ndvar Documentation

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1 3D-var

Let $\mathbf{X}$ be the state vector of the model atmosphere and $\mathbf{X}_B$ be the background state vector from a previous forecast. $\mathbf{Y}$ shall denounce the vector of observations valid at the analysis time. $\mathbf{Y}$ differs from $\mathbf{X}$ in size and the elements of $\mathbf{Y}$ are not observed at model grid points nor are they the prognostic model variables. Meta data must be available about the elements of $\mathbf{Y}$ containing information about the points of observation and what has actually been measured. The procedure *3D-var* is based on the minimization of a cost function $J = J(\mathbf{X}) \geq 0$ containing three terms:

- The distance between the analysis and the background state.
- The distance between the analysis and the observations.
- The distance between the analysis and the balance equation. This term vanishes at high resolutions.

Thus, one arrives at

$$\boxed{J(\mathbf{X}) = (\mathbf{X} - \mathbf{X}_B)^T \overleftrightarrow{A}_B (\mathbf{X} - \mathbf{X}_B)^T + \left(\mathbf{Y} - \overleftrightarrow{H}\mathbf{X}\right)^T \overleftrightarrow{A}_O \left(\mathbf{Y} - \overleftrightarrow{H}\mathbf{X}\right)^T} \tag{1}$$

Becasue of Eq. it is indeed

$$J \geq 0. \tag{2}$$

At the minimum of J,

$$\nabla J = 0 \tag{3}$$

holds. From Eq. one obtains

$$\nabla J = 2\overleftrightarrow{A}_B (\mathbf{X} - \mathbf{X}_B) + 2\overleftrightarrow{A}_O \left(\mathbf{Y} - \overleftrightarrow{H}\mathbf{X}\right). \tag{4}$$

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are all taken to be valid at the analysis time. $T = 3$ h is a typical value..

2 4D-var

4D-Var is based on 3D-var but also takes the time dimension into account. The cost function J is generalized as

$$J = J_{\text{3D-Var}} + \sum_{n=1}^{N} J_n, \tag{5}$$

where N is the number of time steps involved (excluding the analysis time itself) and J_n is the cost function at the time step n. $J_{\text{3D-Var}}$ is defined as in Eq. (1). Let $\mathbf{Y}_n$ be the vector of observations valid at the time step n. At every time step this vector might be of different size and of different meaning. $\mathbf{X}_n$ shall be the state vector valid at the time step n, which is deduced by the initial state vector $\mathbf{X}$ via

$$\mathbf{X}_n = M_n(\mathbf{X}), \tag{6}$$

where M_n is the model integration to step n. From M_n one deduces the so-called *tangent linear model operator* $\overleftrightarrow{M}_n$, which is a matrix. Thus, one obtains

$$\boxed{J_n = \left(\mathbf{Y}_n - \overleftrightarrow{H}_n \overleftrightarrow{M}_n \mathbf{X}\right)^T \overleftrightarrow{A}_{O,n} \left(\mathbf{Y}_n - \overleftrightarrow{H}_n \overleftrightarrow{M}_n \mathbf{X}\right).} \tag{7}$$

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are taken to be valid at individual time steps $n\Delta t$. $T = 6$ h and $\Delta t = 15$ min are typical values.