`ndvar` Documentation

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Chapter 1

Optimum interpolation

$$\left(\overleftrightarrow{H}\overleftrightarrow{B}\right)_{i,j} = \sum_{k=1}^{M} \overleftrightarrow{H}_{i,k}\overleftrightarrow{B}_{k,j}. \tag{1.1}$$

In the special case, where $\overleftrightarrow{B}$ has diagonal form, this reduces to

$$\left(\overleftrightarrow{H}\overleftrightarrow{B}\right)_{i,j} = \sum_{k=1}^{M} \overleftrightarrow{H}_{i,k}\overleftrightarrow{B}_{k,j}\delta_{k,j} = \sum_{k=1}^{M} \overleftrightarrow{H}_{i,k}\overleftrightarrow{B}_{j,j}\delta_{k,j} = \overleftrightarrow{H}_{i,j}\overleftrightarrow{B}_{j,j}. \tag{1.2}$$

For $\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T$, one obtains

$$\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T\right)_{i,j} = \left[\left(\overleftrightarrow{H}\overleftrightarrow{B}\right)\overleftrightarrow{H}^T\right]_{i,j} = \sum_{k=1}^{M}\left(\overleftrightarrow{H}\overleftrightarrow{B}\right)_{i,k}\overleftrightarrow{H}^T_{k,j} = \sum_{k=1}^{M}\left(\overleftrightarrow{H}\overleftrightarrow{B}\right)_{i,k}\overleftrightarrow{H}_{j,k} = \sum_{k=1}^{M}\left(\sum_{l=1}^{M}\overleftrightarrow{H}_{i,l}\overleftrightarrow{B}_{l,k}\right)\overleftrightarrow{H}_{j,k} = \sum_{k,l=1}^{M}\overleftrightarrow{H}_{i,l}\overleftrightarrow{B}_{l,k}\overleftrightarrow{H}_{j,k}. \tag{1.3}$$

In the special case, where $\overleftrightarrow{B}$ has diagonal form, this reduces to

$$\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T\right)_{i,j} = \sum_{k=1}^{M}\overleftrightarrow{H}_{i,k}\overleftrightarrow{B}_{k,k}\overleftrightarrow{H}_{j,k} = \sum_{k=1}^{M}\overleftrightarrow{B}_{k,k}\left(\overleftrightarrow{H}_{i,k}\overleftrightarrow{H}_{j,k}\right). \tag{1.4}$$

For the final result, we obtain

$$\begin{aligned}
x_i &= x_{B,i} + \sum_{j=1}^{N}\left[\overleftrightarrow{B}\overleftrightarrow{H}^T\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)\right]_{i,j}\left(y_j-h_j\right) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{B}\overleftrightarrow{H}^T\right)_{i,k}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\right]\left(y_j-h_j\right)\\
&= x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\left(\overleftrightarrow{B}\overleftrightarrow{H}^T\right)_{i,k}\right]\left(y_j-h_j\right) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\left(\sum_{l=1}^{M}\overleftrightarrow{B}_{i,l}\overleftrightarrow{H}^T_{l,k}\right)\right]\left(y_j-h_j\right)\\
&= x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\left(\sum_{l=1}^{M}\overleftrightarrow{B}_{i,l}\overleftrightarrow{H}_{k,l}\right)\right]\left(y_j-h_j\right).
\end{aligned} \tag{1.5}$$

In the special case, where $\overleftrightarrow{B}$ has diagonal form, this reduces to

$$x_i = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\left(\overleftrightarrow{B}_{i,i}\overleftrightarrow{H}_{k,i}\right)\right]\left(y_j-h_j\right) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overleftrightarrow{H}\overleftrightarrow{B}\overleftrightarrow{H}^T+\overleftrightarrow{R}\right)_{k,j}\overleftrightarrow{B}_{i,i}\overleftrightarrow{H}_{k,i}\right]\left(y_j-h_j\right). \tag{1.6}$$

Chapter 2

3D-Var

2.1 Technicalities

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are all taken to be valid at the analysis time. $T = 3$ h is a typical value..

Chapter 3

4D-Var

3.1 Technicalities

The observations used come from a time window $\left[-\frac{T}{2}, \frac{T}{2}\right]$ around the analysis time and are taken to be valid at individual time steps $n\Delta t$. $T = 6$ h and $\Delta t = 15$ min are typical values.