Burnside's Lemma (Counting Theorem)

Wednesday, April 15

How many ways can you place blue and gold placemats around a 10 seat round table?

How many ways can you color the faces of a cube using 3 colors?

$G =$ Symmetries of a cube

$= \{ r_0, r_1, r_2, r_3, \rho_1, \rho_2, \rho_3, \varphi_1, \varphi_2, \varphi_3, d_1, d_1^2, d_2, d_2^2, d_3, d_3^2, d_4, d_4^2, e_1, e_2, e_3, e_4, e_5, e_6 \}$

r_i = rotation fixing front/back.

ρ_i = fix top/bottom

φ_i = fix left/right

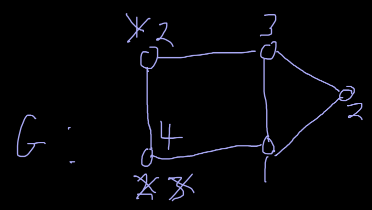
d_1 = fix top right front corner

$$K = \frac{1}{24}\left(3^6 + 6 \cdot 3^3 + 3 \cdot 3^4 + 8 \cdot 3^2 + 6 \cdot 3^3 \right)$$

r_0 $\quad$ $r_1\, r_3\, \rho_1\, \rho_3\, \varphi_1\, \varphi_3$ $\quad$ $r_2\, \rho_2\, \varphi_2$ $\quad$ d_i^j

$$= \frac{1368}{24} = 57$$

Chromatic number of a graph.　　　　4-color Theorem.



$\chi(G) = 3$

proper vertex coloring

How many 3-colorings of the graph are possible?
4-colorings