

GAME-DA Documentation

GAME-DA Development Team

Chapter 1

Optimum interpolation

It is

$$\left(\overleftarrow{H} \overleftarrow{B}\right)_{i,j} = \sum_{k=1}^M \overleftarrow{H}_{i,k} \overleftarrow{B}_{k,j}. \quad (1.1)$$

If \overleftarrow{B} is diagonal, this reduces to

$$\left(\overleftarrow{H} \overleftarrow{B}\right)_{i,j} = \sum_{k=1}^M \overleftarrow{H}_{i,k} \overleftarrow{B}_{k,j} \delta_{k,j} = \sum_{k=1}^M \overleftarrow{H}_{i,k} \overleftarrow{B}_{j,j} \delta_{k,j} = \overleftarrow{H}_{i,j} \overleftarrow{B}_{j,j}. \quad (1.2)$$

For $\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T$, one obtains

$$\left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T\right)_{i,j} = \left[\left(\overleftarrow{H} \overleftarrow{B}\right) \overleftarrow{H}^T\right]_{i,j} = \sum_{k=1}^M \left(\overleftarrow{H} \overleftarrow{B}\right)_{i,k} \overleftarrow{H}_{k,j}^T = \sum_{k=1}^M \left(\overleftarrow{H} \overleftarrow{B}\right)_{i,k} \overleftarrow{H}_{j,k} = \sum_{k=1}^M \left(\sum_{l=1}^M \overleftarrow{H}_{i,l} \overleftarrow{B}_{l,k}\right) \overleftarrow{H}_{j,k} = \sum_{k,l=1}^M \overleftarrow{H}_{i,l} \overleftarrow{B}_{l,k} \overleftarrow{H}_{j,k}. \quad (1.3)$$

If \overleftarrow{B} is diagonal, this reduces to

$$\left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T\right)_{i,j} = \sum_{k=1}^M \overleftarrow{H}_{i,k} \overleftarrow{B}_{k,k} \overleftarrow{H}_{j,k} = \sum_{k=1}^M \overleftarrow{B}_{k,k} \left(\overleftarrow{H}_{i,k} \overleftarrow{H}_{j,k}\right). \quad (1.4)$$

For the final result, we obtain

$$\begin{aligned} x_i &= x_{B,i} + \sum_{j=1}^N \left[\overleftarrow{B} \overleftarrow{H}^T \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right) \right]_{i,j} (y_j - h_j) = x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{B} \overleftarrow{H}^T \right)_{i,k} \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \right] (y_j - h_j) \\ &= x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \left(\overleftarrow{B} \overleftarrow{H}^T \right)_{i,k} \right] (y_j - h_j) = x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \left(\sum_{l=1}^M \overleftarrow{B}_{i,l} \overleftarrow{H}_{l,k}^T \right) \right] (y_j - h_j) \\ &= x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \left(\sum_{l=1}^M \overleftarrow{B}_{i,l} \overleftarrow{H}_{k,l} \right) \right] (y_j - h_j). \end{aligned} \quad (1.5)$$

If \overleftarrow{B} is diagonal, this reduces to

$$x_i = x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \left(\overleftarrow{B}_{i,i} \overleftarrow{H}_{k,i} \right) \right] (y_j - h_j) = x_{B,i} + \sum_{j=1}^N \left[\sum_{k=1}^N \left(\overleftarrow{H} \overleftarrow{B} \overleftarrow{H}^T + \overleftarrow{R} \right)_{k,j} \overleftarrow{B}_{i,i} \overleftarrow{H}_{k,i} \right] (y_j - h_j). \quad (1.6)$$

1.1 Inclusion of moisture

So far, moisture is taken into account in a separate assimilation process for efficiency.

Chapter 2

3D-Var

2.1 Technicalities

The observations used come from a time window $[-\frac{T}{2}, \frac{T}{2}]$ around the analysis time and are all taken to be valid at the analysis time. $T = 3$ h is a typical value..

Chapter 3

4D-Var

3.1 Technicalities

The observations used come from a time window $[-\frac{T}{2}, \frac{T}{2}]$ around the analysis time and are taken to be valid at individual time steps $n\Delta t$. $T = 6$ h and $\Delta t = 15$ min are typical values.