MATH 322 **Homework 6** Due: Wednesday, March 27

Instructions: Same rules as usual. Work together, write-up alone, no internet!

(6pts) 1. For any prime p, a *p-group* is a group of order p^n for some n.

(a) Explain why every element of a p group has order that is a power of p.

(b) Prove that for any group G, if every element has order some power of p, then G is a p-group. Hint: apply Cauchy's theorem.

(6pts) 2. Prove, using inner direct products, that $\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n$ if and only if $\gcd(m, n) = 1$. Note that the textbook has a proof of this using other methods, but you must use inner direct products for credit here.

(6pts) 3. Consider the group $U_{35} = \{1, 2, 3, 4, 6, 8, 9, 11, 12, 13, 16, 17, 18, 19, 22, 23, 24, 26, 27, 29, 31, 32, 33, 34\}$ under the operation of multiplication modulo 35. The orders of the elements are:

g	1	2	3	4	6	8	9	11	12	13	16	17	18	19	22	23	24	26	27	29	31	32	33	34
$\text{ord}(g)$	1	12	12	6	2	4	6	3	12	4	3	12	12	6	4	12	6	6	4	2	6	12	12	2

(a) Find two p-groups H and K such that U_{35} is the internal direct product of H and K. Briefly explain why your groups work.

(b) Let H be the larger of the two groups above. Show how to decompose it as the internal direct product of $\langle a \rangle$ and H' where a is of maximal order and H' is some other subgroup of H.

(c) Using the decompositions above (perhaps repeating the second step as needed), write U_{35} as the direct product of groups of the form $\mathbb{Z}_{p^k}$ (p prime).

(6pts) 4. Describe all abelian groups of order 200 (up to isomorphism). Explain how you know you have them all.

(6pts) 5. Let G, H, and K be finite abelian groups. Suppose $G \times H \cong G \times K$. Prove that $H \cong K$.