GAME-DA Documentation

GAME-DA Development Team

Contents

Chapter 1

Introduction

GAME-DA provides *data assimilation (DA)* funcitonality for the GAME model. The theoretical derivations are presented in [1]. This documentation contains the technical details of the implementations of the data assimilation algorithms. GAME-DA uses optimum interpolation (OI). As an option, a simple spatial interpolation can also be used.

The basic workflow is as follows:

1. Downloading the observations. This is done by the bash scripts residing in the directory `obs_collector`.
2. Bringing the observations into a standardized format. This is done by the separate executable `formatter` residing in the directory of the same name.
3. Executing the data assimilation itself, resulting in an input file for the model.

Chapter 2

Optimum interpolation

Let $M \geq 1$ be the number of degrees of freedom of the model, $N \geq 1$ the number of observations, $\mathbf{h} \in \mathbb{R}^N$ the observations as reconstructed from the background state, $\mathbf{y} \in \mathbb{R}^N$ the actual observations, $\overrightarrow{H} \in \mathbb{R}^{N \times M}$ the Jacobian of the observations operator, $\overrightarrow{B} \in \mathbb{R}^{M \times M}$ the background error covariance matrix and $\overrightarrow{R} \in \mathbb{R}^{N \times N}$ the observations error covariance matrix. $\overrightarrow{R}$ is usually assumed to be diagonal, which is an excellent approximation if the observation erros are of statistical nature. This does not lead to useful simplifications, however.

It is

$$\left(\overrightarrow{H}\,\overrightarrow{B}\right)_{i,j} = \sum_{k=1}^{M} \overrightarrow{H}_{i,k}\,\overrightarrow{B}_{k,j}. \tag{2.1}$$

For $\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T$, one obtains

$$\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T\right)_{i,j} = \left[\left(\overrightarrow{H}\,\overrightarrow{B}\right)\overrightarrow{H}^T\right]_{i,j} = \sum_{k=1}^{M}\left(\overrightarrow{H}\,\overrightarrow{B}\right)_{i,k}\overrightarrow{H}^T_{k,j} = \sum_{k=1}^{M}\left(\overrightarrow{H}\,\overrightarrow{B}\right)_{i,k}\overrightarrow{H}_{j,k} = \sum_{k=1}^{M}\left(\sum_{l=1}^{M}\overrightarrow{H}_{i,l}\,\overrightarrow{B}_{l,k}\right)\overrightarrow{H}_{j,k} = \sum_{k,l=1}^{M}\overrightarrow{H}_{i,l}\,\overrightarrow{B}_{l,k}\,\overrightarrow{H}_{j,k}. \tag{2.2}$$

For the final result, we obtain

$$\begin{aligned} x_i &= x_{B,i} + \sum_{j=1}^{N}\left[\overrightarrow{B}\,\overrightarrow{H}^T\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T + \overrightarrow{R}\right)^{-1}\right]_{i,j}(y_j - h_j) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overrightarrow{B}\,\overrightarrow{H}^T\right)_{i,k}\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T + \overrightarrow{R}\right)^{-1}_{k,j}\right](y_j - h_j) \\ &= x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T + \overrightarrow{R}\right)^{-1}_{k,j}\left(\overrightarrow{B}\,\overrightarrow{H}^T\right)_{i,k}\right](y_j - h_j) = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T + \overrightarrow{R}\right)^{-1}_{k,j}\left(\sum_{l=1}^{M}\overrightarrow{B}_{i,l}\,\overrightarrow{H}^T_{l,k}\right)\right](y_j - h_j) \end{aligned}$$

$$\Rightarrow x_i = x_{B,i} + \sum_{j=1}^{N}\left[\sum_{k=1}^{N}\left(\overrightarrow{H}\,\overrightarrow{B}\,\overrightarrow{H}^T + \overrightarrow{R}\right)^{-1}_{k,j}\left(\sum_{l=1}^{M}\overrightarrow{B}_{i,l}\,\overrightarrow{H}_{k,l}\right)\right](y_j - h_j). \tag{2.3}$$

2.1 Inclusion of moisture

So far, moisture is taken into account in a separate assimilation process for efficiency.

Bibliography

[1] M. H. Balsmeier. *Kompendium Theoretische Meteorologie*. 2021. URL: https://raw.githubusercontent.com/MHBalsmeier/kompendium/master/kompendium.pdf.