Math 322 **Homework 8** Due: Friday, May 3

Instructions: Same rules as always. Turn this in early for a 5% bonus for every day it is early (before Friday, May 3rd).

(6pts) 1. Let G be the additive group of real numbers, $(\mathbb{R}, +)$ and let $X = \mathbb{R}^2$ be the real plane. For each point $P \in X$ and $r \in G$, define (r, P) to be the point P lands on after rotating the plane counterclockwise about the origin through r radians.

(a) Prove that X is a G-set. That is, prove that the procedure above defines an action of G on X.

(b) Pick a point P other than the origin. Describe geometrically the orbit containing P.

(c) Find the group G_P.

(9pts) 2. Let $X = \{1, 2, 3, 4, 5, 6\}$ and let $G = \{(1), (123), (132), (45), (123)(45), (132)(45)\}$. Let G act on X in the obvious way.

(a) For each $x \in X$ and $g \in G$, find $\mathcal{O}_x$, G_x and X_g. Label these clearly.

(b) Verify the *orbit-stabilizer theorem* and *Burnside's lemma* for this example and explain (i.e., demonstrate that you know what these are and mean).

(c) To thank your professors for doing such an amazing job all semester, you decide to bake 6 pies. You will give 3 to your favorite abstract algebra teacher, 2 to your next favorite teacher, and 1 to your third favorite. You know how to make 4 different types of pies. How many different pie-to-professor combinations can you create? Use Burnside's lemma and explain how this relates to the X and G in this problem.

(12pts) 3. There are 24 different rotations of a cube (think about where the axis of rotation could be). Use these, together with Burnside's lemma to count the number of ways to color the cube as described below. In each case, explain how your arrived at your answer (make a table relating the sorts of rotations to size of their fixed sets).

(a) Describe the 24 rotations carefully (break them into groups).

(b) Color the faces of a cube using up to 4 colors.

(c) Color the vertices of a cube using up to 3 colors.

(d) Color the edges of a cube using up to 3 colors.

(3pts) 4. How many ways can you label a 6-sided die (number cube) with the numbers 1 through 6, giving each side a different number?