We will consider a new sort of operation: an interaction between a group and a set (which *could* be another group, but doesn't have to be). Here are some relevant definitions.

- We will call the group G and the set X. An *action* of G on X is a function that sends pairs in $G \times X$ to things in X. We write $(g, x) \mapsto gx$, satisfying

  1. $ex = x$ for all $x \in X$
  2. $(g_1 g_2)x = g_1(g_2 x)$ for all $x \in X$ and all $g_1, g_2 \in G$.

  We call X a *G-set*.

- Two elements $x, y \in X$ are *G-equivalent* (written $x \sim y$) provided there is some $g \in G$ such that $gx = y$.

- The *orbit* of an element $x \in X$ (written $\mathcal{O}_x$) is the set of all elements $y \in X$ that are G-equivalent to x.

- The *fixed point set* of an element $g \in G$ (written X_g) is the set of all $x \in X$ such that $gx = x$.

- The *stabilizer subgroup* of an element $x \in X$ (written G_x) is the set of all $g \in G$ such that $gx = x$.

To get used to these new definitions, let's work a few examples.

1. Let $X = \{1, 2, 3, 4, 5, 6\}$ and $G = \{(1), (12)(3456), (35)(46), (12)(3654)\}$ (a subgroup of S_6). G acts on X by $(\sigma, x) \mapsto \sigma(x)$.

   (a) For each $x \in X$, find $\mathcal{O}_x$, the orbit of x in G.

   (b) For each $g \in G$, find X_g, the fixed point set of g in X.

   (c) For each $x \in X$, find G_x, the stabilizer subgroup of x in G.

   (d) Do you notice anything about the sizes of the sets you found, how many different sets you found, etc?