Name: _____

(10pts)   1. Consider the polynomial $x^3 - 7$ in $\mathbb{Q}[x]$. Outside of $\mathbb{Q}$, this has a root $\sqrt[3]{7}$, so let's consider the extension field $\mathbb{Q}(\sqrt[3]{7})$. We can also consider the quotient ring $\mathbb{Q}[x]/\langle x^3 - 7 \rangle$.

      (a) In $\mathbb{Q}[x]/\langle x^3 - 7 \rangle$, find the inverse of the coset $\langle x^3 - 7 \rangle + x^2 - x$. You should use the Euclidean algorithm (show all your steps) and explain why your answer is correct.

      (b) Use your solution above to find the inverse of the element $-\sqrt[3]{7} + \sqrt[3]{7}^2$ in $\mathbb{Q}(\sqrt[3]{7})$. Briefly explain how you know your answer must be correct, even if you don't do any calculations in the field.