Overlapping Generations Models for Policy Analysis: Theory and Computation

Richard W. Evans

Jason DeBacker

University of Chicago

University of South Carolina

2020

(version 2020.08.a)

Contents

P	Preface		iii	
Ι	In	troduction and Motivation	1	
1	Inti	roduction	3	
	1.1	A Simple 2-period-lived Example	4	
	1.2	Equilibrium Solution	5	
	1.3	Perfect Foresight versus Uncertainty	6	
	1.4	Outline	7	
II 2		imple OG Model with Exogenous Labor Supply eriod-lived Agents with Exogenous Labor Supply	9	
	2.1	Households	12	
	2.2	Firms	17	
	2.3	Market clearing	18	
	2.4	Equilibrium	18	
	2.5	Solution method: time path iteration (TPI)	21	
	2.6	Calibration	24	
	2.7	Exercises	24	
3	S-p	eriod-lived Agent Problem with Exogenous labor Supply	27	
	3.1	Households	27	

X CONTENTS

	3.2	Firms	32
	3.3	Market clearing	33
	3.4	Equilibrium	33
	3.5	Solution method: time path iteration (TPI)	36
	3.6	Calibration	38
	3.7	Exercises	38
Π	Ι]	Endogenous Labor and Heterogeneous Ability	43
4	S-p	eriod-lived Agents with Endogenous Labor	45
	4.1	Disutility of labor	45
	4.2	Households	50
	4.3	Firms	51
	4.4	Market Clearing	52
	4.5	Equilibrium	52
	4.6	Solution Method	55
	4.7	Calibration	65
	4.8	Exercises	68
5	S-p	eriod-lived Agents with Endogenous Labor and Dynamic Firms	73
	5.1	Households	73
	5.2	Firms	75
	5.3	Market Clearing	80
	5.4	Equilibrium	80
	5.5	Solution Method	82
	5.6	Calibration	91
	5.7	Exercises	95
6	S-p	eriod Lives, Endogenous Labor, and Heterogeneous Abilities	99
	6.1	Heterogeneous lifetime ability paths	99
	6.2	Households	101

CONTENTS	xi

	6.3	Firms	
	6.4	Market clearing	
	6.5	Equilibrium	
	6.6	Solution Method	
	6.7	Calibration	
	6.8	Exercises	
7	Sim	ple Stochastic Ability 121	
	7.1	Households	
	7.2	Firms	
	7.3	Market clearing	
	7.4	Equilibrium	
	7.5	Solution Method	
	7.6	Exercises	
		Population Dynamics, Productivity Growth, and Station-	
IV ar 8	$^{\prime}$ I $_{ m izat}$	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic	
ar	$^{\prime}$ I $_{ m izat}$	Population Dynamics, Productivity Growth, and Stationion 137	
ar	$^{\prime}$ I $_{ m izat}$	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic	
ar	[/] І izat S-р Dyr	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139	
ar	7 I izat S-p Dyr 8.1	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139 Population Dynamics	
ar	 I izat S-p Dyr 8.1 8.2 	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139 Population Dynamics	
ar	7 I izat S-p Dyr 8.1 8.2 8.3	Population Dynamics, Productivity Growth, and Station- ion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139 Population Dynamics 140 Households 141 Firms 144 Market clearing 144 Equilibrium 146	
ar	7 I izat S-p Dyr 8.1 8.2 8.3 8.4	Population Dynamics, Productivity Growth, and Stationion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139 Population Dynamics 140 Households 141 Firms 144 Market clearing 144 Equilibrium 146 Solution method: time path iteration (TPI) 152	
ar	7 I izat S-p Dyr 8.1 8.2 8.3 8.4 8.5	Population Dynamics, Productivity Growth, and Station- ion 137 eriod-lived Agent Problem with Exogenous Labor Supply, Demographic namics, and Productivity Growth 139 Population Dynamics 140 Households 141 Firms 144 Market clearing 144 Equilibrium 146	

xii CONTENTS

V	В	Bequests	173
9	S-p	eriod Lives, Exogenous Labor, Intended Bequests	175
	9.1	Bequest Data	175
	9.2	Households	177
	9.3	Firms	181
	9.4	Market clearing	182
	9.5	Equilibrium	182
	9.6	Solution method: time path iteration (TPI)	185
	9.7	Calibration	188
	9.8	Exercises	188
		period Lives, Exogenous Labor, Bequests with Population Dynamics quests with Population Dynamics, Endogenous Labor, and Heteroge	193
V	1 1	Household taxes and government budget constraint	197
12	Hou	usehold taxes	199
	12.1	Incorporating Tax Functions into DGE Model	
	12.2		199
	12.3	Households	199 219
	19.4	Households	219
	12.4		219
		Firms	219 222
	12.5	Firms	219 222 222
	12.5 12.6	Firms	219222222223
	12.5 12.6 12.7	Firms Government balanced budget Market Clearing Equilibrium	219222222223223
	12.5 12.6 12.7 12.8	Firms Government balanced budget Market Clearing Equilibrium Solution Method	219222223223226232
13	12.5 12.6 12.7 12.8 12.9	Firms Government balanced budget Market Clearing Equilibrium Solution Method Calibration	219222223223226232

CONTENTS		xiii
13.2 Firms	 	235
13.3 Government	 	236
13.4 Market Clearing	 	238
13.5 Equilibrium	 	238
13.6 Solution Method	 	242
13.7 Calibration	 	252
13.8 Exercises	 	254
14 Modeling Social Security and pension systems		257
VII Multiple industries, multiple goods		259
15 Two Industries, Two Goods, Three-period-lived agents		261
15.1 Households	 	261
15.2 Firms	 	267
15.3 Market clearing	 	269
15.4 Equilibrium	 	271
15.5 Solution method	 	272
15.6 Calibration	 	276
15.7 Exercises	 	276
16 Multiple Static Industries		277
16.1 Household Industry-specific Consumption	 	277
16.2 Household Composite Consumption	 	279
16.3 Industries and Firms	 	281
16.4 Market Clearing	 	282
16.5 Equilibrium	 	283
16.6 Solution Method	 	285
16.7 Calibration	 	289
16.8 Exercises	 	289

xiv CONTENTS

17 Mu	ıltiple Static Industries, Endogenous Labor	291
17.	1 Household Industry-specific Consumption	291
17.	2 Household Composite Consumption and Labor	293
17.	3 Industries and Firms	296
17.	4 Market Clearing	298
17.	5 Equilibrium	299
17.	6 Solution Method	301
17.	7 Calibration	313
17.	8 Exercises	313
18 Mı	ultiple industries, Multiple Goods, and S -period-lived individuals	315
18.	1 Households	315
18.	2 Firms	320
18.	3 Market clearing	321
18.	4 Equilibrium	323
19 Mı	ultiple dynamic industries, Multiple Goods, and S -period-lived individ	l-
ual	${f ls}$	325
VIII	International markets	327
20 Sm	nall Open Economy OG Model	329
20.	1 Households	329
20.	2 Firms	330
20.	3 Market Clearing	331
20.	4 Equilibrium	331
20.	5 Solution Method	334
20.	6 Calibration	339
20	7 Exercises	342

CONTENTS	XV
CONTENTS	AV

21 Multi-country (Large Open Economy) OG Model	345
21.1 Households	346
21.2 Firms	351
21.3 Market Clearing	357
21.4 Equilibrium	359
21.5 Solution Method	360
21.6 Calibration	362
21.7 Exercises	363
IX Aggregate Shocks, Asset Pricing, and Adaptive E	xpecta-
tions	365
22 Aggregate Shocks and Business Cycles	367
22.1 Two-period model	367
22.2 Three-period model	367
23 Asset Pricing	375
24 Adaptive Expectations	377
X Monetary Policy	379
25 Monetary Policy	381
XI Calibration	383
26 Labor supply distribution	385
27 Wealth distribution	387
Appendices	391

xvi CONTENTS

A	Der	ivations	391
	A.1	Properties of the CES Production Function	391
В	End	ogenous Grid Method	393
	B.1	Final two periods of life solutions: ages S and $S-1$	393
\mathbf{C}	Usiı	ng Python	397
	C.1	Installing Python	397
	C.2	Learning Python	398
	C.3	Principles of Writing Good Code	399
	C.4	Running Python	400
	C.5	Debugging	402
	C.6	Python cheat sheet	402
	C.7	Optimization: Root Finders and Minimizers	402
D	Usii	ng Git and GitHub.com	405
	D.1	Why Not Use Dropbox or Google Docs?	406
	D.2	Installing Git and Settings	408
	D.3	Git and GitHub Structure, Workflow	408
	D.4	Git Cheat Sheet Commands	417
	D.5	Using GitHub for Collaborative Issue Tracking	418
${f E}$	Cali	bration	421
	E.1	Initial period labor supply Euler equations	421
	E.2	Alternative Explanation/Derivation	424
\mathbf{F}	Tax	Data with Tax-Calculator	429
	F.1	TaxBrain Web Application	429
	F.2	Command Line Interface (CLI)	429
	F.3	Python Application Program Interface (API)	429
	FΛ	Evereises	430