

# GAME-DA Documentation

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# Chapter 1

## Introduction

GAME-DA provides *data assimilation (DA)* functionality for the GAME model. The theoretical derivations are presented in [1]. This documentation contains the technical details of the implementations of the data assimilation algorithms. Three different methods of DA are implemented in GAME-DA:

- optimum interpolation (OI)
- three-dimensional variational data assimilation (3D-Var)
- four-dimensional variational data assimilation (4D-Var)

Depending on the resolution and the type and amount of observations, certain methods are more suitable than others.

In any of the three cases, the basic workflow is as follows:

1. Downloading the observations. This is done by the bash scripts residing in the directory `obs_collector`.
2. Bringing the observations into a standardized format. This is done by the separate executable formatter residing in the directory of the same name.
3. Executing the data assimilation itself, resulting in an input file for the model.

## Chapter 2

# Optimum interpolation

Let  $M \geq 1$  be the number of degrees of freedom of the model,  $N \geq 1$  the number of observations,  $\mathbf{h} \in \mathbb{R}^N$  the observations as reconstructed from the background state,  $\mathbf{y} \in \mathbb{R}^N$  the actual observations,  $\overleftarrow{\mathbf{H}} \in \mathbb{R}^{N \times M}$  the Jacobian of the observations operator,  $\overleftarrow{\mathbf{B}} \in \mathbb{R}^{M \times M}$  the background error covariance matrix and  $\overleftarrow{\mathbf{R}} \in \mathbb{R}^{N \times N}$  the observations error covariance matrix.  $\overleftarrow{\mathbf{R}}$  is usually assumed to be diagonal, which is an excellent approximation if the observation errors are of statistical nature. This does not lead to useful simplifications, however.

It is

$$\left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}}\right)_{i,j} = \sum_{k=1}^M \overleftarrow{\mathbf{H}}_{i,k} \overleftarrow{\mathbf{B}}_{k,j}. \quad (2.1)$$

For  $\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T$ , one obtains

$$\left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T\right)_{i,j} = \left[\left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}}\right) \overleftarrow{\mathbf{H}}^T\right]_{i,j} = \sum_{k=1}^M \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}}\right)_{i,k} \overleftarrow{\mathbf{H}}_{k,j}^T = \sum_{k=1}^M \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}}\right)_{i,k} \overleftarrow{\mathbf{H}}_{j,k} = \sum_{k=1}^M \left(\sum_{l=1}^M \overleftarrow{\mathbf{H}}_{i,l} \overleftarrow{\mathbf{B}}_{l,k}\right) \overleftarrow{\mathbf{H}}_{j,k} = \sum_{k,l=1}^M \overleftarrow{\mathbf{H}}_{i,l} \overleftarrow{\mathbf{B}}_{l,k} \overleftarrow{\mathbf{H}}_{j,k}. \quad (2.2)$$

For the final result, we obtain

$$\begin{aligned} x_i &= x_{B,i} + \sum_{j=1}^N \left[ \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T + \overleftarrow{\mathbf{R}}\right)^{-1} \right]_{i,j} (y_j - h_j) = x_{B,i} + \sum_{j=1}^N \left[ \sum_{k=1}^N \left(\overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T\right)_{i,k} \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T + \overleftarrow{\mathbf{R}}\right)^{-1}_{k,j} \right] (y_j - h_j) \\ &= x_{B,i} + \sum_{j=1}^N \left[ \sum_{k=1}^N \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T + \overleftarrow{\mathbf{R}}\right)^{-1}_{k,j} \left(\overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T\right)_{i,k} \right] (y_j - h_j) = x_{B,i} + \sum_{j=1}^N \left[ \sum_{k=1}^N \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T + \overleftarrow{\mathbf{R}}\right)^{-1}_{k,j} \left(\sum_{l=1}^M \overleftarrow{\mathbf{B}}_{i,l} \overleftarrow{\mathbf{H}}_{l,k}^T\right) \right] (y_j - h_j) \end{aligned}$$

$$\Rightarrow x_i = x_{B,i} + \sum_{j=1}^N \left[ \sum_{k=1}^N \left(\overleftarrow{\mathbf{H}} \overleftarrow{\mathbf{B}} \overleftarrow{\mathbf{H}}^T + \overleftarrow{\mathbf{R}}\right)^{-1}_{k,j} \left(\sum_{l=1}^M \overleftarrow{\mathbf{B}}_{i,l} \overleftarrow{\mathbf{H}}_{k,l}\right) \right] (y_j - h_j). \quad (2.3)$$

### 2.1 Inclusion of moisture

So far, moisture is taken into account in a separate assimilation process for efficiency.

## Chapter 3

# 3D-Var

### 3.1 Technicalities

The observations used come from a time window  $[-\frac{T}{2}, \frac{T}{2}]$  around the analysis time and are all taken to be valid at the analysis time.  $T = 3$  h is a typical value..

# Chapter 4

## 4D-Var

### 4.1 Technicalities

The observations used come from a time window  $[-\frac{T}{2}, \frac{T}{2}]$  around the analysis time and are taken to be valid at individual time steps  $n\Delta t$ .  $T = 6$  h and  $\Delta t = 15$  min are typical values.

# Bibliography

- [1] M. H. Balsmeier. *Kompendium Theoretische Meteorologie*. 2021. URL: <https://raw.githubusercontent.com/MHBalsmeier/kompendium/master/kompendium.pdf>.