GAME-DA Documentation

GAME-DA Development Team

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Chapter 1

Introduction

GAME-DA provides *data assimilation (DA)* funcitonality for the GAME model. The theoretical derivations are presented in [1]. This documentation contains the technical details of the implementations of the data assimilation algorithms. Three different methods of DA are implemented in GAME-DA:

- optimum interplation (OI)
- three-dimensional variational data assimilation (3D-Var)
- four-dimensional variational data assimilation (4D-Var)

Depending on the resolution and the type and amount of observations, certain methods are more suitable than others.

In any of the three cases, the basic workflow is as follows:

1. Downloading the observations. This is done by the bash scripts residing in the directory `obs_collector`.
2. Bringing the observations into a standardized format. This is done by the separate executable `formatter` residing in the directory of the same name.
3. Executing the data assimilation itself, resulting in an input file for the model.

Chapter 2

Optimum interpolation

Let $M \geq 1$ be the number of degrees of freedom of the model, $N \geq 1$ the number of observations, $\mathbf{h} \in \mathbb{R}^N$ the observations as reconstructed from the background state, $\mathbf{y} \in \mathbb{R}^N$ the actual observations, $\overleftrightarrow{H} \in \mathbb{R}^{N\times M}$ the Jacobian of the observations operator, $\overleftrightarrow{B} \in \mathbb{R}^{M\times M}$ the background error covariance matrix and $\overleftrightarrow{R} \in \mathbb{R}^{N\times N}$ the observations error covariance matrix. $\overleftrightarrow{R}$ is usually assumed to be diagonal, which is an excellent approximation if the observation erros are of statistical nature. This does not lead to useful simplifications, however.

It is

$$\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\right)_{i,j} = \sum_{k=1}^{M} \overleftrightarrow{H}_{i,k}\overleftrightarrow{B}_{k,j}. \tag{2.1}$$

For $\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T$, one obtains

$$\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T\right)_{i,j} = \left[\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\right)\overleftrightarrow{H}^T\right]_{i,j} = \sum_{k=1}^{M}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\right)_{i,k}\overleftrightarrow{H}^T_{k,j} = \sum_{k=1}^{M}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\right)_{i,k}\overleftrightarrow{H}_{j,k} = \sum_{k=1}^{M}\left(\sum_{l=1}^{M}\overleftrightarrow{H}_{i,l}\overleftrightarrow{B}_{l,k}\right)\overleftrightarrow{H}_{j,k} = \sum_{k,l=1}^{M}\overleftrightarrow{H}_{i,l}\overleftrightarrow{B}_{l,k}\overleftrightarrow{H}_{j,k}. \tag{2.2}$$

For the final result, we obtain

$$\begin{aligned} x_i &= x_{B,i} + \sum_{j=1}^{N}\left[\overleftrightarrow{B}\,\overleftrightarrow{H}^T\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T + \overleftrightarrow{R}\right)^{-1}\right]_{i,j}\left(y_j - h_j\right) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{B}\,\overleftrightarrow{H}^T\right)_{i,k}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T + \overleftrightarrow{R}\right)^{-1}_{k,j}\right]\left(y_j - h_j\right) \\ &= x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T + \overleftrightarrow{R}\right)^{-1}_{k,j}\left(\overleftrightarrow{B}\,\overleftrightarrow{H}^T\right)_{i,k}\right]\left(y_j - h_j\right) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T + \overleftrightarrow{R}\right)^{-1}_{k,j}\left(\sum_{l=1}^{M}\overleftrightarrow{B}_{i,l}\overleftrightarrow{H}^T_{l,k}\right)\right]\left(y_j - h_j\right) \end{aligned}$$

$$\boxed{\Rightarrow x_i = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\,\overleftrightarrow{B}\,\overleftrightarrow{H}^T + \overleftrightarrow{R}\right)^{-1}_{k,j}\left(\sum_{l=1}^{M}\overleftrightarrow{B}_{i,l}\overleftrightarrow{H}_{k,l}\right)\right]\left(y_j - h_j\right).} \tag{2.3}$$

2.1 Inclusion of moisture

So far, moisture is taken into account in a separate assimilation process for efficiency.

Chapter 3

3D-Var

3.1 Technicalities

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are all taken to be valid at the analysis time. $T = 3$ h is a typical value..

Chapter 4

4D-Var

4.1 Technicalities

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are taken to be valid at individual time steps $n\Delta t$. $T = 6$ h and $\Delta t = 15$ min are typical values.

Bibliography

[1] M. H. Balsmeier. *Kompendium Theoretische Meteorologie*. 2021. URL: https://raw.githubusercontent.com/MHBalsmeier/kompendium/master/kompendium.pdf.