supset H\supset \{e\}$$

and that $G/H \cong \mathbb{Z}_{10}$. Assume also that H is simple.

(a) Explain how we know that the above series is not a composition series.

(b) Explain how we could find two different composition series for G.

(c) Prove that if H is abelian, then G is solvable.

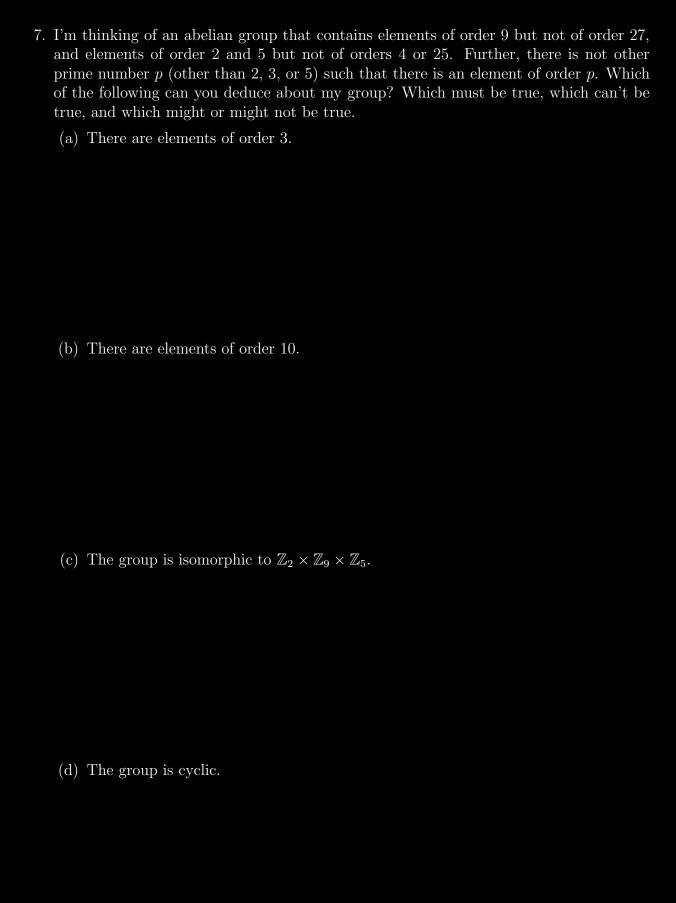
(d) If G happens to be the Galois group for some field E over \mathbb{Q} , what can you say about subfields of E?

Consider the polynomial $p(x) = x^3 + 5x^2 - 10x + 15$. Let E be the splitting field for $p(x)$ and G be the Galois group of E over \mathbb{Q} .
(a) Prove that G contains an element of order 3.
(b) Prove that G contains an element of order 2.
(c) Explain how we know that there is a intermediate field I strictly between $\mathbb Q$ and I that is the splitting field for a polynomial. What can you say about this field?
(d) Explain how you know that $G \cong S_3$ and not to \mathbb{Z}_6 .
(e) Does the argument above prove that $p(x)$ is not solvable by radicals? Is $p(x)$ solvable by radicals?

- 5. Consider the number $n = 1643 = 31 \cdot 53$.
 - (a) What is 42^{1560} congruent to modulo 1643? Explain, using group theory. What if we replaced 42 with another number?

(b) Note that E=7 is relatively prime to 1643. Find an integer D such that $(a^7)^D\equiv a\pmod{1643}$ for any a relatively prime to 1643. Explain how you know your D works.

6.	What is the difference Illustrate with an exam	between an inner nple.	r direct product	and an external	direct product?



8. Find all abelian groups of order 480.

9. Let $X = \{1, 2, 3, 4, 5, 6\}$ and $G = \{(1), (12), (345), (354), (12)(345), (12)(354)\}$. Find X_g , G_x and \mathcal{O}_x for each $g \in G$ and $x \in X$. Then verify the orbit-stabilizer theorem and Burnside's theorem.

10.	three colors?	ways could	tne	vertices	or an	equilateral	triangle	be colored	using

11. Use Burnside's theorem to explain why $\binom{7}{3} = \frac{1}{3!}P(7,3)$. That is, why are there 6 times as many ways to make three scoop ice-cream cones chosen from 7 flavors as there are to make three scoop milkshakes (cones and shakes not allowing for repeated flavors).