Math 322 **Exam 2 – In Class** April 15, 2019

Name: ______________________________

For the problems on this page, let G be a finite group containing an element a.

(6pts) 1. Prove that if $\mathrm{ord}(a) = n$ and $a^k = e$, then k is a multiple of n. Your proof should use the division algorithm.

(6pts) 2. What can you say about the orders of a^m? Use the previous question to prove that for any m we have that $\mathrm{ord}(a^m)$ is a divisor of $\mathrm{ord}(a)$. (Another way to say this is that $\mathrm{ord}(a)$ is a multiple of $\mathrm{ord}(a^m)$.)

(6pts) 3. Find all abelian groups of order $1125 = 3^2 \cdot 5^3$ which contain at least one element of order 25. Briefly explain how you know you have them all.

(4pts) 4. Of the groups (of order 1125) described in the previous question, how many are cyclic? Explain.

(16pts) 5. For each of the statements below, decide whether they are **TRUE** or **FALSE**. Then justify your choices either with a brief explanation (if true) or counterexample (if false).

(a) Every element in S_8 can be written as the product of 8 transpositions.

(b) Every element of A_8 can be written as the product of 8 transpositions.

(c) For any group G, all subnormal series of G are the same length.

(d) If a has order 5 then the cyclic group $\langle a \rangle$ is isomorphic to a subgroup of S_5.

(12pts) 6. Suppose $p(x)$ is a degree 7 polynomial that is irreducible over $\mathbb{Q}$. Let E be the splitting field for $p(x)$, and let $G = \mathrm{Gal}(E/\mathbb{Q})$ be the Galois group.

(a) Carefully explain (citing any appropriate theorem) why G has an element of order 7.

(b) Suppose $p(x)$ has at least one non-real complex root. Carefully explain why G has an element of order 2.