MATH 322 **Quiz 4 Solutions** Monday, February 25

Name: ____________________

1. Let G be a group with elements $\{e, a, b, c, d, f, g, h\}$. Here is part of the table for G:

$$\begin{array}{c|cccccccc}
 & e & a & b & c & d & f & g & h \\
\hline
\vdots & & & & \vdots & & & & \\
c & c & b & e & a & h & g & d & f \\
\vdots & & & & \vdots & & & &
\end{array}$$

(3pts) (a) Find the left regular representation of c as in Cayley's theorem. That is, find the permutation λ_c in S_8 corresponding to c.

Solution: $\lambda_c = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 1 & 2 & 8 & 7 & 5 & 6 \end{pmatrix}$

(3pts) (b) Suppose $\lambda_d = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 2 & 1 & 3 & 4 \end{pmatrix}$ is the left regular representation of d. Write the row for d in the group table.

Solution: The row will read $d\ f\ g\ h\ a\ e\ b\ c$

(4pts) (c) If you compute $\lambda_c\lambda_d$ you get another permutation in S_8. Of which element in G is it the left regular representation? That is, find $x \in G$ such that $\lambda_x = \lambda_c\lambda_d$. Briefly explain how you know you are right.

Solution: $\lambda_c\lambda_d = \lambda_{cd}$ by the way we define regular representations. But because G is isomorphic to the group of regular representations, we know λ_{cd} corresponds to cd which in G equals h. Thus $\lambda_c\lambda_d = \lambda_h$.

(3bn-pts) 2. Bonus: Which of the permutations you found above are even? Briefly explain how you know