

Recall that last semester we saw that $\mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$. When does this sort of thing happen?

- 1

For any n , the group U_n is the set of all positive integers less than and relatively prime to n , under multiplication modulo n . For example we saw that $U_8 = \{1, 3, 5, 7\}$ is a group under multiplication modulo 8.

Consider the group U_{28} . The table below gives the twelve elements with their orders:

g	1	3	5	9	11	13	15	17	19	23	25	27
$\text{ord}(g)$	1	6	6	3	6	2	2	6	6	6	3	2

4. Let $G(n)$ be the set of all elements of order some power of n . Find $G(2)$ and $G(3)$ for U_{28} .

5. Are $G(2)$ and $G(3)$ subgroups of U_{28} ?

6. Do $G(2)$ and $G(3)$ have the property that $G(2) \cap G(3) = \{1\}$ and $U_{28} = G(2)G(3)$?

7. Is $U_{28} \cong G(2) \times G(3)$? Is $U_{28} \cong \mathbb{Z}_m \times \mathbb{Z}_n$ for some values of m and n ?