Chapter 1

Fake chapter

1.1 Functions

1.1.1 Definition of Function

We often want to predict values of one variable from the values of a related variable. For example, when a physician prescribes a drug in a certain dosage, she needs to know how long the dose will remain in the bloodstream. A sales manager needs to know how the price of his product will affect its sales. A **function** is a special type of relationship between variables that allows us to make such predictions.

Suppose it costs \$800 for flying lessons, plus \$30 per hour to rent a plane. If we let C represent the total cost for t hours of flying lessons, then

$$C = 800 + 305t \quad (t \geq 0)$$

Thus, for example

$$\begin{array}{lll} \text{when} & t = \mathbf{0}, & C = 800 + 30(\mathbf{0}) = 800 \\ \text{when} & t = \mathbf{4}, & C = 800 + 30(\mathbf{4}) = 920 \\ \text{when} & t = \mathbf{10}, & C = 800 + 30(\mathbf{10}) = 1100 \end{array}$$

The variable t is called the **input** or **independent** variable, and C is the **output** or **dependent** variable, because its values are determined by the value of t. We can display the relationship between two variables by a table or by ordered pairs. The input variable is the first component of the ordered pair, and the output variable is the second component.

t	C	(t, C)
0	800	$(0, 800)$
4	920	$(4, 920)$
10	1100	$(10, 1100)$

For this relationship, we can find the value C of associated with any given value of t. All we have to do is substitute the value of t into the equation and solve for C. The result has no ambiguity: Only one value for C corresponds to each value of t. This type of relationship between variables is called a **function**. In general, we make the following definition.

Definition of Function

A **function** is a relationship between two variables for which a unique value of the **output** variable can be determined from a value of the **input** variable.

What distinguishes functions from other variable relationships? The definition of a function calls for a *unique value*—that is, *exactly one value* of the output variable corresponding to each value of the input variable. This property makes functions useful in applications because they can often be used to make predictions.

Example 1.1.

a The distance, d, traveled by a car in 2 hours is a function of its speed, r. If we know the speed of the car, we can determine the distance it travels by the formula $d = r \cdot 2$.

b The cost of a fill-up with unleaded gasoline is a function of the number of gallons purchased. The gas pump represents the function by displaying the corresponding values of the input variable (number of gallons) and the output variable (cost).

c Score on the Scholastic Aptitude Test (SAT) is not a function of score on an IQ test, because two people with the same score on an IQ test may score differently on the SAT; that is, a person's score on the SAT is not uniquely determined by his or her score on an IQ test.

Exercise 1.2.

a As part of a project to improve the success rate of freshmen, the counseling department studied the grades earned by a group of students in English and algebra. Do you think that a student's grade in algebra is a function of his or her grade in English? Explain why or why not.

b Phatburger features a soda bar, where you can serve your own soft drinks in any size. Do you think that the number of calories in a serving of Zap Kola is a function of the number of fluid ounces? Explain why or why not.

1.1.2 Functions Defined by Tables

When we use a table to describe a function, the first variable in the table (the left column of a vertical table or the top row of a horizontal table) is the input variable, and the second variable is the output. We say that the output variable is a function of the input.

Example 1.3.

a Table 1.4 shows data on sales compiled over several years by the accounting office for Eau Claire Auto Parts, a division of Major Motors. In this example, the year is the input variable, and total sales is the output. We say that total sales, S, is a function of t.

Year (t)	Total sales (S)
2000	\$612,000
2001	\$663,000
2002	\$692,000
2003	\$749,000
2004	\$904,000

Table 1.4

b Table 1.5 gives the cost of sending printed material by first-class mail in 2016.

Weight in ounces (w)	Postage (P)
$0 < w \leq 1$	\$0.47
$1 < w \leq 2$	\$0.68
$2 < w \leq 3$	\$0.89
$3 < w \leq 4$	\$1.10
$4 < w \leq 5$	\$1.31
$5 < w \leq 6$	\$1.52
$6 < w \leq 7$	\$1.73

Table 1.5

If we know the weight of the article being shipped, we can determine the required postage from Table 1.5. For instance, a catalog weighing 4.5 ounces would require \$1.31 in postage. In this example, w is the input variable and p is the output variable. We say that *p is a function of w*.

c Table 1.6 records the age and cholesterol count for 20 patients tested in a hospital survey.

Age	Cholesterol count		Age	Cholesterol count
53	217		**51**	**209**
48	232		53	241
55	198		49	186
56	238		**51**	**216**
51	**227**		57	208
52	264		52	248
53	195		50	214
47	203		56	271
48	212		53	193
50	234		48	172

Table 1.6

According to these data, cholesterol count is *not* a function of age, because several patients who are the same age have different cholesterol levels. For example, three different patients are 51 years old but have cholesterol counts of 227, 209, and 216, respectively. Thus, we cannot determine a *unique* value of the output variable (cholesterol count) from the value of the input variable (age). Other factors besides age must influence a person's cholesterol count.

Exercise 1.7. Decide whether each table describes y as a function of x. Explain your choice.

a

x	3.5	2.0	2.5	3.5	2.5	4.0	2.5	3.0
y	2.5	3.0	2.5	4.0	3.5	4.0	2.0	2.5

b

x	-3	-2	-1	0	1	2	3
y	17	3	0	-1	0	3	17

1.1.3 Functions Defined by Graphs

A graph may also be used to define one variable as a function of another. The input variable is displayed on the horizontal axis, and the output variable on the vertical axis.

Example 1.8. Figure 1.9 shows the number of hours, H, that the sun is above the horizon in Peoria, Illinois, on day t, where January 1 corresponds to $t = 0$.

a Which variable is the input, and which is the output?

b Approximately how many hours of sunlight are there in Peoria on day 150?

c On which days are there 12 hours of sunlight?

d What are the maximum and minimum values of H, and when do these valuesoccur?

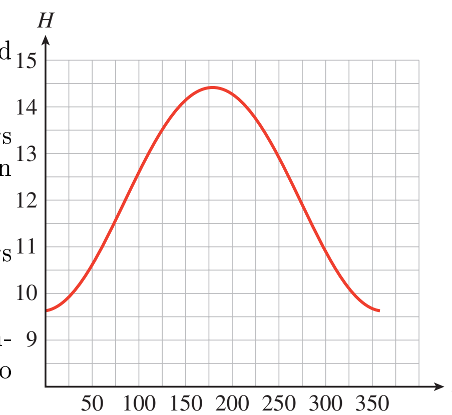

Figure 1.9

Solution.

a The input variable, t, appears on the horizontal axis. The number of daylight hours, H, is a function of the date. The output variable appears on the vertical axis.

b The point on the curve where $t = 150$ has $H \approx 14.1$, so Peoria gets about 14.1 hours of daylight when $t = 150$, which is at the end of May.

c $H = 12$ at the two points where $t \approx 85$ (in late March) and $t \approx 270$ (late September).

d The maximum value of 14.4 hours occurs on the longest day of the year, when $t \approx 170$, about three weeks into June. The minimum of 9.6 hours occurs on the shortest day, when $t \approx 355$, about three weeks into December.

Exercise 1.10. Figure 1.11 shows the elevation in feet, a, of the Los Angeles Marathon course at a distance d miles into the race. (Source: *Los Angeles Times*, March 3, 2005)

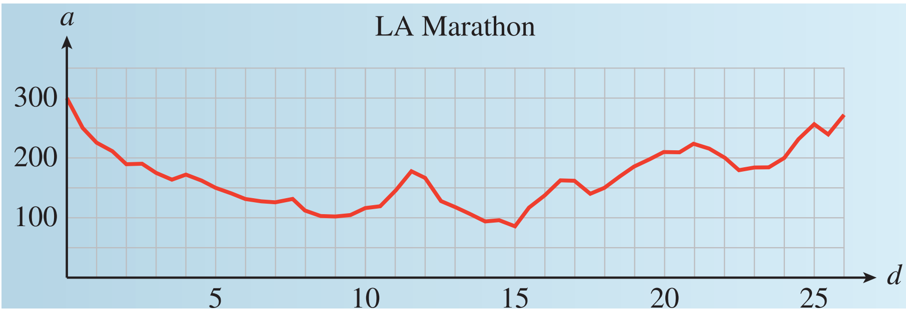

Figure 1.11

a Which variable is the input, and which is the output?

b What is the elevation at mile 20?

c At what distances is the elevation 150 feet?

d What are the maximum and minimum values of a, and when do these values occur?

e The runners pass by the Los Angeles Coliseum at about 4.2 miles into the race. What is the elevation there?

1.1.4 Functions Defined by Equations

Example 1.12 illustrates a function defined by an equation.

Example 1.12. As of 2016, One World Trade Center in New York City is the nation's tallest building, at 1776 feet. If an algebra book is dropped from the top of the Sears Tower, its height above the ground after t seconds is given by the equation

$$h = 1776 - 16t^2$$

Thus, after **1** second the book's height is

$$h = 1776 - 16(\mathbf{1})^2 = 1760 \text{ feet}$$

After **2** seconds its height is

$$h = 1776 - 16(\mathbf{2})^2 = 1712 \text{ feet}$$

For this function, t is the input variable and h is the output variable. For any value of t, a unique value of h can be determined from the equation for h. We say that *h is a function of t.*

Exercise 1.13. Write an equation that gives the volume, V, of a sphere as a function of its radius, r.