Activity: Geometric Constructions

Our first application of algebra (to other mathematics) will be to questions of classical geometry. We will look at geometry as it was done in ancient Greece, except that we will use GeoGebra for our constructions.

Our main question is, what can you *construct* using reasonable, fundamental tools. The tools are: an *unmarked straightedge* and a *compass*.

In GeoGebra, there are many more tools than these. Make sure you only use the "new point" tool (to place points at the intersections of lines and circles), the "line through two points" tool, and the "compass" tool (under the circle menu). You can also use the arrow to drag things around if you need to.

To get a feel for the sorts of things you can construct, and maybe things you cannot, here are a few challenges.

1. Can you construct a 60° angle? A 30° angle? If you have constructed any angle at all, can you construct an angle half its measure? That is, can you *bisect* an given angle?

2. Can you construct a square? Can you double the square? That is, if you can construct a square, can you construct a square of twice the area? Careful: this is not a square whose side length is twice the side length of the original.

3. Can you double the circle? That is, can you construct a circle and then construct a second circle of twice the area?

4. Here are three much harder, but related challenges. For each, play around enough to convince yourself these are really hard, if not impossible:

(a) Can you *trisect* and angle? That is, given a constructed angle, can you construct an angle 1/3 its measure?

(b) Can you double the *cube*? That is, if you can constructed a cube (or at least a line segment which is the length of the edge of a cube), can you construct cube with twice the volume of the original?

(c) Can you square the circle? That is, if you have constructed a circle, can you construct a square that has the same area as the circle?