Activity: Working with S_5 (part 1)

To prove that there is a degree 5 polynomial that is not solvable by radicals, we need to understand how S_5 works. Today will will prove that S_5 is not a *solvable* group.

   To do this, we can show that A_5 is simple, because then the only normal series we can get for S_5 will be

$$S_5 \supset A_5 \supset \{(1)\}$$

but A_5 is not abelian, so S_5 is not solvable.

1. Show that A_5 is generated by the set of 3-cycles. You can do this by showing that every pair of transpositions can be written as a product of 3-cycles.

2. Show that if a normal subgroup N of A_5 contains even one 3-cycle, then it is all of A_5. Remember, normal subgroups are closed under conjugates (here the conjugate would be aba^{-1} where $a \in A_5$ and $b \in N$).

3. Finally, show that every non-trivial normal subgroup of A_n contains a 3-cycle.