

## Networks Worksheet

**Basic laws for electrical circuits.** Monday, January 7This two-page worksheet was generously donated to the sample article by Virgil Pierce at a CuratedCourses workshop in August 2018. It has default (skinny) left and right margins, but we have specified longer top and bottom margins, with the top being the larger of the two.

**Theorem 0.1 Ohms Law.** *The current through a resistor is proportional to the ratio of the Voltage to the Resistance*

$$I = \frac{V}{R}$$

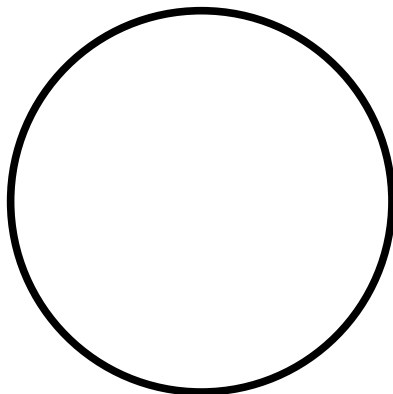
*Or for our purposes*

$$IR = V$$

**Theorem 0.2 Kirchoffs Current Law.** *The sum of the currents in a network meeting at a point is zero.*

$$\sum_{k=1}^n I_k = 0$$

**Example 0.3 Kirchhoff's Current Law.** For the circuit below  $I_1 + I_2 = I_3$ .



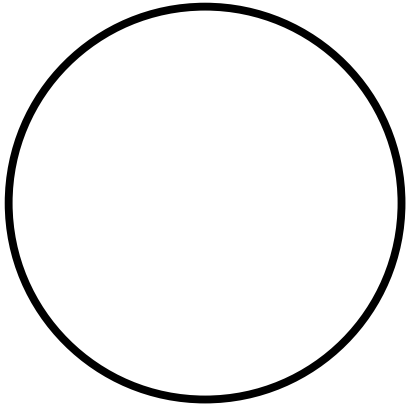
□

**Theorem 0.4 Kirchoffs Voltage Law.** *The sum of the voltages around any closed circuit (or subcircuit) is zero.*

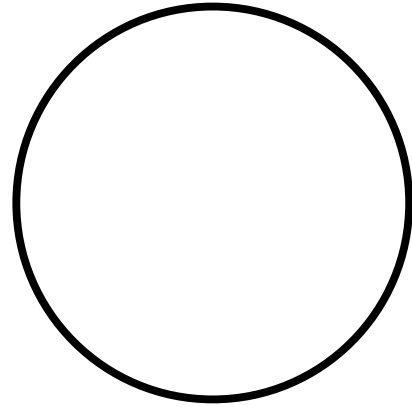
$$\sum_{k=1}^n V_k = 0$$

Kirchoffs Current Law and Kirkoffs Voltage Law combined with Ohms Law gives for any circuit of resistors and sources a linear system that may (or may not) determine the currents.

1. For the simple network pictured, calculate the amperage in each part of the network by setting up a system of linear equations for the amperages.

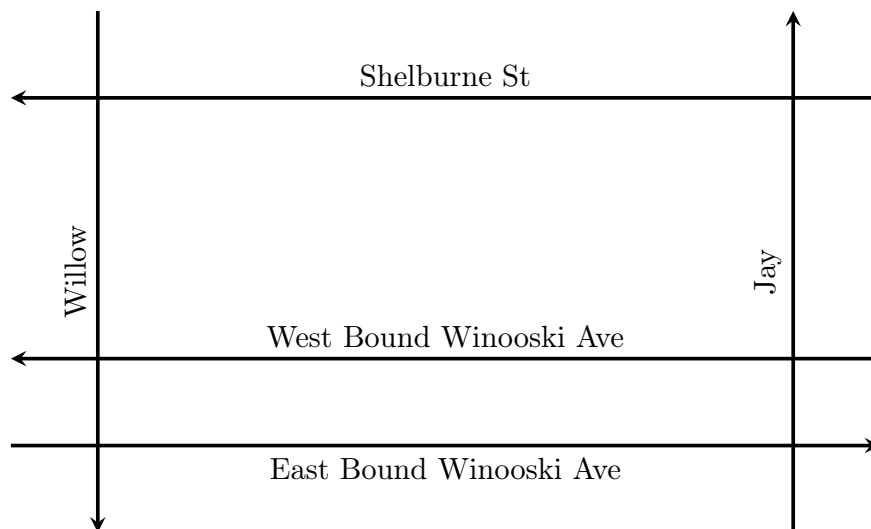


2. Compare it with a parallel circuit network. Calculate the amperage in each part of the network by setting up a system of linear equations for the amperages.



3. Now for a more complicated network. Calculate the amperage in each part of the network by setting up a system of linear equations for the amperages.

4. Now generalize these ideas to a context outside of electrical circuits. Consider the network of streets given in the diagram (with one-way directions as indicated).



A traffic engineer counts the hourly flow of cars into and out of this network at the entrances. They get (EB = East Bound; WB = West Bound):

	EB Winooski	WB Winooski	Shelburne St	Willow	Jay
into	50	400	0	10	50
out of	55	390	20	15	30

**Table 5:** Estimated hourly traffic flow for the road network.

Use a variable for each segment inside of the network and set up a system of linear equations restricting the flow. Solve the system. Note that you should not get a unique solution as traffic should be able to flow through the network in various ways.