INTRODUCTION

An abstract algebra consists of a set of objects (integers, real numbers, permutations, polynomials, matrices, …), various binary operations, along with some properties (closure, inverses, commutativity,…). Examples of abstract algebras include groups, rings, integral domains, and fields. Operations include rotations of regular geometrical figures, ordinary and modular addition and multiplication, addition and multiplication of matrices and of polynomials, composition of permutation cycles, direct products and others.

In one sense the core ideas of algebra are abstracted out and viewed from a much larger lens. For example, the problem of finding analogues of the quadratic formula, around the mid 1500’s, led to the study of the symmetric groups which shed light on the nonsolvability of the general quintic.

Applications are plentiful. Among the many fields of study making significant use of algebraic structures we include cryptography, genetics, mineralogy, the study of molecular structures in chemistry, elementary particle theory in physics, Latin squares in statistical experiments, and finally, architecture and art.

Important contributors over the past several centuries include Joseph Lagrange, Niels Abel, Arthur Cayley, Emmy Noether, Gauss, Galois, Sylow among many others.